Communication Theory
Spring-2025

Assignment 1 **Deadline: February 9, 11:59 PM**

Instructions:

- All questions are compulsory.
- Clearly state the assumptions *(if any)* made that are not specified in the questions.
- Submission format: `Rollnumber.pdf`

Cautions:

(a) One late homework assignment is allowed without penalty.

(b) 2 marks will be deducted on other late assignments.

Questions

1. Consider the following two passband signals:

$$u_p(t) = \text{sinc}(2t)\cos(100\pi t)$$

and

$$v_p(t) = \text{sinc}(t)\sin\left(101\pi t + \frac{\pi}{4}\right)$$

   (a) Find the complex envelopes $u(t)$ and $v(t)$ for $u_p(t)$ and $v_p(t)$, respectively, with respect to the frequency reference $f_c = 50$ Hz.

   (b) What is the bandwidth of $u_p(t)$? What is the bandwidth of $v_p(t)$?

   (c) Find the inner product $\langle u_p, v_p \rangle$ using the result in (a).

   (d) Find the convolution $y_p(t) = (u_p * v_p)(t)$ using the result in (a).

2. Find the Fourier transforms of the signals shown in the below figure :

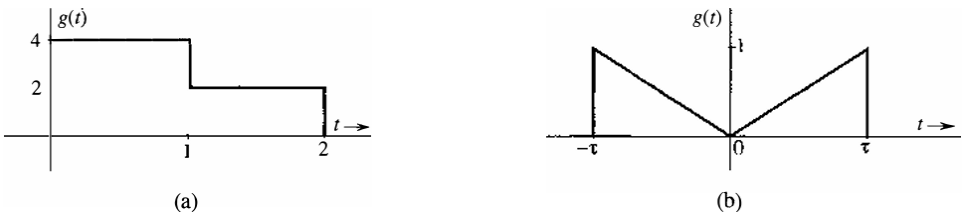

Figure 1

3. (a) Determine the Fourier transform of each of the following signals:

(i) $\text{sinc}^3(t)$

(ii) $te^{-|\alpha|t}\cos(\beta t)$

(b) Using the properties of the Fourier transform, evaluate the following integrals:

(i) $\int_0^\infty e^{-\alpha t}\,\text{sinc}^2(t)\,dt$

(ii) $\int_0^\infty e^{-\alpha t}\cos(\beta t)\,dt$

4. Consider the signal:

$$s(t) = \mathbb{I}_{[-1,1]}(t)\cos(400\pi t).$$

(a) Find and sketch the baseband signal $u(t)$ that results when $s(t)$ is downconverted as shown in the upper branch of Figure 2.

(b) The signal $s(t)$ is passed through a bandpass filter with impulse response:

$$h(t) = \mathbb{I}_{[0,1]}(t)\sin\left(400\pi t + \frac{\pi}{4}\right).$$

Find and sketch the baseband signal $v(t)$ that results when the filter output $y(t) = (s * h)(t)$ is downconverted as shown in the lower branch of Figure 2.

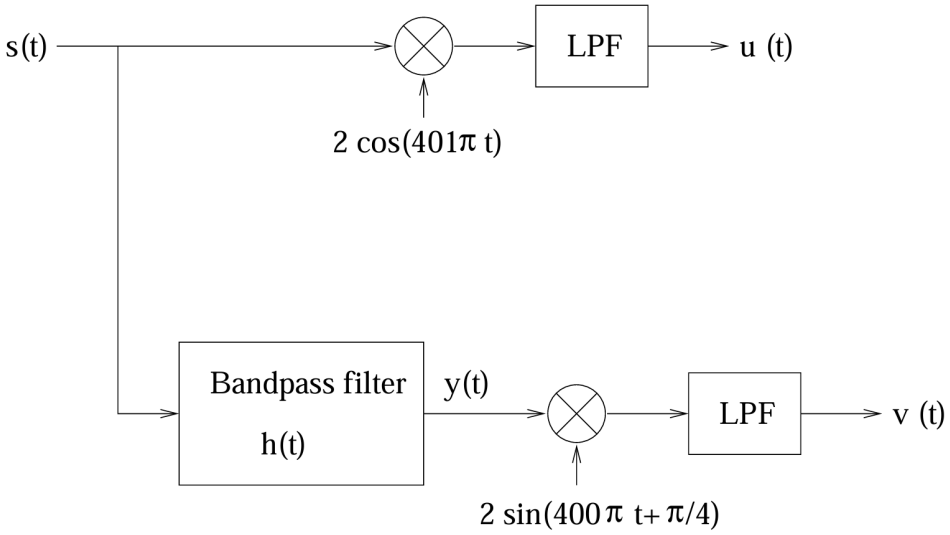

Figure 2

5. Find the inverse Fourier transforms of the spectra shown in the figure:

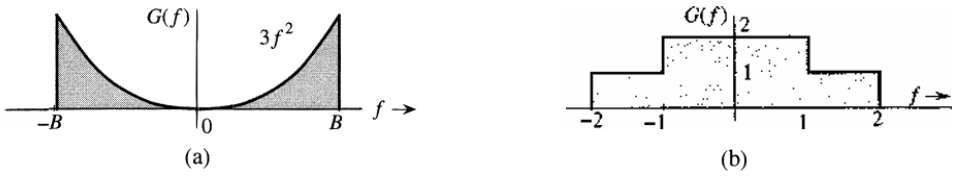

Figure 3

6. Consider the tent signal:

$$s(t) = (1 - |t|)\mathbb{I}_{[-1,1]}(t)$$

where $\mathbb{I}_{[-1,1]}(t)$ is the indicator function for $t \in [-1, 1]$.

(a) Find and sketch the Fourier transform $S(f)$.

(b) Compute the 99% energy containment bandwidth in kHz, assuming that the unit of time is milliseconds.