Foundational Magnetic Susceptibility

Johnny Pribyl

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Abstract

This lab collects data from 19 samples. We measured the changes in mass that were a result of each sample's magnetic moment. Then, after taking the measurements, we used the standard relationship between mass (force) and volume magnetic susceptibility χ_m to compare observed values for χ_m with those in the literature.

1 Introduction

Almost every material has some degree of magnetism present. However, magnetic forcs have a tendency to be quite weak (to the point of being largely imperceptible). Unlike gravitational forces, electromagnetic forces can be attractive or repulsive. Additionally, any time you place an object inside a magnetic field, the charges inside it tend to move around. Weasel's are funny, so let's pretend that we have a weasel sitting between a couple of magnets.

As the weasel's charges move around, they create *another* magnetic field. Sometimes this second 'induced' magnetic field is in the same direction as the original field, and other times it is in the opposite direction. I've never heard of an induced field that goes off in some random direction.. but magnets (and weasels) tend to have minds of their own, so maybe it's possible. At any rate, if the weasel's induced field is in the same direction as the original, we would call the weasel Paramagnetic. If it is in the opposite direction, we would call the weasel Diamagnetic.

In this lab, we examine the magnetic properties of several substances to figue out how many charges like to move around and what direction they tend to move.

Just like last time, you can find all of my code at:

`jpribyl/cautious-palm-tree`

2 Objectives

This lab has three main objectives. If you're not familiar with the methods and procedures of this lab, then I would suggest reviewing the manual. It lives in:

`lab2/lab_descrip/Foundational_Magnetic_Susceptibility_Manual.pdf`

2.1 B Field Calibration

Before we can start doing any kind of analysis on the data that we collected, we had to calibrate the magnetic field between the magnets. Neodymium magnets can be quite strong. We expected a result on the order of .5 Tesla.

It is not possible (or at least not feasible) to measure the magnetic field directly. Instead we measured the mass of the magnet. Then, we placed a copper sheet between the magnet and ran a strong current through it. We measured the change in mass of the magnet and used this equation to calculate B:

$$\vec{F} = \frac{A\chi_m(B_b^2 - B_t^2)}{2\mu_0}$$

We found that keeping the copper sheet over the top of the magnet has a negligible effect upon the mass of the system. It did not register at all on the Guoy Balance. So, we were able to conclude that B_t is zero and solve for B_b:

$$B_b = \frac{2\mu_0(m_1 - m_0)g}{A\chi_m}$$

Where m_1 is the mass of the system with the current running through it and m_0 is the mass of the magnet by itself. Note that when there is no mass change, the magnetic field vanishes.

The data that we collected is available in data/magnetic.xlsx and my lab notebook, so I will not recopy it here. But, here is what it looks like:

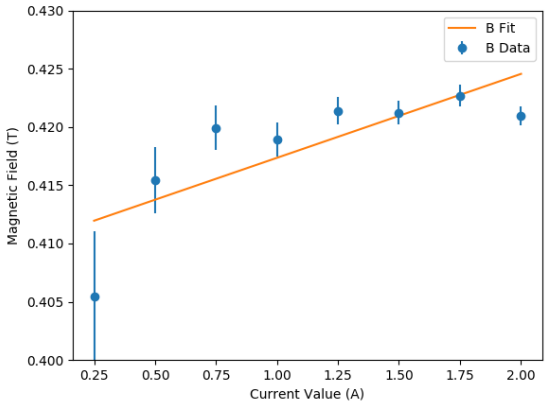

Figure 1: Plotting the Measured values for Magnetic Field

I chose to read my data into python as a Pandas DataFrame. In my experience, pandas is just about the best library to work with heterogeneous data. It's even able to read an excel file:

```
xl = pd.ExcelFile('data/magnetic.xlsx')
df = xl.parse('Sheet1')
```

After that, I dropped the non-existent entries off the bottom of the DataFrame because they're problematic for model fitting. Then, I used the uncerainties package and a lambda function to propagate error through all of our mass and current measurements. I'm not going to copy all of the code here, but the general syntax follows this form:

```
<Measurement>= \
    df[<Measurement>].dropna().apply(
        lambda x: ufloat(x, <error>)
    )
```

Next, for the current menasurements, I looked up the specs for Keithley's model 2000 6 1/2 digit multimeter. I stared at them for a while. Then I pestered Brian for a while. Then I stared at the specs some more. Eventually, I pestered Brian enough that he showed how to read the table. In our case, the current has an uncertainty of:

$$(1000 \times I + 3 \times 15) \times 10^{-6}$$

In python, we are able to make use of the pandas data structure and uncertainties library to propagate this:

```
current_error = \
    1000 * df['current'].dropna() * 10 ** -6 + 3 * 15 * 10**-6

b_cal_current = \
    pd.Series(uarray(df['current'].dropna(), current_error))
```

Notice that I have to explicitly turn the result back into a pandas object. The uarray() method returns a NumPy Array. It would be totally fine to leave the result as a NumPy object, but syntactically NumPy is slightly different and Pandas, so it's beneficial to have all objects be the same type.

I was able to fit the curve using the same method as in the data analysis lab. Specifically, I assumed linearity and fit it with:

```
def lin_fit(x, a, b):
    return a*x + b

popt, pcov = curve_fit(
                lin_fit,
                current_values,
                b_cal_values
            )

b_fit = lin_fit(
    current_values,
    *popt
)
```

We learned last lab that residuals are a pretty decent sanity check on the accuracy of data and models. So let's go ahead and plot the residuals from this fit:

```
r_i = b_cal_values - b_fit

plt.errorbar(
    current_values,
    r_i,
    yerr=b_cal_error,
    fmt='o')
```

And showing this plot, we find that the residuals are actually quite reasonable. They are clustered around zero and their error bars are easily visible. Notice that the size of the error bars around zero is quite large. This makes sense because the current error ought to be similar in magnitude, but its fractional error will increase:

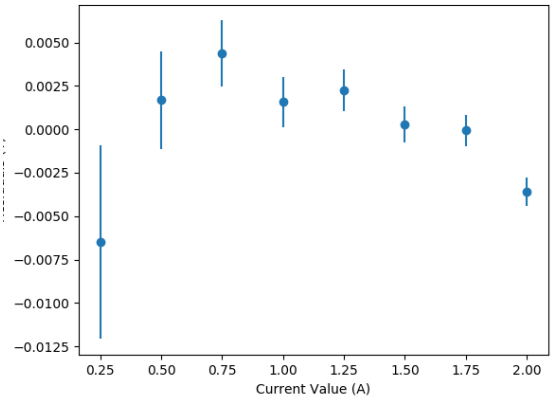

Figure 2: Plotting the residuals as a sanity check

2.2 Magnetic Susceptibility of Samples

Now, hopefully I've convinced you that the environment between our magnets houses a magnetic field that is approximately .4 Teslas. If I haven't convinced you yet, try this:

```
def understand(paper, confused=True):
    if confused:
        read(section_1)
        read(section_2)
        understand(paper)

    return
```

Moving right along, We massed all our samples using the Guoy balance. Then, we did some math to determine how much of our sample was empty space and how much was really the substance in question. For example, if you have a vial full of pebbles, then a good bit of the vial is actually just air. The actual area is given by:

$$A = (w \times l)(\%real)$$

Where % real is:

$$\frac{m_{measured}}{m_{theoretical}}$$

And, for a substance of known density ρ, theoretical mass is:

$$volume \times \rho$$

Putting all of that together with the equation that we derived in lecture, we find the magnetic susceptibilty with:

$$\chi_m = \frac{2 \times \mu_0 \times g(m_1 - m_0)}{AB^2}$$

As before, m_1 refers to the mass of the magnet apparatus with the sample sitting in between the magnet and m_0 is the mass of the magnet without a sample in between it.

One peculiarity that I had not considered prior to doing my data analysis was the possibility of getting a value larger than 1 for % real. This does not physically make sense (because our samples were not pressurized). For most of the samples, this was not an issue.

However, for the two wire samples, things got a little interesting. The wires were too small for us to measure very accurately which led us to accumulate immensely large errors. After thinking on the issue for a while, eventually I decided to accept the measurements on these wires that are provided in the lab to be exact. I also decided to refuse negatve percentages and cap the maximum % real at 1.

In python, we can impose reaonability on percentages with:

```
percent_real = (real_mass / theory_mass)

percent_real.apply(
    lambda x: ufloat(min(abs(x.n), 1), x.s)
)
```

Then, we can drop the error from cobalt:

```
area[4] = area[4].n
percent_real[4] = percent_real[4].n
```

And now we're off to the races! Doing all the math things and putting them in a table thing that compares measurements and uncertainties to accepted χ_m values, we see that most of our results are actually quite good:

	Sample Name	Volume X_m in SI	X_m Uncertainty	Literature Volume X_m Values
0	Cu_110_Alloy	-0.000014	0.000002	-0.000010
1	Al_1100_Alloy	0.000016	0.000002	0.000021
2	Ti_Grade_2	0.000180	0.000005	0.000181
3	Bi_Pellets	-0.000160	0.000005	-0.000170
4	co	2.236249	0.018296	NaN
5	grap	-0.000864	0.000282	-0.000800
6	NdCl_3	0.000750	0.000016	NaN
7	Gd_2_O3	0.012894	0.000266	0.013666
8	Er_2_O_3	0.021730	0.000448	0.020982
9	NH42FeSO46H2O	0.000755	0.000016	NaN
10	Iron Alum	0.000699	0.000015	NaN
11	Cu_Sulphate	0.000144	0.000004	0.000336
12	Cu_Acetate	0.000061	0.000004	NaN
13	Mn_Oxide	0.004504	0.000093	0.007053
14	Mn Chloride	0.002188	0.000045	0.002529
15	Ni_Zn	0.003793	0.000078	NaN
16	h2o_distilled	0.000024	0.000002	-0.000009
17	stainless_steel	0.008420	0.134091	NaN
18	soda_lime_glass	0.000033	0.000004	-0.000011

Figure 3: A table with our results

However, there were a couple of surprises! I was shocked at the lack of literature data for comparison. Most of the data that I *was* able to find was not available in SI units. The conversion from χ_{molar} in cgs to volume χ_m in SI must be done in 2 steps. First, you convert to volume χ_m in cgs and second, you multiply by the conversion factor of 4π. These units are very picky and I still don't fully understand why.

Also, it seems as though something went a bit wrong when we prepared our own samples! While the majority of our data falls within 3 σ of the literature values, we did not even get the correct sign for Water or Soda Lime Glass. Maybe we should have been a bit more suspicious of the bottle labeled "distilled water."

3 Questions

3.1 Units Of χ_m

There are a few common units for χ_m because there are a few different flavors of susceptibility. In class we discussed that Volume χ_m is actually unitless. This is true in both cgs and SI - however, there *is still a conversion factor between*

them. I find that pretty strange, but I learned to stop questioning units when I started measuring the mass of the sun in kilometers.

You might also encounter a molar χ_m. In CGS this has units of $\frac{cm^3}{mol}$ but Brian told us that in SI, molar χ_m has units of $\frac{kg}{mol}$. That's pretty neat, but Wikipedia disagrees with Brian. Wikipedia says that molar χ_m has units of $\frac{m^3}{mol}$ in SI.

The last type of χ_m that Brian mentioned is mass χ_m. In SI this has units of $\frac{m^3}{kg}$ while in cgs it is $\frac{cm^3}{g}$.

3.2 Compare Results to Literature Values

It's tough to compare our results to literature values, because quite a few of the literature values don't exist. And, when they do exist they're in the wrong units. So, in order to do any comparison I had to start by collecting as many literature values for χ_m as I could find. I converted the cgs values of χ_{mol} into volume χ_m in SI like this:

$$\chi_{sivol} = 4\pi \frac{\chi_{cgsmol}}{M_{cgs}/\rho_{cgs}}$$

Where M is the molar mass. If we plot this with the literature values along the x axis and measure values along the y axis, we would expect the line y = x to intersect most of them. I went one step further and ran a linear fit on our data points using the same curve_fit method that I describe in section 2.1. Plotting the results, we see that, overall, our data agrees with the literature:

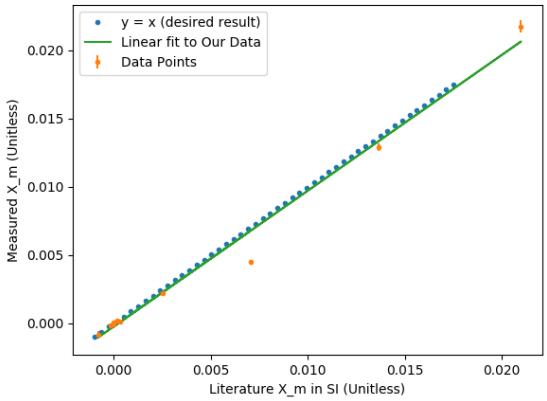

Figure 4: Plotting accepted values for χ_m against our measurements

3.3 Levitation!!

Brian gave this one away. I'm going to go out on a limb here and say that pyrolitic graphite might work, just maybe. Here's what it looks like:

Figure 5: Photo credits to the kind folks over at: http://sci-toys.com

Sci-toys has a really great explanation of the phenomenon. It's worth reading if you like science. Here's an excerpt that will (hopefully) get you excited enough to open up a browser:

> "We can do that by using four magnets. The poles of the magnets push on the diamagnetic material more strongly than other parts of the magnet. With four magnets, the four edges of the square of pyrolytic graphite will be pushed away from the four poles."
> - sci-toys.com

4 Conclusion and Sources of Error

I must say, I'm pretty impressed with our results. I fully expected them to wildly disagree with the accepted values of χ_m. During the experiment, we noticed that the instruments were incredibly sensitive. Things like leaning on the table, or rotating the balance made noticeable impacts upon results. We also noticed that the scale's calibration had a tendency to walk. And, paralax made it quite difficult to ensure that the bottom of our samples was precisely in the middle of the magnetic field.

The only significant source of error seemed to occur during the preparation of our own samples. It's possible that the vials were not adequately cleaned. It's even possible that the samples we prepared were not quite exactly what we

thought they were. Lastly, it's possible that the literature values for χ_m are wrong.

If I were to do this experiment again, I would be more careful to monitor the walking of the scale's calibration and more careful while preparing my own samples.