CS6650: Smart Sensing for Internet of Things

Submitted by: **Madhan Sanikommu, Archana K, Prashasth R B**

Roll no: **EE18B063,CS18B023,CS18B039**

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1 Introduction

The Euclidean Distance Geometry problem, aims at reconstructing the configuration of points given the partial information on pairwise inter-point distances.

1.0.1 Mathematical Setup

Consider a collection of n points in a r-dimensional Euclidean space, ascribed to the columns of the matrix P = $[p_1, p_2, ..., p_n]^T \in \mathbb{R}^{n\times r}$

The Squared distance between any two points p_i and p_j is given by, $d_{i,j}^2 = \|p_i - p_j\|_2^2 = p_i^T p_i + p_j^T p_j - 2p_i^T p_j$.

The Euclidean Distance Matrix D = $[d_{i,j}^2]$

$$D = \mathbf{1} diag(\mathbf{PP}^T)^T + diag(\mathbf{PP}^T)\mathbf{1}^T - 2\mathbf{PP}^T$$

X = $\mathbf{PP}^T$ is the inner product matrix well known as the Gram matrix.

1.0.2 Nuclear Norm Minimization

Assuming that the entries of D are sampled uniformly at random. By definition, **X** is **symmetric** and **positive semidefinite**.

Considering the constraint that the centroid of the points is located at the origin, $\sum_{k=1}^{n}$

Since the nuclear norm of a positive semidefinite matrix equate to its trace the above optimization problem can be written as,

$$\min_{\bar{X}\in\mathbb{R}^{n\times n}} Trace\left(\bar{\mathbf{X}}\right) subject\ to\ R_\Omega\left(\bar{X}\right) = R_\Omega\left(M\right) \bar{\mathbf{X}}\cdot\mathbf{1} = \mathbf{0};\ \ \bar{\mathbf{X}} \geq \mathbf{0}$$

Let a matrix, $\mathbf{C} \in \mathbb{R}^{n\times(n-1)}$ satisfying $\mathbf{C}^T\mathbf{C} = I$ and $\mathbf{C}^T\mathbf{1} = \mathbf{0}$

Rewriting the above optimization problem, by a change of variable $\bar{\mathbf{X}} = \mathbf{CXC}^T$.

$$\min_{X\in\mathbb{R}^{(n-1)\times(n-1)}} Trace\left(\mathbf{CXC}^T\right) subject\ to\ R_\Omega\left(CXC\right) = R_\Omega\left(M\right)\mathbf{X} \geq \mathbf{0}$$

To enforce that **X** is positive semidefinite, let $\mathbf{X} = \bar{\mathbf{P}}\bar{\mathbf{P}}^T$ where $\bar{\mathbf{P}} \in \mathbb{R}^{(n-1)\times q}$. Since, $\bar{\mathbf{P}}\bar{\mathbf{P}}^T$ has at most rank q, a good estimate of q should be a reasonable of the rank. The minimization problem now reduces to

$$\min_{\bar{P}\in\mathbb{R}^{(n-1)\times q}} Trace\left(\mathbf{C}\bar{\mathbf{P}}\bar{\mathbf{P}}^T\mathbf{C}^T\right) \textit{ subject to } R_\Omega\left(\mathbf{C}\bar{\mathbf{P}}\bar{\mathbf{P}}^T\mathbf{C}^T\right) = R_\Omega(M)$$

With **P = C$\bar{\mathbf{P}}$**, the simplified minimization problem,

$$\min_{P\in\mathbb{R}^{n\times q}} Trace\left(\mathbf{P}\mathbf{P}^T\right) \textit{ subject to } R_\Omega\left(\mathbf{P}\mathbf{P}^T\right) = R_\Omega(\mathbf{M})$$

Once $\bar{\mathbf{X}}$ is obtained, classical MDS can then be used to find the point coordinates. Therefore, solving for $\mathbf{P}$ above and then using MDS we can retrieve the topology.

We use the method of augmented Lagrangian to solve minimization problem. The constraint $R_\Omega\left(\mathbf{P}\mathbf{P}^T\right) = R_\Omega(\mathbf{M})$ can be written using the linear operator $\mathbf{A}$ defined as,

$$A : \mathbb{R}^{n\times n} \to \mathbb{R}^{|\Omega|\times 1} with, A(X) = f \in \mathbb{R}^{|\Omega|\times 1}, f_i = \langle X, w_{\alpha_i}\rangle \; for \; \alpha_i \in \Omega$$

For $y \in \mathbb{R}^{|\Omega|\times 1}$, $\langle AX, y\rangle = \sum_i \langle X, w_{\alpha_i}\rangle y_i = \langle X, \sum_i y_i w_{\alpha_i}\rangle$. From this, the adjoint of A,A^* is as follows, $A^* y = \sum_i y_i w_{\alpha_i}$

In practical settings, the available partial information is not exact but noisy. Considering an additive Gaussian noise $\mathbf{Z}$ with mean μ and variance σ. The modified minimization problem can be written as

$$\min \|\mathbf{X}\|_* \textit{subject to } \|R_\Omega(\mathbf{X}) - R_\Omega(\mathbf{M})\|_F \le \delta$$

Using the operator A introduced earlier, the above equation can be rewritten as,

$$\min_P Trace\left(\mathbf{P}\mathbf{P}^T\right) + \frac{\lambda}{2}\|A(\mathbf{P}\mathbf{P}^T - \mathbf{b}\|_F^2$$

where $\mathbf{b} = A(\mathbf{M})$. The augmented Lagrangian is given by

$$\mathbf{L}(\mathbf{P};\Lambda) = Trace\left(\mathbf{P}\mathbf{P}^T\right) + \frac{\lambda}{2}\|A(\mathbf{P}\mathbf{P}^T - \mathbf{b}\|_2^2$$

where $\Lambda \in \mathbb{R}^{n\times 1}$ denotes the Lagrangian multiplier and λ is the penalty term. $\mathbf{P}$ is computed using the Brazilai-Borwein steepest descent method with,

objective function $Trace\left(\mathbf{P}\mathbf{P}^T\right) + \frac{\lambda}{2}\|A(\mathbf{P}\mathbf{P}^T - \mathbf{b}\|_2^2$

and gradient $2\mathbf{P} + 2\lambda A^*\left(A\left(\mathbf{P}\mathbf{P}^T\right) - \mathbf{b}\right)\mathbf{P}$.

Pseudo Code Initialization: Set q = 10, $\mathbf{P}^0 = rand(n,q)$, $E^0_{Total} = 0$, maxiterations,bbiterations,λ,Tol

for k=1:maxiterations **do**

$Barzilai - Borwein(BB)\ descent\ for\ \mathbf{P}_k$

$$E^k_{Total} = Trace\left(\mathbf{P}^k\left(\mathbf{P}^k\right)^T\right) + \frac{\lambda}{2}\|A\left(\mathbf{P}^k\left(\mathbf{P}^k\right)^T\right) - \mathbf{b}\|_2^2$$

if $E^k_{Total} < Tol$ **then**

2 Importing Libraries

[1]:
```
import os
import time
import h5py
import math
import scipy
import random
import psutil
import platform
import scipy.io
import numpy as np
import pandas as pd
import seaborn as sns
from math import sin,cos
import plotly.express as px
import matplotlib.pyplot as plt
import plotly.graph_objects as go
from mpl_toolkits.mplot3d import Axes3D
from plotly.subplots import make_subplots

%matplotlib inline

import plotly.io as pio
pio.renderers.default='notebook'

import warnings
warnings.filterwarnings('ignore')
```

2.0.1 Lat and Long to 3D Coordinates

[2]:
```
def getXYZ(lat,lon,R):
    lat=(lat*math.pi/180)
    lon=(lon*math.pi/180)
    x = R * cos(lat) * cos(lon)
    y = R * cos(lat) * sin(lon)
    z = R *sin(lat)
    return (x, y, z)
```

2.0.2 3D Coordinates to Lat and Long

[3]:
```
def getLatLon(x,y,z,R) :
    lat  = math.asin(z/R)
    lon = math.atan(y/x)
    return (lat/(math.pi)*180 , lon/(math.pi)*180)
```

3 EDM Solver Class

[4]:
```
class EDMSolver():
    """
    EDM SOlver Using Low rank Matrix Completion
    """
    def __init__(self,points,opts,lsopts):

        """
        Initializing Variables

        Parameters
        -----------
        points : 3D array of shape (...,3)
            3D Point Cloud
        opts : Params Class obj
            general options
        lsopts - Params Class obj
            options for BB gradient method

        """

        self.points = points
        self.opts = opts
        self.lsopts = lsopts
        self.pts = self.points.T

    def getEDM(self,points):
        """
        Computes the Euclidean Distance Matrix

        Parameters
        ----------
        points : 3D array of shape (n,3)
            3D point Cloud

        Returns
        -------
        An array of Distances of shape n x n
```

```python
        """

        num_pt = points.shape[0]
        pts = points.T
        PP = np.matmul(pts.T,pts)
        Dist = PP.diagonal().reshape(-1,1) + PP.diagonal().reshape(-1,1).T - 2*PP
        for i in range(num_pt):
            Dist[i,i] = 0

        return Dist

    def getWeight(self):
        """
        Generating Weight for random missing distances

        Parameters
        ----------
        None

        Returns
        -------
            Mask/Weight matrix, Weight == 1 means available distance
        """
        num_pt = self.points.shape[0]
        rate = self.opts.rate
        Weight = np.random.rand(num_pt,num_pt)
        Weight[Weight > 1-rate] = 1
        Weight[Weight < 1] = 0
        Weight[Weight > 0] = 1

        for i in range(num_pt):
            Weight[i,i] = 1
            for j in range(i+1,num_pt):
                Weight[i,j] = Weight[j,i]

        self.ratio = ((len(np.where(Weight == 1)[0])-num_pt)/(num_pt**2))*100
        print('%f percent of the EDM Distances are known\n'%(self.ratio))

        return Weight

    def AOp(self,X):
        """
        The linear operator A for:
        R_{\Omega}(X) = R_{\Omega}(M)

        Parameters
        ----------
```

```
        X: array_like
            The array input to the linear operator

        Returns
        -------
        y : array_like
            y = A(X)
        """
        # diagonal of XX'
        X_diag = np.sum(np.multiply(X,X),axis=1,keepdims=True)
        # off diagonal of XX'
        X_offdiag = np.sum(np.multiply(X[self.I,:],X[self.J,:
↪]),axis=1,keepdims=True)
        Y = X_diag[self.I] + X_diag[self.J] - 2*X_offdiag;
        return Y

    def AOpstar(self,y):
        """
        The Adjoint operator of the linear operator A, A*
        A*(y) = \sum_{i} y^{1}_{alpha_i} w_{alpha_i}

        Parameters
        ----------
        y : array_like

        Returns
        -------
        X : array_like
        """
        X = np.zeros((self.dim,self.dim))
        X = X.reshape(-1,1)
        X[self.edgeind] = -2*y
        X_copy = X.reshape(self.dim,self.dim)
        X[self.diagind] = -np.sum(X_copy,1,keepdims=True)
        X = X.reshape((self.dim,self.dim))
        return X

    def gradient(self,P):
        """
        The gradient function for the BBGradient

        Parameters
        ----------
        P : array_like
            Input to the BB Gradient Descent

        Returns
```

```
        -------
        F : int
            The value of the objective function
        G : array_like
            The gradient
        """
        tmp1 = self.AOp(P) - self.b;
        # The Objective Function
        F = np.sum(np.sum(np.multiply(P,P),axis=1)) + 0.5*self.lambda_*np.linalg.
↪norm(tmp1,'fro')**2
        # The Gradient
        G = 2*P + np.matmul(2.0*self.lambda_*self.AOpstar(tmp1),P)
        return F,G

    def addNoise(self):
        """
        Helper function to add noise to the distance matrix
        """
        # Gram Matrix(Inner Product Matrix)
        self.lambda_ = self.opts.lambda_
        self.Rk = self.opts.rank
        self.dim,_ = self.Dist.shape

        # Gram Matrix(Inner Product Matrix)
        self.Gram_Truth = self.Dist - np.matmul(np.mean(self.
↪Dist,axis=1,keepdims=True),np.ones((1,self.dim)))
        self.Gram_Truth = -0.5*(self.Gram_Truth - np.matmul(np.ones((self.
↪dim,1)),np.mean(self.Gram_Truth,axis=0,keepdims=True)))

        # Obtaining Omega i.e., indices of randomly choosen entries of D
        self.I,self.J = np.where(self.Weight == 1)

        # Diagonal indices
        self.diagind = np.array(range(1,self.dim+1)) + self.dim*np.
↪array(range(self.dim)) - 1

        # Edge indices of choosen entries of D (includes symmetrical indices)
        self.edgeind = self.I + (self.J)*self.dim


        M_noisefree = self.Dist.reshape(-1,1)[self.edgeind]
        # Minimum sampled distance
        mindist = np.min(M_noisefree[M_noisefree > 0])

        # Adding noise in the range of the minimum measured distance
        #and ensuring all distances are positive
```

```
        M = M_noisefree + 4*mindist + mindist*np.random.
↪standard_normal(M_noisefree.shape);
        M[M_noisefree == 0] = 0
        self.b = M

    def BBGradient(self,x):
        """
        The Brazilai-Borwein steepest descent method
        """
        # The Objective and the gradient initialization
        [f,g] = self.gradient(x)

        # Norm of the Gradient
        gnorm = np.linalg.norm(g,'fro')

        # Initialize Q, C and alpha
        Q = 1;
        C = f;
        alpha = self.lsopts.alpha

        # Initializing xmin and fmin
        xmin = x
        fmin = f

        n = x.shape[0]

        # Maximum and Minimum values of alpha
        MAXalpha = 1e16
        MINalpha = 1e-16

        for it in range(self.lsopts.maxit):
            xold = x
            fold = f
            gold = g

            numlinesearch = 1
            wolfe_factor = self.lsopts.rho*gnorm**2

            while True:
                x = xold - alpha*gold
                [f,g] = self.gradient(x)

                if f <= C - alpha*wolfe_factor or numlinesearch >= 5:
                    break
                alpha = self.lsopts.sigma*alpha
                numlinesearch = numlinesearch + 1
```

```
            if f < fmin:
                xmin = x
                fmin = f

            # Stopping Conditions
            gnorm = np.linalg.norm(g,'fro')
            s = x - xold
            xstop = np.linalg.norm(s,'fro')/np.sqrt(n)
            fstop = abs(fold - f)/(abs(fold) + 1)

            if (xstop < self.lsopts.xtol and self.lsopts.ftol) or (gnorm < self.
↪lsopts.gtol):
                break

            # Updates
            y = g - gold
            normy = np.linalg.norm(y,'fro')
            sy = abs(np.trace(np.matmul(s.T,y)))

            if (it+1)%2 == 0:
                alpha = (np.linalg.norm(s,'fro')**2)/sy
            else:
                alpha = sy/(normy**2)

            # To make sure alpha lies in bound
            alpha = min(alpha,MAXalpha)
            alpha = max(alpha,MINalpha)

            # Updating Cost
            Qold = Q
            Q = self.lsopts.eta*Qold + 1
            C = (self.lsopts.eta*Qold*C + f)/Q

        x = xmin

        return x

    def matrixCompletion(self):
        """
        Performs the BB method to solve for P, getting the gram matrix and
        reconstructed Gram matrix and performing canonical Multidimensional
        scaling(MDS)
        """
        ## Initializing Values
        # P, lagrangian Multipliers
        P = np.random.random((self.dim,self.Rk))
        # Energies
```

```
        E = np.zeros((self.opts.maxit,1))
        # Iteration counter, relative error
        num_it = 0
        cre = 1

        # Barzilai-Borwein(BB) descent to solve for P

        for i in range(self.opts.maxit):
            num_it = num_it + 1
            # Gradient descent for P
            P = self.BBGradient(P)
            # Calculating total energy
            E[i] = np.sum(np.multiply(P,P)) + 0.5*self.lambda_*np.linalg.
↪norm(self.AOp(P)-self.b,'fro')**2

            # To print energy
            if self.opts.printenergy == 1:
                print("Iteration %d, TotalE = %f\n" % (i,E[i]))

            # Calculating relative error
            if i > 0:
                cre = abs(E[i] - E[i-1])/E[i]

            # stopping condition
            if cre < 1e-5:
                break

        # Plotting Energies with respect to iteration number
        if self.opts.plotting == 1:
            plt.figure(figsize=(12,8))
            plt.semilogy(range(num_it),E[:num_it],linewidth=2)
            plt.xlabel("Iteration number",fontsize=18)
            plt.ylabel("$\log(E)$",fontsize=18)
            plt.grid()
            plt.show()

        # Obtaining the Gram Matrix
        Gram = np.matmul(P,P.T)

        # Reconstructing the Gram Matrix and Multidimensional Scaling (MDS)
        self.Gram_Recon = Gram - (1/self.dim)*np.tile(np.
↪sum(Gram,axis=0,keepdims=True),(self.dim,1)) + (1/(self.dim**2))*np.tile(np.
↪sum(Gram),(self.dim,self.dim))
        self.Gram_Recon = (self.Gram_Recon + self.Gram_Recon.T)/2

        self.Gram_err = np.linalg.norm(self.Gram_Truth - self.Gram_Recon,'fro')/
↪np.linalg.norm(self.Gram_Truth,'fro')
```

```
        D,V = scipy.sparse.linalg.eigs(self.Gram_Recon,self.opts.rank,which='LM')
        D = D.reshape(-1,1)

        D,IJ = -np.sort(-D,axis=0),np.argsort(-D,axis=0)
        V = V[:,IJ]
        temp = np.real(V[:,0:3])
        temp = temp.reshape(temp.shape[0],temp.shape[1]*temp.shape[2])
        Global_Coordinate = np.matmul(temp,np.diag(np.sqrt(D[0:3]).reshape(-1)))

        output = dict()

        output['E'] = E[num_it-1]
        output['ReconError'] = self.Gram_err
        output['numit'] = num_it

        return Global_Coordinate,self.Gram_Recon,output

    # helper function 1
    def plot3d(self,coords,title=None):
        """
        Helper function to plot the 3D plots

        Parameters
        ----------
        coords : array_like
            The 3D point cloud locations (x,y,z)
        """
        x, y, z = coords[:,0],coords[:,1],coords[:,2]
        data = go.Scatter3d(x=x,y=y,z=z,mode='markers',
                marker=dict(size=5,color=z,colorscale='Viridis')) # set color to␣
↪an array/list of desired values
        fig = go.Figure(data=[data])

        # tight layout
        fig.update_layout(margin=dict(l=0, r=0, b=0, t=0),title=title)
        fig.show()

    # helper function 2
    def rigid_transform_3D(self,A, B):
        """
        Helper function to fix the orientation of the reconstructed poit cloud

        Parameters
        ----------
        A : array_like of shape (3,...)
        B : array_like of shape (3,...)
```

```
    Returns
    ---------
    R : array_like of shape (3,3)
        Rotation Matrix

    t : array_like of shape(3,1)
        Translation Matrix

    Refrence
    ---------
    This function has been directly used from the below github repository

    https://github.com/nghiaho12/rigid_transform_3D
    """
    assert A.shape == B.shape

    num_rows, num_cols = A.shape
    if num_rows != 3:
        raise Exception(f"matrix A is not 3xN, it is {num_rows}x{num_cols}")

    num_rows, num_cols = B.shape
    if num_rows != 3:
        raise Exception(f"matrix B is not 3xN, it is {num_rows}x{num_cols}")

    # find mean column wise
    centroid_A = np.mean(A, axis=1)
    centroid_B = np.mean(B, axis=1)

    # ensure centroids are 3x1
    centroid_A = centroid_A.reshape(-1, 1)
    centroid_B = centroid_B.reshape(-1, 1)

    # subtract mean
    Am = A - centroid_A
    Bm = B - centroid_B

    H = Am @ np.transpose(Bm)

    # sanity check
    #if linalg.matrix_rank(H) < 3:
    #    raise ValueError("rank of H = {}, expecting 3".format(linalg.
↪matrix_rank(H)))

    # find rotation
    U, S, Vt = np.linalg.svd(H)
    R = Vt.T @ U.T
```

```
        # special reflection case
        if np.linalg.det(R) < 0:
            print("det(R) < R, reflection detected!, correcting for it ...")
            Vt[2,:] *= -1
            R = Vt.T @ U.T

        t = -R @ centroid_A + centroid_B

        return R, t

    # helper function 4
    def run(self):
        """
        Helper function to combine all the above functions and run it in a
        sequential manner

        Returns
        --------
        output : dict
            keys:
                rate         : smapling rate
                points       : actual 3D point cloud
                tpoints      : transformed reconstructed point cloud
                system       : system parameters
                release      : system parameters
                Version      : system parameters
                Machine      : system parameters
                Processor    : system parameters
                error        : Obtained error
                numit        : Number of BB iterations
                timeelapsed  : Total time elapsed in computing
        """
        # Recording starting time
        start = time.time()

        # Obtaining the EDM Matrix
        self.Dist = self.getEDM(self.points)
        self.scaler = np.max(self.Dist)

        # Normalizing the entries of EDM matrix
        self.Dist = self.Dist/self.scaler

        # Getting the mask
        self.Weight = self.getWeight()

        # Adding noise to the randomly choosen distances
        self.addNoise()
```

```
        # Matrix Completion and Multi Dimensional Scaling(MDS)
        self.GCor, ipm, _output = self.matrixCompletion()

        self.GCor = np.real(self.GCor)

        # Rescaling the points to original ranges
        self.GCor = self.GCor*np.sqrt(self.scaler)

        if self.opts.plotting:
            # 3D View of Constructed points
            self.plot3d(self.GCor,'Constructed Points')
            # 3D View of Actual points
            self.plot3d(self.pts.T,'Actual Points')

        # 3D transformation to original orientation
        R,t = self.rigid_transform_3D(self.GCor.T,self.points.T)
        self.transformed_points = (R @ self.GCor.T) + t
        self.transformed_points = self.transformed_points.T

        if self.opts.plotting:
            # plotting the 3D transformed points
            self.plot3d(self.transformed_points,'Transformed Reconstructed␣
↪Points')

        # Record finish time
        end = time.time()

        # Error Valuation
        error = np.linalg.norm(self.points-self.transformed_points,'fro')/np.
↪linalg.norm(self.points,'fro') # relative error
        output = dict()

        uname = platform.uname()
        output['ratio'] = self.ratio
        output['rate'] = self.opts.rate
        output['points'] = self.points
        output['tpoints'] = self.transformed_points
        output['System'] = uname.system
        output['Release'] = uname.release
        output['Version'] = uname.version
        output['Machine'] = uname.machine
        output['Processor'] = uname.processor
        output['error'] = error
        output['numit'] = _output['numit']
        output['timeelapsed'] = end - start
```

```
        return output
```

3.1 Parameters Class

[5]:
```
class params():
    lambda_ = None
    printenergy = None
    plotting = None
    rank = None
    rate = None
    maxit = None
    tol = None
    maxit = None
    xtol = None
    gtol = None
    ftol = None
    alpha = None
    rho = None
    sigma = None
    eta = None
```

4 Dataset Generation

4.1 Dataset 1: Mapping the major cities of a country

4.1.1 Loading Countries Location Data

[6]:
```
WorldData = pd.read_csv('./data/worldcitiespop.csv')
WorldData
```

[6]:
```
        Country          City   AccentCity Region  Population   Latitude  \
0            ad         aixas        Aixàs      6         NaN  42.483333
1            ad   aixirivali   Aixirivali      6         NaN  42.466667
2            ad   aixirivall   Aixirivall      6         NaN  42.466667
3            ad    aixirvall    Aixirvall      6         NaN  42.466667
4            ad     aixovall     Aixovall      6         NaN  42.466667
...         ...           ...          ...    ...         ...        ...
3173953      zw   zimre park   Zimre Park      4         NaN -17.866111
3173954      zw  ziyakamanas  Ziyakamanas      0         NaN -18.216667
3173955      zw   zizalisari   Zizalisari      4         NaN -17.758889
3173956      zw      zuzumba      Zuzumba      6         NaN -20.033333
3173957      zw   zvishavane   Zvishavane      7     79876.0 -20.333333

         Longitude
0         1.466667
1         1.500000
```

```
2          1.500000
3          1.500000
4          1.483333
...             ...
3173953   31.213611
3173954   27.950000
3173955   31.010556
3173956   27.933333
3173957   30.033333

[3173958 rows x 7 columns]
```

4.1.2 Selecting A Country: India

[7]:
```
Data = WorldData[WorldData['Country'] == 'in']
Data = Data.reset_index(drop = True)
Data.shape
```

[7]:
```
(39813, 7)
```

4.1.3 Extracting and saving 3D Point Cloud

[8]:
```
points1 = []
for i in random.sample(range(0, Data.shape[0]), 2500):
    points1.append(getXYZ(Data['Latitude'][i],Data['Longitude'][i],6371))

points1 = np.array(points1)
print(points1.shape)

np.savetxt('./data/india_latlon.csv', points1, delimiter=',')
```

```
(2500, 3)
```

4.1.4 Viewing the 3D point Cloud

[9]:
```
x, y, z = points1[:,0],points1[:,1],points1[:,2]
data1 = go.Scatter3d(x=x,y=y,z=z,mode='markers',
        marker=dict(size=5,color=z,colorscale='Viridis')) # set color to an␣
 ↪array/list of desired values
fig = go.Figure(data=[data1])

# tight layout
fig.update_layout(margin=dict(l=0, r=0, b=0, t=0))
fig.show()
```

4.2 Dataset 2: Sewage Pipes Data

4.2.1 Extracting and saving 3D Point Cloud

[10]:
```
path = './data/training_pointcloud_hdf5_real.h5'
with h5py.File(path, 'r') as hdf:
    Data = np.asarray(hdf['Training/PointClouds'][:])
Data = np.array(Data)
print(Data.shape)

np.savetxt('./data/pipe.csv',Data[0,:,:],delimiter=',')

points2 = Data[0,:,:]
```

```
(274, 1024, 3)
```

4.2.2 Viewing the 3D point Cloud

[11]:
```
x, y, z = points2[:,0],points2[:,1],points2[:,2]
data2 = go.Scatter3d(x=x,y=y,z=z,mode='markers',
        marker=dict(size=5,color=z,colorscale='Viridis')) # set color to an␣
 ↪array/list of desired values
fig = go.Figure(data=[data2])

# tight layout
fig.update_layout(margin=dict(l=0, r=0, b=0, t=0))
fig.show()
```

5 Dataset 3 : Location data collected from our peers

[12]:
```
Data = pd.read_csv('./data/dist.csv')
Data.insert(3,"Color",␣
 ↪['act','act','act','act','act','act','act','act','act','act','act','act','act','act','act'],␣
 ↪True)
points3 = []
for i in range(Data.shape[0]):
    points3.append(getXYZ(Data['lat'][i],Data['lng'][i],6371))

points3 = np.array(points3)
```

5.1 Visualisation

[13]:
```
fig = px.scatter_geo(Data,lat='lat',lon='lng',hover_name="City/
 ↪Place",text="Name",color='Color',scope='asia',center={'lat':
 ↪Data['lat'][3],'lon':Data['lng'][3]})
fig.update_layout(title = 'World map', title_x=0.5)
```

```
fig.show()
```

5.2 Dataset 4 : Intramolecular Data to identify protein structure

[14]:
```
f = open("./data/atom.txt", "r")
Lines = f.readlines()
count = 0
points4 = []
for line in Lines:
    count += 1
    x=line.split()
    try:
        t=[float(x[6]),float(x[7]),float(x[8])]
        points4.append(t)
    except:
        continue
points4 = np.array(points4)
```

5.3 Visualisation

[15]:
```
x, y, z = points4[:,0],points4[:,1],points4[:,2]
data4 = go.Scatter3d(x=x,y=y,z=z,mode='markers',
        marker=dict(size=3,color=z,colorscale='Viridis')) # set color to an␣
 ↪array/list of desired values
fig = go.Figure(data=[data4])

# tight layout
fig.update_layout(margin=dict(l=0, r=0, b=0, t=0))
fig.show()
```

5.4 Running the Solver

5.4.1 Defining EDMSolver Class Parameters

[16]:
```
opts = params()
lsopts = params()

# Matrix Completion parameters
opts.lambda_ = 5
opts.rate = 0.08
opts.printenergy = 0
opts.plotting = 0
opts.rank = 10
opts.maxit = 30
opts.tol = 1e-5
```

```
# BBGradient parameters
lsopts.maxit = 20
lsopts.xtol = 1e-8
lsopts.gtol = 1e-8
lsopts.ftol = 1e-10
lsopts.alpha  = 1e-3
lsopts.rho  = 1e-4
lsopts.sigma  = 0.1
lsopts.eta  = 0.8
```

5.4.2 Defining and Running the Solver

[17]:
```
solver1 = EDMSolver(points1,opts,lsopts)
solver2 = EDMSolver(points2,opts,lsopts)
solver3 = EDMSolver(points3,opts,lsopts)
solver4 = EDMSolver(points4,opts,lsopts)

output1 = solver1.run()
output2 = solver2.run()
output3 = solver3.run()
output4 = solver4.run()

outputs = [output1,output2,output3,output4]
solvers = [solver1,solver2,solver3,solver4]
for i in range(len(outputs)):
    outputs[i]['Dataset'] = 'Dataset '+str(i+1)
```

```
8.003296 percent of the EDM Distances are known

7.968140 percent of the EDM Distances are known

10.666667 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
7.986740 percent of the EDM Distances are known
```

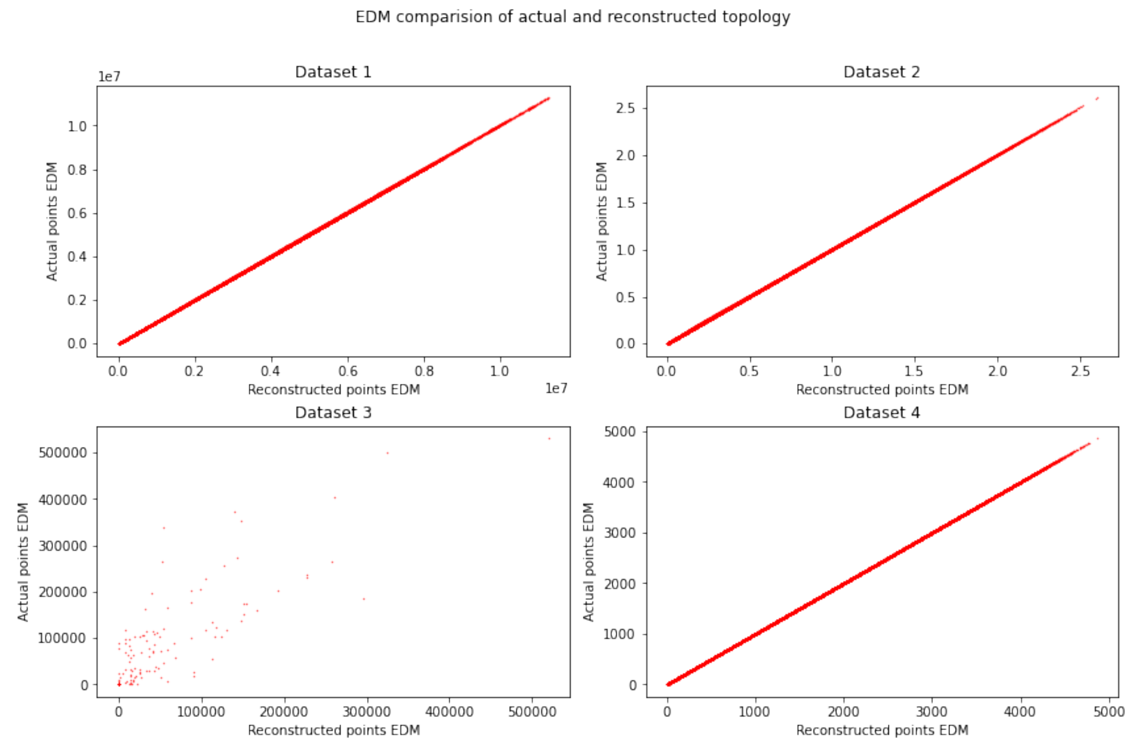
[18]:
```
# Actual points
points1,ponts2,points3,points4 =␣
 ↪output1['points'],output2['points'],output3['points'],output4['points']

fig = make_subplots(
    rows=2, cols=2,
    specs=[[{"type": "scene"}, {"type": "scene"}],
           [{"type": "scene"}, {"type": "scene"}]],
)
```

```
fig.add_trace(go.Scatter3d(x=points1[:,0], y=points1[:,1], z=points1[:
 ↪,2],mode='markers',marker=dict(size=2,color=points1[:
 ↪,2],colorscale='Viridis')),
              row=1, col=1)

fig.add_trace(go.Scatter3d(x=points2[:,0], y=points2[:,1], z=points2[:
 ↪,2],mode='markers',marker=dict(size=2,color=points2[:
 ↪,2],colorscale='Viridis')),
              row=1, col=2)

fig.add_trace(go.Scatter3d(x=points3[:,0], y=points3[:,1], z=points3[:
 ↪,2],mode='markers',marker=dict(size=2,color=points3[:
 ↪,2],colorscale='Viridis')),
              row=2, col=1)

fig.add_trace(go.Scatter3d(x=points4[:,0], y=points4[:,1], z=points4[:
 ↪,2],mode='markers',marker=dict(size=2,color=points4[:
 ↪,2],colorscale='Viridis')),
              row=2, col=2)

fig.update_layout(height=1000, showlegend=False)
fig.show()
```

[19]:
```
# Transformed points
tpoints1,tpoints2,tpoints3,tpoints4 =␣
 ↪output1['tpoints'],output2['tpoints'],output3['tpoints'],output4['tpoints']

fig = make_subplots(
    rows=2, cols=2,
    specs=[[{"type": "scene"}, {"type": "scene"}],
           [{"type": "scene"}, {"type": "scene"}]],
)

fig.add_trace(go.Scatter3d(x=tpoints1[:,0], y=tpoints1[:,1], z=tpoints1[:
 ↪,2],mode='markers',marker=dict(size=2,color=tpoints1[:
 ↪,2],colorscale='Viridis')),
              row=1, col=1)

fig.add_trace(go.Scatter3d(x=tpoints2[:,0], y=tpoints2[:,1], z=tpoints2[:
 ↪,2],mode='markers',marker=dict(size=2,color=tpoints2[:
 ↪,2],colorscale='Viridis')),
              row=1, col=2)
```

```
fig.add_trace(go.Scatter3d(x=tpoints3[:,0], y=tpoints3[:,1], z=tpoints3[:
↪,2],mode='markers',marker=dict(size=2,color=tpoints3[:
↪,2],colorscale='Viridis')),
              row=2, col=1)

fig.add_trace(go.Scatter3d(x=tpoints4[:,0], y=tpoints4[:,1], z=tpoints4[:
↪,2],mode='markers',marker=dict(size=2,color=tpoints4[:
↪,2],colorscale='Viridis')),
              row=2, col=2)

fig.update_layout(height=1000, showlegend=False)
fig.show()
```

[20]:
```
fig,ax = plt.subplots(2,2,figsize=(12,8))
for i in range(len(outputs)):
    solver,output = solvers[i],outputs[i]
    dact = np.tril(solver.getEDM(output['points'])).ravel()
    dObs = np.tril(solver.getEDM(output['tpoints'])).ravel()
    ax[int(i/2),int(i%2)].scatter(dObs,dact,color='r',s=0.1)
    ax[int(i/2),int(i%2)].set_title('{}'.format(output['Dataset']))
    ax[int(i/2),int(i%2)].set_xlabel('Reconstructed points EDM')
    ax[int(i/2),int(i%2)].set_ylabel('Actual points EDM')

plt.tight_layout()
fig.subplots_adjust(top=0.88)
fig.suptitle('EDM comparision of actual and reconstructed topology')
plt.show()
```



Here we have seen different datasets and sampled them to obtain our dataset. The EDM after construction has been sampled at certain rate and the following errors and properties have been noted.

```
[21]: print(' Dataset     Rate    Ratio     Error(%)    Num Iterations   Num Points Time␣
↪Elapsed(s)')
for output in outputs:
    print(output['Dataset'],"  %0.2f    %0.2f    %2.6f          %0.2f           %5d    ␣
↪%0.
↪4f"%(output['rate'],output['ratio'],output['error']*100,output['numit'],output['points'].
↪shape[0],output['timeelapsed']))
```

```
 Dataset     Rate    Ratio     Error(%)    Num Iterations   Num Points Time
Elapsed(s)
Dataset 1    0.08    8.00    0.189246        26.00            2500     79.2980
Dataset 2    0.08    7.97    0.311351        17.00            1024     8.7557
Dataset 3    0.08    10.67    2.217214        6.00              15     0.0190
Dataset 4    0.08    7.99    0.027376        27.00            4384     258.3121
```

5.5 Error Analysis: Dataset 1

5.5.1 Defining EDMSolver Class Parameters

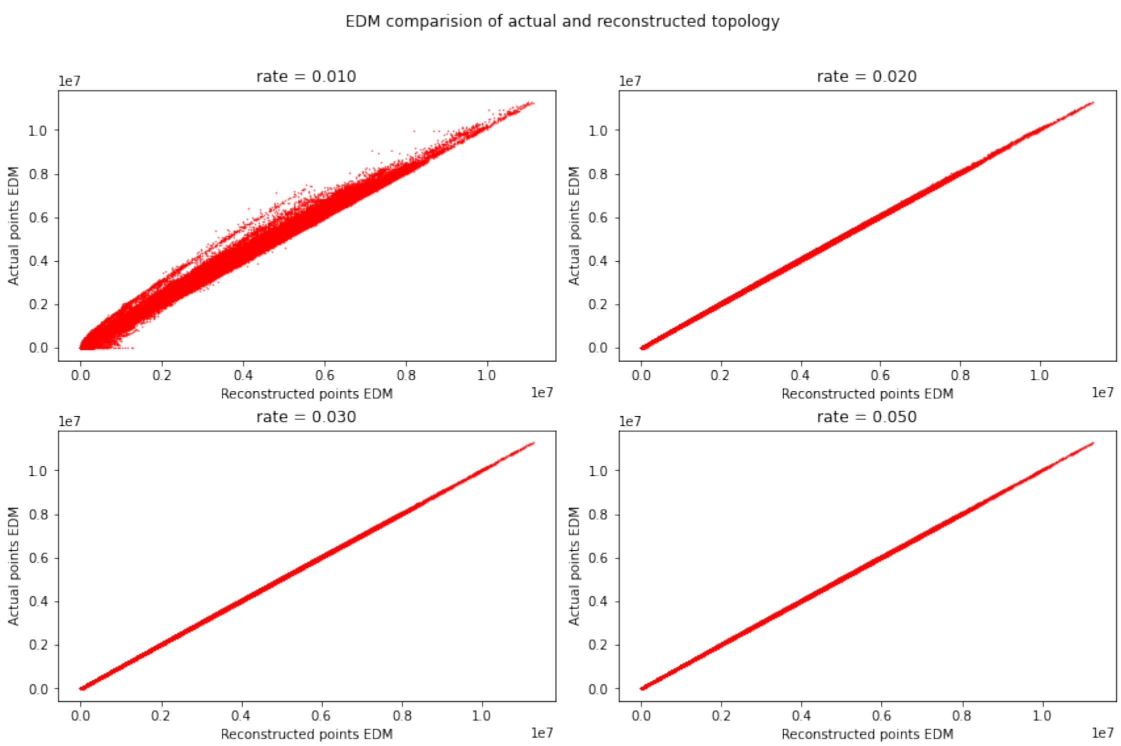
[22]:
```
opts = params()
lsopts = params()

# Matrix Completion parameters
opts.lambda_ = 5
# opts.rate = 0.02
opts.printenergy = 0
opts.plotting = 0
opts.rank = 10
opts.maxit = 30
opts.tol = 1e-5

# BBGradient parameters
lsopts.maxit = 20
lsopts.xtol = 1e-8
lsopts.gtol = 1e-8
lsopts.ftol = 1e-10
lsopts.alpha  = 1e-3
lsopts.rho  = 1e-4
lsopts.sigma  = 0.1
lsopts.eta  = 0.8
```

5.5.2 Defining and Running the Solver

[23]:
```
outputs1 = []
rates1 = [0.01,0.02,0.03,0.05]

points = np.genfromtxt('./data/india_latlon.csv', delimiter=',')

for rate in rates1:
    opts.rate = rate
    solver = EDMSolver(points,opts,lsopts)
    output = solver.run()
    outputs1.append(output)
```

```
0.994080 percent of the EDM Distances are known

1.981792 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
2.994368 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
5.016000 percent of the EDM Distances are known
```

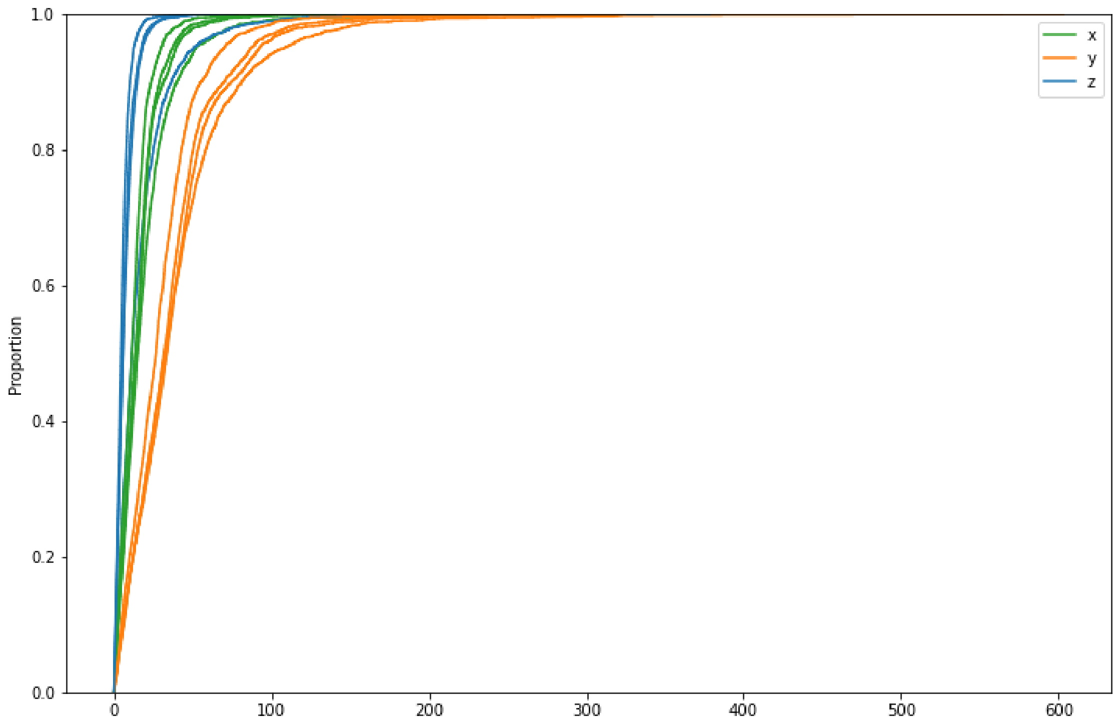
[24]:
```
fig,ax = plt.subplots(2,2,figsize=(12,8))
for i in range(len(outputs1)):
    output = outputs1[i]
    dact = np.tril(solver.getEDM(output['points'])).ravel()
    dObs = np.tril(solver.getEDM(output['tpoints'])).ravel()
    ax[int(i/2),int(i%2)].scatter(dObs,dact,color='r',s=0.1)
    ax[int(i/2),int(i%2)].set_title('rate = %.3f'%output['rate'])
    ax[int(i/2),int(i%2)].set_xlabel('Reconstructed points EDM')
    ax[int(i/2),int(i%2)].set_ylabel('Actual points EDM')

plt.tight_layout()
fig.subplots_adjust(top=0.88)
fig.suptitle('EDM comparision of actual and reconstructed topology')
plt.show()
```



5.5.3 Coordinate Error CDF

[25]:
```
plt.figure(figsize=(12,8))
for i in range(len(outputs1)):
    sns.ecdfplot((abs(outputs1[i]['points'] - outputs1[i]['tpoints'])))
plt.legend(['x','y','z'])
```

```
plt.show()
```



Here we have taken India cities Latitudes and Longitudes and sampled them to obtain our dataset. The EDM after construction has been sampled at various rates and the following errors and properties have been noted.

[26]:
```
print('Rate    Ratio     Error(%)    Num Iterations Time Elapsed(s)')
for output in outputs1:
    print("%0.2f    %0.2f    %0.7f        %0.2f          %0.
 ↪4f"%(output['rate'],output['ratio'],output['error']*100,output['numit'],output['timeelapsed']
```

```
Rate    Ratio     Error(%)    Num Iterations Time Elapsed(s)
0.01    0.99    1.0994404       19.00          29.2304
0.02    1.98    0.7978092       16.00          32.7008
0.03    2.99    0.8518338       16.00          34.3210
0.05    5.02    0.6650077       19.00          57.8823
```

5.6 Error Analysis: Dataset 2

5.6.1 Defining EDMSolver Class Parameters

[27]:
```
opts = params()
lsopts = params()
```

```python
# Matrix Completion parameters
opts.lambda_ = 5
# opts.rate = 0.02
opts.printenergy = 0
opts.plotting = 0
opts.rank = 10
opts.maxit = 30
opts.tol = 1e-5

# BBGradient parameters
lsopts.maxit = 20
lsopts.xtol = 1e-8
lsopts.gtol = 1e-8
lsopts.ftol = 1e-10
lsopts.alpha  = 1e-3
lsopts.rho  = 1e-4
lsopts.sigma  = 0.1
lsopts.eta  = 0.8
```

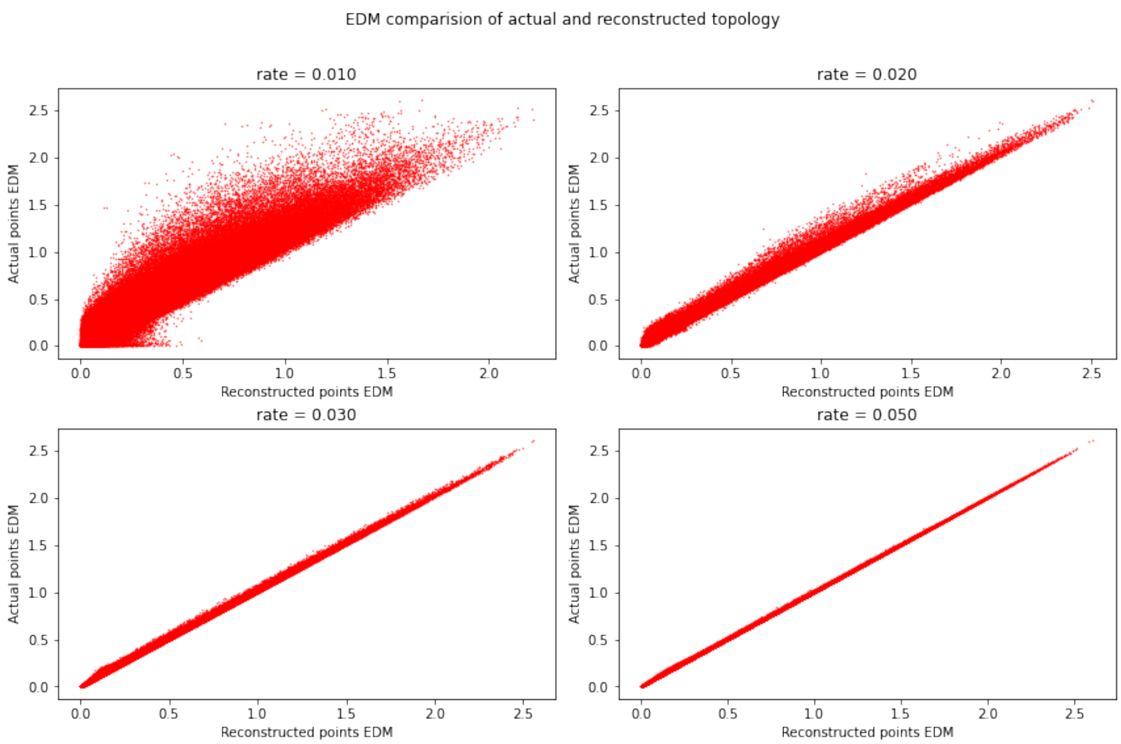
5.6.2 Defining and Running the Solver

[28]:
```python
outputs2 = []
rates2 = [0.01,0.02,0.03,0.05]

points = np.genfromtxt('./data/pipe.csv', delimiter=',')

for rate in rates2:
    opts.rate = rate
    solver = EDMSolver(points,opts,lsopts)
    output = solver.run()
    outputs2.append(output)
```

```
1.002312 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
1.998138 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
3.017235 percent of the EDM Distances are known

5.010223 percent of the EDM Distances are known
```

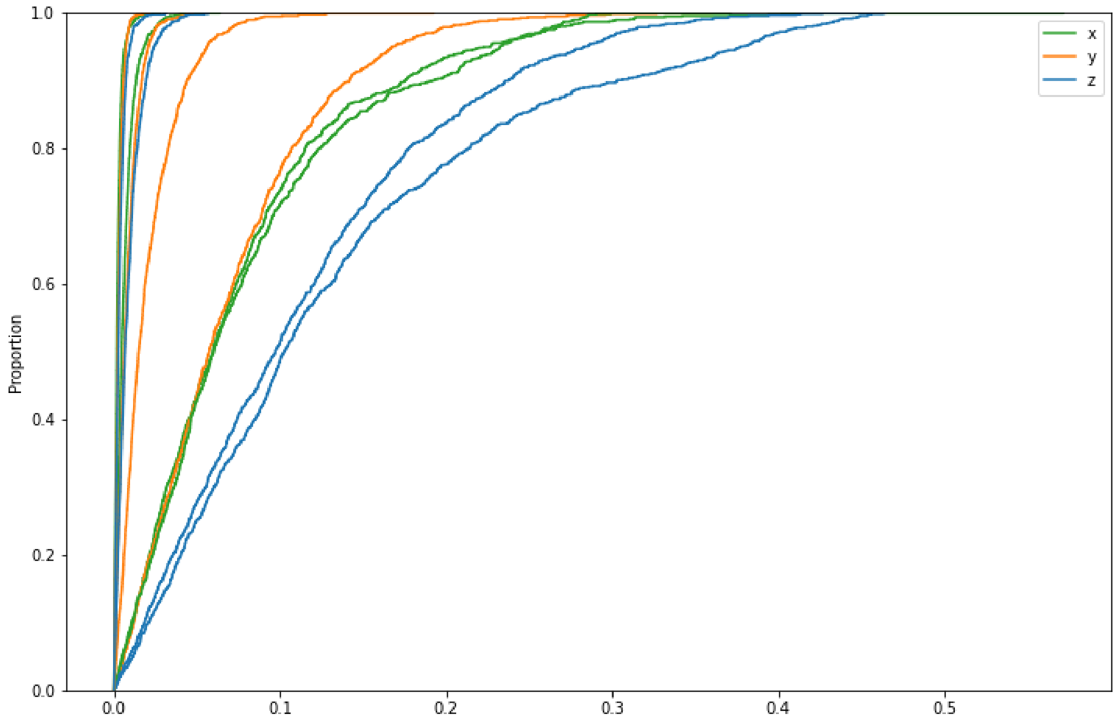
[29]:
```python
fig,ax = plt.subplots(2,2,figsize=(12,8))
for i in range(len(outputs2)):
    output = outputs2[i]
    dact = np.tril(solver.getEDM(output['points'])).ravel()
```

```
    dObs = np.tril(solver.getEDM(output['tpoints'])).ravel()
    ax[int(i/2),int(i%2)].scatter(dObs,dact,color='r',s=0.1)
    ax[int(i/2),int(i%2)].set_title('rate = %.3f'%output['rate'])
    ax[int(i/2),int(i%2)].set_xlabel('Reconstructed points EDM')
    ax[int(i/2),int(i%2)].set_ylabel('Actual points EDM')

plt.tight_layout()
fig.subplots_adjust(top=0.88)
fig.suptitle('EDM comparision of actual and reconstructed topology')
plt.show()
```



5.6.3 Coordinate Error CDF

[30]:
```
plt.figure(figsize=(12,8))
for output in outputs2:
    sns.ecdfplot((abs(output['points'] - output['tpoints'])))
plt.legend(['x','y','z'])
plt.show()
```



[31]:
```
print('Rate   Ratio    Error(%)   Num Iterations Time Elapsed(s)')
for output in outputs2:
    print("%0.2f   %0.2f   %0.7f       %0.2f          %0.
↪4f"%(output['rate'],output['ratio'],output['error']*100,output['numit'],output['timeelapsed']
```

```
Rate   Ratio    Error(%)   Num Iterations Time Elapsed(s)
0.01   1.00   21.3511732      30.00         7.6700
0.02   2.00   22.1612297      28.00         8.1524
0.03   3.02   2.0046797      19.00         6.7776
0.05   5.01   0.7753925      15.00         6.2720
```

5.7 Error Analysis: Dataset 3

5.7.1 Defining EDMSolver Class Parameters

[32]:
```
opts = params()
lsopts = params()

opts.lambda_ = 5
# opts.rate = 0.4
opts.printenergy = 0
opts.plotting = 0
opts.rank = 10
opts.maxit = 30
```

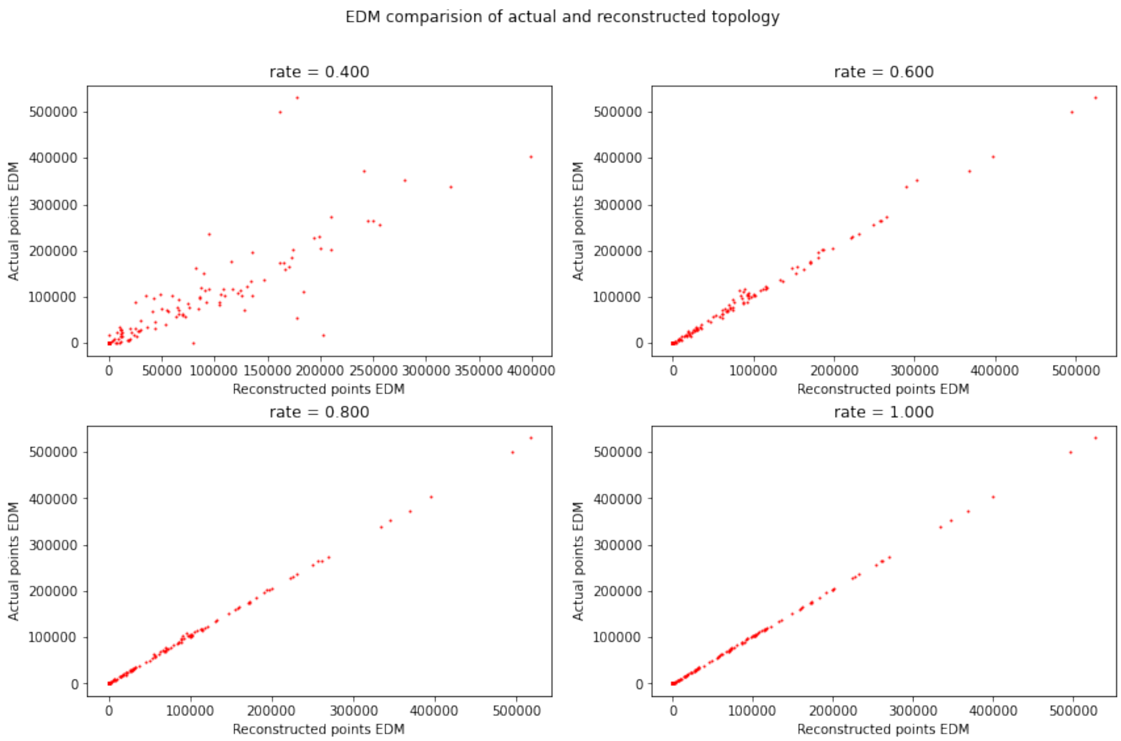
```
opts.tol = 1e-5

lsopts.maxit = 20
lsopts.xtol = 1e-8
lsopts.gtol = 1e-8
lsopts.ftol = 1e-10
lsopts.alpha  = 1e-3
lsopts.rho  = 1e-4
lsopts.sigma  = 0.1
lsopts.eta  = 0.8
```

5.7.2 Defining and Running the Solver

[33]:
```
outputs3 = []
outputdata = []
rates3 = [0.4,0.6,0.8,1]

for rate in rates3:
    opts.rate = rate
    solver = EDMSolver(points3,opts,lsopts)
    output = solver.run()
    outputs3.append(output)
```

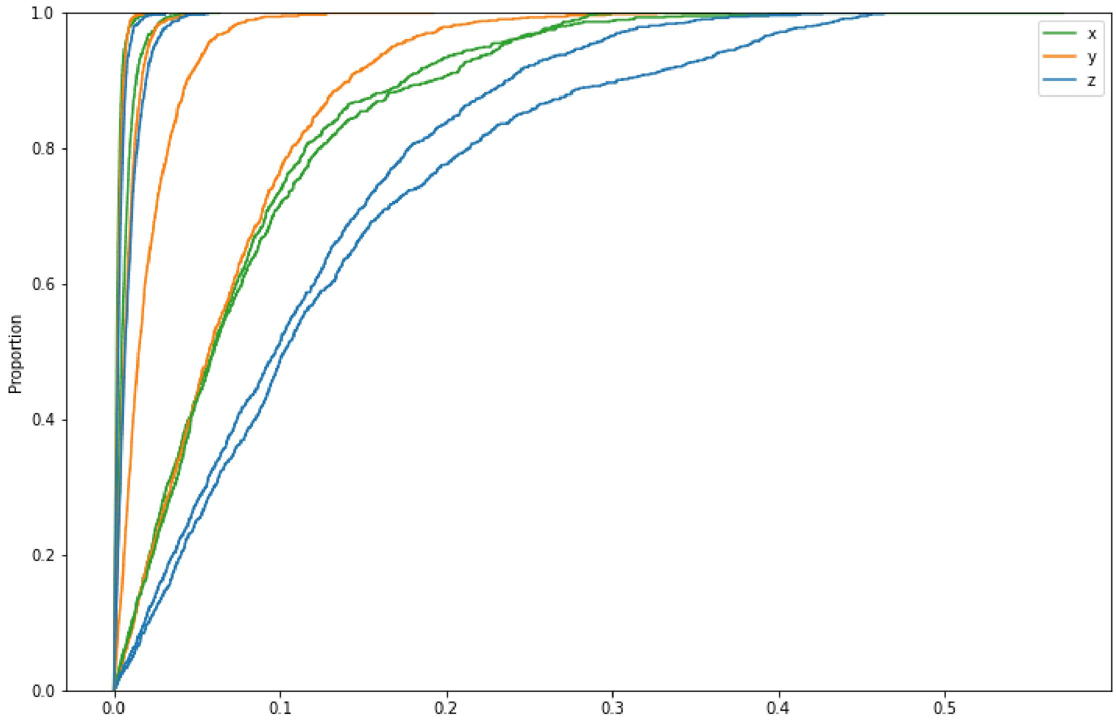
```
41.777778 percent of the EDM Distances are known

58.666667 percent of the EDM Distances are known

74.666667 percent of the EDM Distances are known

93.333333 percent of the EDM Distances are known
```

[34]:
```
fig,ax = plt.subplots(2,2,figsize=(12,8))
for i in range(len(outputs3)):
    output = outputs3[i]
    dact = np.tril(solver.getEDM(output['points'])).ravel()
    dObs = np.tril(solver.getEDM(output['tpoints'])).ravel()
    ax[int(i/2),int(i%2)].scatter(dObs,dact,color='r',s=1)
    ax[int(i/2),int(i%2)].set_title('rate = %.3f'%output['rate'])
    ax[int(i/2),int(i%2)].set_xlabel('Reconstructed points EDM')
    ax[int(i/2),int(i%2)].set_ylabel('Actual points EDM')

plt.tight_layout()
fig.subplots_adjust(top=0.88)
fig.suptitle('EDM comparision of actual and reconstructed topology')
plt.show()
```



5.7.3 Coordinate Error CDF

[35]:
```
plt.figure(figsize=(12,8))
for output in outputs2:
    sns.ecdfplot((abs(output['points'] - output['tpoints'])))
plt.legend(['x','y','z'])
plt.show()
```



[36]:
```
print('Rate    Ratio     Error(%)    Num Iterations Time Elapsed(s)')
for output in outputs3:
    print("%0.2f    %0.2f    %0.7f         %0.2f           %0.
 ↪4f"%(output['rate'],output['ratio'],output['error']*100,output['numit'],output['timeelapsed']
```

```
Rate    Ratio     Error(%)    Num Iterations Time Elapsed(s)
0.40    41.78    2.2919492       5.00          0.0140
0.60    58.67    0.3994958       11.00          0.0299
0.80    74.67    0.1712731       8.00          0.0219
1.00    93.33    0.0687895       5.00          0.0150
```

5.8 Error Analysis: Dataset 4

5.8.1 Defining EDMSolver Class Parameters

[37]:
```
opts = params()
lsopts = params()

opts.lambda_ = 5
# opts.rate = 0.4
opts.printenergy = 0
opts.plotting = 0
opts.rank = 10
opts.maxit = 30
```

```
opts.tol = 1e-5

lsopts.maxit = 20
lsopts.xtol = 1e-8
lsopts.gtol = 1e-8
lsopts.ftol = 1e-10
lsopts.alpha  = 1e-3
lsopts.rho  = 1e-4
lsopts.sigma  = 0.1
lsopts.eta  = 0.8
```

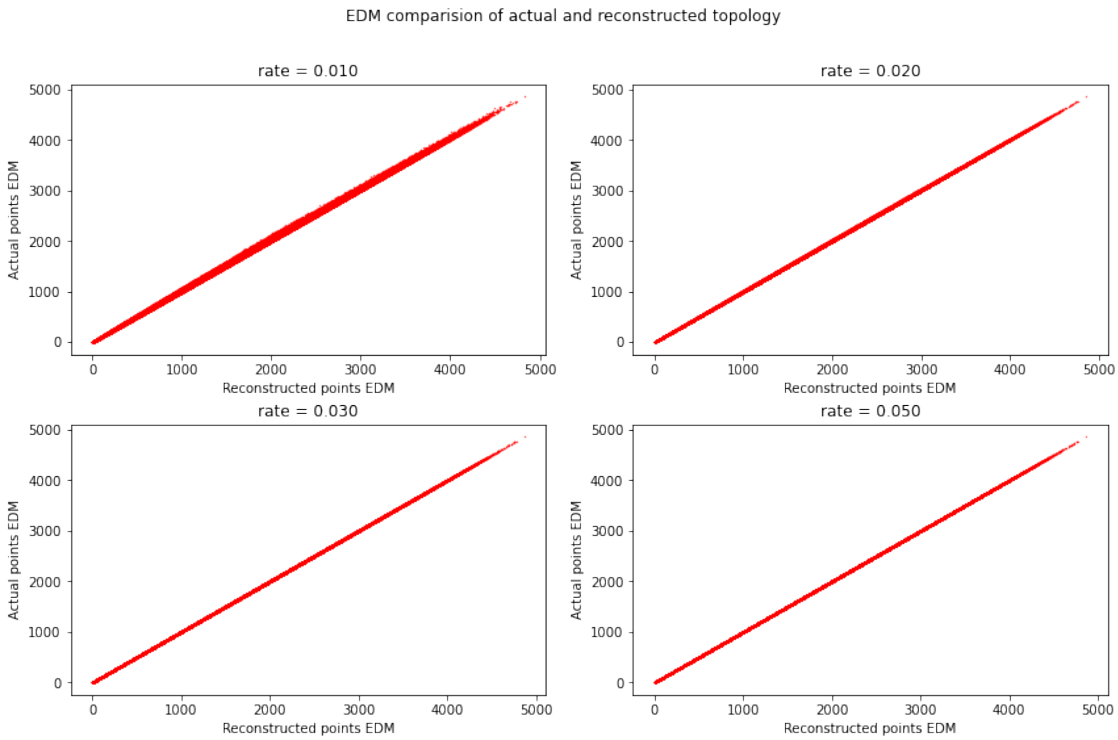
5.8.2 Defining and Running the Solver

[38]:
```
outputs4 = []
rates4 = [0.01,0.02,0.03,0.05]

for rate in rates4:
    opts.rate = rate
    solver = EDMSolver(points4,opts,lsopts)
    output = solver.run()
    outputs4.append(output)
```

```
0.998259 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
2.001472 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
3.002135 percent of the EDM Distances are known

4.999413 percent of the EDM Distances are known

det(R) < R, reflection detected!, correcting for it ...
```

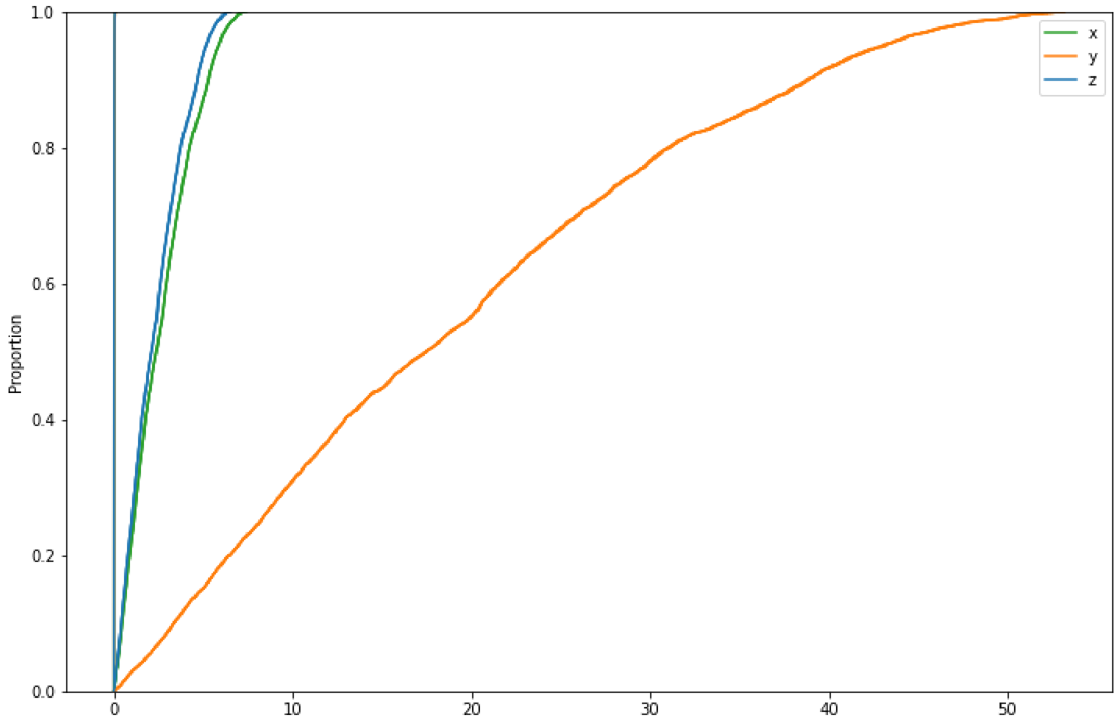
[39]:
```
fig,ax = plt.subplots(2,2,figsize=(12,8))
fig.suptitle('EDM comparision of actual and reconstructed topology')
for i in range(len(outputs4)):
    output = outputs4[i]
    dact = np.tril(solver.getEDM(output['points'])).ravel()
    dObs = np.tril(solver.getEDM(output['tpoints'])).ravel()
    ax[int(i/2),int(i%2)].scatter(dObs,dact,color='r',s=0.1)
    ax[int(i/2),int(i%2)].set_title('rate = %.3f'%output['rate'])
    ax[int(i/2),int(i%2)].set_xlabel('Reconstructed points EDM')
    ax[int(i/2),int(i%2)].set_ylabel('Actual points EDM')

plt.tight_layout()
fig.subplots_adjust(top=0.88)
```

```
plt.show()
```



5.8.3 Coordinate Error CDF

[40]:
```
plt.figure(figsize=(12,8))
for output in outputs4:
    sns.ecdfplot((abs(output['points'] - output['tpoints'])))
plt.legend(['x','y','z'])
plt.show()
```



[41]:
```
print('Rate   Ratio    Error(%)   Num Iterations Time Elapsed(s)')
for output in outputs4:
    print("%0.2f   %0.2f   %0.7f       %0.2f          %0.
 ↪4f"%(output['rate'],output['ratio'],output['error']*100,output['numit'],output['timeelapsed']
```

```
Rate    Ratio    Error(%)    Num Iterations Time Elapsed(s)
0.01    1.00    47.7290209       14.00          66.8547
0.02    2.00    47.8567700       12.00          69.5083
0.03    3.00    0.0577544       15.00         103.2109
0.05    5.00    47.8917195       22.00          179.7898
```

6 observations

We can see that the errors plotted for the 4 datasets in the above run is tabularised as follows

Table 1 : Dataset 1

Rate	Ratio	Error(%)	Num Iterations	Time Elapsed(s)
0.01	0.99	1.0994404	19.00	29.2304
0.02	1.98	0.7978092	16.00	32.7008
0.03	2.99	0.8518338	16.00	34.3210
0.05	5.02	0.6650077	19.00	57.8823

It is evident that as the sampling rate, the number of entries from the EDM matrix increased , the performance of our algorithm has increased considerably. However there is a improvement in error as we move from 0.03 to 0.05 sampling rate. This could be due to randomised algorithm which we are using to label some our EDM matrix entries as missing. It could be possible that the data available was not representative enough of the spatial orientation. This introduces another problem of identifying the crucial edges of our topology which play an important role in our algorithm.

Table 2 : Dataset 2

Rate	Ratio	Error(%)	Num Iterations	Time Elapsed(s)
0.01	1.00	21.3511732	30.00	7.6700
0.02	2.00	22.1612297	28.00	8.1524
0.03	3.02	2.0046797	19.00	6.7776
0.05	5.01	0.7753925	15.00	6.2720

We see a perfect decrease in error as the sample rating increases, almost near ideal accuracies as we approach a sampling rate of 5%. This is in line with our expected observations.

Table 3 : Dataset 3

Rate	Ratio	Error(%)	Num Iterations	Time Elapsed(s)
0.40	41.78	2.2919492	5.00	0.0140
0.60	58.67	0.3994958	11.00	0.0299
0.80	74.67	0.1712731	8.00	0.0219
1.00	93.33	0.0687895	5.00	0.0150

Since this is a very small dataset in comparison to our previous ones, we had to use 10-50 times the sampling rate in the previous datasets. This is because as the number of points decreases, the total number of connections also decrease and thus our solver has much lesser data to work with. Reasonable accuracies were obtained with about 40% of the data which is acceptable in most real life applications.

Table 4 : Dataset 4

Rate	Ratio	Error(%)	Num Iterations	Time Elapsed(s)
0.01	1.00	47.7290209	14.00	66.8547
0.02	2.00	47.8567700	12.00	69.5083
0.03	3.00	0.0577544	15.00	103.2109
0.05	5.00	47.8917195	22.00	179.7898

We found this dataset interesting because it is one of the state-of-the-art applications of our algorithm. From NMR spectral data we derived the molecular conformation of "HAEMOGLOBIN IN THE DEOXY QUATERNARY STATE WITH LIGAND BOUND AT THE ALPHA HAEMS". However, we can see that the error is not that fancy, this could be because the intra molecular distance is very very small (~10^{-10}) and any small deviations due to our calculations of the euclidean distance will results in huge margin for our relative error that is being calculated in this case.

7 Conclusions

Our error that is being calculated is not a precise measure of our algorithm because of a few inherent flaws in it.

One of the major issues is that it is always possible that some set of distance matrix entries are more important and representative our of data than another one. Hence we are leaving the performance of our algorithm at random when we start from a point cloud (All these datasets). However, in most real life applications we start with an EDM matrix and hence this should not be a problem for our algorithm.

Another important issue we are dealing with is the normalising of EDM matrix and subsequent scaling of the output point matrix obtained. These rounding off may seem small to the naked eye but we can see that we are using the kilometers as a basic unit for our algorithm (For datasets 1 and 3). A small variation in this about $\sim 10^{-2}$ could results in deviations of about 10 metres.

8 Future extensions for our project

Identify the crucial edges for our topology by generating the performance data of our algorithm with different sets of entries in our EDM matrix and its parameters and training an ML model using this data to assign an importance factor to each of these EDM matrix entries

Dynamic reconstruction of the topology of moving nodes using IMU data obtained.

9 References

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Rigid Transform 3D - https://github.com/nghiaho12/rigid_transform_3D

Dataset 1 - https://www.kaggle.com/max-mind/world-cities-database

Dataset 2 - https://www.kaggle.com/aalborguniversity/sewerpointclouds

Dataset 4 - https://www.rcsb.org/structure/1COH