Exploring the Relationship between the Suspension Length and Period of a Torsional Pendulum for the determination of the Shear Modulus of Copper

1 Introduction

Torsional pendulums are a class of pendulums that use the principles of rotational dynamics to govern their motion. Unlike simple pendulums which use spherical bobs, torsional pendulums typically use a flat cylindrical disk that oscillates about the axis of the suspension wire when the disk is displaced by an angle θ.

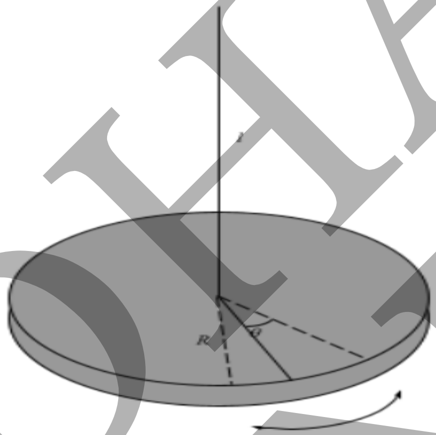

Figure 1: Typical torsional pendulum; drawn on Google Drawings by author

Torsional pendulums have a wide range of applications from characterising the properties of materials to studying the phenomena of torsion, both of which can be done via a process called "Torsional Braid Analysis" (Gillham and Roller). Torsion is the practice of a body twisting under applied torques ("Mechanics of Materials"). The idea of torsion and its importance manifests itself in more relatable scenarios such as the torsion bar suspension, which is used in vehicles to "provide spring action" ("Torsion bar"). Given that neither torsion nor rotational motion is covered in-depth in the core syllabus, I was interested in studying the simplest system that exhibits torsion: the torsional pendulum and how it can be used to investigate the properties of materials. Thus, this led to the research question:

How does the suspension length (0.4800, 0.5000, 0.5200, 0.5400, 0.5600, 0.5800, 0.6000, 0.6200, 0.6400, 0.6000) m of a torsional pendulum affect its period of oscillation in s, and how can this relationship be used to measure the shear modulus of copper?

2 Theoretical Framework and Hypothesis

For a given angular displacement θ that is sufficiently small, torsional pendulums execute angular simple harmonic motion (SHM) (Davis). Hence, we can use the mathematics of SHM to model the pendulum's motion. When the disc is displaced by θ, a shear stress is applied to the copper wire as a torque is applied since one end is fixed to the wire chuck while the other end rotates by some angle called the angle of twist. Stress is defined as the force per unit area tangential to a surface (Young and Freedman 374), and is responsible for deformation and torsion; shear stress is therefore stress related to the twisting angle (Krousgrill). As the disc is executing SHM, there will be a restoring torque τ provided by the thin copper wire the disc is suspended on that restores the disc to equilibrium. This restoring torque is given by (Davis),

$$\tau = -\kappa\theta \tag{2.1}$$

where κ is the torsional constant of the suspension wire with units $\mathrm{N\,m\,rad^{-1}}$. The torsional constant can be thought of as an analogue to the spring constant (Davis) and it is therefore a measure of the wire's resistance to twisting. The net torque on the disk can be expressed using Newton's second law of rotation,

$$\tau = I\alpha \tag{2.2}$$

where I is the moment of inertia of the disc with units $\mathrm{kg\,m^2}$ and α is the disc's angular acceleration at a particular point in its oscillation, with units $\mathrm{rad\,s^{-2}}$. As the pendulum mass used is effectively a solid cylinder suspended at its centre of mass, the moment of inertia is given by (Young and Freedman 313),

$$I = \frac{1}{2}MR^2 \tag{2.3}$$

where M is the mass of the disk in kg and R is the radius of the disk in m. When equating eq. (2.2) and eq. (2.1), angular acceleration is more commonly expressed as the second derivative of angular displacement with respect to time to ensure that the resulting equation is in terms of one variable only. Thus,

$$\begin{aligned} I\frac{d^2\theta}{dt^2} &= -\kappa\theta \\ \therefore \frac{d^2\theta}{dt^2} &= \frac{-\kappa}{I}\theta \end{aligned} \tag{2.4}$$

As the wire that suspends the mass is a thin copper wire, its torsional constant can be expressed as (Tatum),

$$\kappa = \frac{\pi G r^4}{2l} \tag{2.5}$$

where r is the radius of the wire with units m, l is the length—in this case the suspension length of the wire—with units m and G is a constant known as the shear modulus with units Pa. The shear modulus is a measure of how rigid a material is. It is defined as the ratio of shear stress to shear strain (Young and Freedman 378). Deformations typically cause changes in length and strain is defined as the ratio of the deformed length to the original length (Holmes). Shear strain is strain caused by twisting, and the angle of twist is considered instead of the deformed length.

Substituting eq. (2.3) and eq. (2.5) into eq. (2.4) gives,

$$\frac{d^2\theta}{dt^2} = -\frac{\pi G r^4}{MR^2 l}\theta \tag{2.6}$$

The defining equation for SHM is given by $\frac{d^2x}{dt^2} = -\omega^2 x$ where $\frac{d^2x}{dt^2}$ is the acceleration of the system at a given point, ω^2 is the square of the angular frequency and x is the position of the body executing SHM at some arbitrary time. As angular displacement is the rotational analogue of linear displacement, we can express the defining equation for angular SHM as,

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \tag{2.7}$$

By comparing terms in eq. (2.6) and eq. (2.7) and taking the square root, we obtain the angular frequency, ω, in rad $\mathrm{s^{-1}}$ as,

$$\omega = \sqrt{\frac{\pi G r^4}{MR^2 l}} \tag{2.8}$$

As the period of oscillation, measured in s, is given by $T = \frac{2\pi}{\omega}$, we can substitute eq. (2.8) and simplify to get the period of oscillation to be,

$$T = 2\pi\sqrt{\frac{MR^2 l}{\pi G r^4}} \tag{2.9}$$

By squaring eq. (2.9) and simplifying we get,

$$T^2 = \frac{4\pi MR^2}{Gr^4}l \tag{2.10}$$

Thus, we obtain the relationship,

$$T^2 \propto l \tag{2.11}$$

According to relationship 2.11 we can see that the square of the period of a torsional pendulum is directly proportional to suspension length. **Hence, it is hypothesised that greater suspension lengths will result in a longer period.** Furthermore, plotting T^2 against l will result in a directly proportional relationship with the gradient of the resultant straight line being,

$$\frac{4\pi MR^2}{Gr^4}$$

As M, R and r can be measured experimentally, the gradient will be used to calculate the value of G for the copper wire used in the experiment. This will then be compared with literature values to verify that eq. (2.9) holds.

Key assumptions were made to simplify the mathematical model of the system:

1. The copper wire has negligible mass and thus does not contribute to the moment of inertia of the pendulum
2. There is no air resistance (see table 1)

3 Experimental Design

3.1 Variables

Independent variable: Suspension length
The suspension length is defined as the length of wire between the ends of the two wire chucks in the pendulum system (see fig. 2). The range of lengths in m that was investigated was 0.4800, 0.5000, 0.5200, 0.5400, 0.5600, 0.5800, 0.6000, 0.6200, 0.6400, 0.6600. Lengths beyond 0.6600 m could not be investigated as the retort stand was not tall enough to ensure the disc was above the table, and lengths below 0.4800 m could not be investigated without neglecting damping. The lengths were measured using a tape measure that was placed between the two wire chucks. The length on the tape measure was then read at eye-level to avoid parallax.
Dependent variable: Period of oscillation
To ensure an accurate period is measured, the time taken for 20 oscillations is measured. It was found during the trial investigation that measuring the time taken via a stopwatch led to significant random errors. Thus, the oscillations were recorded using a 60 frames-per-second (FPS) smartphone camera. The recordings were then uploaded on an editing software with a 60 FPS track to count the exact time an oscillation starts and ends. 20 full oscillations were measured to minimise random error in counting a frame on the software.
Controlled variables:

Table 1: Controlled variables

Quantity controlled	Reason for control	Method of control
Mass of disc	As the mass of disc is present in eq. (2.3), different masses lead to different moments of inertia thereby affecting period.	The same disc was used throughout the experiment (See section 4.1 for parameters of the system).
Radius of disc	As the radius of disc is present in eq. (2.3), different radii lead to different moments of inertia thereby affecting period.	

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Table 1: Controlled variables (Continued)

Material of wire	Shear moduli is dependent on the material of a body, hence different materials will have different shear moduli which can affect the torsional constant and hence period. This is further corroborated by G being present in eq. (2.5).	The same copper wire was used throughout the experiment (See section 4.1 for parameters of the system).
Radius of wire	As the radius of wire is present in eq. (2.5), different radii lead to different torsional constants thereby affecting period.	
Initial angular displacement	Although SHM oscillations are isochronous, large angular displacements could introduce damping. Constant angular displacements help maintain consistency and reduce random errors due to changes in initial conditions.	The angle was kept constant at $(10.0 \pm 0.5)^\circ$ by marking a piece of tape using a protractor from a mass-slot on the disc. It was ensured that the mass-slot always faced the retort stand at rest and the disc was displaced such that the tape marking was in front of the retort stand. This angle was chosen as the trial investigation revealed that smaller angles made it difficult to discern oscillations while larger angles introduced damping.
Airflow	Airflow could introduce damping and affect the period of oscillation as drag force opposes the rotation of the disc.	The experiment was conducted in a closed room with all air-conditioning and fan units turned off.

3.2 Apparatus

1. Steel torsional pendulum disc
2. Retort stand
3. Wire chuck × 2
4. 90 cm copper wire
5. 5 kg digital laboratory balance
6. (15 × 1) mm micrometer screw gauge
7. 60 FPS smartphone camera
8. 150 m tape measure
9. Protractor
10. 50 cm wool thread
11. Tripod with adjustable height feature
12. C-Clamp

3.3 Experimental Considerations

Safety considerations:

1. Injuries due to falling disc
   In the event the disc was to break from the system and fall, it could result in serious injuries if it were to come in contact with a person. Hence, the retort stand was set up over a tub of sand to prevent it from landing on the ground. A C-clamp was also used to clamp the base of the retort stand to prevent it from falling.

2. Wire cuts due to flickback

Two wire chucks were used to set up the pendulum system, this ensured that it wasn't possible for there to be protruding sharp ends that could cause a wire cut. When adjusting the suspension length, goggles were used to minimise potential injuries to the face.

There were no ethical or environmental issues to consider for this experiment.

3.4 Method

1. Measure the suspension length of the copper wire using the measuring tape, and adjust the amount through the wire chucks until the desired length is achieved; ensure that the disc is aligned such that one of the mass slots (see fig. 3) aligns with the retort stand.
2. Start recording on the 60 FPS camera placed on the tripod.
3. Ensure that the disc is at rest.
4. Displace the pendulum such that the 10° mark is aligned with the retort stand.
5. Release the pendulum and count 20 oscillations.
6. Stop the recording and export the video onto the editing software.
7. Ensure the track of the editing software is set to 60 FPS.
8. Cut out frames before the pendulum starts oscillating and after the pendulum finishes its 20th oscillation.
9. Identify the frame in which the pendulum finishes its oscillation and record this time.
10. Repeat steps 3 to 9 for a total of five trials each length.
11. Repeat steps 1 to 10 for the remaining lengths.

3.5 Diagrams of Experimental Setup

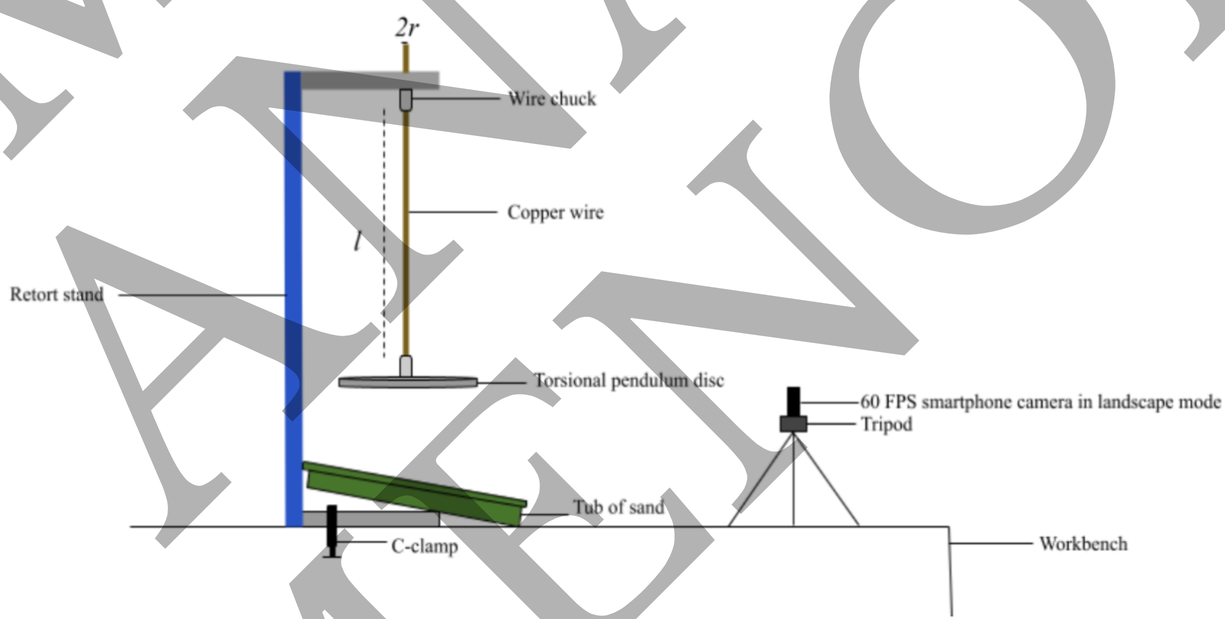

Figure 2: Front view; drawn on Google Drawings by author

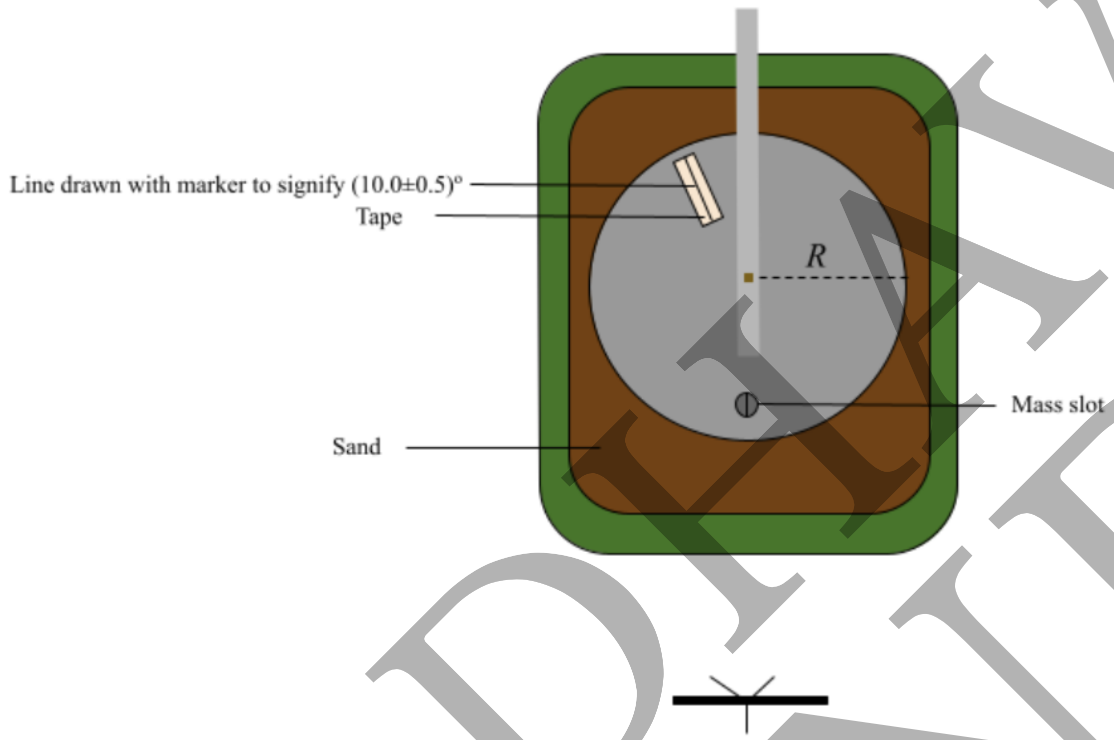

Figure 3: Top view; drawn on Google Drawings by author

4 Experimental Data

4.1 Relevant parameters of the system

Mass of disk, $M = (1.115 \pm 0.001)$ kg
Radius of disk, $R = (6.62 \pm 0.05)$ cm
Radius of copper wire, $r = (0.46 \pm 0.01)$ mm

The mass of the disk was measured using a digital laboratory balance that quoted responses to three decimal places. Thus, the uncertainty in mass is the smallest division: 0.001 kg. Due to presence of the wire chuck and mass slots on the disk, its radius could not be measured directly using a ruler. Hence, a piece of woolen thread was looped around the disk and its length was measured using a metre ruler. This length was the circumference, C, of the disk. The radius was thus $R = \frac{C}{2\pi} = \frac{41.6\,\text{cm}}{2\pi} \approx 6.62$ cm. The smallest division of the ruler was 1 mm, hence the uncertainty is $\frac{1\,\text{mm}}{2} = 0.5$ mm $= 0.05$ cm. The radius of the wire was measured using a 15×1 mm micrometer screw-gauge with a least count of 0.01 mm.

4.2 Qualitative Observations

It could be observed that greater suspension lengths resulted in larger periods of oscillation. It was also found that at smaller suspension lengths, there was greater "wobbling" as the disc no longer tended to oscillate in the same plane. Potential strategies to prevent this in the future is outlined in section 6.

4.3 Raw Data

Table 2: Time taken (s) for 20 oscillations for each suspension length l (m)

l (m) ± 0.0005 m	Time taken for 20 oscillations (s) ± 0.01 s					Standard Deviation σ (s)
	T_1	T_2	T_3	T_4	T_5	
0.4800	74.32	74.10	74.12	74.33	74.25	0.09728
0.5000	76.17	76.00	75.83	75.83	75.93	0.1266
0.5200	76.88	76.73	76.93	76.73	76.90	0.08639
0.5400	78.53	78.87	78.63	78.67	78.62	0.1127
0.5600	79.57	79.48	79.38	79.53	79.63	0.08471
0.5800	80.80	80.82	80.80	80.77	81.03	0.09436
0.6000	82.37	82.28	82.23	82.23	82.13	0.07808
0.6200	83.47	83.58	83.47	83.65	83.45	0.07787
0.6400	84.98	85.07	85.05	84.97	84.88	0.06723
0.6600	86.33	86.00	86.30	86.25	86.23	0.1165

The smallest scale division on the tape measure was 1 mm, thus the uncertainty in length measurements was half the smallest scale division: 0.5 mm = 0.0005 m. The editing software and recording had a framerate of 60 FPS = 60 s^{-1}. Thus, the number of seconds per frame is $\frac{1}{60\,\mathrm{s}^{-1}} = \frac{1\,\mathrm{s}}{60}$. However, the movement of the disc could also have occurred in between frames. Thus, the uncertainty is considered to be half of the number of seconds per frame: $\frac{1}{2} \times \frac{1\,\mathrm{s}}{60} = \frac{1\,\mathrm{s}}{120} \approx 0.008$ s. Since the software only quoted time to two decimal places, the uncertainty in the time measurement was rounded to the nearest hundredths to give 0.01 s.

4.4 Processed Data

To obtain the period of one oscillation for a given length, the mean of the five trials for each length were calculated and divided by 20. Each value was then rounded off to four significant figures as all measured times had four significant figures.

Sample calculation for $l = 0.4800$ m:

$$\overline{T} = \frac{1}{20} \times \frac{(74.32 + 74.10 + 74.12 + 74.33 + 74.25)\,\mathrm{s}}{5} \approx 3.711\,\mathrm{s}$$

As the standard deviation for period was different for each length, the uncertainty for the period of one oscillation for all the lengths also had to be calculated. This was done by taking the half the difference between the upper and lower bounds of the time recorded for each length in table 2 and dividing it by 20. This was then rounded off to one significant figure.

Sample calculation for $l = 0.4800$ m:

$$\Delta\overline{T} = \frac{1}{20} \times \frac{([74.33 + 0.01] - [74.10 - 0.01])\,\mathrm{s}}{2} \approx 0.006\,\mathrm{s}$$

Table 3: Mean period $\overline{T}$ (s) for each suspension length l (m)

l (m) ± 0.0005 m	$\overline{T}$ (s)	$\Delta\overline{T}$ (s)
0.4800	3.711	0.006
0.5000	3.798	0.009
0.5200	3.842	0.006
0.5400	3.933	0.009
0.5600	3.976	0.007
0.5800	4.042	0.007
0.6000	4.112	0.007
0.6200	4.176	0.006
0.6400	4.250	0.005
0.6600	4.311	0.009

Relationship 2.11 tells us that a plot of T^2 against l will be linear. Hence, we must calculate the respective values of $\overline{T}^2$ and their uncertainties $\Delta\overline{T}^2$ to be able to plot a graph. Sample calculation for $l = 0.4800$ m:

$$\overline{T}^2 = (3.711\,\mathrm{s})^2 \approx 13.77\,\mathrm{s}^2$$
$$\frac{\Delta\overline{T}^2}{\overline{T}^2} = 2\frac{\Delta\overline{T}}{\overline{T}} \Rightarrow \Delta\overline{T}^2 = 2\overline{T}\Delta\overline{T}$$
$$\Delta\overline{T}^2 = 2(3.711\,\mathrm{s})(0.006\,\mathrm{s}) \approx 0.04\,\mathrm{s}^2$$

$\Delta\overline{T}^2$ was rounded to one significant figure as its calculation involved $\Delta\overline{T}$ which is the least precise quantity with only one significant figure.

Table 4: Mean period squared $\overline{T}^2$ (s^2) for each suspension length l (m)

l (m) ± 0.0005 m	$\overline{T}^2$ (s^2)	$\Delta\overline{T}^2$ (s^2)
0.4800	13.77	0.04
0.5000	14.42	0.07
0.5200	14.76	0.05
0.5400	15.47	0.07
0.5600	15.81	0.06
0.5800	16.34	0.06
0.6000	16.91	0.06
0.6200	17.44	0.05
0.6400	18.06	0.04
0.6600	18.58	0.08

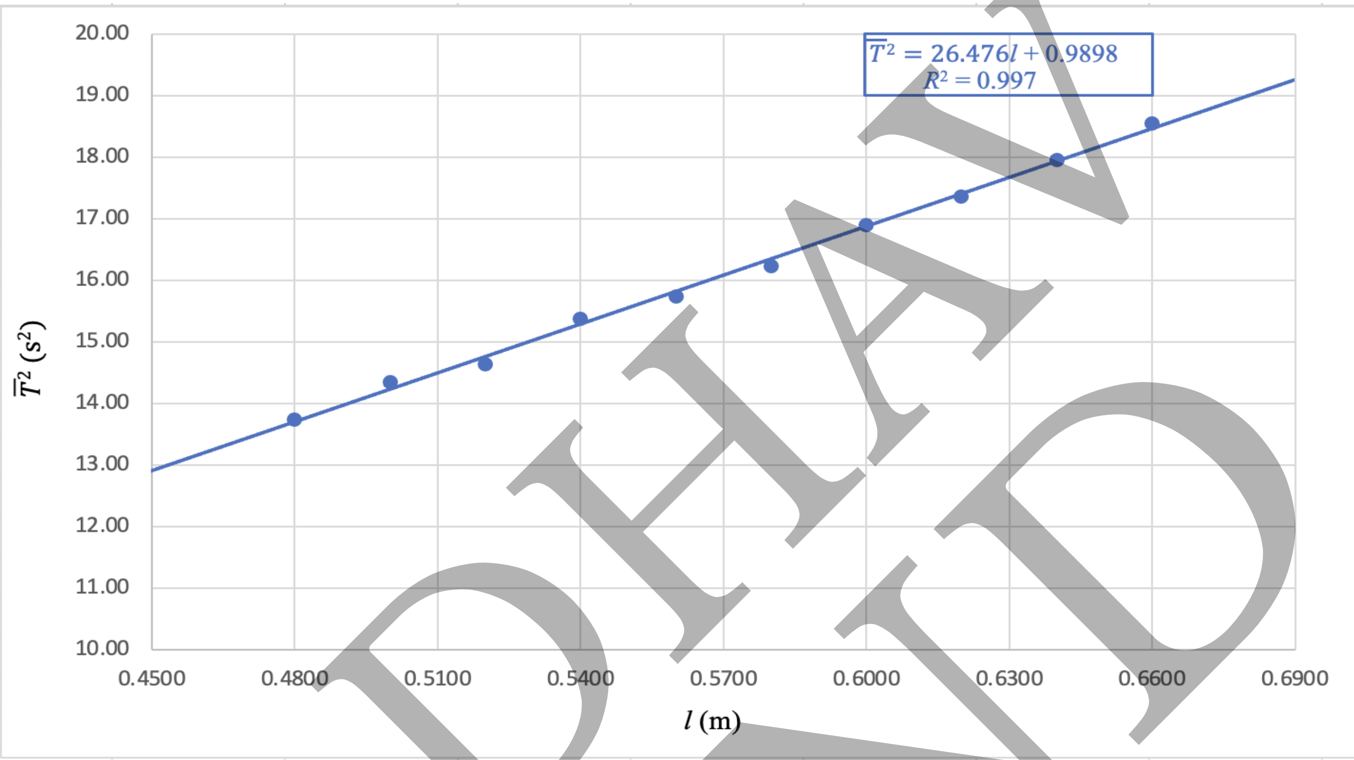

Figure 4: Graph of $\overline{T}^2$ (s^2) against l (m); Plotted on Microsoft Excel by author

It should be noted that the error bars for $\overline{T}^2$ and l are not visible as their uncertainties are negligibly small. Maximum and minimum lines are also not displayed due to this reason.

5 Analysis

It was hypothesised that there would be a positive, directly proportional relationship between the square of the period and suspension length of a torsional pendulum. Fig. 4 clearly shows a positive linear relationship between $\overline{T}^2$ and l, thus partly confirming the hypothesis. However, the two quantities are not proportional as there is a non-zero $\overline{T}^2$ intercept of 0.9898. This indicates that there are systematic errors in this experiment. Potential systematic errors include friction between the copper wire and the wire chuck and the copper wire having non-negligible mass. As the wire chuck was extremely narrow and tightened over the pendulum, there was significant contact between the wire and wire chuck, thus potentially resulting in non-negligible friction. Furthermore, it was assumed that the copper wire had negligible mass; as the copper wire did have mass, it would have contributed to the moment of inertia of the system. These are sensible sources of systematic error as despite there being friction, damped oscillations did not take place and despite the copper wire contributing to the moment of inertia of the system, there was a low percentage error in the calculation of shear modulus (see Determining shear modulus). Hence, the magnitude of the proposed systematic errors are shown to be low which is confirmed by the $\overline{T}^2$ intercept being less than 1.000 and thus, relatively close to the origin. Furthermore, the error bars being small to the point of them being negligible demonstrates a high precision in the data collected. This is further shown as the measured time for each trial in table 1 is within a fraction of a second of the other trials. This is also confirmed by the line of best fit touching every single data point in fig. 4. As no other mathematical model can be used as a line of best fit in fig. 4, it can be said that random errors are low.

Determining shear modulus

As the data in table 3 was only quoted to four significant figures, the gradient was rounded to four significant figures as the calculation of the gradient would have involved computing $gradient = \frac{\overline{T_2}^2 - \overline{T_1}^2}{l_2 - l_1}$. As each quantity has four significant figures, the gradient would have four significant figures. The theoretical gradient is outlined in section 2. Thus,

$$\frac{4\pi M R^2}{G r^4} = 26.48\ \mathrm{s^2\,m^{-1}} \tag{5.1}$$

Rearranging for the shear modulus,

$$G = \frac{4\pi M R^2}{26.48 r^4} \tag{5.2}$$

Substiting in the relevant parameters after converting into SI units,

$$\begin{aligned} G &= \frac{4\pi \cdot (1.115)\ \mathrm{kg} \cdot (0.0662)^2\ \mathrm{m}^2}{26.48\ \mathrm{s^2 m^{-1}} \cdot (0.00046)^4\ \mathrm{m}^4} \\ &\approx 5.2 \times 10^{10}\ \mathrm{Pa} \quad (2\ \mathrm{s.f}) \end{aligned} \tag{5.3}$$

The equation for the absolute uncertainty of G is,

$$\begin{aligned} \frac{\Delta G}{G} &= \frac{\Delta M}{M} + 2\frac{\Delta R}{R} + 4\frac{\Delta r}{r} \\ \implies \Delta G &= G\left(\frac{\Delta M}{M} + 2\frac{\Delta R}{R} + 4\frac{\Delta r}{r}\right) \end{aligned} \tag{5.4}$$

Substituting the obtained value and the parameter of the systems and their uncertainties into eq. (5.4),

$$\begin{aligned} \Delta G &= (5.2 \times 10^{10}\ \mathrm{Pa}) \times \left(\left[\frac{0.001}{1.115}\right] + 2\left[\frac{0.0005}{0.0662}\right] + 4\left[\frac{0.00001}{0.00046}\right]\right) \\ &\approx 5 \times 10^9\ \mathrm{Pa} \quad (1\ \text{s.f as it is an absolute uncertainty}) \end{aligned}$$

Therefore the shear modulus and its absolute uncertainty is,

$$G = (50 \pm 5) \times 10^9\ \mathrm{Pa} \tag{5.5}$$

The percentage uncertainty is,

$$\frac{\Delta G}{G} \times 100\% = \frac{5}{50} \times 100\% = 10\%$$

According to literature values, the shear modulus for copper is 46.0 GPa ("Copper, Cu"). Therefore, the percentage error between the obtained shear modulus value and the literature value is given by,

$$\begin{aligned} Percentage\ \ error &= \left|\frac{(50 - 46.0) \times 10^9}{46.0 \times 10^9}\right| \times 100\% \\ &\approx 9\% \quad (1\ \mathrm{s.f}) \end{aligned}$$

Thus, the shear modulus of copper has been experimentally obtained to a relatively high degree of accuracy with a percentage uncertainty of 10% and a percentage error with respect to literature values of 9%. The percentage uncertainty could have been lower if equipment with finer divisions were available. For example, since only a manual (15× 1) screw-gauge was available, the percentage uncertainty for r was increased as its fractional uncertainty was multiplied by 4 since r was raised to the fourth power. The same holds for R: the method of using a string to determine the circumference would not be ideal due to string fibers that extended beyond the mark made on the ruler. This discrepancy from the true value would have been further exacerbated as R was raised to the second power. Nevertheless, the obtained value is still remarkably accurate.

Given the low percentage error in the value of G, it is shown that it is close to its true value; combined with the fact that the graph is still close to the origin, it is reasonable to say that $\overline{T}^2$ is directly proportional to l.

6 Evaluation

Table 5: Strengths

Strength	Impact on study
Oscillations were subjected to video analysis	Subjecting the oscillations to video analysis helped reduce random errors due to reaction time. Video analysis helped ensure that the approximate frame at which an oscillation ended could be seen, thereby giving an accurate estimate of the period of oscillations. This is shown by a low absolute uncertainty in the measured periods as well as a high precision in the data.
A wide range of lengths was chosen with a suitable number of trials for each length	A total of 10 lengths were chosen with 5 trials for each length, giving way to a total of 50 trials for the entire experiment. Hence, it was possible to establish that the equation for the period of a torsional pendulum holds over the range of 0.4800 to 0.6600 m.
Environmental conditions helped minimise damping	Oscillatory systems like the torsional pendulum can be subjected to damping, thereby making undamped analysis difficult. Luckily, this was not the case as the experiment was conducted in a room with minimal airflow. Hence, air resistance had a negligible effect on the range of lengths that were investigated.

Table 6: Limitations and possible improvements

Limitation	Explanation & Impact on study	Possible improvements
Copper wire was not uniform in shape	The only copper wire available was not straight and had bends throughout. Hence, the radius of the wire had to be measured by taking readings with screw-gauge at different points in the wire and averaging them, thereby resulting in only an approximate. Furthermore, the bends could have increased damping effects as small crevices could have allowed for drag forces to have a greater effect on oscillations. Nevertheless, the damping effect was negligible and so the main inaccuracy contributed by the deformations is in the measurement of the radius of the wire. As the wire was not newly purchased, there was no manufacturing reference label to compare the experimentally obtained radius to that quoted by the manufacturer.	The copper wire could have been heated prior to the experiment to help flatten it. However, this will cause the copper to expand and may alter the shear modulus as properties of materials are typically temperature dependent.
Method of measuring the radius of the pendulum disc	The length of the wool thread that wrapped around the disc was measured using a metre ruler. A smaller ruler may have been more appropriate. Furthermore, limitations included thin fibers of the thread extending from the end. Therefore, it was difficult to mark where the thread ended, leading to inaccuracies in the measurement of the circumference of the disc, thereby leading to further inaccuracies in the measurements of period and shear modulus. As the disc was not newly purchased, there was no manufacturing reference label to compare to.	A large spherometer could be used to measure the diameter of the disc. Alternatively, the manufacturing specification could also be saved for reference.

Human error in video analysis	Although 60 FPS is considered to be suitable for video analysis, there was still an element of human error involved as the frame in which an oscillation was completed had to be determined by eye. As the difference between frames was mere pixels, it is possible that the "wrong" frame had been selected to determine period. Nevertheless, the difference between each frame was 0.01 s (rounded to 1 s.f), and since an approximately "correct" frame was chosen, it is unlikely that this error is significant. Framerate lags were also present in some of the videos, likely introduced upon uploading into the editing software, which made it harder to determine the frame an oscillation ended.	A camera and a video editor that supports a higher frame rate could be used. A video analysis tracker could also have been employed to help facilitate the tracking of an oscillation.
Oscillations not taking place in the same plane	At smaller suspension lengths, the pendulum disc "wobbled"; it did not always oscillate in the same plane. Trials that had more wobbling than others for the same suspension length resulted in approximately the same time taken for 20 oscillations, and therefore it is reasonable to say that this limitation did not significantly hinder the investigation. Trials that had significant wobbling were repeated. The wobbling was likely caused by displacing the pendulum by hand, it is possible that the disc was displaced at an angle with respect to the vertical, thereby resulting in tension forces acting at an angle to the center of the mass of the disc.	A motor system could be used to ensure that the suspension length is not displaced with an angle to the vertical.

7 Conclusion and Extension

This investigation focused on the relationship between the **suspension length** l and **period** T of a torsional pendulum. It has been shown that the square of the period is directly proportional to the suspension length theoretically, and has been confirmed experimentally through the calculation of the shear modulus of copper. Thus, one of the aims of the experiment, to verify eq. (2.9) has been achieved.

Future work could involve using a greater range of lengths. For example, the lengths in the range 0.4000 to 0.7000 m that were not tested could be used. This will help ensure that the equation holds for a greater range. Furthermore, wires of different lengths could be tested for their shear modulus and compared to literature values to further confirm if the method of using the relationship between suspension length and period to determine shear modulus holds. Alternatively, the shear modulus of copper could be recalculated using a different method. A variation of the experiment involves slotting laboratory masses on the mass slot and varying the suspension length and distance between the laboratory masses to extrapolate the shear modulus. This method was used by Kumavat et al. and resulted in a shear modulus value of 5.3438×10^{10} Pa, which happens to be an overestimate. Alternatively, different variables like radius of the wire, mass of the disk, or radius of the disc could be changed. This investigation was also constrained using the small angle approximation. Future investigations could look at how the period is affected by the initial angular displacement. As the motion will not be simple-harmonic, they will no longer be isochronous. It would be interesting to see how the mathematical equation for the period changes.

It is fascinating how a system as simple as a disc suspended on a wire can be leveraged to unearth information about different materials. This goes to show how Physics is not only a subject that serves its own purpose, but also has practical applications in other fields. For example, information about shear moduli and torsional constants can be considered when deciding on the material of a certain object. Large gusts of wind can cause bridges to twist (BridgeMasters), and hence information about these constants will be crucial in determining suitable material to build the beams and trusses of a bridge. While these values are often determined using advanced machinery on an industrial scale, this investigation shows that these values can be recalculated to a relatively high degree of accuracy using simple equipment found in a school laboratory. In fact, reasonable overestimates may be better for such quantities as unpredictable weather events may cause unanticipated torsion in bridges.

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Appendix A Screenshot from Editing Software

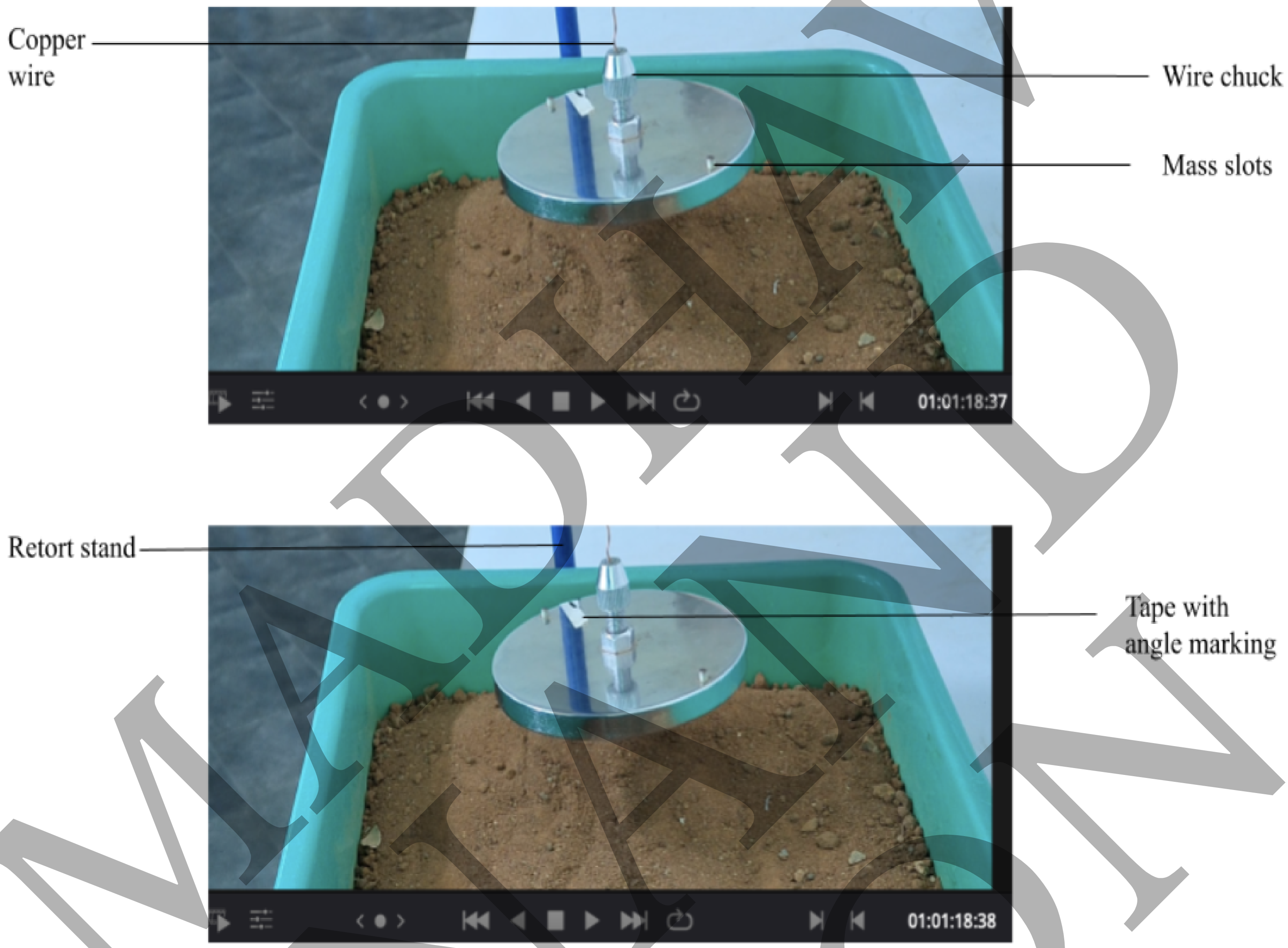

Figure 5: Screenshot from DaVinci Resolve Editing Software

The two images here are one frame apart and taken mid-oscillation for l = 0.6000m. This image shows how close two frames are, and therefore hard to determine the exact frame in which an oscillation ended; thus illustrating the limitation of human error in video analysis.