# Edge Detection

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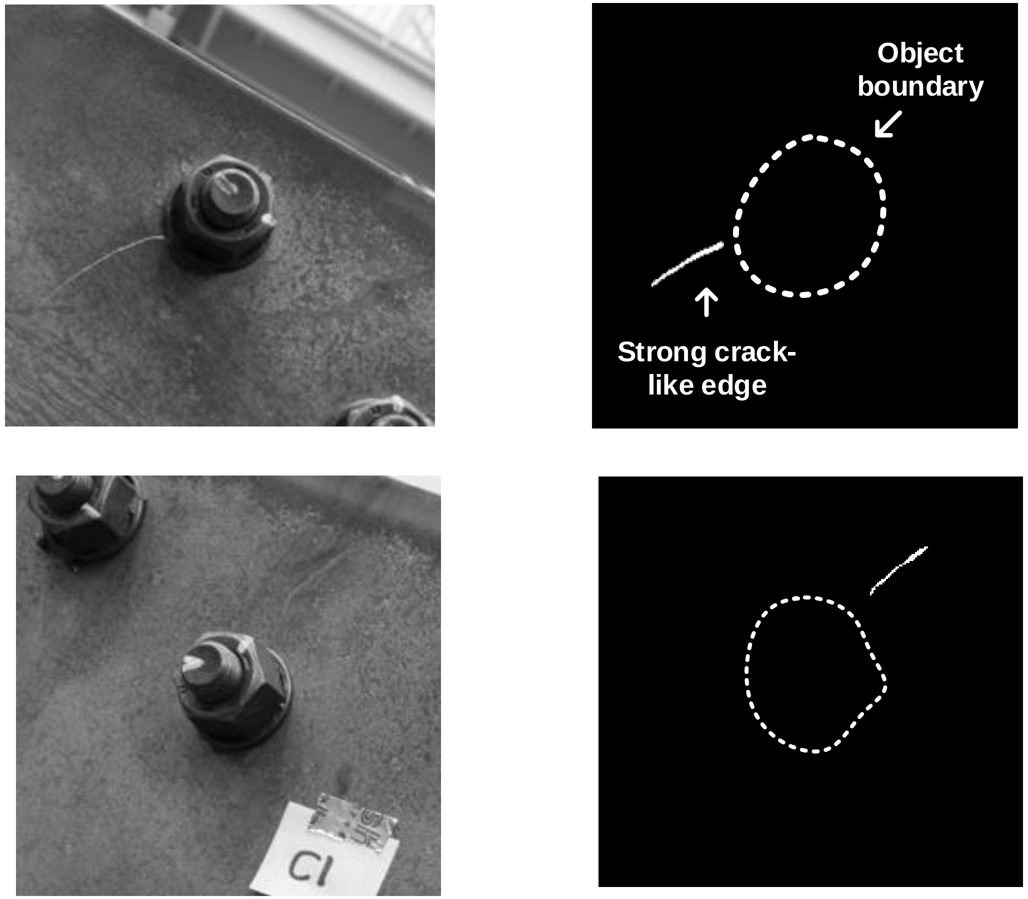
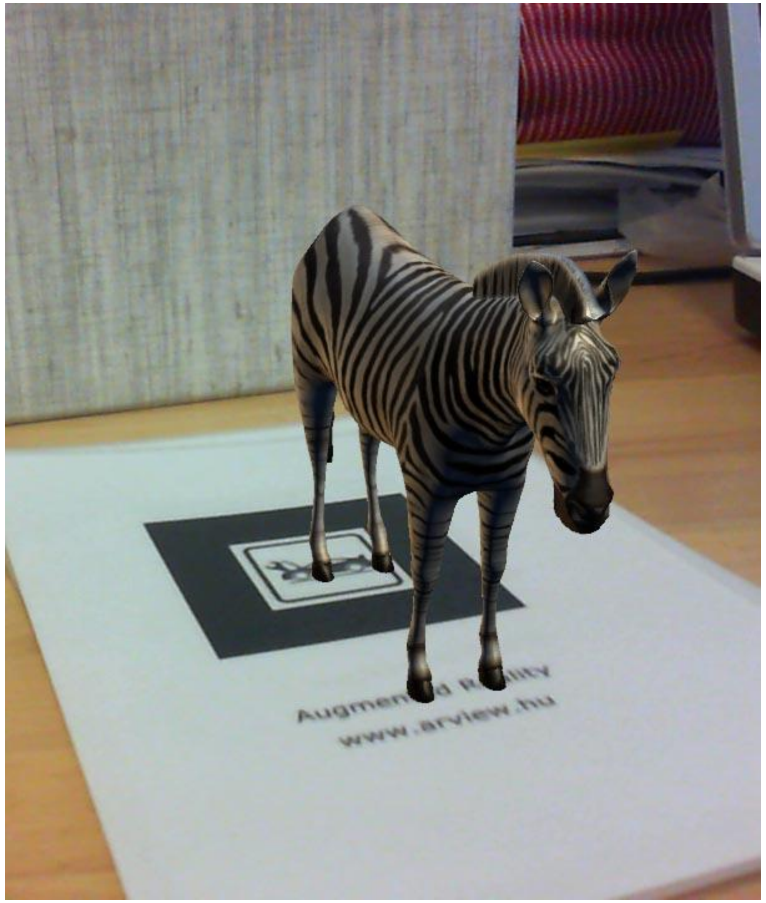
# Edge Detection Applications

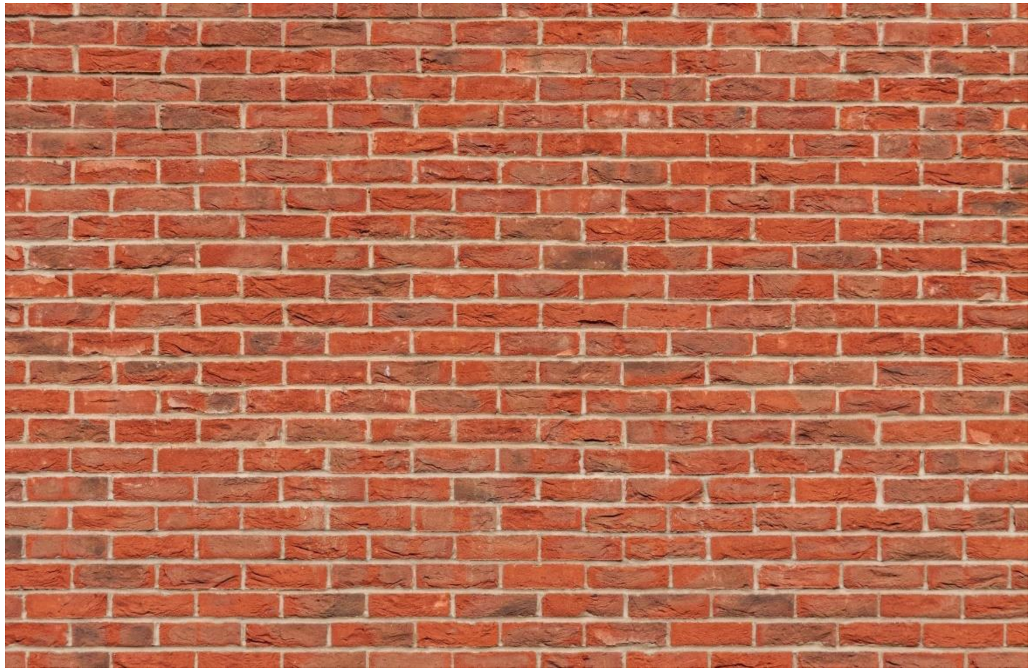


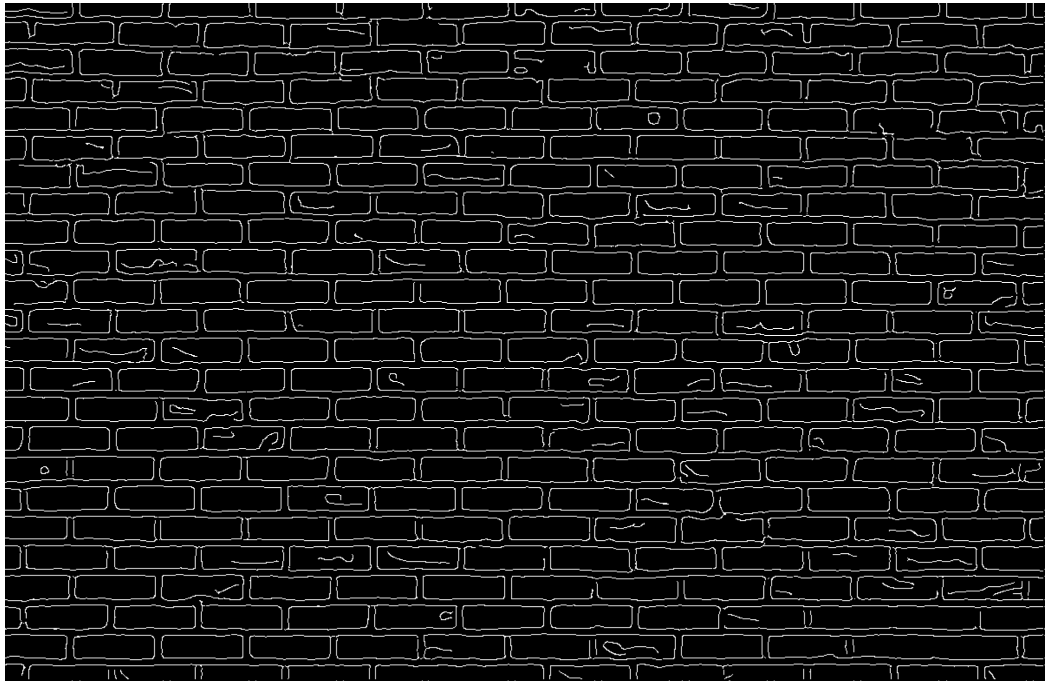

Crack detection




Augmented reality

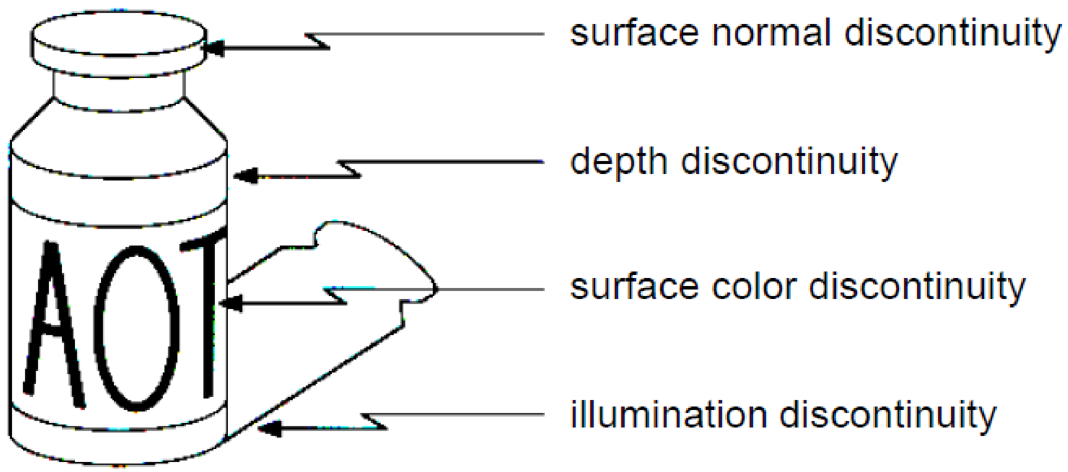
# Edge Detection



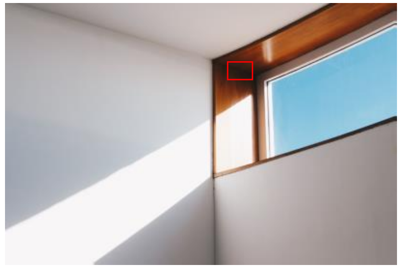


- Convert a 2D image into a set of curves
  - Semantic and shape information
  - More compact than pixels

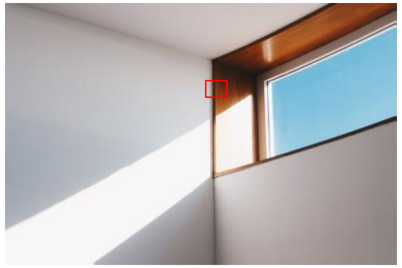
# Origin of Edges



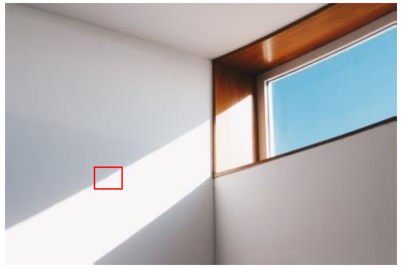

## Surface normal discontinuity



## Material discontinuity

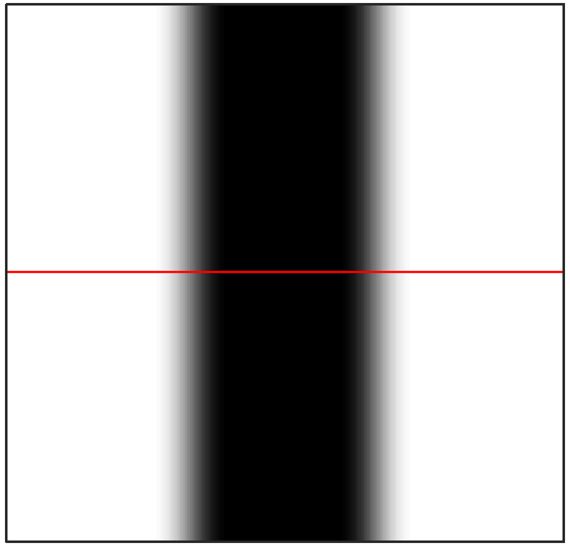


## Lighting discontinuity

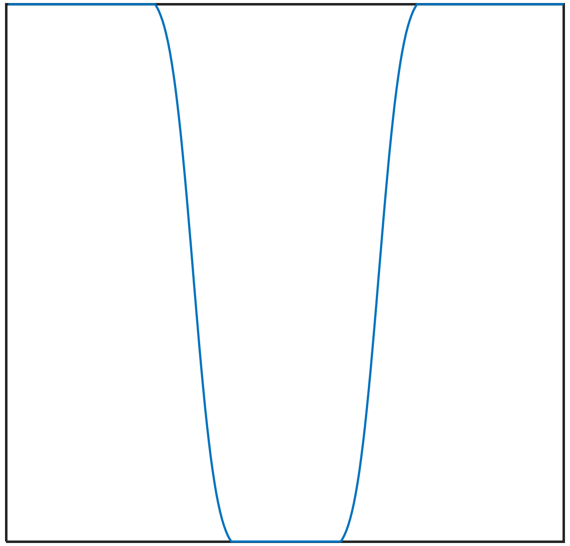


# Characterizing Edge by Differentiation

- An edge is a place of rapid change in the image intensity function in a single direction.

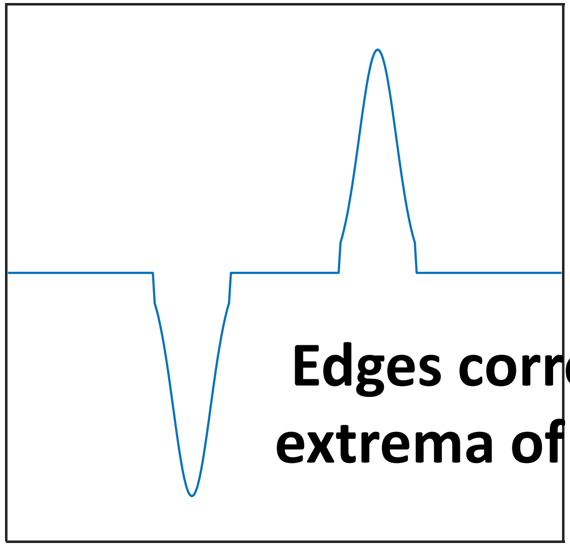


Image



Intensity function along a red line

$$\frac{d}{dx}$$



Edges correspond to extrema of derivative

First derivative

# Image Derivatives

How can we differentiate a digital image $F[x, y]$?

Option 1: Reconstruct a continuous image, f, then compute the derivative

Option 2: Take discrete derivative (finite difference)

$$\frac{\partial}{\partial x} f[x, y] \approx F[x+1, y] - F[x, y]$$

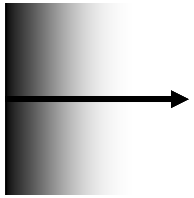
$\Delta x = 1$ pixel

| | | |
|---|---|---|
| | | |
| | | |
| | | |

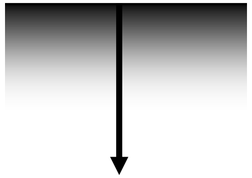
How would you implement this as a kernel?

# Image Gradient

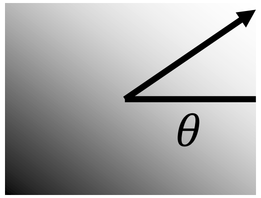
The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial x}\right]$$




$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid increase in intensity

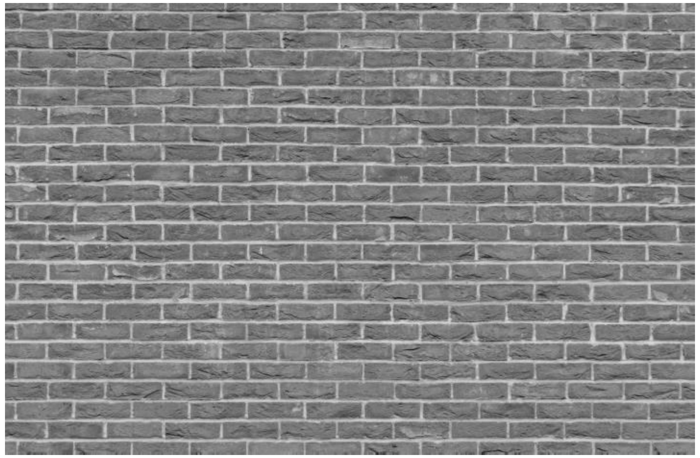
The gradient direction is given by

$$\theta = tan^{-1}\left(\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}\right)$$

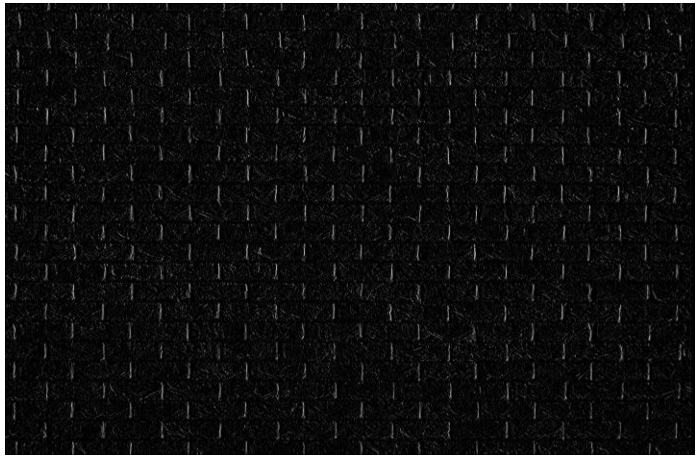
The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial Y}\right)^2}$$

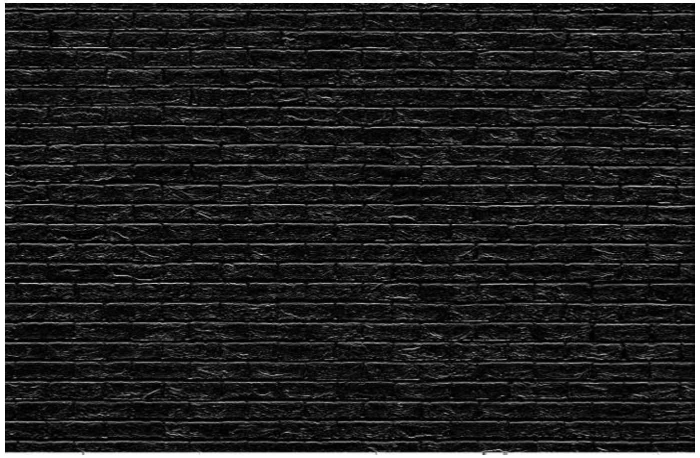
# Example: Image Gradient



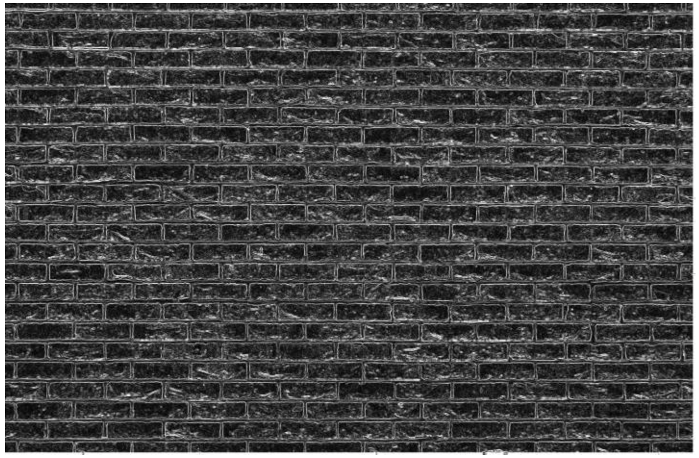
$f$



$\partial f / \partial x$

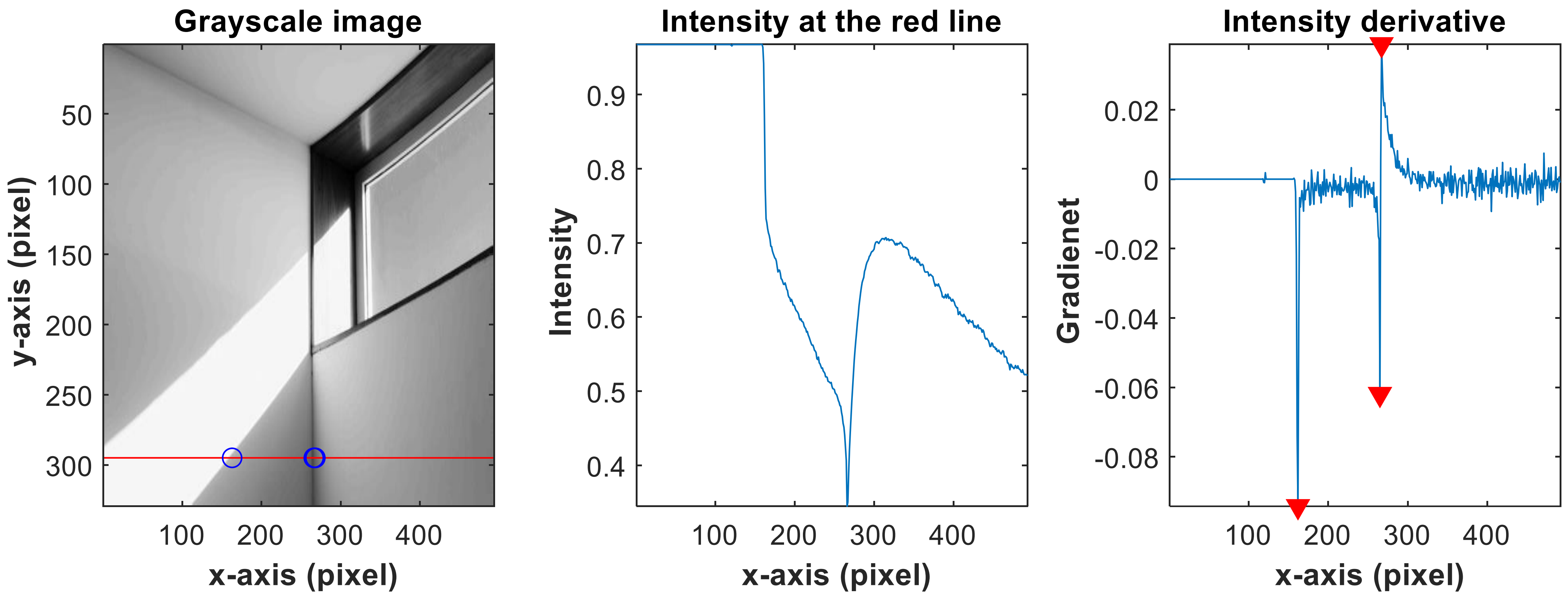


$\partial f / \partial y$



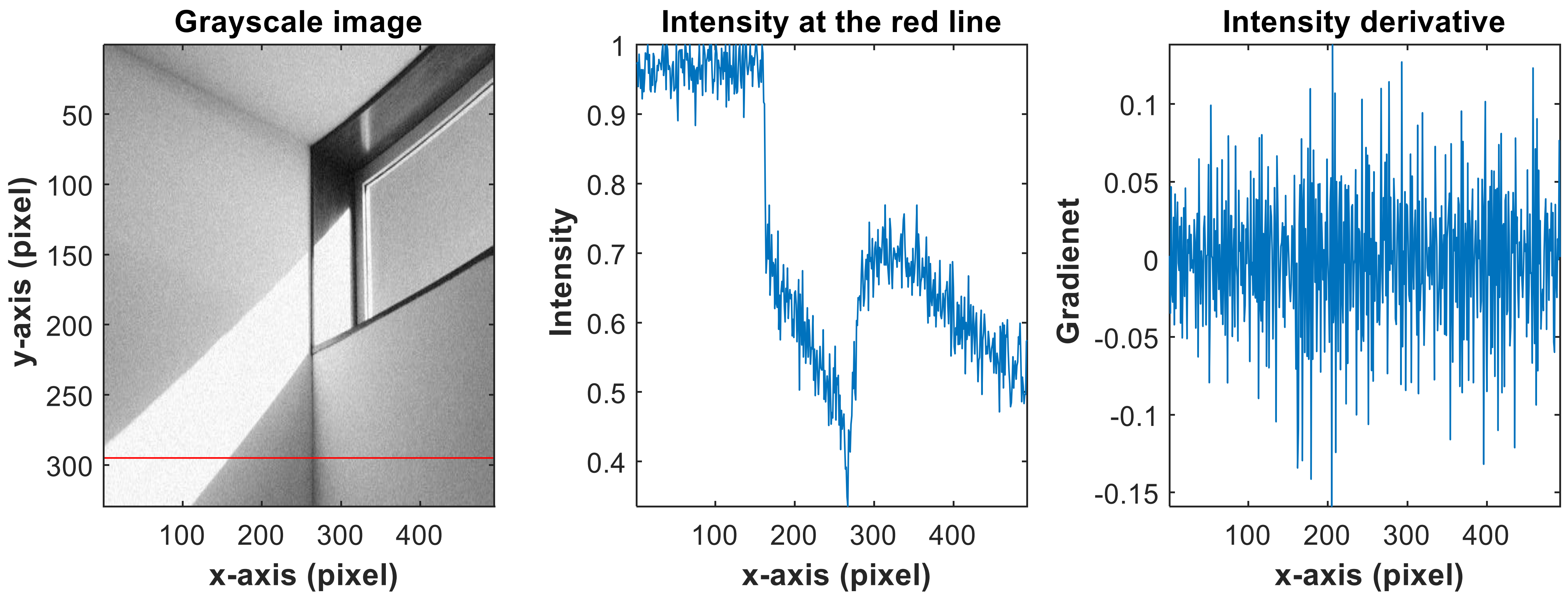
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial Y}\right)^2}$$

# Intensity Changes on an Image

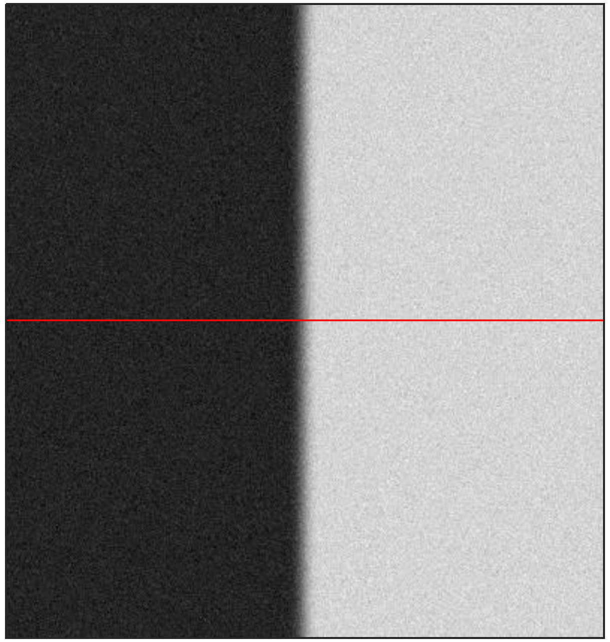



**See Tutorial !**

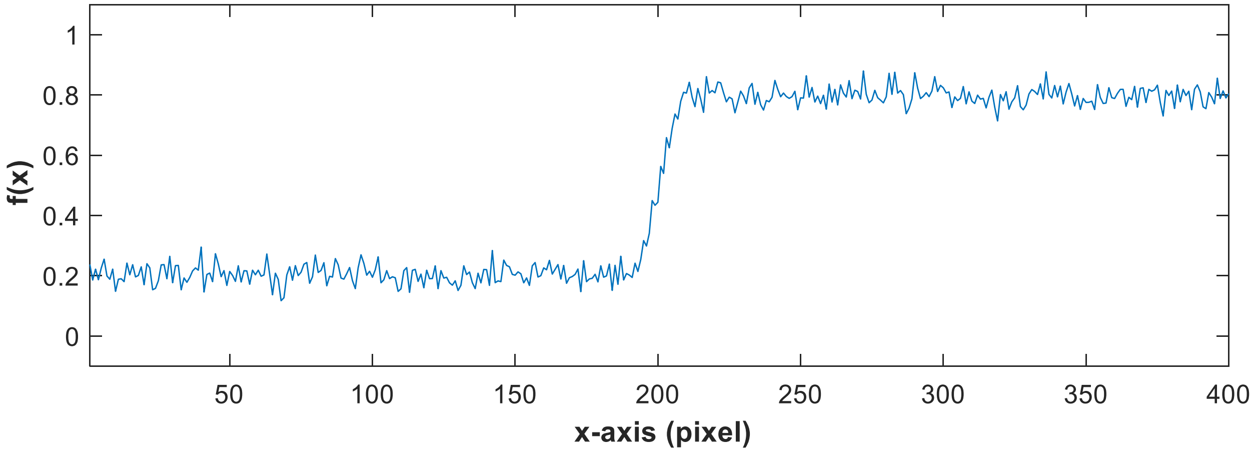
# Intensity Changes on an Image (with Noise)

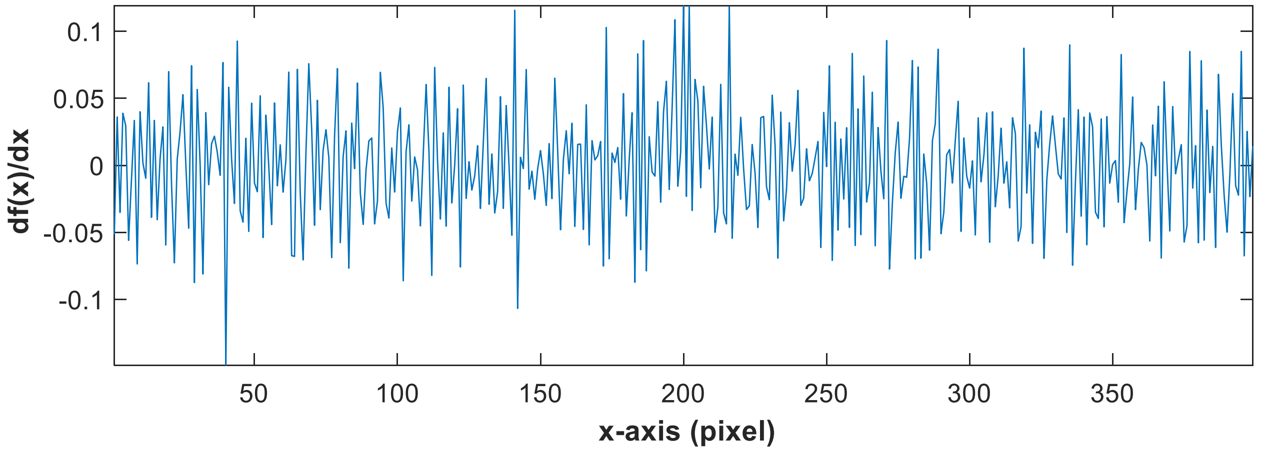


# Effects of Noise



Noisy input image




**Where is the edge?**

# (Review) Properties of Fourier Transforms

## 1. Time scaling

$$F\big(x(at)\big) = \frac{1}{|a|}X(\frac{f}{a})$$

*Inverse spreading relationship*

## 2. Time reversal

$$F\big(x(-t)\big) = X(-f)$$

## 3. Differentiation

$$F\big(\dot{x}(t)\big) = i2\pi f X(f)$$

## 4. Time shifting

$$F\big(x(t-t_0)\big) = e^{-i2\pi f t_0}X(f)$$

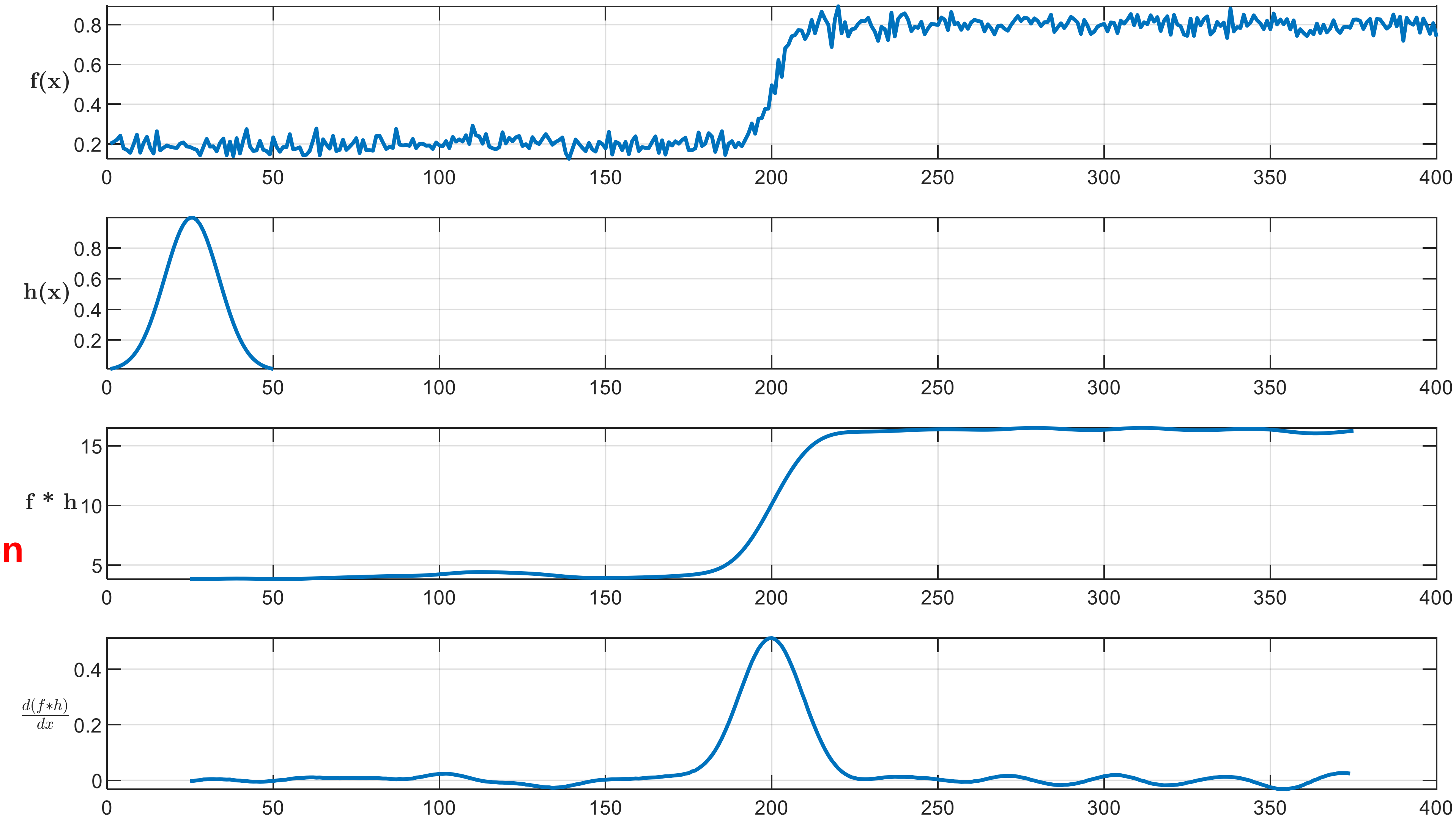
*Only phase shift !* **Sine wave**

## 5. Modulation

$$F\big(x(t)e^{i2\pi f_0 t}\big) = X(f - f_0)$$

$$F(x(t)\cos(2\pi f_0 t)\,)$$
$$= \frac{1}{2}[X(f - f_0)\ + X(f + f_0)\,]$$

# Smoothing First and Applying a Derivative Operator

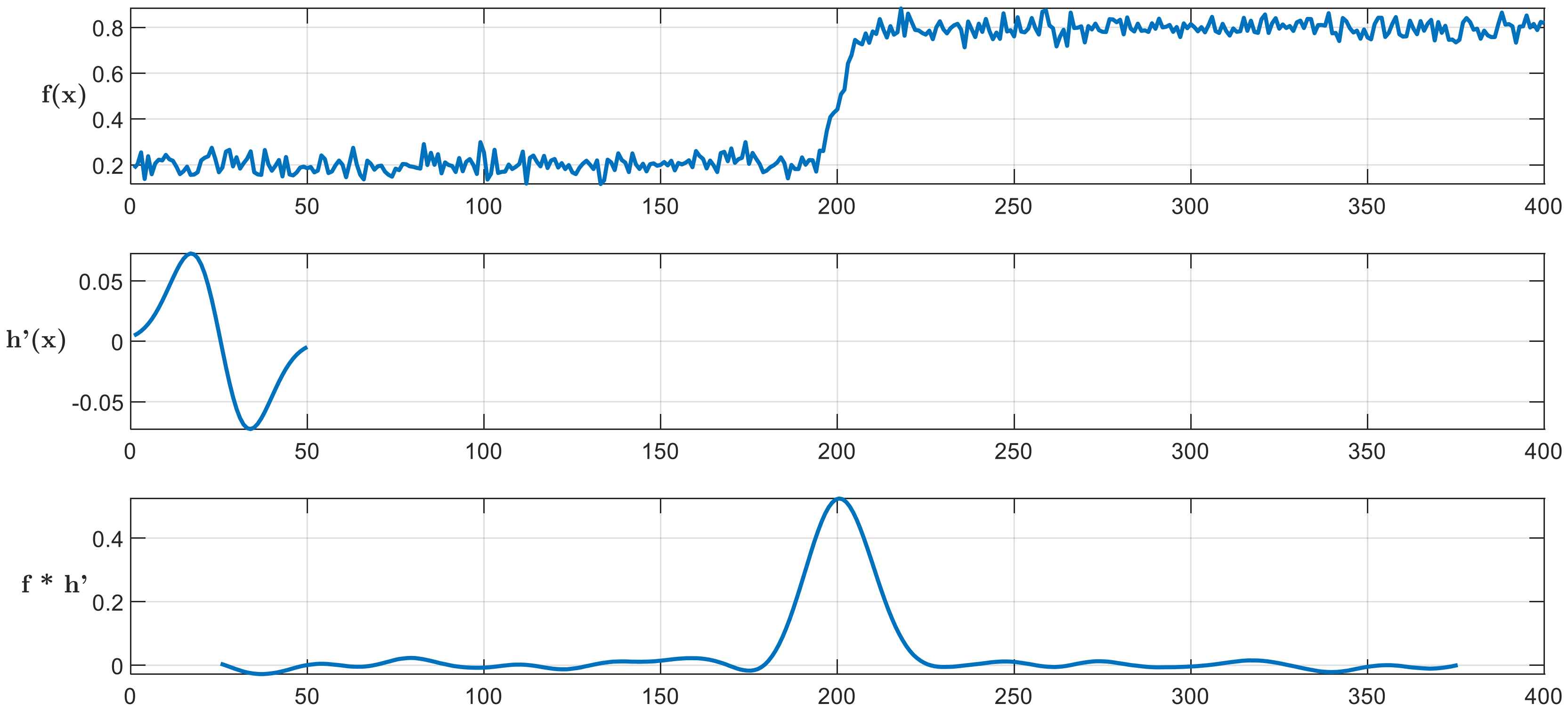



convolution

# Smoothing First and Applying a Derivative Operator (Continue)

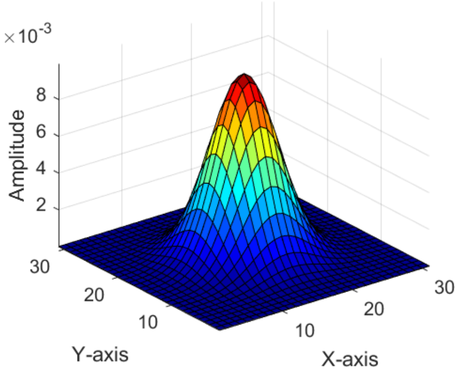
**Relationship with differentiation:** $(F * H)' = F' * H = F * H'$

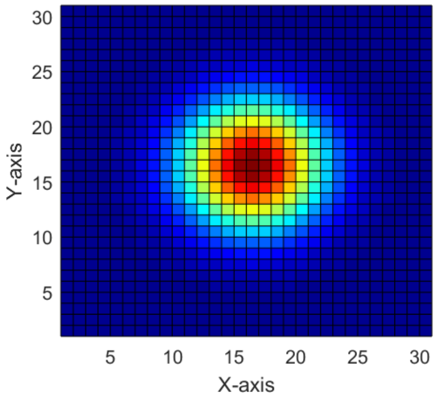
$$(f * h)' = f * h'$$




**What is the advantage?**

# 2D Edge Detection Filter

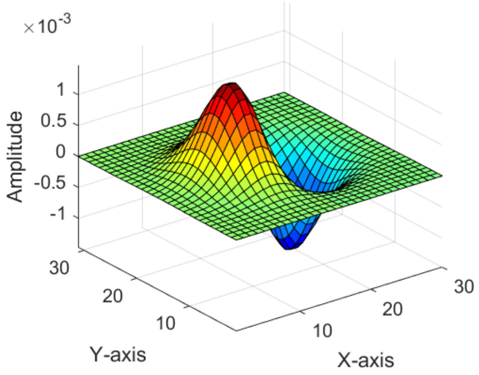


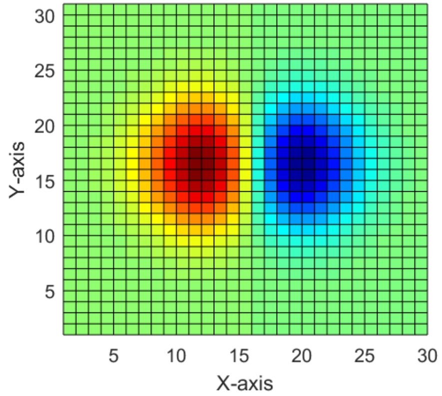




$$h_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

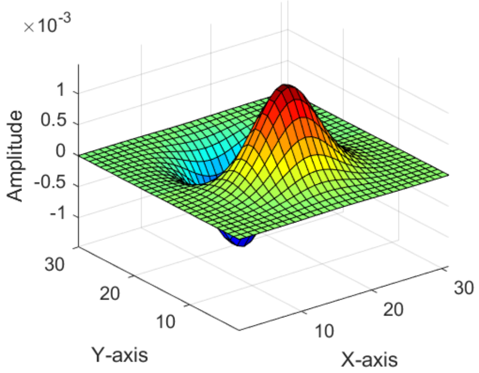
**Gaussian kernel**

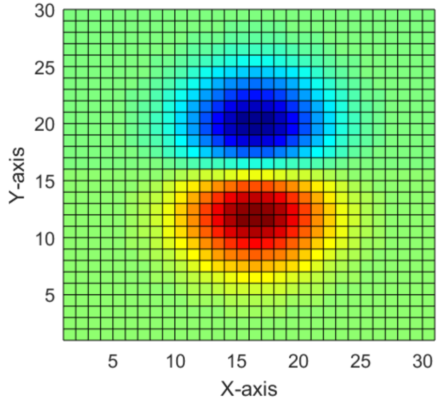
$$\frac{\partial \boldsymbol{h_\sigma(x,y)}}{\partial \boldsymbol{x}}$$



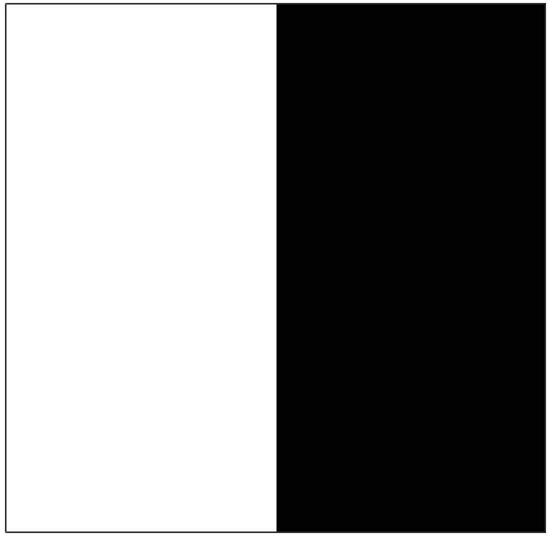

$$\frac{\partial \boldsymbol{h_\sigma(x,y)}}{\partial \boldsymbol{y}}$$

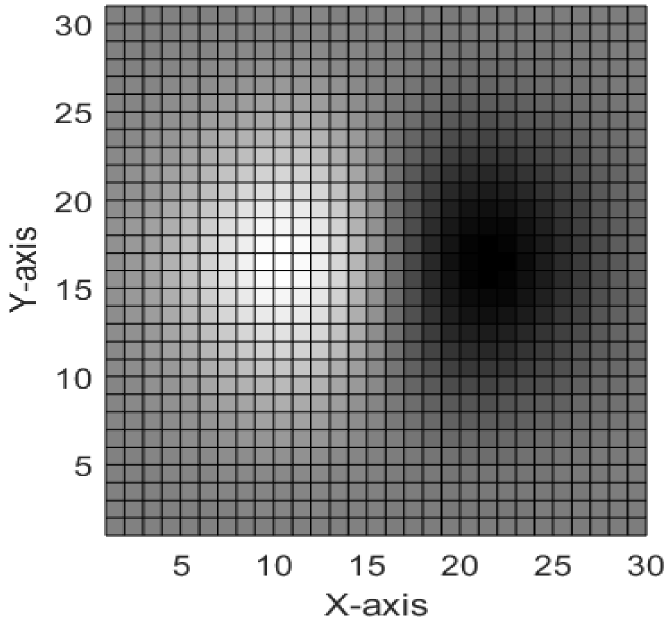



**Derivative of Gaussian filter**

# 2D Edge Detection Filter (Continue)






| | | |
|---|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

**Prewitt operator**

**?**

# Sobel Operator

Common approximation of derivative of Gaussian

$$\frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

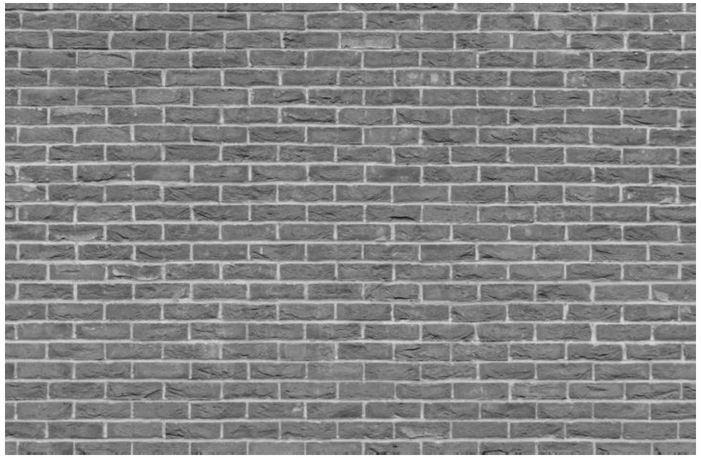
$$s_x$$

$$\frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

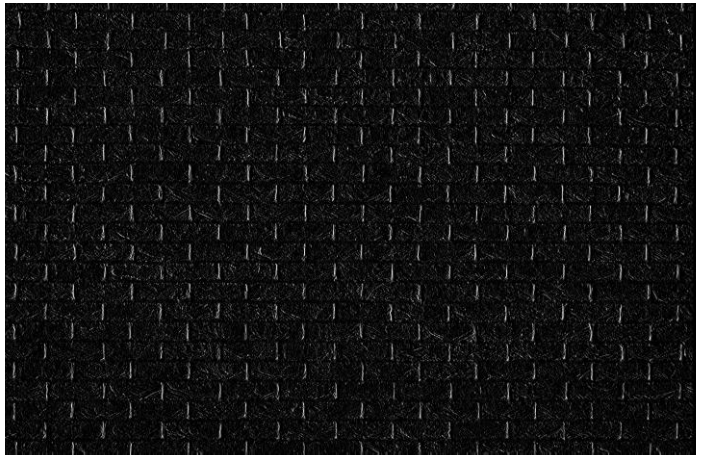
$$s_y$$

The Sobel operator omits the 1/8 term because it does not make a difference for edge detection

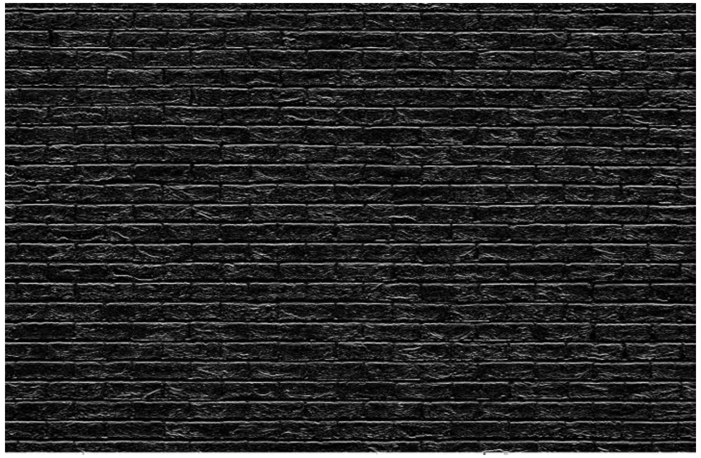
# Sobel Operator (Example)



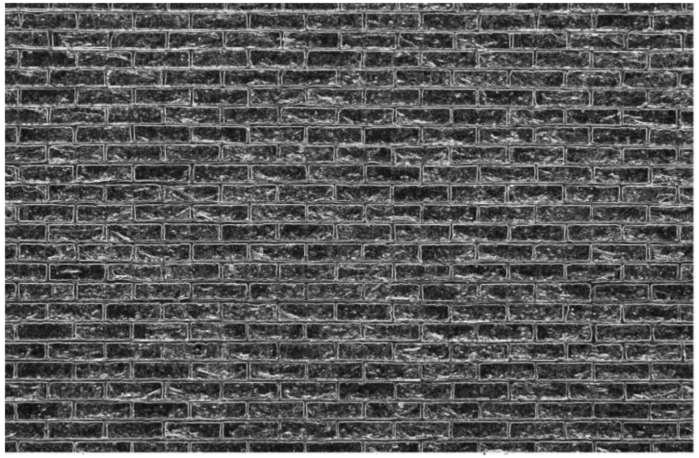
$f$



$s_x$

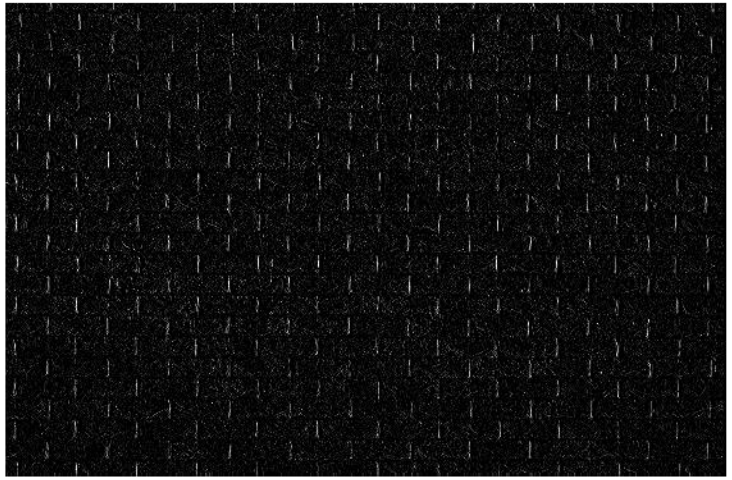


$s_y$

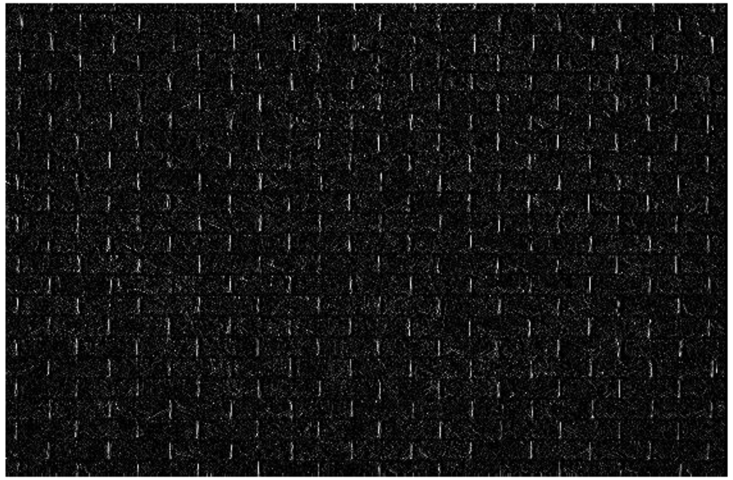


$\|\nabla f\| = \sqrt{(s_x)^2 + (s_y)^2}$

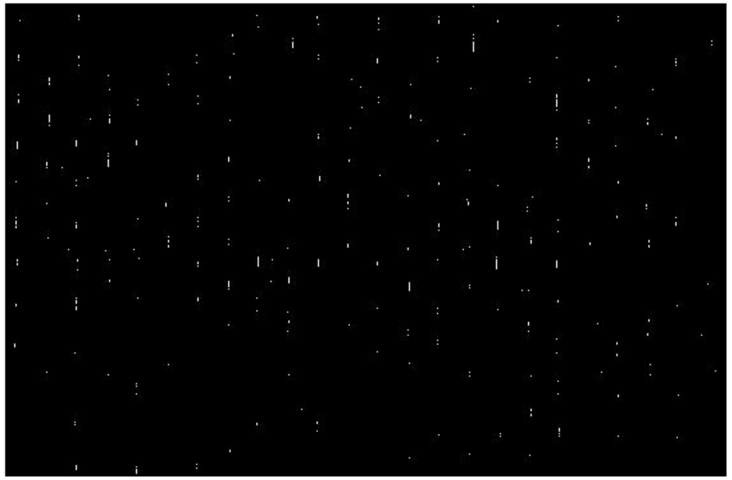
# Comparison of Derivative Operators (Prewitt and Sobel)



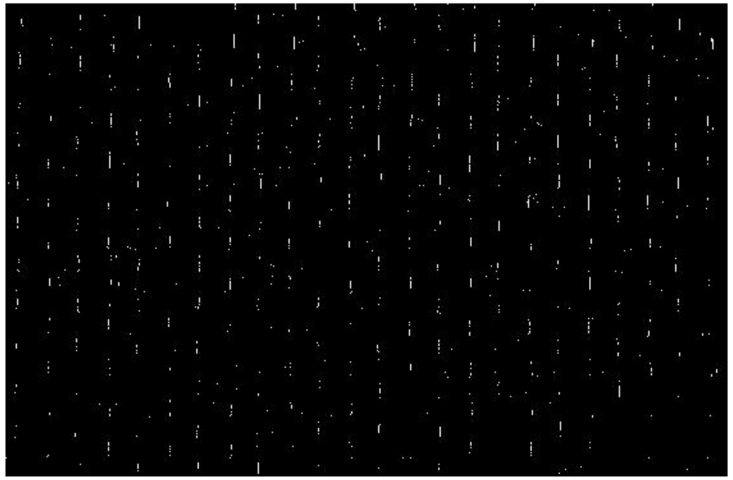
$f_x$



$s_x$

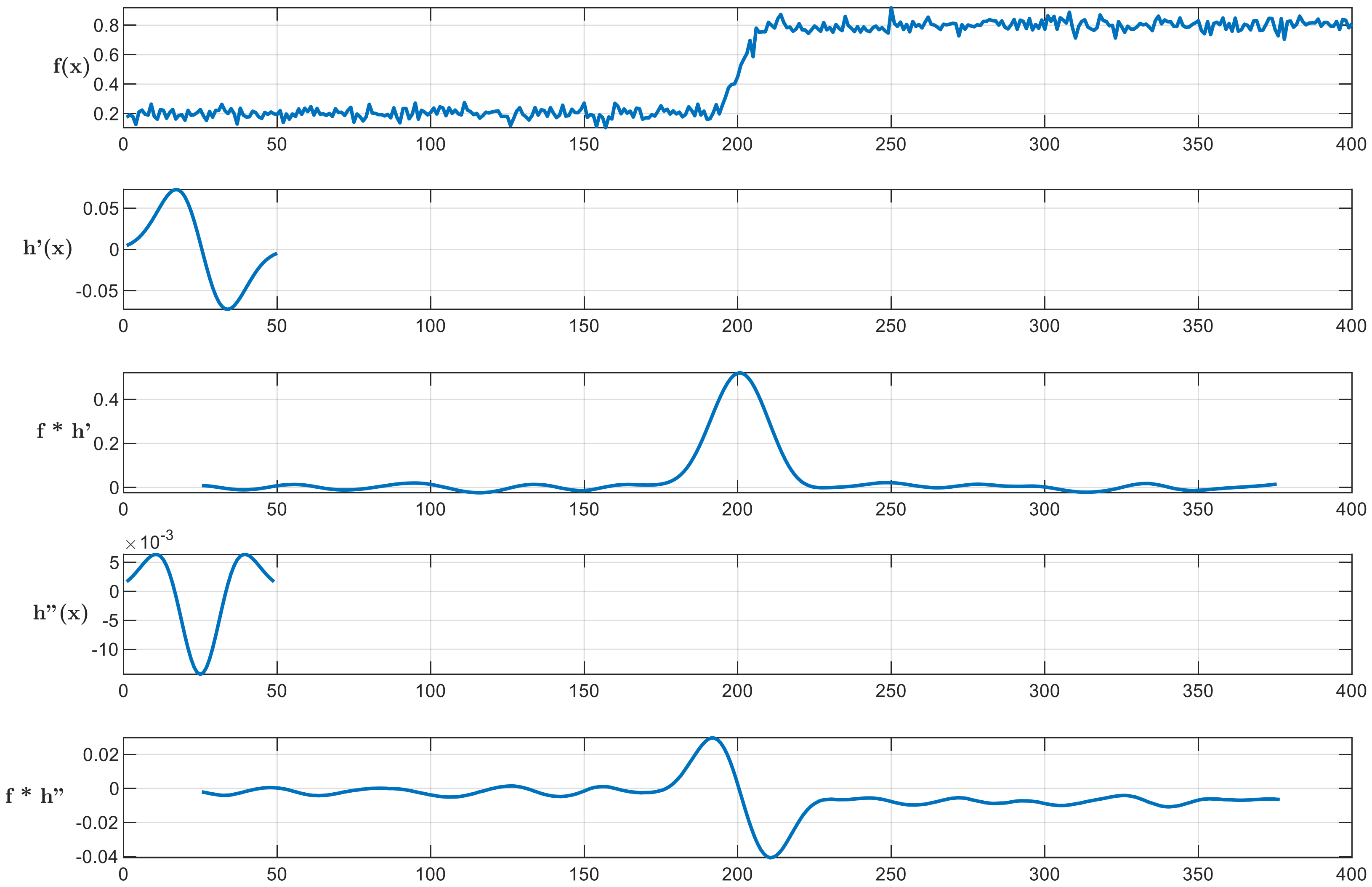


$f_x > 0.75$



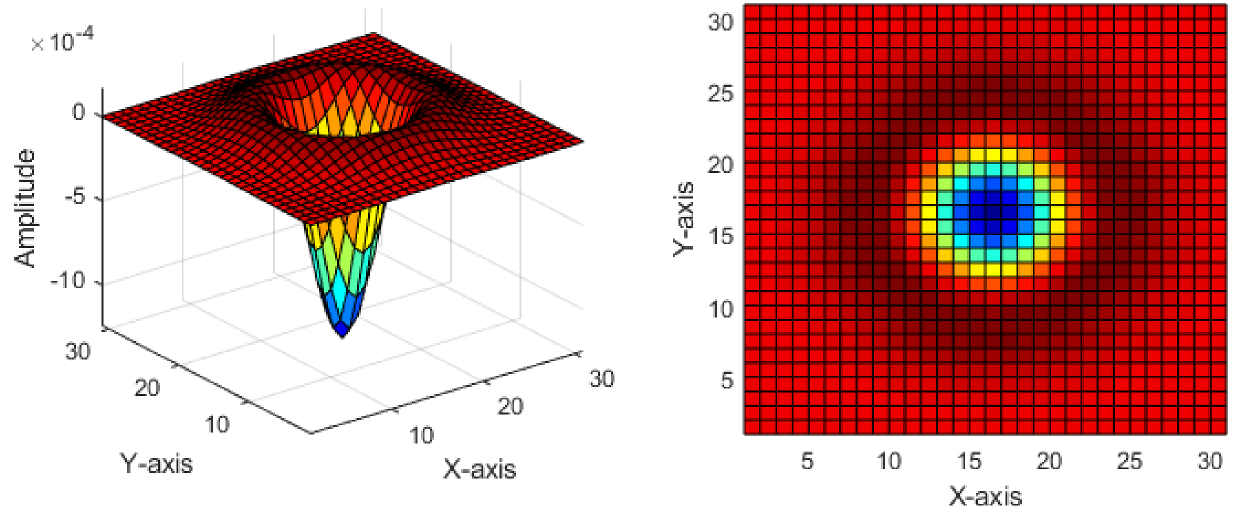
$s_x > 0.75$

# Laplacian of Gaussian (1D Example)



# Laplacian of Gaussian

$$\nabla^2 h_\sigma(x, y)$$




| | | |
|---|---|---|
| 0 | -1 | 0 |
| -1 | 4 | -1 |
| 0 | -1 | 0 |

$$\nabla^2 h_\sigma(x, y) = \frac{\partial^2 h_\sigma}{\partial x^2} + \frac{\partial^2 h_\sigma}{\partial y^2}$$

$$h_\sigma(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

# Implementation Issues for Gradient-based Edge Detection

- The gradient magnitude is large along a thick “trail” or “ridge,” so how do we identify the actual edge points?
- How do we link the edge points to form curves?

# Canny Edge Detector

- This is probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

**Steps**

- Filter image with x, y derivatives of Gaussian
- Find magnitude and orientation of gradient
- Non-maximum suppression: Thin multi-pixel wide “ridges” down to single pixel width
- Thresholding and linking (hysteresis): Define two thresholds: **low and high** and use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: edge(image, ‘canny’)

# Original Image and Its Gradient

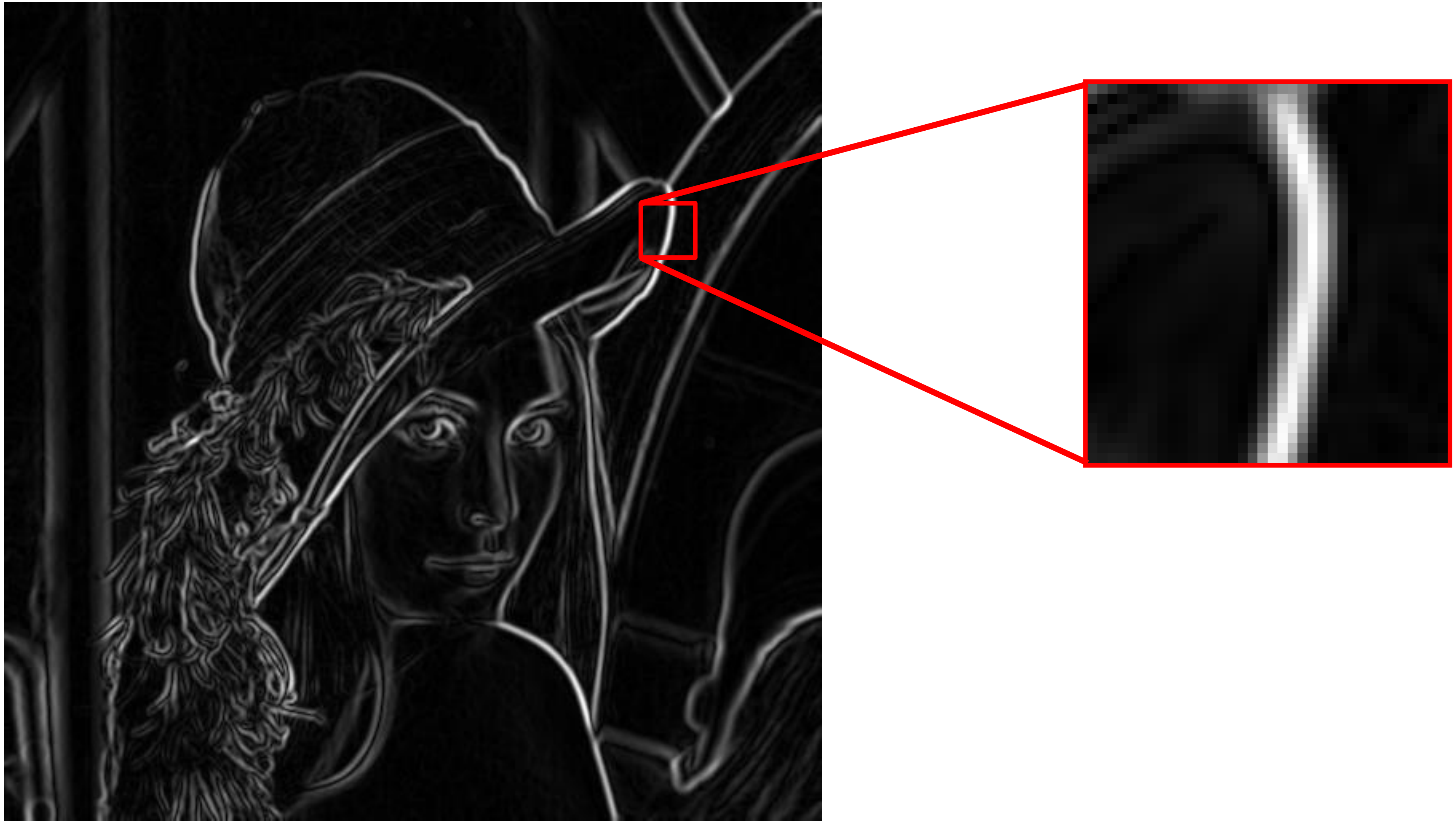
original image (Lena)

X-Derivative of Gaussian

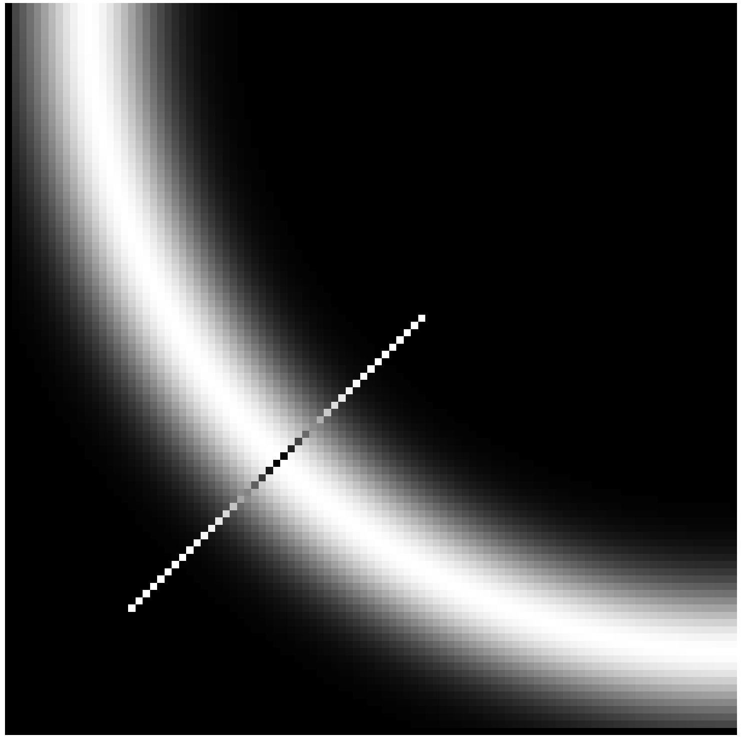
Y-Derivative of Gaussian

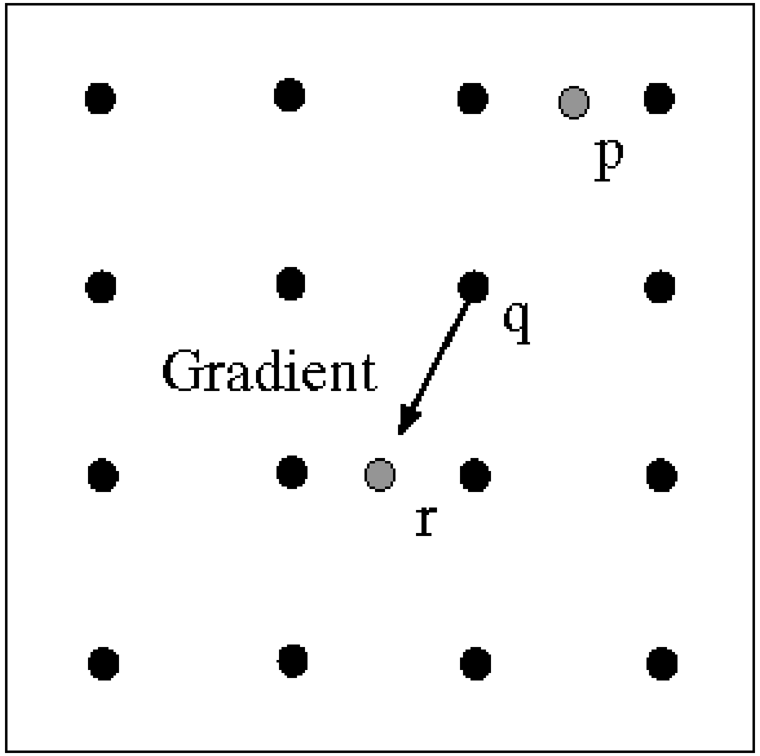
Gradient Magnitude

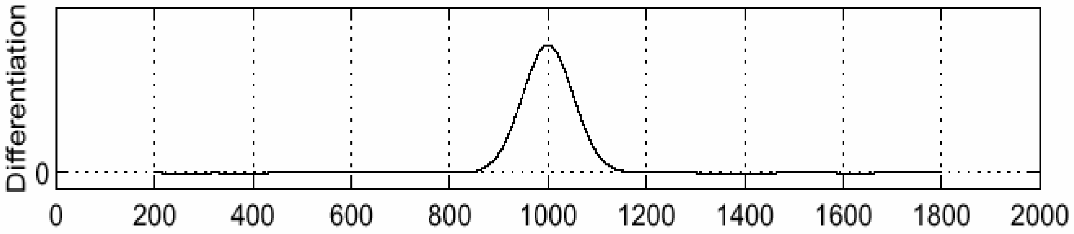
# Finding Edges



# Non-maximum Suppression






- Check if pixel is local maximum along gradient direction
  - Requires interpolating pixels p and r

# After Non-maximum Suppression

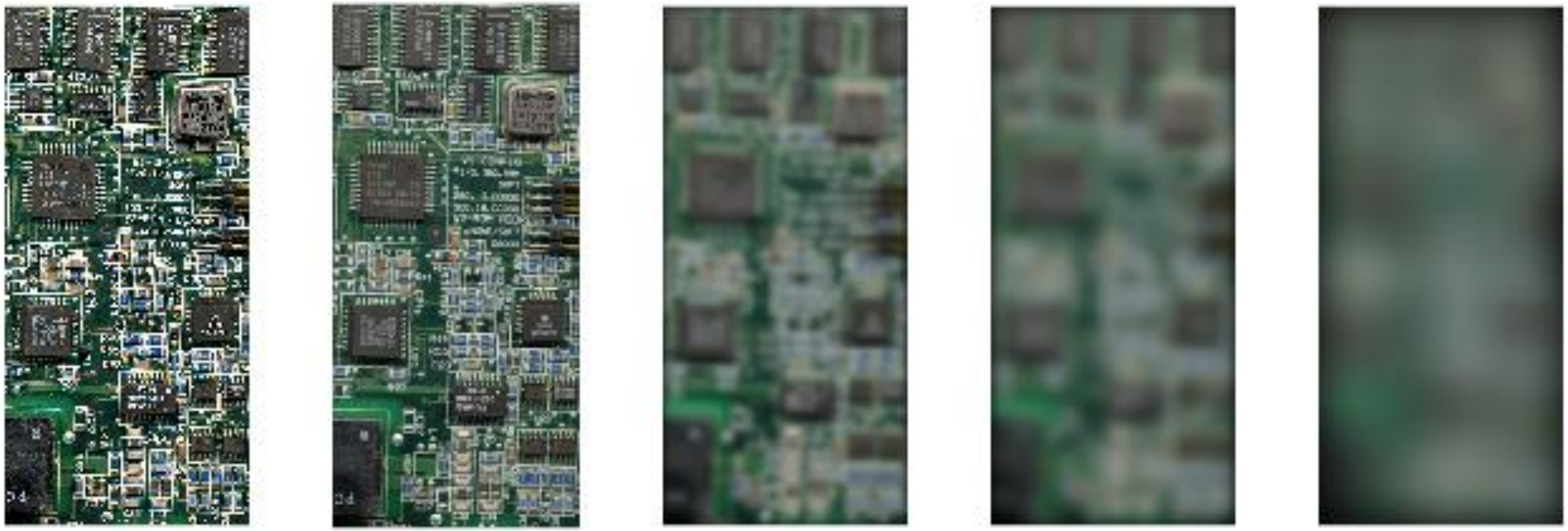
# Hysteresis Thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

*Finalize the detection of edges by suppressing all the other edges that are weak and not connected to strong edges.*

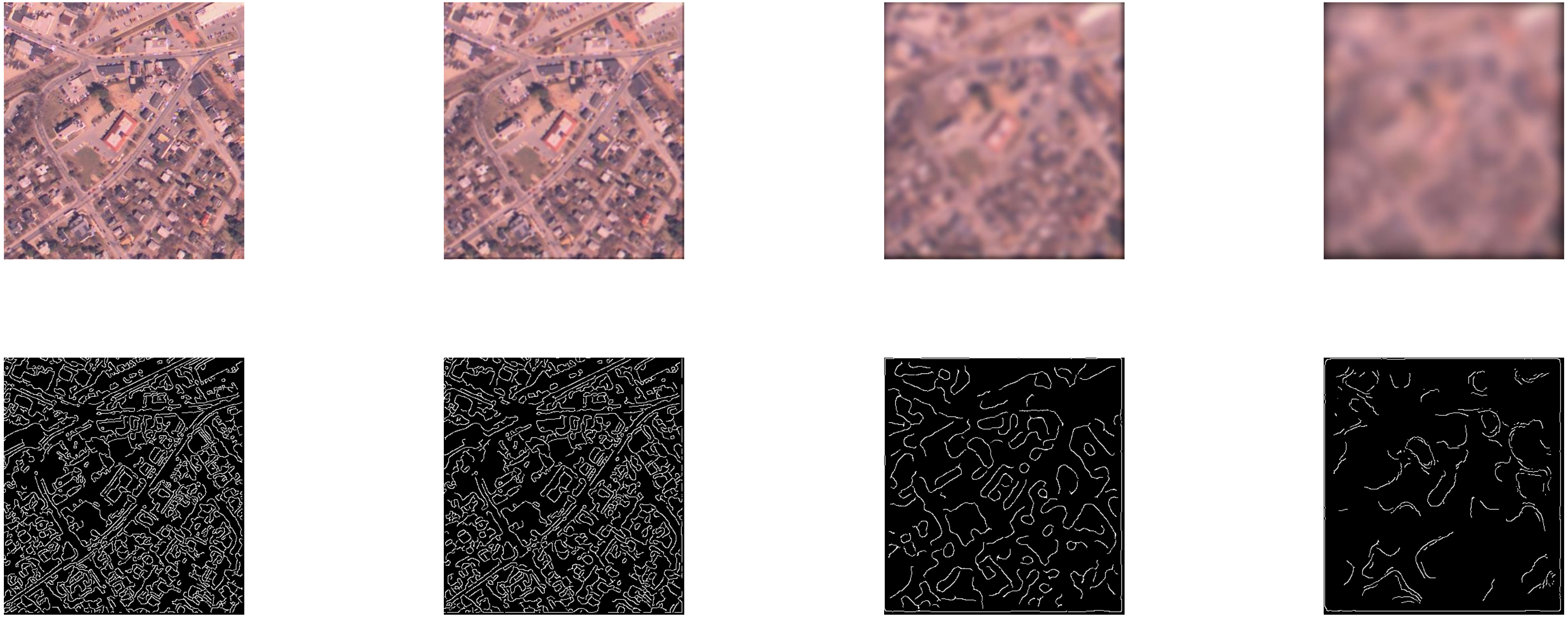
# Final Canny Edges

# (Revisit) Effect of Gaussian Window Sizes



```
f1 =  fspecial('gaussian', 101,1);
f2 =  fspecial('gaussian', 101,5);
f3 =  fspecial('gaussian', 101,10);
f4 =  fspecial('gaussian', 101,30);
```

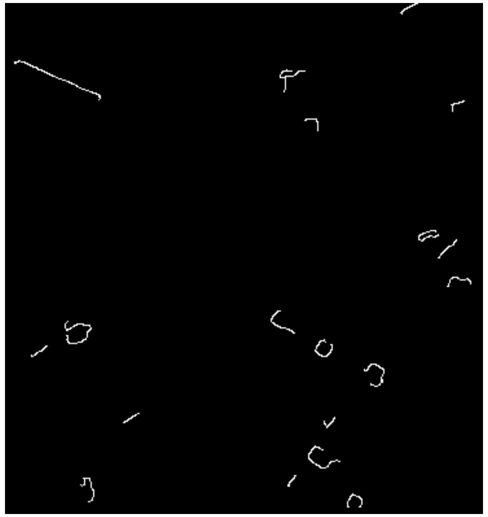
# Effect of σ (Gaussian Kernel Size)



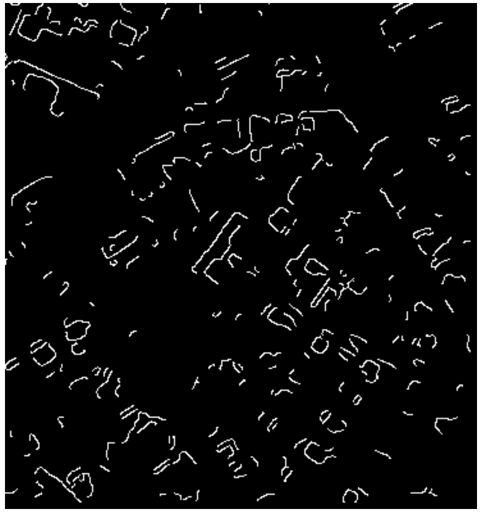
The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

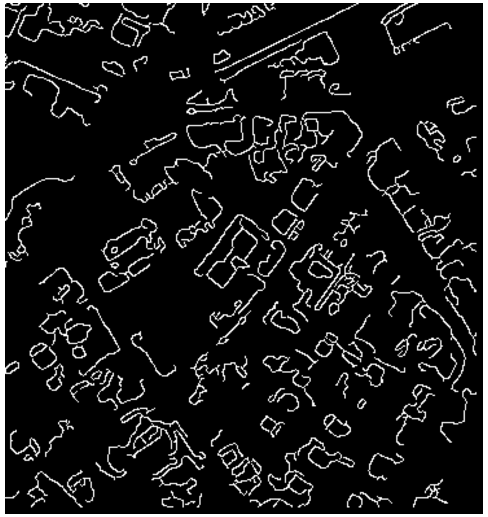
# Effect of High and Low Thresholds



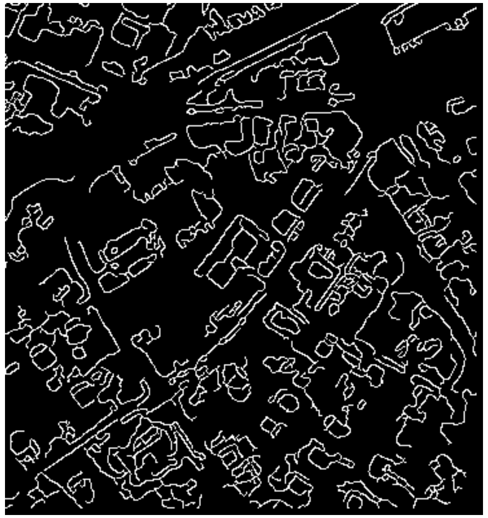
[0.4 0.8]



[0.4 0.5]



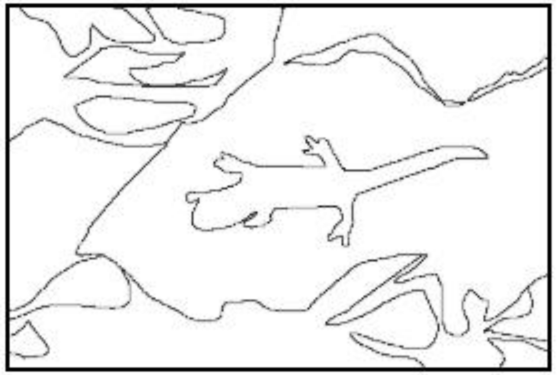
[0.2 0.5]



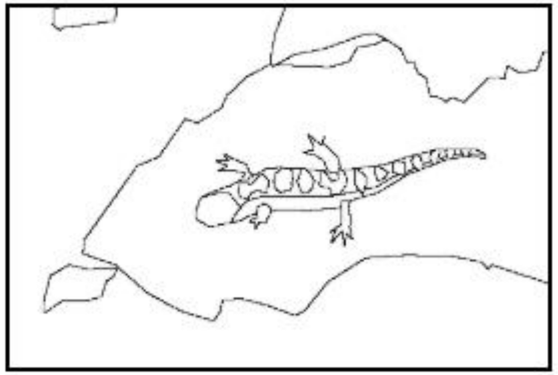
[0.1 0.5]

# Contour Detection and Image Segmentation Resources (Computer Vision Group in Berkeley)

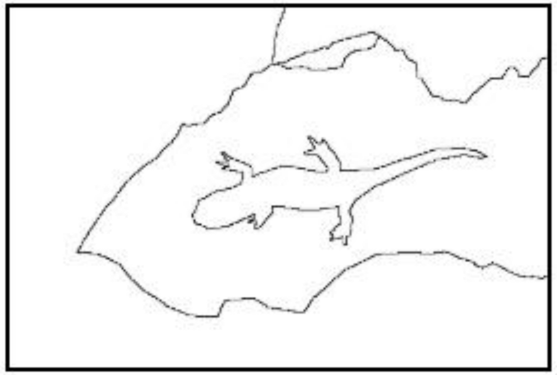
Original Image



Subject 1

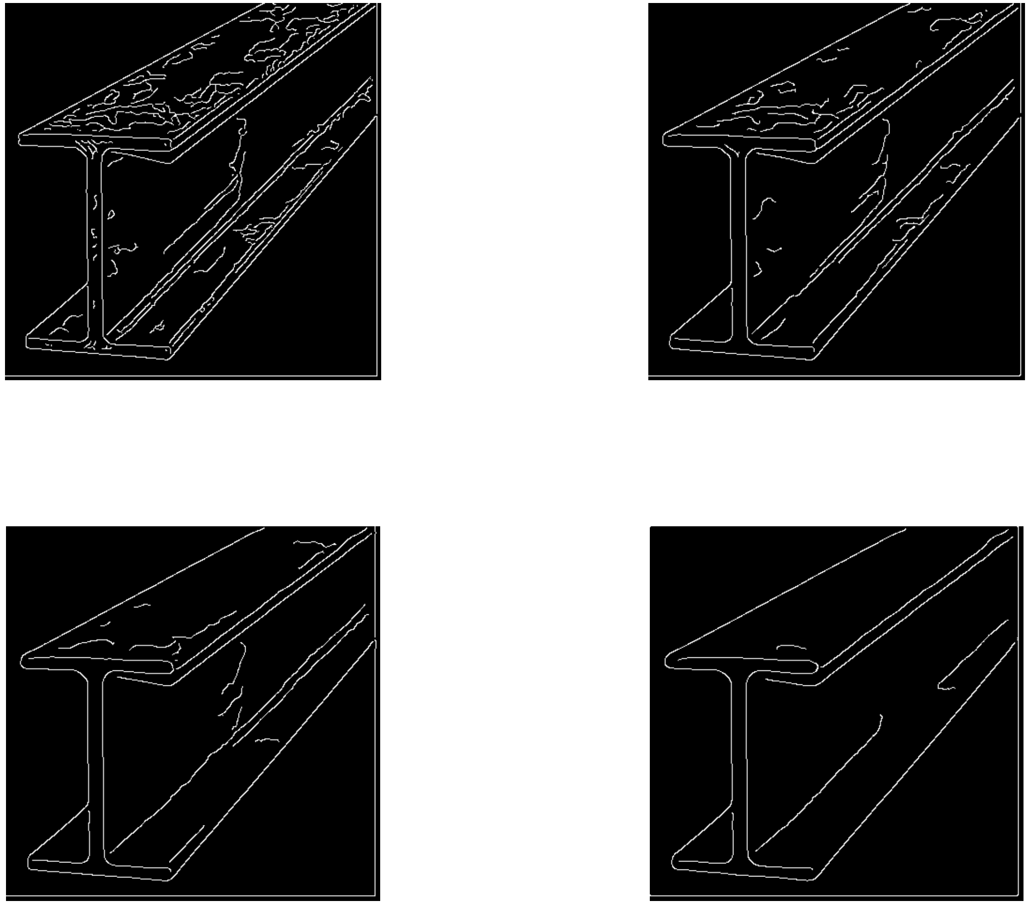


Subject 2



Subject 3

# Hough Transform



The basic idea is to examine the parameter space for lines, rather than the image spaces.

# Image and Parameter Spaces

Equation of Line: $y = mx + c$

Find: $(m, c)$

Consider point: $(x_i, y_i)$

$$y_i = mx_i + c \quad or \quad c = -x_i m + y_i$$

Parameter space also called Hough Space

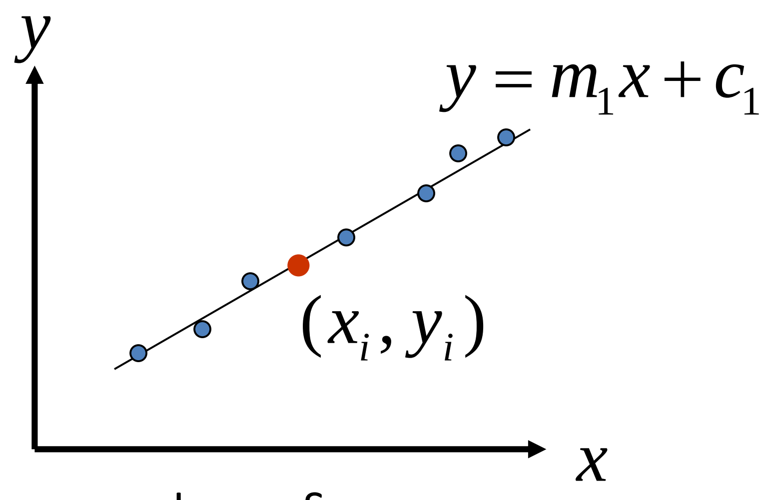



Image Space

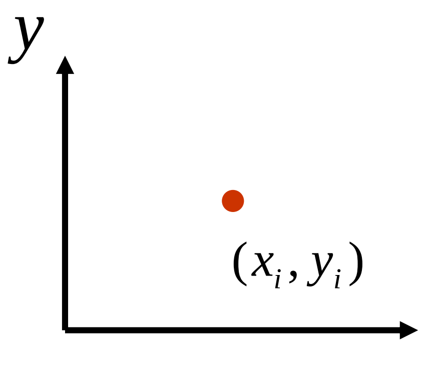
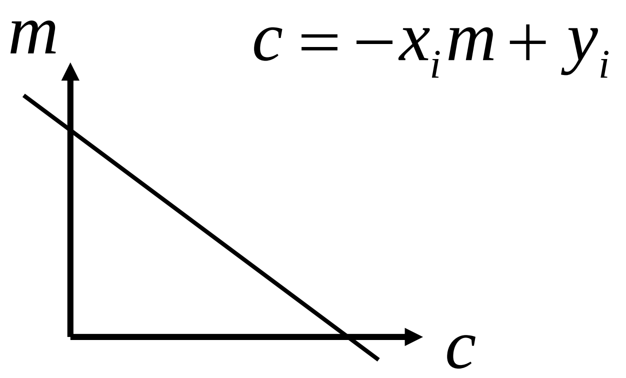
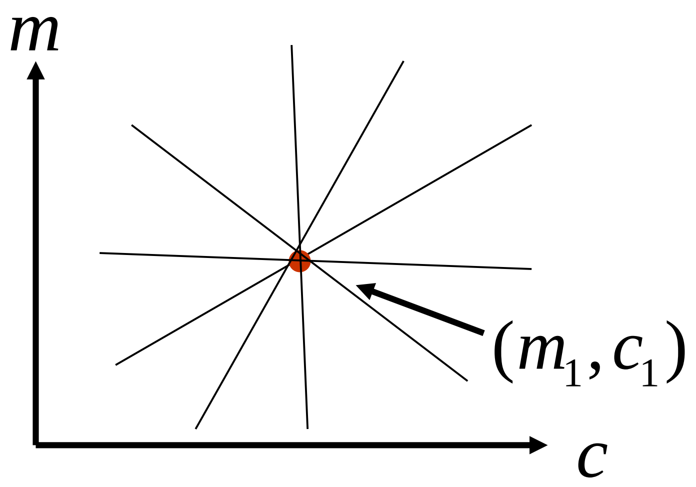



Image Space




Parameter Space




Parameter Space

# Line Detection by Hough Transform

Algorithm:

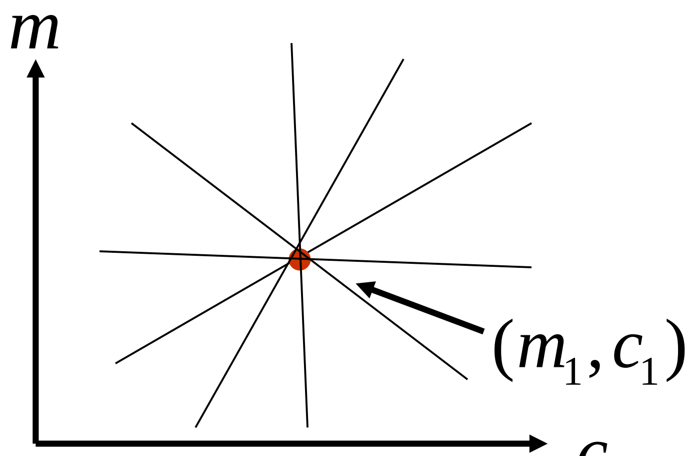
- Quantize Parameter Space $(m, c)$
- Create Accumulator Array $A(m, c)$
- Set $A(m, c) = 0 \quad \forall m, c$
- For each image edge $(x_i, y_i)$ increment:

$$A(m, c) = A(m, c) + 1$$

- If $(m, c)$ lies on the line:

$$c = -x_i m + y_i$$

- Find local maxima in $A(m, c)$

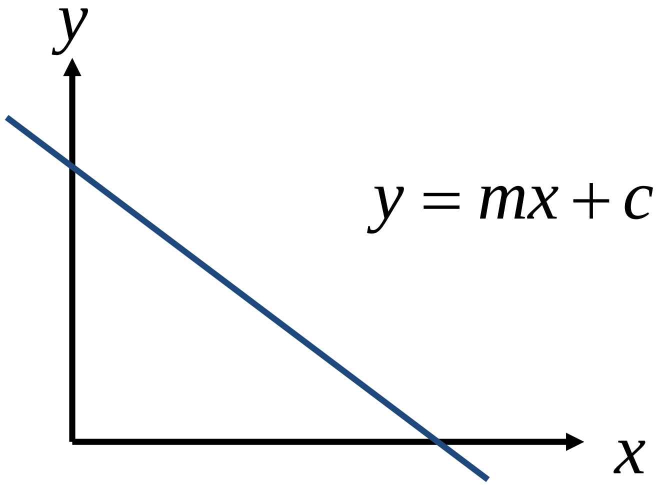



Parameter Space

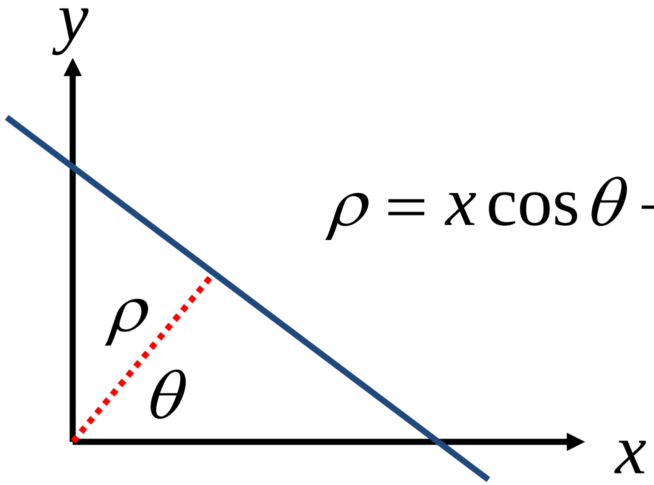
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | | | | 1 | | |
| | | 1 | | | | 1 | | | |
| | | | 1 | | 1 | | | | |
| | | | | 2 | | | | | |
| | | | 1 | | 1 | | | | |
| | | 1 | | | | 1 | | | |
| | 1 | | | | | | 1 | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$A(m, c)$

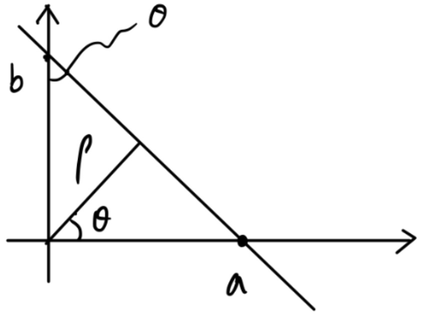
# Another Representation of a Line Equation




$$y = mx + c$$




$$\rho = x\cos\theta + y\sin\theta$$




$$a = \frac{\rho}{\cos\theta}, \quad b = \frac{\rho}{\sin\theta}$$

$$m = -\frac{b}{a} = -\frac{\cos\theta}{\sin\theta}, \quad c = \frac{\rho}{\sin\theta}$$

$$\therefore \; y = -\frac{\cos\theta}{\sin\theta}x + \frac{\rho}{\sin\theta}$$

$$\rho = x\cos\theta + y\sin\theta$$

# Better Parameterization

NOTE: $-\infty \leq m \leq \infty$

Large Accumulator

More memory and computations

Improvement: (Finite Accumulator Array Size)

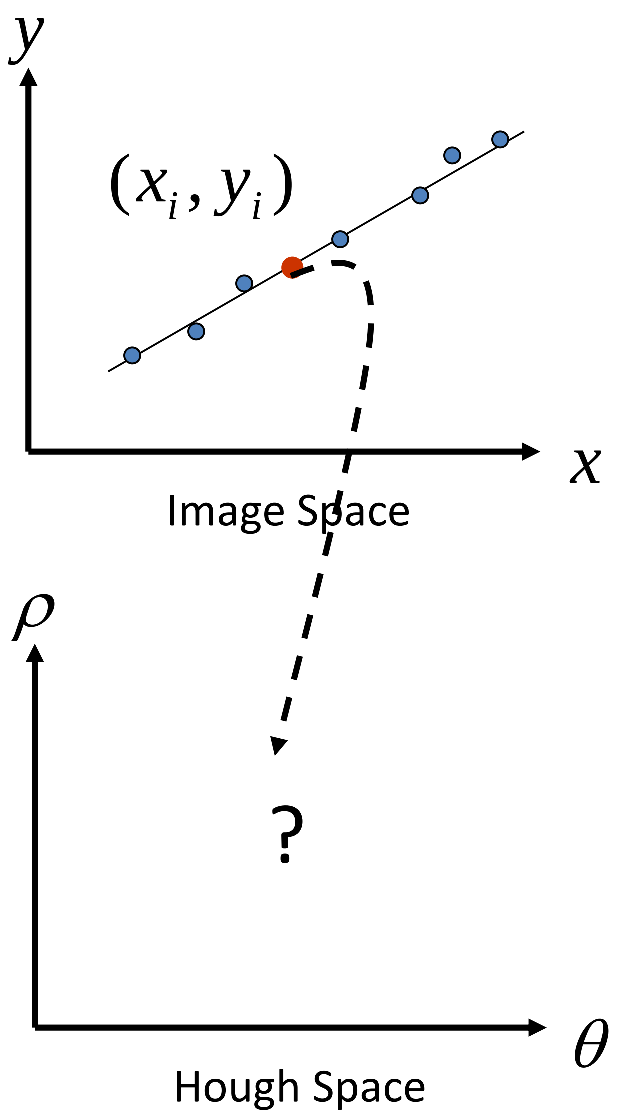
Line equation: $\rho = x\cos\theta + y\sin\theta$

Here $0 \leq \theta \leq 2\pi$

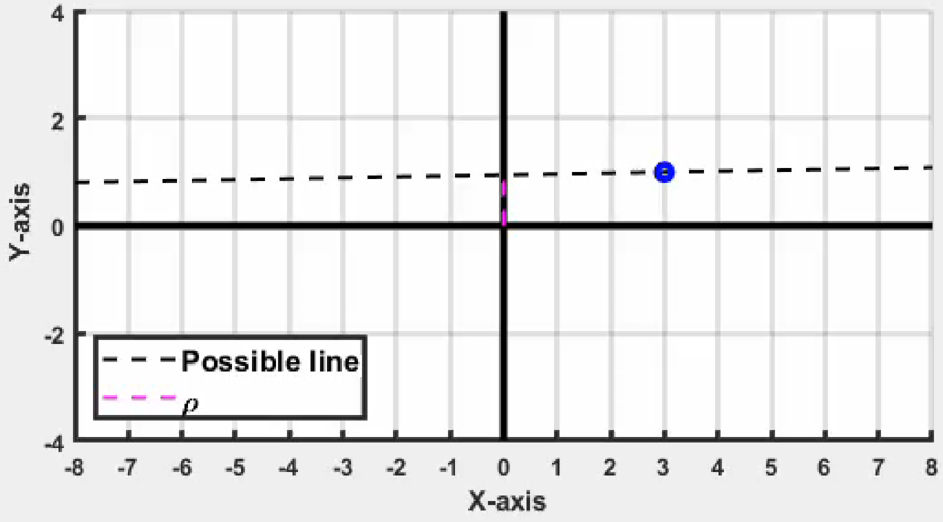
$0 \leq \rho \leq \rho_{\max}$

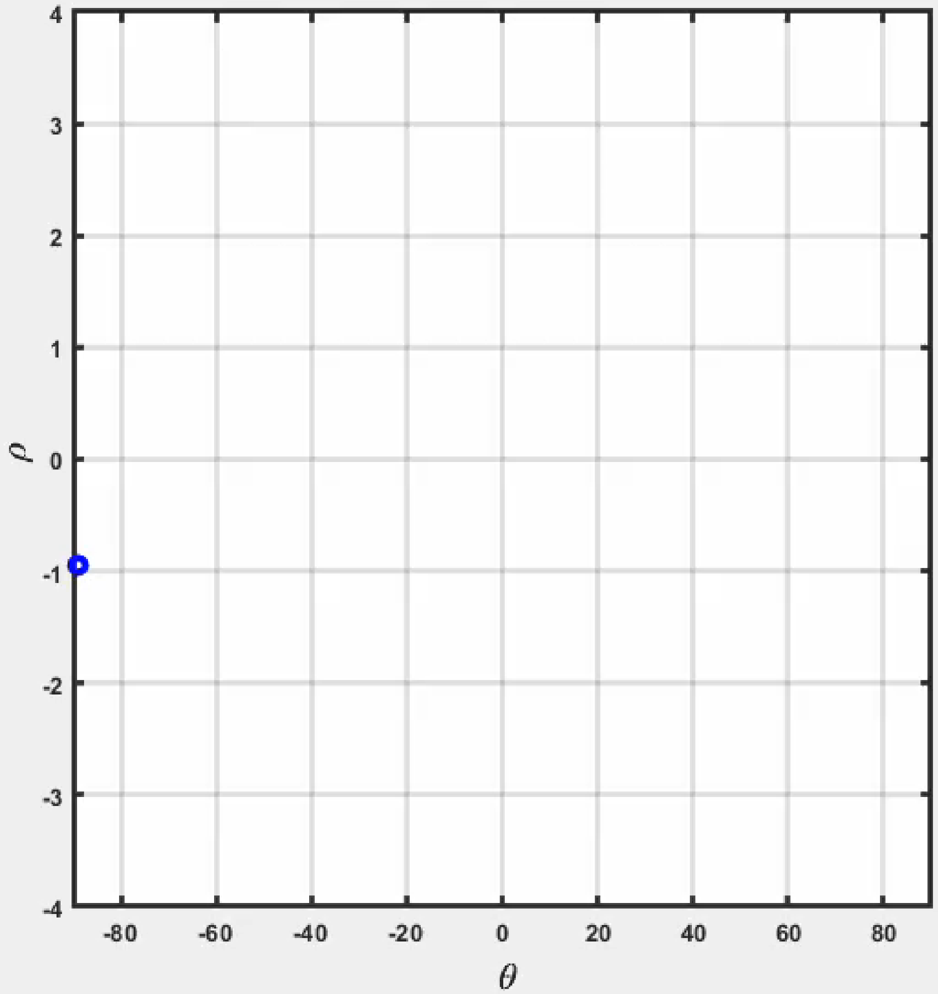
Given points $(x_i, y_i)$ find $(\rho, \theta)$

Hough Space Sinusoid

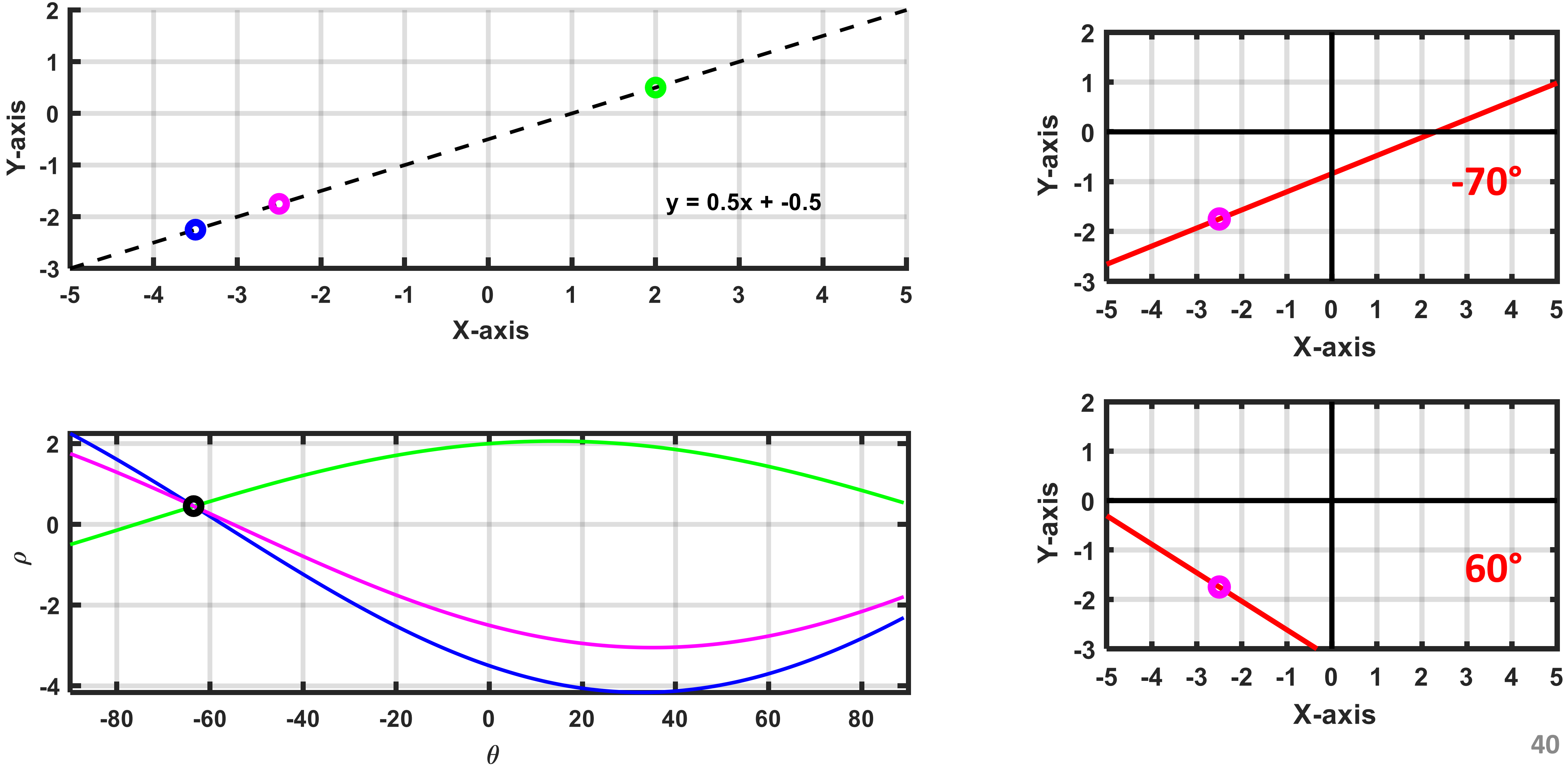


# Hough Transform Demo

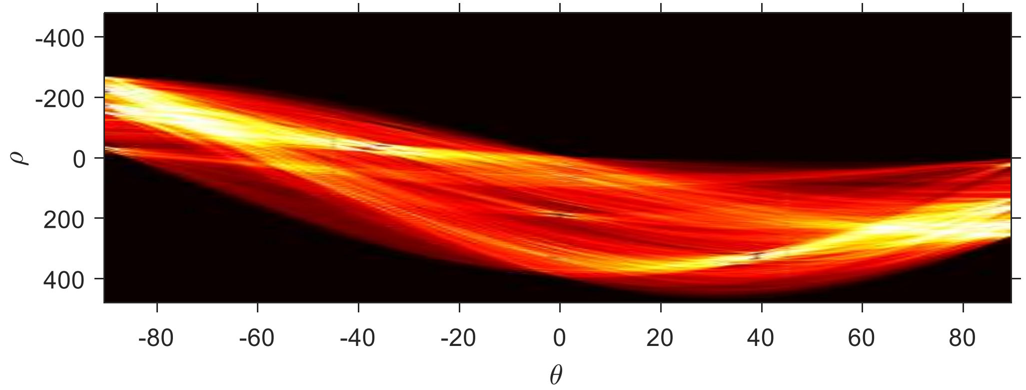


# Example: Line Passing Through Three Dots



# Example: Line Detection




$1^{st}$ Peak

# Principles of the Hough Transform

- Difficulties
  - how big should the cells be? (too big, and we merge quite different lines; too small, and noise causes lines to be missed)

- How many lines?
  - Count the peaks in the Hough array
  - Treat adjacent peaks as a single peak
- Which points belong to each line?
  - Search for points close to the line
  - Solve again for line and iterate

# Slide Credits and References

- Lecture notes: S. Narasimhan
- Lecture notes: Gordon Wetzstein
- Lecture notes: Noah Snavely
- Lecture notes: L. Fei-Fei
- Lecture notes: D. Frosyth
- Lecture notes: James Hayes
- Lecture notes: Yacov Hel-Or