Communication Theory

Spring-2025

Assignment 4 **Deadline: 11:59 pm 30 March**

Instructions:

- All questions are compulsory.
- Clearly state the assumptions *(if any)* made that are not specified in the questions.
- Submission format: `Rollnumber.pdf`

Cautions:

- One late homework assignment is allowed without penalty.
- 2 marks will be deducted on other late assignments.

Theory:

1. Consider linear modulation with a signaling pulse $p(t) = \text{sinc}(at)\text{sinc}(bt)$, where a and b are to be determined.

   (a) How should a and b be chosen so that $p(t)$ is Nyquist with 50% excess bandwidth for a data rate of 40 Mbps using 16QAM? Specify the occupied bandwidth.

   (b) How should a and b be chosen so that $p(t)$ can be used for Nyquist signaling both for a 16QAM system with a 40 Mbps data rate, and for an 8PSK system with an 18 Mbps data rate? Specify the occupied bandwidth.

2. Let $R(f) = I_{[-\frac{1}{2},\frac{1}{2}]}(f)$ denote an ideal boxcar transfer function, and let

$$C(f) = \frac{\pi}{2a}\cos\left(\frac{\pi}{a}f\right) I_{[-\frac{a}{2},\frac{a}{2}]}(f)$$

   denote a cosine transfer function.

   (a) Sketch $R(f)$ and $C(f)$, for $0 < a < 1$.

   (b) Show that the frequency domain raised cosine pulse can be written as:

$$S(f) = (R * C)(f)$$

   (Note: Where '$*$' means the convolution operator).

   (c) Find the time domain pulse $s(t) = r(t)c(t)$. Identify where the zeros of $s(t)$ are. Conclude that $s(t/T)$ is a Nyquist pulse with duration T.

   (d) Sketch an argument that shows that if the pulse $s(t/T)$ is used for BPSK signaling at rate $1/T$, then the magnitude of the transmitted waveform is always finite.

3. A 16QAM system transmits at 50 Mbps using an excess bandwidth of 50%. The transmit power is 100 mW.

   (a) Assuming that the carrier frequency is 5.2 GHz, specify the frequency interval occupied by the passband modulated signal.

   (b) Using the same frequency band in (a), how fast could you signal using QPSK with the same excess bandwidth?

   (c) Estimate the transmit power needed in the QPSK system, assuming the same range and reliability requirements as in the 16QAM system.

4. Consider the pulse:

$$p(t) = \begin{cases} \frac{t}{a}, & 0 \leq t \leq a \\ 1, & a \leq t \leq 1-a \\ \frac{1-t}{a}, & 1-a \leq t \leq 1 \\ 0, & \text{else} \end{cases} \tag{1}$$

   where $0 \leq a \leq \frac{1}{2}$.

   (a) Sketch $p(t)$ and find its Fourier transform $P(f)$. (b) Consider the linearly modulated signal:

$$u(t) = \sum_n b[n]p(t-n), \tag{2}$$

   where $b[n]$ take values independently and with equal probability in a 4-PAM alphabet $\{\pm 1, \pm 3\}$. Find an expression for the PSD of u as a function of the pulse shape parameter a.

   (c) Numerically estimate the 95% fractional power containment bandwidth for u and plot it as a function of $0 \leq a \leq \frac{1}{2}$. For concreteness, assume the unit of time is 100 picoseconds and specify the units of bandwidth in your plot.

MATLAB:

1. Write the following functions mapping bits to symbols for different signal constellations. These functions should allow for vector inputs and outputs. The mapping is said to be a **Gray code** or **Gray labelling**, if the bit map for the nearest neighbors in the constellation differs by exactly one bit. In all of the following, choose the bit map to be a Gray code.

   (a) **bpskmap**: Input a 0/1 bit, output a ± 1 symbol.

   (b) **qpskmap**: Input two 0/1 bits, output a symbol taking one of four values in $\pm 1 \pm j$.

   (c) **fourpammap**: Input two 0/1 bits, output a symbol taking one of four values in $\{\pm 1, \pm 3\}$.

   (d) **sixteenqammap**: Input four 0/1 bits, output a symbol taking one of 16 values in:

$$\{b_c + jb_s : b_c, b_s \in \{\pm 1, \pm 3\}\}.$$

   (e) **eightpskmap**: Input three 0/1 bits, output a symbol taking one of 8 values in:

$$e^{j2\pi i/8}, \quad i = 0, 1, \ldots, 7.$$

2. (a) Random Bit Generation Write a MATLAB function `randbit` that generates random bits (0 or 1) with equal probability.

   (b) BPSK Mapping Write a function `bpskmap` that performs BPSK mapping as follows:

   - Input: A vector of bits (0s and 1s)
   - Output: A vector where 0 is mapped to +1 and 1 is mapped to -1.

   Ensure that the function supports vectorized operations.

   (c) BPSK Symbol Generation

   i. Use `randbit` to generate **12,000 random bits**.

   ii. Map these bits to BPSK symbols using `bpskmap`.

   (d) Adding AWGN (Noise) We introduce additive white Gaussian noise (AWGN):

   - Add independent and identically distributed (i.i.d.) complex Gaussian noise.
   - The noise variance per dimension is given by:

$$\sigma^2 = \frac{N_0}{2} \tag{3}$$

   - The noise variance σ^2 is adjusted for a given E_b/N_0.
   - Add the noise to the BPSK symbols.

   (e) Ideal Bit Error Probability Plot The theoretical bit error probability for BPSK is given by:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{4}$$

   where $Q(x)$ is the tail probability of the standard normal distribution.

   - Plot P_b on a **log scale** as a function of E_b/N_0 (in dB) over the range **0–30 dB**.
   - Determine the E_b/N_0 values that correspond to error probabilities of 10^{-2} and 10^{-5}.
   - Provide interpretations of the observed values.