

MA2.101: Linear Algebra (Spring 2022)

Exam

Wednesday, 28 March 2024

Course outcomes: CO1, CO3, CO6.

1. ([4 marks]) Solve one of the following.

- The system of equations

$$\begin{aligned}x + y + z &= 6 \\x + 4y + 6z &= 20 \\x + 4y + \lambda z &= \phi.\end{aligned}$$

Find the values of λ and ϕ for which this system of equations has no solutions.

- If $Ax = b$ always has at least one solution, show that the only solution to $A^T y = 0$ is $y = 0$. Here A^T denotes the transposition of matrix A .
2. ([3 marks]) V is a finite-dimensional vector space and let $T : V \rightarrow V$ be a linear operator on V . Suppose that $\text{rank}(T^2) = \text{rank}(T)$. Prove that the range and nullspace of T have only the zero vector 0 in common.
3. ([4 marks]) Two vector spaces are called *isomorphic* if there exists an invertible linear transformation from one vector space onto the other one. Prove that two finite-dimensional vector spaces over F are isomorphic if and only if they have the same dimension.
4. ([4 marks]) Solve one of the following.
- (a) Prove both of the following statements.
- The image or the range of a linear transformation $T : V \rightarrow W$ is a subspace of W .
 - A linear transformation $T : V \rightarrow W$ is one-to-one if and only if the nullspace of T only contains $0 \in V$.

(b) Consider the ordered bases $\mathcal{A} = \{(1, 2), (-2, -3)\}$ and $\mathcal{B} = \{(2, 1), (1, 3)\}$ for a vector space V . Then find the following

- Matrix P that changes coordinates of any vector $\vec{\alpha} \in V$ w.r.t. the ordered basis \mathcal{A} to coordinates w.r.t. the ordered basis \mathcal{B} .
 - Matrix Q that changes coordinates of any vector $\vec{\alpha} \in V$ w.r.t. the ordered basis \mathcal{B} to coordinates w.r.t. the ordered basis \mathcal{A} .
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Best wishes for all your endeavours. Stay healthy and happy!