

# Monte Carlo Methods

Univariate

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June 13, 2023

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# Introduction

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## General Form

The general form of Monte Carlo methods is: The expectation of a function  $f(x)$  with respect to a distribution  $p(x)$  is given by:

$$\mathbb{E}_{x \sim p(x)}[f(x)] = \int f(x)p(x)dx \quad (1)$$

Using Monte Carlo methods, we can estimate the above expectation by sampling  $x_i$  from  $p(x)$  and computing the average of  $f(x_i)$ .

$$\mathbb{E}_{x \sim p(x)}[f(x)] \approx \frac{1}{N} \sum_{i=1}^N f(x_i) \quad (2)$$

where  $x_i \sim p(x)$ .

## Estimating Pi using Monte Carlo (Part 1)

We can estimate the value of pi using Monte Carlo methods by considering a unit square with a quarter circle inscribed within it.

- Let  $p(x)$  be defined over the unit square using the uniform distribution in two dimensions, i.e.,  $p(x) = U(x) = 1$  for  $x \in [0, 1]^2$ .
- Let  $f(x)$  be the indicator function defined as follows:

$$f(x) = \begin{cases} \text{Green}(1), & \text{if } x \text{ falls inside the quarter circle,} \\ \text{Red}(0), & \text{otherwise.} \end{cases}$$

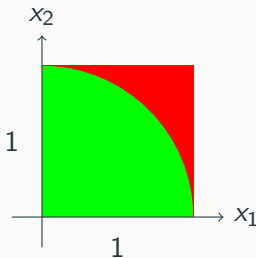
## Estimating Pi using Monte Carlo (Part 1)

- Or, we can write  $f(x)$  to be the following:

$$f(x) = \begin{cases} 1, & \text{if } x_1^2 + x_2^2 \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- Or, using the indicator function, we can write  $f(x)$  to be the following:

$$f(x) = \mathbb{I}(x_1^2 + x_2^2 \leq 1)$$



$$\frac{\pi}{4} \approx \frac{\text{Green area}}{\text{Green area} + \text{Red area}}$$

## Estimating prior predictive distribution

- Let  $p(\theta)$  be the prior distribution of parameter  $\theta \in R^2$ . Say, for example,  $p(\theta_i) = \mathcal{N}(0, 1) \forall i$ .
- Let  $p(y|\theta, x)$  be the likelihood function. Say, for example,  $p(y|\theta, x) = \mathcal{N}(\theta_0 + \theta_1 x, 1)$ .
- Then, the prior predictive distribution is given by:

$$p(y|x) = \int p(y|\theta, x)p(\theta)d\theta \quad (3)$$

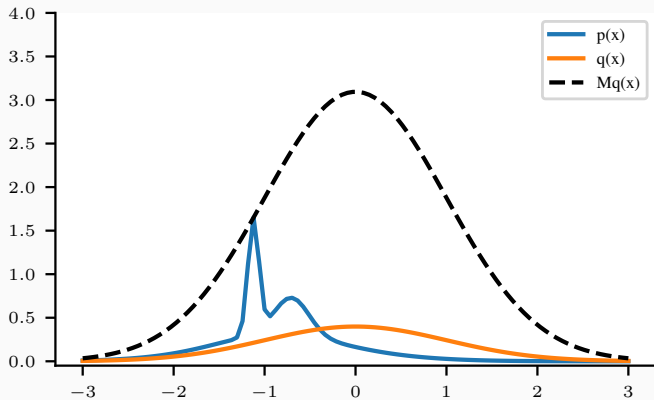
$$p(y|x) \approx \frac{1}{N} \sum_{i=1}^N p(y|\theta_i, x) \quad (4)$$

where  $\theta_i \sim p(\theta)$ .

# Rejection Sampling

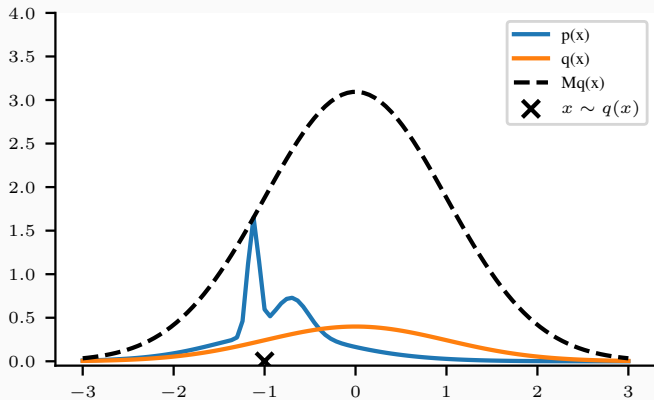
- Let  $p(x)$  be the target distribution from which we want to sample.
- Let  $q(x)$  be a proposal distribution from which we can sample.
- Let  $M$  be a constant such that  $M \geq \frac{p(x)}{q(x)} \forall x$ .
- Then, we can sample from  $p(x)$  by sampling from  $q(x)$  and accepting the sample with probability  $\frac{p(x)}{Mq(x)}$ .

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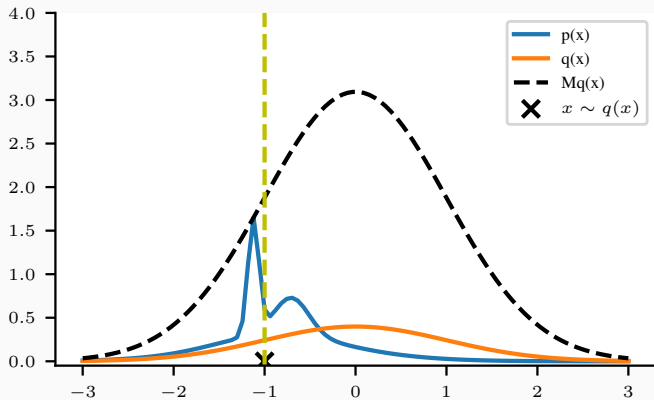




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