Artificial Neural Network

Saturday, February 25, 2023 10:14 AM

ARTIFICIAL NEURAL NETWORKS

INPUT LAYER — 1

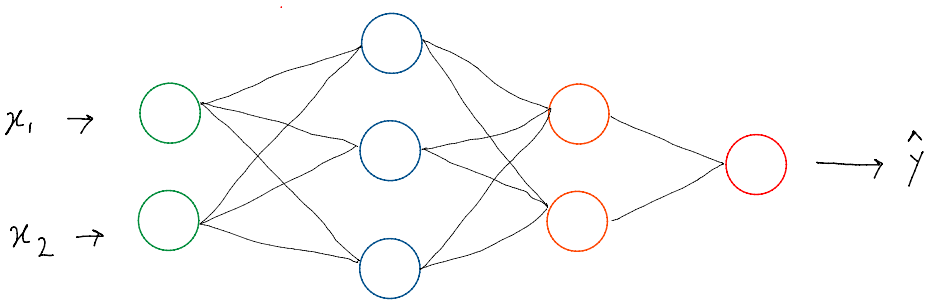
HIDDEN LAYER(S) — 0 or more

OUTPUT LAYER — 1

	ACTIVATION (HIDDEN L.)	ACTIVATION (OUTPUT L.)	LOSS FUNCTION	#Neurons OUTPUT LAYER
REGRESSION	ReLU	Linear	MSE	1
B. CLASSIFICATION	ReLU	Sigmoid	Binary Crossentropy	1
M. CLASSIFICATION	ReLU	Softmax	Categorical Crossentropy	n

Ex: Loan default classification.

Income	Spend	Default
–	–	Yes
–	–	No
–	–	Yes
⋮	⋮	⋮



* HIDDEN LAYER 1 :-

$$z_1 = w_{11}x_1 + w_{12}x_2 + b_1, \quad a_1 = ReLU(z_1)$$
$$z_2 = w_{21}x_1 + w_{22}x_2 + b_2, \quad a_2 = ReLU(z_2)$$
$$z_3 = w_{31}x_1 + w_{32}x_2 + b_3, \quad a_3 = ReLU(z_3)$$

* HIDDEN LAYER 2 :-

$$z_1 = w_{11}a_1 + w_{12}a_2 + w_{13}a_3 + b_1, \quad a_1 = ReLU(z_1)$$
$$z_2 = w_{21}a_1 + w_{22}a_2 + w_{23}a_3 + b_2, \quad a_2 = ReLU(z_2)$$

* OUTPUT LAYER :-

$z_1 = w_{11} a_1 + w_{12} a_2 + b_1$, $a_1 = \text{Sigmoid}(z_1)$

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* MATRIX :- $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

* VECTOR :-

* ROW VECTOR :- $[1\ 2\ 3]$

* COLUMN VECTOR :- $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(i) ADDITION :- $(m, n) = (p, q)$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 5 & 2 \\ 5 & 4 & 4 \end{bmatrix}$$

$(3,3)$ $(3,3)$

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$$

$(3,1)$ $(3,1)$

* MULTIPLICATION :- (M,N) , (P,Q)

$N = P$ (condition for multi.)

(M, Q) (Shape of output)

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 7 \\ 3 & 1 & 3 \\ 7 & 1 & 6 \end{bmatrix}$$

$(3,3)$ $(3,3)$ $(3,3)$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 7 & 3 \\ 7 & 5 \end{bmatrix}$$

$(3,3)$ $(3,2)$ $=$ $(3,2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

$(3,3)$ $(3,1)$ $(3,1)$

* Input to a neural network is a vector. $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

* Weights of each neuron can be represented as a vector.

$$w_1' = \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix}$$

* All weights belonging to a layer can be represented as a matrix. (Row wise).

$$W1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

* Output of a layer can be represented as a vector.

$$A1 = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

* HIDDEN LAYER 1 :-

$$\text{Input } X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad W1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}, \quad B1 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$Z1 = W1 \cdot X + B1, \quad A1 = ReLU(Z1)$$

$$\text{OUTPUT} = A1 = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

* HIDDEN LAYER 2 :-

$$\text{INPUT} = A1 = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad W2 = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix}, \quad B1 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$Z2 = W2 \cdot A1 + B2, \quad A2 = ReLU(Z2)$$

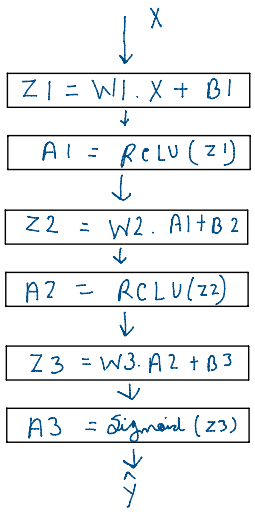
$$\text{OUTPUT} = A2 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

* OUTPUT LAYER :-

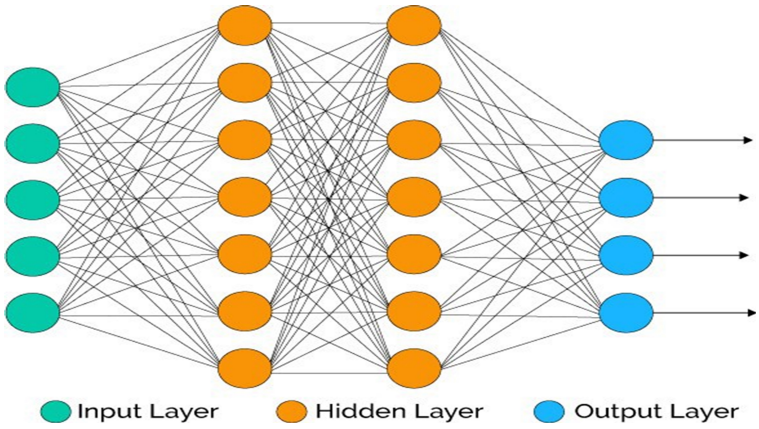
$$\text{INPUT} = A2 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad W3 = \begin{bmatrix} w_{11} & w_{12} \end{bmatrix}, \quad B3 = [b_1]$$

$$Z3 = W3 \cdot A2 + B3, \quad A3 = \text{Sigmoid}(Z3)$$

* COMPUTATION GRAPH :-



(Q.1)



	HL1	HL2	OL (5)
ACTIVATION	Relu	Relu	Softmax
Input	5,1	7,1	7,1
Weight matrix	7,5	7,7	4,7
Bias Vector	7,1	7,1	4,1
Weighted Sum	7,1	7,1	4,1
OUTPUT	7,1	7,1	4,1
Parameter	42	56	32