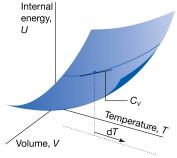
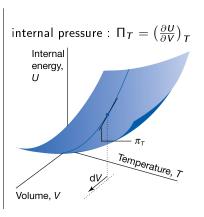
### Real systems:

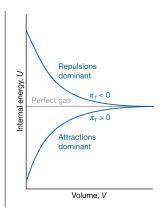
heat capacity at constant volume

$$: C_V = \left(\frac{\partial U}{\partial T}\right)_V$$





If  $\Delta U>0$  as  $\Delta V>0$  isothermally when there are attractive forces between the particles then a plot of U against V slopes upwards and  $\pi_T>0$ 



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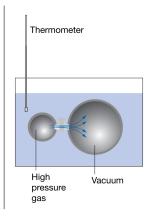
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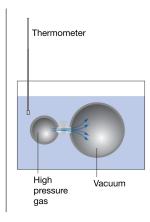
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experimental limitations



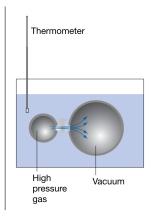
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No energy entered or left the system as heat because the temperature of the bath did not change, so q=0

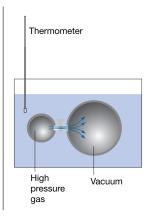
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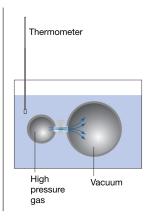
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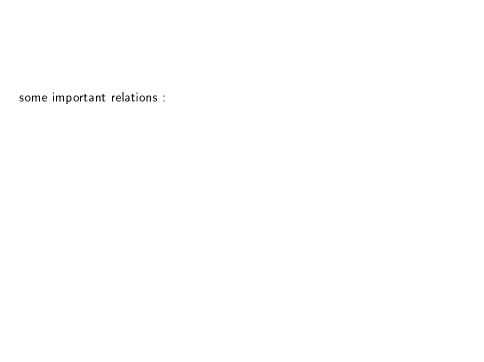


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some important relations :  $U \equiv U(V, T)$ 

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$$\therefore dU = \underbrace{\left(\frac{\partial U}{\partial V}\right)_{T}}_{\pi_{T}} dV + \underbrace{\left(\frac{\partial U}{\partial T}\right)_{V}}_{C_{V}} dT$$

$$\underbrace{\left(\frac{\partial U}{\partial T}\right)_{p} = \pi_{T} \left(\frac{\partial V}{\partial T}\right)_{p} + C_{V}}_{}$$

 $U \equiv U(V, T)$ 

$$\underbrace{\frac{U}{V}}_{T} \underbrace{\int_{T}^{T} dV} + \underbrace{\left(\frac{\partial U}{\partial T}\right)_{V}}_{C_{V}} dT$$

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$$dV + \underbrace{\left(\frac{\partial U}{\partial T}\right)_{V}}_{C_{V}} dT$$

def. : expansion coefficient,  $\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p}$ , then,  $\left( \frac{\partial U}{\partial T} \right)_{p} = \alpha \pi_{T} V + C_{V}$ 

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show :  $\alpha = \frac{1}{T}$  for perfect gas

$$\underbrace{\frac{\partial V}{\partial T}}_{\pi_T} \underbrace{\frac{\partial T}{\partial V}}_{C_V}$$

$$= \pi_T \left(\frac{\partial V}{\partial T}\right) + C_V$$

Also, for a perfect gas,  $\pi_T = 0$ ,  $\therefore \left(\frac{\partial U}{\partial T}\right)_n = C_V$ 

def.: expansion coefficient,  $\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{n}$ , then,  $\left( \frac{\partial U}{\partial T} \right)_{n} = \alpha \pi_{T} V + C_{V}$ 

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$$C_p$$

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$$C_n$$

$$C_p$$

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$$C_{p}-C_{V}=\left(\frac{\partial H}{\partial T}\right)_{p}-\left(\frac{\partial U}{\partial T}\right)_{V}=\left(\frac{\partial [U+\rho V]}{\partial T}\right)_{p}-\left(\frac{\partial U}{\partial T}\right)_{V}=\left(\frac{\partial U}{\partial T}\right)_{p}+\rho\left(\frac{\partial V}{\partial T}\right)_{p}-\left(\frac{\partial U}{\partial T}\right)_{V}$$

$$C_{p} - C_{V} = \left(\frac{\partial T}{\partial T}\right)_{p} - \left(\frac{\partial V}{\partial T}\right)_{V} = \left(\frac{\partial V}{\partial T}\right)_{p} - \left(\frac{\partial V}{\partial T}\right)_{V} =$$

$$= \left(\alpha \pi_{T} V + C_{V}\right) + p \left(\frac{\partial V}{\partial T}\right)_{p} - C_{V} = \alpha \pi_{T} V + p \left(\frac{\partial V}{\partial T}\right)_{p}$$

$$\left(\frac{\partial Q}{\partial T}\right)$$

$$\begin{aligned} & \left(\frac{\partial U}{\partial T}\right)_{p} = \pi_{T} \left(\frac{\partial V}{\partial T}\right)_{p} + C_{V}; \quad \alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_{p} \\ & C_{p} - C_{V} = \left(\frac{\partial H}{\partial T}\right)_{p} - \left(\frac{\partial U}{\partial T}\right)_{V} = \left(\frac{\partial [U + pV]}{\partial T}\right)_{p} - \left(\frac{\partial U}{\partial T}\right)_{V} = \left(\frac{\partial U}{\partial T}\right)_{p} + p \left(\frac{\partial V}{\partial T}\right)_{p} - \left(\frac{\partial U}{\partial T}\right)_{V} \end{aligned}$$

 $= (\alpha \pi_T V + C_V) + \rho \left(\frac{\partial V}{\partial T}\right)_p - C_V = \alpha \pi_T V + \rho \left(\frac{\partial V}{\partial T}\right)_n$ 

For perfect gas,  $C_p - C_V = p \left( \frac{\partial V}{\partial T} \right)_n = nR$ 

$$C_p$$
 -

$$C_p$$
 –

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$$=$$
 ( $\epsilon$ 

$$= (\alpha \pi_T V +$$

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$$(\partial V + G) = (\partial V)$$

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For perfect gas,  $C_p - C_V = p \left( \frac{\partial V}{\partial T} \right)_p = nR$ 

Generally,  $p\left(\frac{\partial V}{\partial T}\right)_p = \alpha p V$  and  $C_p - C_V = \alpha \left(p + \pi_T\right) V$ 

$$z = f(x, y) \implies dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

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for constant 
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,  $dz = 0$ ,  $\Longrightarrow$   $\left(\frac{\partial z}{\partial x}\right)_{y} dx_{z} = -\left(\frac{\partial z}{\partial y}\right)_{z} dy_{z}$ 

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- for constant z, dz = 0,  $\Longrightarrow$   $\left(\frac{\partial z}{\partial x}\right)_{y} dx_{z} = -\left(\frac{\partial z}{\partial y}\right) dy_{z}$ or,  $\left(\frac{\partial z}{\partial x}\right)_{y}\left(\frac{\partial x}{\partial y}\right)_{z}=-\left(\frac{\partial z}{\partial y}\right)_{y}, \quad \Longrightarrow \quad \left(\frac{\partial z}{\partial x}\right)_{y}\left(\frac{\partial x}{\partial y}\right)_{z}\left(\frac{\partial y}{\partial z}\right)_{y}=-1$

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 $-\left(\frac{\partial T}{\partial p}\right)_{H}\left(\frac{\partial H}{\partial T}\right)_{p}$ 

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 $-\left(\frac{\partial T}{\partial p}\right)_{\mu}\left(\frac{\partial H}{\partial T}\right)_{p} = -\mu C_{p}$ 

$$\uparrow$$
J T coefficient

$$z = f(x, y)$$
for constant

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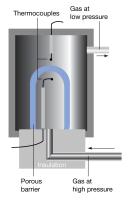
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 $z = f(x, y) \implies dz = \left(\frac{\partial z}{\partial x}\right)_{y} dx + \left(\frac{\partial z}{\partial y}\right) dy$ 

 $\therefore dH = -\mu C_p dp + C_p dT$ 

 $-\left(\frac{\partial T}{\partial p}\right)_{\mu}\left(\frac{\partial H}{\partial T}\right)_{p} = -\mu C_{p}$ 

#### Joule-Thomson coefficient - apparatus



let a gas expand through a porous barrier from one constant pressure to another

monitor the difference of temperature that arises from expansion

whole apparatus is insulated : adiabatic

$$q=0 \implies \Delta U=w$$

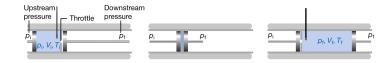
observed lower temperature on low pressure side,  $\Delta \mathcal{T} \propto$  pressure difference

Joule-Thomson effect : cooling by isenthalpic expansion

# Joule-Thomson coefficient - thermodynamic basis

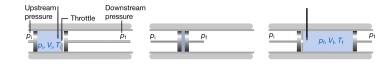


# Joule-Thomson coefficient - thermodynamic basis



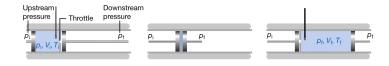
$$\Delta U = U_f - U_i = w = w_1 + w_2$$

# Joule-Thomson coefficient - thermodynamic basis



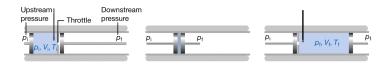
$$\Delta U = U_f - U_i = w = w_1 + w_2 = -p_i (0 - V_i) - p_f (V_f - 0)$$

## Joule-Thomson coefficient - thermodynamic basis



$$\Delta \textit{U} = \textit{U}_f - \textit{U}_i = \textit{w} = \textit{w}_1 + \textit{w}_2 = -\textit{p}_i \left( 0 - \textit{V}_i \right) - \textit{p}_f \left( \textit{V}_f - 0 \right) = \textit{p}_i \textit{V}_i - \textit{p}_f \textit{V}_f$$

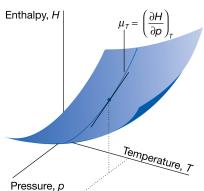
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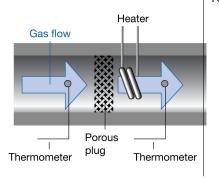


$$\Delta U = U_f - U_i = w = w_1 + w_2 = -p_i (0 - V_i) - p_f (V_f - 0) = p_i V_i - p_f V_f$$

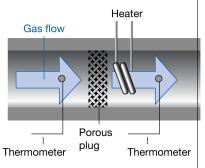
$$\therefore U_f + p_f V_f = U_i + p_i V_i = \text{const } H$$

isothermal Joule-Thomson coefficient, 
$$\mu_T = \left(\frac{\partial H}{\partial \rho}\right)_T = -\mathcal{C}_{\rho}\mu$$



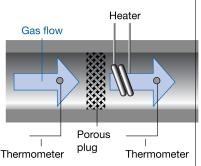


- measurement gas pumped continuously at steady pressure through heat exchanger (which brings it to required temperature)



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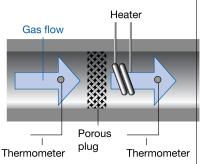
then passed through a porous plug inside a thermally insulated container



gas pumped continuously at steady pressure through heat exchanger (which brings it to required temperature)

then passed through a porous plug inside a thermally insulated container steep pressure drop is measured, and cooling effect is exactly effect by an electric

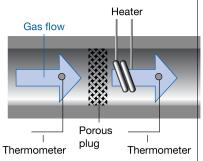
cooling effect is exactly offset by an electric heater placed immediately after the plug



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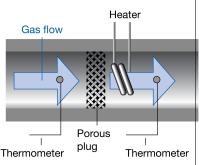
thermally insulated container
steep pressure drop is measured, and
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heater placed immediately after the plug
energy provided by the heater is monitored

then passed through a porous plug inside a



 measurement gas pumped continuously at steady pressure through heat exchanger (which brings it to required temperature)

then passed through a porous plug inside a thermally insulated container steep pressure drop is measured, and cooling effect is exactly offset by an electric heater placed immediately after the plug energy provided by the heater is monitored energy transferred as heat  $= \Delta H_{\rm p}$ 



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then passed through a porous plug inside a thermally insulated container steep pressure drop is measured, and cooling effect is exactly offset by an electric heater placed immediately after the plug energy provided by the heater is monitored energy transferred as heat  $= \Delta H_p$  find  $\mu_T = \underset{p \to 0}{\mathcal{L}} \frac{\Delta H}{\Delta p}$  and convert to  $\mu$ 

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- ► If speed is reduced to the point that neighbours can capture each other by intermolecular attractions, then it condenses to a liquid
  - $\blacktriangleright$  Sign of  $\mu$  depends on the gas, p, relative magnitudes of attractive and repulsive forces, and T

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- ▶ speed of molecules ∝ T
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- ► Real gases have nonzero J T coefficients
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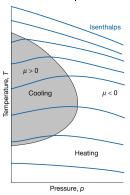
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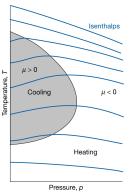
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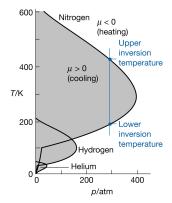
- in order to travel more slowly  $\blacktriangleright$  when repulsions are dominant, Z>1: heating effect
- ▶ inversion temperature : Where the shift in behaviour occurs (cooling to heating with lowering pressure).
- typically there are two inversion temperatures, one at high temperature and other at low :  $\mu < 0$  at some temperatures
  - lacktriangle and cooling effect  $\mu>0$  when T< upper inversion temperature

# Inversion temperature :



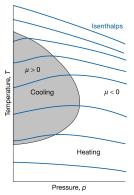
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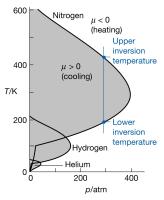




Inversion temperature corresponds to the boundary at a given pressure

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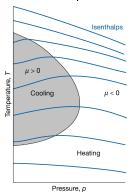


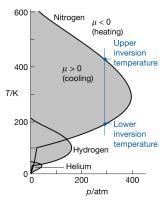
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For a given pressure, temperature must be below a certain value if cooling is required

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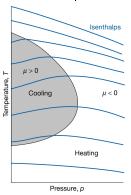


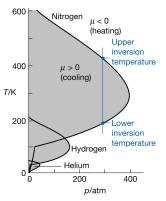
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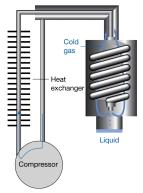


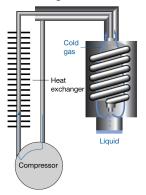
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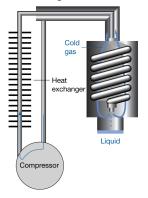
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inversion temperature curve runs through points of isenthalps where their slope changes from negative to positive



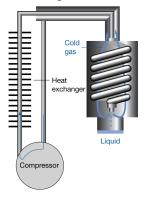


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There comes a stage when the circulating gas becomes so cold that it condenses to a liquid.

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- ightharpoonup The coefficient depends on derivatives and not on p, V and T themselves

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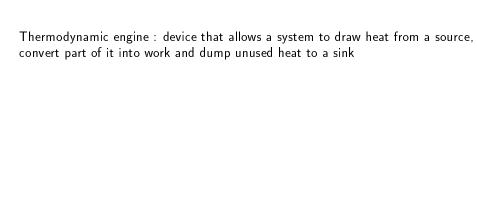
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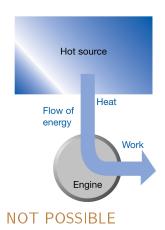
  Clausius inequality:  $\Delta S \geq \int \frac{dq}{T}$
- ightharpoonup Caratheodory axiom: In the neighbourhood of any arbitrary initial state of a physical system,  $P_0$ , there exist neighbouring states that are not accessible from  $P_0$  along reversible adiabatic paths



Thermodynamic engine: device that all	ows a system to draw heat from a source,
convert part of it into work and dump u	nused heat to a sink

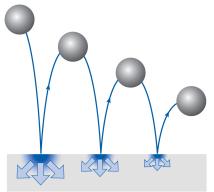
Second law: complete conversion of heat into work impossible

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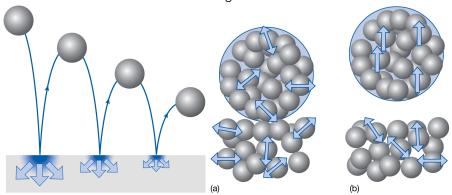


Second law: complete conversion of heat into work impossible

Heat and work: molecular understanding



Heat and work: molecular understanding

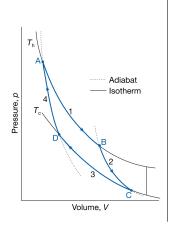


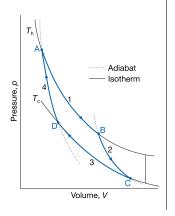
spontaneous change: direction of change leading to dispersal of total energy of isolated systems

Entropy : 
$$dS = \int rac{dq_{
m rev}}{T}$$
 and  $\Delta S = \int\limits_{i}^{f} rac{dq_{
m rev}}{T}$ 

isothermal expansion :  $\Delta S = \frac{1}{T} \int\limits_{\cdot}^{f} dq_{\rm rev} = \frac{q_{\rm rev}}{T} = \frac{-w_{\rm rev}}{T} = nR \ln \frac{V_f}{V_i}$ 

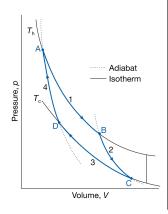
adiabatic process :  $\Delta S = 0$ 





1. Reversible isothermal expansion from A to B at  $T_h$ ;  $\Delta S = \frac{q_h}{T_L}$ ;

where  $q_h = \text{energy supplied from hot source} > 0$ 



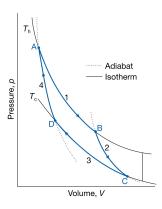
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- 2. Reversible adiabatic expansion from B to C  $\Delta S = 0$ ;  $\Delta T = -(T_h T_c)$
- 3. Reversible isothermal compression from C to D at  $T_c$ ;  $\Delta S = \frac{q_c}{T_c}$

Energy released as heat to the cold sink < 0

4. Reversible adiabatic compression from D to A  $\Delta S = 0$ ;  $\Delta T = T_h - T_c$ 



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4. Reversible adiabatic compression from D to A  $\Delta S = 0$ ;  $\Delta T = T_h - T_c$ 

Total  $\Delta S = \oint dS = \frac{q_h}{T_h} + \frac{q_c}{T_c}$