

Prove the correctness of the following “silly” program

```
method M (x0 : int) returns (x : int)
ensures (x0 < 3 ==> x == 1) && (x0 >= 3 ==> x < x0);
{
    x := x0 - 3;
    if (x < 0) {
        x := 1;
    }
    else {
        if (true){
            x := x + 1;
        }
        else {
            x := 10;
        }
    }
}
```

Prove m1 correct!

```
wp(x := e , R) = R[x → e]
wp(S1 ; S2 , R) = wp(S1, wp(S2, R))
wp(assert B, R) = B && R
wp(if B {S1} else {S2}, R) =
  ( B ==> wp(S1, R) ) &&
  (!B ==> wp(S2, R))
```

```
wp(while B I D S, R) =
  I
  && (B && I ==> wp(S, I))
  && (!B && I ==> R)

  && (I ==> D >= 0)
  && (B && I ==>
    wp(tmp := D ; S, tmp > D))
```

```
method m1(n : nat) returns (i : nat)
requires n >= 0
ensures i == 2*n

{
  i := 0;
  while (i < n)
    invariant i <= n
    variant n-i
    { i := i + 1; }
  i := 2*i;
}
```

Prove fib correct!

```
wp(x := e , R) = R[x → e]
wp(S1 ; S2 , R) = wp(S1, wp(S2, R))
wp(assert B, R) = B && R
wp(if B {S1} else {S2}, R) =
  ( B ==> wp(S1, R) ) &&
  (!B ==> wp(S2, R))
```

```
wp(while B I D S, R) =
  I
  && (B && I ==> wp(S, I))
  && (!B && I ==> R)

  && (I ==> D >= 0)
  && (B && I ==>
    wp(tmp := D ; S, tmp > D))
```

```
function fib(n : nat) : nat
{ if n <= 1 then n else fib(n-1) + fib(n - 2) }

method fibFast(n : nat) returns (c : nat)
requires n >= 1
ensures c == fib(n)
{
  var p := 0;
  c := 1;
  var i := 1;
  while i < n
  invariant 1 <= i <= n
  invariant p == fib(i - 1) && c == fib(i)
  decreases (n - i)
  { var new := p + c;
    p := c;
    c := new;
    i := i + 1;
  }
}
```