

Neural Network II

Chul Min Yeum

Assistant Professor

Civil and Environmental Engineering

University of Waterloo, Canada

CIVE 497 – CIVE 700: Smart Structure Technology

Last updated: 2021-04-05



UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING

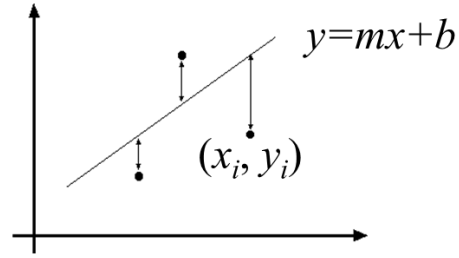
Recall: Linear Regression

Data (measurement): $(x_1, y_1), \dots, (x_n, y_n)$

Model: Line $f(x_i, m, b) = mx_i + b$

Task: Find (m, b)

Minimize $E = J(m, b) = \sum_{i=1}^n (y_i - f(x_i, m, b))^2 = \sum_{i=1}^n (y_i - mx_i - b)^2$



$$J(\theta) = J(\theta^1, \theta^2) = \sum_{i=1}^n (y_i - \theta^1 x_i - \theta^2)^2$$

$$\frac{\partial J(\theta^1, \theta^2)}{\partial \theta^1} = -2 \sum_{i=1}^n [y_i - \theta^1 x_i - \theta^2] x_i \quad \theta_{j+1}^1 \leftarrow \theta_j^1 - \alpha \frac{\partial}{\partial \theta_j^1} J(\theta)$$

$$\frac{\partial J(\theta^1, \theta^2)}{\partial \theta^2} = -2 \sum_{i=1}^n [y_i - \theta^1 x_i - \theta^2] \quad \theta_{j+1}^2 \leftarrow \theta_j^2 - \alpha \frac{\partial}{\partial \theta_j^2} J(\theta)$$

Recall: Linear Regression (Continue)

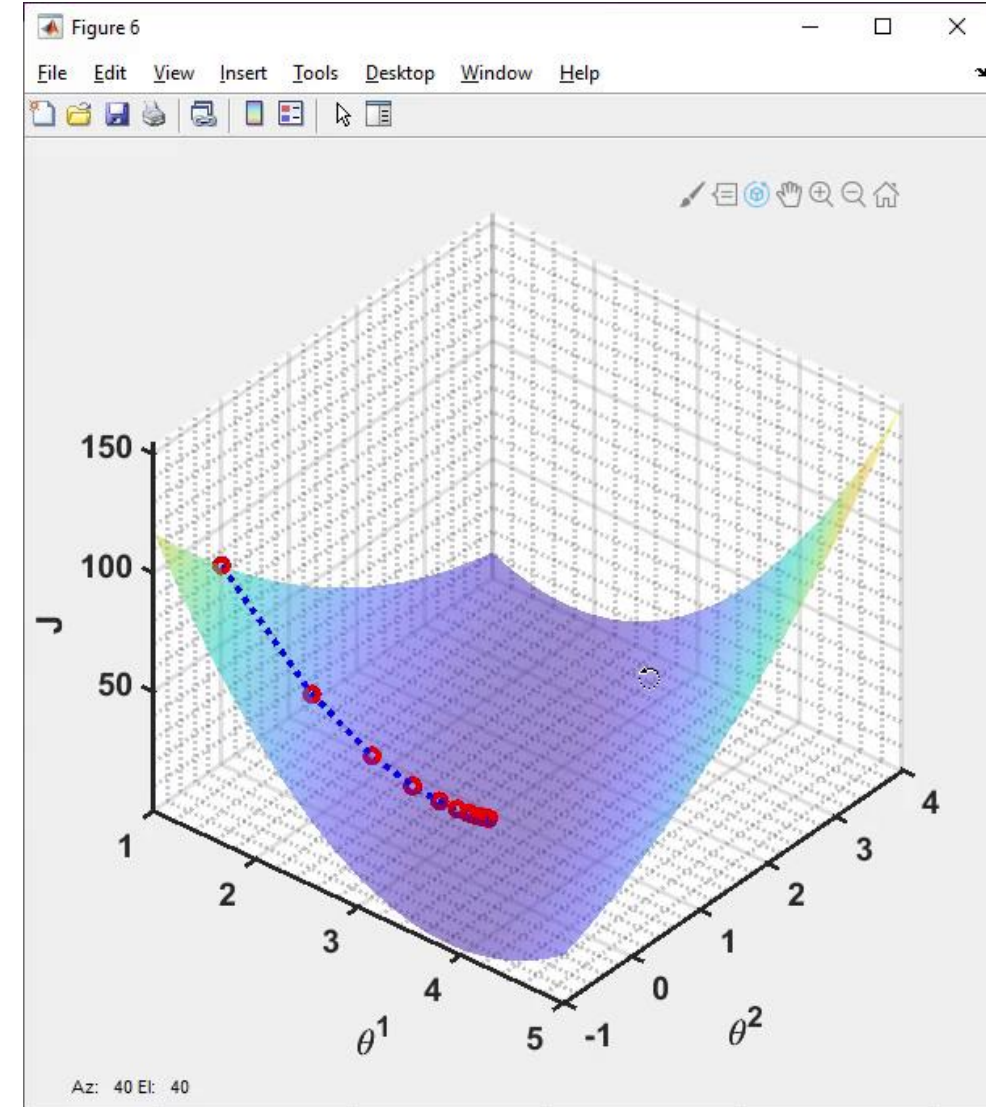
Gradient Descent

$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

Repeat until convergence

$$\frac{\partial J(\theta^1, \theta^2)}{\partial \theta^1} = -2 \sum_{i=1}^n [y_i - \theta^1 x_i - \theta^2] x_i \quad \theta_{j+1}^1 \leftarrow \theta_j^1 - \alpha \frac{\partial}{\partial \theta_j^1} J(\theta)$$

$$\frac{\partial J(\theta^1, \theta^2)}{\partial \theta^2} = -2 \sum_{i=1}^n [y_i - \theta^1 x_i - \theta^2] \quad \theta_{j+1}^2 \leftarrow \theta_j^2 - \alpha \frac{\partial}{\partial \theta_j^2} J(\theta)$$



Backward Pass

Our goal with backpropagation is to update each of the weights in the network so that they cause the actual output to be closer the target output, thereby minimizing the error for each output neuron and the network as a whole.

$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

How to find $\frac{\partial}{\partial \theta_j} J(\theta)$ to update the parameter θ ?

Chain Rule

Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

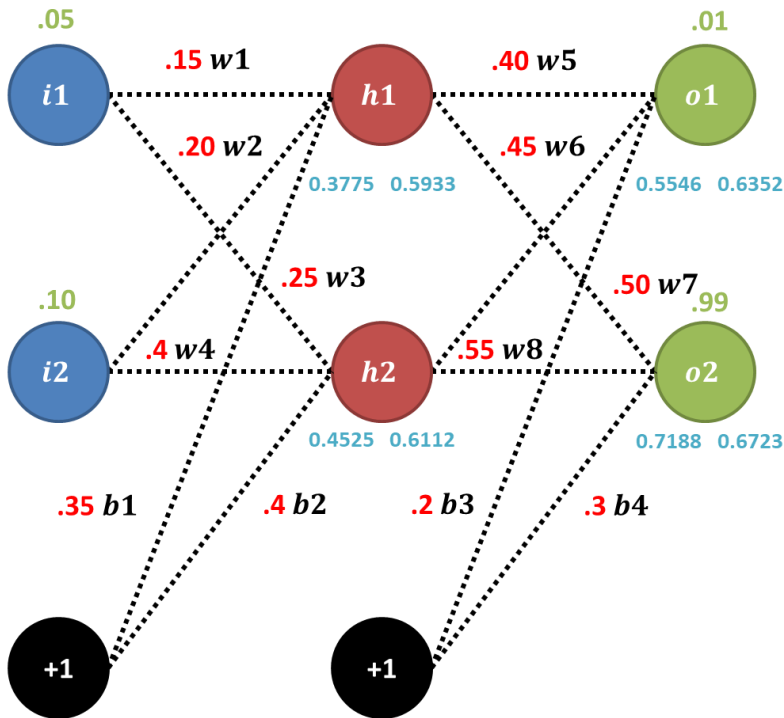
$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

Derivative of composition function

The single variable chain rule tells you how to take the derivative of the composition of two functions:

$$\frac{d}{dt} f(g(t)) = \frac{df}{dg} \frac{dg}{dt} = f'(g(t))g'(t)$$

Backpropagation (w_5)



$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

$$w_5 \leftarrow w_5 - \alpha \frac{\partial E}{\partial w_5}$$

$$E = J(\mathbf{w}, \mathbf{b})$$

$$= (o_1 - f(i_1, i_2, \mathbf{w}, \mathbf{b})_1)^2 + (o_2 - f(i_1, i_2, \mathbf{w}, \mathbf{b})_2)^2$$

$$= (o_1 - out_{o1})^2 + (o_2 - out_{o2})^2$$

$$out_{o1} = f(net_{o1}) = \frac{1}{1 + e^{-net_{o1}}}$$

$$net_{o1} = w_5 out_{h1} + w_6 out_{h2} + b_3$$

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_5}$$

$$= \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5}$$

0

Backpropagation (w_5)

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5}$$

$$\begin{aligned} E &= J(\mathbf{w}, \mathbf{b}) \\ &= (o_1 - f(i_1, i_2, \mathbf{w}, \mathbf{b})_1)^2 \\ &\quad + (o_2 - f(i_1, i_2, \mathbf{w}, \mathbf{b})_2)^2 \\ &= (o_1 - out_{o1})^2 + (o_2 - out_{o2})^2 \end{aligned}$$

$$out_{o1} = f(net_{o1}) = \frac{1}{1 + e^{-net_{o1}}}$$

$$net_{o1} = w_5 out_{h1} + w_6 out_{h2} + b_3$$

$$\frac{\partial E}{\partial out_{o1}} = -2(o_1 - out_{o1})$$

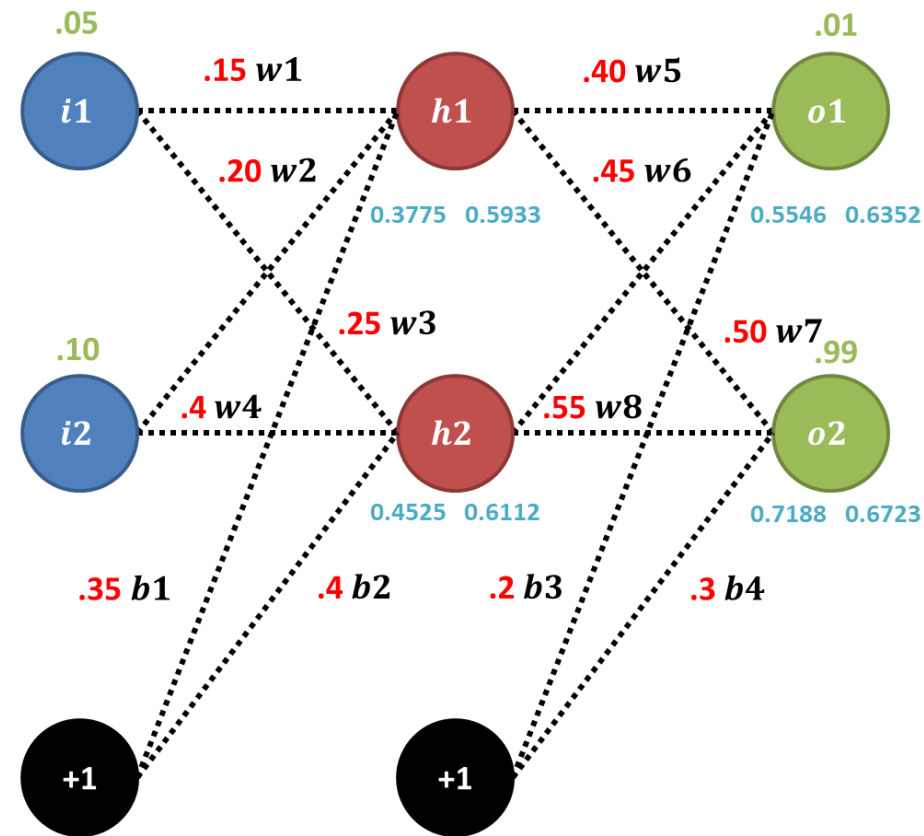
$$\frac{\partial out_{o1}}{\partial net_{o1}} = f(net_{o1}) * f(-net_{o1})$$

$$\frac{\partial net_{o1}}{\partial w_5} = out_{h1}$$

$$f(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{1 + e^x},$$

$$\frac{d}{dx} f(x) = \frac{e^x \cdot (1 + e^x) - e^x \cdot e^x}{(1 + e^x)^2} = \frac{e^x}{(1 + e^x)^2} = f(x)(1 - f(x)) = f(x)f(-x).$$

Backpropagation (w_5)



$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

$$w_5 \leftarrow w_5 - \alpha \frac{\partial E}{\partial w_5}$$

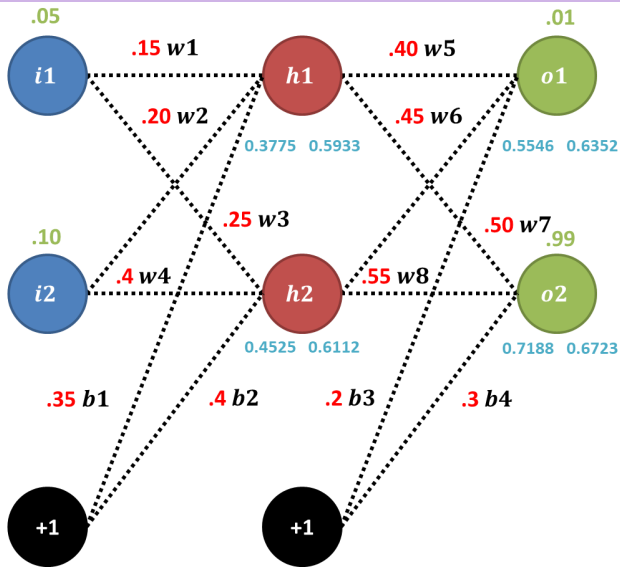
$$\frac{\partial E}{\partial out_{o1}} = -2(o_1 - out_{o1}) = -2(0.01 - 0.6352) = 1.2504$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = f(net_{o1}) * f(-net_{o1}) = f(0.5546) * f(-0.5546) = 0.2317$$

$$\frac{\partial net_{o1}}{\partial w_5} = out_{h1} = 0.5933$$

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5} = 1.2504 * 0.2317 * 0.5933 = 0.1719$$

Backpropagation (w_1)



$$E = J(\mathbf{w}, \mathbf{b}) = (o_1 - \text{out}_{o1})^2 + (o_2 - \text{out}_{o2})^2$$

$$\text{out}_{o1} = f(\text{net}_{o1}) = \frac{1}{1 + e^{-\text{net}_{o1}}}$$

$$\text{net}_{o2} = w_7 \text{out}_{h1} + w_8 \text{out}_{h2} + b_4$$

$$\text{net}_{o1} = w_5 \text{out}_{h1} + w_6 \text{out}_{h2} + b_3$$

$$\text{out}_{o2} = f(\text{net}_{o2}) = \frac{1}{1 + e^{-\text{net}_{o2}}}$$

$$\text{net}_{h1} = w_1 i_1 + w_2 i_2 + b_1$$

$$\text{net}_{h2} = w_3 i_1 + w_4 i_2 + b_2$$

$$\text{out}_{h2} = f(\text{net}_{h2}) = \frac{1}{1 + e^{-\text{net}_{h2}}}$$

$$\text{out}_{h1} = f(\text{net}_{h1}) = \frac{1}{1 + e^{-\text{net}_{h1}}}$$

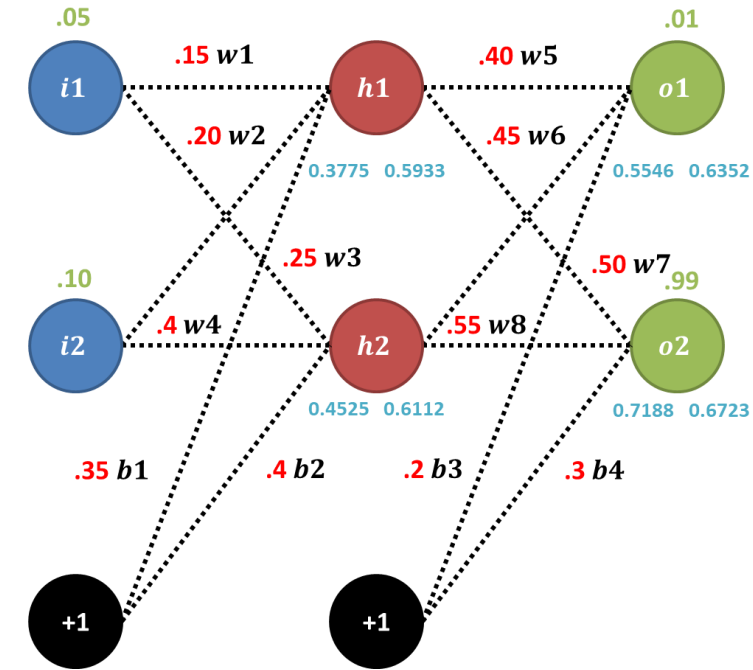
$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial \text{out}_{o1}} \frac{\partial \text{out}_{o1}}{\partial \text{net}_{o1}} \frac{\partial \text{net}_{o1}}{\partial w_1} + \frac{\partial E}{\partial \text{out}_{o2}} \frac{\partial \text{out}_{o2}}{\partial \text{net}_{o2}} \frac{\partial \text{net}_{o2}}{\partial w_1}$$

$$= -2(o_1 - \text{out}_{o1}) * f(\text{net}_{o1}) * f(-\text{net}_{o1}) * \frac{\partial \text{net}_{o1}}{\partial w_1} + -2(o_2 - \text{out}_{o2}) * f(\text{net}_{o2}) * f(-\text{net}_{o2}) * \frac{\partial \text{net}_{o2}}{\partial w_1}$$

$$\frac{\partial \text{net}_{o1}}{\partial w_1} = \frac{\partial \text{net}_{o1}}{\partial \text{out}_{h1}} \frac{\partial \text{out}_{h1}}{\partial \text{net}_{h1}} \frac{\partial \text{net}_{h1}}{\partial w_1} + \frac{\partial \text{net}_{o1}}{\partial \text{out}_{h2}} \frac{\partial \text{out}_{h2}}{\partial \text{net}_{h2}} \frac{\partial \text{net}_{h2}}{\partial w_1} = \frac{\partial \text{net}_{o1}}{\partial \text{out}_{h1}} \frac{\partial \text{out}_{h1}}{\partial \text{net}_{h1}} \frac{\partial \text{net}_{h1}}{\partial w_1} = w_5 * f(\text{net}_{h1}) * f(-\text{net}_{h1}) * i_1$$

$$\frac{\partial \text{net}_{o2}}{\partial w_1} = \frac{\partial \text{net}_{o2}}{\partial \text{out}_{h1}} \frac{\partial \text{out}_{h1}}{\partial \text{net}_{h1}} \frac{\partial \text{net}_{h1}}{\partial w_1} + \frac{\partial \text{net}_{o2}}{\partial \text{out}_{h2}} \frac{\partial \text{out}_{h2}}{\partial \text{net}_{h2}} \frac{\partial \text{net}_{h2}}{\partial w_1} = \frac{\partial \text{net}_{o2}}{\partial \text{out}_{h1}} \frac{\partial \text{out}_{h1}}{\partial \text{net}_{h1}} \frac{\partial \text{net}_{h1}}{\partial w_1} = w_7 * f(\text{net}_{h1}) * f(-\text{net}_{h1}) * i_1$$

Backpropagation (w_1)



$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_1} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_1}$$

$$= -2(o_1 - out_{o1}) * f(net_{o1}) * f(-net_{o1}) * \frac{\partial net_{o1}}{\partial w_1} +$$

$$-2(o_2 - out_{o2}) * f(net_{o2}) * f(-net_{o2}) * \frac{\partial net_{o2}}{\partial w_1}$$

$$= -2(0.01 - 0.6352) * f(0.5546) * f(-0.5546) * 0.0048 - 2(0.99 - 0.6723) * f(0.7188) * f(-0.7188) * 0.0006 = 5.5090e - 04$$

$$\frac{\partial net_{o1}}{\partial w_1} = \frac{\partial net_{o1}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1} + \frac{\partial net_{o1}}{\partial out_{h2}} \frac{\partial out_{h2}}{\partial net_{h2}} \frac{\partial net_{h2}}{\partial w_1} = \frac{\partial net_{o1}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1}$$

$$= w_5 * f(net_{h1}) * f(-net_{h1}) * i_1 = 0.4 * f(0.3775) * f(-0.3775) * 0.05 = 0.0048$$

$$\frac{\partial net_{o2}}{\partial w_1} = \frac{\partial net_{o2}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1} + \frac{\partial net_{o2}}{\partial out_{h2}} \frac{\partial out_{h2}}{\partial net_{h2}} \frac{\partial net_{h2}}{\partial w_1} = \frac{\partial net_{o2}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1}$$

$$= w_7 * f(net_{h1}) * f(-net_{h1}) * i_1 = 0.5 * f(0.3775) * f(-0.3775) * 0.05 = 0.0006$$

$$\theta^{j+1} \leftarrow \theta^j - \alpha \frac{\partial}{\partial \theta^j} J(\theta)$$

$$w_1 \leftarrow w_1 - \alpha \frac{\partial E}{\partial w_1}$$

Efficient Computation Forward and Backward Propagation

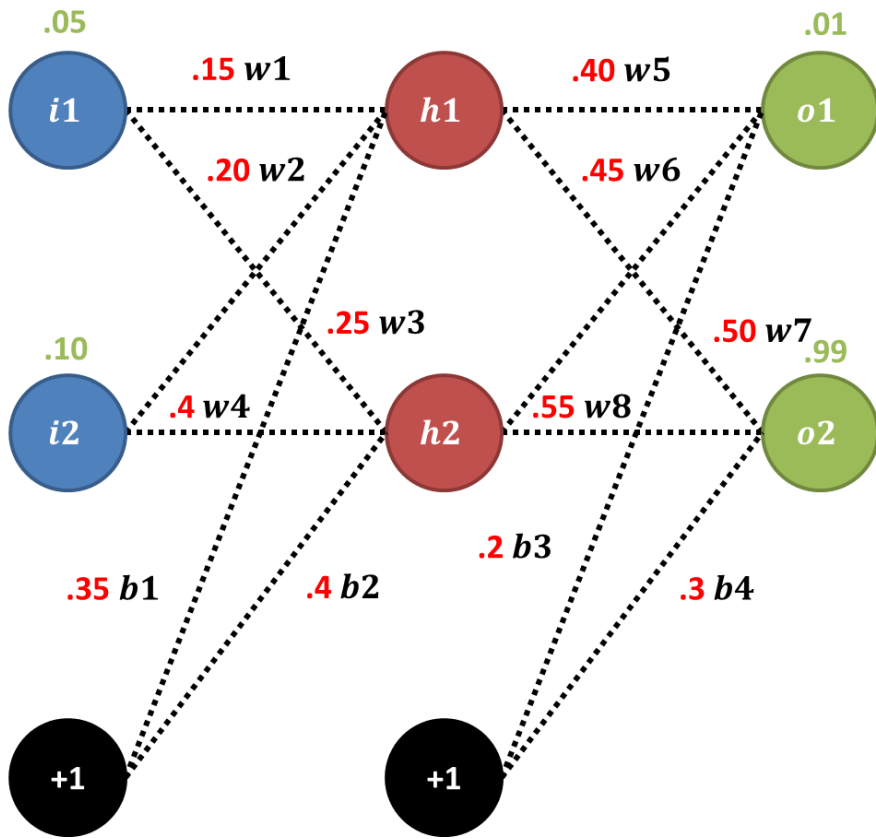
$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5}$$

$$\frac{\partial E}{\partial w_7} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_7} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_7} = \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_7}$$

$$\begin{aligned} \frac{\partial E}{\partial w_1} &= \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_1} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_1} \\ &= \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1} \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial w_4} &= \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_4} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_4} \\ &= \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial out_{h2}} \frac{\partial out_{h2}}{\partial net_{h2}} \frac{\partial net_{h2}}{\partial w_4} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial out_{h2}} \frac{\partial out_{h2}}{\partial net_{h2}} \frac{\partial net_{h2}}{\partial w_4} \end{aligned}$$

Matrix Representation of Network Parameters



$$Input = [i_1 \ i_2] \quad Output = \begin{bmatrix} o_1 \\ o_2 \end{bmatrix}$$

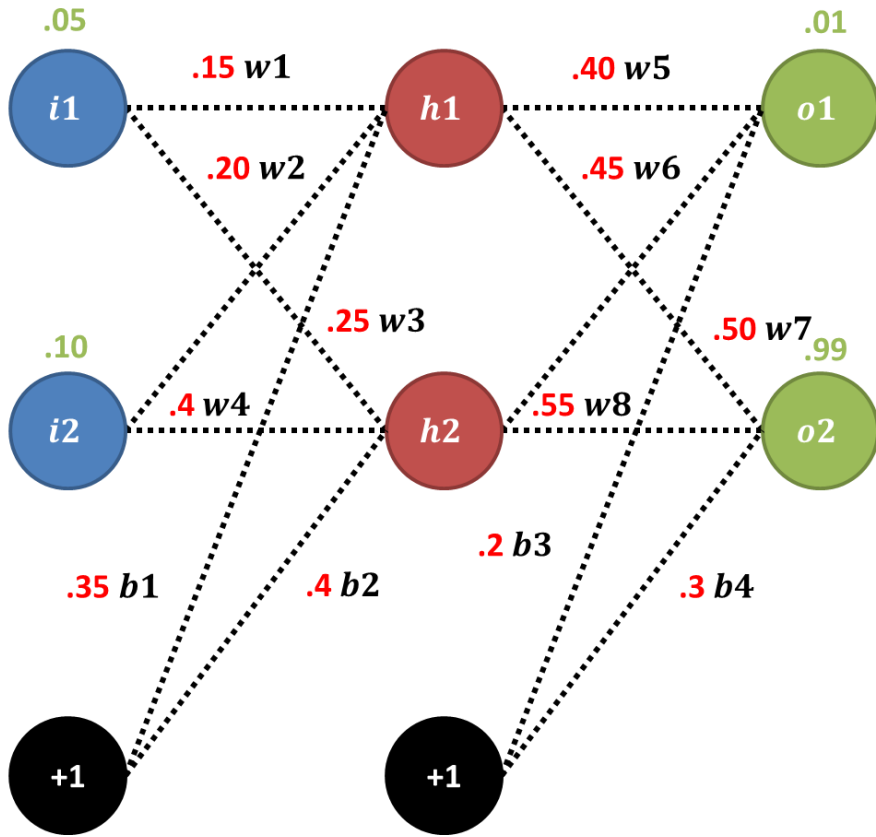
$$W_{ih} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} \quad b_{ih} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$W_{ho} = \begin{bmatrix} w_5 & w_7 \\ w_6 & w_8 \end{bmatrix} \quad b_{oh} = \begin{bmatrix} b_3 \\ b_4 \end{bmatrix}$$

$$h_{net} = \begin{bmatrix} net_{h1} \\ net_{h2} \end{bmatrix} \quad h_{out} = \begin{bmatrix} out_{h1} \\ out_{h2} \end{bmatrix}$$

$$O_{net} = \begin{bmatrix} net_{o1} \\ net_{o2} \end{bmatrix} \quad O_{out} = \begin{bmatrix} out_{o1} \\ out_{o2} \end{bmatrix}$$

Forward Propagation (Matrix Operation)



$$h_{net} = Input * W_{ih} + b_{ih} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} net_{h1} \\ net_{h2} \end{bmatrix}$$

$$h_{out} = f \left(\begin{bmatrix} net_{h1} \\ net_{h2} \end{bmatrix} \right) = \begin{bmatrix} out_{h1} \\ out_{h2} \end{bmatrix}$$

$$o_{net} = h_{out} * W_{ho} + b_{ho} = \begin{bmatrix} out_{h1} \\ out_{h2} \end{bmatrix} \begin{bmatrix} w_5 & w_7 \\ w_6 & w_8 \end{bmatrix} + \begin{bmatrix} b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} net_{o1} \\ net_{o2} \end{bmatrix}$$

$$o_{out} = f \left(\begin{bmatrix} net_{o1} \\ net_{o2} \end{bmatrix} \right) = \begin{bmatrix} out_{o1} \\ out_{o2} \end{bmatrix}$$

$$E = \left[\begin{bmatrix} out_{o1} \\ out_{o2} \end{bmatrix} - Ouptut \right]^T \left[\begin{bmatrix} out_{o1} \\ out_{o2} \end{bmatrix} - Ouptut \right]$$

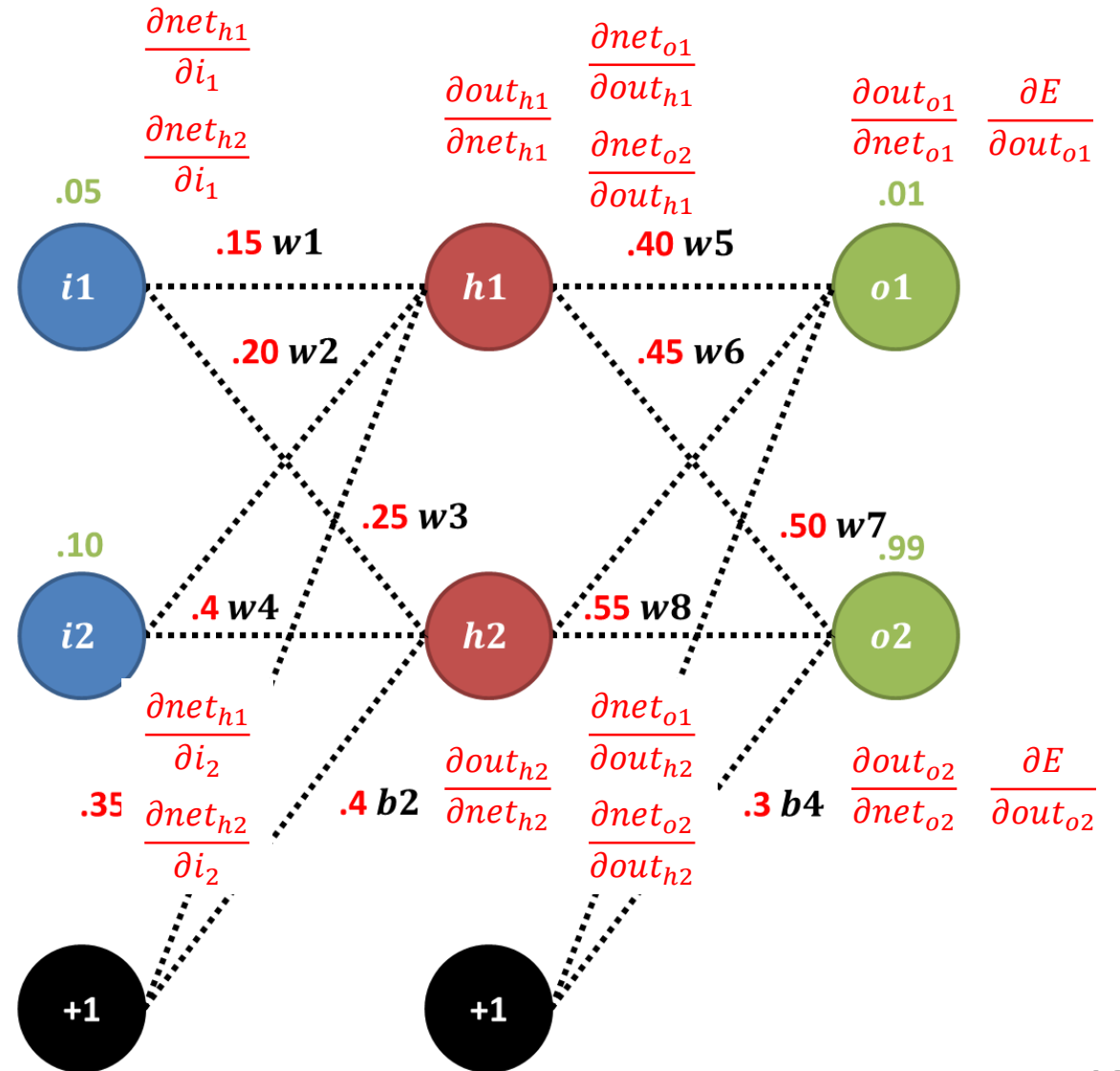
Backward Propagation (Matrix Operation)

$$O_{ON} = \begin{bmatrix} \frac{\partial out_{o1}}{\partial net_{o1}} & \frac{\partial out_{o1}}{\partial net_{o2}} \\ \frac{\partial out_{o2}}{\partial net_{o1}} & \frac{\partial out_{o2}}{\partial net_{o2}} \end{bmatrix} \quad O_{EO} = \begin{bmatrix} \frac{\partial E}{\partial out_{o1}} & \frac{\partial E}{\partial out_{o2}} \\ \frac{\partial E}{\partial out_{o1}} & \frac{\partial E}{\partial out_{o2}} \end{bmatrix}$$

$$O_{NW} = \begin{bmatrix} \frac{\partial net_{o1}}{\partial w_5} & \frac{\partial net_{o2}}{\partial w_7} \\ \frac{\partial net_{o1}}{\partial w_6} & \frac{\partial net_{o2}}{\partial w_5} \end{bmatrix}$$

$$H_{ON} = \begin{bmatrix} \frac{\partial out_{h1}}{\partial net_{h1}} & \frac{\partial out_{h1}}{\partial net_{h2}} \\ \frac{\partial out_{h2}}{\partial net_{h1}} & \frac{\partial out_{h2}}{\partial net_{h2}} \end{bmatrix} \quad H_{EO} = \begin{bmatrix} \frac{\partial net_{o1}}{\partial out_{h1}} & \frac{\partial net_{o1}}{\partial out_{h2}} \\ \frac{\partial net_{o2}}{\partial out_{h1}} & \frac{\partial net_{o2}}{\partial out_{h2}} \end{bmatrix}$$

$$H_{NW} = \begin{bmatrix} \frac{\partial net_{h1}}{\partial w_1} & \frac{\partial net_{h1}}{\partial w_3} \\ \frac{\partial net_{h1}}{\partial w_2} & \frac{\partial net_{h1}}{\partial w_4} \end{bmatrix} \quad I_{EO} = \begin{bmatrix} \frac{\partial net_{h1}}{\partial i_1} & \frac{\partial net_{h1}}{\partial i_2} \\ \frac{\partial net_{h2}}{\partial i_1} & \frac{\partial net_{h2}}{\partial i_2} \end{bmatrix}$$



Backward Propagation (Matrix Operation)

$$O_{ON} = \begin{bmatrix} \frac{\partial out_{o1}}{\partial net_{o1}} & \frac{\partial out_{o1}}{\partial net_{o1}} \\ \frac{\partial out_{o2}}{\partial net_{o1}} & \frac{\partial out_{o2}}{\partial net_{o1}} \\ \frac{\partial out_{o1}}{\partial net_{o2}} & \frac{\partial out_{o1}}{\partial net_{o2}} \\ \frac{\partial out_{o2}}{\partial net_{o2}} & \frac{\partial out_{o2}}{\partial net_{o2}} \end{bmatrix}$$

$$O_{ON} = \begin{bmatrix} f(net_{o1})f(-net_{o1}) & f(net_{o1})f(-net_{o1}) \\ f(net_{o2})f(-net_{o2}) & f(net_{o2})f(-net_{o2}) \end{bmatrix}$$

$$O_{NW} = \begin{bmatrix} \frac{\partial net_{o1}}{\partial w_5} & \frac{\partial net_{o2}}{\partial w_7} \\ \frac{\partial net_{o1}}{\partial w_6} & \frac{\partial net_{o2}}{\partial w_8} \end{bmatrix}$$

$$O_{NW} = \begin{bmatrix} out_{h1} & out_{h1} \\ out_{h2} & out_{h2} \end{bmatrix}$$

$$E = J(\mathbf{w}, \mathbf{b}) = (o_1 - out_{o1})^2 + (o_2 - out_{o2})^2$$

$$out_{o1} = f(net_{o1}) = \frac{1}{1 + e^{-net_{o1}}}$$

$$net_{o1} = w_5 out_{h1} + w_6 out_{h2} + b_3$$

$$net_{h1} = w_1 i_1 + w_2 i_2 + b_1$$

$$out_{h1} = f(net_{h1}) = \frac{1}{1 + e^{-net_{h1}}}$$

$$net_{o2} = w_7 out_{h1} + w_8 out_{h2} + b_4$$

$$out_{o2} = f(net_{o2}) = \frac{1}{1 + e^{-net_{o2}}}$$

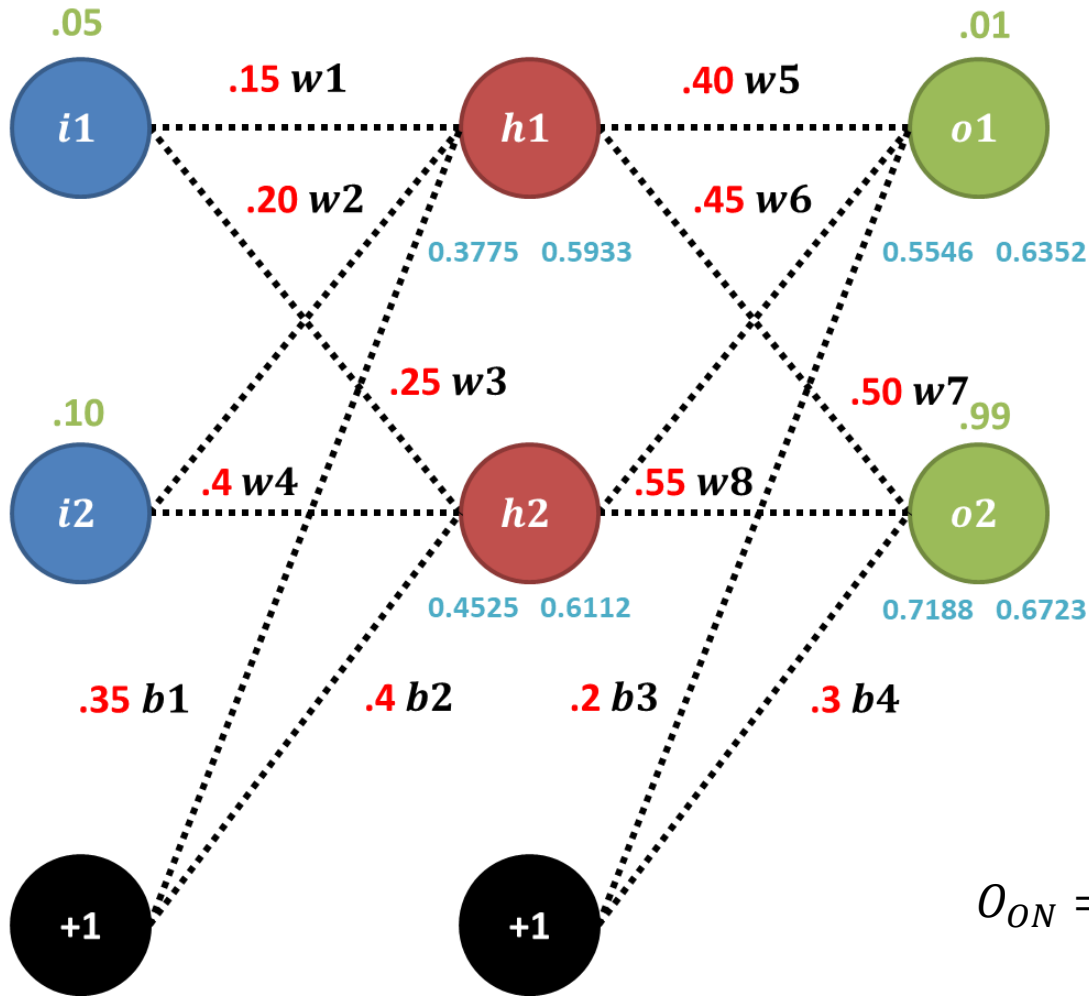
$$net_{h2} = w_3 i_1 + w_4 i_2 + b_2$$

$$out_{h2} = f(net_{h2}) = \frac{1}{1 + e^{-net_{h2}}}$$

$$O_{EO} = \begin{bmatrix} \frac{\partial E}{\partial out_{o1}} & \frac{\partial E}{\partial out_{o1}} \\ \frac{\partial E}{\partial out_{o2}} & \frac{\partial E}{\partial out_{o2}} \end{bmatrix}$$

$$O_{EO} = \begin{bmatrix} -2(o_1 - out_{o1}) & -2(o_1 - out_{o1}) \\ -2(o_2 - out_{o2}) & -2(o_2 - out_{o2}) \end{bmatrix}$$

Backward Propagation (Matrix Operation)



$$O_{EO} = \begin{bmatrix} \frac{\partial E}{\partial out_{o1}} & \frac{\partial E}{\partial out_{o1}} \\ \frac{\partial E}{\partial out_{o2}} & \frac{\partial E}{\partial out_{o2}} \end{bmatrix} = \begin{bmatrix} -2(o_1 - out_{o1}) & -2(o_1 - out_{o1}) \\ -2(o_2 - out_{o2}) & -2(o_2 - out_{o2}) \end{bmatrix}$$

$$O_{NW} = \begin{bmatrix} \frac{\partial net_{o1}}{\partial w_5} & \frac{\partial net_{o2}}{\partial w_7} \\ \frac{\partial net_{o1}}{\partial w_6} & \frac{\partial net_{o2}}{\partial w_8} \end{bmatrix} = \begin{bmatrix} out_{h1} & out_{h1} \\ out_{h2} & out_{h2} \end{bmatrix}$$

$$O_{ON} = \begin{bmatrix} \frac{\partial out_{o1}}{\partial net_{o1}} & \frac{\partial out_{o1}}{\partial net_{o1}} \\ \frac{\partial out_{o2}}{\partial net_{o2}} & \frac{\partial out_{o2}}{\partial net_{o2}} \end{bmatrix} = \begin{bmatrix} f(net_{o1})f(-net_{o1}) & f(net_{o1})f(-net_{o1}) \\ f(net_{o2})f(-net_{o2}) & f(net_{o2})f(-net_{o2}) \end{bmatrix}$$

Backward Propagation (Matrix Operation)

$$H_{ON} = \begin{bmatrix} \frac{\partial out_{h1}}{\partial net_{h1}} & \frac{\partial out_{h1}}{\partial net_{h2}} \\ \frac{\partial out_{h2}}{\partial net_{h1}} & \frac{\partial out_{h2}}{\partial net_{h2}} \end{bmatrix} = \begin{bmatrix} f(net_{h1})f(-net_{h1}) & f(net_{h1})f(-net_{h1}) \\ f(net_{h2})f(-net_{h2}) & f(net_{h2})f(-net_{h2}) \end{bmatrix}$$

$$H_{EO} = \begin{bmatrix} \frac{\partial net_{o1}}{\partial out_{h1}} & \frac{\partial net_{o1}}{\partial out_{h2}} \\ \frac{\partial net_{o2}}{\partial out_{h1}} & \frac{\partial net_{o2}}{\partial out_{h2}} \end{bmatrix} = \begin{bmatrix} w_5 & w_6 \\ w_7 & w_8 \end{bmatrix} \quad H_{NW} = \begin{bmatrix} \frac{\partial net_{h1}}{\partial w_1} & \frac{\partial net_{h1}}{\partial w_3} \\ \frac{\partial net_{h1}}{\partial w_2} & \frac{\partial net_{h1}}{\partial w_4} \end{bmatrix} = \begin{bmatrix} i_1 & i_1 \\ i_2 & i_2 \end{bmatrix}$$

$$E = J(\mathbf{w}, \mathbf{b}) = (o_1 - out_{o1})^2 + (o_2 - out_{o2})^2$$

$$out_{o1} = f(net_{o1}) = \frac{1}{1 + e^{-net_{o1}}}$$

$$net_{o2} = w_7 out_{h1} + w_8 out_{h2} + b_4$$

$$net_{o1} = w_5 out_{h1} + w_6 out_{h2} + b_3$$

$$out_{o2} = f(net_{o2}) = \frac{1}{1 + e^{-net_{o2}}}$$

$$net_{h1} = w_1 i_1 + w_2 i_2 + b_1$$

$$net_{h2} = w_3 i_1 + w_4 i_2 + b_2$$

$$out_{h1} = f(net_{h1}) = \frac{1}{1 + e^{-net_{h1}}}$$

$$out_{h2} = f(net_{h2}) = \frac{1}{1 + e^{-net_{h2}}}$$

$$I_{EO} = \begin{bmatrix} \frac{\partial net_{h1}}{\partial i_1} & \frac{\partial net_{h1}}{\partial i_2} \\ \frac{\partial net_{h2}}{\partial i_1} & \frac{\partial net_{h2}}{\partial i_2} \end{bmatrix} = \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix}$$

Backward Propagation (Matrix Operation)

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_5} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_5} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_5}$$

$$W_{ih} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix}$$

$$\frac{\partial E}{\partial w_7} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_7} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_7} = \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_7} + \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_7}$$

$$\frac{\partial E}{\partial w_{ih}} = \begin{bmatrix} \frac{\partial E}{\partial w_5} & \frac{\partial E}{\partial w_7} \\ \frac{\partial E}{\partial w_6} & \frac{\partial E}{\partial w_8} \end{bmatrix} = \begin{bmatrix} \frac{\partial E}{\partial out_{o1}} & \frac{\partial E}{\partial out_{o1}} \\ \frac{\partial E}{\partial out_{o2}} & \frac{\partial E}{\partial out_{o2}} \end{bmatrix} .* \begin{bmatrix} \frac{\partial out_{o1}}{\partial net_{o1}} & \frac{\partial out_{o1}}{\partial net_{o1}} \\ \frac{\partial out_{o2}}{\partial net_{o2}} & \frac{\partial out_{o2}}{\partial net_{o2}} \end{bmatrix} .* \begin{bmatrix} \frac{\partial net_{o1}}{\partial w_5} & \frac{\partial net_{o2}}{\partial w_7} \\ \frac{\partial net_{o1}}{\partial w_6} & \frac{\partial net_{o2}}{\partial w_8} \end{bmatrix} = O_{EO} .* O_{ON} .* O_{NW}$$

.*: Element-wise multiplication

Backward Propagation (Matrix Operation)

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial w_1} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial w_1}$$

$$W_{ho} = \begin{bmatrix} w_5 & w_7 \\ w_6 & w_8 \end{bmatrix}$$

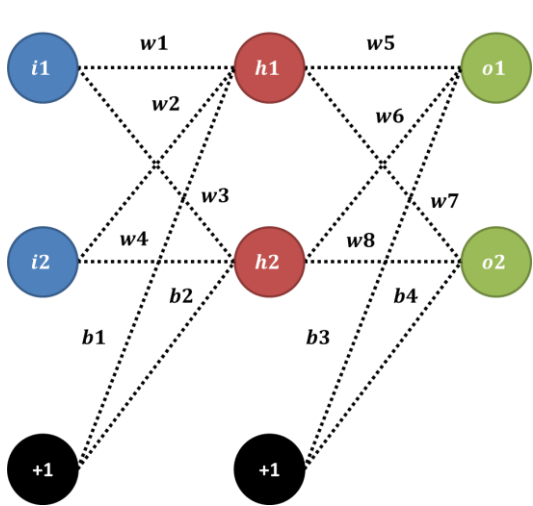
$$= \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1} + \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial out_{h1}} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1}$$

$$\frac{\partial E}{\partial w_{ho}} = \begin{bmatrix} \frac{\partial E}{\partial w_1} & \frac{\partial E}{\partial w_3} \\ \frac{\partial E}{\partial w_2} & \frac{\partial E}{\partial w_4} \end{bmatrix} = (O_{EO}.* O_{ON} * H_{EO} + flipud(O_{EO}.* O_{ON} * H_{EO})).* H_{ON}.* H_{NW}$$

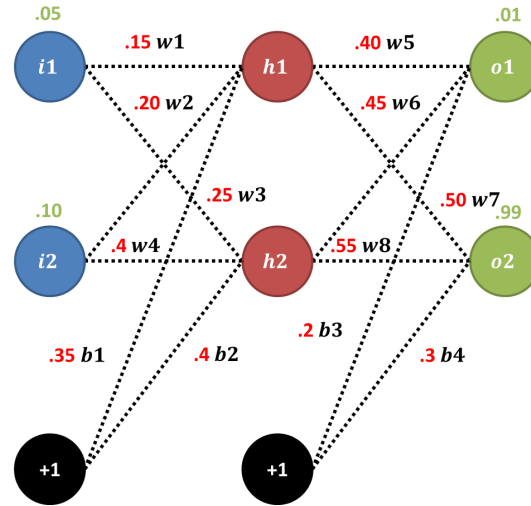
$$O_{EO}.* O_{ON} * H_{EO} = \begin{bmatrix} \frac{\partial E}{\partial out_{o1}} & \frac{\partial E}{\partial out_{o1}} \\ \frac{\partial E}{\partial out_{o2}} & \frac{\partial E}{\partial out_{o2}} \end{bmatrix} .* \begin{bmatrix} \frac{\partial out_{o1}}{\partial net_{o1}} & \frac{\partial out_{o1}}{\partial net_{o1}} \\ \frac{\partial out_{o2}}{\partial net_{o2}} & \frac{\partial out_{o2}}{\partial net_{o2}} \end{bmatrix} .* \begin{bmatrix} \frac{\partial net_{o1}}{\partial out_{h1}} & \frac{\partial net_{o1}}{\partial out_{h2}} \\ \frac{\partial net_{o2}}{\partial out_{h1}} & \frac{\partial net_{o2}}{\partial out_{h2}} \end{bmatrix} = \begin{bmatrix} \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial out_{h1}} & \frac{\partial E}{\partial out_{o1}} \frac{\partial out_{o1}}{\partial net_{o1}} \frac{\partial net_{o1}}{\partial out_{h2}} \\ \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial out_{h1}} & \frac{\partial E}{\partial out_{o2}} \frac{\partial out_{o2}}{\partial net_{o2}} \frac{\partial net_{o2}}{\partial out_{h2}} \end{bmatrix}$$

$$H_{ON}.* H_{NW} = \begin{bmatrix} \frac{\partial out_{h1}}{\partial net_{h1}} & \frac{\partial out_{h1}}{\partial net_{h1}} \\ \frac{\partial out_{h2}}{\partial net_{h1}} & \frac{\partial out_{h2}}{\partial net_{h1}} \end{bmatrix} .* \begin{bmatrix} \frac{\partial net_{h1}}{\partial w_1} & \frac{\partial net_{h1}}{\partial w_3} \\ \frac{\partial net_{h1}}{\partial w_2} & \frac{\partial net_{h1}}{\partial w_4} \end{bmatrix} = \begin{bmatrix} \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_1} & \frac{\partial out_{h1}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_3} \\ \frac{\partial out_{h2}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_2} & \frac{\partial out_{h2}}{\partial net_{h1}} \frac{\partial net_{h1}}{\partial w_4} \end{bmatrix}$$

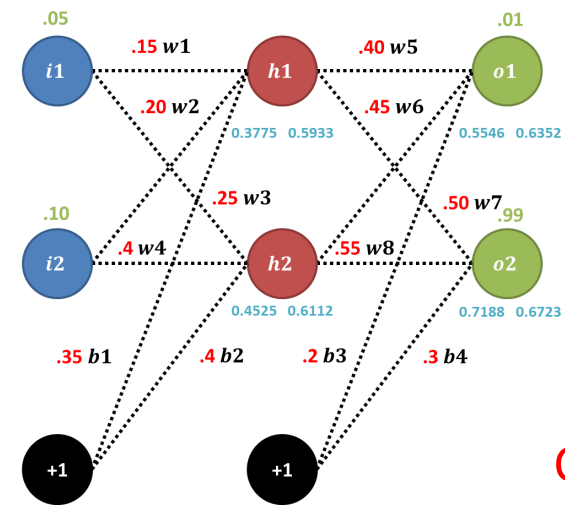
Summary of Neural Network



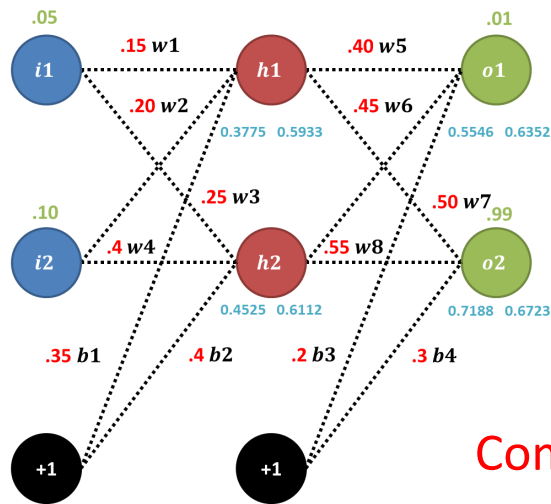
S1. Design Neural Network



S2. Initialization of NN



S3. Forward Propagation



S4. Backward Propagation

$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

S5. Update NN

$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

Batch (Vanilla) Gradient Descent

$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(x^i, y^j; \theta)$$

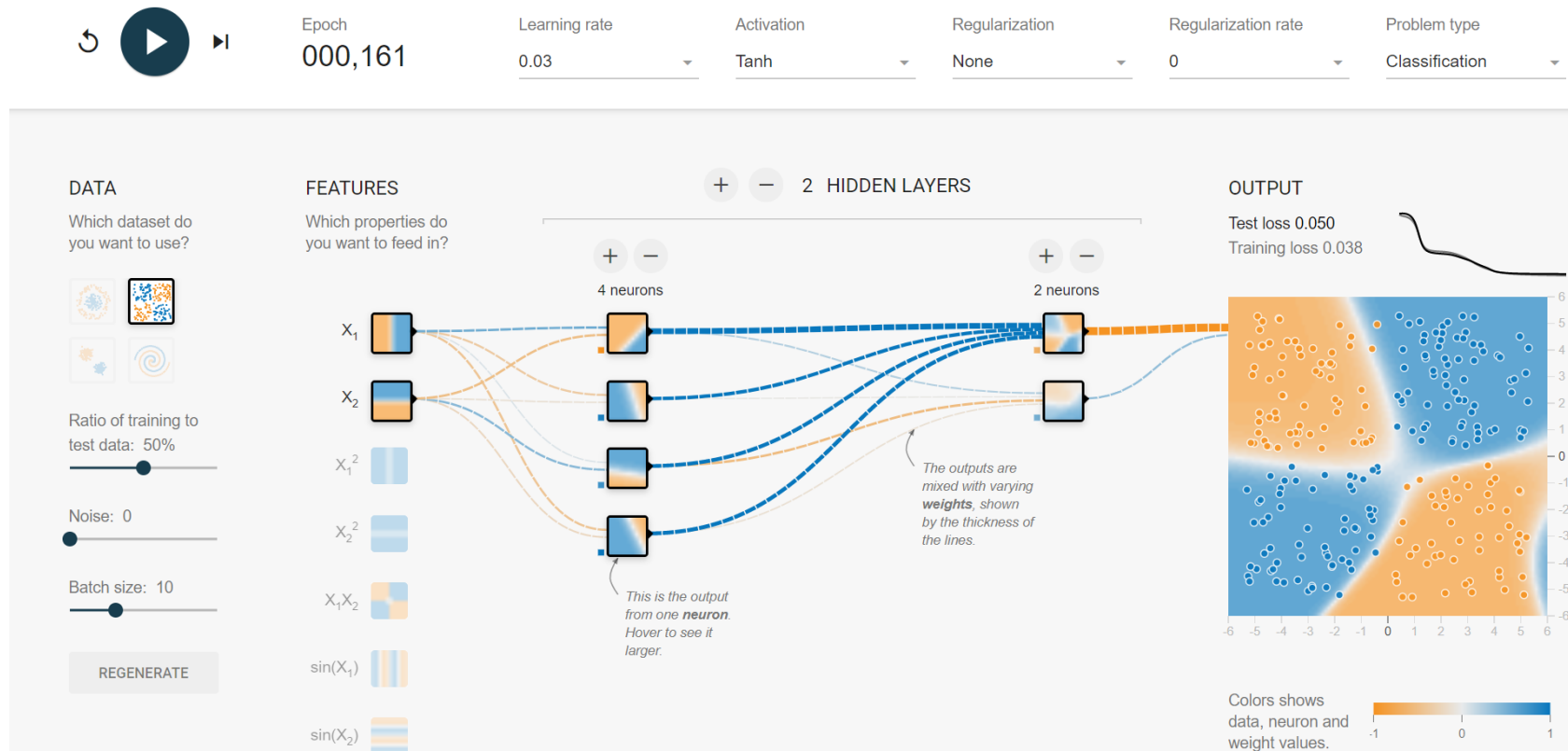
Stochastic Gradient Descent

$$\theta_{j+1} \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(x^{\{i:i+b\}}, y^{\{i:i+b\}}; \theta)$$

Mini-batch Gradient Descent

Neural Network Playground

Tinker With a **Neural Network** Right Here in Your Browser.
Don't Worry, You Can't Break It. We Promise.



Binary Classification (Circle)

The image shows the MATLAB R2018b interface with the following components:

- Editor:** Displays the script `nn_demo_run.m` with the following content:

```
1 function nn_demo_run
2 % Simple neural network demonstration
3 % This code shows how to implement a multi-layer neural network (having a
4 % single hidden layer). You can change some basic parameters including
5 % learning rate, number of neurons, number of epoch, etc. You can also see
6 % similar demo from here:
7 % https://github.com/tensorflow/playground
8
9 % Author: Chul Min Yeum (cmyeum@uwaterloo.ca)
10 % Last update: 03/25/2019
11
12 % dataset 1: demonstration using gaussian dataset
13 % dataset = nn_generate_dataset('gaussian');
14 % test_nn(dataset);
15
16 % dataset 2: demonstration using circle dataset
17 dataset = nn_generate_dataset('circle');
18 test_nn(dataset);
19
20 % dataset 3: demonstration using xor dataset
21 % dataset = nn_generate_dataset('xor');
22 % test_nn(dataset);
23
24 end
25
26 function net = set_nn(...)
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48 function test_nn(dataset) ...
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94 function show_acc(cur_ax, epoch, accuracy) ...
95
96
97
98
99
100
101
102
103
104
105
106
107
```

- Current Folder:** Lists files in the current directory:
 - `nn_activation.m`
 - `nn_demo_run.m`
 - `nn_generate_dataset.m`
 - `nn_loss.m`
 - `nn_test_grad_net.m`
 - `nn_test_net.m`
 - `nn_train_net.m`
- Workspace:** Shows the current workspace with the following variables:

Name	Value
dataset	1x1000 double
net	1x1 struct
test_nn	function handle
show_acc	function handle
- Command Window:** Displays the prompt `>>` and a message: "New to MATLAB? See resources for [Getting Started](#)."