Information and Communication (EC4.102)

**Assignment 4**

Spring-24

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1. Consider the linear code defined by the generator matrix $G = \big(I_k|\mathbf{1}\big)$, where $\mathbf{1}$ denotes an all-one column vector of length $k$.
   (a) What is the dimension and rate of this code ?
   (b) Describe the set of all codewords of this code. Obtain the codewords corresponding to the message vector $(1, 0, 1, 0, \ldots, 0)$ of length $k$. (Note: The ellipsis '...' here denotes 0 sequence, as it is sandwiched between two 0s).
   (c) What are the set of parity check equations for this code? Obtain a parity check matrix of this code.
   (d) How many erasures can this code correct surely? How many bit-flip errors can this code correct surely? Analyze and obtain the answer. (Note: Your argument has to be valid for any choice of the message vector.)
   (e) Demonstrate one erasure or error pattern which this code cannot correct.

2. Consider a linear code defined by the parity check matrix $H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$.
   (a) What is the rate of this code?
   (b) How many maximum bit-flip errors can this code surely correct? How many erasures can this code correct?
   (c) Find a generator matrix for this code.
   (d) Estimate the codeword transmitted if the received vector on a BEC was $(1, 1, e, e, e, 1)$ (where $e$ stands for the erasure symbol). Estimate the transmitted codeword if the received vector on a BSC was $(1, 0, 1, 1, 0, 1, 0)$.

3. Calculate the probability of error when using the code generated by the matrix $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ on $BEC(0.2)$.

4. Calculate the capacity of the Binary-Input Channel with output alphabet $\mathcal{Y} = \{1, 2, 3, 4\}$, with transition probabilities $p_{Y|X}$ as follows.
   (a) $\begin{bmatrix} 0.3 & 0.7 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \end{bmatrix}$.
   (b) $\begin{bmatrix} 0.3 & 0.6 & 0.1 & 0 \\ 0 & 0.1 & 0.6 & 0.3 \end{bmatrix}$.