TRANSMISSION LINES

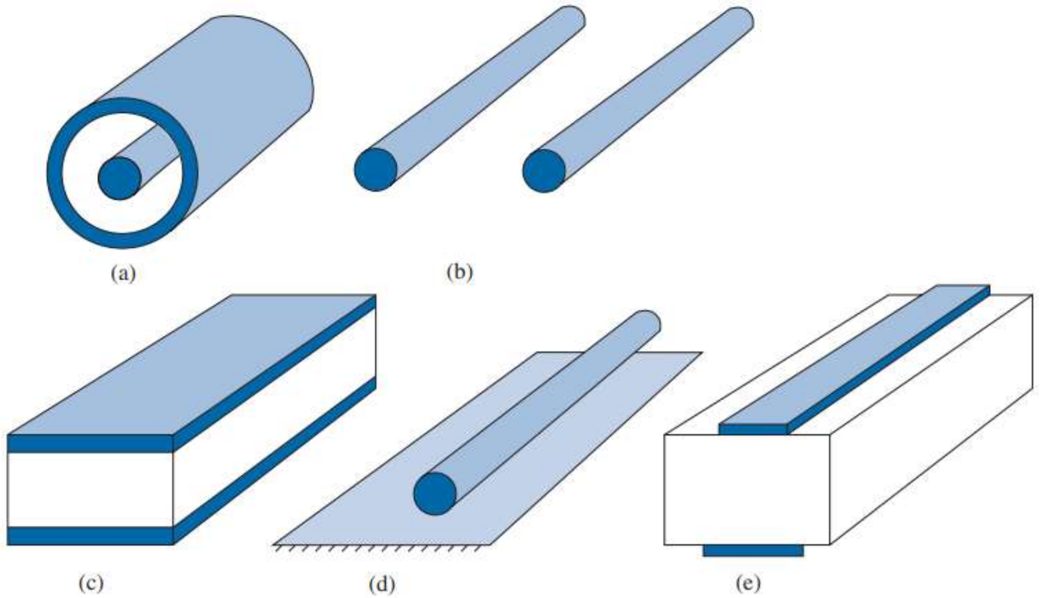


FIGURE 11.1 Typical transmission lines in cross-sectional view: **(a)** coaxial line, **(b)** two-wire line, **(c)** planar line, **(d)** wire above conducting plane, **(e)** microstrip line.

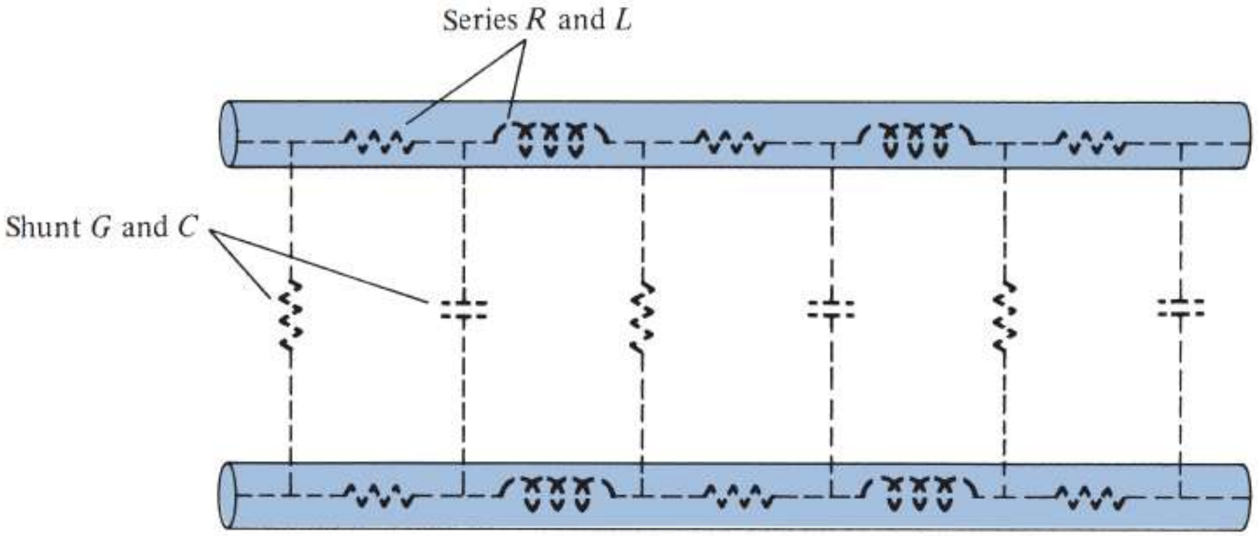


FIGURE 11.3 Distributed parameters of a two-conductor transmission line.

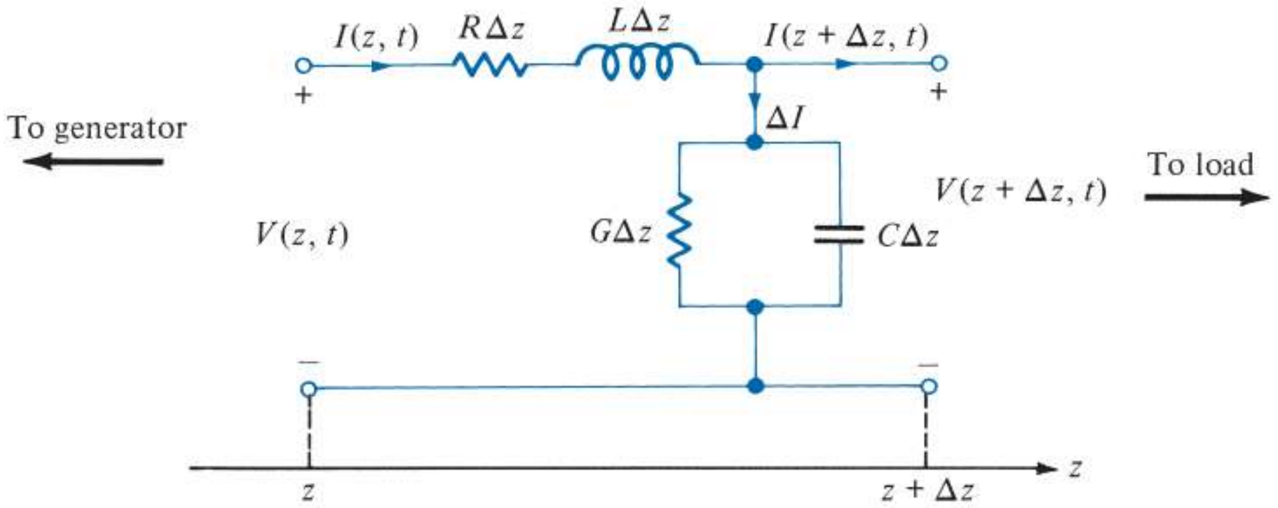


FIGURE 11.5 An *L*-type equivalent circuit model of a two-conductor transmission line of differential length Δz.

$$\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0$$

$$V_s(z) = \underset{\longrightarrow +z}{V_o^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{V_o^- e^{\gamma z}} \tag{11.15}$$

and

$$I_s(z) = \underset{\longrightarrow +z}{I_o^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{I_o^- e^{\gamma z}} \tag{11.16}$$

where V_o^+, V_o^-, I_o^+, and I_o^- are wave amplitudes; the $+$ and $-$ signs, respectively, denote waves traveling along $+z$- and $-z$-directions, as is also indicated by the arrows. We obtain the instantaneous expression for voltage as

$$\begin{aligned} V(z, t) &= \mathrm{Re}[V_s(z)\, e^{j\omega t}] \\ &= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z) \end{aligned} \tag{11.17}$$

The **characteristic impedance** Z_o of the line is the ratio of the positively traveling voltage wave to the current wave at any point on the line.

The characteristic impedance Z_o is analogous to η, the intrinsic impedance of the medium of wave propagation. By substituting eqs. (11.15) and (11.16) into eqs. (11.8) and (11.9) and equating coefficients of terms $e^{\gamma z}$ and $e^{-\gamma z}$, we obtain

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} \tag{11.18}$$

or

$$\boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o} \tag{11.19}$$

where R_o and X_o are the real and imaginary parts of Z_o. Do not mistake R_o for R—while R is in ohms per meter, R_o is in ohms. The propagation constant γ and the characteristic impedance Z_o are important properties of the line because both depend on the line parameters R, L, G, and C and the frequency of operation. The reciprocal of Z_o is the characteristic admittance Y_o, that is, $Y_o = 1/Z_o$.

1. The phase velocity is independent of frequency because the phase constant β linearly depends on frequency. We have shape distortion of signals unless α and u are independent of frequency.

TABLE 11.2 Transmission Line Characteristics

Case	Propagation Constant $\gamma = \alpha + j\beta$	Characteristic Impedance $Z_o = R_o + jX_o$
General	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\dfrac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\dfrac{L}{C}} + j0$

2. Both u and Z_o remain the same as for lossless lines.
3. A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

A summary of our discussion in this section is in Table 11.2. For the greater part of our analysis, we shall restrict our discussion to lossless transmission lines.

An air line has a characteristic impedance of 70 Ω and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

A transmission line operating at 500 MHz has $Z_o = 80\ \Omega$, $\alpha = 0.04$ Np/m, $\beta = 1.5$ rad/m. Find the line parameters R, L, G, and C.

A distortionless line has $Z_o = 60\ \Omega$, $\alpha = 20$ mNp/m, $u = 0.6c$, where c is the speed of light in a vacuum. Find R, L, G, C, and λ at 100 MHz.

A telephone line has $R = 30\ \Omega$/km, $L = 100$ mH/km, $G = 0$, and $C = 20\ \mu$F/km. At $f = 1$ kHz, obtain:

(a) The characteristic impedance of the line

(b) The propagation constant

(c) The phase velocity

INPUT IMPEDANCE, STANDING WAVE RATIO, AND POWER

Let the transmission line extend from $z = 0$ at the generator to $z = \ell$ at the load. First of all, we need the voltage and current waves in eqs. (11.15) and (11.16), that is,

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \tag{11.24}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \tag{11.25}$$

where eq. (11.18) has been incorporated. To find V_o^+ and V_o^-, the terminal conditions must be given. For example, if we are given the conditions at the input, say

$$V_o = V(z = 0), \quad I_o = I(z = 0) \tag{11.26}$$

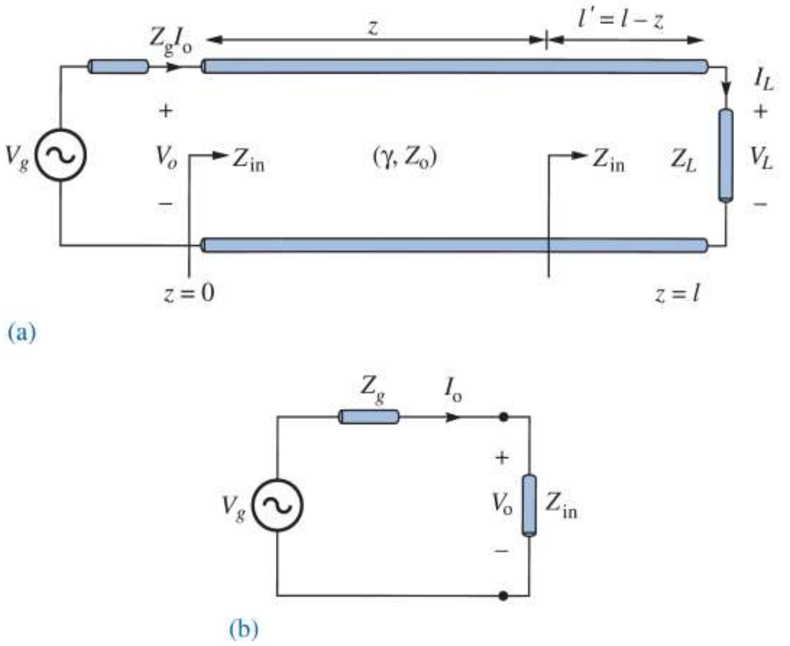


FIGURE 11.6 **(a)** Input impedance due to a line terminated by a load. **(b)** Equivalent circuit for finding V_o and I_o in terms of Z_{in} at the input.

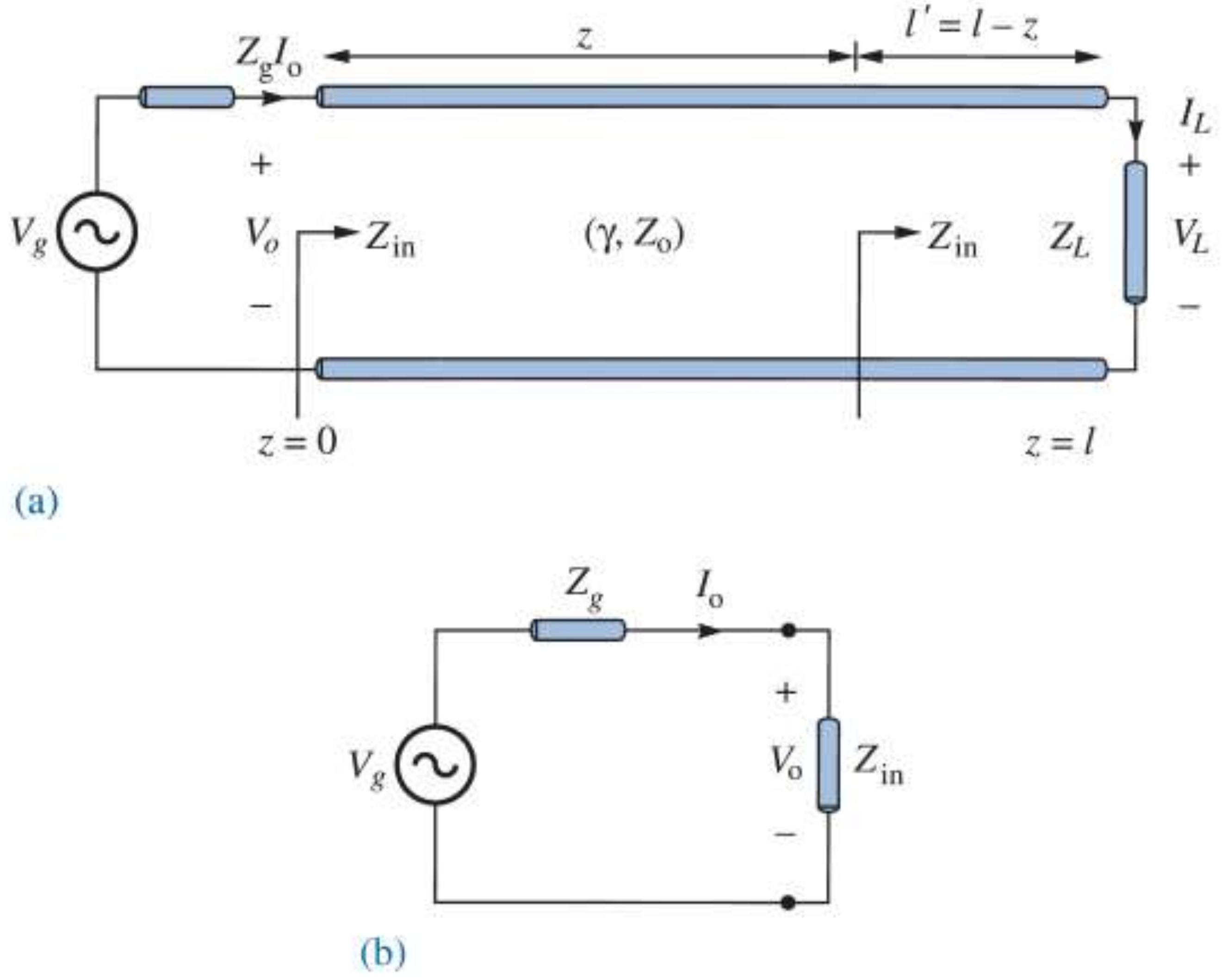


FIGURE 11.6 **(a)** Input impedance due to a line terminated by a load. **(b)** Equivalent circuit for finding V_o and I_o in terms of Z_{in} at the input.

$$\boxed{Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell}\right]} \quad \text{(lossy)}$$

$$\boxed{Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell}\right]} \quad \text{(lossless)}$$

The **voltage reflection coefficient** at any point on the line is the ratio of the reflected voltage wave to that of the incident wave.

That is,

$$\Gamma(z) = \frac{V_o^- e^{\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{2\gamma z}$$

But $z = \ell - \ell'$. Substituting and combining with eq. (11.35), we get

$$\Gamma(z) = \frac{V_o^-}{V_o^+} e^{2\gamma \ell} e^{-2\gamma \ell'} = \Gamma_L e^{-2\gamma \ell'} \tag{11.37}$$

The **current reflection coefficient** at any point on the line is the negative of the voltage reflection coefficient at that point.

Thus, the current reflection coefficient at the load is $I_o^- e^{\gamma \ell} / I_o^+ e^{-\gamma \ell} = -\Gamma_L$.

Just as we did for plane waves, we define the *standing wave ratio s* (otherwise denoted by SWR) as

$$\boxed{s = \frac{V_{\max}}{V_{\min}} = \frac{I_{\max}}{I_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}} \tag{11.38a}$$

$$\boxed{|\Gamma_L| = \frac{S-1}{S+1}} \tag{11.38b}$$

It is easy to show that $I_{\max} = V_{\max}/Z_o$ and $I_{\min} = V_{\min}/Z_o$. The input impedance Z_{in} in eq. (11.34) has maxima and minima that occur, respectively, at the maxima and minima of the voltage standing wave. It can also be shown that

$$\boxed{|Z_{in}|_{\max} = \frac{V_{\max}}{I_{\min}} = sZ_o} \tag{11.39a}$$

and

$$\boxed{|Z_{in}|_{\min} = \frac{V_{\min}}{I_{\max}} = \frac{Z_o}{s}} \tag{11.39b}$$

As a way of demonstrating these concepts, consider a lossless line with characteristic impedance of $Z_o = 50\ \Omega$. For the sake of simplicity, we assume that the line is terminated in a pure resistive load $Z_L = 100\ \Omega$ and the voltage at the load is 100 V (rms). The conditions on the line are displayed in Figure 11.7. Note from Figure 11.7 that conditions on the line repeat themselves every half-wavelength.

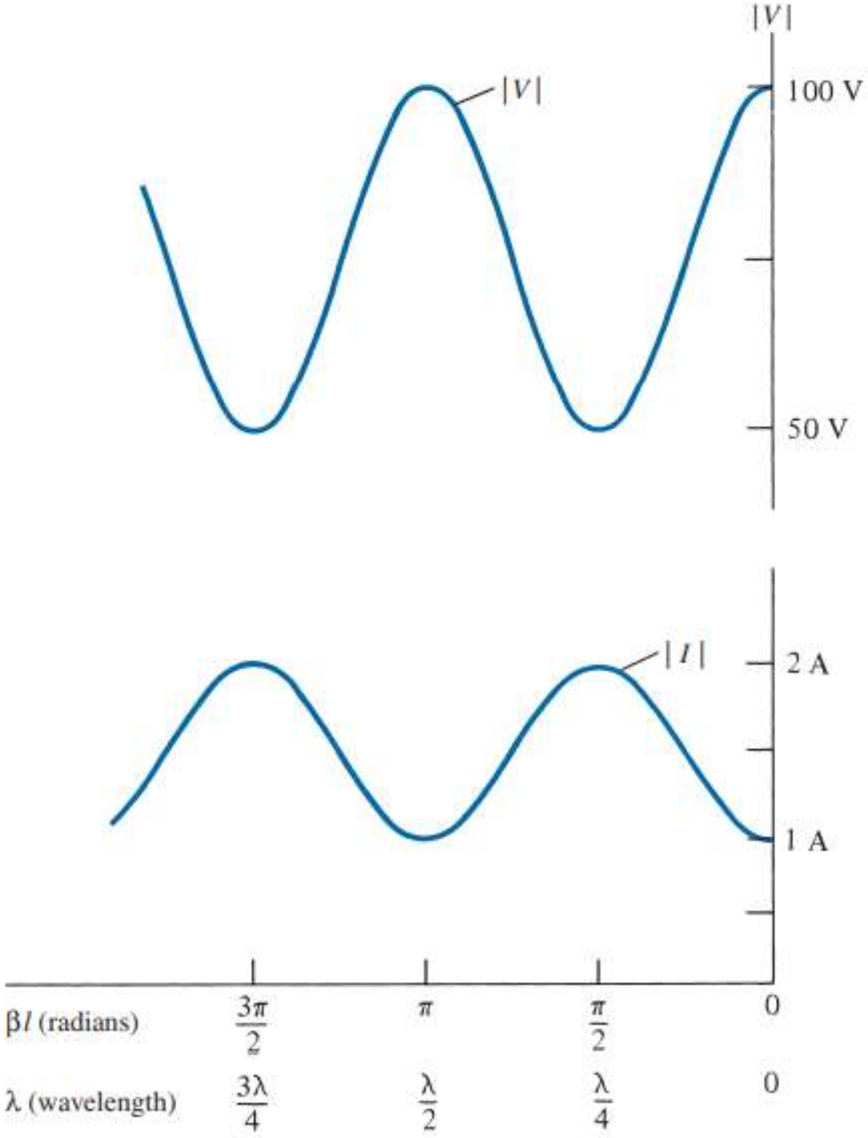


FIGURE 11.7 Voltage and current standing wave patterns on a lossless line terminated by a resistive load.

We now consider special cases when the line is connected to load $Z_L = 0$, $Z_L = \infty$, and $Z_L = Z_o$. These special cases can easily be derived from the general case.

A. Shorted Line ($Z_L = 0$)

For this case, eq. (11.34) becomes

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = jZ_o \tan \beta\ell \tag{11.41a}$$

Also, from eqs. (11.36) and (11.38)

$$\Gamma_L = -1, \quad s = \infty \tag{11.41b}$$

We notice from eq. (11.41a) that Z_{in} is a pure reactance, which could be capacitive or inductive depending on the value of ℓ. The variation of Z_{in} with ℓ is shown in Figure 11.8(a).

B. Open-Circuited Line ($Z_L = \infty$)

In this case, eq. (11.34) becomes

$$Z_{oc} = \lim_{Z_L\to\infty} Z_{in} = \frac{Z_o}{j \tan \beta\ell} = -jZ_o \cot \beta\ell \tag{11.42a}$$

and from eqs. (11.36) and (11.38),

$$\Gamma_L = 1, \quad s = \infty \tag{11.42b}$$

The variation of Z_{in} with ℓ is shown in Figure 11.8(b). Notice from eqs. (11.41a) and (11.42a) that

$$Z_{sc}Z_{oc} = Z_o^2 \tag{11.43}$$

C. Matched Line ($Z_L = Z_o$)

The most desired case from the practical point of view is the matched line i.e., $Z_L = Z_o$. For this case, eq. (11.34) reduces to

$$Z_{in} = Z_o \qquad (11.44a)$$

and from eqs. (11.36) and (11.38),

$$\Gamma_L = 0, \quad s = 1 \qquad (11.44b)$$

that is, $V_o^- = 0$; the whole wave is transmitted, and there is no reflection. The incident power is fully absorbed by the load. Thus maximum power transfer is possible when a transmission line is matched to the load.

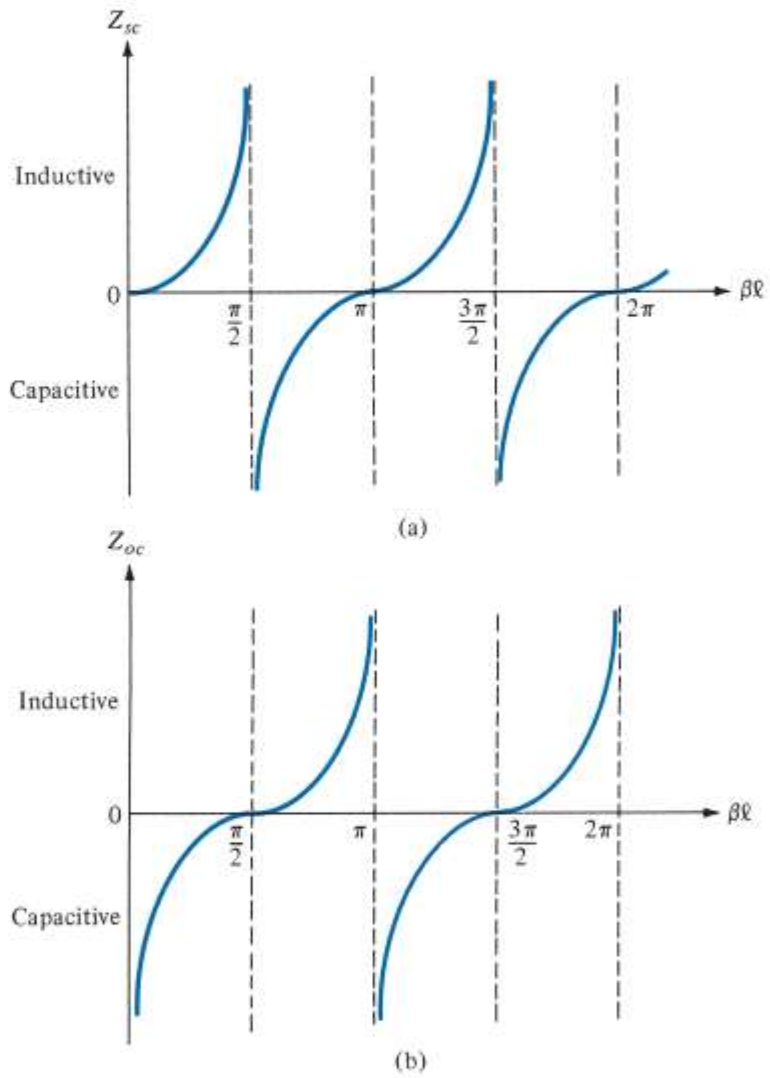


FIGURE 11.8 Input impedance of a lossless line: **(a)** when shorted, **(b)** when open.

A certain transmission line 2 m long operating at $\omega = 10^6$ rad/s has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_o = 60 + j40\ \Omega$. If the line is connected to a source of $10\angle 0°$ V, $Z_g = 40\ \Omega$ and terminated by a load of $20 + j50\ \Omega$, determine

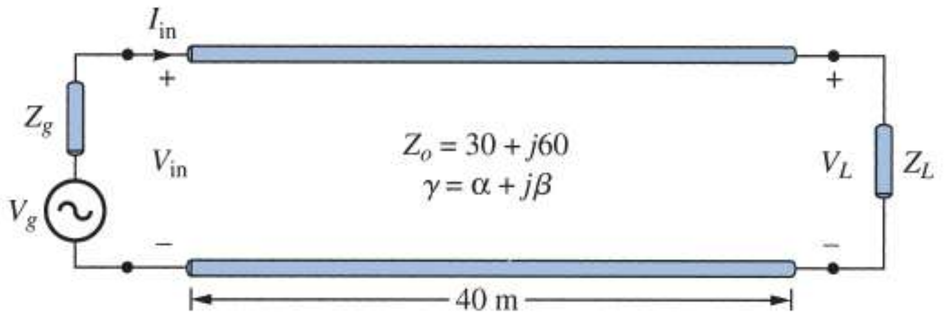
(a) The input impedance
(b) The sending-end current
(c) The current at the middle of the line

PRACTICE EXERCISE 11.3

The transmission line shown in Figure 11.9 is 40 m long and has $V_g = 15\angle 0°\ V_{rms}$, $Z_o = 30 + j60\ \Omega$, and $V_L = 5\angle -48°\ V_{rms}$. If the line is matched to the load and $Z_g = 0$, calculate:

(a) The input impedance Z_{in}
(b) The sending-end current I_{in} and voltage V_{in}
(c) The propagation constant γ

Answer: (a) $30 + j60\ \Omega$, (b) $0.2236\angle -63.43°$ A, $7.5\angle 0°\ V_{rms}$, (c) $0.0101 + j0.02094$ /m.



A lossless transmission line with $Z_o = 50\ \Omega$ is 30 m long and operates at 2 MHz. The line is terminated with a load $Z_L = 60 + j40\ \Omega$. If $u = 0.6c$ on the line, find

(a) The reflection coefficient Γ

(b) The standing wave ratio s

(c) The input impedance

Transmission lines are used to serve different purposes. Here we consider how transmission lines are used for load matching and impedance measurements.

A. Quarter-Wave Transformer (Matching)

When $Z_o \neq Z_L$, we say that the load is *mismatched* and a reflected wave exists on the line. However, for maximum power transfer, it is desired that the load be matched to the transmission line $(Z_o = Z_L)$ so that there is no reflection $(|\Gamma| = 0$ or $s = 1)$. The matching is achieved by using shorted sections of transmission lines.

We recall from eq. (11.34) that when $\ell = \lambda/4$ or $\beta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2$,

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan \pi/2}{Z_o + jZ_L \tan \pi/2}\right] = \frac{Z_o^2}{Z_L} \tag{11.56}$$

that is,

$$\frac{Z_{in}}{Z_o} = \frac{Z_o}{Z_L}$$

or

$$z_{in} = \frac{1}{z_L} \rightarrow y_{in} = z_L \tag{11.57}$$

Thus by adding a $\lambda/4$ line on the Smith chart, we obtain the input admittance corresponding to a given load impedance.

Also, a mismatched load Z_L can be properly matched to a line (with characteristic impedance Z_o) by inserting prior to the load a transmission line $\lambda/4$ long (with characteristic impedance Z_o') as shown in Figure 11.17. The $\lambda/4$ section of the transmission line is called a *quarter-wave transformer* because it is used for impedance matching like an ordinary transformer. From eq. (11.56), Z_o' is selected such that $(Z_{in} = Z_o)$

$$Z_o' = \sqrt{Z_o Z_L} \tag{11.58}$$

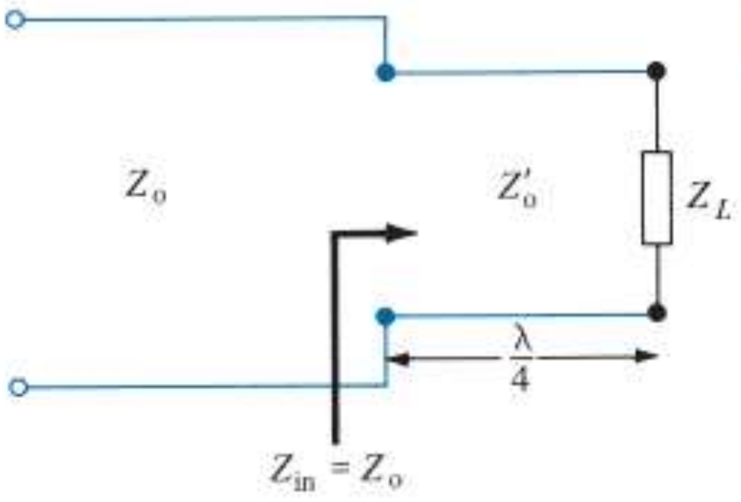
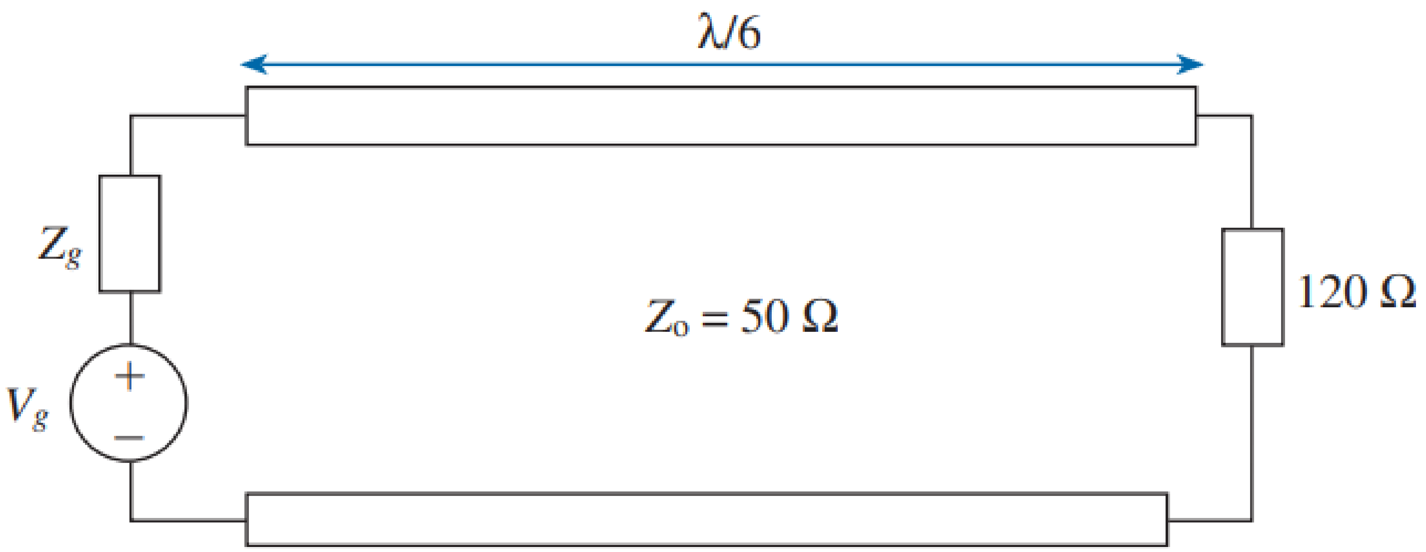
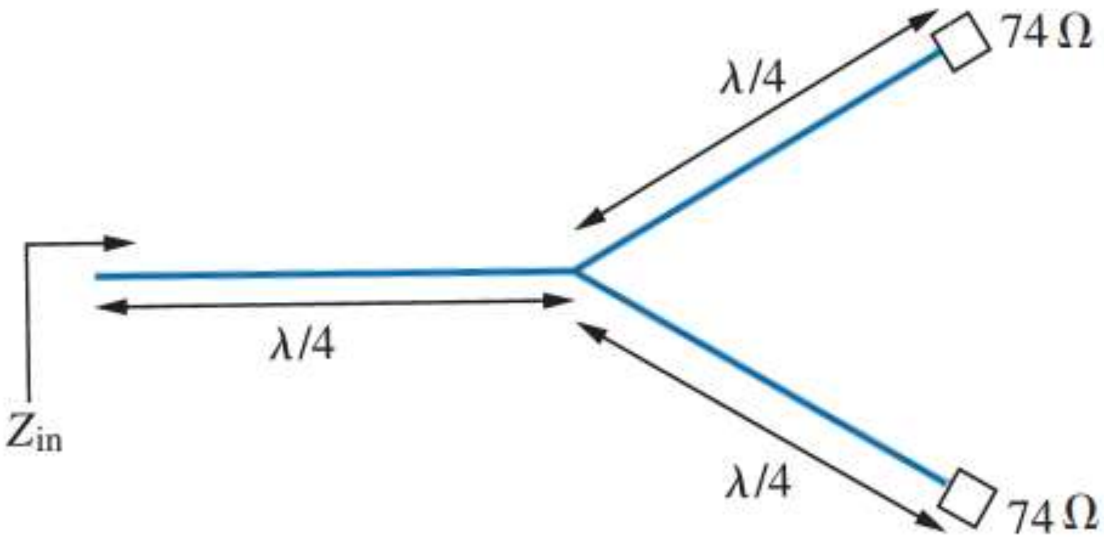


FIGURE 11.17 Load matching using a $\lambda/4$ transformer.

11.32 Refer to the lossless transmission line shown in Figure 11.49. (a) Find Γ and s. (b) Determine Z_{in} at the generator.



11.54 Two identical antennas, each with input impedance 74 Ω, are fed with three identical 50 Ω quarter-wave lossless transmission lines as shown in Figure 11.53. Calculate the input impedance at the source end.



11.56 Consider the three lossless lines in Figure 11.54. If $Z_o = 50\ \Omega$, calculate:

(a) Z_{in} looking into line 1

(b) Z_{in} looking into line 2

(c) Z_{in} looking into line 3