Madelyn Caufield

1 $R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

$R_t = \frac{4676 - 4489}{44.89} = .041657$

1 mo investment yields a 4.17% monthly return

$FV = PV(1+r)^n$

$FV = \$10000(1+R_{Apt})$

$= \$10000(1 + .041657)$

$= \$10000(1.041657) = 10,416.57$

---

2 $r_t = \ln(1 + R_t)$

$r_t = \ln(1.041657) = .0408127$

1 mo investment yields a 4.08% cc monthly return

$e^{r_t} = \ln(1 + R_t)$

$e^{.0408127} = 1 + R_t$

$R_t = e^{.0408127} - 1 = .041657$

$= 4.17\%$

3 $R_A = (1 + R_t)^{12} - 1$

$R_A = (1 + .041657)^{12} - 1$

$R_A = (1.041657)^{12} - 1 = .6319124$

per year

$\approx 63.19\%$

---

4 $R_A = (1 + r_t)^{12} - 1$

$R_A = (1 + .0408127)^{12} - 1$

$R_A = (1\ .0408127)^{12} - 1 = .6161$

per year

$\approx 61.61\%$

---

5. $R_t^2 = \frac{53.10 - 44.89}{44.89} = .1828915$

$\approx 18.28\%$

$FV = \$10{,}000(1 + .1828915)$

$= \$11{,}828.915$

simple annual

Here, we have the realistic return, which is less than the simple annual return with monthly compounding

It is much less because the simple annual return with monthly compounding in 3 assumes that r is the same over all 12 months

---

6. $r_t^{real} = \ln(1 + R_t^R)$

$r_t^R = \ln(1 + .18289151)$

$= .16796187 33$

$\approx 16.80\%$

$e^{r_t^R} = (1 + R_t^R)$

$e^{.16796187 33} = 1 + R_t^R$

$R_t^R = e^{16796187 33} - 1$

$= .18289151$

Here, we have the actual (REAL) annual continuously compounded return (16.80%) which is less than the continuously compounded annual return (61.61%) The same assumption for 4 holds for above

Part IV

1. $f(0) = \ln(1+0) - 0 = 0$

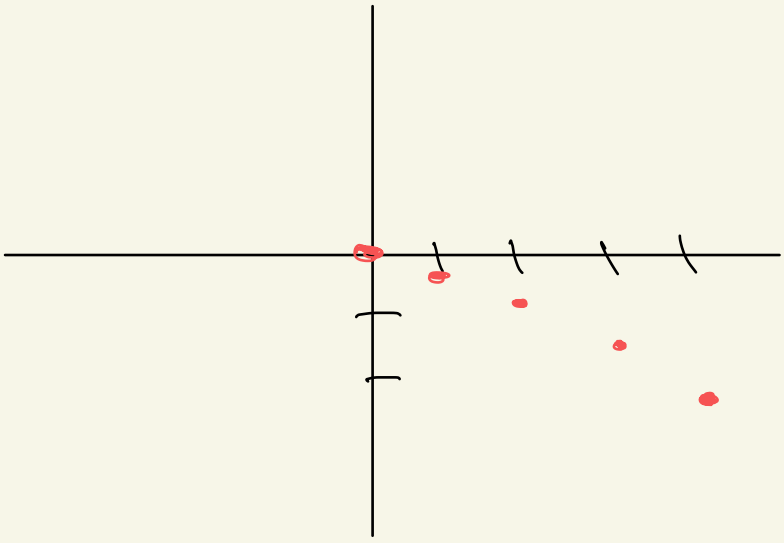
$f(1) = \ln(1+1) - 1 = -.3068528$

$f(2) = \ln(1+2) - 2 = -.9013877$

$f(3) = \ln(1+3) - 3 = -1.613705$

$f(4) = \ln(1+4) - 4 = -2.390562088$

x	y
0	0
1	-.3068528
2	-.9013877
3	-1.613705
4	-2.390562088



$f(x) \leq 0$ for any $x > -1$

2. We need to use proof by induction

We consider X_1, X_2 as a portfolio

$r_1 \cdot X_1 (r_2 \cdot X_2)$

Then treat X_3 as a separate entity

$$\underbrace{r_1 \cdot X_1 + r_2 \cdot X_2}_{y} + r_3 \cdot X_3$$

$$= (1 - X_3)\, y + \underbrace{X_3 \cdot r_3}_{\text{3rd asset}}$$

y: combined (cc return of first 2 assets)

$$y = \underbrace{\frac{X_1}{(1-X_3)}}_{\text{get } X_1 \text{ back}} \cdot r_1 + \underbrace{\frac{X_2}{(1-X_3)}}_{\text{get } X_2 \text{ back}} \cdot r_2$$

$$f(x+1) = \frac{x+1}{(1-(x+1))} + x+1 \cdot r_1 + \frac{x+1}{(1-(x+1))} + x+1 \cdot r_2$$

$$\frac{x+1}{(1-(x+1))} + \frac{(1-(x+1))(x+1)}{(1-(x+1))} \cdot r_1$$

$$+ \frac{x+1}{(1-(x+1))} + \frac{(1-(x+1))(x+1)}{(1-(x+1))} \cdot r_2$$

$$= \frac{(x+1)(1+(1-(x+1))}{(1-(x+1))} \circ r_1$$

$$+ \frac{(x+1)(1+(1-(x+1))}{(1-(x+1))} \circ r_2$$

$$y = x_1 \circ r_1 + x_2 \circ r_2$$

ECON 432 Homework 1

Madelyn Caufield

Jan 20,2021

Contents


```
getwd()
```

```
## [1] "/Users/madelyncaufield/Desktop/ECON432 Data Science for Financial Engineering/Homework 1"
```

```
setwd("/Users/madelyncaufield/Desktop/ECON432 Data Science for Financial Engineering/Homework 1")
getwd()
```

```
## [1] "/Users/madelyncaufield/Desktop/ECON432 Data Science for Financial Engineering/Homework 1"
```

```
df <- read.csv("sbuxPrices.csv")
head(df)
```

```
##         Date     Open     High      Low    Close Adj.Close    Volume
## 1 1998-03-01 2.472656 2.902344 2.382813 2.832031  2.367074 297006400
## 2 1998-04-01 2.835938 3.062500 2.656250 3.007813  2.513996 330811200
## 3 1998-05-01 3.015625 3.117188 2.781250 3.000000  2.507466 233537600
## 4 1998-06-01 2.976563 3.421875 2.910156 3.339844  2.791515 362950400
## 5 1998-07-01 3.351563 3.746094 2.601563 2.617188  2.187503 594307200
## 6 1998-08-01 2.636719 2.703125 1.929688 1.972656  1.648790 652120000
```

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
sbux.df = df %>%
          select(Date, Adj.Close)
```

Part II

Question 1

```
head(sbux.df)
```

```
##         Date Adj.Close
## 1 1998-03-01  2.367074
## 2 1998-04-01  2.513996
## 3 1998-05-01  2.507466
## 4 1998-06-01  2.791515
## 5 1998-07-01  2.187503
## 6 1998-08-01  1.648790
```

This gives us the first 6 rows of the data.

```
tail(sbux.df)
```

```
##           Date Adj.Close
## 176 2012-10-01  19.94681
## 177 2012-11-01  22.54120
## 178 2012-12-01  23.40302
## 179 2013-01-01  24.48960
## 180 2013-02-01  23.93541
## 181 2013-03-01  24.94520
```

This gives us the last 6 rows of the data.

```
sbuxPrices.df = sbux.df[, "Adj.Close", drop=FALSE]
sbuxPrices.df
```

```
##     Adj.Close
## 1    2.367074
## 2    2.513996
## 3    2.507466
## 4    2.791515
## 5    2.187503
## 6    1.648790
## 7    1.890395
## 8    2.265861
## 9    2.409518
## 10   2.931906
```

```
## 11   2.719687
## 12   2.762131
## 13   2.931906
## 14   3.859147
## 15   3.852618
## 16   3.924447
## 17   2.429108
## 18   2.389929
## 19   2.589089
## 20   2.840489
## 21   2.775190
## 22   2.533586
## 23   3.343288
## 24   3.669783
## 25   4.681910
## 26   3.158820
## 27   3.552243
## 28   3.989744
## 29   3.917916
## 30   3.826499
## 31   4.185639
## 32   4.668849
## 33   4.760267
## 34   4.623142
## 35   5.217360
## 36   4.975753
## 37   4.433774
## 38   4.043289
## 39   4.078811
## 40   4.805977
## 41   3.769558
## 42   3.525080
## 43   3.121796
## 44   3.577317
## 45   3.702692
## 46   3.980602
## 47   4.966872
## 48   4.808066
## 49   4.833141
## 50   4.768364
## 51   5.073440
## 52   5.192544
## 53   4.101796
## 54   4.200006
## 55   4.317021
## 56   4.981499
## 57   4.542691
## 58   4.258514
## 59   4.747469
## 60   4.900007
## 61   5.382694
## 62   4.912546
## 63   5.154932
## 64   5.129857
```

```
## 65   5.710755
## 66   5.932247
## 67   6.017919
## 68   6.602994
## 69   6.722100
## 70   6.928964
## 71   7.649861
## 72   7.814933
## 73   7.913146
## 74   8.132547
## 75   8.483591
## 76   9.087473
## 77   9.818820
## 78   9.035233
## 79   9.499116
## 80  11.049570
## 81  11.755837
## 82  13.030468
## 83  11.283600
## 84  10.825988
## 85  10.794641
## 86  10.347478
## 87  11.448672
## 88  10.794641
## 89  10.980611
## 90  10.245090
## 91  10.468672
## 92  11.818524
## 93  12.725391
## 94  12.541508
## 95  13.247780
## 96  15.178531
## 97  15.725989
## 98  15.575548
## 99  14.898525
## 100 15.780325
## 101 14.305095
## 102 12.959421
## 103 14.229874
## 104 15.776139
## 105 14.748080
## 106 14.802410
## 107 14.601812
## 108 12.913448
## 109 13.105693
## 110 12.963603
## 111 12.040015
## 112 10.965987
## 113 11.149868
## 114 11.513449
## 115 10.949268
## 116 11.149868
## 117  9.774938
## 118  8.554641
```

```
## 119  7.902697
## 120  7.514042
## 121  7.313445
## 122  6.782696
## 123  7.601803
## 124  6.577919
## 125  6.139113
## 126  6.502696
## 127  6.214338
## 128  5.487173
## 129  3.731944
## 130  3.953439
## 131  3.945081
## 132  3.823885
## 133  4.642992
## 134  6.042995
## 135  6.013739
## 136  5.804784
## 137  7.397027
## 138  7.936127
## 139  8.629864
## 140  7.931952
## 141  9.152253
## 142  9.637027
## 143  9.106283
## 144  9.574342
## 145 10.142697
## 146 10.857329
## 147 10.864536
## 148 10.197306
## 149 10.428110
## 150  9.643379
## 151 10.778244
## 152 12.048012
## 153 12.908585
## 154 13.611482
## 155 13.357297
## 156 13.971573
## 157 15.716125
## 158 15.397117
## 159 15.648073
## 160 16.856457
## 161 17.112574
## 162 16.485100
## 163 15.973932
## 164 18.145773
## 165 18.625546
## 166 19.786406
## 167 20.607794
## 168 20.883020
## 169 24.120111
## 170 24.758835
## 171 23.688551
## 172 23.081621
```

```
## 173 19.601198
## 174 21.475607
## 175 22.037100
## 176 19.946812
## 177 22.541204
## 178 23.403025
## 179 24.489603
## 180 23.935408
## 181 24.945196
```

This gives us just the column for adjusted closing prices and drops the column Date column to make a new data frame called sbuxPrices.df.

```
rownames(sbuxPrices.df) = sbux.df$Date
rownames(sbuxPrices.df)
```

```
##   [1] "1998-03-01" "1998-04-01" "1998-05-01" "1998-06-01" "1998-07-01"
##   [6] "1998-08-01" "1998-09-01" "1998-10-01" "1998-11-01" "1998-12-01"
##  [11] "1999-01-01" "1999-02-01" "1999-03-01" "1999-04-01" "1999-05-01"
##  [16] "1999-06-01" "1999-07-01" "1999-08-01" "1999-09-01" "1999-10-01"
##  [21] "1999-11-01" "1999-12-01" "2000-01-01" "2000-02-01" "2000-03-01"
##  [26] "2000-04-01" "2000-05-01" "2000-06-01" "2000-07-01" "2000-08-01"
##  [31] "2000-09-01" "2000-10-01" "2000-11-01" "2000-12-01" "2001-01-01"
##  [36] "2001-02-01" "2001-03-01" "2001-04-01" "2001-05-01" "2001-06-01"
##  [41] "2001-07-01" "2001-08-01" "2001-09-01" "2001-10-01" "2001-11-01"
##  [46] "2001-12-01" "2002-01-01" "2002-02-01" "2002-03-01" "2002-04-01"
##  [51] "2002-05-01" "2002-06-01" "2002-07-01" "2002-08-01" "2002-09-01"
##  [56] "2002-10-01" "2002-11-01" "2002-12-01" "2003-01-01" "2003-02-01"
##  [61] "2003-03-01" "2003-04-01" "2003-05-01" "2003-06-01" "2003-07-01"
##  [66] "2003-08-01" "2003-09-01" "2003-10-01" "2003-11-01" "2003-12-01"
##  [71] "2004-01-01" "2004-02-01" "2004-03-01" "2004-04-01" "2004-05-01"
##  [76] "2004-06-01" "2004-07-01" "2004-08-01" "2004-09-01" "2004-10-01"
##  [81] "2004-11-01" "2004-12-01" "2005-01-01" "2005-02-01" "2005-03-01"
##  [86] "2005-04-01" "2005-05-01" "2005-06-01" "2005-07-01" "2005-08-01"
##  [91] "2005-09-01" "2005-10-01" "2005-11-01" "2005-12-01" "2006-01-01"
##  [96] "2006-02-01" "2006-03-01" "2006-04-01" "2006-05-01" "2006-06-01"
## [101] "2006-07-01" "2006-08-01" "2006-09-01" "2006-10-01" "2006-11-01"
## [106] "2006-12-01" "2007-01-01" "2007-02-01" "2007-03-01" "2007-04-01"
## [111] "2007-05-01" "2007-06-01" "2007-07-01" "2007-08-01" "2007-09-01"
## [116] "2007-10-01" "2007-11-01" "2007-12-01" "2008-01-01" "2008-02-01"
## [121] "2008-03-01" "2008-04-01" "2008-05-01" "2008-06-01" "2008-07-01"
## [126] "2008-08-01" "2008-09-01" "2008-10-01" "2008-11-01" "2008-12-01"
## [131] "2009-01-01" "2009-02-01" "2009-03-01" "2009-04-01" "2009-05-01"
## [136] "2009-06-01" "2009-07-01" "2009-08-01" "2009-09-01" "2009-10-01"
## [141] "2009-11-01" "2009-12-01" "2010-01-01" "2010-02-01" "2010-03-01"
## [146] "2010-04-01" "2010-05-01" "2010-06-01" "2010-07-01" "2010-08-01"
## [151] "2010-09-01" "2010-10-01" "2010-11-01" "2010-12-01" "2011-01-01"
## [156] "2011-02-01" "2011-03-01" "2011-04-01" "2011-05-01" "2011-06-01"
## [161] "2011-07-01" "2011-08-01" "2011-09-01" "2011-10-01" "2011-11-01"
## [166] "2011-12-01" "2012-01-01" "2012-02-01" "2012-03-01" "2012-04-01"
## [171] "2012-05-01" "2012-06-01" "2012-07-01" "2012-08-01" "2012-09-01"
## [176] "2012-10-01" "2012-11-01" "2012-12-01" "2013-01-01" "2013-02-01"
## [181] "2013-03-01"
```

This converted the Date column to row names for the sbuxPrices.df data frame.

```
head(sbuxPrices.df)
```

```
##            Adj.Close
## 1998-03-01  2.367074
## 1998-04-01  2.513996
## 1998-05-01  2.507466
## 1998-06-01  2.791515
## 1998-07-01  2.187503
## 1998-08-01  1.648790
```

This gave us the first 6 rows of the adjusted closing price column with the new row names (dates from above) from the sbuxPrices.df data frame.

```
sbux.df[101:132,]
```

```
##           Date Adj.Close
## 101 2006-07-01 14.305095
## 102 2006-08-01 12.959421
## 103 2006-09-01 14.229874
## 104 2006-10-01 15.776139
## 105 2006-11-01 14.748080
## 106 2006-12-01 14.802410
## 107 2007-01-01 14.601812
## 108 2007-02-01 12.913448
## 109 2007-03-01 13.105693
## 110 2007-04-01 12.963603
## 111 2007-05-01 12.040015
## 112 2007-06-01 10.965987
## 113 2007-07-01 11.149868
## 114 2007-08-01 11.513449
## 115 2007-09-01 10.949268
## 116 2007-10-01 11.149868
## 117 2007-11-01  9.774938
## 118 2007-12-01  8.554641
## 119 2008-01-01  7.902697
## 120 2008-02-01  7.514042
## 121 2008-03-01  7.313445
## 122 2008-04-01  6.782696
## 123 2008-05-01  7.601803
## 124 2008-06-01  6.577919
## 125 2008-07-01  6.139113
## 126 2008-08-01  6.502696
## 127 2008-09-01  6.214338
## 128 2008-10-01  5.487173
## 129 2008-11-01  3.731944
## 130 2008-12-01  3.953439
## 131 2009-01-01  3.945081
## 132 2009-02-01  3.823885
```

This gave us only rows 101-132 for the sbux.df data frame.

Question 2

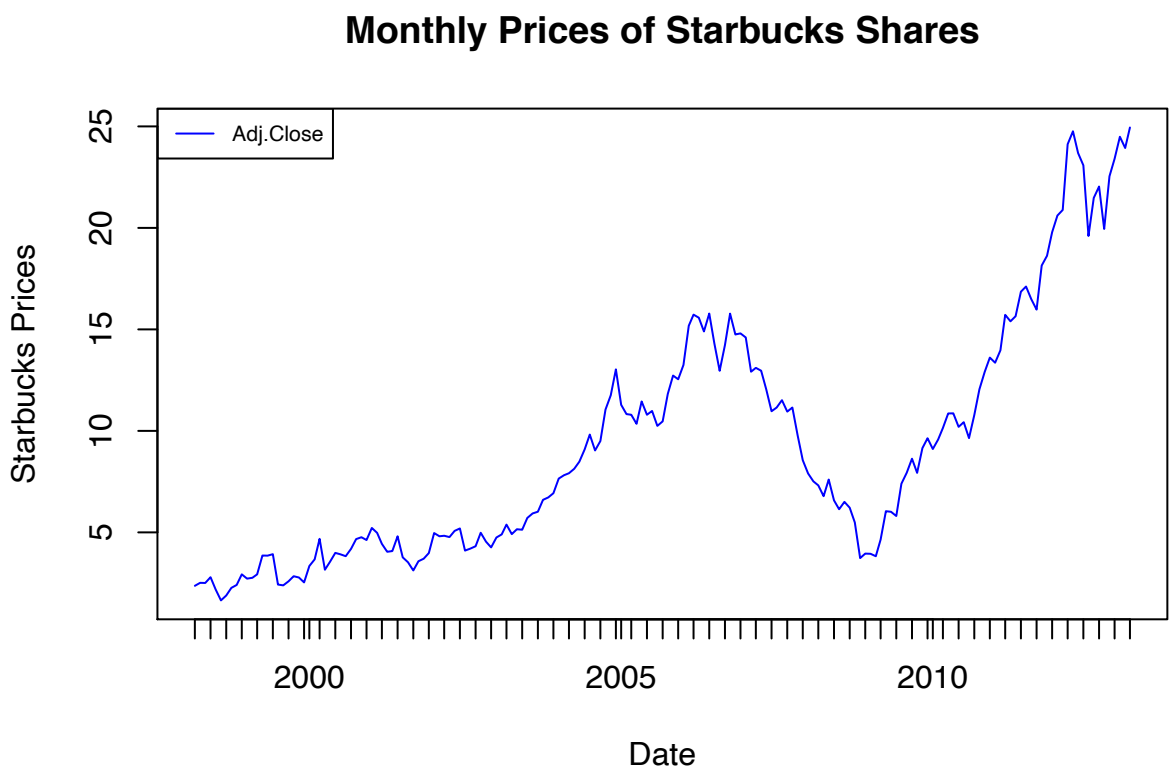
```
class(sbux.df$Date)
```

```
## [1] "character"
```

```
sbux.df$Date <- as.Date(sbux.df$Date,format = '%Y-%m-%d')
class(sbux.df$Date)
```

```
## [1] "Date"
```

```
plot(Adj.Close~Date, sbux.df, type='l',col="blue",xlab = 'Date',ylab='Starbucks Prices',
     main='Monthly Prices of Starbucks Shares')
T <- length(sbux.df$Date)
ticks <- seq(sbux.df$Date[1], sbux.df$Date[T], by = "quarters")
axis(1, at = ticks, labels = FALSE)
legend('topleft', names(sbuxPrices.df), col= "blue", lty=1, cex=.65)
```



Question 3

Monthly Simple Return

```
sbux.df$ret[2:T] <-  sbux.df$Adj.Close[2:T]/sbux.df$Adj.Close[1:(T-1)] - 1
head(sbux.df$ret)
```

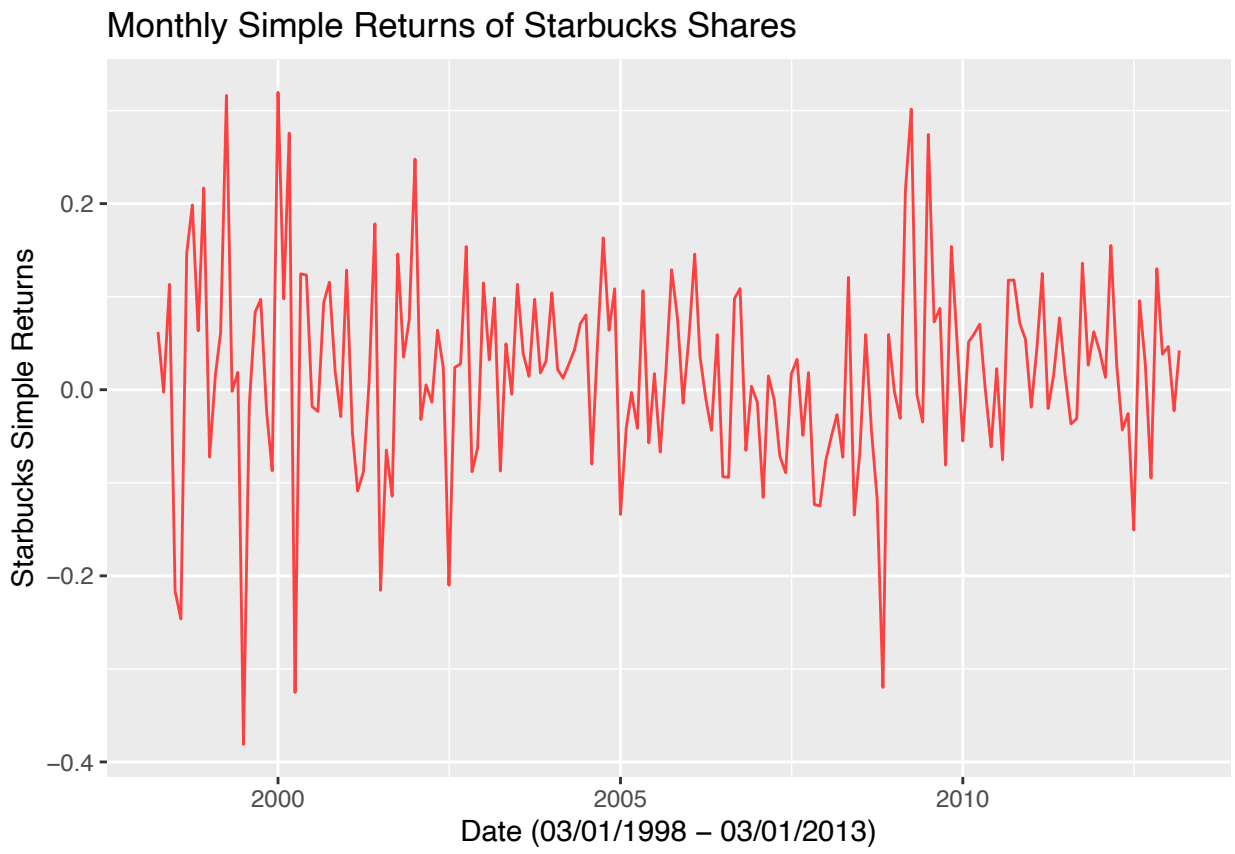
```
## [1]           NA  0.062069035 -0.002597458  0.113281297 -0.216374263
## [6] -0.246268462
```

```
library(ggplot2)

df.g <- sbux.df[,c('Date','ret')]
df.g <- df.g[-c(1),]

head(df.g)
```

```
##         Date          ret
## 2 1998-04-01  0.062069035
## 3 1998-05-01 -0.002597458
## 4 1998-06-01  0.113281297
## 5 1998-07-01 -0.216374263
## 6 1998-08-01 -0.246268462
## 7 1998-09-01  0.146534732
```

Monthly Simple Return Plot

```
g <- ggplot(df.g, aes(x = Date, y = ret)) +
  geom_line(color="brown1") +
  ylab("Starbucks Simple Returns") +
  xlab("Date (03/01/1998 - 03/01/2013)") +
  ggtitle("Monthly Simple Returns of Starbucks Shares")
g
```



Monthly Coninuously Compounded Return

```
sbux.df$cc.ret[2:T] <-  log(sbux.df$Adj.Close[2:T]/sbux.df$Adj.Close[1:(T-1)])
head(sbux.df$cc.ret)
```

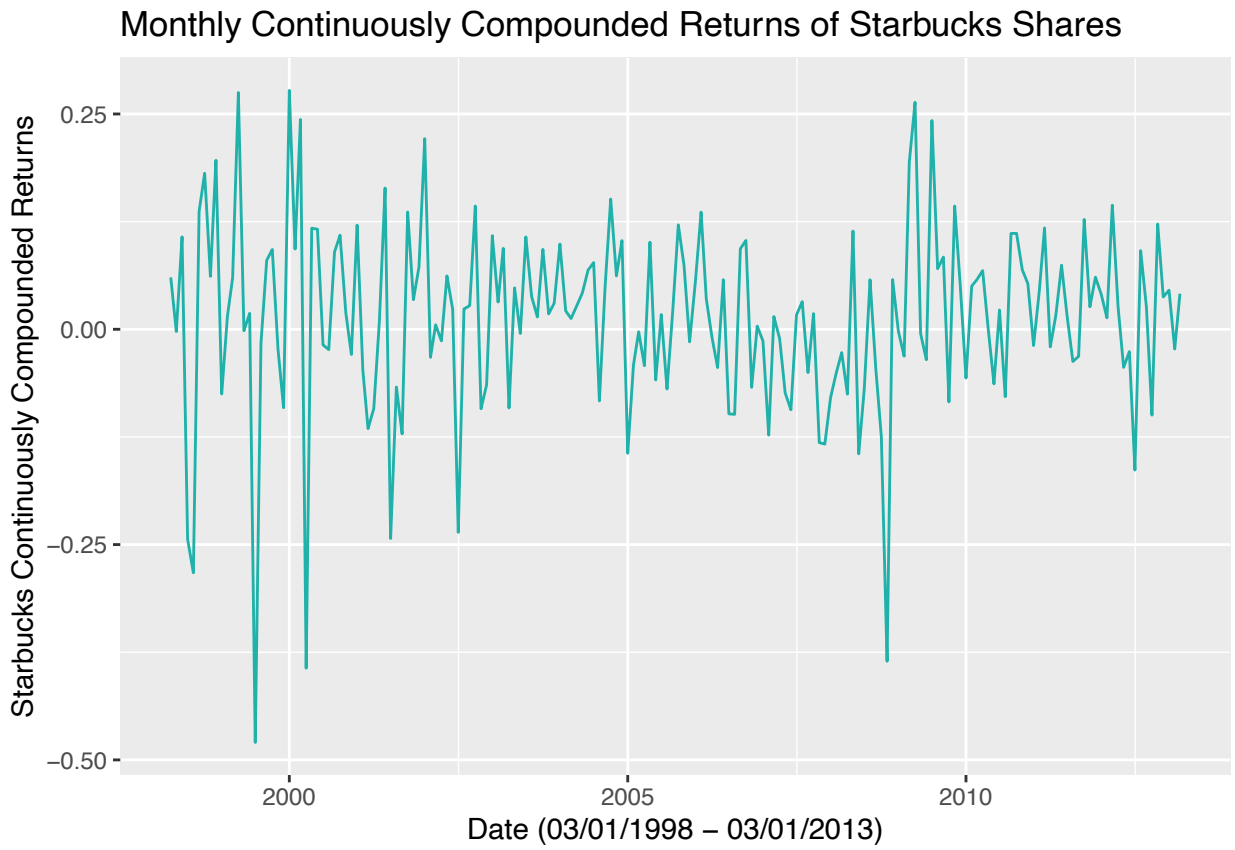
```
## [1]           NA  0.060218926 -0.002600838  0.107311778 -0.243823749
## [6] -0.282719025
```

```
df.g$cc.ret <- sbux.df$cc.ret[2:T]
head(df.g)
```

```
##         Date          ret       cc.ret
## 2 1998-04-01  0.062069035  0.060218926
## 3 1998-05-01 -0.002597458 -0.002600838
## 4 1998-06-01  0.113281297  0.107311778
## 5 1998-07-01 -0.216374263 -0.243823749
## 6 1998-08-01 -0.246268462 -0.282719025
## 7 1998-09-01  0.146534732  0.136744116
```

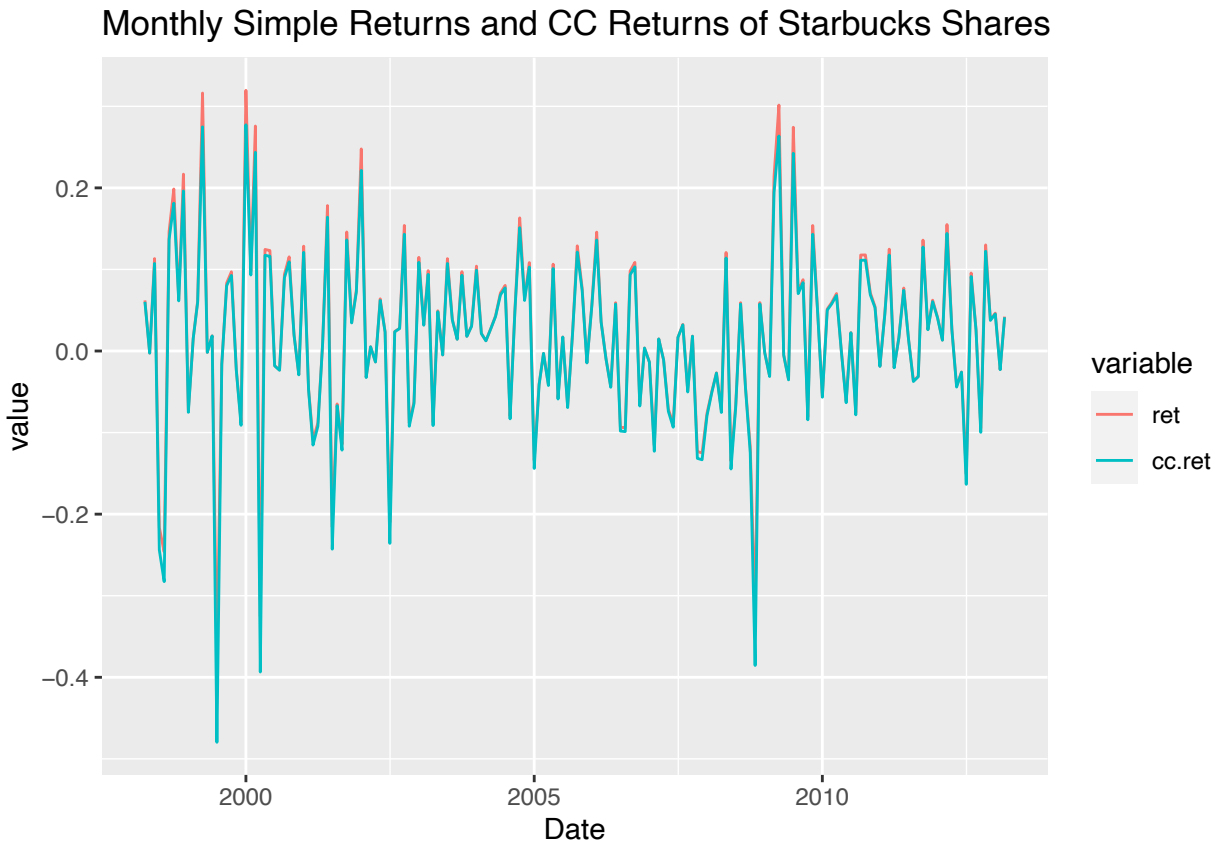
Monthly Continuously Compounded Return Plot

```
g <- ggplot(df.g, aes(x = Date, y = cc.ret)) +
  geom_line(color="lightseagreen") +
  ylab("Starbucks Continuously Compounded Returns") +
  xlab("Date (03/01/1998 - 03/01/2013)") +
  ggtitle("Monthly Continuously Compounded Returns of Starbucks Shares")
g
```



Plot of the Monthly Simple Return and Continuously Compounded Return

```
library(reshape2)
d <- melt(df.g, id.vars="Date")

ggplot(d, aes(Date, value, col=variable)) +
  geom_line() +
  ggtitle("Monthly Simple Returns and CC Returns of Starbucks Shares")
```


Monthly Simple Returns and CC Returns of Starbucks Shares
value
0.2
0.0
−0.2
−0.4
2000
2005
2010
Date
variable
ret
cc.ret