Chapter 3

Logic and Proof Method

Logic:

This term came from Greek word logos, which is sometime translated as 'discours', 'reason', etc.

More generally it is the science dealing with the method of reasoning.

In AI field, it is used to represent the knowledge.

There are various application areas of logic in the field of computer science:

- Circuit Design,
- Programming,
- AI automata theory,
- Program verification,
- Computability, etc.

On the basis of logic representation capability it has two types:

1. Propositional logic
2. Predicate logic

1. Propositional logic:

Also known as sentential logic that is the branch of logic which studies the way of joining or modifying proposition to form more complicated propositions as well as logical relationship. Here sentence are classified as simple and companded in propositional

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logic.
Simple sentences express atomic proposition about the world.
Compound sentences express logical relationships between the simpler sentences which they are composed.

Propositions:
Declarative sentence that is either true or false but not both.
For example,
- 2+2 = 5 (False), is a proposition.
- 3-2 = 1 (True), is a proposition.
- Get out (not a proposition)

Simple Propositions:
Any statement whose true value doesnot depend on another proposition.
For example, Kathmandu is the capital city of Nepal.

Compound Propositions:
Formed from simpler propositions and express relationships among the constituent sentences. There are 6 types of compound sentences: negation, conjunction, disjunction, implication, reductions and equivalences.

Truth table:
A table which consists of all possible input and output truth values of any

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propositions.

Logical operators or Connectives:
Any mathematical statements having one or more propositions by combining propositions are constructed by logical operators.

Negation ($\neg$):

p	$\neg p$
T	F
F	T

Conjunction ($\wedge$) and disjunction ($\vee$):

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Exclusive OR: $\oplus$

Implication ($\rightarrow$):
Let p and q are two propositions, the proposition implication $p \rightarrow q$ is the proposition that is false when p is true and q is false, otherwise true.

$p \rightarrow$ hypothesis / antecedent / premise
$q \rightarrow$ conclusion / consequence

- if p, then q,
- p is sufficient for q,
- q follows from p,
- p implies q,

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- q provides p, etc.

For example:

p = Today is Saturday.

q = It is holiday.

The implication can be,

If today is Saturday then it is holiday.

OR,

Today is Saturday only if it is holiday.

Inverse of implication:

When we add 'not' to the hypothesis and conclusion of implication $p \to q$ then it becomes $\neg p \to \neg q$.

For example:

If it is raining, then the road is muddy.

→ If it is not raining, then the road is not muddy.

Converse of implication:

If we flip/interchange the normal implication form then it become from $p \to q$ to $q \to p$, which is called as converse form of implication.

For example:

If it is raining, then the road is moddy.

→ The road is muddy if it is raining.

Contrapositive of Implication:

If we flip/interchange the hypothesis and conclusion of inverse statement of implication $p \to q$ then the resulting state-

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ment $\neg q \rightarrow \neg p$ is known as contrapositive form of implication.
For example:
If it is raining, then The road is muddy.
→ If it is not raining, then the road is not muddy.

Biconditional ($\leftrightarrow$):
Given proposition p and q. The biconditional $p \leftrightarrow q$ is a proposition that is true when p and q have same truth values.
More generally,

$$p \leftrightarrow q \text{ is true if } p \rightarrow q \text{ and } q \rightarrow p.$$

- p iff q
- if p then q and conversely.
- p is necessary and sufficient for q.

For example,

p: You may take the class.
q: You want to study DS.
$p \leftrightarrow q$: You may take the class if and only if you want to study DS.

Truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Representing English Sentence in propositional Logic:

It is important to translate normal english sentence propositional logic. The ambiguity is present in almost every natural language. So, in order to remove it we translate the sentences. Propositional logic are logical expression from where we can analyze the statement, determine their truth values, manipulate them and we can use rules of interference to reason about them.

For example,

If it is sunny, it is hot.

→ It is sunny.

→ It is hot.

Represent each atomic sentence by prepositional variables.

p = It is sunny.

q = It is hot.

- Connect with appropriate connectives logical expression of above sentence is $p \rightarrow q$.

i.e. Sentences are connected with implication.

Translate the following simple declarative sentences:

Let,

p : It is raining

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q: Sita is sick.
r: Ram stayed up late last night.
t: Kathmandu is capital of Nepal.
u: Ashok is a loud mouth.

a) Translating Negation:

(i) It is not raining.
$\neg p$, where p: it is raining.

(ii) It is not the case that Sita isn't sick.
Since: q: "Sita is sick"
$\neg q$: Sita isn't sick.
And: $\neg(\neg q)$: it is not the case that Sita isn't sick.

b) Translating Conjunction:

(i) It is raining and Sita is sick.
$(p \wedge q)$

(ii) Kathmandu isn't Capital of Nepal and it isn't raining.
$(\neg t \wedge \neg p)$

(iii) It is not the case that it is raining and Sita is sick.
Translation 1: It is not the case that both it is raining and Sita is sick.
$\neg(p \wedge q)$
Translation 2: Sita is sick and it is not the case that it is raining.
$(\neg p \wedge q)$

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c) Translating Disjunction:

(i) Kathmandu is capital of Nepal and it is raining or Ashok is a loud mouth.
$[(t \wedge p) \vee u]$ or $[t \wedge (p \vee u)]$

(ii) Sita is sick or Sita isn't sick.
$(q \vee \neg q)$

(iii) Ram stayed up at last night or Kathmandu isn't capital of Nepal.
$(r \vee \neg t)$

d) Translating Implications:

(i) If it is raining then Sita is sick.
$(p \rightarrow q)$

(ii) It is raining when Ashok is a loudmouth.
$(u \rightarrow p)$

(iii) Sita is sick and it is raining implies that Ram stayed up late last night.
$(q \wedge p) \rightarrow r$

e) Translating Biconditional:

(i) It is raining if an only if Sita is sick.
$p \leftrightarrow q$

(ii) Kathmandu is capital of Nepal is equivalent to Ram stayed up late night.
$p \leftrightarrow r$

Toutology:

A compound proposition that is always true no matter what the truth values of the atomic proposition that contains in it is called a tautology. Show that

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$p \vee \neg p$ is tautology.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

$\neg(p \to q) \to \neg q$

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Contradiction:

A compound proposition that is always false, no matter what the truth values of the atomic proposition that contain in it is called contradiction.

$(p \vee q) \wedge ((\neg p) \wedge (\neg q))$

p	q	$\neg p$	$\neg q$	$p \vee q$	$(\neg p) \wedge (\neg q)$	$(p \vee q) \wedge [(\neg p) \wedge (\neg q)]$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Contingency:

Any compound proposition that is neither a toutology nor a contradiction is called as contingency.

Showing up $\neg p \wedge \neg q$ is contingency.

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p	q	¬p	¬q	¬p ∧ ¬q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

__Propositional Equivalences:__

If their exist two propositions which are semantically identical then we can say those to proposition are equivalent.

Let propositions $P(p, q, r, \ldots\ldots)$ and $Q(p, q, r, \ldots\ldots)$ where p, q, r are the proportional variables have the same truth values in every possible case, the propositional are called logically equivalent and denoted as

$$P(p, q, r) \equiv Q(p, q, r)$$

Q. Prove $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$.

→ Sol:

p	q	r	p→r	q→r	(p→r)∨(q→r)	(p∧q)	(p∧q)→r
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$\therefore (p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

proved

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Laws of propositional equivalence

1. Identity law:

$p \wedge T \Leftrightarrow p$

p		$p \wedge T$
T	T	T
F	T	F

$p \vee F \Leftrightarrow p$

p		$p \vee F$
T	F	T
F	F	F

2. Domination law:

$p \wedge F \Leftrightarrow F$

p		$p \wedge F$
T	F	F
F	F	F

$p \vee T \Leftrightarrow T$

p		$p \vee T$
T	T	T
F	T	T

3. Idempotent law:

$p \wedge p \Leftrightarrow p$

p	$p \wedge p$
T	T
F	F

$p \vee p \Leftrightarrow p$

p	$p \vee p$
T	T
F	F

4. Double Negation law:

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

5. Commutative law:

$p \wedge q \Leftrightarrow q \wedge p$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \vee q \Leftrightarrow q \vee p$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

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6. Associative law:

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

7. Distributive law:

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F

F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

$p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$

p	q	r	$q \land r$	$p \lor (q \land r)$	$p \lor q$	$p \lor r$	$(p \lor q) \land (p \lor r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

8. De-morgan's law:

$\neg(p \land q) \Leftrightarrow \neg p \lor \neg q$

p	q	$\neg p$	$\neg q$	$p \land q$	$\neg(p \land q)$	$\neg p \lor \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\neg(p \lor q) \Leftrightarrow \neg p \land \neg q$

p	q	$\neg p$	$\neg q$	$p \lor q$	$\neg(p \lor q)$	$\neg p \land \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

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Q. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent using law of equivalent.

→ Sol:

Here,

$(p \rightarrow r) \wedge (q \rightarrow r)$

$\Rightarrow (\neg p \vee q) \rightarrow (\neg q \vee r)$ [implication]

Let, A represent $\neg p$ and B represent $\neg q$ and C represent r.

$(A \vee C) \wedge (B \vee C) = (C \vee A) \wedge (C \vee B)$

$= C \vee (A \wedge B)$ [∵ Distributive law]

$= (A \wedge B) \vee C$ [∵ Commutative law]

$= (\neg p \wedge \neg q) \vee r$ [∵ Substituting]

$= \neg (p \vee q) \vee r$ [∵ De-morgan's law]

Predicate:

In propositional logic, we have seen that how to represent statements which includes only facts, which are either true or false. Propositional logic is not sufficient to represent the complex sentences or natural language statement. Propositional logic have very limited expressive power. Consider following sentences, which cannot represent using propositional logic.

- Some of you are stupid.
- Rohit likes cricket.

To represent the above statements, we need some more powerful logic, such as predicate logic or first order logic.

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It is,

- Another way or knowledge representation.

Any declarative statements involving variables often found in mathematical assertion and in computer programs, which are neither true or false when the values of variable are not specified is called predicate. The predicate "$x > 4$" has two parts, first part, variable x is subject of statement and another is relation part "> 4" called predicate, refers to a property that the subject of statement value.

The predicate is a sentence that contain finite number of variables and becomes a proposition when specific values are substituted for the variables.

Q. Let, $P(x) : x + 3 < 10$, find the truth value of $P(6)$ and $P(8)$.

→ Sol;

Here,

$$P(x) : x + 3 < 10$$

When, $x = 6$

$$P(6) : 9 < 10 \text{ (true)}$$

When, $x = 8$

$$P(8) : 11 < 10 \text{ (false)}$$

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Quantifiers:

Quantifiers are the tools that change the propositional function into a proposition. Simply the construction of propositions from the predicates using quantifier is called quantification. There are two types of quantifiers and they are:

1. Universal Quantifier:

The phrase "for all" denoted by $\forall$, is called universal quantifiers. The process of converting predicate function into proposition using universal quantifier is called universal quantification. So, the universal quantification of $P(x)$, denoted by $\forall x P(x)$, is a proposition where "$P(x)$ is true for all the values of x in the universe of discourse."

For example,
Let, $P(x)$ be the statement "$x + 1 > x$". What is the truth value of quantification $\forall x P(x)$ where domain consists of all real numbers.

→ Sol:
The quantification $\forall x P(x)$ is true, because $P(x)$ is true for all real number x.

2. Existential Quantifier:

The phrase "there exist" denoted by $\exists$, is called existential quantifier. The process of converting predicate function into proposition

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using existential quantifier is called exist ential quantification.
Thus, existential quantification of $P(x)$, denoted by $\exists x P(x)$ is a proposition, wher "$P(x)$ is true for some values of x is the universe of discorse."
For example,
English sentence, "Some students of Vedas college likes discrete structure."
Some students of Vedas college: $\exists x$
likes discrete structure: $P(x)$
So, the quantification $\exists x P(x)$ is true proposition for above.

Binding variable:
When a quantifier is used on variable, then it is called binding variable. Any occurance of a variable that is not bounded by a quantifier or set equal to a particular value of said to be free.
For example,
In the statement $\forall y\, Q(x,y)$, the variable y is bounded by universal quantifier $\forall y$ but variable x is free because it is not bounded by any quantifier.

Translating sentences into Logical Expression
- Not every integer is even.
Let $E(x)$ denotes x is even.
So, $\neg \forall x\, E(x)$.

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- Every man is mortal.
  Let $M(x)$ denotes x is mortal.
  So, $\forall x\, M(x)$

Q. Some student of this college passed CSIT entrance exam. We have to translate this as, for some x, x is student of this college and x has passed CSIT entrance exam.

→ Sol;
$C(x)$ denotes x is student of this college.
$E(x)$ denotes x passed CSIT entrance exam.
So, $\exists x\, C(x) \wedge E(x)$

<u>Nested Quantifier:</u>
When we have more than one quantifier in a sequence while representing any sentences this form of representation contains nested quantifier.
For example,

- Everyone loves someone.
  Let (x,y): x loves y.
  $\forall x \exists y, L(x,y)$

- Some one loves sombody.
  $\exists x \exists y, L(x,y)$

- Everyone loves everybody.
  $\forall x \forall y, L(x,y)$

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- No[illegible] boys in this room is active.
  $L(x)$: x is active.
  $\neg \forall x, L(x)$

Rule of Inference

In order to reach the conclusion or to draw the conclusion of the given premise we need to be able to apply some well defined steps. Those steps of reaching the conclusion are provided by the rules of inference.

Here are some rules of inference.

1. Modus Pones (Law of detachment):
Whenever two propositions p and $p \to q$ are both true then we confirm that q is true. We write this rule as,
$\dfrac{p \to q, p}{\therefore q}$, this rules is valid rules of inference because the implication, $[p \wedge (p \to q)] \to q$ is a tautology.
For example, Ram is hard working and if Ram is hard working then he is intellegent.

2. Hypothetical Syllogism (Transctive rule):
Whenever two propositions $p \to q$ and $q \to r$ are both true then we confirm that implication $p \to r$ is true. We write this rule as $\dfrac{p \to q, q \to r}{\therefore p \to r}$, this rule is valid rule of inference because the implication

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$[(q \to r) \wedge (p \to q)] \longrightarrow (p \to r)$ is tautology.

3. Addition:

Due to the tautology $p \to (p \vee q)$ the rule $\dfrac{p}{\therefore q \vee p}$ is valid rule of inference.

4. Simplification:

Due to the tautology $(p \wedge q) \to p$, the rule $\dfrac{p}{\therefore p \vee q}$ is valid rule of inference.

5. Conjunction:

Due to the tautology $[p \wedge q] \longrightarrow p \wedge q$, the rule $\dfrac{p, q}{\therefore q \wedge p}$ is valid rule of inference.

6. Modus tollens:

Due to tautology $[\neg q \wedge (p \longrightarrow q)] \longrightarrow \neg p$, the rule $\dfrac{\neg q, p \to q}{\therefore \neg p}$ is valid rule of inference.

7. Resolution:

Due to tautology $[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$, the rule $\dfrac{p \vee q, \neg p \vee r}{\therefore q \vee r}$ is valid rule of inference.

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Q. Determine whether the following argument is valid or not.
"If Ram is human, humans are mortal then Ram is mortal."

→ Sol:
Here,

p = Ram is human.
q = Ram is mortal.

The given argument in symbolic form, we've got as $\frac{p, p \to q}{\therefore q}$, this is valid argument if $[p \wedge (p \to q)] \to q$ is tautologity.

Proof Method:
We know any mathematical statement can be consider as a theorem that can be shown to be true. A proof of given theorem is said to be well founded if its steps of mathematical statement can be present on argument that makes the theorem true.
This method of understanding correctness of statement by applying sequence of logical argument is known as proof of statement.

Direct Proofs:
The implication $p \to q$ can be proved bey showing that if p is true then q must be true. To carry out such a proof,

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we assume that hypothesis p is true and using information already available if conclusion q becomes true argument becomes valid.

Q. If a and b are odd integers then $a+b$ is an even integer.

→ We know that if a number is even then we can represent it as $2k$, when k is an integer and, if the number is odd then it can be written as $2l+1$ where l is an integer. Assume that $a = 2k+1$ and $b = 2l+1$, for some k and m, then,

$$a+b = 2k+1+2l+1$$
$$= 2(k+l+1)$$

Here, $k+l+1$ is an integer.
Hence, $a+b$ is even integer.

Q. If n is odd integer then n^2 is an odd integer.

→ We know that if n is odd integer then it can be written as $2l+1$ where l is an integer. Assume that $n = 2l+1$, for some l, then,

$$n^2 = (2l+1)^2$$
$$= (2l+1)(2l+1)$$
$$= 4l^2 + 2l + 2l + 1$$
$$= 4l^2 + 4l + 1$$
$$= 4l(l+1) + 1$$

Hence, n^2 is an odd integer.

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Q. Using direct method, prove that for every positive integer n, n^3+n is even.

→ Soln:

Case 1:

Suppose n is even, then $n=2k$ for some k.

Then,

$$n^3+n=(2k)^3+2k$$
$$=8k^3+2k$$
$$=2k(4k^2+1)$$

Case 2:

Suppose n is odd, then $n=2k+1$ for some k.

Then,

$$n^3+n=(2k+1)^3+2k+1$$
$$=8k^3+3.4k^2.1+3.2k.1+1+2k+1$$
$$=8k^3+12k^2+6k+2k+2$$
$$=8k^3+12k^2+8k+2$$
$$=2(4k^3+6k^2+4k+1)$$

Hence, n^3+n is even integer.

Indirect Proofs:

We have $p \to q \equiv \neg q \to \neg p$ i.e. contrapositive of implication is equivalent to the implication. So, the implication $p \to q$ can be proved by showing that it contrapositive $\neg q \to \neg p$ is true.

We prove the implication $p \to q$ by assuming that the conclusion (q) is false and using the known facts we show that the hypothesis (p) is also false.

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Q. If the product of two integers a and b is even, then a is even or b is even.

→ Suppose both a and b are odd, then we have $a = 2k+1$ and $b = 2l+1$ for some k and l.

So,

$$ab = (2k+1)(2l+1)$$
$$= 4kl + 2k + 2l + 1$$
$$= 2(k + 2kl + l) + 1$$

i.e. ab is odd number.

Hence, both a and b being odd implies ab is also odd. This is indirect proof.

Q. Using indirect proof, show that if $3n+2$ is odd then n is odd.

→ Suppose n is even, then we have $n = 2k$ for some k.

So,

$$3n+2 = 3 \times 2k + 2$$
$$= 6k+2$$
$$= 2(3k+1)$$

i.e. $3n+2$ is even number.

Hence, n being even implies $3n+2$ is also even. This is indirect proof.

Proof by Contradiction:

The steps in proof of implication $p \to q$ by contradiction are

→ Assume $p \wedge \neg q$ is true.

→ Try to show that above assumption

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$(p \wedge \neg q)$ is false.

→ When the assumption is found to be false the $p \to q$ is true.

→ Since, $p \to q$ is equivalent to $\neg p \vee q$ and negation of $\neg p \vee q$ is $p \wedge \neg q$ (By De-Morgan's law). So, if our assumption is false then its negation is true.

8. Prove that if n^3+5 is odd, then n is even.

→ Let $p \to q$: if n^3+5 is odd, then n is even.

Suppose n^3+5 is odd and n is also odd.

∴ n can be expressed as $n = 2k+1$ for some positive k.

So,

$$\begin{aligned} n^3+5 &= (2k+1)^3+5 \\ &= 8k^3+12k^2+6k+1+5 \\ &= 8k^3+12k^2+6k+6 \\ &= 2(4k^3+6k^2+3k+3) \end{aligned}$$

i.e. n^3+5 is even.

Due to contradict or assumption. Hence it is odd.

Proof by Counter Example:

Counter example is an exception to a proposed general rule on low and appears as an example which disproves a universal statement.

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For example,
$4n + 4$ is always multiple of 8 for all positive integer n.

→ Sol;
When $n=2$,
$$4n+4 = 4 \times 2 + 4 = 12$$
Which is not multiple of 8.

Existence Proofs:

A proof of a proposition of the form $\exists x P(x)$ is called existence proof.

- Constructive existence proof:
Sometime some element "a" is found to show P(a) to be true
- Non-Constructive existence proof:
Do not provide "a" such that P(a) is true but prove that $\exists x P$ is true in different way.

Uniqueness Proofs:

To prove the theorem that assert the existence of unique element with particular property we must show that element with this property exists and no other element has this property.
There are two parts:

- Existence: The element with desire property exists.
- Uniqueness: If $y \neq x$, then y doesnot have desire property.

The above two steps can be proved if

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we can prove, $\exists x(P(x) \land \forall y(y \neq x \rightarrow \neg P(y)))$.

Q. If n is an integer, then n^2+3n+2 is even.

→ Soln;

Case 1:

Let $2k$ is an even integer.

Then,

$$n^2+3n+2 = (2k)^2+3\cdot 2k+2$$
$$= 4k^2+6k+2$$
$$= 4k^2+k(4+2)+2$$
$$= 4k^2+4k+2k+2$$
$$= 4k(k+1)+2(k+1)$$
$$= (k+1)(4k+1)$$

Hence, n^2+3n+2 is even.

Case 2:

Let $2k+1$ is an odd integer.

Then,

$$n^2+3n+2 = (2k+1)^2+3(2k+1)+2$$
$$= (2k+1)(2k+1)+6k+3+2$$
$$= 4k^2+4k+1+6k+3+2$$
$$= 4k^2+10k+6$$
$$= 2(2k^2+5k+3)$$

Hence, n^2+3n+2 is even.

Q. If n is even integer, then $3n+2$ is even. Prove from indirect proof and contradiction proof.

→ Soln;

DATE:

Indirect Proof:

Let n is even, then $n = 2k$ for some k.

So,

$$3n + 2 = 3 \times 2k + 2$$
$$= 6k + 2$$
$$= 2(3k + 1)$$

Hence $3n + 2$ is even.

Contradiction Proof:

Let n is odd, then $n = 2k + 1$ for some k.
When n is odd, we can assume n to be even i.e. $n = 2k$.

So,

$$3n + 2 = 3 \times 2k + 2$$
$$= 6k + 2$$
$$= 2(3k + 1)$$

Irrespective of the value of $3k + 1$, we will always get the value of $3n + 2$ to be even.

Exhaustive Proofs:

Some statements can be proved by examining a relatively small number of examples such proofs are called exhaustive proof, because these proofs proceeds by exhausting all possibilities. It is special type of proof by cases where each case involves checking a single example.

DATE:

Q. Prove that $(n+1)^3 \geq n^3$ if n is a positive integer with $n \leq 4$.

→ Sol:

We verify the inequality when $n = 1, 2, 3$ and 4.

Case I: When $n = 1$,

$$(n+1)^3 \geq 3^n$$
$$(1+1)^3 \geq 3^1$$
$$8 \geq 3$$

Case II: When $n = 2$,

$$(n+1)^3 \geq 3^n$$
$$(2+1)^3 \geq 3^2$$
$$27 \geq 9$$

Case III: When $n = 3$,

$$(n+1)^3 \geq 3^n$$
$$(3+1)^3 \geq 3^3$$
$$64 \geq 27$$

Case IV: When $n = 4$,

$$(n+1)^3 \geq 3^n$$
$$(4+1)^3 \geq 3^4$$
$$125 \geq 81$$

Hence, each of the above cases we can see the statements are valid and true.