Introduction to Coding Theory - Spring 2025
Tutorial 1

1. Show that $\forall \underline{y} \in \mathbb{F}_2^7, \exists$ at max 1 codeword $\underline{c}$ in hamming codes, such that $d_H(\underline{y}, \underline{c}) = 1$.

2. Show that an (n, M, d) code can correct $t_c = \left\lfloor \frac{d-1}{2} \right\rfloor$ errors and can detect $t_d = d - 1$ errors.

3. Show that for Binary symmetric channel (BSC), Maximum likelihood decoder is equivalent to nearest codeword decoder.
   Assume a codeword (01010) of the code $\{00000, 01010, 10101, 11111\}$ is transmitted through a BSC channel,and maximum likelihood decoder is applied to received codeword (00010),show that maximum likelihood decoder have 50% chances to decode it correctly.

4. Let $\mathcal{C}$ be a code containing M codewords.This code,$\mathcal{C}$, with bipolar representation is used on AWGN channel. Consider the MAP decoding of this code. For two real vectors $\mathbf{x}, \mathbf{y}$, let $\langle \mathbf{x}, \mathbf{y} \rangle$ denote their inner product. Let $\mathbf{y}$ denote the received vector when a codeword is transmitted over the AWGN channel. Show that the MAP decoding of the code $\mathcal{C}$ on the AWGN channel is essentially equivalent to the maximum inner-product rule, defined as follows.
$$\hat{\mathbf{c}}_{\text{MAP}} = \arg \max_{\mathbf{c} \in \mathcal{C}} \langle \mathbf{y}, \mathbf{c} \rangle$$
   where in the RHS inner products, we consider the codewords in their bipolar form.

5. A codeword is transmitted randomly from repetition code over a BSC(p) channel. Compute the probability of error assuming the receiver does nearest codeword decoding. Be careful while handling cases of even n. Is there another decoding strategy that can lower the probability of error?

6. Let $V_2(\underline{x}, n, t) = \left\{ \underline{y} \in \mathbb{F}_2^n : d_H(\underline{x}, \underline{y}) \leq t \right\}$. Compute $|V_2(\underline{x}, n, t)|$. What is the dependence of this value on $\underline{x}$?

7. Let $V_2(n, t) = \left\{ \underline{y} \in \mathbb{F}_2^n : w_H(\underline{y}) \leq t \right\}$. Show that for an (n, M, d) code
$$M \leq \frac{2^n}{V_2(n, t_c)}$$
   Do any of the codes you know achieve this bound with equality?