Introduction to Coding Theory - Spring 2025
Tutorial 2

1. Let $\mathcal{C}$ be an $[n, k]$ linear code over $\mathbb{F}_2$, with no all zero columns in generator matrix. Prove that $\forall i \in \{0, 1, \ldots, n-1\}$ and $\alpha \in \mathbb{F}_2$, the number of codewords $\mathbf{c}$ such that $c_i = \alpha$ is 2^{k-1}

2. Consider the following matrices over $\mathbb{F}_2$

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{G}' = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

   Show that these two matrices are the generators of the same code.

3. Consider a code and the corresponding mapping:

	Message	**Codeword**		**Message**	**Codeword**
1	0000	00000000	9	0001	01010101
2	1000	11111111	10	1001	10101010
3	0100	00001111	11	0101	01011010
4	1100	11110000	12	1101	10100101
5	0010	00110011	13	0011	01100110
6	1010	11001100	14	1011	10010110
7	0110	00111100	15	0111	01101001
8	1110	11000011	16	1111	10011001

Table 1: Message and Codeword Mapping

   Prove that the code is linear. Can you make a generator matrix for the code that does this mapping?

4. Let $\mathcal{C}_1$ be an $[n, k_1, d_1]$ binary linear code and $\mathcal{C}_2$ an $[n, k_2, d_2]$ binary linear code. Let $\mathcal{C} = \{(\mathbf{u}|\mathbf{v}) : \mathbf{u} \in \mathcal{C}_1, \mathbf{v} \in \mathcal{C}_2\}$. Find the dimension and minimum distance of $\mathcal{C}$.

5. Let $\mathcal{C}_1$ be an $[n, k_1, d_1]$ binary linear code and $\mathcal{C}_2$ an $[n, k_2, d_2]$ binary linear code. Let $\mathcal{C} = \{(\mathbf{u}|\mathbf{u} + \mathbf{v}) : \mathbf{u} \in \mathcal{C}_1, \mathbf{v} \in \mathcal{C}_2\}$. Find the dimension and minimum distance of $\mathcal{C}$.