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Review Questions

1a. $SE_{bt}(\hat{\theta}) = \left((B-1)^{-1} \sum_{b=1}^{B} (\hat{\theta}^{*,b} - \bar{\hat{\theta}}^{*})^2\right)^{1/2}$

$SE_{bt}(\hat{\theta}) = \left(((10{,}000-1)^{-1}\left((0.047-.0000341)^2 + (0.049-.000841)^2 + (0.053-.0000341)^2 + (0.060-.0000341)^2 + (0.065-.0000341)^2 + (0.067-.0000341)^2\right)\right)^{1/2}$

$SE_{bt}(\hat{\theta}) = \left((9{,}999)^{-1}(0.0022 + 0.00239 + 0.002805 + 0.003596 + 0.00422 + 0.004484)\right)^{1/2}$

$\left((9{,}999)^{-1}(0.019695)\right)^{1/2}$

$\left(\frac{0.019695}{9999}\right)^{1/2}$

$SE_{bt}(\hat{\theta}) = 0.00140$

1b. $CI = [\hat{\theta} - SE_{bt}(\hat{\theta})Z_{1-\alpha/2}, \hat{\theta} + SE_{bt}(\hat{\theta})Z_{1-\alpha/2}]$

$[0.057 - 0.00140(1.645), 0.057 + 0.00140(1.645)]$

90% $CI_{bt} = [0.054697, 0.059303]$

$[0.057 - 0.00140(1.96), 0.057 + 0.00140(1.96)]$

95% $CI_{bt} = [0.054256, 0.059744]$

$\frac{\hat{\theta} - \theta}{SE(\hat{\theta})} \sim N(0,1) \quad \frac{.056 - .057}{0.00140} = -0.7143$

1c. $SE_{et}(\hat{\theta}) = \frac{.053 - .060}{-.677789 - .677789} = \frac{0.007}{1.355578}$

$SE_{et}(\hat{\theta}) = 0.00516385$

90% $Cl_{et} = [0.057 - 0.00516385(1.645), 0.057 + 0.00516385(1.645)]$

90% $Cl_{et} = [0.04850, 0.06549]$

95% $Cl_{et} = [0.057 - 0.00516385(1.96), 0.057 + 0.00516385(1.96)]$

95% $Cl_{et} = [0.04678854, 0.06712146]$

2a. $bias(\hat{\theta}) = E(\hat{\theta}) - \theta$

$0.68431 - 0.69119$

$bias(\hat{\theta}) = -0.00688$

2b. $SD = 0.11293$

2c. $MSE = var(\hat{\theta}) + bias^2(\theta)$

$var(\hat{\theta}) = 0.11293^2 = 0.01275318$

$MSE = 0.01275318 + (-0.00688)^2$

$MSE = 0.01280051$

2d. $\frac{0.00004733}{0.01280051} = \frac{369751}{100000000}$ of MSE is due to bias.

Ideally, we want our bias to be as small as possible, but there are also times in which we want to increase our bias (if we have a large variance) in order to minimize MSE as much as possible. The ratio of the MSE that is the bias is <50% so it does not dominate the bias, and it is not very serious here.

ECON 432 Homework 4 R Excercises

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Contents

1A

```
set.seed(432)
```

```
options(digits=4, width=70)
# Round off a number to 4 decimal places
# The width option says how many characters R should put on a line.

library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```r
# look at help on get.hist.quote
# ?get.hist.quote

# get the adjusted closing prices from Yahoo!
AAPL.prices = get.hist.quote(instrument="AAPL", start="2010-01-01",
                             end="2021-01-31", quote="AdjClose",
                             provider="yahoo",
                             compression="w", retclass="zoo")
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```

```
## time series ends   2021-01-29
```

```r
AAPL.return = diff(log(AAPL.prices))
T = length(AAPL.return)

mu.hat = mean(AAPL.return)
sigma.hat = sd(AAPL.return)
variance = sigma.hat^2
SE = sigma.hat/sqrt(T-1)
cbind(mu.hat,variance,SE)
```

```
##        mu.hat variance       SE
## [1,] 0.005212 0.001375 0.001544
```

1B

```r
library(bootstrap)
nboot = 1000
sam_m = function(x){mean(x)}
Ret <- as.numeric(AAPL.return)
results = bootstrap(Ret, nboot, sam_m)
B_SE = sd(results$thetastar)
IQR_SE = IQR(results$thetastar)/(qnorm(0.75)-qnorm(0.25))
cbind(SE, B_SE, IQR_SE)
```

```
##            SE     B_SE   IQR_SE
## [1,] 0.001544 0.001522 0.001519
```

The SE, bootstrap SE, and IQR SE are all fairly close to eachother with the SE and IQR SE being the closest.

1C

```
library(bootstrap)
nboot = 1000
sam_m = function(x){mean(x)}
Ret <- as.numeric(AAPL.return)
results = bootstrap(Ret, nboot, sam_m)
B_SE = sd(results$thetastar)
B_SE
```

```
## [1] 0.001484
```

```
IQR_SE = IQR(results$thetastar)/(qnorm(0.75)-qnorm(0.25))
IQR_SE
```

```
## [1] 0.001472
```

1D

```
B_bias = mean(results$thetastar)-mu.hat
MSE = mean((results$thetastar-mu.hat)^2)
cbind(B_bias,MSE)
```

```
##         B_bias       MSE
## [1,] 5.686e-05 2.205e-06
```

1E

```
alpha = 0.05
q_sym = quantile(abs(results$thetastar-mu.hat),1-alpha)
q_et_1 = quantile(mu.hat-results$thetastar,alpha/2)
q_et_2 = quantile(mu.hat-results$thetastar,1-alpha/2)

CI_et = c(mu.hat+q_et_1, mu.hat+q_et_2)
CI_sym = c(mu.hat-q_sym, mu.hat+q_sym)
cbind(CI_et,CI_sym)
```

```
##          CI_et   CI_sym
## 2.5%  0.002450 0.002359
## 97.5% 0.008075 0.008065
```

1F

```r
W0 <- 1000
alpha <- 0.1
L1 <- W0*(exp(AAPL.return)-1)
VaR_Para_Est <- W0*(exp(qnorm(alpha,mu.hat,sigma.hat))-1)

VaR_NonP_Est <- W0*(exp(quantile(AAPL.return,alpha))-1)

library(cvar)
L1_qf <- function(x){ W0*(exp(qnorm(x, mu.hat, sigma.hat)) - 1)}
para_ES <- -1*ES(L1_qf, x = alpha, dist.type = "qf")

ind <- as.numeric(L1 <= VaR_NonP_Est)
nonp_ES <- mean(L1*ind)/mean(ind)

cbind(VaR_Para_Est, para_ES,VaR_NonP_Est,nonp_ES)
```

```
##     VaR_Para_Est para_ES VaR_NonP_Est nonp_ES
## 10%       -41.42  -57.99        -38.2  -62.54
```

1G

```r
set.seed(123)
nboot <-  1000
VaR_Est  <- function(x, p = alpha){ W0*(exp(quantile(x, p)) - 1) }

Ret <- as.numeric(AAPL.return)
results_VaR <- bootstrap(Ret, nboot, VaR_Est)

B_SE_VaR <- sd(results_VaR$thetastar)

Q_VaR <- quantile(results_VaR$thetastar, c(0.25, 0.75))
B_IQRSE_VaR <- (Q_VaR[2] - Q_VaR[1])/(qnorm(0.75) - qnorm(0.25))

ES_Est   <- function(x, p=alpha){
var_est  = W0*(exp(quantile(x, p)) - 1)
L1       = W0*(exp(x) - 1)
ind      = as.numeric(L1 <= var_est)
f_val    = mean(L1*ind)/mean(ind)
return(f_val)
}

set.seed(123)
results_ES <- bootstrap(Ret,nboot,ES_Est)

Q_ES <- quantile(results_ES$thetastar, c(0.25, 0.75))
B_SE_ES <- sd(results_ES$thetastar)
B_IQRSE_ES <- (Q_ES[2] - Q_ES[1])/(qnorm(0.75) - qnorm(0.25))

cbind(B_SE_VaR,B_IQRSE_VaR,B_SE_ES,B_IQRSE_ES)
```

```
##     B_SE_VaR B_IQRSE_VaR B_SE_ES B_IQRSE_ES
## 75%    2.531        2.73   4.482      4.384
```

1H

```
CI_B_SE1.ES <- c(nonp_ES - B_SE_ES*1.96,    nonp_ES + B_SE_ES*1.96)
CI_B_SE1.VaR <- c(VaR_NonP_Est - B_SE_VaR*1.96,    VaR_NonP_Est + B_SE_VaR*1.96)
cbind(CI_B_SE1.ES,CI_B_SE1.VaR)
```

```
##     CI_B_SE1.ES CI_B_SE1.VaR
## 10%      -71.32       -43.16
## 10%      -53.75       -33.23
```