IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Charging Infrastructure

Lecture-22

Pulse width Modulation

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Recap

- Boost based PFC converter: operation, Design, small signal model, closed loop control, CCM, DCM operation.
- Flyback based PFC converter: operation, DCM operation.

derived buck-boost

Half-bridge Configuration

- Half-bridge is the fundamental building block of a voltage source converter.
- Comprises of two switches (S1 and S2).

S_1 & S_2 → blocks voltage of single polarity & allows current in either direction

State	S1	S2	V_{AO}	V_{AN}
A	ON	OFF	$V_{dc}/2$	V_{dc}
B	OFF	ON	$-V_{dc}/2$	0

→ pole voltage

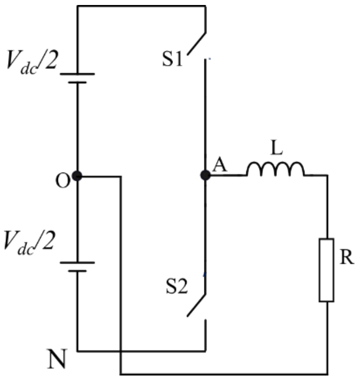
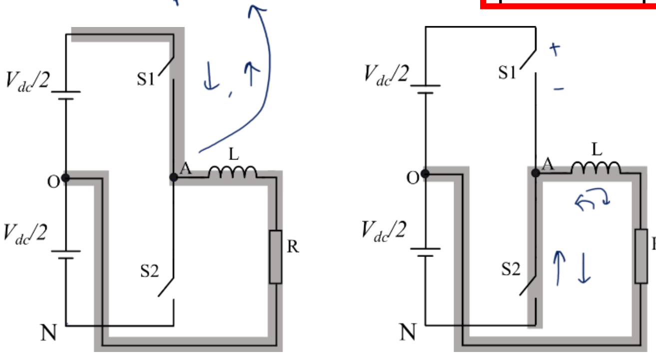


Fig.a: Half bridge configuration

Pole of the half-bridge



State A State B

Fig.b: Different switching States

Pulse Width Modulation

- In pulse width modulation, the output voltage of the inverter, is controlled by changing the width of the pole voltage (VAO) pulse.
- A high frequency carrier (triangle) signal, is compared with low frequency modulating (or reference) signal. The low frequency modulating signal can be DC or sinusoidal with frequency very less than the carrier frequency.

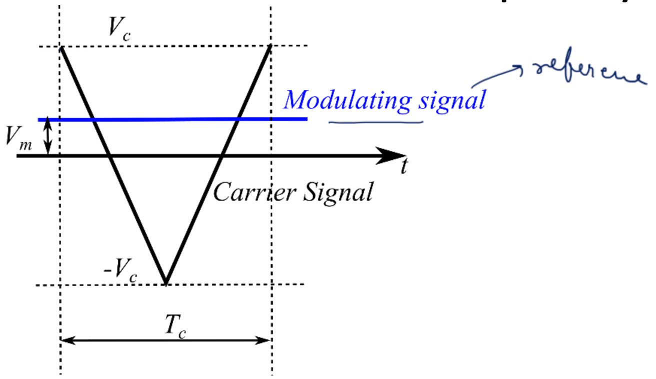


Fig.a: One cycle of carrier

Pulse Width Modulation Cont.

- For example in the Fig.a, a cycle of carrier signal is shown
  - Carrier is varying between $+V_c$ to $-V_c$ at a high frequency, $f_c = \frac{1}{T_c}$ carrier frequency
  - The modulating signal is a DC quantity or can be low frequency signal (frequency very less than carrier signal frequency), which in a carrier cycle frequency appear as DC.

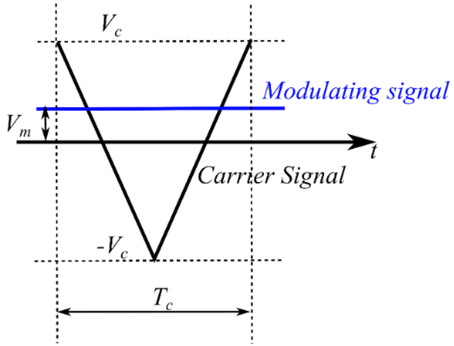
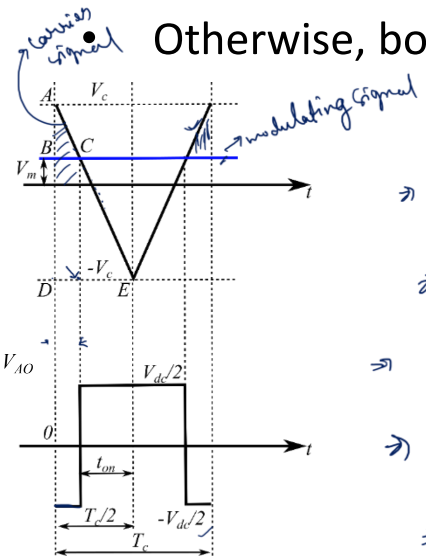


Fig.a: One cycle of carrier

Average Pole Voltage Variation

- In half-bridge configuration, the logic of switching is considered as
  - If modulating wave > carrier wave, then Top switch (S1) is ON and applies $V_{dc}/2$ voltage across the load.
  - Otherwise, bottom switch (S2) is ON and applies $-V_{dc}/2$ voltage across the load



One cycle of carrier

$\Rightarrow$ slope of line AC $= \dfrac{V_c - V_m}{\left(\frac{T_c}{2} - t_{on}\right)}$

$\Rightarrow$ slope of line AE $= \dfrac{V_c - (-V_c)}{T_c/2}$

$\Rightarrow$ slope of AC = slope of AE

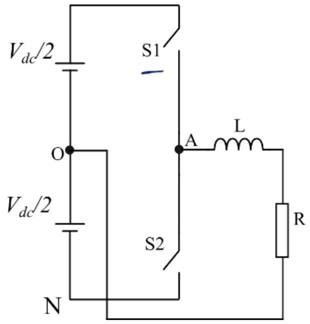
$$\Rightarrow \frac{V_c - V_m}{\frac{T_c}{2} - t_{on}} = \frac{2V_c}{T_c/2}$$

$$\Rightarrow \frac{V_m}{2V_c} + \frac{1}{2} = \frac{t_{on}}{T_c/2} = D \longrightarrow (1)$$

duty ratio

$$\frac{V_c - V_m}{2V_c} = \frac{T_c/2 - t_{on}}{T_c/2}$$

$$\Rightarrow \frac{1}{2} - \frac{V_m}{2V_c} = 1 - \frac{t_{on}}{T_c/2}$$



Half bridge configuration

Average Pole Voltage Variation Cont.

- The average variation of pole voltage (V_{AO}) over a carrier cycle is given as

$$V_{AO}(av) = \frac{-\frac{V_{dc}}{2}\left(\frac{T_c}{2}-t_{on}\right)+\frac{V_{dc}}{2}t_{on}}{\frac{T_c}{2}} = -\frac{V_{dc}}{2} + \frac{V_{dc}t_{on}}{\frac{T_c}{2}}$$

$D = \frac{t_{on}}{T_c/2}$

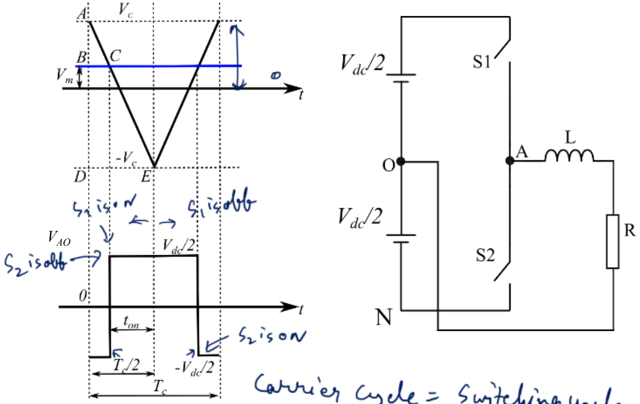
$$V_{AO}(av) = -\frac{V_{dc}}{2} + V_{dc}D$$

- Substitute value of $D = \frac{V_m}{2V_c} + \frac{1}{2}$ (eq. (1)) gives,

$$V_{AO}(av) = -\frac{V_{dc}}{2} + V_{dc}\left(\frac{V_m}{2V_c} + \frac{1}{2}\right)$$

$$\boxed{V_{AO}(av) = \frac{V_m}{2V_c}V_{dc}}$$

- The average variation of pole voltage over a carrier (switching) cycle is proportional to the amplitude of modulating signal (sampled during carrier cycle). ⇒



Carrier cycle = Switching cycle
⇒ carrier freq. = Switching freq.

Sinusoidal Pulse Width Modulation

- If the amplitude of modulating signal varies sinusoidal, the average pole voltage variation will also varies sinusoidal over a cycle of modulating wave.
- The width of pole voltage pulses varies sinusoidally. This method of pulse width modulation is called as Sinusoidal Pulse width Modulation (SPWM).
- It is one of the commonly used high frequency pulse width modulation used in inverters to generate an AC voltage from the DC bus.

f_m f_c $\Rightarrow f_c >> f_m$

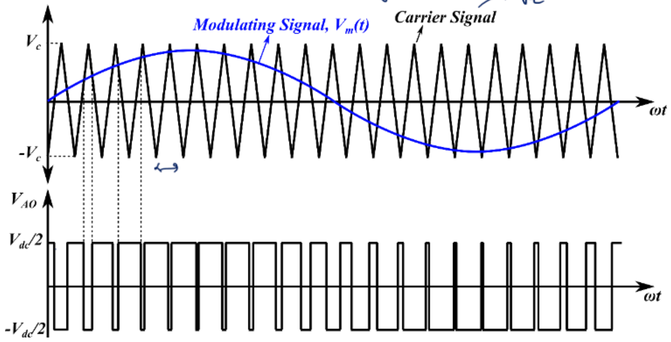


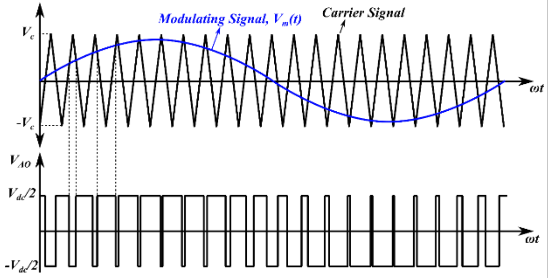
Fig.a: Modulating, carrier signal and pole voltage

Sinusoidal Pulse Width Modulation Cont.

- The modulating signal is represented as

$$V_m(t) = V_m sin\omega t$$

- Where, $\omega = 2\pi f_m$ and f_m is the frequency of modulating signal also called as fundamental frequency.



Modulating, carrier signal and pole voltage

- Thus, substitute value of $V_m(t)$ in $V_{AO}(av) = \frac{V_m}{2V_c} V_{dc}$ gives,

$$V_{AO}(t) = \frac{V_m sin\omega t}{2V_c} V_{dc}$$

(av)

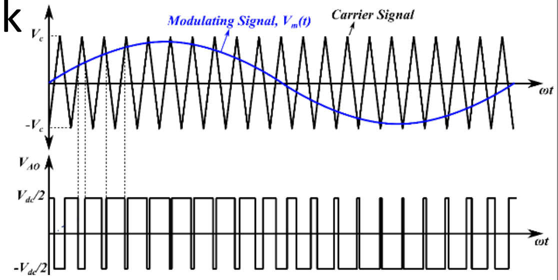
Sinusoidal Pulse Width Modulation Cont.

- Let us define, the ratio of peak of modulating signal to the peak of carrier signal as modulation index, $m = \frac{V_m}{V_c}$
- Thus, substitute value of *m* gives

$$V_{AO}(t) = \frac{mV_{dc}}{2} sin\omega t$$

'f_m'

- The pole voltage expression shows the fundamental component.
- Additionally harmonics are also generated.



Modulating, carrier signal and pole voltage

Sinusoidal Pulse Width Modulation Cont.

- The SPWM modulated half bridge output has
  - Fundamental component (at f_m), $\frac{mV_{dc}}{2}$ (based on the modulation index)
  - Harmonics
    - Present at carrier frequency and its sidebands, $h = im_f \pm j$, where, $m_f = \frac{f_c}{f_m}$

      $f_c \rightarrow$ Carrier freq

      $f_m \rightarrow$ modulating signal freq.
      - If i is odd, then j is even *(i =1, j =0, 2,4,6….)*
      - If i is even, then j is odd *(i =2, j =1,3,5….)*

  $\Rightarrow f_c = 21 \times 50 = 1050$ Hz
- Example, for $f_m = f_s$ = 50Hz, m_f = 21, the harmonics will present at
  - Fundamental $\rightarrow$ at 'f_m' frequency
  - Sidebands of carrier frequency h = 17, 19, 21, 23, 25……,

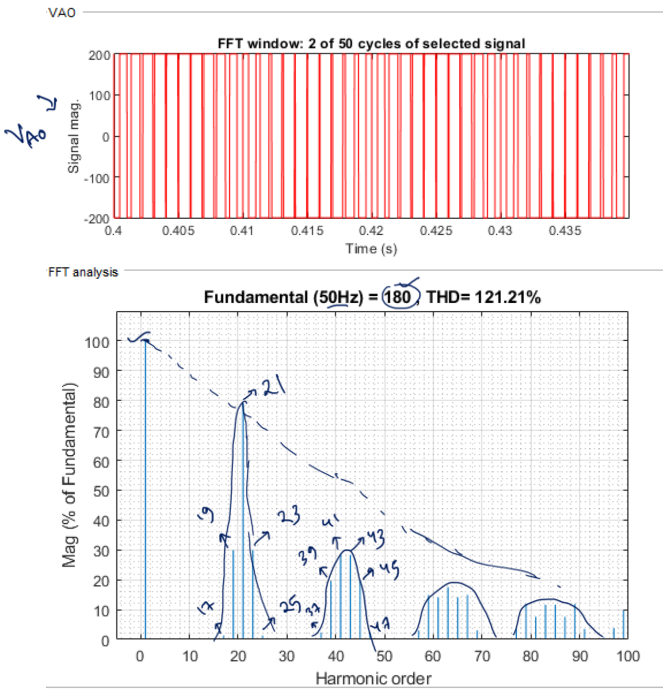
    $i=1, j=-4$; $i=1, j=-2$; $i=1, j=0$; $i=1, j=2$; $i=1, j=4$

    39, 41, 43, 45, 47, ……

    $i=2, j=-3$; $i=2, j=-1$; $i=2, j=1$; $i=2, j=3$; $i=2, j=5$

Source: "Pulse Width Modulation for Power Converters: Principles and Practice" by D. Grahame Holmes; Thomas A. Lipo

Sinusoidal Pulse Width Modulation: Harmonics Spectrum



Harmonic Spectrum for SPWM modulated half bridge

- V_{dc} = 400V, m =0.9, m_f =21. , $f_m = f_s = 50\,Hz$

  $\frac{V_m}{V_c}$ $\quad$ $\frac{f_c}{f_m} = 21$ $\quad$ $\Rightarrow f_c = 1050\,Hz$

- The V_{AO} contains:
  - Fundamental component = $\frac{mV_{dc}}{2} = \frac{0.9*400}{2} =$ 180V.
  - Harmonics exist at 17, 19, 21, 23, 25……39, 41, 43, 45……..

Thank You