IIIT-H
Information and Communication
Spring-2024

Arti Yardi & Prasad Krishnan

Exam: End Semester
Marks: 100

Date: 30-Apr-2024
Time:3 hrs

Instructions:

- Answering all the questions is compulsory. The topic of each question is mentioned in **bold**, at the beginning of the question.
- All steps should be justified in detail.
- Clearly state the assumptions (*if any*) made that are not specified in the questions.

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1. (22 marks) (**Channel Coding:**) Consider a linear code $\mathcal{C}$ generated by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$.

   (a) (1+3+1) What is the length of this code? Determine all the codewords in the code. What is the rate of this code?

   (b) (3+3 marks) Write down the parity check equations for this code carefully, by observing the structure of an arbitrary codeword $\boldsymbol{m}G$ (where $\boldsymbol{m}$ is an arbitrary message vector). Using these equations, find a parity check matrix for $\mathcal{C}$.

   (c) (4 marks) Show that the code $\mathcal{C}$ can correct a single erasure at any arbitrary location.

   (d) (3 marks) Show that this code is actually linear, i.e., show that any linear combination (with coefficients from $\{0, 1\}$ and mod-2 operations) of *any* two codewords in $\mathcal{C}$ lies in $\mathcal{C}$. [Note: Examples are not proofs. If you show that, for linear combination of some example codewords is a codeword, and then say 'similarly, we can show', **that is not a proof.** Your answer should cover all choices of linear combinations of all possible codewords, without actually going through one-by-one. Think how arbitrary codewords of $\mathcal{C}$ look like, and then think how an arbitrary linear combination of these codewords look like, and then show this.]

   (e) (4 marks) Consider a G' such that $G' = [I_3|P]$, for some matrix P of size 3×3 over $\{0, 1\}$(mod 2). Suppose it is given that G and G' are both valid generator matrices of $\mathcal{C}$ (i.e., both G and G' have the same rowspace, which is $\mathcal{C}$). Find P and hence write down what G' is.

2. (11 marks) (**Digital Communication Pipeline:**) See the conceptual digital communication picture shown in Fig. 1. The following details are given to you.

   - The quantizer we wish to use has 8 levels. The bit-conversion block after the quantizer simply converts each of the 8 levels into its bit-representation (think how many bits per level is required). We do not use any source coding or anti-aliasing filter. Note their absence in the figure.

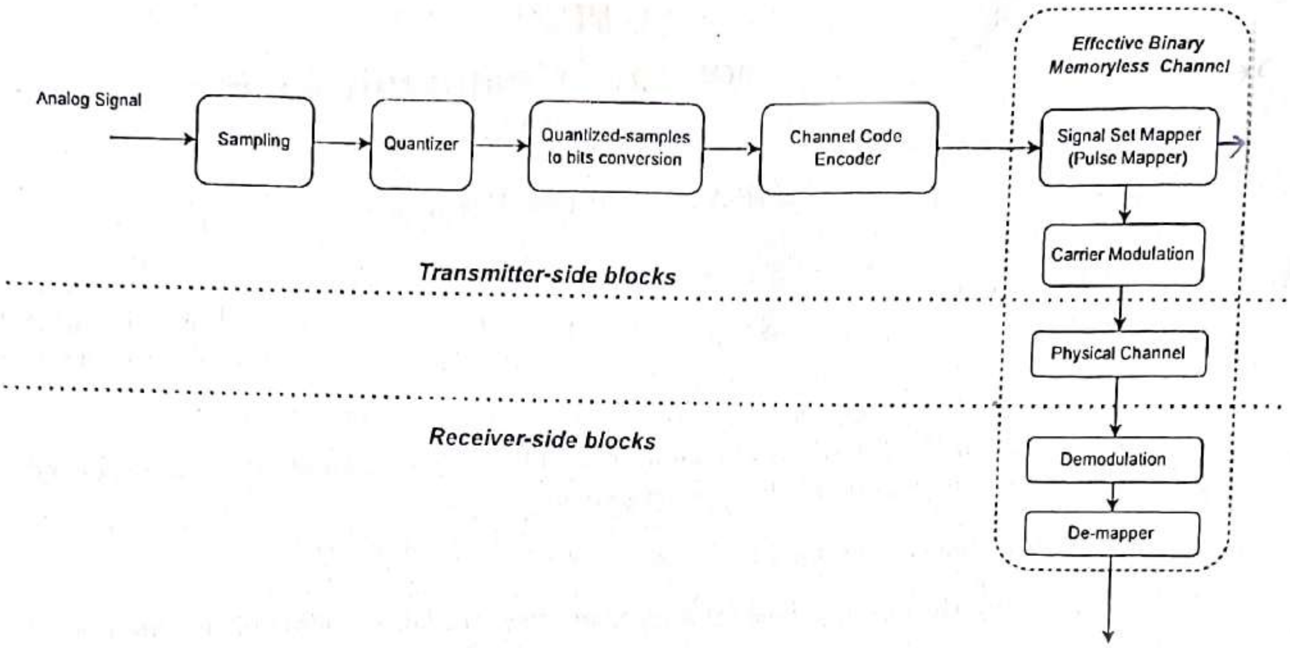

Figure 1: Digital Communication Pipeline (Q2)

- A channel code of block length 1000 bits is used in this channel.
- The *effective binary memoryless channel* shown has a capacity of 0.3 bits/channel-use.
- One use of the binary channel is equivalent to sending a positive or negative pulse with time-width $0.1\mu s$ (μs means microseconds) through the *physical channel* (i.e., every bit of transmission through the channel occupies $0.1\mu s$).

Answer the following questions.

(a) (6 marks) What can be the maximum frequency of the input analog signal supported on this communication channel? Answer this with careful appropriate reasoning and calculations.

(b) (2 marks) Suppose we were to use source coding and anti-aliasing filter blocks. Redraw the figure with these included in the appropriate locations.

(c) (3 marks) Describe how the carrier modulation and demodulation used in this channel would work. As per your estimates, what should be the minimum carrier frequency that should be used for this channel? Argue with reasons.

3. (9 marks) **(Channel Capacity:)**

(a) (2 marks) Define the Shannon's notion of capacity of a memoryless discrete communication channel.

(b) (4 marks) Derive the capacity of $BSC(p)$ (i.e., the Binary Symmetric Channel with bit-flip probability p).

(c) (3 marks) Viewing the capacity as a function of p, obtain the values of p for which the capacity is maximum and minimum, as well as the values of the maximum and minimum capacity. Intuitively explain the reasons for the same. [It is O.K. to give the maximizing and minimizing capacity values without formal proof, but with only intuitive arguments. But you *should* give the intuitive understanding of the *channel behaviour* in these maximum/minimum capacity scenarios.]

4. (10 marks) **[Basic concepts of Information Theory]** A pack of cards has 52 cards in it, each being in one of 4 suits, the 4 suits being Diamonds, Clubs, Spades, and Hearts. The suits Diamonds and Hearts are red in colour, and the other two are black. Each suit has 13 types of cards in it, $\{\text{Ace}, \text{King}, \text{Queen}, \text{Jack}, 2, 3, ^{4567}\ldots, 8, 9^{10}\}$. A card X is drawn uniformly at random from the pack. Let S denote its suit, T denote its type, and C denote its colour. Calculate the following. (Hint: Note that in each problem, you may need to compute relevant probability distribution(s). To calculate these distributions correctly, you may use formal definitions of conditional probability. If you are unable to do this, at least you should be give the correct intuitive thinking behind why you arrive at such expression(s) for the distribution(s)).

   (a) (1.5 marks) $H(X)$.

   (b) (4 marks) $H(X|T)$.

   (c) (3 marks) $I(S;T)$.

   (d) (1.5 marks) $I(X;C)$.

5. (10 marks) **[Signals and Systems]** Recall that the output of transmitting a signal $x(t)$ through a LTI system with impulse response $h(t)$ results in the output being the convolution $h(t) \star x(t)$. Determine the output of a pulse $p(t) = rect\left(\frac{t-50\mu s}{200\mu s}\right)$ which is sent as input through the following filters. Show corresponding figures to illustrate the input, filtering operation and the output. [Note: In each case, you are free to use either the time domain or frequency domain to represent your output. However, do not leave your answer as an integral. You *should* also draw figures as required.]

   (a) (4 marks) Ideal Low-Pass-Filter with cut-off frequency 7500 Hz.

   (b) (3 marks) Ideal High pass filter with cut-off frequency 10 KHz.

   (c) (3 marks) Ideal Bandpass filter with centre-frequency at 10 KHz, and bandwidth 5KHz around the centre frequency.

6. (12 marks) **Source coding:** A zero-memory source emits messages m_1, m_2, and m_3 with probabilities 0.5, 0.3 and 0.2 respectively.

   (a) (7 marks) Find binary Huffman codes for its first and second order extensions. Determine code efficiency in each case.

   (b) (2 marks) In part (a), would the constructed Huffman codes be unique? Yes/No? Justify your answer.

   (c) (3 marks) Write down the formal statement of the source coding theorem.

7. (12 marks) **Kraft inequality:**

   (a) (2+5 marks) Give the complete precise statement for the Kraft inequality for binary source coding and prove the same.

   (b) (5 marks) Can there exist a source code that satisfies Kraft inequality with equality but does not satisfy the prefix condition? If yes, provide an example with at least 5 codewords. If no, justify your answer.

8. (14 marks) **Probability and random variables:**

   (a) Let $F_X(.)$ denotes the cumulative distribution function (CDF) of a random variable X. Show that $F_X(.)$ satisfies the following properties.

(i) (3 marks) $F_X(.)$ is monotonically nondecreasing.

(ii) (2 marks) $F_X(-\infty) = 0$ and $F_X(\infty) = 1$.

(iii) (3 marks) $F_X(.)$ is right continuous.

(b) Given that $G(x)$ is a valid CDF, which of the following would also be a valid CDF? Justify your answer in each case. [Note: Clearly, you should argue each property holds, in case it is a valid CDF. If invalid, showing that it does not satisfy any one property is good enough as a proof.]

(i) (2 marks) $G_1(x) = \alpha G(x)$, where α is any real number.

(ii) (2 marks) $G_2(x) = 1 - G(x)$

(iii) (2 marks) $G_4(x) = G(x) + (1 - G(x)) \log_e(1 - G(x))$
