EC5.201: Signal Processing | End-Semester Exam | Monsoon 2024

Marks: 60 | Date: 22nd Nov 2024 | Time: 3 hours

Instructions:

a) There are *9 questions* for a total of *60 marks.*
b) Clearly show the steps used to arrive at the solutions.

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1. 10 Marks [1+2+3+4]

A causal FIR filter with impulse response $h[n] = \left\{\underset{\uparrow}{1}, 0, -1\right\}$ is to be implemented.

a. Find and plot the magnitude spectrum of this filter.
b. Show that the filter has linear phase and plot it. Hint: use trigonometric identities.
c. If the input to this filter is the signal $x[n] = 1 + \sin\left(\frac{\pi n}{6}\right) + \sin\left(\frac{\pi n}{2}\right) + \cos(\pi n)$, find the output of this filter using frequency domain analysis. Give exact analytical expression.
d. Due to a mistake, the filter $h_1[n] = h[n-1]$ gets implemented.
   i. Show that $h_1[n]$ is also a linear phase filter.
   ii. How are its magnitude and phase spectrum related to that of $h[n]$?
   iii. If the above signal $x[n]$ is given as input to this filter, what is the output?

2. 4 Marks [2+2]

A non-zero signal $x[n]$ is passed through an ideal low pass filter (cutoff frequency ω_1) to give output $y[n]$. The signal $y[n]$ is then passed through an ideal high pass filter (cutoff frequency ω_2) to give output $z[n]$. If it is observed that $z[n] = x[n]$, then answer the following:

a. What must be the relation between ω_1 & ω_2 for this to happen? Justify.
b. The Fourier transform of input signal $x[n]$ is necessarily zero in which frequency regions?

3. [6 Marks]

An FIR system with transfer function $H(z)$ is known to be causal, stable, and has linear phase characteristics. What can you say about the causality and stability of the following systems? If they are causal & stable, comment on their linear phase nature with reasons.

a. $\frac{1}{H(z)}$
b. $H(z^{-1})$
c. $H(-z)$
d. $-H(z)$

4. 10 Marks [3+3+4]

Shiva and Madhuri are designing digital IIR filters using different methods. Shiva is using the method of derivative approximation with $T = 1$, i.e., $s = 1 - z^{-1}$. Madhuri is using a modified version of the bilinear transformation given by $s = \frac{z}{z+1}$.

a. For both, find how does the imaginary axis, i.e., $s = j\omega$, maps in the z-plane.
b. For both, show that a stable and causal filter in continuous-time gives a stable and causal filter in the discrete-time.
c. If they both start with the same analog low pass filter with transfer function $H(s) = \frac{1}{s+2}$, find the corresponding digital filter, the pole-zero plots, and the nature of the filter (low pass, high pass, etc.) obtained by each of them.

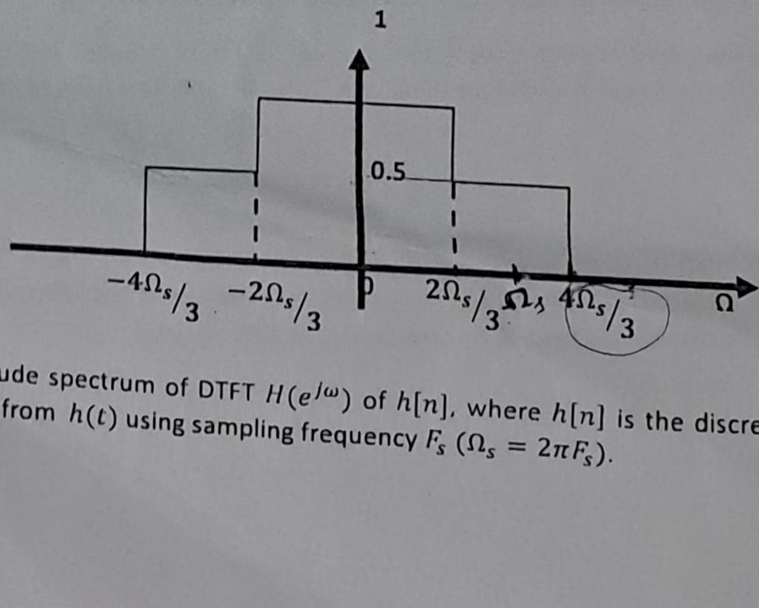
5. 10 Marks [3+2+3+2]

Let $x[n] = \{\underset{\uparrow}{1}, 1,1,1,1,1,1,1\}$, then do the following and show the necessary calculations.

a. Draw the magnitude spectrum ($|X(e^{j\omega})|$) of $x[n]$
b. Compute $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega$
c. Compute $y(n)$ whose DTFT is $X_R(e^{j\omega})e^{j2\omega}$, where $X_R(e^{j\omega})$ is the real part of the spectrum $X(e^{j\omega})$.
d. Compute the 8-point DFT of $x[n]$.

6. [3 Marks]

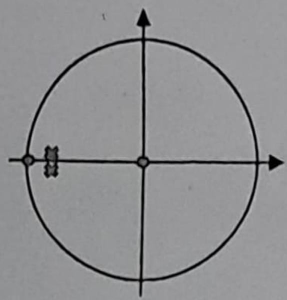
Let's consider $H(J\Omega)$ is a CTFT of $h(t)$ with a magnitude spectrum as shown below with zero phase spectrum then



a. Plot the amplitude spectrum of DTFT $H(e^{j\omega})$ of $h[n]$, where $h[n]$ is the discrete time signal obtained from $h(t)$ using sampling frequency F_s ($\Omega_s = 2\pi F_s$).

7. [5 Marks]

Let the pole-zero plot of a systems is shown below then compute and precisely plot the magnitude and phase spectrum of the system. Explain necessary calculations and assumptions.



8. [5 Marks]

A signal $x[n] = \{x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7]\}$ of length 8 i.e., there are 8 non-zero consecutive values and the remaining values are zero. For this signal, it is required to obtain following four DFT values $X[1], X[3], X[5], X[7]$ using FFT chip. Unfortunately, there is no 8-point FFT chip, only one 4-point FFT chip is available. Let's assume that the 4-point chip takes any arbitrary signal $y[n] = \{y[0], y[1], y[2], y[3]\}$ as the input in the normal sequence order, compute 4-point DFT and produce the output in normal sequence order $Y[k] = \{Y[0], Y[1], Y[2], Y[3]\}$.

a. Using this 4-point FFT chip, can the required DFT values $(X[1], X[3], X[5], X[7])$ computed for any $x[n]$? if yes derive the necessary equations and draw the complete connections else argue the reasons by deriving the necessary equations.

9. [7 Marks]

Let $x_1[n] = x_2[n] = \{1,1,1\}$ then compute $y[n]$. Please note that the 5-point FFT chip takes any arbitrary signal as the input in the normal sequence order, compute a 5-point DFT and produce the output in normal sequence order.