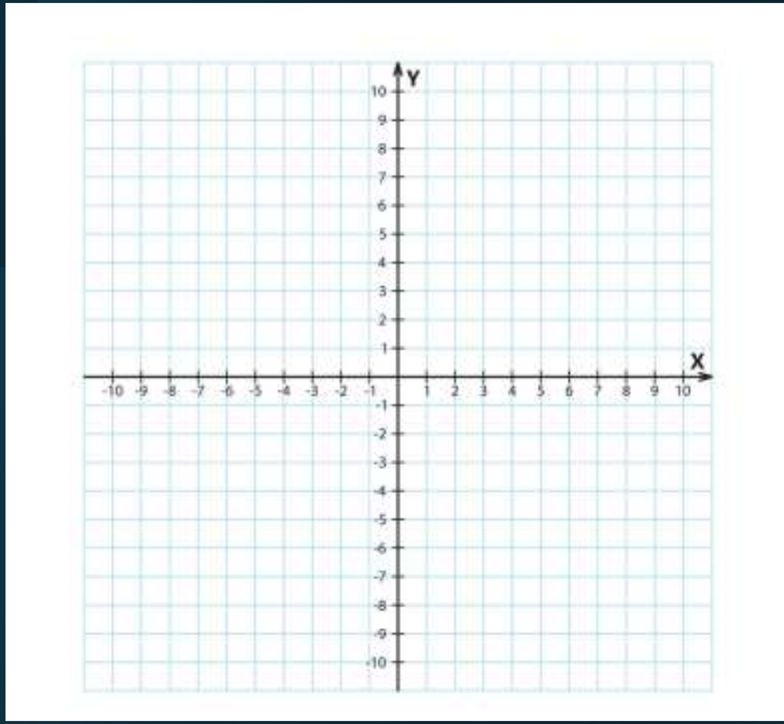
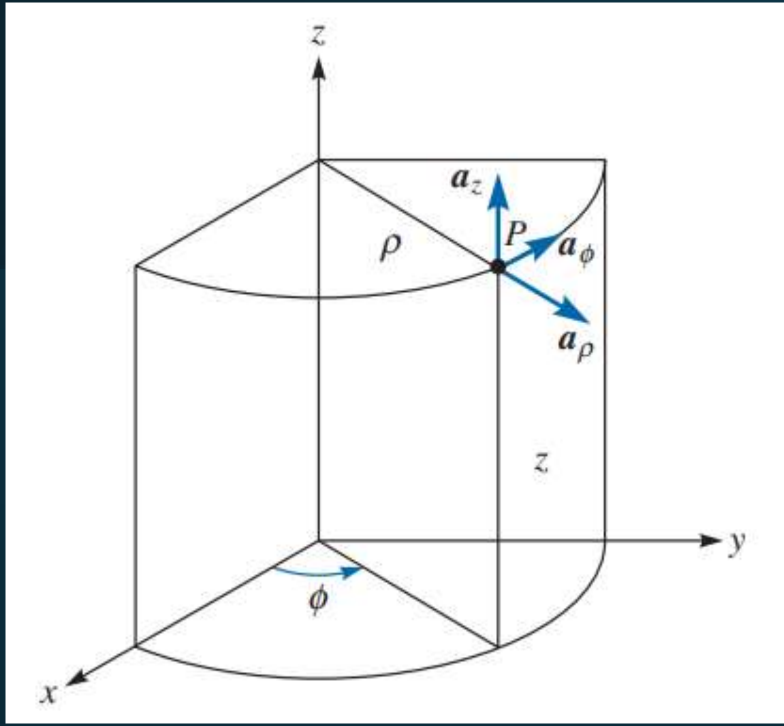
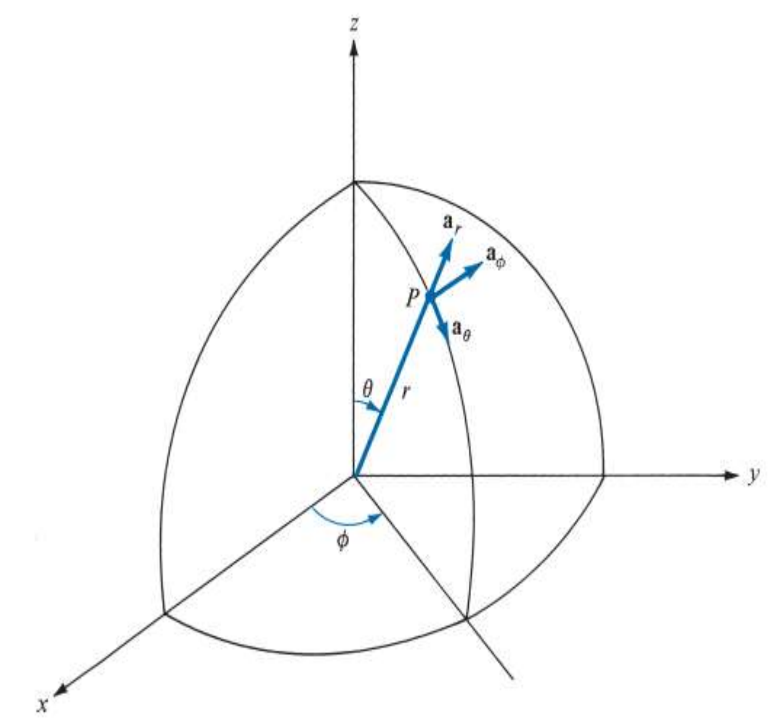
Radio Frequency Based Sensors design
Principles and Applications

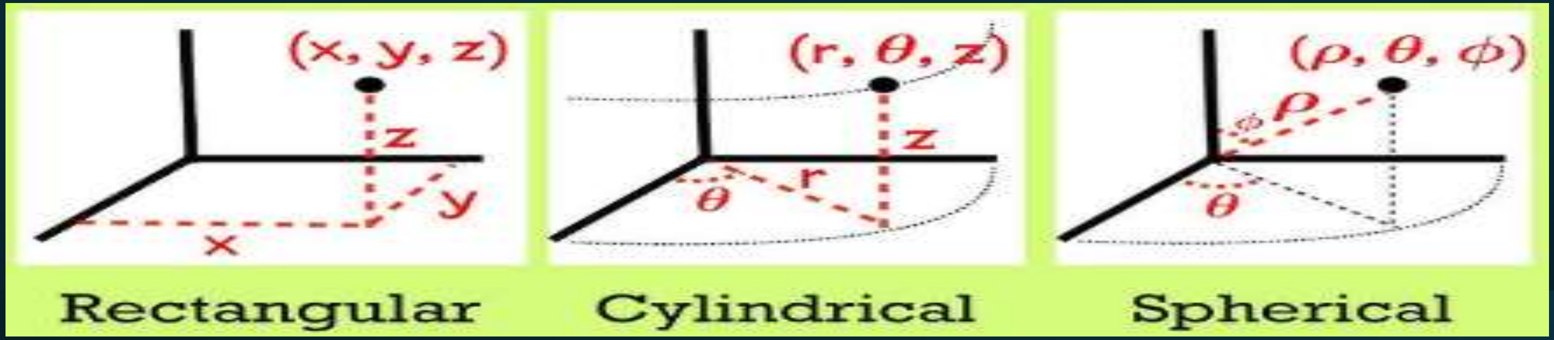
Tutorial 1

Cartesian, Cylindrical and Spherical coordinates









Conversion Formulae

Cartesian $\leftrightarrow$ Cylindrical Coordinates

(a) Cartesian $\rightarrow$ Cylindrical

Given (x, y, z), the cylindrical coordinates (ρ, ϕ, z) are:

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

(b) Cylindrical $\rightarrow$ Cartesian

Given (ρ, ϕ, z), the Cartesian coordinates (x, y, z) are:

$$x = \rho \cos\phi$$

$$y = \rho \sin\phi$$

$$z = z$$

Cartesian $\leftrightarrow$ Spherical Coordinates

(a) Cartesian $\rightarrow$ Spherical

Given (x, y, z), the spherical coordinates (r, θ, ϕ) are:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

(b) Spherical $\rightarrow$ Cartesian

Given (r, θ, ϕ), the Cartesian coordinates (x, y, z) are:

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

Cylindrical $\leftrightarrow$ Spherical Coordinates

(a) Cylindrical $\rightarrow$ Spherical

Given (ρ, ϕ, z), the spherical coordinates (r, θ, ϕ) are:

$$r = \sqrt{\rho^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$$

$$\phi = \phi$$

(b) Spherical $\rightarrow$ Cylindrical

Given (r, θ, ϕ), the cylindrical coordinates (ρ, ϕ, z) are:

$$\rho = r \sin\theta$$

$$\phi = \phi$$

$$z = r \cos\theta$$

1. Convert points $P(1, 3, 5)$, $T(0, -4, 3)$, and $S(-3, -4, -10)$ from Cartesian to cylindrical and spherical coordinates.

2. Given point $T(10, 60^\circ, 30^\circ)$ in spherical coordinates, express T in Cartesian and cylindrical coordinates.

Gradient, Divergence and Curl

1. Gradient (∇f)

The **gradient** of a scalar function represents the direction and rate of the fastest increase of the function.

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{a_x} + \frac{\partial f}{\partial y}\mathbf{a_y} + \frac{\partial f}{\partial z}\mathbf{a_z}$$

2. Divergence ($\nabla \cdot \mathbf{A}$)

The **divergence** of a vector field measures the extent to which the field is spreading out from a point.

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergence Theorem:

$$\iiint_V (\nabla \cdot \mathbf{A}) dV = \oiint_S \mathbf{A} \cdot d\mathbf{S}$$

3. Curl ($\nabla \times \mathbf{A}$)

The **curl** of a vector field represents the rotational tendency or swirling motion of the field.

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a_x} & \mathbf{a_y} & \mathbf{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Stokes' Theorem:

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \iint_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

3. Find the gradient of the following scalar fields:

   (a) $V = e^{-z} \sin 2x \cosh y$

   (b) $U = \rho^2 z \cos 2\Phi$

   (c) $W = 10r \sin^2 \theta \cos \Phi$

4. Determine the divergence of the following vector field and evaluate it at the specified point: $\mathbf{A} = yz\mathbf{a_x} + 4xy\mathbf{a_y} + y\mathbf{a_z}$ at $(1, -2, 3)$.

5. Evaluate the curl of the following vector field: $\mathbf{A} = xy\mathbf{a_x} + y^2\mathbf{a_y} - xz\mathbf{a_z}$.

Coulomb's Law (Electric Force, $\mathbf{F}$)

The force between two point charges q_1 and q_2 separated by a distance r is given by:

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

where:

- ϵ_0 is the permittivity of free space (8.85×10^{-12} F/m),
- $\hat{r}$ is the unit vector pointing from one charge to the other.
- The direction of force follows attraction (opposite charges) or repulsion (like charges).
- The net force is found by vector addition of forces due to multiple charges.

Electric Field Intensity ($\mathbf{E}$)

The electric field at a point due to a charge q is given by:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- The total electric field at a point due to multiple charges is the vector sum of individual fields.

Point charges 1 mC and -2 mC are located at $(3, 2, -1)$ and $(-1, -1, 4)$, respectively. Calculate the electric force on a 10 nC charge located at $(0, 3, 1)$ and the electric field intensity at that point.

Dielectric Constant (ε_r)

- The **dielectric constant** (also called **relative permittivity**) is the ratio of a material's permittivity (ε) to the permittivity of free space (ε_0):

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

- It determines how much a material can store electrical energy in an electric field.
- **For vacuum**: $\varepsilon_r = 1$, **for air**: ≈ 1.0006, **for water**: ≈ 80.

Volume Charge Density (ρ_v)

- **Definition**: Volume charge density represents **charge per unit volume** in a given region.
- It is obtained using **Gauss's law in differential form**:

$$\rho_v = \nabla \cdot \mathbf{D}$$

- Where:
  - $\mathbf{D}$ is the **electric flux density** (C/m^2),
  - $\nabla \cdot$ is the **divergence operator**.

Electric Flux (Φ)

- **Flux** represents the total electric field passing through a surface.
- Mathematically, flux through a surface S is given by:

$$\Phi = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

Total Charge Enclosed (Q_{enc})

- Using **Gauss's Law**:

$$Q_{\text{enc}} = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

- Alternatively, by integrating the volume charge density:

$$Q_{\text{enc}} = \int_V \rho_v \, dV$$

- This equation calculates the charge enclosed within a volume V by integrating ρ_v over the volume.

Relation Between Flux and Total Charge

Gauss's Law states that the total **electric flux** (Φ) through a closed surface is **equal to the total charge enclosed** divided by the permittivity (ϵ):

$$\Phi = \oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$$

7. Given the vector field:

$$\mathbf{D} = (2y^2 + z)\mathbf{a_x} + 4xy\mathbf{a_y} + x\mathbf{a_z} \quad \text{C/m}^2$$

find:

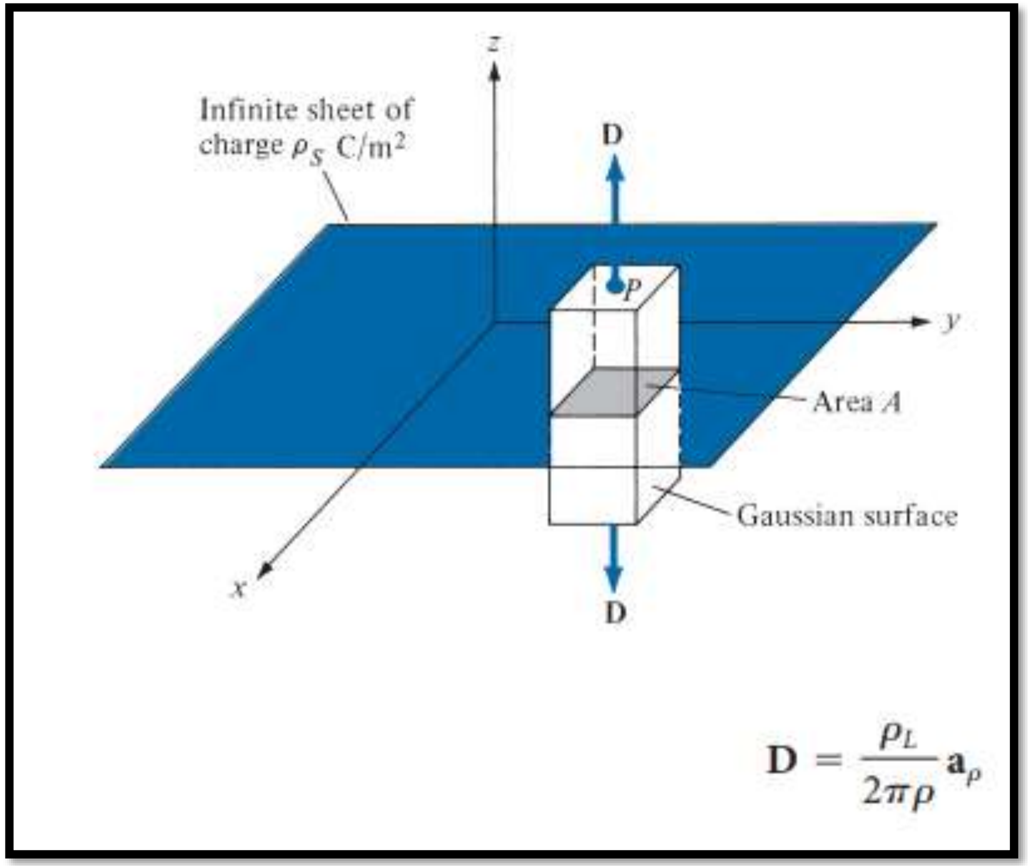
(a) The volume charge density at $(-1, 0, 3)$.

(b) The flux through the cube defined by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

(c) The total charge enclosed by the cube.

8. The electric field intensity in polystyrene ($\varepsilon_r = 2.55$) filling the space between the plates of a parallel-plate capacitor is 10 kV/m. Calculate the dielectric constant.

Infinite Sheet of Charge



Gaussian surface about an infinite line sheet of charge

Consider an infinite sheet of uniform charge ρ_S C/m^2 lying on the $z = 0$ plane. To determine **D** at point P, we choose a rectangular box that is cut symmetrically by the sheet of charge and has two of its faces parallel to the sheet as shown in Figure 4.15. As **D** is normal to the sheet, $\mathbf{D} = D_z\mathbf{a}_z$, and applying Gauss's law gives

$$\rho_S \int_S dS = Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = D_z\left[\int_{\text{top}} dS + \int_{\text{bottom}} dS\right] \tag{4.48}$$

Note that $\mathbf{D} \cdot d\mathbf{S}$ evaluated on the sides of the box is zero because **D** has no components along $\mathbf{a}_x$ and $\mathbf{a}_y$. If the top and bottom area of the box each has area A, eq. (4.48) becomes

$$\rho_S A = D_z(A + A) \tag{4.49}$$

and thus

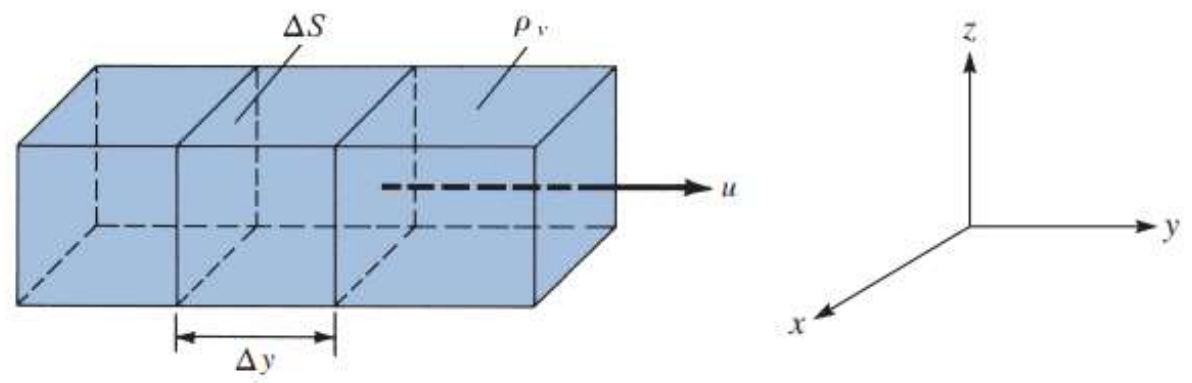
$$\mathbf{D} = \frac{\rho_S}{2}\mathbf{a}_z$$

or

$$\mathbf{E} = \frac{\mathbf{D}}{\varepsilon_o} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a}_z \tag{4.50}$$

CONVECTION CURRENT

Convection current, as distinct from conduction current, does not involve conductors and consequently does not satisfy Ohm's law. It occurs when current flows through an insulating medium such as liquid, rarefied gas, or a vacuum. A beam of electrons in a vacuum tube, for example, is a convection current.

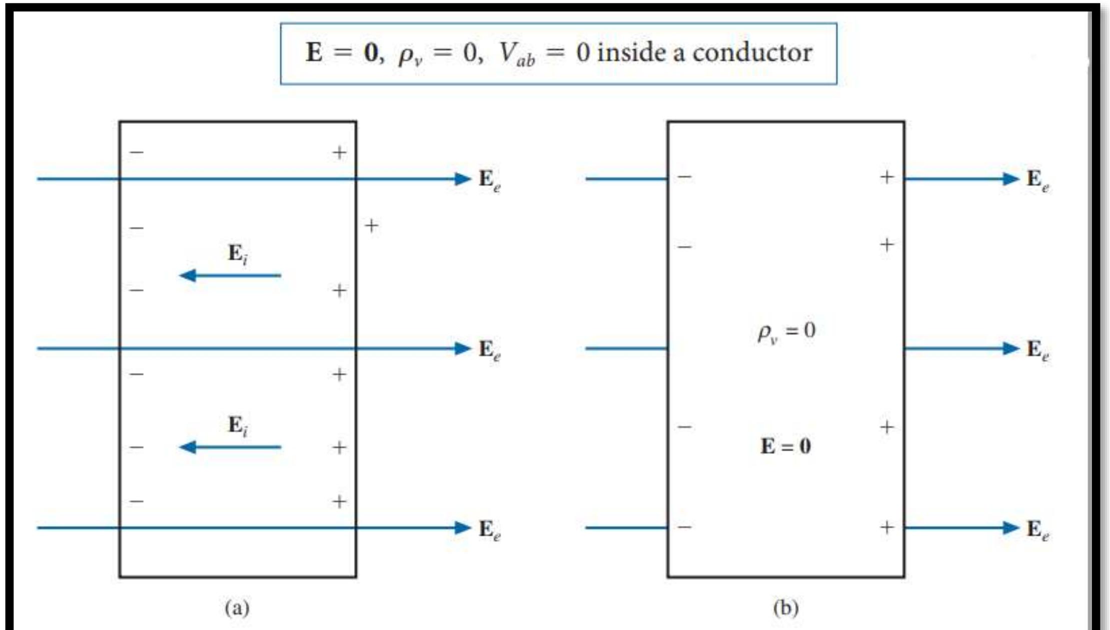


Current in a filament.

CONDUCTION CURRENT

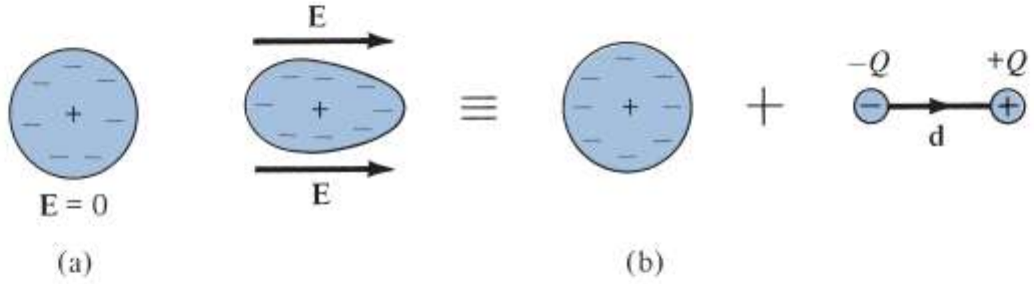
Conduction current requires a conductor. A conductor is characterized by a large number of free electrons that provide conduction current due to an impressed electric field. When an electric field **E** is applied, the force on an electron with charge $-e$ is

$$\mathbf{F} = -e\mathbf{E}$$



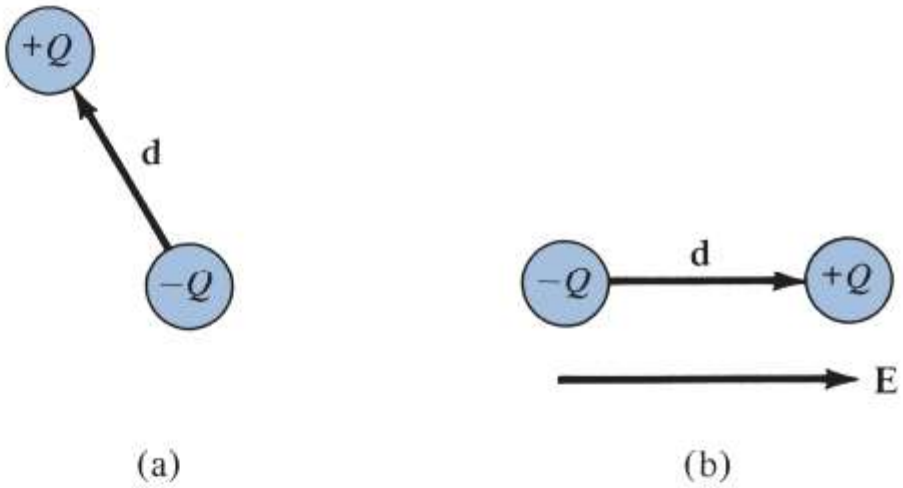
(**a**) An isolated conductor under the influence of an applied field. (**b**) A conductor has zero electric field under static conditions.

POLARIZATION in DIELECTRICS



Polarization of a nonpolar atom or molecule.

ELECTRIC FIELDS IN MATERIAL SPACE



Polarization of a polar molecule: (**a**) permanent dipole (**E** = **0**), (**b**) alignment of permanent dipole (**E** ≠ **0**).

$$\boxed{\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}}$$

$$\boxed{\mathbf{P} = \chi_e \varepsilon_o \mathbf{E}}$$

where:

- $\mathbf{D}$ = **Electric flux density** (C/m²)
- ε_0 = **Permittivity of free space** (8.85×10^{-12} F/m)
- $\mathbf{E}$ = **Electric field** (V/m)
- $\mathbf{P}$ = **Polarization vector** (C/m²), representing the dipole moment per unit volume
- χ_e = **Electric susceptibility** (unitless, material-dependent)

Dipole Moment and Polarization

$$\mathbf{p} = Q\mathbf{d} \tag{5.21}$$

where $\mathbf{d}$ is the distance vector from $-Q$ to $+Q$ of the dipole as in Figure 5.6(b). If there are N dipoles in a volume Δv of the dielectric, the total dipole moment due to the electric field is

$$Q_1\mathbf{d}_1 + Q_2\mathbf{d}_2 + \cdots + Q_N\mathbf{d}_N = \sum_{k=1}^{N} Q_k\mathbf{d}_k \tag{5.22}$$

As a measure of intensity of the polarization, we define *polarization* $\mathbf{P}$ (in coulombs per meter squared) as the dipole moment per unit volume of the dielectric; that is,

$$\boxed{\mathbf{P} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} Q_k\mathbf{d}_k}{\Delta v}} \tag{5.23}$$

Thus we conclude that the major effect of the electric field $\mathbf{E}$ on a dielectric is the creation of dipole moments that align themselves in the direction of $\mathbf{E}$. This type of dielectric

The **dielectric constant** (or **relative permittivity**) ε_r is the ratio of the permittivity of the dielectric to that of free space.

$$\boxed{\mathbf{D} = \varepsilon \mathbf{E}}$$

$$\boxed{\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}}$$

$$\boxed{\varepsilon = \varepsilon_o \varepsilon_r}$$

The **dielectric strength** is the maximum electric field that a dielectric can tolerate or withstand without electrical breakdown.

HFSS