week5_ipynb

February 2, 2021

1 Week 3: Classification

In this section we will apply what we have learned about the logistic regression model to fit a model and make predictions. We will be using the `penguins` dataset from `seaborn` and try to predict where or not a penguin is of Adelie species.

We will build a Logistic Regression model from scratch.

[55]:
```python
# Code for Week 5
import pandas as pd
import numpy as np
import seaborn as sns

# Import penguins
penguins = (sns.load_dataset("penguins")).dropna()
penguins["One"] = 1
penguins["Adelie"] = 1*(penguins["species"] == "Adelie")

print("Data Shape, ",penguins.shape)

# Take a look at the columns
print(penguins.head())

# What percentage of our data is Adelie
print(np.mean(penguins.Adelie))

# What features do we have
print(penguins.columns)
```

```
Data Shape,  (333, 9)
  species     island  bill_length_mm  bill_depth_mm  flipper_length_mm  \
0  Adelie  Torgersen            39.1           18.7              181.0
1  Adelie  Torgersen            39.5           17.4              186.0
2  Adelie  Torgersen            40.3           18.0              195.0
4  Adelie  Torgersen            36.7           19.3              193.0
5  Adelie  Torgersen            39.3           20.6              190.0

   body_mass_g    sex  One  Adelie
0       3750.0   MALE    1       1
```

```
1        3800.0  FEMALE    1        1
2        3250.0  FEMALE    1        1
4        3450.0  FEMALE    1        1
5        3650.0    MALE    1        1
0.43843843843843844
Index(['species', 'island', 'bill_length_mm', 'bill_depth_mm',
       'flipper_length_mm', 'body_mass_g', 'sex', 'One', 'Adelie'],
      dtype='object')
```

1.1 Setting up our Model

In this example, we will use the features "bill_length_mm", "bill_depth_mm", "flipper_length_mm", and "body_mass_g" to predict whether or not the species is Adelie.

[56]:
```
X = penguins[['One','bill_length_mm',␣
↪'bill_depth_mm','flipper_length_mm','body_mass_g']]
Y = penguins['Adelie']
```

Recall that in logistic regression, we model the probability as

$$\pi(\mathbf{X}_i; \boldsymbol{\beta}) = \frac{1}{1 + \exp\left(-\beta_0 - \beta_1 X_{i,1} - \cdots - \beta_p X_{i,p}\right)}$$

To make predictions with this model (and evaluate the gradient) we will first need to write a function that takes in our feature matrix and a guessed value of $\boldsymbol{\beta}$ and returns a vector of probabilities.

[57]:
```
beta_initial = np.array((-0.001,0.001,0.001,-0.001,0.001))
def LogitReg(Xtrain, beta):
    power = -1*np.dot(Xtrain,beta)
    pHat = 1/(1 + np.exp(power))
    return pHat
test = LogitReg(X,beta_initial)
# print(test)
```

We will also want a function that takes in the predicted probabilitis and returns the evaluates the log-likelihood:

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n} Y_i \ln \pi\left(\mathbf{X}_i; \boldsymbol{\beta}\right) + (1 - Y_i) \ln(1 - \pi(\mathbf{X}_i; \boldsymbol{\beta}))$$

[58]:
```
def LogitLikelihood(Ytrain, pHat):
    return 1/(len(Ytrain))*np.sum(Ytrain*np.log(pHat) + (1 - Ytrain)*np.log(1␣
↪-pHat))

test2 = LogitLikelihood(Y,test)
print(test2)
```

```
-2.5221444759399567
```

We will find the parameters $\beta_0, \beta_1, ..., \beta_p$ to maximize the (simplified) log-likelihood:

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n} \left[\ln \left\{ 1 + e^{\boldsymbol{\beta}\mathbf{X}_i} \right\} - Y_i \boldsymbol{\beta}\mathbf{X}_i \right]$$

The gradient of $\ell(\boldsymbol{\beta})$ is given:

$$\nabla \ell(\boldsymbol{\beta}) = [\tilde{\pi}(\mathbf{X}; \boldsymbol{\beta}) - \mathbf{Y}] \cdot \mathbf{X}$$

where:

$$\tilde{\pi}(\mathbf{X}; \boldsymbol{\beta}) = (\pi(\mathbf{X}_i; \boldsymbol{\beta}), \ldots, \pi(\mathbf{X}_n; \boldsymbol{\beta}))'$$

denotes our vector of predicted probabilities at guess $\boldsymbol{\beta}$

In order to implement this, we will need to write a gradient descent function. We can use what we have above:

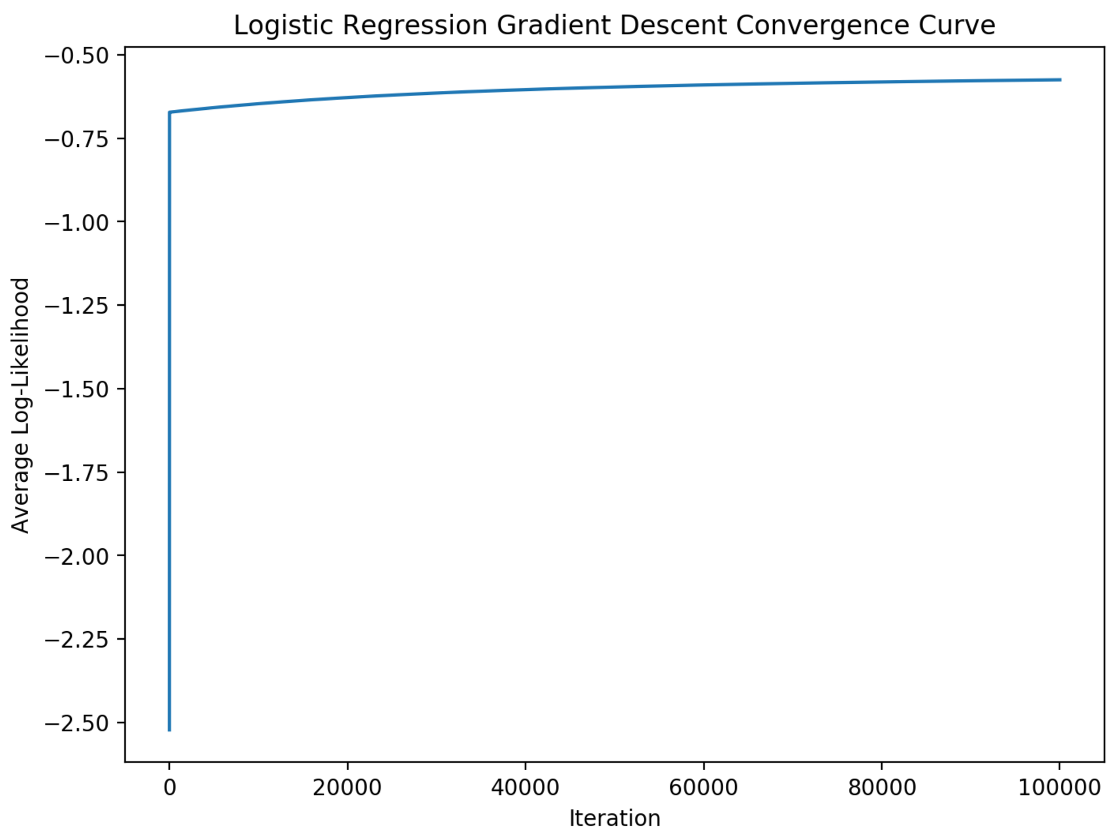
[86]:
```
def LogitGradientDescent(beta_initial, num_iterations, gamma, Xtrain, Ytrain):
    # Set up for gradient descent
    beta = beta_initial
    likelihoods = []
    # Do the gradient descent (updating each time)
    for i in range(num_iterations):
        # Find the vector of probabilities
        pHat = LogitReg(Xtrain, beta)
        # Evaluate the log-likelihood function
        likelihood = LogitLikelihood(Ytrain, pHat)
        # Add the likelihood to the list
        likelihoods.append(likelihood)
        # Compute the gradient
        grad = (1/len(Ytrain))*np.dot(Ytrain - pHat, X)
        # Update Beta
        beta = beta + gamma*grad
    # Compute the likelihood for the final value of beta
    pHat = LogitReg(Xtrain, beta)
    likelihood = LogitLikelihood(Ytrain, pHat)
    likelihoods.append(likelihood)
    # Return the last value of beta and the likelihoods
    return beta, np.array(likelihoods)

from sklearn.model_selection import train_test_split
Xtrain, Xtest, Ytrain, Ytest = train_test_split(X,Y, random_state = 0)


num_iterations = 50000
gamma = 0.0000001

betaFinal, likelihoods = LogitGradientDescent(beta_initial, num_iterations,␣
 ↪gamma, Xtrain, Ytrain)
```

```python
print(betaFinal)
```

```
[-0.00069094 -0.00481325  0.01057491  0.02554648 -0.00130612]
```

Now we plot the convergence curve:

[91]:
```python
import matplotlib.pyplot as plt
x = np.arange(len(likelihoods))
a = plt.figure(num=None, figsize=(8, 6), dpi=200, facecolor='w', edgecolor='k')
a = plt.title("Logistic Regression Gradient Descent Convergence Curve")
a = plt.ylabel("Average Log-Likelihood")
a = plt.xlabel("Iteration")
a = plt.plot(x,likelihoods)
a = plt.savefig("Convergence Curve")
```



And assess the preformance of our model

[88]:
```python
pHatFinal = LogitReg(Xtest,betaFinal)
Yhat = 1*(pHatFinal >= 0.5)
accuracy = np.mean(Yhat == Ytest)
print(accuracy)
```

```
0.6428571428571429
```

How does this compare to the linear model?

[89]:
```python
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(Xtrain,Ytrain)
test = model.predict(Xtest)
YHatLinear = 1*(test >= 0.5)
linearAccuracy = np.mean(YHatLinear == Ytest)
print(linearAccuracy)
```

```
0.9761904761904762
```

Machine Learning is hard