Notation	Agda notation	Predicate logic notation	Arithmetic notation
Interpretation	Functions/pairs	For all/there exists	Products/sums
Dependent function types	$(x : A) \to B(x)$	$\Pi x : A.B(x)$	$\prod_{x:A} B(x)$
Dependent pair types	$(x : A) \times B(x)$	$\Sigma x : A.B(x)$	$\sum_{x:A} B(x)$

Contents

1 Idea

This project is to formalize much of the high school mathematics curriculum in cohesive homotopy type theory. This includes the theory of equations in the real numbers, elementary functions over the real numbers, and analytic Euclidean planar and volumetric geometry, as well as miscellaneous topics such as elementary combinatorics, number theory, probability, and statistics.

We work in cohesive homotopy type theory rather than plain homotopy type theory, because major topics in high school geometry, such as the perimeter and area of planar geometric figures, requires proving theorems about the topological properties of the real numbers, subtypes of the Euclidean plane, and continuous functions thereof. Due to the synthetic treatment of topological spaces and continuous functions in cohesive homotopy type theory, the proofs in cohesive homotopy type theory tend to be shorter than in plain homotopy type theory: whereas in plain homotopy type theory the theorems need to be appended with statements that types have the proper topology based off the Euclidean metric, and functions are continuous in a suitable way, in cohesive homotopy type theory, all types which are cohesive already have the proper topology, and all functions which are elements of cohesive function types are already continuous.

2 Foundations and synthetic topology

We use agda notation, predicate logic notation [insert article reference to Charles Sanders Pierce], and arithmetic notation interchangeably for dependent pair and dependent function types:

- dependent type theory introduced as follows: types, terms, dependent types, identity types, pair types, function types, dependent pair types, dependent function types, sum types, empty type, unit type, equivalences, function extensionality, transport, actions on identities, and heterogeneous identity tyeps. Also, a type of all propositions, which means that the theory is globally impredicative, as well as propositional truncation, which allows for predicate logic to be done. We use := informally to define terms and types, and relegate a formal definition of := to the appendix.

- the theory of univalent Tarski universes, predicativity in impredicative mathematics (using types of locally small propositions instead of the type of all propositions), propositional resizing (local impredicativity), local excluded middle, local "global" choice, universe of finite types and arithmetic of finite types, and modalities

- the Tarski universes $(U_{\text{crisp}}, \text{El}_{\text{crisp}})$ representing crisp types and $(U_{\text{cohesive}}, \text{El}_{\text{cohesive}})$ representing cohesive types, as well as functions
$$\text{disc} : U_{\text{crisp}} \hookrightarrow U_{\text{cohesive}}$$
$$\text{codisc} : U_{\text{crisp}} \hookrightarrow U_{\text{cohesive}}$$
$$\text{underlying} : U_{\text{cohesive}} \to U_{\text{crisp}}$$
$$\text{fundamental} : U_{\text{cohesive}} \to U_{\text{crisp}}$$
necessary to formalize the modalities
$$\sharp : U_{\text{cohesive}} \to U_{\text{cohesive}}$$
$$\flat : U_{\text{crisp}} \to U_{\text{crisp}}$$
$$\text{shape} : U_{\text{cohesive}} \to U_{\text{cohesive}}$$
This approach is more similar to the earlier approach of Urs Schreiber and Michael Shulman in [insert article reference], compared to the later approach of Michael Shulman in [insert article reference].

- the abstract unit interval $\mathbf{I} : U_{\text{cohesive}}$, which we shall assume for the time being to only be a type with two non-equal terms, and the associated axiom $\mathbf{I}\flat$ (basically Shulman's axiom C2 for a family indexed by the unit type). This axiom already implies that $\mathbf{I}$ is compact connected and that it's shape is contractible.

3 Real numbers

We follow Peter Freyd [insert article reference] in defining $\mathbf{I}$ to be the terminal interval coalgebra, and then define the real numbers accordingly.

4 Elementary functions and the theory of equations

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5 Euclidean geometry

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6 Miscellaneous topics

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7 Appendix

Formal type theory. Natural deduction.

Formal definitions: two approaches to formally defining the symbol := in type theory: propositional definitions, using identities of elements and equivalences of types, and judgmental definitions, using additional judgments representing judgmental equality of terms and of types.