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Chapter 1

Basic Discrete Structure

1. Set and Subset

A set is an unordered collection of distinct objects. Those objects in a set are called as elements or members.
Generally, any set can be defined by simply listing its members inside curly braces.
For eg: Prime number less than 10 can be represented as $\{2, 3, 5, 7\}$.

A subset is a set that contains only few elements from another set.
For example: $\{2, 3\}$ is the subset of the set of prime number less than 10.

A is said to be the subset of B iff every element of A is also an element of B, thus we can write $A \subseteq B$.
Two sets are said to be equal iff they have same elements while we write $A = B$ more formally $A = B$ if $A \subseteq B$ & $B \subseteq A$.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$
$A \cap B = \{x : x \in A \text{ and } x \in B\}$
$A \neq B = \{x : x \in A \text{ and } x \notin B\}$

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Power set:

A power set of a set is the set of all possible subset of that set.

For eg: The power set of the set {1, 2} would be {{∅}, {1}, {2}, {1, 2}}.

Cartesian product:

The cartesian product of two sets A and B is the set of all the ordered pairs where the first elements comes from set A and the second element comes from set B.

It can be noted as $A \times B$.

If $A = \{1, 2\}$ and $B = \{a, b\}$,

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

If M = {apple, banana} and N = {mango, guava},

M×N = {(apple, mango), (apple, guava), (banana, mango), (banana, guava)}

Set operations:

1. Union:

The union of two sets A and B is the set that contains all the elements present in either A or B (or both).

For eg: $A = \{1, 2, 3\}$ and $B = \{3, 4\}$,

$A \cup B = \{1, 2, 3, 4\}$

2. Intersection:

The intersection of any two sets is the set that contains the elements that are present in both of the set.

For eg: $A = \{1, 2, 3\}$ and $B = \{3, 4\}$,

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$A \cap B = \{3\}$.

3. Difference:

The difference between two sets A and B is the set that contains the elements present in A but not in B.

For eg: $A = \{1, 2, 3\}$ and $B = \{3, 4\}$

$\therefore A - B = \{1, 2\}$

$B - A = \{4\}$

4. Complement:

The complement of the set A inside of its universal set U can be defined as the set of those elements which are not present in A.

For eg: $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 2, 3\}$

$\therefore \bar{A} = \{4, 5, 6, 7\}$

5. Symmetric difference:

The symmetric difference of two sets A and B can be defined as the set that contains the elements which either present in A or B but not the common.

For eg: $A = \{1, 2, 3\}$ and $B = \{3, 4\}$

$\therefore A \Delta B = \{1, 2, 4\}$

Laws of sets:

- Identity law:

Union identity:

For any set A, the union of A with the empty set $\{\emptyset\}$ is A.

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$A \cup \emptyset = A$

Intersection identity:

For any set A, the intersection of A with the universal set U is equal to A.

$A \cap U = A$

- Idempotent law:

Union idempotent:

The union of a set A with itself is equal to A.

$A \cup A = A$

Intersection idempotent:

The intersection of a set A with itself is equal to A.

$A \cap A = A.$

- Commutative law:

The union of set A and B is equal to the union of set B and A.

$A \cup B = B \cup A$

Similarly, $A \cap B = B \cap A$

- Associative law:

Union associative:

The union of set A, B and C is associative.

$(A \cup B) \cup C = A \cup (B \cup C)$

Intersection associative:

The intersection of set A, B and C is associative.

$(A \cap B) \cap C = A \cap (B \cap C)$

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- Distributive law:

Union distributive over intersection:
The union of set A with the intersection of set B and C is equal to the intersection of the union of A with B and union of A with C.

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$$

Intersection distributive over union:
The intersection of set A with the union of set B and C is equal to the union of the intersections of A with B and intersection of A with C.

$$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$$

- De-Morgans law:

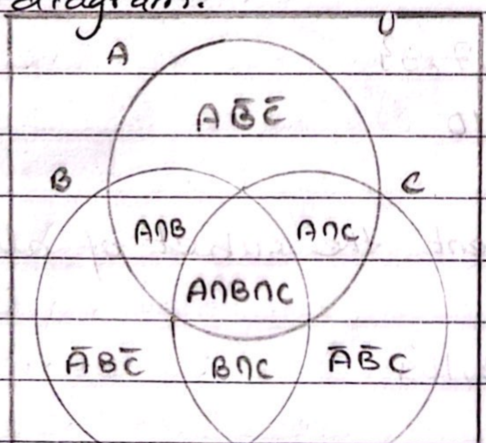
For union:
The complement of the union of two sets is equal to the intersection of their complement.

$$(\overline{A\cup B})=\bar{A}\cap\bar{B}$$

For intersection:
The complement of the intersection of two sets is equal to the union of their complements.

$$(\overline{A\cap B})=\bar{A}\cup\bar{B}$$

Venn-diagram:



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The Inclusion-Exclusion Principle:

It is a counting technique used to calculate the cardinality (size) of the union of multiple sets. It provides a formula for counting elements that belongs to at least one of the sets while avoiding double counting.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Computer Representation of sets:

It is the method for storing elements using arbitary ordering of the elements of the universal set. Specify an arbitary ordering of the element of U, for eg. $a_1, a_2, \ldots\ldots, a_n$. Represent a subset A of U with the bit string of length n, where the i^{th} bit in this string is 1 if a_i belongs to A and is 0 if a_i does not belong to A.

For eg:

Let, $U = \{1, 2, 3, 4, 5, \ldots\ldots, 10\}$

1. What bit string represent the subset of all odd integer in U?

→ Odd integer $= \{1, 3, 5, 7, 9\}$

bit string = 1010101010

2. What bit string represent the subset of all even integer in U?

→ Even integer $= \{2, 4, 6, 8, 10\}$

bit string = 0101010101

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3. What bit string represent the subset of integer not exceeding 5 in U?

→ Integer not exceeding $5 = \{1, 2, 3, 4, 5\}$

bit string = 1111100000

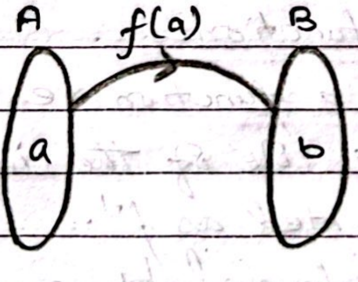
1.2 **Function.**

In mathematics, a function is a relation between two sets that assigns a unique output value to each input value. It represents a rule or mapping that connects elements from the input set (domain) to the elements of the output set (co-domain).

Given two sets A and B, a function $f: A \rightarrow B$ is a subset of $A \times B$, • if $x \in A$, there exist $y \in B$ such that $(x, y) \in f$.

• if $(x, y) \in f$ and $(x, z) \in f$, then $(x, y, z) \in f$.

A function is sometimes called map or mapping. The set A in above defination is the domain and B is the co-domain of f.



Relates:

Every $x \in A$ to exactly one element of B.

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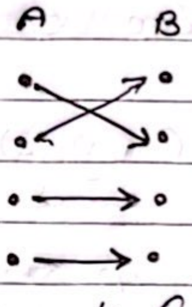
Two functions are equal when they have the same domain and the co-domain. While mapping is exist there, if we change either domain or co-domain of the function we obtain a different function. Also on changing the mapping of element we get different function.

Injective and Bijective functions:

Injective: also known as one to one function

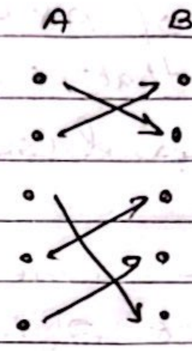
- A function that each input value has a unique output value.



(B can't have many A)

Bijective: both injective and subjective (onto) function

- Every input has a unique output and every output has a corresponding input



(A to B perfectly)

Inverse and Composite function:

The inverse function of a function rerverse the mapping i.e. it swap the role of the input and output. Generally, represented as f^{-1}.

Composite function are formed by combining two or more functions. Output of one function becomes the input for another succesive function.

Let g be the function in between A and B.

Also, f be the function in between B and C.

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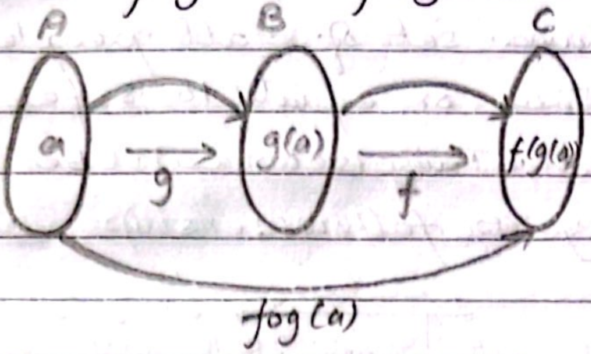
The composite function between f and g represented as $(f \circ g)(a) = f(g(a))$



Graph of functions:

The graph of a function represents the relationship between the input and the output values geometrically. In two-dimensional cartesian co-ordinate system, the graph of a function can be visualized as a curve or set of points.

Functions for Computer Science:

- Ceiling function
- Floor function
- Boolean function
- Exponential function

Fuzzy Sets:

These are generalization of classical or crisp sets that allow for degrees of membership rather than strict membership.

In fuzzy sets, elements have partial membership values ranging from 0 to 1, indicating the degree to which an element belongs to the set fuzzy sets are useful when

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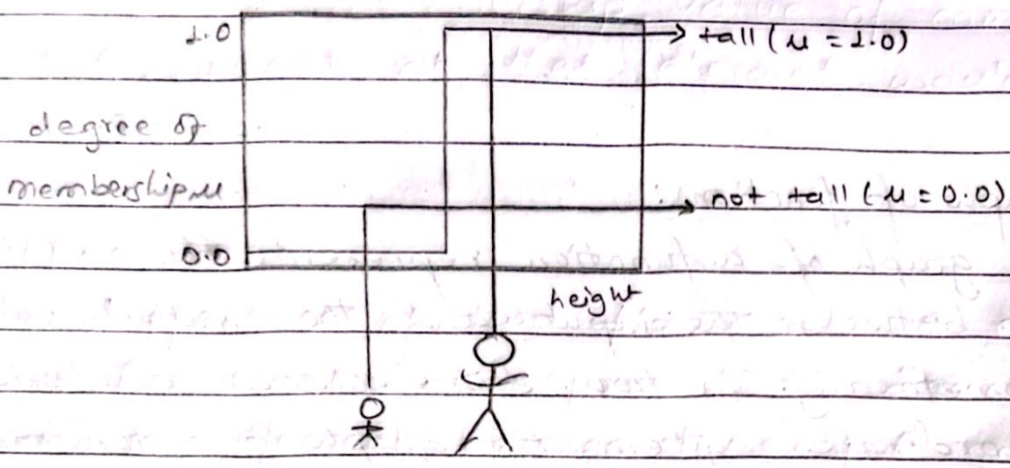
dealing with uncertainity, vagueness and gradations of membership.

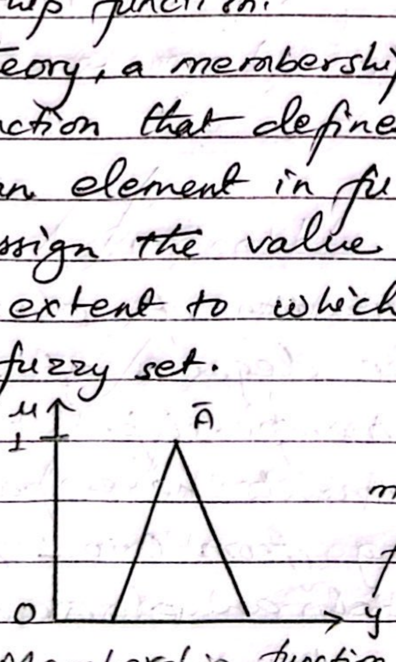
For eg: Let we have set of tall people where people taller than or equal to 5 feet are considered as tall. This set can be represented graphically as follows:



Membership function:

We know that the fuzzy logic is not a logic that is fuzzy but logic that is used to describe the fuzziness. This fuzziness is characterize by the membership function.

In fuzzy theory, a membership function is a mathematical function that defines the degree of membership of an element in fuzzy set. It's major role is to assign the value between 0 to 1, indicating the extent to which the element belongs to the fuzzy set.



membership function of fuzzy set Ā

Fig.: Membership function

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Fuzzy Set Operations:

It includes union, intersection and complements. These operations are performed on fuzzy set to combine certain values as per some rules under the criteria of fuzzy set.

1. Union:

The union of two fuzzy sets A and B is fuzzy set that represent the degree to which an element belongs to either A or B.

For example: set A represent tall.
set B represent short.

If element have the membership value of 0.7 in A and 0.4 in B. The membership value of union fuzzy set at that element would be max (0.7, 0.4) = 0.7.

2. Intersection:

The intersection of two fuzzy sets A and B is fuzzy set that represent the degree to which an element belongs to both A and B.

For example: set A represent tall.
Set B represent short.

If element have the membership value of 0.7 in A and 0.4 in B. The membership value of intersection fuzzy set at that element would be min(0.7, 0.4) = 0.4.

3. Complement:

The membership function of the complement of a fuzzy set A with membership function

$\mu_{\bar{A}}$ is defined as the negation of the specified membership function. This is called the negation criterion.

$$\mu_{\bar{A}} = 1 - \mu_A$$

For example: Set A represents tall.
If element have the membership value of 0.7 in A. The membership value of complement fuzzy set at that element would be $1 - 0.7 = 0.3$.

4. Difference:
The difference of two fuzzy sets A and B is fuzzy set that represent the degree to which an element belongs to only A or only B.

For example: Set A represents tall.
Set B represents short.
If element have the membership ~~for~~ value of 0.7 in A and 0.4 in B. The membership value of difference fuzzy set at that element would be $\min(0.7, 1-0.4) = \min(0.7, 0.6)$ $= 0.6$.

1.3 Sequence and Summations

Sequences:
In the context of discrete structure, sequences refer to ordered lists of elements that follows certain patterns or rules. Here are some basic concept related to sequences.

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- Finite or infinite sequence: A finite sequence has a specific number of elements, while an infinite sequence continue indefinitely.
- Arithmetic sequence
- Geometric sequence
- Permutation and Combination

Geometric progression:

In geometric progression, each term is obtained by multiplying the previous term by a constant factor called the common ratio (r). The generic form is,

$a, ar, ar^2, ar^3, \ldots\ldots$

a = first term

r = common ratio (non-zero real number)

n^{th} term, $a^n = ar^{n-1}$

Sum of term, $S_n = \frac{a(1-r^n)}{1-r}$

Arithmetic Progression:

In arithmetic progression, each term is obtained by adding a constant difference (d) to previous term. The generic form is,

$a, a+d, a+2d, \ldots\ldots$

a = first term

d = common difference

n^{th} term, $a^n = a + (n-1)d$

Sum of term, $S_n = \frac{n}{2}(2a+(n-1)d)$

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Application of G.P. and A.P.:

Understanding and utilizing the properties of A.P. and G.P. is crucial in discrete structure. They provide the framework for analyzing, predicting and solving the probleme related to patterns, growth rates, series and sequence.

1. Modeling patterns
2. Time complexity
3. Finance and Investment
4. Data Structure

Single Summation:

In discrete structure, a single summation is the sum of sequence in terms. It is denoted using the sigma (Σ) and expressed as,

$$\Sigma(\text{expression}) \text{ or } \Sigma(\text{term, index})$$

Components of Single Summation:

1. Index:
   Range value over which the summation is performed.

2. Term:
   The expression or formula involving the index that is being summed up.

3. Upper and lower limit:
   Indicates the starting and ending point of the summation.

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Index takes on values within this (lower-upper limit) range and the terms associated with those values are summed up.

For example: $\sum_{x=0}^{5} x^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 55$

Double Summation:

In discrete structure, a double summation is the sum of sequence of terms over two indices. It is used to sum up values from a two-dimensional array or to perform nested iterations. It is denoted using two sigma (Σ) notation and expressed as,

$\Sigma\Sigma$ (expression)

Components of Double Summation:

1. Term:
The expression or formula involving the index that is being summed up.

2. Upper and lower limit:
Indicates the starting and ending point of the summation.

3. Outer index:
The outer index represents the variables that takes on different values for outer loop of the double summation.

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4. Inner index:
The inner index represent the variable that takes in different values for inner loop of the double summation. Provide the range over which the inner summation is performed.

For example: $\sum_{i=0}^{m} \sum_{j=0}^{n} i^3 j^2$

Single and double summation are widely used in discrete structure, mathematics and computer science for calculating sum, analyzing series, evaluating algorithms and solving problems involving sequence and array.

Exercises

Q. Given, $A=\{2,4,6\}$, $B=\{0,1,2,3\}$ and $C=\{0,2,4\}$, verify the distributive laws.

→ Sol:

Here,

$A=\{2,4,6\}$

$B=\{0,1,2,3\}$

$C=\{0,2,4\}$

The distributive laws for union and intersections are:

(i) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(ii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Now,

(i) L.H.S. $= A\cup(B\cap C)$

$= \{2,4,6\}\cup[\{0,1,2,3\}\cap\{0,2,4\}]$

$= \{2,4,6\}\cup\{0,2\}$

$= \{0,2,4,6\}$

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R.H.S. $= (A\cup B)\cap(A\cup C)$

$= [\{2,4,6\}\cup\{0,1,2,3\}]\cap[\{2,4,6\}\cup\{0,2,4\}]$

$= \{0,1,2,3,4,6\}\cap\{0,2,4,6\}$

$= \{0,2,4,6\}$

$\therefore$ L.H.S. = R.H.S.

(ii) L.H.S. $= A\cap(B\cup C)$

$= \{2,4,6\}\cap[\{0,1,2,3\}\cup\{0,2,4\}]$

$= \{2,4,6\}\cap\{0,1,2,3,4\}$

$= \{2,4\}$

R.H.S. $= (A\cap B)\cup(A\cap C)$

$= [\{2,4,6\}\cap\{0,1,2,3\}]\cup[\{2,4,6\}\cap\{0,2,4\}]$

$= \{2\}\cup\{2,4\}$

$= \{2,4\}$

$\therefore$ L.H.S. = R.H.S.

Q. Prove that: $\overline{A\cup B} = \bar{A}\cap\bar{B}$

→ Sol^n:

$\overline{A\cup B} = \{x : x\in(\overline{A\cup B})\}$

$= \{x : \neg(x\in(A\cup B))\}$

$= \{x : \neg(x\in A \vee x\in B)\}$

$= \{x : \neg(x\in A)\wedge\neg(x\in B)\}$

$= \{x : x\notin A \wedge x\notin B\}$

$= \{x : x\in\bar{A}\wedge x\in\bar{B}\}$

$= \{x : x\in(\bar{A}\cap\bar{B})\}$

$= \bar{A}\cap\bar{B}$

proved

$\neg$ → negation
$\vee$ → conjuction
$\wedge$ → disjunction

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Q. Find the inverse of function $f(x) = 3x - 7$.

→ Soln;

Here, $f(x) = 3x - 7$.

Let, $y = 3x - 7$

Interchanging the value of x and y, we get

$x = 3y - 7$

or, $x + 7 = 3y$

or, $y = \frac{x+7}{3}$

$\therefore f^{-1}(x) = \frac{x+7}{3}$,,

Q. Given, $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find gof and fog.

→ Soln;

Here,

$f(x) = 2x + 1$

$g(x) = x^2 - 2$

Now,

$gof = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2 - 2$

$= 4x^2 + 4x + 1 - 2$

$= 4x^2 + 4x - 1$,,

$fog = f(g(x))$

$= f(x^2 - 2)$

$= 2(x^2 - 2) + 1$

$= 2x^2 - 4 + 1$

$= 2x^2 - 3$,,

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Q. Find the value of $\sum_{i=1}^{2}\sum_{j=1}^{3}(3i+2j)$.

→ Sol;

$$\sum_{i=1}^{2}\sum_{j=1}^{3}(3i+2j) = \sum_{i=1}^{2}(3i+2)+(3i+4)+(3i+6)$$

$$= \sum_{i=1}^{2}(9i+12)$$

$$= (9+12)+(18+12)$$

$$= 51$$