Foundational Magnetic Susceptibility

January 23, 2018

1 Introduction

1.1 Background Reading

1. "Paramagnetic and Diamagnetic Materials", F. Thompson, Phys. Educ. 46, 328 (2011).
2. "Classical and Semiclassical Diamagnetism: A Critique of Treatment in Elementary Texts", S. L. O'Dell, and R. K. P. Zia, Am. J. Phys. 54, 32 (1986).
3. "Foundational Magnetic Susceptibility Instructor's Manual", TeachSpin, Inc.
4. "Measurements and their Uncertainties", Ifan G. Hughes and Thomas P. A. Hase.

1.2 Motivation

Anyone's first play with permanent magnets allows a sorting of materials into the categories 'magnetic' vs. 'non-magnetic'. But it turns out that even materials we generally call nonmagnetic do display a magnetic response; and even if it is a much weaker response than we associate with ferromagnetism, it is nevertheless a highly revealing response. This experiment allows this response to be demonstrated, quantified, and explained. Michael Faraday was the first to show that non-ferromagnetic materials nevertheless showed non-zero magnetic response. He found that some materials (labeled paramagnetic) were attracted into regions of larger magnetic field, while others (labeled diamagnetic) we repelled out of those high-field regions. Both phenomena are described in electromagnetism by assigning to a medium a 'magnetic susceptibility' χ, which describes how much magnetization M (magnetic moment per unit volume) arises when the material is immersed in a pre-existing magnetic field H. For materials that are linear and isotropic, χ is a scalar, and the response M is given by $\vec{M} = \chi\vec{H}$. The susceptibility χ is positive for paramagnetic, and negative for diamagnetic, materials. It is also dimensionless, and quite small for most non-ferromagnetic materials.

The susceptibility is interesting not just because its sign allows a sorting into dia- vs. paramagnetic materials, but because it is so readily measured, and so easily connected to an underlying quantum-mechanical description of a material. It is a striking consequence of the Bohr-van Leeuwen theorem that if materials were made of charged particles obeying the laws of classical mechanics, then they would always display $\chi = 0$ at equilibrium! So measuring a value of $\chi \neq 0$ is already a hint that matter needs to be described quantum-mechanically.

Better still, there are quite straightforward quantum-mechanical derivations that describe what the susceptibility of a material ought to be. From a perturbation-theory calculation it emerges that all materials ought to possess a diamagnetic response, whose size is small, and predictable from the mean-square size of electronic wavefunctions, $\sum\langle r^2\rangle$. But the same calculation shows that materials with non-zero electron spin content ought also to display Curie paramagnetism, and this in general outweighs the weak underlying diamagnetism of the material.

So the tabletop measurement of magnetic susceptibility provides an easily, non-invasive, and non-destructive way to look inside a material for unpaired electron spins! You will see that the Gouy method has adequate sensitivity, precision, and accuracy.

2 Objectives

These objectives are intended to guide your experiments. They are not a step-by-step procedure.

1. Measure the magnetic field to calibrate the Guoy balance. State your calibration and uncertainty, paying careful attention to all sources of uncertainty. Your approach should include a linear fit as a function of calibration current, and an analysis of the residuals left from the fit.

2. Measure the magnetic susceptibility of all 16 samples that have been pre-prepared. Give the uncertainty along with each value. Discuss any surprises.

3. Measure the magnetic susceptibility of at least 3 more samples which you prepare on your own. Give the uncertainty along with each value. Discuss any surprises.

4. OPTIONAL: Check the linearity of magnetic susceptibility.

3 Questions

These questions should be specifically answered in your lab notebook, and can also serve as a guide for discussion in your lab report analysis.

1. What are the commonly used units of magnetic susceptibility (cgs and mks)?

2. Compare the magnetic susceptibilities that you measured to accepted values (consider making a plot with the accepted values on the horizontal axis and your measured values on the vertical axis). Watch out for units, and don't assume that the accepted values are necessarily correct.

3. Is it plausible to use a magnetic field gradient to levitate a diamagnetic material? Based on your measurements, what combinations of materials and magnetic field gradients could succeed?

4. OPTIONAL: Try to predict the magnetic susceptibilities of the diamagnetic and paramagnetic materials you measured, and check whether you at least got the trends right by plotting with the measured values on the horizontal axis and your predicted values on the vertical axis.