EEB319H1S – Population Ecology
LAB 2 - Theoretical Population Growth

Theoretical population ecology is based on a suite of fairly simple models, which are used to capture certain features of population dynamics and to enhance our general understanding of the functioning of populations. Many computer programs exist to develop and explore these population models, but we can explore the properties of three basic population models using a simple spreadsheet like *Excel*. The reason for using a spreadsheet to introduce you to modelling is to (hopefully) make these models more concrete to you, and to help you understand how they work. As you will see, you can do a lot with very simple means! We will go through these models in class, but this lab gives you a chance to "play" with them and understand them more thoroughly.

In this lab, you will explore the properties of three simple models:

1) Exponential population growth model
2) Logistic population growth curve
3) Time-lag population growth model

I introduce these models in the "Population Dynamics" lecture, but to really understand them, there is nothing like working with them yourself.

1. Exponential Population Growth Model *(in populations with continuous generations)*

The simplest model of population growth is the Exponential Population Growth Model:

$$dN / dt = r \cdot N \quad \text{(differential equation)}$$

or

$$N_t = N_0 \cdot e^{rt}, \qquad \text{(integral form of the equation)}$$

where N is population size (N_0 and N_t are population size at time *0* and time *t*, respectively), r is the per-capita rate of population growth, and t is time.

Task 1A) In *Excel*, use the integral form of the exponential population growth model (above) to show the pattern of population growth at five different per-capita rate of population growth: r = -1.5, -0.5, 0, 0.5, 1.5. <u>Plot the dynamics of these five populations on one graph</u>, with Population size (N_t) on the Y-axis and Time (t) on the X-axis. Adjust the axes so we can actually see the important features of each dynamics, and use a graph legend to clearly label each model (*in Excel, name the Data Source Series with r=-1.5, r=-0.5, etc*). Include an informative title on your graph (after a while, all graphs look the same…).

To get started building your model, I suggest you set up 2 columns in an *Excel* spreadsheet (see Table 1 below).

- The first column is Time (t) and you will initially fill the column with values from 0 to 25 units (*of course, when plotting your results, you should choose a range of time (t) that is appropriate to show the important features of the dynamics – you may need to add more time units*).
- The second column is where you enter the formula to calculate N_t (I suggest you include in your equation the references to values of N_0 and r that you enter in separate rows at the top of the column, see Table 1 – this allows you to easily change these values as you explore the behavior of your model). If you have anchored your references properly (i.e. using "$" to anchor the column and/or the row of your reference – as learnt in Lab 1, Part 1), you may then copy the formula to the other rows (down to the 25th row) to follow N_t through 25 time units (*<u>always</u> double-check your formulae to make sure they copied properly!!*). To change the parameters of the model, you can simply change the values of N_0 and r at the top of the column, or if you want to keep a copy of your work, copy the whole column (column B in Table 1) and then change the values of N_0 and r in the new column.

Table 1. Suggested set-up to explore the Geometric Population Growth Model in *Excel* (with formulae). <u>Note</u>*: the formula was entered in the first row and copied to the other rows with Time data.*

	A	B	C
1	**Geometric Population Growth**		
2			
3	**N0=**	100	
4	**r=**	0.5	
5			
6	**Time (t)**	**N(t)**	
7	0	=B$3*EXP(B$4*$A7)	
8	1	=B$3*EXP(B$4*$A8)	
9	2	=B$3*EXP(B$4*$A9)	
10	3	=B$3*EXP(B$4*$A10)	
11	4	=B$3*EXP(B$4*$A11)	
12	5	=B$3*EXP(B$4*$A12)	
13	6	=B$3*EXP(B$4*$A13)	
14	7	=B$3*EXP(B$4*$A14)	
15	8	=B$3*EXP(B$4*$A15)	

Question 1B) How many time units it will take each of these five populations (i.e., with r = -1.5, -0.5, 0, 0.5, 1.5) to double in number? Show your calculations for one population and present all results in a table. (*Answer this question mathematically; <u>Hint</u>: work with N_t and N_0 in the exponential growth equation*).

Question 1C) When and where would you expect to find exponential population growth in natural populations?

2. Logistic Population Growth Model *(in populations with discrete generations)*

One important finding in theoretical population ecology is that density-dependent changes in net reproductive rates (through intraspecific competition) can destabilize populations and generate a wider range of possible dynamics. In populations with discrete generations, where we assume that the net reproductive rate (R_0) decreases linearly with increasing population density, we can derive the following simple logistic population growth model:

$$N_{t+1} = R_0 N_t = (1.0 - B (N_t - N_{eq})) N_t ,$$

where N_t is population size at time *t*, B is the slope of a linear decline in net reproductive rate (R_0) with increasing population density (i.e. the magnitude of the negative density-dependent effect), and N_{eq} is the equilibrium population size (when $R_0 = 1$).

This simple model can result in a surprisingly wide range of population dynamics (monotonic increase to equilibrium, damped oscillations, stable limit cycle, chaotic fluctuations). In this part of the lab, you will explore, again using a spreadsheet, the dynamics of this model.

First, choose different values of B and N_{eq} to run the model (see suggested set-up for the *Excel* spreadsheet in Table 2), until you find combinations that give you each of the four types of population dynamics (i.e., monotonic increase to equilibrium, damped oscillations, stable limit cycle, chaotic fluctuations). You may use Table 3 as a guideline to choose B and N_{eq}, but explore broadly (try positive and negative values, large and very small values), thinking about what each parameter is doing.

Table 2. Suggested set-up to explore the Logistic Population Growth Model in *Excel* (with formulae).

	A	B	C
1	**Logistic Population Growth Model**		
2			
3	**B=**	0.018	
4	**Neq=**	100	
5			
6	**Time (t)**	**N(t)**	
7	0	10	
8	1	=(1-B$3*(B7-B$4))*B7	
9	2	=(1-B$3*(B8-B$4))*B8	
10	3	=(1-B$3*(B9-B$4))*B9	
11	4	=(1-B$3*(B10-B$4))*B10	
12	5	=(1-B$3*(B11-B$4))*B11	
13	6	=(1-B$3*(B12-B$4))*B12	

Table 3. Types of population dynamics observed with logistic model (populations with discrete generations). (Krebs 2001)

$B \times N_{eq}$	**Type of dynamics**
0 – 1	Monotonic increase to equilibrium
1 – 2	Damped oscillations
2 – 2.57	Stable limit cycles (continue indefinitely)
> 2.57	Chaotic fluctuations

After you have figured out a useful range of parameter values to explore, choose 5 values of B and 5 values of N_{eq}, and figure out the type of population dynamics resulting from each of these 25 models (i.e., for each value of B, run the model with the 5 values of N_{eq} you have chosen). The *Excel*-based method you are using here is coarse, but this general approach is commonly used to figure out what ranges of parameters result in what types of population dynamics. More efficient algorithms are used to explore ranges of parameters in more details. *[<u>Hint</u>: if you set your Excel worksheet in the format shown in Table 2, you will simply need to copy your formulae in column B to 24 other columns, change the parameters at the top of each column and plot the dynamics]*

Task 2A) Plot the results of your analysis on a graph of B (X-axis) vs N_{eq} (Y-axis), with different symbols for each type of population dynamics (make sure to include a legend that clearly labels each type of symbol; see Fig. 1). For e.g., if you run the equation with B=0.0023, N_{eq}=1000 and you find stable limit cycles, you will put (say) a black triangle on your graph at B=0.0023 and N_{eq}=1000; if you then run the equation with B=0.05, N_{eq}=10 and you find a monotonic increase to equilibrium, you will put (say) an open diamond on your graph at B=0.05 and N_{eq}=10. You will have a total of 25 such points on your graph, with 4 different types of symbols (for each type of dynamics). To produce this graph in *Excel*, you will be plotting 4 variables (one for each type of dynamics) on the same graph (your graph should look like Fig. 1 below). *Please prepare your graph electronically; we will not accept hand-drawn graphs.*

Question 2B) Describe in writing the graph that you prepared in 2A. What are the main features shown by the graph and what are your conclusions?

Question 2C) Based on these results, explain in words in what types of populations you would expect each type of dynamics.

Question 2D) How might each type of dynamic affect the long-term viability of these populations? Explain your reasoning.

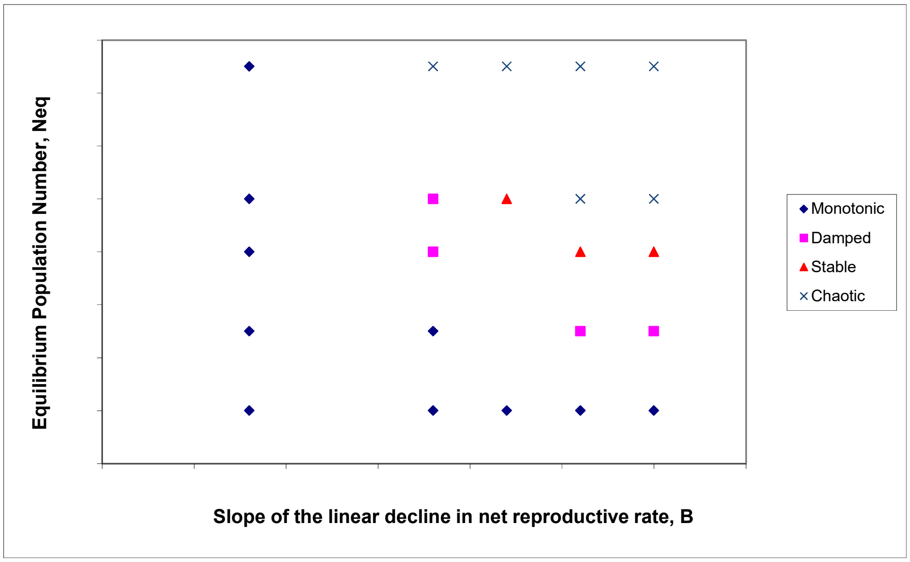

Fig. 1. Layout of graph exploring the dynamics of populations using the discrete Logistic Population Growth Model. There should be 25 points representing 4 different types of population dynamics (use 4 different symbols). Note that the values on the axes have been omitted; it is up to you to figure out appropriate values for B and Neq and to include them in your graph.

3. Time-Lag Models

Another group of simple models, with time-lags, have been shown to produce a wide diversity of population growth dynamics. The following model is modified from a logistic population growth model that assumes linear density-dependence of the net reproductive rate (R_0):

$$N_{t+1} = (1.0 - B\,(N_{t-x} - N_{eq}))\,N_t\ ,$$

where N is population size, t is time and B is the slope of the decline in R_0 with increasing population density (i.e. the intensity of density-dependence). A time-lag ***x*** is included in this model through N_{t-x}, where *x*=1 (i.e. N_{t-1}) for a 1 generation time-lag, *x*=2 (i.e. N_{t-2}) for a 2 generations time-lag, *x*=0 (i.e. N_t) for no time-lag. *(Again, we have or will soon discuss this model in lecture)*

Task 3A) Using the above model, compare the dynamics of populations without time-lag, with a 1 generation time-lag and with a 2 generations time-lag. Plot the population dynamics from these 3 models on the same graph, with clear labels for each model. Run these comparisons with 3 sets of parameters: N_{eq} = 100, B = 0.011; N_{eq} = 100, B=0.0075 and N_{eq} = 100, B=0.001, and provide a graph for each (i.e. you should have a total of 3 graphs).

Question 3B) What is the effect of time-lags on the dynamics and long-term viability of these populations? Describe your findings in a brief paragraph.

LAB 2 - Theoretical Population Growth

<u>Individual</u> Lab Assignment: *(marked out of 100, worth 5% of final grade)*
Assemble all graphs and short answers into a single Word document, and include your *name, student number, "EEB319-Lab 2" and date* at the top.

Make sure to include the following (tasks and questions in the text above):

1) <u>Exponential Population Growth Model</u> (30%)
   - Task 1A: One graph comparing dynamics of 5 populations
   - Answer questions 1B-C

2) <u>Logistic Population Growth Model</u> (50%)
   - Task 2A: Graph of B vs N_{eq} showing ranges for different types of dynamics
   - Answer questions 2B-D

3) <u>Time-Lag Models</u> (20%)
   - Task 3A: graphs showing the effects of time lags on population dynamics
   - Answer question 3B

Submit on Quercus before February 14 11:59pm. Late penalty is -10% per day including week-ends.