

Instructions:

- There are 6 questions for a total of 40 marks.
- Mention any additional assumptions you make that is not given in the question.
- Clearly show the steps used to arrive at the solutions.

1. Let the $x_1[n] = \cos(\frac{\pi}{9}n)$, $x_2[n] = \cos(\frac{2\pi}{3}n)$ and $x[n] = x_1[n] + x_2[n]$ then [2+3+3+2]

- Compute DTFT of $x[n]$.
- A design engineer would like to extract $x_1[n]$ from $x[n]$ using a discrete LTI system, so an impulse response $h[n]$ is chosen with spectrum $H(\omega) = 1 \forall 0 \leq \omega < \frac{\pi}{3}$ and $0 \forall \frac{\pi}{3} \leq \omega < 2\pi$. Compute the impulse response $h[n]$.
- The same engineer mistakenly considers only real part of the $h[n]$ i.e., $h_R[n]$ in the discrete LTI system and obtain output $y[n]$. Is $y[n]$ is equal to $x_1[n]$? Justify your answer.
- Also, the engineer sends $h_{AM}[n] = e^{j\frac{\pi}{2}n} h_R[n]$, which is a modulated impulse response of the $h_R[n]$ with carrier frequency of $\frac{\pi}{2}$, via a communication channel. At the receiving end, another engineer computes the output $y[n]$ of the discrete LTI system considering input as $x[n]$ and impulse response as $h_{AM}[n]$. Is $y[n]$ is equal to $x_1[n]$? If yes justify your answer else find $y[n]$

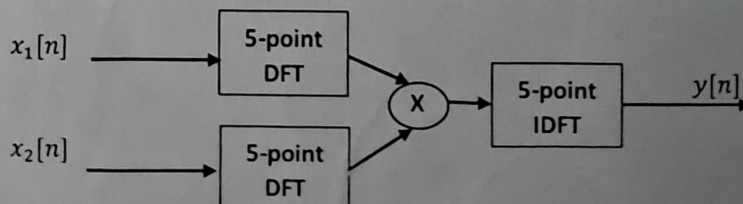
2. Let $X[k]$ and $H_1[k]$ are the sampled spectrums of $X(\omega)$ and $H_1(\omega)$ with N_x and N_h points respectively, where $X(\omega)$ and $H_1(\omega)$ are the DTFT of $x[n]$ and $h_R[n]$ defined in problem 1. Then [1+1+2+2]

- Compute the minimum value of N_x that can preserve the spectrums of both $x_1[n]$ and $x_2[n]$?
- $H_2[k]$ is N_h -point DFT of $h_R[n]$, then, is $H_2[k]$ equal to $H_1[k]$, Justify your answer?
- Is IDFT of $H_1[k]$ equal to $h_R[n]$, justify your answer.
- Compute the signal whose DFT is $X[k - N_x]$.

3. Answer the below questions

[4+2+2]

- Let $X(\omega)$ and $X_1(\omega)$ are respective DTFTs of $x[n]$ and $x_1[n]$, where $x_1[n] = x[2n]$, then, express $X_1(\omega)$ in terms of $X(\omega)$. Also, verify the result by computing $X(\omega)$ and $X_1(\omega)$ for the case $x[n] = u[n] - u[n - k]$
- Let $x_1[n] = x_2[n] = \{1, 1, 1\}$ then compute $y[n]$.



- Let $x_3[n] = \{3, 2, 1, 0, 1, 2\}$ then clearly deduce the relation between $x_3[n]$ and $y[n]$ in 3.b.

4. [4+1+3]

It is known that the impulse response of an LTI system (S_1) is of the form $h[n] = \alpha\delta[n] + \beta\delta[n-1]$, where α and β are unknown constants. When the input to this system is the step signal $u[n]$, the output is the impulse signal $\delta[n]$.

- Find the impulse response of the system S_1 , i.e., find α and β , using Z-transform analysis.
- If the input to this system is any constant signal, what is the output?
- Find the impulse response of the non-causal system (S_2) which acts as inverse of the system S_1 . Verify using convolution that the systems S_1 and S_2 are indeed inverse systems.

5. [2+3]

A second order LTI system is described using the difference equation

$$y[n] = \alpha y[n-1] + \alpha^2 y[n-2] + x[n],$$

where $\alpha \in \mathbb{R}$ is an unknown real constant.

- For what values of α is the above system not stable?
- For what range of values of α is the above system causal and stable?

6. [3]

A first order LTI system is described using the difference equation

$$y[n] = \alpha y[n-1] + x[n],$$

where $\alpha \in \mathbb{R}$ is an unknown real constant. We know that all complex sinusoid signals of the form $e^{j\omega n}$ are eigensignals for any LTI system. Find the range of values of α such that all complex sinusoid signals (irrespective of the frequency ω), when given as input to this system, undergo reduction in amplitude at the output.