The line l tangent to C a point on C is given by $l = Cx$.

Let's assume that $l = Cx$ passes through x,

$$l^T x = (Cx)^T x = x^T C^T x = x^T C x = 0$$

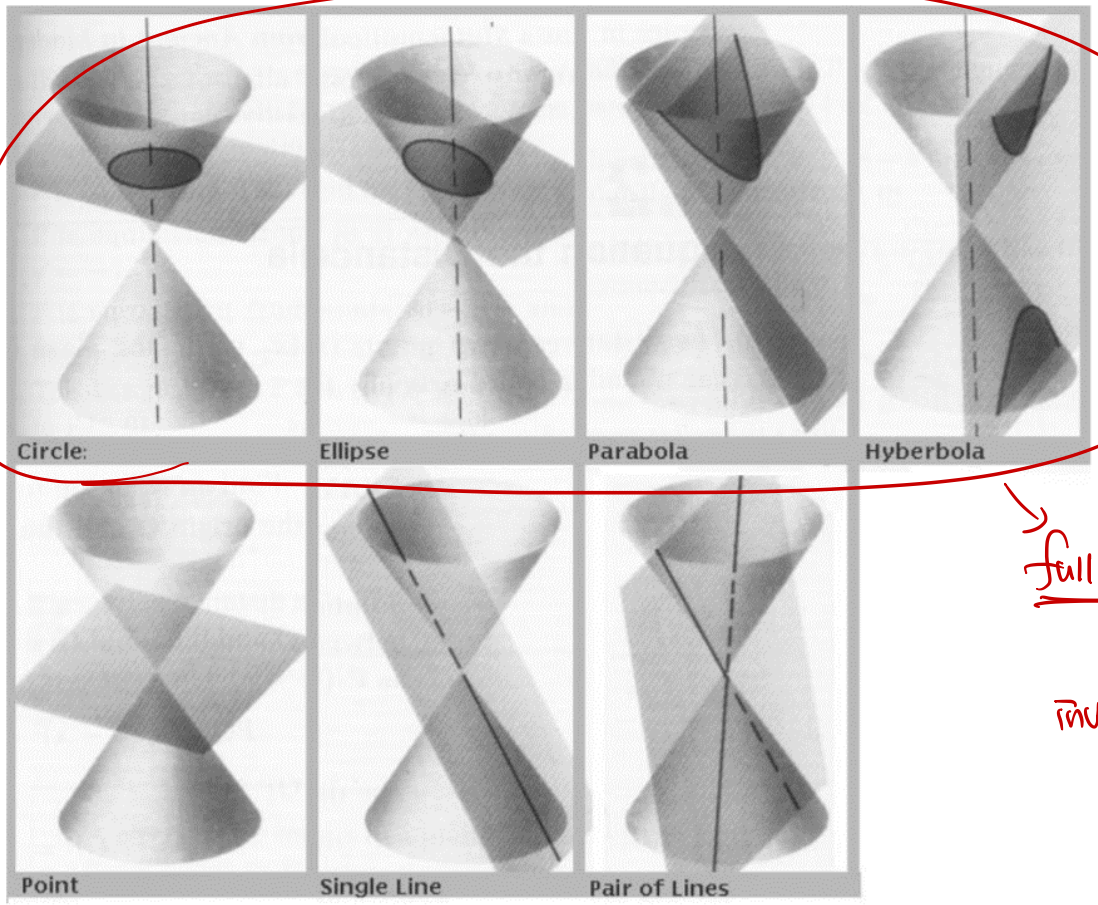
$\because$ C is a symmetrix matrix.

If l Contact one point x, l is a tagential line.

Otherwise, assume that l passess trough two points, x and y

$x^T C x = 0$ $\quad l = Cx$ $\quad C^T l = x$ $\quad$ C is invetiable

$y^T C y = 0$ $\quad l = Cy$ $\quad C^T l = y$ $\quad x = y$.



full rank

↓

Invertible.