$$X_s(f + r/\Delta) = \sum_{n=-\infty}^{\infty} x(n\Delta)e^{-i2\pi(f+r/\Delta)n\Delta} = \sum_{n=-\infty}^{\infty} x(n\Delta)e^{-i2\pi f n\Delta - i2\pi r n} = \sum_{n=-\infty}^{\infty} x(n\Delta)e^{-i2\pi f n\Delta} = X_s(f)$$

$X_s(f)$: Fourier transform of a sampled (discretized) signal (sequence)

$$X_s(f + r/\Delta) = X_s(f + r f_s) = X_s(f)$$

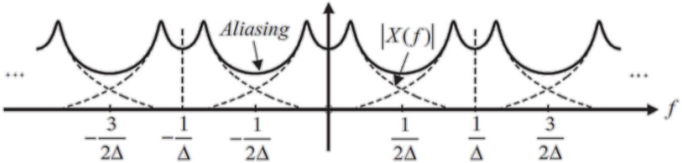
$\Rightarrow$ periodic function.

* This equation does not tell anything about the relationship with original signals.

Fourier Transform of a discrete sequence, $x_s(t)$

$$X_s(f) = I(f) * X(f) = \int_{-\infty}^{\infty} I(g)X(f-g)dg = \int_{-\infty}^{\infty} \frac{1}{\Delta}\sum_{n=-\infty}^{\infty} \delta\left(g-\frac{n}{\Delta}\right)X(f-g)dg = \frac{1}{\Delta}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} \delta\left(g-\frac{n}{\Delta}\right)X(f-g)dg$$

$$= \frac{1}{\Delta}\sum_{n=-\infty}^{\infty} X\left(f-\frac{n}{\Delta}\right) = \frac{1}{\Delta}\left(\dots + X\left(f-\frac{2}{\Delta}\right) + X\left(f-\frac{1}{\Delta}\right) + X(f) + X\left(f+\frac{1}{\Delta}\right) + \dots\right)$$



$X(f)$: fourier transform of the original signal $\Rightarrow$ a true Fourier spectrum of your signal.

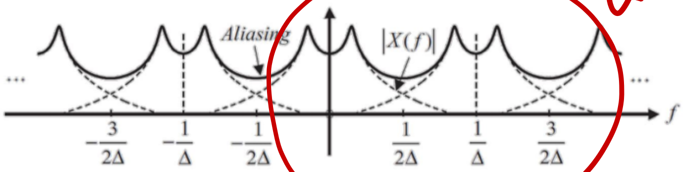
$$X_s(f) = \frac{1}{\Delta}\left(\dots + X\left(f-\frac{2}{\Delta}\right) + X\left(f-\frac{1}{\Delta}\right) + \dots \right)$$

$X_s(f)$ is the summation of $X(f)$ and shifted $X(f)$ with $n\frac{1}{\Delta}$
n: integer.

Fourier Transform of a discrete sequence, $x_s(t)$

$$X_s(f) = I(f) * X(f) = \int_{-\infty}^{\infty} I(g)X(f-g)dg = \int_{-\infty}^{\infty} \frac{1}{\Delta}\sum_{n=-\infty}^{\infty} \delta\left(g-\frac{n}{\Delta}\right)X(f-g)dg = \frac{1}{\Delta}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} \delta\left(g-\frac{n}{\Delta}\right)X(f-g)dg$$

$$= \frac{1}{\Delta}\sum_{n=-\infty}^{\infty} X\left(f-\frac{n}{\Delta}\right) = \frac{1}{\Delta}\left(\dots + X\left(f-\frac{2}{\Delta}\right) + X\left(f-\frac{1}{\Delta}\right) + X(f) + X\left(f+\frac{1}{\Delta}\right) + \dots\right)$$



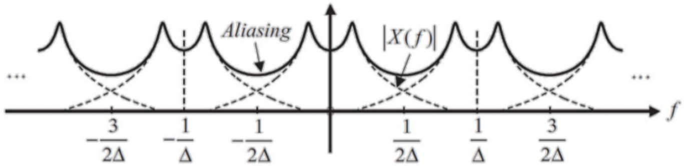
If There is no frequency component of above $1/2\Delta$, there is no overlap between $X(f)$ and shifted $X(f)$ (here, $X(f-\frac{1}{\Delta})$)
(Not aliasing)

$\Longrightarrow$ There is no aliasing !!!!

Fourier Transform of a discrete sequence, $x_s(t)$

$$X_s(f) = I(f) * X(f) = \int_{-\infty}^{\infty} I(g)X(f-g)dg = \int_{-\infty}^{\infty} \frac{1}{\Delta} \sum_{n=-\infty}^{\infty} \delta\left(g - \frac{n}{\Delta}\right) X(f-g)dg = \frac{1}{\Delta} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta\left(g - \frac{n}{\Delta}\right) X(f-g)dg$$

$$= \frac{1}{\Delta} \sum_{n=-\infty}^{\infty} X\left(f - \frac{n}{\Delta}\right) = \frac{1}{\Delta}\left(\ldots + X\left(f - \frac{2}{\Delta}\right) + X\left(f - \frac{1}{\Delta}\right) + X(f) + X\left(f + \frac{1}{\Delta}\right) + \ldots\right)$$



If there are frequency components at above $1/2\Delta$, there is overlaps between $X(f)$ and shifted $X(f)$

$\Longrightarrow$ This is aliasing!!!

Why Wrap-around ??