Madelyn Caufield

Part 1

1) Let $X \sim U[0,1]$ & $Y \sim Ber(0.5)$

pdf of X is

$$f(x)=1 \quad 0 \leq x \leq 1$$

pmf of y is

$$p(y=1)=(0.5)^y(0.5)^{1-y} \quad y=(0,1)$$

$$E(x)=\int_0^1 x\,dx$$

$$=\left[\frac{x^2}{2}\right]_0^1$$

$$=\frac{1}{2}[1-0]$$

$$E(x)=\frac{1}{2}$$

$$E(x^2)=\int_0^1 x^2\,dx$$

$$=\left[\frac{x^3}{3}\right]_0^1$$

$$=\frac{1}{3}[1-0]$$

$$E(x^2)=\frac{1}{3}$$

$$Var(x)=E(x^2)-[E(x)]^2$$

$$=\frac{1}{3}-\frac{1}{4}$$

$$Var(x)=\frac{1}{12}$$

$$E(y)=\sum_{y=0}^{1} y(0.5)^y(0.5)^{1-y}$$

$$=0(0.5)^0(0.5)^{1-0}+1(0.5)^1(0.5)^{1-1}$$

$$E(y)=0.5$$

$$E(y^2)=\sum_{y=0}^{1} y^2(0.5)^y(0.5)^{1-y}$$

$$=0(0.5)^0(0.5)^{1-0}+1(0.5)^1(0.5)^{1-1}$$

$$=0.5$$

$Var(y) = E(y^2) - [E(y)]^2$

$= 0.5 - 0.25$

$Var(y) = 0.25$

$Pr(x < 0.1) = \int_0^{0.1} 1 dx$

$= [x]_0^{0.1}$

$= 0.1 - 0$

$Pr(x < 0.1) = 0.1$

$Pr(y < 0.1) = p\left(\frac{1 - E(y)}{\sqrt{var(y)}} < \frac{0.1 - 0.5}{\sqrt{0.25}}\right)$

$= p(z < \frac{-0.9}{0.5}$

$= p(z < -0.8)$

$= p(z > 0.8)$

$Pr(y < 0.1) = 0.21186$

$E(0.3x + 0.7y) = 0.3E(x) + 0.7E(y)$

$= 0.3(1/2) + 0.7(0.5)$

$= 0.15 + 0.35$

$E(0.3x + 0.7y) = 0.50$

$E(0.5x + 0.5y) = 0.5E(x) + 0.5E(y)$

$= 0.5(1/2) + 0.5(0.5)$

$= 0.25 + 0.25$

$E(0.5x + 0.5y) = 0.50$

for $\alpha \in [0,1]$

$E[\alpha x + (1-\alpha)y] = \alpha E(x) + (1-\alpha)E(y)$

$= \alpha\left(\frac{1}{2}\right) + (1-\alpha)(0.5)$

$= 0.5[\alpha + 1 - \alpha]$

$E[\alpha x + (1-\alpha)y] = 0.5$

The variance of $\alpha X+(1-\alpha)Y$ is:

$$\alpha^2 var(X)+(1-\alpha)^2 var(Y)$$

$$\alpha^2 \cdot var(X)+\alpha^2-2\alpha+1 \cdot var(y)$$

$$=\frac{\alpha^2}{12}+\frac{\alpha^2-2\alpha+1}{4} \quad (3)$$

$$=\frac{4\alpha^2-6\alpha+3}{12}$$

Since the function $4\alpha^2-6\alpha+3$ is minimized at $\alpha=3/4$, the variance of $\alpha X+(1-\alpha)Y$ is minimized at $\alpha=3/4$. Therefore, we should put 3/4 of \$100 on X and 1/4 of \$100 on Y. (Given $E(X)=E(Y)=0.5$)

ECON 432 Homework 2 (2-6)

Madelyn Caufield

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Contents

Part 1: Review Questions (2-6)

Question 2

```
pnorm(-1.96, mean = 0, sd = 1)
```

```
## [1] 0.0249979
```

```
1-pnorm(1.64, mean = 0, sd = 1)
```

```
## [1] 0.05050258
```

```
val <- pnorm(0.5, mean = 0, sd = 1) - pnorm(-0.5, mean = 0, sd = 1)
cat("The target probablity is: ", val)
```

```
## The target probablity is:  0.3829249
```

```
qnorm(0.010)
```

```
## [1] -2.326348
```

```r
qnorm(0.99)
```

```
## [1] 2.326348
```

```r
qnorm(.05)
```

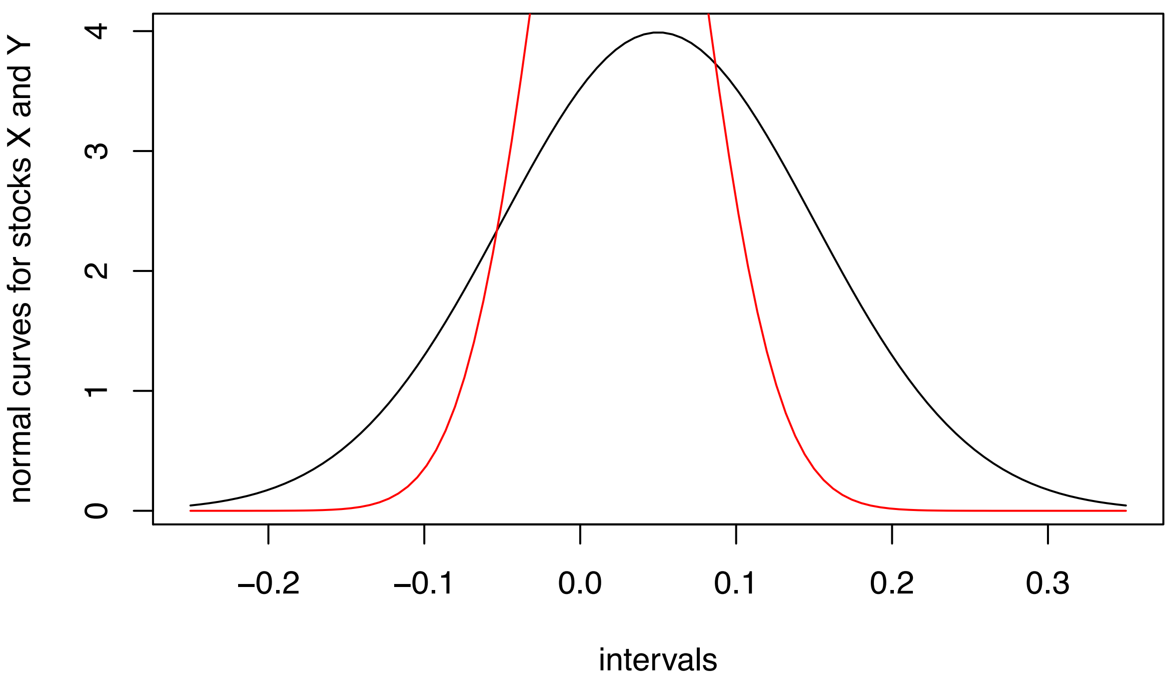
```
## [1] -1.644854
```

```r
qnorm(.95)
```

```
## [1] 1.644854
```

Question 3

```r
intervals <- seq(-0.25, 0.35,length=100)
mrst <- dnorm(intervals,0.05,0.1)
sbux <- dnorm(intervals,0.025,0.05)
plot(intervals, mrst, ylab='normal curves for stocks X and Y', type="l", col="black")
lines(intervals, sbux, type="l", col="red")
```



The Microsoft stock has the highest risk because it has the highest deviation from the mean. If I was a risk-averse investor I would want to invest in Starbucks.

Question 4

```r
library(scales)
W0 = 100000
mu.r = 0.05
sigma.r = 0.12
alpha = 0.01
VaR.01 = qnorm(alpha, mu.r, sigma.r) * W0
sprintf("The %s VaR is %.3f", percent(alpha), VaR.01)
```

```
## [1] "The 1% VaR is -22916.174"
```

```r
Integrand <- function(u, alpha_value, mu, sigma) {
  W0*(qnorm(u, mu, sigma))/alpha_value
  }
res <- integrate(Integrand, lower = 0, upper = alpha,
                 alpha_value=alpha,
                 mu = mu.r,
                 sigma = sigma.r)

sprintf("The %s ES is %#.3f", percent(alpha), res[1])
```

```
## [1] "The 1% ES is -26982.571"
```

```r
alpha = 0.05
VaR.05 = qnorm(0.05, mu.r, sigma.r) * W0
sprintf("The %s VaR is %.3f", percent(alpha), VaR.05)
```

```
## [1] "The 5% VaR is -14738.244"
```

```r
Integrand <- function(u, alpha_value, mu, sigma) {
  W0*(qnorm(u, mu, sigma))/alpha_value
  }
res <- integrate(Integrand, lower = 0, upper = alpha,
                 alpha_value=alpha,
                 mu = mu.r,
                 sigma = sigma.r)

sprintf("The %s ES is %#.3f", percent(alpha), res[1])
```

```
## [1] "The 5% ES is -19752.554"
```

Question 5

```r
library(scales)
W0 <- 100000
alpha = 0.01
mu.r = 0.05
```

```r
sigma.r = 0.12
VaR.01 = W0*(exp(qnorm(alpha, mu.r, sigma.r))-1)
sprintf("The %s VaR is %.3f", percent(alpha), VaR.01)
```

```
## [1] "The 1% VaR is -20480.010"
```

```r
Integrand <- function(u, alpha_value, mu, sigma) {
  W0*(exp(qnorm(u, mu, sigma))-1)/alpha_value
  }
res <- integrate(Integrand, lower = 0, upper = alpha,
                 alpha_value=alpha,
                 mu = mu.r,
                 sigma = sigma.r)

sprintf("The %s ES is %#.3f", percent(alpha), res[1])
```

```
## [1] "The 1% ES is -23596.520"
```

```r
W0 <- 100000
alpha = 0.05
mu.r = 0.05
sigma.r = 0.12
VaR.05 = W0*(exp(qnorm(alpha, mu.r, sigma.r))-1)
sprintf("The %s VaR is %.3f", percent(alpha), VaR.05)
```

```
## [1] "The 5% VaR is -13703.611"
```

```r
Integrand <- function(u, alpha_value, mu, sigma) {
  W0*(exp(qnorm(u, mu, sigma))-1)/alpha_value
  }
res <- integrate(Integrand, lower = 0, upper = alpha,
                 alpha_value=alpha,
                 mu = mu.r,
                 sigma = sigma.r)

sprintf("The %s ES is %#.3f", percent(alpha), res[1])
```

```
## [1] "The 5% ES is -17844.193"
```

```r
mu = 12*mu.r
abs.sd = sqrt(12*(sigma.r)^2)
alpha = 0.01
VaR.01 = W0*(exp(qnorm(alpha, mu, abs.sd))-1)
sprintf("The %s VaR over the year investment is %.3f", percent(alpha), VaR.01)
```

```
## [1] "The 1% VaR over the year investment is -30722.129"
```

```r
Integrand <- function(u, alpha_value, mu, sigma) {
  W0*(exp(qnorm(u, mu, sigma))-1)/alpha_value
  }
res <- integrate(Integrand, lower = 0, upper = alpha,
                 alpha_value=alpha,
                 mu = mu,
                 sigma = abs.sd)

sprintf("The %s ES over the year investment is %#.3f", percent(alpha), res[1])
```

```
## [1] "The 1% ES over the year investment is -39351.696"
```

```r
alpha = 0.05
VaR.05 = W0*(exp(qnorm(alpha, mu, abs.sd))-1)
sprintf("The %s VaR over the year investment is %.3f", percent(alpha), VaR.05)
```

```
## [1] "The 5% VaR over the year investment is -8034.144"
```

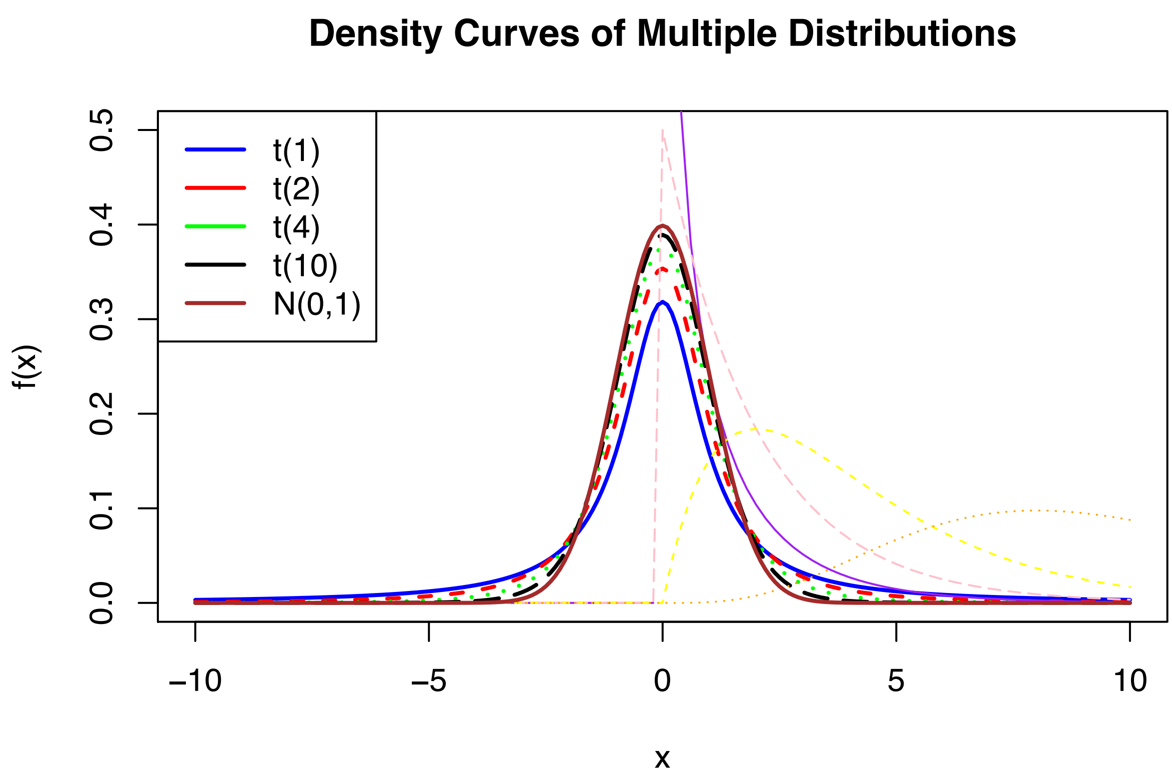
```r
Integrand <- function(u, alpha_value, mu, sigma) {
  W0*(exp(qnorm(u, mu, sigma))-1)/alpha_value
  }
res <- integrate(Integrand, lower = 0, upper = alpha,
                 alpha_value=alpha,
                 mu = mu,
                 sigma = abs.sd)

sprintf("The %s ES over the year investment is %#.3f", percent(alpha), res[1])
```

```
## [1] "The 5% ES over the year investment is -21836.387"
```

Question 6

```r
x <- seq(-10,10,by=0.1)

x.norm <- dnorm(x) # standard norm density

x.t1 <- dt(x,df=1)
x.t2 <- dt(x,df=2)
x.t4 <- dt(x,df=4)
x.t10 <- dt(x,df=10)

plot(x.t1~x, type="l", col="blue",
     lwd=2, ylim=c(0,0.5), xlab = "x", ylab = "f(x)",
     main = "Density Curves of Multiple Distributions")

lines(x.t2~x, lty = "dashed", col="red",
     lwd=2)

lines(x.t4~x, lty = "dotted", col = "green", lwd = 2)
```

```
lines(x.t10~x, lty ="longdash", col = "black", lwd = 2)

curve( dchisq(x, df=1), col='purple', add=TRUE)

curve( dchisq(x, df=2), col='pink', add=TRUE, lty = "longdash")

curve( dchisq(x, df=4), col='yellow', add=TRUE, lty = "dashed")

curve( dchisq(x, df=10), col='orange', add=TRUE, lty = "dotted")

lines(x.norm~x, type="l", col = "brown", lwd = 2)

legend(x="topleft", legend=c("t(1)","t(2)","t(4)","t(10)","N(0,1)"),
       col=c("blue","red", "green","black", "brown"), lwd=2)
```



5% VaR of t-distribution with 2 degrees of freedom is greater because the number of observations is less.

Part 1

7) Bivariate pdf

X \ Y	-1	1	
-1	1/8	0	1/8
0	1/8	1/8	2/8
1	1/8	3/8	4/8
2	0	1/8	1/8
	3/8	5/8	

$E(x) = -1(1/8) + 0(2/8) + 1(4/8) + 2(1/8)$

$E(x) = 5/8$

$E(x^2) = -1^2(1/8) + 0^2(2/8) + 1^2(4/8) + 2^2(1/8)$

$E(x^2) = 7/8$

$Var(x) = E(x^2) - [E(x)]^2 = 7/8 - (5/8)^2$

$Var(x) = 31/64$

$SD(x) = \sqrt{Var(x)} = \sqrt{31/64}$

$SD(x) = \sqrt{31}/8$

$E(y) = -1(3/8) + 1(5/8)$

$E(y) = 1/4$

$E(y^2) = -1^2(3/8) + 1^2(5/8)$

$E(y^2) = 1/4$

$Var(y) = E(y^2) - [E(y)]^2 = 1/4 - (1/4)^2$

$Var(y) = 3/16$

$SD(y) = \sqrt{3/16}$

$SD(y) = \sqrt{3}/4$

$\operatorname{cov}(x,y) = E[XY] - E[X]E[Y]$

$(2-5/8)(-1-1/4)\cdot 0 + (1-5/8)(-1-1/4)\cdot 1/8 + (0-5/8)(-1-1/4)\cdot 1/8$
$+ (-1-5/8)(-1-1/4)\cdot 1/8 + (2-5/8)(1-1/4)\cdot 1/8 + (1-5/8)(1-1/4)\cdot 3/8$
$+ (0-5/8)(1-1/4)\cdot 1/8 + (-1-5/8)(1-1/4)\cdot 0$

$\operatorname{cov}(x,y) = 31/128$

$$\operatorname{Cor}(x,y) = \frac{\operatorname{cov}(x,y)}{SD(x)\cdot SD(Y)} = \frac{(31/128)}{\sqrt{31}/8 \cdot \sqrt{3}/4}$$

$\operatorname{Cor}(x,y) = \sqrt{93}/12$

$$f(x \mid Y=1) = \frac{P(X=x, y=1)}{P(y=1)} = \begin{cases} 0 \cdot 5/8 = 0, & x=-1 \\ 1/8 \cdot 5/8 = 5/64, & x=0 \\ 3/8 \cdot 5/8 = 15/64, & x=1 \\ 1/8 \cdot 5/8 = 5/64, & x=2 \end{cases}$$

$$f(Y \mid X=2) = \begin{cases} 0 \cdot 1/8 = 0, & Y=-1 \\ 1/8 \cdot 1/8 = 1/64, & Y=1 \end{cases}$$

$E[X \mid Y=1] = -1\cdot 0 + 0\cdot 5/64 + 1\cdot 15/64 + 2\cdot 5/64$

$E[X \mid y=1] = 25/64$

$E[y \mid x=2] = -1\cdot 0 + 1\cdot 1/64$

$E[y \mid x=2] = 1/64$

$\operatorname{var}[x \mid y=1] = (-1-25/64)^2\cdot 0 + (0-25/64)^2\cdot 5/64 + (1-25/64)^2\cdot 15/64$
$+ (2-25/64)^2 \cdot 5/64$

$\operatorname{Var}[x \mid y=1] = 78985/262144$

$\operatorname{var}[y \mid x=2] = (-1-1/64)^2\cdot 0 + (1-1/64)^2\cdot 1/64$

$\operatorname{Var}[y \mid x=2] = 3969/262144$

X & Y are not independent since
$f(x \mid y=1) \neq f(x)$ and $f(y \mid x=2) \neq f(y)$

ECON 432 Homework 2 Pt. II

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Contents

Part 2: R Excercises

Question 1

```
set.seed(123) # for reproducible randomness
runs <- 1
mu.x <- 0
sigma.x <- 0.1

X <- rnorm(runs, mu.x, sigma.x) # the random sample of 5000 observations
head(X)
```

```
## [1] -0.05604756
```

```
sum(X >= 0.5)/runs
```

```
## [1] 0
```

Question 2

```
set.seed(321)
X <- matrix(rbinom(runs*3,size = 1, prob = 0.5), ncol = 3)
X
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    0
```

```
Y <- ifelse(X[,1]==1 & X[,2]==1, -3,
            ifelse(X[,1]==0 & X[,2]==0, -1, 2))
head(cbind(X,Y))
```

```
##             Y
## [1,] 1 1 0 -3
```

```
sum(Y)/runs
```

```
## [1] -3
```

Question 3

```
set.seed(321)
X <- matrix(rbinom(runs*3,size = 1, prob = 0.64), ncol = 3)
X
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    1
```

```
Y <- ifelse(X[,1]==1 & X[,2]==1, -3,
            ifelse(X[,1]==0 & X[,2]==0, -1, 2))
head(cbind(X,Y))
```

```
##             Y
## [1,] 0 0 1 -1
```

```
sum(Y)/runs
```

```
## [1] -1
```

Question 4

```
set.seed(321)
X <- matrix(rbinom(runs*3,size = 1, prob = 0.64), ncol = 3)
X
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    1
```

```
Y <- ifelse(X[,1]==1 & X[,2]==1, -3,
            ifelse(X[,1]==0 & X[,2]==0, -1, 2))
head(cbind(X,Y))
```

```
##               Y
## [1,] 0 0 1 -1
```

```r
sum(Y>0)/runs
```

```
## [1] 0
```