

Tribhuvan University

Institute of Science and Technology

2080

Bachelor Level / third-semester / Science

Computer Science and Information Technology(CSC212)

Numerical Method

Full Marks: 60 + 20 + 20

Pass Marks: 24 + 8 + 8

Time: 3 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Section A

Attempt any two questions.

1

How secant methods differs from Newton Rhapson method? Derive the formula for Secant Method. Solve the equation $\cos x + 2\sin x - x^2 = 0$ using Secant method. Assume error precision as 0.01. Discuss the drawbacks of the Newton Rhapson method.

2

Define the terms interpolation and extrapolation. Write down the algorithm and program for Newton's divided difference interpolation.

3

How Gauss Jordan method differs from Gauss Elimination method? Solve the following system of equations using Gauss Jordan method. How can we use Gauss Jordan method to find the inverse of a matrix? Discuss.

$$2x - y + 4z = 15$$

$$2x + 3y - 2z = 4$$

$$3x + 2y - 4z = -4$$

Section B

Attempt any eight questions.

4 Define the terms approximate error and relative approximate error? Discuss the working of Half Interval method for finding the roots of non-linear equation.

5

Construct Newton's backward difference table for given data points and approximate the value of $f(x)$ at $x=45$.

x	10	20	30	40	50
f(x)	0.985	0.934	0.866	0.766	0.643

6

Fit the quadratic curve through the following data points and estimate the value of $f(x)$ at $x=2$.

x	1	3	4	5	6
y	2	7	8	7	5

7

Factorise the following matrix using Cholesky method.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

8

How can we calculate derivatives of discrete (tabulated) functions? Write down its algorithm.

9

Find the following integral using composite trapezoidal rule for using 2 segments ($k=2$) and 4 segments ($k=4$).

$$\int_2^4 (x^3+2) dx$$

10

Approximate the solution of $y'=3x^2$, $y(1)=1$ using Taylor's series method using first four terms. Approximate the value of $y(2)$.

11

Solve the Poisson's equation $\nabla^2 f = xy$ and $f=2$ on boundary by assuming square domain $0 \leq x \leq 3$ and $0 \leq y \leq 3$ and $h=1$.

12

Write down the program for solving ordinary differential equation using Heun's method.

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2079

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Time: 3 Hours

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Section A

Attempt any two questions.

1

How secant method can approximate the root of a non-linear equation? Explain with necessary derivation. Estimate a real root of following equation using secant method. Assume error precision of 0.01.

$$x^3 + 2x - \cos(x) = 4$$

2

How spline interpolation differs with the Lagrange's interpolation? Estimate the value of $f(0)$ and $f(4)$ using cubic spline interpolation from the following data.

x	-1	1	2	3
f(x)	-10	-2	14	86

3

What is pivoting? Why is it necessary? Write an algorithm and program to solve the set of n linear equations using Gaussian elimination method.

Section B

Attempt any eight questions.

4. Calculate a real root of the following function using bisection method correct upto 3 significant figures.

$$x^2 - e^{-x} = 3$$

5

What is fixed point iteration method? How can it converge to the root of a non-linear equation? Also explain the diverging cases with suitable examples.

6

Write down the program for solving ordinary differential equation using Heun's method.

7

Fit the quadratic function for the data given below using least square method.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
f(x)	2.7	4	5.8	8.3	11.2	15	19

8

Estimate the integral value of following function from $x = 1.2$ to 2.4 using Simpson's $1/3$ rule.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
f(x)	1.53	2.25	3.18	4.32	5.67	7.23	8.98	10.94	13.08

9

What is Gaussian integration formula? Evaluate the following integration using Gaussian integration three ordinate formula

$$\int_0^1 \sin x / x \, dx$$

10

Solve the following set of equations using Gauss Siedal method.

$$x + 2y + 3z = 4$$

$$6x + 4y + 5z = 16$$

$$5x + 2y + 3z = 12$$

11

Solve the following differential equation for $0 \leq x \leq 1$ taking $h=0.5$ using Runge Kutta 4th order method.

$$y'(x) + y = 3x \text{ with } y(0)=2$$

12. Solve the Poisson's equation $\nabla^2 f = 3x^2y$ over the square domain $0 \leq x \leq 3, 0 \leq y \leq 3$ with $f=0$ on the boundary and $h=1$.

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2078

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Numerical Method

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Time: 3 Hours

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Section A

Attempt any TWO questions:

1

How can Horner's rule be used to evaluate the $f(x)$ and $f'(x)$ of a polynomial at a given point? Explain. Write an algorithm and program to calculate a real root of a polynomial using Horner's rule.

2

Write matrix factorization? How can be used to solve a system of linear equations? Factorize the given matrix A and solve the system of equations $Ax = b$ for given b using L and U matrices.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 11 \\ 3 & 22 & 36 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 12 \\ 28 \end{bmatrix}$$

3

What is a higher-order differential equation? How can you solve the higher-order differential equation? Explain. Solve the following differential equation for $1 \leq x \leq 2$, taking $h = 0.25$

$d^2y/dx^2 + 3dy/dx + 5y = 0$, with $y(1) = 1$ and $y'(1) = 2$

Section B

Attempt any EIGHT questions:

4

How the half-interval method can be estimate a root of a non-linear equation? Find a real root of the following equation using the half-interval method to correct up to two decimal places.

$$x^2 - e^{-x} - x = 1$$

5

Calculate the real root of the given equation using fixed point iteration correct up to 3 significant figures.

$$2x^3 - 2x = 5$$

6

What is Newton's interpolation? Obtain the divided difference table from the following data set and estimate the $f(x)$ at $x = 2$ and $x = 5$.

x	3.2	2.7	1.0	4.8	5.6
f(x)	22.0	17.8	14.2	38.3	51.7

7

What is linear regression? Fit the linear function to the following data

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
f(x)	2.0	2.6	3.9	6.0	9.3	15	20.6	30.4

8

What are the problems with polynomial interpolation for a large number of data set? How such problems are addressed? Explain with an example.

9

Evaluate the following integration using Romberg integration.

$$\int_0^1 10 \sin 2x \, dx$$

10

Solve the following set of linear equations using the Gauss-Jordan method.

$$x_2 + 2x_3 + 3x_4 = 9$$

$$7x_1 + 6x_2 + 5x_3 + 4x_4 = 33$$

$$8x_1 + 9x_2 + x_4 = 27$$

$$2x_1 + 5x_2 + 4x_3 + 3x_4 = 23$$

11

Solve the following differential equation for $1 \leq x \leq 2$, taking $h = 0.25$ using Heun's method.

$$y'(x) + x^2y = 3x, \text{ with } y(1) = 1$$

12

Consider a metallic plate of size 90cm by 90cm. The two adjacent sides of the plate are maintained at a temperature of 100°C and the remaining two adjacent sides are held at 200°C. Calculate the steady-state temperature at interior points assuming a grid size of 30 cm by 30 cm.

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2077

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Group A

Attempt any Two questions:(10 x 2 = 20)

1

Derive the formula for Newton Raphson Method. Solve the equation $x^2+4x-9=0$ using Newton Raphson method. Assume error precision is 0.01. Discuss drawbacks of the Newton Raphson method.

2

How interpolation differs from regression? Write down algorithm and program for Lagrange interpolation.

3

Why partial pivoting is used with Naive Gauss Elimination method? Solve the following system of equations using Gauss Elimination method with partial pivoting? How Gauss Jordan method differs from Gauss elimination method?

$$2x + 2y - z = 6$$

$$4x + 2y + 3z = 4$$

$$x + y + z = 0$$

Group B

Attempt any eight questions:(5 x 8 = 40)

4

Define the terms true error and relative error? Use Horner' method to evaluate polynomial $2x^3 - 3x^2 + 5x - 2$ at $x = 3$ and write down its algorithm.

5

Construct Newton's forward difference table for the given data points and approximate the value of $f(x)$ at $x = 15$.

x	10	20	30	40	50
F(x)	0.173	0.342	0.5	0.643	0.766

6

Fit the curve $y = ae^{bx}$ through the following data points.

x	1	2	3	4
y	1.65	2.70	4.50	7.35

7

Discuss the Doolittle LU decomposition method for matrix factorization.

8

Write down algorithm and program for the differentiating continuous function using three point formula.

9

How Simpson's 1/3 rule differs from Trapezoidal rule? Drive the formula for Simpson's 1/3 rule.

10

Appropriate the solution of $y' = 2x + y$, $y(0) = 1$ using Euler's method with step size 0.1. Approximate the value of $y(0.4)$.

11

A plate of dimension 18cm x 18cm is subjected to temperatures as follows: left side at 1000c, right side at 2000c. Upper part at 500c, and lower at 1500c. If square grid length of 6cm x 6cm is assumed, what will be the temperature at the interior nodes?

12

How boundary value problems differs from initial value problems? Discuss shooting method for solving boundary value problem.

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2075

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Time: 3 Hours

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Group A

Attempt any Two questions: (10 x 2 = 20)

1

What is non-linear equation? Derive the required expression to calculate the root of non-linear equation using secant method. Using this expression find a root of following equation.

$$x^2 + \cos(x) - e^{-x} - 2 = 0$$

2

What is matrix factorization? Factorize the given matrix A into LU using Doolittle algorithm and also solve $Ax = b$ for given b using L and U matrices.

$$A = \begin{bmatrix} 2 & 4 & -4 & 0 \\ 1 & 5 & -5 & -3 \\ 2 & 3 & 1 & 3 \\ 1 & 4 & -2 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 12 \\ 18 \\ 8 \\ 8 \end{bmatrix}$$

3

What is initial value problem and boundary value problem? Write an algorithm and program to solve the boundary value problem using shooting method.

Group B

Attempt any Eight questions:(5 x 8 = 40)

4

Calculate a real negative root of following equation using Newton's method for polynomial.

$$x^4 + 2x^3 + 3x^2 + 4x = 5x^4 + 2x^3 + 3x^2 + 4x = 5$$

5

What is least squares approximation of fitting a function? How does it differ with polynomial interpolation? Explain with suitable example.

6

Find the lowest degree polynomial, which passes through the following points:

X	-2	-1	1	2	3	4
F(x)	-19	0	2	-3	-4	5

Using this polynomial estimate $f(x)$ at $x = 0$

7

The fit function of type $y = a + bx$ for the following points using the least square method.

X	-1	1.2	2	2.7	3.6	4
F(x)	1	20	27	33	41	45

8

Calculate the integral value of the function given below from $x = 1.8$ to $x = 3.4$ using Simpson's 1/3 rule.

X	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.4
F(x)	0.003	0.778	1.632	2.566	3.579	4.672	7.097	8.429

9

Evaluate the following integration using Romberg integration.

$$\int_0^1 \sin x dx$$

10

Solve the following set of equations using Gauss Seidel method.

$$x + 2y + 3z = 4$$

$$6x - 4y + 5z = 10$$

$$5x + 2y + 2z = 25$$

11

From the following differential equation estimate $y(1)$ using RK 4th order method.

$$dy/dx + 2x^2y = 4 \text{ with } y(0) = 1$$

[Take $h = 0.5$]

12

Solve the Poisson's equation over the square domain $0 \leq x \leq 1.5$, $0 \leq y \leq 1.5$ with $f = 0$ on the boundary and $h = 0.5$.

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Model Set I

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Numerical Method

Full Marks: 60 + 20 + 20

Pass Marks: 24 + 8 + 8

Time: 3 Hours

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Group A

Attempts any two questions

1

Derive the formula for Newton Raphson method. Solve the equation $x^4 - x - 10 = 0$ using Newton Raphson method assuming error precision is 0.01. Discuss drawback for this method.

2

How interpolation differs from regression? Write down algorithm and program for Lagrange interpolation.

3

Why partial pivoting is used with Naive Gauss Elimination method? Solve the following system of equations using Gauss Elimination method with partial pivoting? How Gauss Jordan method differs from Gauss elimination method?

$$2x + 2y - z = 6$$

$$4x + 2y + 3z = 4$$

$$x + y + z = 0$$

Group B

Attempts any eight questions

4

Construct Newton's forward difference table for the given data points and approximate the value of $f(x)$ at $x = 12$.

x	10	20	30	40	50
F(x)	0.173	0.342	0.5	0.643	0.766

5

How Simpson's 1/3 rule differs from Trapezoidal rule? Drive the formula for Simson's 1/3 rule.

6

Solve $dy/dx = 3xy$; $y(0) = 1$ for $y(0.3)$ taking $h = 0.1$ by Euler's method.

7

A plate of dimension 18cm x 18cm is subjected to temperatures as follows: left side at 1000c, right side at 2000c. Upper part at 500c, and lower at 1500c. If square grid length of 6cm x 6cm is assumed, what will be the temperature at the interior nodes?

8

Fit the curve $y = ae^{bx}$ through the following data points.

x	0	1	3	5	7	9
y	1.0	0.891	0.708	0.563	0.447	0.355

9

How Doolittle LU decomposition differs from Crout's Decomposition? Factorize

$$e \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} l$$

by Crout's method.

10

Solve a boundary value problem by using shooting method $d^2y/dx^2 = 6x$, $y(1) = 2$, $y(2) = 9$

11

Evaluate the following integration using Romberg integration.

$$\int_0^1 \sin x dx$$

12

Solve the non linear equation $\log x - \cos x = 0$ by using false position method correct to four decimal places.

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Model Set II

Bachelor Level / third-semester / Science

Computer Science and Information Technology(CSC212)

Numerical Method

Full Marks: 60 + 20 + 20

Pass Marks: 24 + 8 + 8

Time: 3 Hours

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SECTION A

Attempt any TWO question.

1

What do you mean by ill condition? Compare Gauss elimination method and Gauss jordan method of solving simultaneous equations.

Using Gauss jordan method solve the given system of equation

$$6x_1 - 2x_2 + x_3 = 11$$

$$x_1 + 2x_2 - 5x_3 = -1$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

2

Define the terms interpolation and extrapolation. Find the Langrange interpolation polynomial to fit the following data and find value of $y(10)$.

x	5	6	9	11
y	12	13	14	16

3

Solve the ordinary differential equation given below by using Euler,s method. And calculate the value of $y' = 3x^2 + 1$; $y(0) = 2$, when

1. $h = 0.5$
2. $h = 0.25$

SECTION B

Attempt any EIGHT question.

4

Write down the program for solving ordinary differential equation using Heun's method.

5

Find the missing term in the following using Newton's divided difference formula.

x	0	1	2	3	4
y	1	3	9	81

6

Using a method of least square find the relation of the form $y = ax+b$

x	0.301	0.4771	0.6021	0.6990
y	1.4440	1.7931	2.0414	2.2068

7

Find the largest eigen value and the corresponding eigen vector of the following matrix :

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

8

Factorize the matrix using Cholesky's decomposition.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}.$$

9

Use Romberg estimate to evaluate $R(2,2)$

$\int_0^2 (1+x)^{11} dx$ from 0 to 2.

10

Calculate the integral value of the following tabulated function from $x = 0$ to $x = 1.6$ using Simpson's $3/8$ rule.

x	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
f(x)	0	0.24	0.55	0.92	1.63	1.84	2.37	2.95	3.56

11

Solve the Poisson's equation over the square domain $0 \leq x \leq 1.5$, $0 \leq y \leq 1.5$ with $f = 0$ on the boundary and $h = 0.5$.

12

Estimate a real root of the following non-linear equation using bisection method correct upto three significant figure $x^2 - e^{-x} = 3$.