Image compression using Principal Component Analysis (PCA)

Submitted By -: Madhav Kaushik

Roll Number – 23

Subject – Machine Learning

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What is PCA?

Principal Component Analysis is a dimensionality-reduction method that is often used to reduce the dimensionality of large data sets, by transforming a large set of variables into a smaller one that still contains most of the information in the large set.

Image Compression using PCA

One of the use cases of PCA is that it can be used for image compression, a technique that minimizes the size in bytes of an image while keeping as much of the quality of the image as possible.

How?

We apply some Transformations. They are applied such that linearly correlated variables get transformed into uncorrelated ones (Correlation denotes redundancy and when taken care of results in Compression).

The Transformed features are known as PC's (Principal Components). The first few PCs are rich in information (explains most of the variance) whereas the last ones express noise (provides negligible information). The idea is to retain first few PCs, thereby reducing number of variables significantly and still suffer minimal loss of information.

The direction of projection is determined using Eigenvalues and Eigenvectors.

We will now observe Two Images to understand the working of Image Compression using PCA.

Observations for the First Image

Original Image

Grayscale Image

Converted the Image into Grayscale to do Principal Component Analysis

Size of the Original Image (Resolution) - (1280, 676)

We now observe the Image after retaining 5%, 10%, 20%, 30%, 40% of its Principal Components.

<u>When 5% components are retained.</u>

Image shape: (1280, 676)

nump_pc: 33

Shape of Z (centered data matrix * W): (1280, 33)

Reduction: 95.11834319526628

(Image after 5% components are retained)

The Image obtained is completely blurred. The texture of the kurta cannot be identified. The contents of the frame hanging on the wall have been completely lost. The facial features are very hazy but two have been degraded tremendously, the eyes cannot be made out easily and the dimple on the right cheek has completely vanished.

When 10% components are retained.

Image shape: (1280, 676)

nump_pc: 67

Shape of Z (centered data matrix * W): (1280, 67)

Reduction: 90.08875739644971

(Image after 10% components are retained)

This image obtained already shows significant improvement over the previous iteration with 5% retention. The texture of the Kurta is more distinct and you can see a dot in the background (Push Pin in Original Image). The facial features are more distinct, hint of the dimple can be seen and the eyes are much clearer. Still the image is very grainy.

When 20% components are retained.

Image shape: (1280, 676)

nump_pc: 135

Shape of Z (centered data matrix * W): (1280, 135)

Reduction: 80.02958579881657

(Image after 20% components are retained)

The image obtained is brighter than the previous one, you can begin to make out the contents of the frame on the wall. The facial features are even more distinct and a shadow of an Adam's Apple can be seen.

When 30% components are retained.

Image shape: (1280, 676)

nump_pc: 202

Shape of Z (centered data matrix * W): (1280, 202)

Reduction: 70.11834319526628

(Image after 30% components are retained)

The image obtained despite being a Grayscale one shows more saturation and is smoother in overall texture, it is far less grainy as compared to the image obtained after 10% retention. The facial features are also smooth and you can observe the dimple leaving a much deeper impression. Adam's apple is more dominant now and rest of the features are pretty much clear. When compared to the first iteration of 5%, this image shows great promise.

<u>When 40% components are retained.</u>

Image shape: (1280, 676)

nump_pc: 270

Shape of Z (centered data matrix * W): (1280, 270)

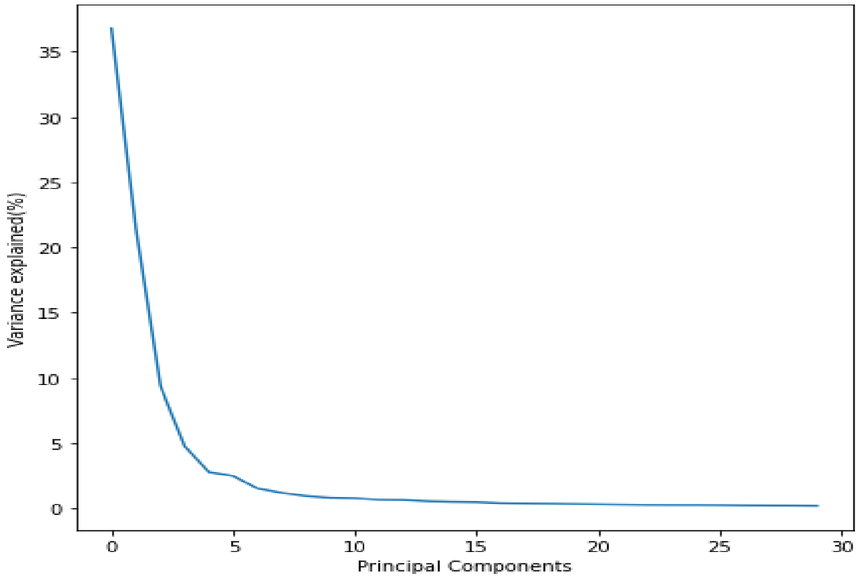
Reduction: 60.05917159763313

(Image after 40% components are retained)

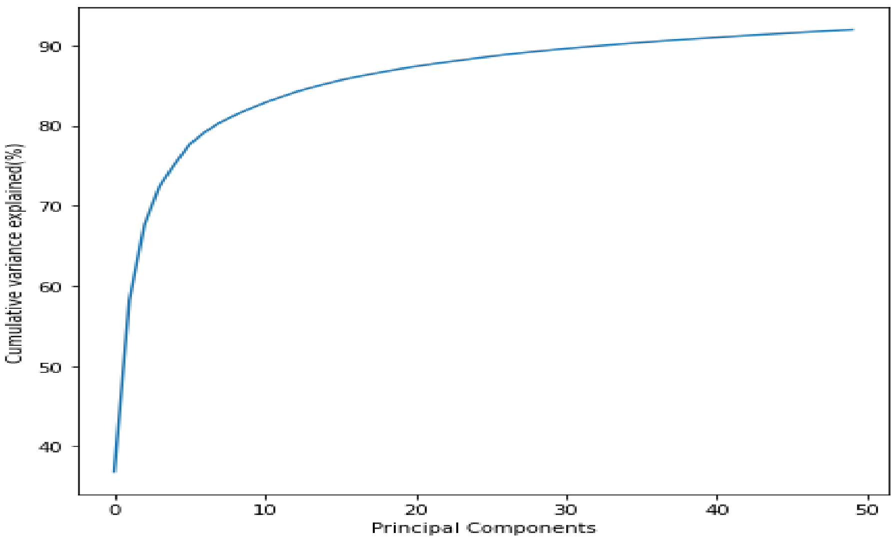
The image obtained is the smoothest one yet, the hair appears darker and all the facial features are very distinct, some shadows can be observed around the features. When compared to the original image, you can clearly map out everything in the background including the photo frame. Although the image is still minutely blurry, it is not grainy at all and does not give an intermixing of pixels effect.

Scree Plots

Plot of Principal Components (X) with Variance Explained % (Y)



Plot of Principal Components (X) with Cumulative Variance Explained % (Y)



The Table below shows the PCs Retained along with the Image Resolution thus obtained. The second column denotes the size of Centered data matrix * W. [Matrix used for Reconstructing the Image]

Shape of the Original Image is (1280, 676).

Principal Components Retained	Image Shape
5%	(1280, 33)
10%	(1280, 67)
20%	(1280, 135)
30%	(1280, 202)
40%	(1280, 270)

- Eigenvalues give us the order of significance of Eigenvectors. Eigenvectors with largest Eigenvalue is the most significant Principal component.
- After looking at the first plot, we can observe a distinct elbow point at around 4-5 value in the X axis (Number of Principal Components).
- After observing the second plot, we can see that for 4-5 Principal Components, almost 75-80% of the cumulative variance is explained which is an excellent value.

After observing all the images with different principal components retained, it can be observed that retaining **30% of the PCs** is a good choice. It maintains a great quality while reducing the variables.

Observations for the Second Image

Original Image

Grayscale Image

Converted the Image into Grayscale to do Principal Component Analysis

Size of the Original Image (Resolution) - (1280, 1043)

We now observe the Image after retaining 5%, 10%, 20%, 30%, 40% of its Principal Components.

When 5% components are retained.

Image shape: (1280, 1043)

nump_pc: 52

Shape of Z (centered data matrix * W): (1280, 52)

Reduction: 95.0143815915628

(Image after 5% components are retained)

The image obtained is very grainy, the leaves at the bottom left have completely vanished. The wrinkles of the jacket are not clear. The facial features have taken a great hit, the left ear has camouflaged with the background, the lip looks badly disfigured and the soul patch has completely vanished. The compression has not been able to handle the effects caused by Sunlight.

When 10% components are retained.

Image shape: (1280, 1043)

nump_pc: 104

Shape of Z (centered data matrix * W): (1280, 104)

Reduction: 90.0287631831256

(Image after 10% components are retained)

The image obtained is less grainy as compared to the previous iteration. The left ear can be distinguished from the background. The darkness under the lower lip (Soul Patch in Original Image) cannot be mistaken for a shadow now and because of that the lip does not look disfigured. You can also begin to notice the pattern on the Sweater.

When 20% components are retained.

Image shape: (1280, 1043)

nump_pc: 208

Shape of Z (centered data matrix * W): (1280, 208)

Reduction: 80.0575263662512

(Image after 20% components are retained)

The image obtained has improved drastically over the last two iterations. A quick glance can lead you to even consider this as the final result. The soul patch is clearly visible, the leaves in the lower left can easily be made out, the trees in the background are much more distinct and the image is a lot less grainy. The facial features are very clear and you can begin to notice the shadows caused due to sunlight.

When 30% components are retained.

Image shape: (1280, 1043)

nump_pc: 312

Shape of Z (centered data matrix * W): (1280, 312)

Reduction: 70.0862895493768

(Image after 30% components are retained)

The image obtained is brighter when compared to the one in obtained after the last iteration. The shadows are much more prominent and the image is more saturated. The facial features are totally clear and you can notice the crease of the Jacket easily.

When 40% components are retained.

Image shape: (1280, 1043)

nump_pc: 417

Shape of Z (centered data matrix * W): (1280, 417)

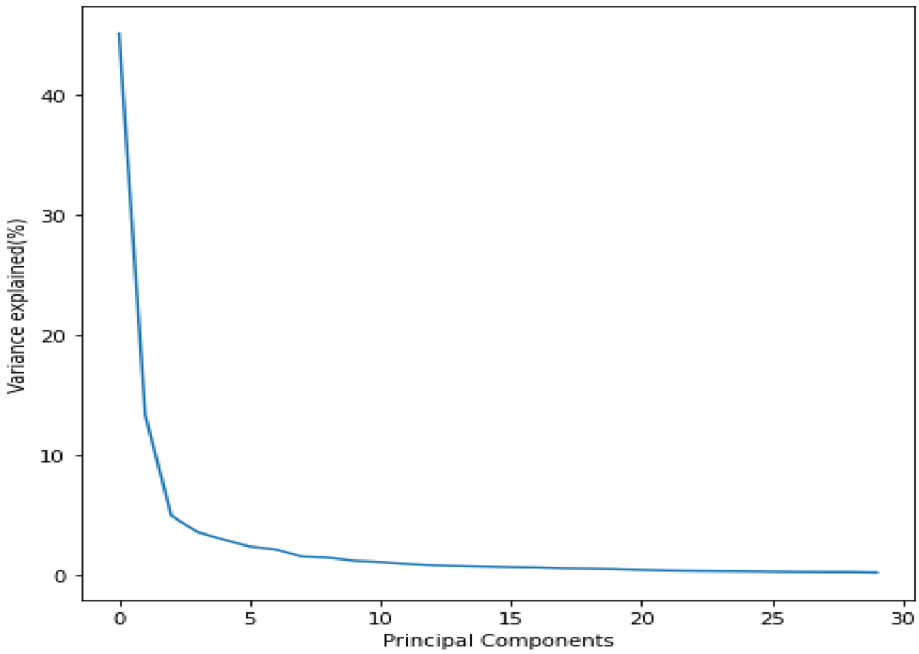
Reduction: 60.01917545541706

(Image after 40% components are retained)

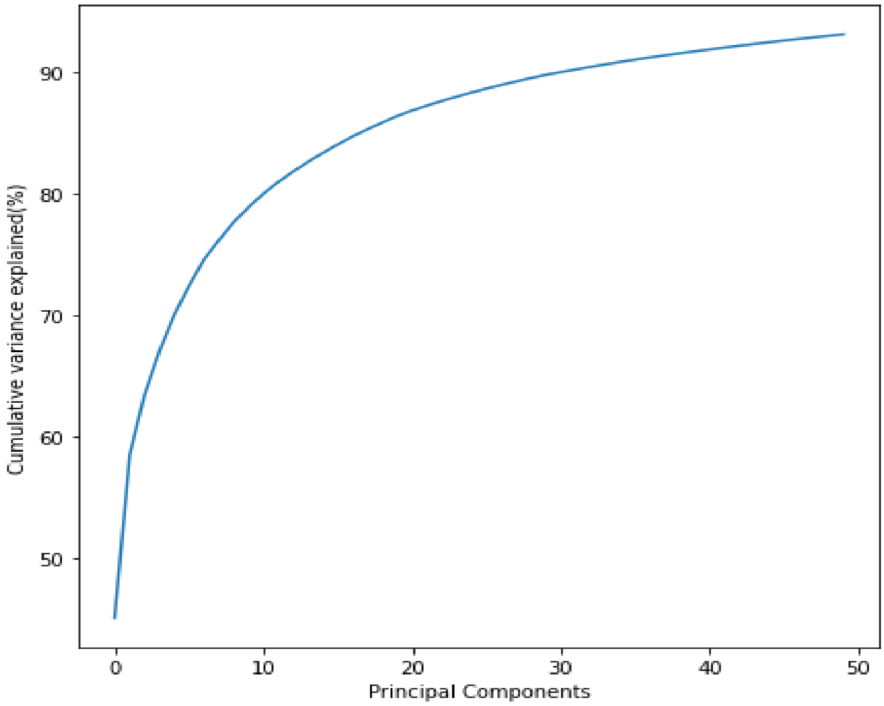
The Image obtained looks a lot like the original Grayscale image. The image is simply more vibrant and some minute features are enhanced even more (The eyebrows are darker compared to the Image obtained after the previous iteration). The shadows are prominent, the features are prominent and the background is clear. The image is not grainy at all.

Scree Plots

Plot of Principal Components (X) with Variance Explained % (Y)



Plot of Principal Components (X) with Cumulative Variance Explained % (Y)



The Table below shows the PCs Retained along with the Image Resolution thus obtained. The second column denotes the size of Centered data matrix * W. [Matrix used for Reconstructing the Image]

Shape of the Original Image is (1280, 1043).

Principal Components Retained	Image Shape
5%	(1280, 52)
10%	(1280, 104)
20%	(1280, 208)
30%	(1280, 312)
40%	(1280, 417)

- Eigenvalues give us the order of significance of Eigenvectors. Eigenvectors with largest Eigenvalue is the most significant Principal component.

- After looking at the first plot, we can observe a distinct elbow point at around 2-3 [most notably 2] value in the X axis (Number of Principal Components).

- After observing the second plot, we can see a steep increase in explained variance in the beginning. For 2-3 Principal Components, almost 70% of the cumulative variance is explained whereas the first 10 components explain 80% variance. So, taking 2 Principal components is an excellent choice.

After observing all the images with different principal components retained, it can be observed that retaining **20% of the PCs** is a good choice. It maintains a great quality while reducing the variables.

Conclusion

Principal Components Analysis is a great technique for Image Compression. It has provided us with great results in terms of quality preservation (information retention) and reduction of variables (compression).