EC5.101 – Network, Signals and Systems – Mid Exam

Date: 21st September, 2023
Exam duration: 90 minutes

Maximum marks: 42

Instructions:

a) There are *6 questions* for a total of *42 marks.*

b) Mention any additional assumptions you make that is not given in the question.

c) Clearly show the steps used to arrive at the solutions.

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1. [9 marks] Recall that the pulse signal is given by

$$p_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & 0 < t \leq \Delta \\ 0, & \text{otherwise,} \end{cases}$$

Consider two pulse signals $p_{\Delta_1}(t)$ and $p_{\Delta_2}(t)$ for some $\Delta_1 \neq \Delta_2$.

   (a) [1] Sketch the pulses $p_{\Delta_1}(t)$ and $p_{\Delta_2}(t)$. If your roll number is odd consider $\Delta_1 < \Delta_2$ and if it is even consider $\Delta_1 > \Delta_2$.

   (b) [5] Find the convolution of these two pulse signals, i.e., $y(t) = p_{\Delta_1}(t) * p_{\Delta_2}(t)$. Clearly plot and label this signal.

   (c) [3] Analyse your answer as simultaneously both $\Delta_1 \to 0$ and $\Delta_2 \to 0$. How can you use this to show that $\delta(t) * \delta(t) = \delta(t)$?

2. [8 marks] Consider a system with input output relation given by

$$y(t) = \int_{t+1}^{\infty} x(\tau) d\tau$$

   (a) [1] Is this system causal? Justify.

   (b) [2] Is this system BIBO stable? Justify.

   (c) [2] Find the impulse response of this system.

   (d) [3] Analyse the system with regards to linearity and time-invariance.

3. [7 marks] Consider the signal given by

$$x(t) = \delta(t+1) + u(t).$$

(a) [1] Sketch this signal.

(b) [3] Find the Laplace transform expression and region of convergence (ROC) for this signal.

(c) [3] Find another signal in time-domain which has the same Laplace transform expression as above. What is the corresponding ROC?

4. [5 marks] Consider the Laplace transform expression given by

$$X(s) = \frac{s+2}{s(s+1)(s-2)}$$

(a) [1] Sketch the pole-zero plot for this.

(b) [2] Indicate all the possible region of convergence (ROC) for this expression.

(c) [2] For each possible ROC, there is an associated time-domain signal. For each such ROC, identify the nature of the time-domain signal (i.e., right-sided, left-sided, or two-sided). You are not required to find the time-domain signals.

5. [8 marks] An LTI system has impulse response given by

$$h(t) = u(t) - u(t-1).$$

Let $x(t)$ and $y(t)$ denote the input and output signals of this system.

(a) [2] Comment on stability and causality for this system. Justify.

(b) [2] How are $x(t)$, $y(t)$ and $h(t)$ related for any LTI system? Using the impulse response given above, find the explicit relation between $x(t)$ and $y(t)$.

(c) [3] Let the input $x(t)$ to this system be a periodic signal with time period T and complex Fourier series (FS) coefficients given by $a_k, k = 0, \pm 1 \ldots \pm\infty$. Show that the output signal $y(t)$ is periodic and find its complex FS coefficients b_k in terms of the input signal coefficients a_k and T.

(d) [1] In above part, how is b_0 related to a_0?

6. [5 marks] Find the convolution of signals $x_1(t)$ and $x_2(t)$ given below. Give expression and sketch the answers.

(a) [1] $x_1(t) = \delta(t-1)$ and $x_2(t) = \sin(2\pi t)$

(b) [2] $x_1(t) = \delta(t) - \delta(t-3)$ and $x_2(t) = tu(t)$

(c) [2] $x_1(t) = \sum_{k=0}^{\infty} \delta(t-k)$ and $x_2(t) = u(t)$