Assignment 3

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Problem 1

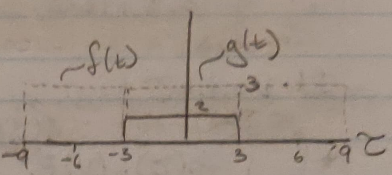
a)

1.a)

$$f(t)=\begin{cases}3, & |t|<3\\ 0, & \text{otherwise}\end{cases} \qquad g(t)=\begin{cases}2, & |t|<3\\ 0, & \text{otherwise}\end{cases}$$

$$y(t)=f(t)*g(t)=\int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau$$

$y(t)$ exists for $-6<t<6$



$$\therefore y(t)=\int_{-6}^{6} f(\tau)g(t-\tau)\,d\tau.$$

$$=f(\tau)\int_{-6}^{6} g(t-\tau)\,d\tau-\int_{-6}^{6} \overset{0}{\cancel{f'(\tau)}}\left[\int_{-6}^{6} g(t-\tau)\,d\tau\right]d\tau.$$

we know that $f(\tau)=3 \therefore f'(\tau)=0$

$$y(t)=3\int_{-6}^{6} g(t-\tau)\,d\tau.$$

Split into $-6<t<0$ & $0<t<6$.

$\underline{-6<t\leq 0}$

$$y(t)=3\int_{-3-t}^{3} 2\,d\tau = 6\tau\Big|_{-3-t}^{3}$$

$$=18-(6(-3-t))$$
$$=18-(-18-6t)$$
$$=36+6t$$

$\underline{0<t<6}$

$$y(t)=3\int_{-3}^{3-t} 2\,d\tau = 6\tau\Big|_{-3}^{3-t} = 6(3-t)+18$$

$$=36-6t.$$

$$\boxed{\therefore y(t)=\begin{cases}36+6t & t\in[-6,0]\\ 36-6t & t\in(0,+6]\\ 0 & \text{otherwise}\end{cases}}$$

b & c)

```
%1b&c
clear; clc ; close all;

syms t

a = 3; b = 3;
sig1 = @(t) a*(abs(t)<=b) + 0;

a = 2; b = 3;
sig2 = @(t) a*(abs(t)<=b) + 0;

start = -10;
finish = 10;
interval = 0.01;

t = start:interval:finish;

fig1 = figure(1);
set(fig1,'Position', [100 100 1100 300]);
plot(t,sig1(t),'-b', 'linewidth', 2);hold on;
plot(t,sig2(t),'-r', 'linewidth', 2);
legend('h(x)', 'x(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 4])
xlabel('\bf Time');xlim([-5 5])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');

y = [];
for ii = t
        sig3 = @(t) sig1(t) .* sig2(t-ii);
        y(end+1) = integral(sig3,-inf,inf);
end


fig2 = figure(2);
set(fig2,'Position', [100 100 1100 300]);
plot(t,y,'-g', 'linewidth', 2);
legend('y(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 40])
xlabel('\bf Time');xlim([-10 10])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');

sig3_conv = conv(sig1(t),sig2(t),'same')*interval;
fig3 = figure(3);
set(fig3,'Position', [100 100 1100 300]);
plot(t,sig3_conv,'-m', 'linewidth', 2);
legend('y(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 40])
xlabel('\bf Time');xlim([-10 10])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
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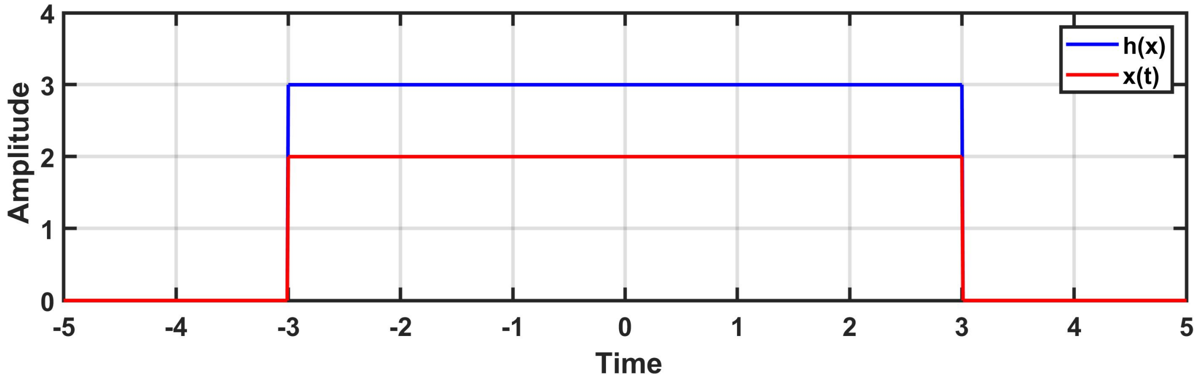
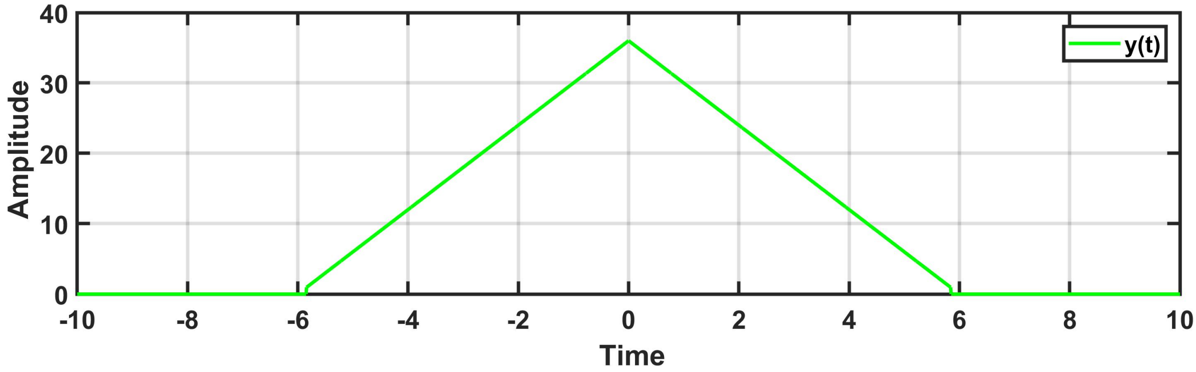
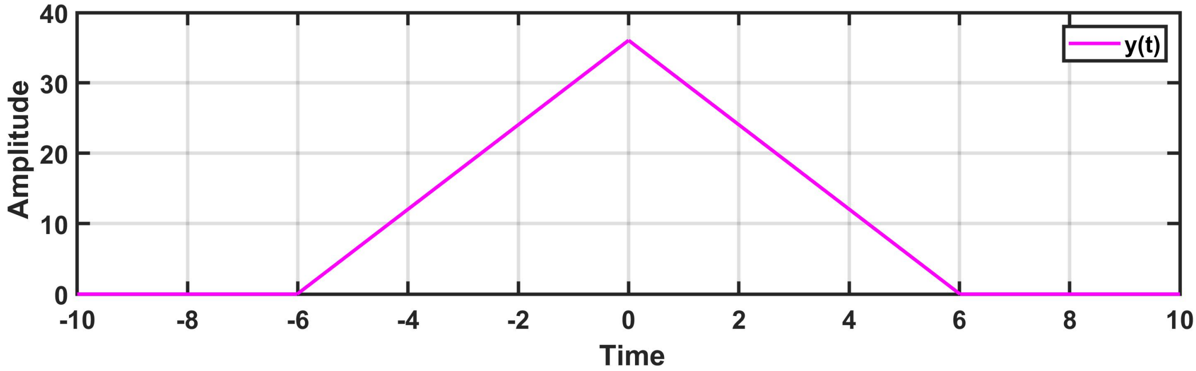






Problem 2

a)

2.a)

$$F\{x(t)*h(t)\} = X(f)\cdot H(f).$$

L.S. $$F\{x(t)*h(t)\} = \int_{-\infty}^{\infty}[x(t)*h(t)]e^{-i2\pi ft}\,dt.$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(t-\tau)e^{-i2\pi ft}\,d\tau\,dt.$$

Let $\nu = t-\tau$.

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(\nu)e^{-i2\pi f(\nu+\tau)}\,d\tau\,d\nu$$

$$=\int_{-\infty}^{\infty} x(\tau)e^{-i2\pi f\tau}\,d\tau\cdot\int_{-\infty}^{\infty} h(\nu)e^{-i2\pi f\nu}\,d\nu$$

$= X(f)\cdot H(f) =$ R.S. QED!

$$F\{x(t)\cdot h(t)\} = X(f)*H(f)$$

L.S.

$$F\{x(t)\cdot h(t)\} = \int_{-\infty}^{\infty} x(t)h(t)e^{-i2\pi ft}\,dt.$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(f_1)e^{i2\pi f_1 t}\cdot H(f_2)e^{i2\pi f_2 t}\cdot e^{-i2\pi ft}\,df_1\,df_2\,dt.$$

$$=\int_{-\infty}^{\infty} X(f_1)\int_{-\infty}^{\infty} H(f_2)\int_{-\infty}^{\infty} e^{i2\pi t(f_1+f_2-f)}\,df_1\,df_2\,dt.$$

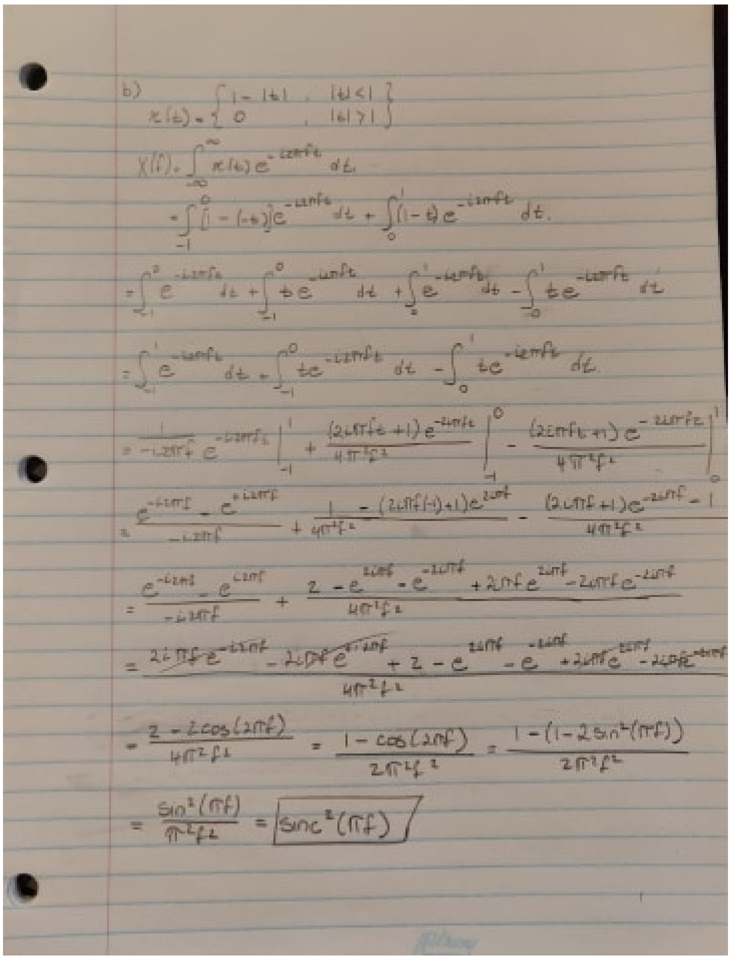
$$=\int_{-\infty}^{\infty} X(f_1)\int_{-\infty}^{\infty} H(f_2)\cdot\delta(f_1+f_2-f)\,df_1\,df_2$$

$= \int_{-\infty}^{\infty} X(f_1)\,H(f-f_1)\,df_1 = X(f)*H(f) =$ R.S. QED!

Convolution.

This relationship means that by performing a convolution of two signals in the time domain and then taking the Fourier transform would result in the same thing as simply taking the Fourier transform of both signals in the frequency domain individually and then multiplying them together. It also shows that one can multiply two signals in the time domain and then take the Fourier transform of the product to get a convolution of the individual Fourier transform of each signal in the frequency domain.

b)



b)

$$x(t) = \begin{cases} 1-|t| , & |t|<1 \\ 0 , & |t|>1 \end{cases}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi f t} \, dt$$

$$= \int_{-1}^{0} [1-(-t)] e^{-i2\pi f t} \, dt + \int_{0}^{1} (1-t) e^{-i2\pi f t} \, dt$$

$$= \int_{-1}^{0} e^{-i2\pi f t} \, dt + \int_{-1}^{0} t e^{-i2\pi f t} \, dt + \int_{0}^{1} e^{-i2\pi f t} \, dt - \int_{0}^{1} t e^{-i2\pi f t} \, dt$$

$$= \int_{-1}^{1} e^{-i2\pi f t} \, dt + \int_{-1}^{0} t e^{-i2\pi f t} \, dt - \int_{0}^{1} t e^{-i2\pi f t} \, dt$$

$$= \frac{1}{-i2\pi f} e^{-i2\pi f t} \Big|_{-1}^{1} + \frac{(2i\pi f t+1) e^{-2i\pi f t}}{4\pi^2 f^2} \Big|_{-1}^{0} - \frac{(2i\pi f t+1) e^{-2i\pi f t}}{4\pi^2 f^2} \Big|_{0}^{1}$$

$$= \frac{e^{-i2\pi f} - e^{+i2\pi f}}{-i2\pi f} + \frac{1 - (2i\pi f(-1)+1) e^{2i\pi f}}{4\pi^2 f^2} - \frac{(2i\pi f+1) e^{-2i\pi f} - 1}{4\pi^2 f^2}$$

$$= \frac{e^{-i2\pi f} - e^{i2\pi f}}{-i2\pi f} + \frac{2 - e^{2i\pi f} - e^{-2i\pi f} + 2i\pi f e^{2i\pi f} - 2i\pi f e^{-2i\pi f}}{4\pi^2 f^2}$$

$$= \frac{\cancel{2i\pi f e^{-i2\pi f}} - \cancel{2i\pi f e^{+i2\pi f}} + 2 - e^{2i\pi f} - e^{-2i\pi f} + \cancel{2i\pi f e^{2i\pi f}} - \cancel{2i\pi f e^{-2i\pi f}}}{4\pi^2 f^2}$$

$$= \frac{2 - 2\cos(2\pi f)}{4\pi^2 f^2} = \frac{1-\cos(2\pi f)}{2\pi^2 f^2} = \frac{1-(1-2\sin^2(\pi f))}{2\pi^2 f^2}$$

$$= \frac{\sin^2(\pi f)}{\pi^2 f^2} = \boxed{\operatorname{sinc}^2(\pi f)}$$

```
%2b
clear; clc ; close all;

syms t
Fs = 100;           % Sampling frequency
start = -4;
finish = 4;
interval = 1/Fs;
t = start:interval:finish;
L = length(t);
n = 2^nextpow2(L);

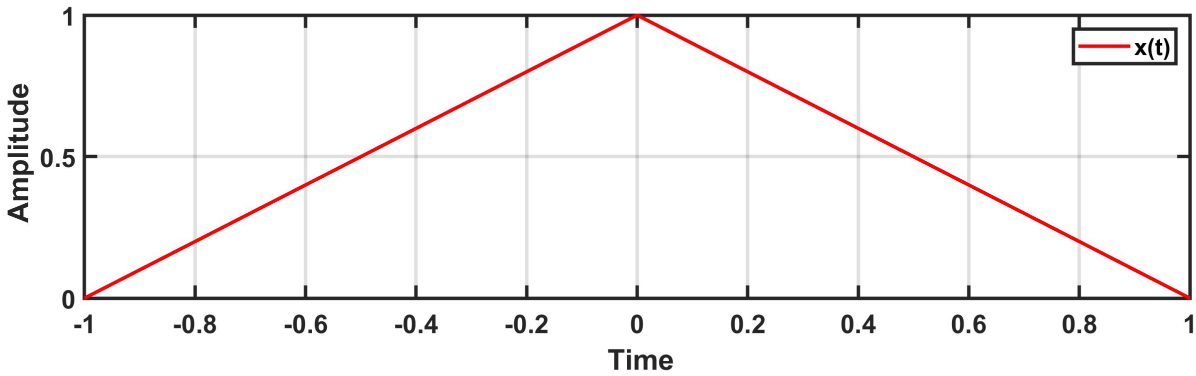
sig1 = @(t) (1-abs(t)).*(abs(t)<1);

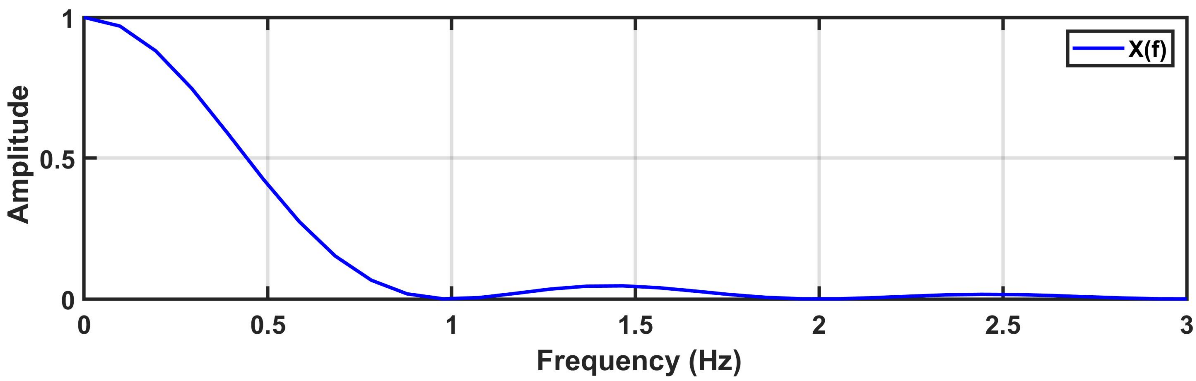
Xf = fft(sig1(t),n);
f = 1/(n*interval) * (0:n/2-1);

fig1 = figure(1);
set(fig1,'Position', [100 100 1100 300]);
plot(t,sig1(t),'-r', 'linewidth', 2);
legend('x(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 1])
xlabel('\bf Time');xlim([-1 1])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_2a.jpg','Resolution',300)

fig2 = figure(2);
set(fig2,'Position', [100 100 1100 300]);
plot(f,1/Fs*abs(Xf(1:n/2)),'-b', 'linewidth', 2);
```

```
legend('X(f)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 1])
xlabel('\bf Frequency (Hz)');xlim([0, 3])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_2b.jpg','Resolution',300)
```





c)

Based on part b we found that the Fourier transform of a triangular signal is sinc2(t). From the lecture slides it was also proven that the Fourier transform of a square signal results in sinc(t). From problem 1 when we find the convolution of two square functions we get a triangular signal. As such we prove that the Fourier transform of the convolution is equal to the product of the individual Fourier transform.

$$F\left(h(t) * x(t)\right) = H(f)X(f)$$

$$LS:\ F(h(t) * x(t)) = F(sinc(t)) = sinc^2(t)$$

$$RS: H(f)X(f) = sinc(t) \times sinc(t) = \ sinc^2(t)$$

Problem 3

a)

X(f) is a Fourier transform of a continuous time signal to a continuous frequency domain.

Xs(f) is a Fourier transform of a discrete time signal to a continuous frequency domain.

X(k) is a Fourier transform of a discrete time signal to a discrete frequency domain.

b)

The graph on the left demonstrates that when a discrete time signal is Fourier transformed, the signal becomes periodic based on the sampling period used to obtain the discrete time signal. The image on the right shows that if the sampling frequency is not high enough such that the Nyquist frequency is below that of the time signals true frequency, overlapping between the period frequency signals (explained above) will occur causing aliasing.

c)

This relationship tells us that the Fourier transform of a discrete time signal sampled at an interval of Δ is the same as the Fourier transform of a discrete time signal shifted by a multiple of the sampling frequency. In other words Xs(f) = Xs(f + r/Δ).

d)

This relationship tells us that the Fourier transform of a discrete time signal is equal to the sum of the Fourier transform of a continuous signal shifted by a multiple of the sampling frerquency and divided by the sampling period. This sum of the shifting signal can result in aliasing when the Nyquist frequency is not larger than the true frequency of the signal resulting in overlaps between neighboring signals. This can be avoided by having a Nyquist frequency higher than the true frequency of the signal.

Problem 4

a) & b)

```
%4a&b
clear; clc ; close all;

syms t
Fs = 50;            % Sampling frequency
start = 0;
finish = 5;
interval = 1/Fs;
t = start:interval:finish;
L = length(t);
n = 2^nextpow2(L);

z1 = @(t) exp(-2.*abs(t)).*(2*cos(2*pi*3.*t)+6*cos(2*pi*6.*t));
z2 = @(t) exp(-0.3.*abs(t)).*(10*cos(2*pi*5.*t)+3*cos(2*pi*8.*t));

fig1 = figure(1);
set(fig1,'Position', [100 100 700 500]);
plot(t,z1(t),'-r', 'linewidth', 2); hold on;
plot(t,z2(t),'-b', 'linewidth', 2);
legend('z1(t)','z2(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([-10 10])
xlabel('\bf Time');xlim([0 5])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_4a.jpg','Resolution',300)

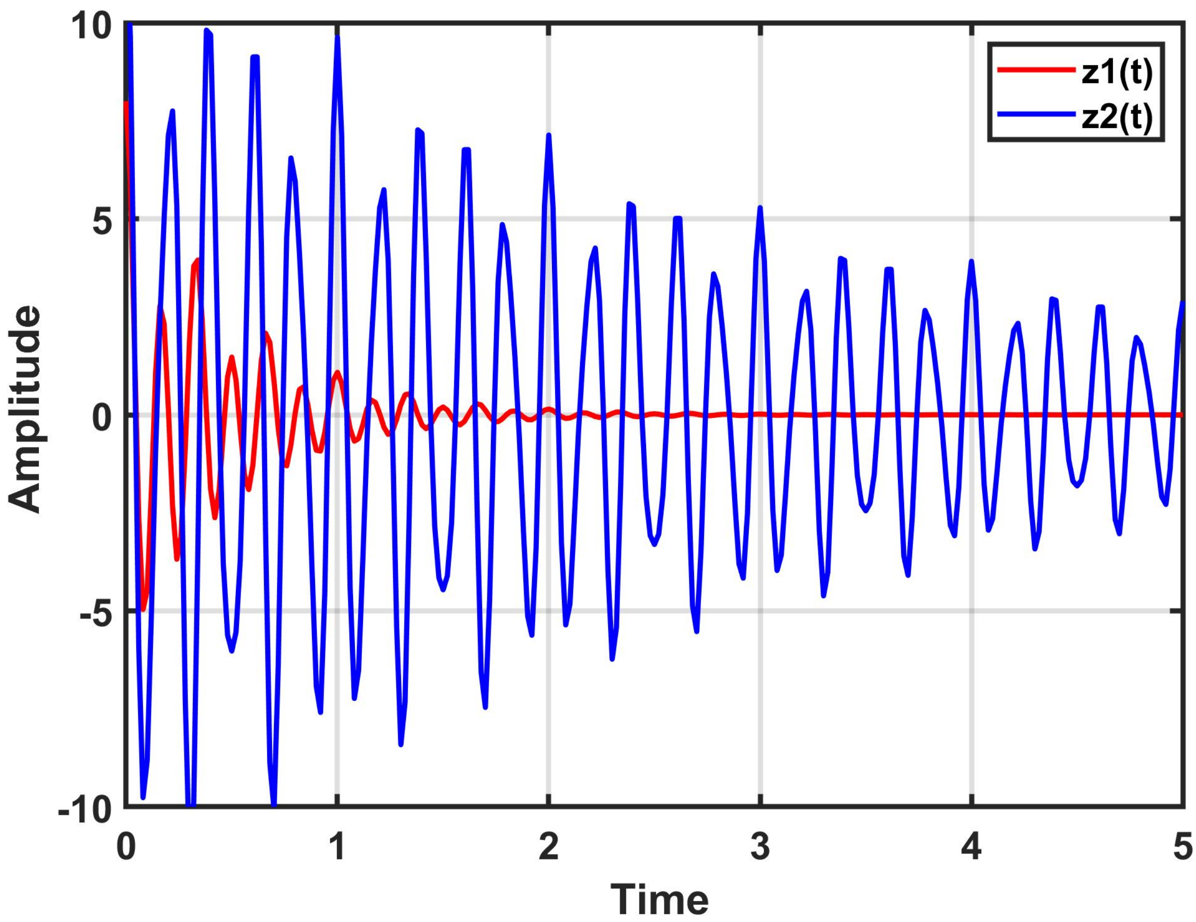
Xk1 = fft(z1(t),n);
```

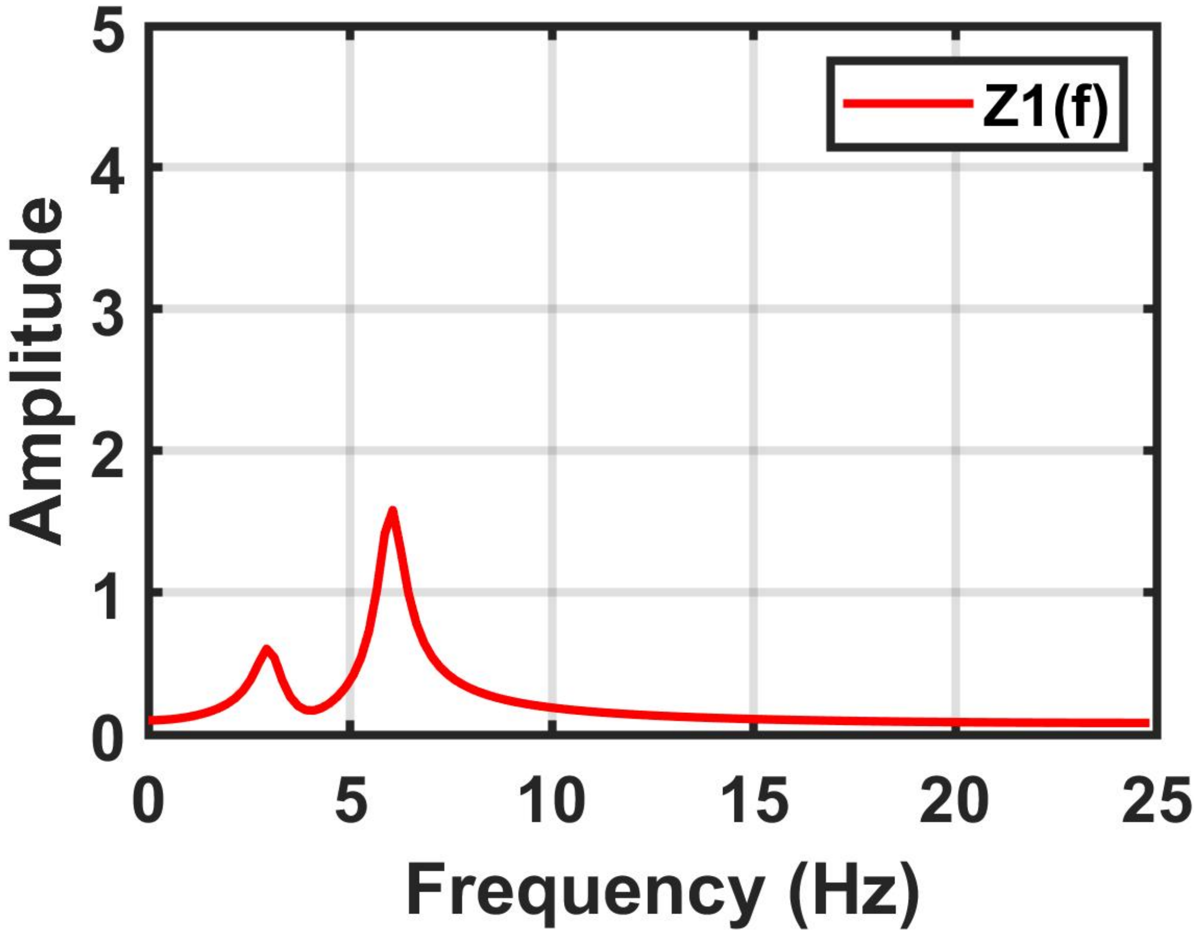
```
Xk2 = fft(z2(t),n);

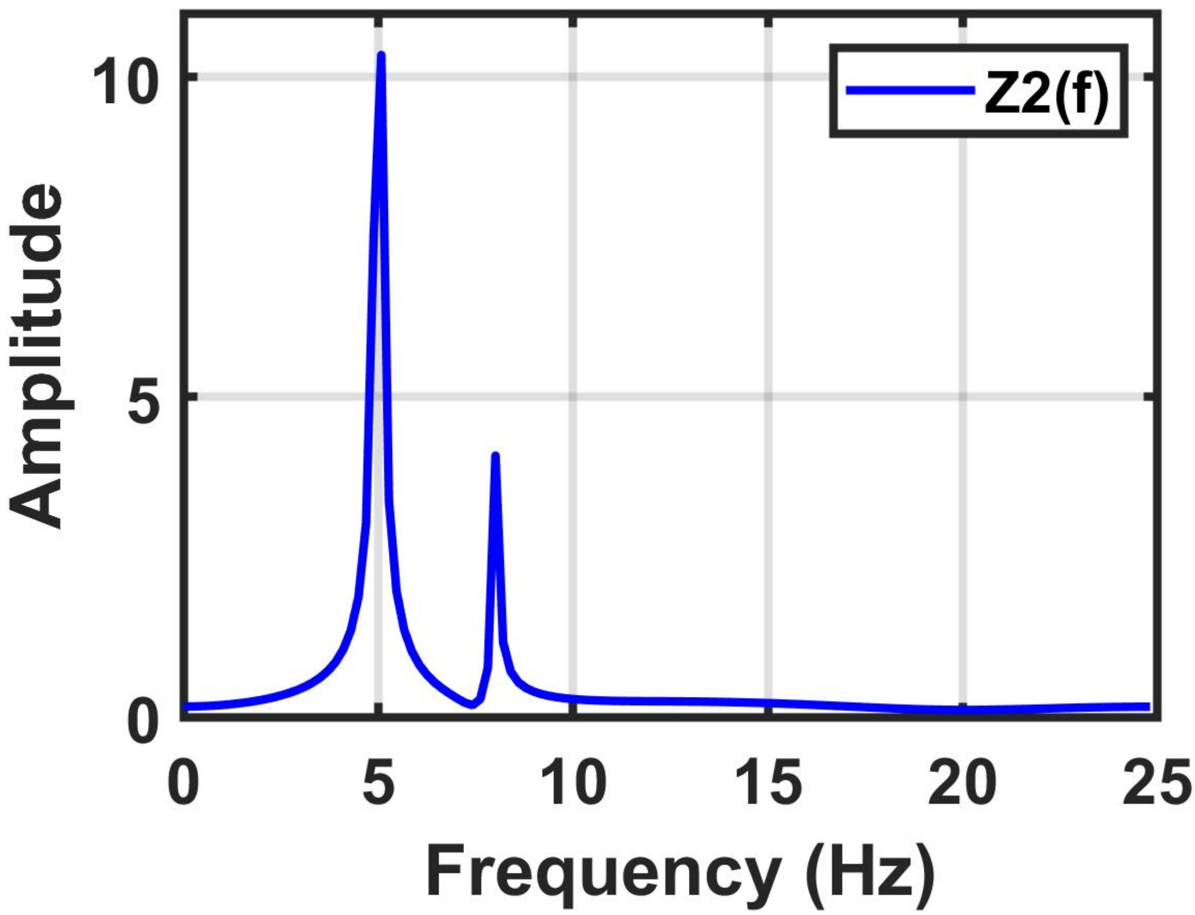
f = 1/(n*interval) * (0:n/2-1);

fig2 = figure(2);
set(fig2,'Position', [100 100 400 300]);
plot(f,1/Fs*abs(Xk1(1:n/2)),'-r', 'linewidth', 2);
legend('Z1(f)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 5])
xlabel('\bf Frequency (Hz)');xlim([0, Fs/2])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_4b.jpg','Resolution',300)

fig3 = figure(3);
set(fig3,'Position', [100 100 400 300]);
plot(f,1/Fs*abs(Xk2(1:n/2)),'-b', 'linewidth', 2);
legend('Z2(f)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 11])
xlabel('\bf Frequency (Hz)');xlim([0, Fs/2])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_4c.jpg','Resolution',300)
```







c)

Z2 is the narrower graph. This difference is made by the "a" coefficient as decreasing that value widens the signal on the time graph and reduces the slope of the exponential function, resulting in a thinner graph on the frequency domain.

Problem 5

a) & b)

```
%5a&b
clear; clc ; close all;

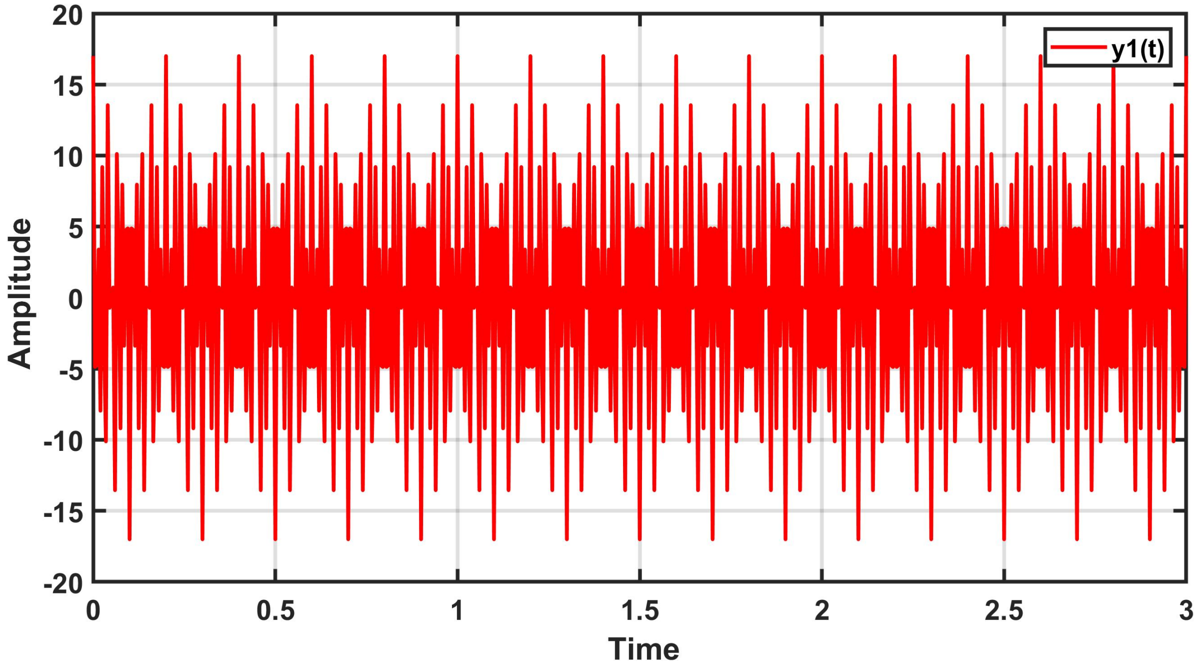
syms t
Fs = 200;            % Sampling frequency
start = 0;
finish = 3;
interval = 1/Fs;
t = start:interval:finish;
L = length(t);
n = 2^nextpow2(L);

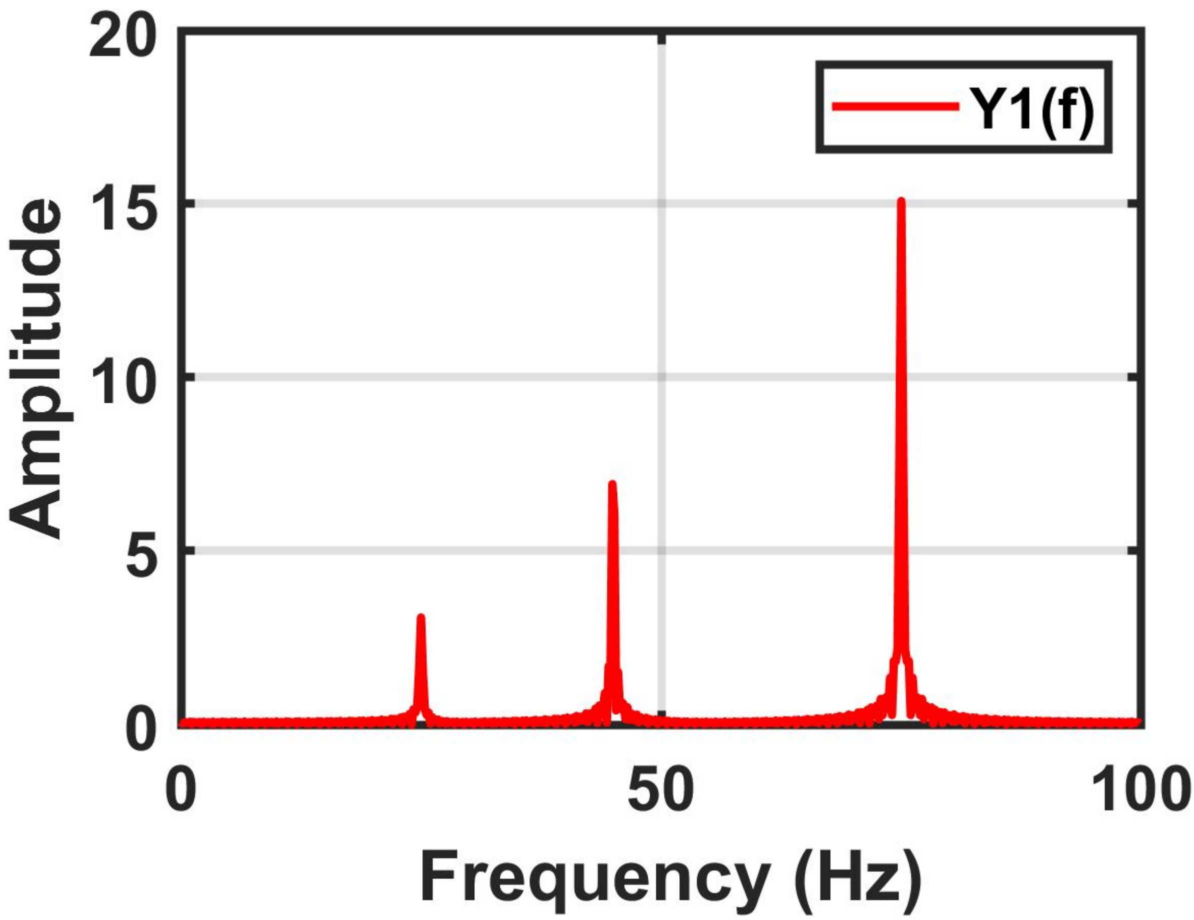
y1 = @(t) 2*cos(2*pi*25.*t)+5*cos(2*pi*45.*t) +10*cos(2*pi*75.*t);

fig1 = figure(1);
set(fig1,'Position', [1000 1000 1000 500]);
plot(t,y1(t),'-r', 'linewidth', 2);
legend('y1(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([-20 20])
xlabel('\bf Time');xlim([0 finish])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_5a.jpg','Resolution',300)

Xk1 = fft(y1(t),n);
f = 1/(n*interval) * (0:n/2-1);

fig2 = figure(2);
set(fig2,'Position', [100 100 400 300]);
plot(f,1/Fs*abs(Xk1(1:n/2)),'-r', 'linewidth', 2);
legend('Y1(f)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 20])
xlabel('\bf Frequency (Hz)');xlim([0, Fs/2])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_5b.jpg','Resolution',300)
```





Yes we can measure all 3 frequencies.

c) & d)

```
%5c&d
clear; clc ; close all;

syms t
Fs = 110;             % Sampling frequency
start = 0;
finish = 3;
interval = 1/Fs;
```

```
t = start:interval:finish;
L = length(t);
n = 2^nextpow2(L);

y1 = @(t) 2*cos(2*pi*25.*t)+5*cos(2*pi*45.*t) +10*cos(2*pi*75.*t);

fig1 = figure(1);
set(fig1,'Position', [1000 1000 1000 500]);
plot(t,y1(t),'-r', 'linewidth', 2);
legend('y1(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([-20 20])
xlabel('\bf Time');xlim([0 finish])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_5c.jpg','Resolution',300)

Xk1 = fft(y1(t),n);
f = 1/(n*interval) * (0:n/2-1);

fig2 = figure(2);
set(fig2,'Position', [100 100 400 300]);
plot(f,1/Fs*abs(Xk1(1:n/2)),'-r', 'linewidth', 2);
legend('y1(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 20])
xlabel('\bf Frequency (Hz)');xlim([0, Fs/2])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_5d.jpg','Resolution',300)
```

No we cannot measure all 3 frequencies, we are missing the 75 Hz frequency because are Nyquist frequency is 55Hz < 75 Hz therefore we get an aliasing frequency at 35 Hz.

e)

No. Increasing your window reduces the smearing error but this cannot compensate for aliasing as that is a result of the sampling frequency.

f)

No as the Nyquist frequency is 60 Hz which is still less than 75 Hz.

g)

Yes we can as the Nyquist frequency is 75.5 Hz > 75 Hz so it can be measured.

Problem 6

a)

```
%6a
clear; clc ; close all;

load("data1.mat");

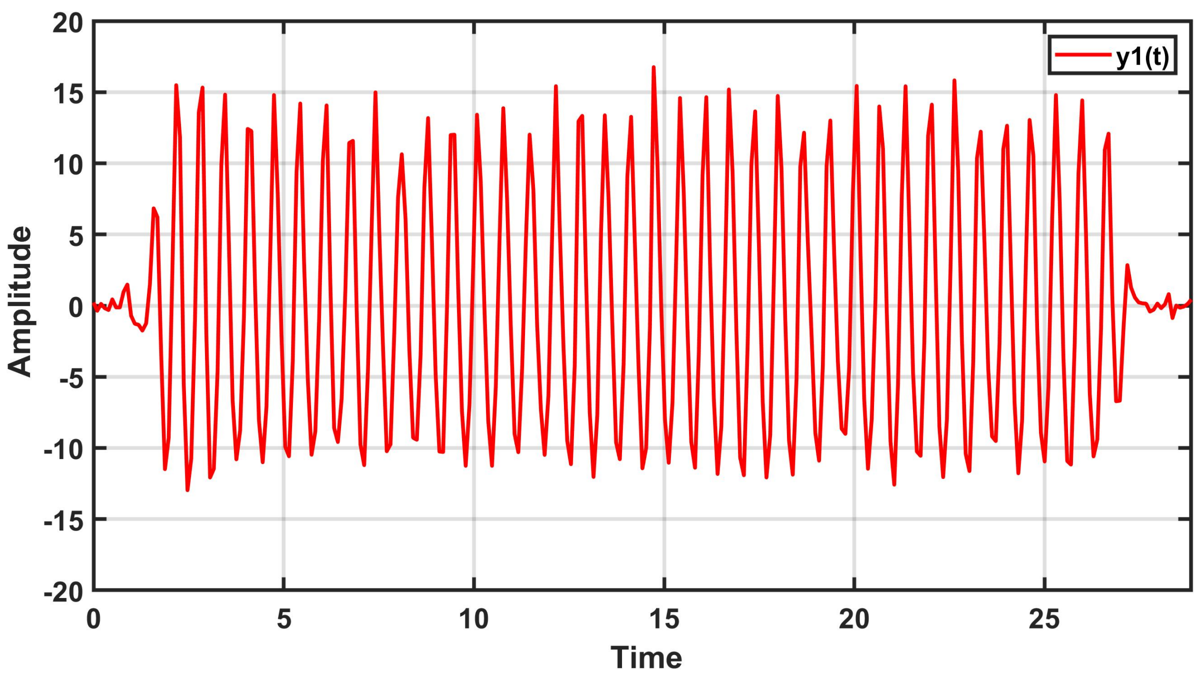
fs = 10.1192;           % Sampling frequency
```

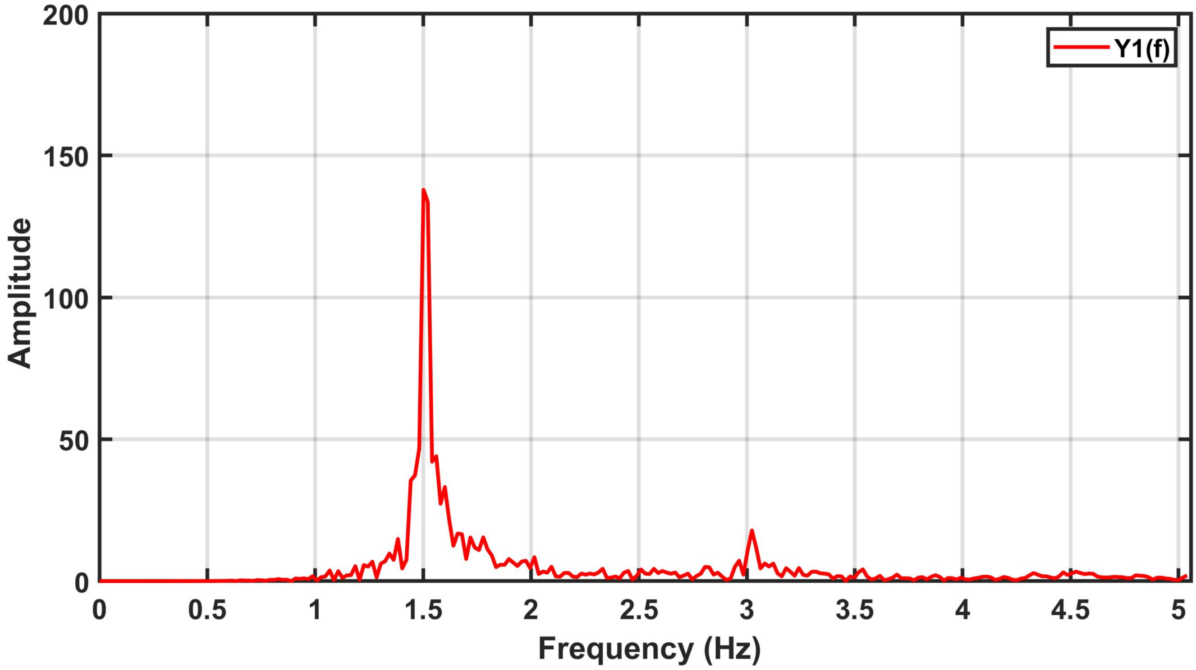
```
start = 0;
finish = 1/fs * numel(zvib)-1/fs;
interval = 1/fs;
t = start:interval:finish;
L = length(t);
n = 2^nextpow2(L);

fig1 = figure(1);
set(fig1,'Position', [1000 1000 1000 500]);
plot(t,zvib','-r', 'linewidth', 2);
legend('y1(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([-20 20])
xlabel('\bf Time');xlim([0 finish])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_6a.jpg','Resolution',300)

Xk1 = fft(zvib',n);
f = 1/(n*interval) * (0:n/2-1);

fig2 = figure(2);
set(fig2,'Position', [100 100 1000 500]);
plot(f,1/fs*abs(Xk1(1:n/2)),'-r', 'linewidth', 2);
legend('Y1(f)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 200])
xlabel('\bf Frequency (Hz)');xlim([0, fs/2])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_6b.jpg','Resolution',300)
```





The main frequency of this graph is 1.5 Hz.

b)

```
%6b
clear; clc ; close all;

load("data2.mat");

fs = 10.1192;

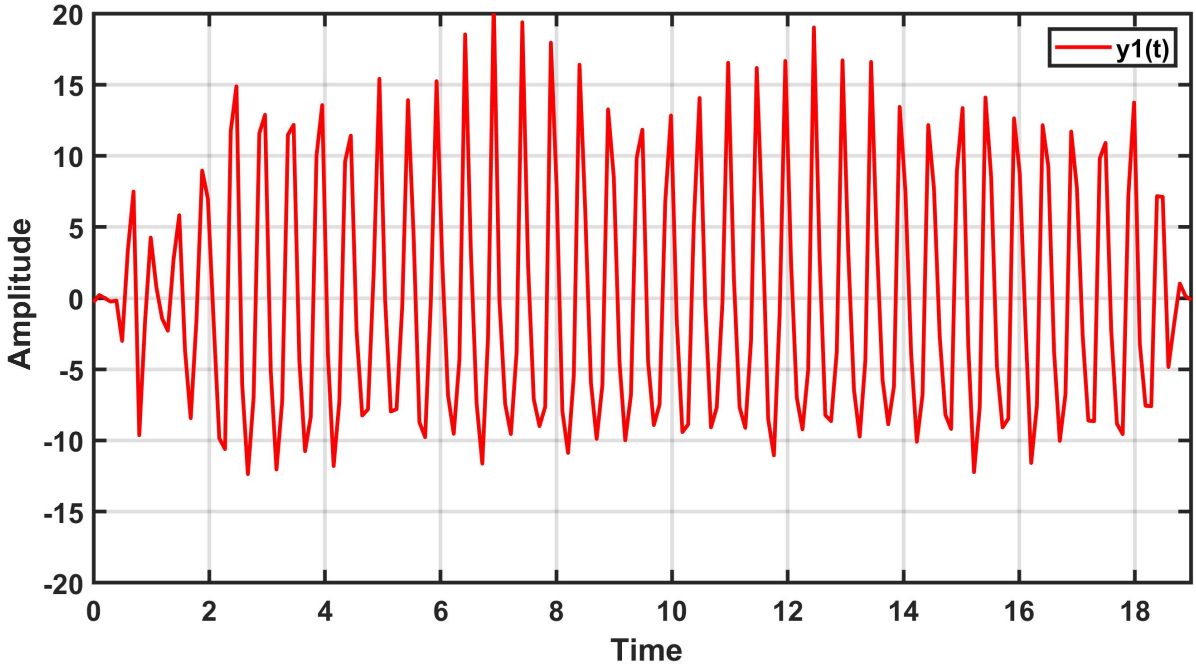
start = 0;
finish = 1/fs * numel(zvib)-1/fs;
interval = 1/fs;
t = start:interval:finish;
L = length(t);
n = 2^nextpow2(L);

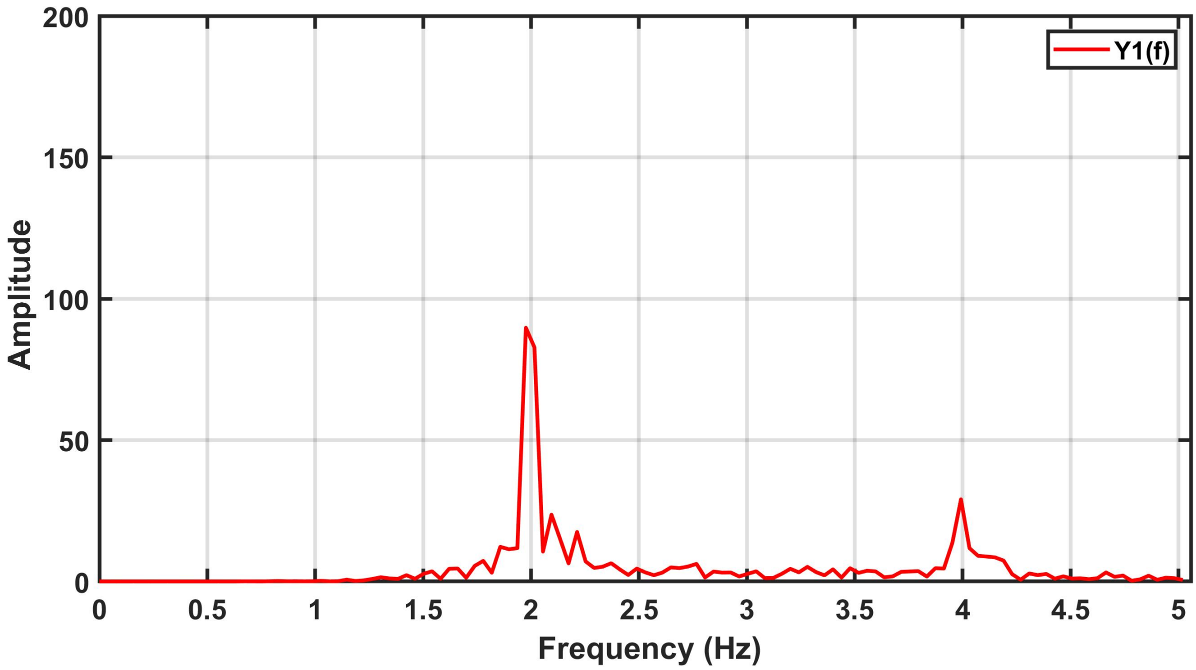
fig1 = figure(1);
set(fig1,'Position', [1000 1000 1000 500]);
plot(t,zvib','-r', 'linewidth', 2);
legend('y1(t)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([-20 20])
xlabel('\bf Time');xlim([0 finish])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
a = gcf;
exportgraphics(a,'Task3_6c.jpg','Resolution',300)

Xk1 = fft(zvib',n);
f = 1/(n*interval) * (0:n/2-1);

fig2 = figure(2);
set(fig2,'Position', [100 100 1000 500]);
plot(f,1/fs*abs(Xk1(1:n/2)),'-r', 'linewidth', 2);
legend('Y1(f)'); axis tight;grid on;
ylabel('\bf Amplitude'); ylim([0 200])
xlabel('\bf Frequency (Hz)');xlim([0, fs/2])
set(gca,'fontsize',15,'linewidth',2,'fontweight','bold');
```

```
a = gcf;
exportgraphics(a,'Task3_6d.jpg','Resolution',300)
```





The main frequency of this graph is 2 Hz.