IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Charging Infrastructure

Lecture-15

Closed Loop Control of Single-phase Boost PFC Converter-III

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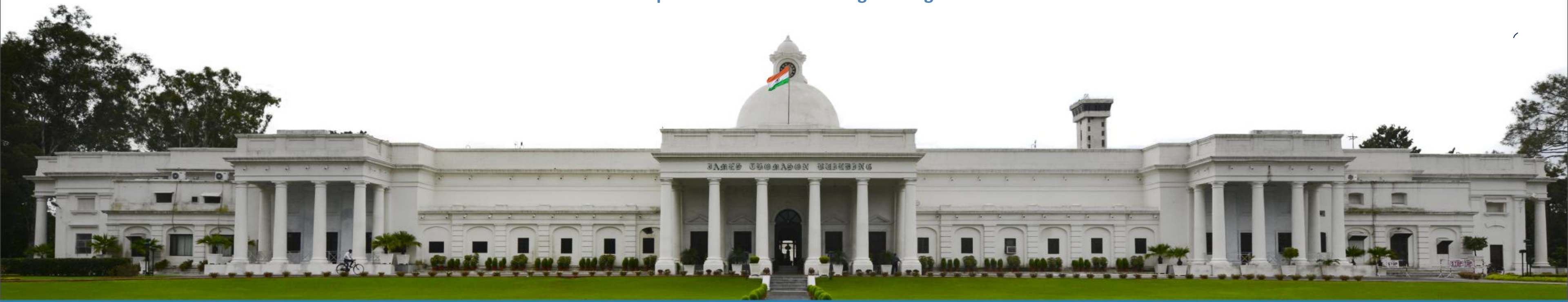
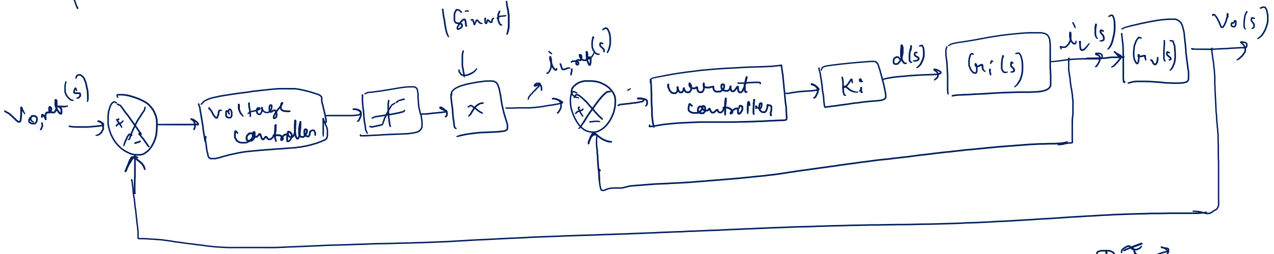


Recap



$$G_i(s) = \frac{i_L(s)}{d(s)}$$

$$G_v(s) = \frac{V_o(s)}{i_L(s)} \approx \frac{v_C(s)}{i_L(s)}$$

$DT_s \rightarrow \;\Rightarrow$ $A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_L C} \end{bmatrix}$; $A_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_L C} \end{bmatrix}$

$B_1 = B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$; $C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$

$D_1 = D_2 = [0]$

$$\underbrace{\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{1-D}{C} & -\frac{1}{R_L C} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_L \\ v_C \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B} \underbrace{|v_s|}_{|v_s|}$$

$$V_o = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} i_L \\ v_C \end{bmatrix}}_{x}$$

$A = A_1 D + A_2(1-D)$

$B = B_1 D + B_2(1-D)$

$C = C_1 D + C_2(1-D)$

$D \Rightarrow D + \tilde{d}$ $\quad (\tilde{d} << D)$

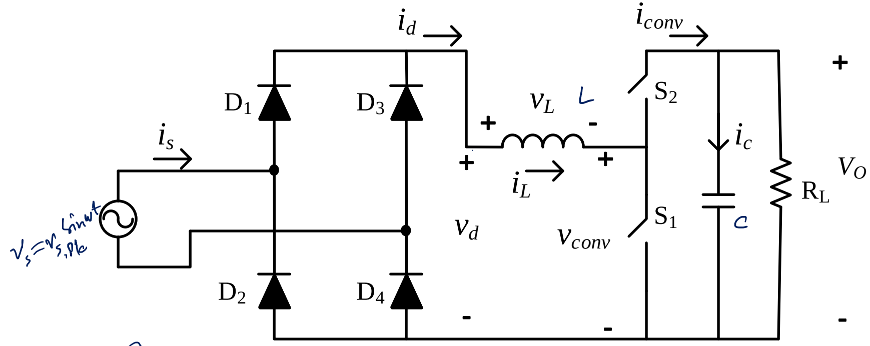
$i_L \Rightarrow i_L + \tilde{i}_L$ $\quad (\tilde{i}_L << i_L)$

$v_c \Rightarrow v_c + \tilde{v}_c$ $\quad (\tilde{v}_c << v_c)$

$|v_s| \Rightarrow |v_s| + |\tilde{v}_s|$ $\quad (|\tilde{v}_s| << |v_s|)$

$v_o \Rightarrow V_o + \tilde{v}_o$ $\quad (\tilde{v}_o << V_o)$

$x = \begin{bmatrix} i_L \\ v_c \end{bmatrix} \Rightarrow x = X + \tilde{x}$



$$\frac{d(X+\tilde{x})}{dt} = \left[A_1(D+\tilde{d}) + A_2(1-D-\tilde{d})\right](X+\tilde{x}) + \left[B_1(D+\tilde{d}) + B_2(1-D-\tilde{d})\right](|v_s| + |\tilde{v}_s|) \longrightarrow (1)$$

$$V_o + \tilde{v}_o = \left[C_1(D+\tilde{d}) + C_2(1-D-\tilde{d})\right]\left[X+\tilde{x}\right] \longrightarrow (2)$$

$\Rightarrow \quad |\tilde{v}_s| \cdot \tilde{d} = 0 \; ; \; \tilde{x} \cdot \tilde{d} = 0 \longrightarrow$ this indicates very small quantities

$$\frac{dx}{dt} + \frac{d\tilde{x}}{dt} = \left(A_1 D + A_2(1-D)\right)x + \left(A_1 D + A_2(1-D)\right)\tilde{x} + \left(B_1 D + B_2(1-D)\right)|v_s| + \left(B_1 D + B_2(1-D)\right)|\tilde{v}_s| + \left[(A_1 - A_2)x + (B_1 - B_2)|v_s|\right]\tilde{d}$$

$$\frac{dx}{dt} + \frac{d\tilde{x}}{dt} = Ax + A\tilde{x} + B|v_s| + B|\tilde{v}_s| + \left[(A_1 - A_2)x + (B_1 - B_2)|v_s|\right]\tilde{d} \longrightarrow (3)$$

$$V_o + \tilde{v}_o = Cx + C\tilde{x} + \left[(C_1 - C_2)x\right]\tilde{d} \longrightarrow (4)$$

$$\begin{bmatrix} |\tilde{v}_s| \cdot \tilde{d} = 0 \\ \tilde{x} \cdot \tilde{d} = 0 \end{bmatrix}$$

linearize the State equation in the Presence of small Perturbation

Steady state terms

$$\frac{dx}{dt} = Ax + B|v_s|$$

$$V_o = Cx$$

$$\frac{dx}{dt} = 0$$

$$\Rightarrow \left.\begin{aligned} x &= -A^{-1}B|v_s| \\ V_o &= -CA^{-1}B|v_s| \end{aligned}\right\} \Rightarrow \text{A matrix must be an invertible matrix}$$

$$A = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{1-D}{C} & -\frac{1}{R_L C} \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{\frac{(1-D)^2}{LC}} \begin{bmatrix} -\frac{1}{R_L C} & \frac{1-D}{L} \\ -\frac{(1-D)}{C} & 0 \end{bmatrix} ; \quad B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} ; \quad x = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$X = \frac{-1}{\frac{(1-D)^2}{LC}} \begin{bmatrix} \frac{-1}{R_L C} & \frac{(1-D)}{L} \\ -\frac{(1-D)}{C} & 0 \end{bmatrix} \begin{bmatrix} 1/L \\ 0 \end{bmatrix} |V_s|$$

$$X = \frac{-1}{\frac{(1-D)^2}{LC}} \begin{bmatrix} \frac{-1}{R_L CL} \\ -\frac{(1-D)}{LC} \end{bmatrix} |V_s|$$

$$\Rightarrow X = \begin{bmatrix} i_L \\ V_c \end{bmatrix} = \begin{bmatrix} \frac{|V_s|}{R_L (1-D)^2} \\ \frac{|V_s|}{1-D} \end{bmatrix} \longrightarrow 5$$

from $\quad V_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{|V_s|}{R_L (1-D)^2} \\ \frac{|V_s|}{1-D} \end{bmatrix}$

$$\Rightarrow V_o = \frac{|V_s|}{1-D}$$

Small-signal terms

$$B_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} ; \quad B_2 \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

$$\frac{d\tilde{x}}{dt} = A\tilde{x} + B|\tilde{v}_s| + \left[(A_1 - A_2)x + (B_1 - B_2)|v_s| \right] \tilde{d}$$

$$\tilde{v}_o = C\tilde{x} + \left[(C_1 - C_2)x \right] \tilde{d}$$

$$\begin{bmatrix} \frac{d\tilde{i}_L}{dt} \\ \frac{d\tilde{v}_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{R_L C} \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} |\tilde{v}_s| + \left[\left(\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_L C} \end{bmatrix} - \begin{bmatrix} 0 & -1/L \\ 1/C & -1/R_L C \end{bmatrix} \right) \begin{bmatrix} \frac{|v_s|}{R_L (1-D)^2} \\ \frac{|v_s|}{1-D} \end{bmatrix} + 0 \right] \tilde{d}$$

$$\underbrace{\begin{bmatrix} \frac{d\tilde{i}_L}{dt} \\ \frac{d\tilde{v}_c}{dt} \end{bmatrix}}_{\frac{d\tilde{x}}{dt}} = \underbrace{\begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{R_L C} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \tilde{i}_L \\ \tilde{v}_o \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B} \underbrace{|\tilde{v}_s|}_{|\tilde{v}_s|} + \underbrace{\begin{bmatrix} \frac{|v_s|}{L(1-D)} \\ -\frac{|v_s|}{R_L C (1-D)^2} \end{bmatrix}}_{K} \underbrace{\tilde{d}}_{\tilde{d}}$$

$$\tilde{v}_o = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} i_L \\ v_o \end{bmatrix}}_{x} \cdot \underbrace{\tilde{d}}_{\tilde{d}}$$

$$\frac{d\tilde{x}}{dt} = A\tilde{x} + B|\tilde{v}_s| + K\tilde{d} \longrightarrow (6)$$

$$\tilde{v}_0 = C\tilde{x} + Q\times\tilde{d} \longrightarrow (7)$$

Apply laplace Transform in eq. (6)

$$s\tilde{x}(s) = A\tilde{x}(s) + B|\tilde{v}_s(s)| + K\tilde{d}(s)$$

$$\Rightarrow (sI - A)\tilde{x}(s) = B|\tilde{v}_s(s)| + K\tilde{d}(s) \longrightarrow (8)$$

from eq. (7)

$$\tilde{v}_0(s) = C\tilde{x}(s) + \cancelto{0}{Q\times\tilde{d}(s)} \longrightarrow (9)$$

from eq. (8)

$$\left.\frac{\tilde{x}(s)}{\tilde{d}(s)}\right|_{|\tilde{v}_s(s)|=0} = (sI-A)^{-1}K \longrightarrow (10)$$

from eq. (9)

$$\left.\frac{\tilde{v}_0(s)}{\tilde{d}(s)}\right|_{|\tilde{v}_s(s)|=0} = C\left.\frac{\tilde{x}(s)}{\tilde{d}(s)}\right|_{|\tilde{v}_s|=0} \longrightarrow (11)$$

Substitute eq-(11) in eq-(10)

$$\Rightarrow \left.\frac{\tilde{v}_0(s)}{\tilde{d}(s)}\right|_{|\tilde{v}_s(s)|=0} = C(SI-A)^{-1}K$$

from eq. (10)

$$\Rightarrow \left.\begin{bmatrix} \dfrac{\tilde{i}_L(s)}{\tilde{d}(s)} \\ \dfrac{\tilde{v}_c(s)}{\tilde{d}(s)} \end{bmatrix}\right|_{|\tilde{v}_s(s)|=0} = (SI-A)^{-1}K$$

$$\Rightarrow \boxed{\left.\frac{\tilde{i}_L(s)}{\tilde{d}(s)}\right|_{|\tilde{v}_s(s)|=0} = \frac{|v_s|}{R_L(1-D)^3}\left[\frac{R_LCs+2}{\dfrac{s^2LC}{(1-D)^2}+\dfrac{sL}{R_L(1-D)^2}+1}\right] = G_i(s)}$$

$$(SI-A)^{-1} = \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -\dfrac{(1-D)}{L} \\ \dfrac{1-D}{C} & \dfrac{-1}{R_LC} \end{bmatrix}\right)^{-1}$$

$$= \begin{bmatrix} s & \dfrac{(1-D)}{L} \\ -\dfrac{(1-D)}{C} & s+\dfrac{1}{R_LC} \end{bmatrix}^{-1}$$

$$(SI-A)^{-1} = \frac{1}{s\left(s+\dfrac{1}{R_LC}\right)+\dfrac{(1-D)^2}{LC}}\begin{bmatrix} s+\dfrac{1}{R_LC} & -\dfrac{(1-D)}{L} \\ \dfrac{1-D}{C} & s \end{bmatrix}_{2\times 2}$$

$$K = \begin{bmatrix} \dfrac{|v_s|}{L(1-D)} \\ -\dfrac{|v_s|}{R_LC(1-D)^2} \end{bmatrix}_{2\times 1}$$

Thank You