UNIT - 2

1-04-21

→ Fourier series

→ Fourier Transform

1. $\mathcal{F}\{f(t)\} = F(\mu)$

$$F(\mu) = \int_{-\infty}^{\infty} f(\cancel{t})\, e^{-j2\pi\mu t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\left[\cos 2\pi\mu t - j\sin 2\pi\mu t\right]$$

Inverse F.T,

$$f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{j2\pi\mu t}\, d\mu$$

$$= \int_{-\infty}^{\infty} F(\mu)\left\{\cos 2\pi\mu t + j\sin 2\pi\mu t\right\}$$

$$f(t) = \mathcal{F}^{-1}(F(\mu))$$ } 1D

2D. FT

$$F(\mu, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t, z)\, e^{-j2\pi(\mu t + vz)}\, dt.dz$$

Inverse,

$$f(t, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\mu, v)\, e^{j2\pi(\mu t + vz)}\, d\mu\, dv$$

7-4-21

⇒ Relationship b/w spatial & Frequency

⇒ Translation & Rotation:

$$f(x,y)\, e^{j2\pi(u_0 x/M + v_0 y/N)} \Leftrightarrow F(u-u_0, v-v_0) \quad \text{--- (5)}$$

$$f(x-x_0, y-y_0) \Leftrightarrow F(u,v)\, e^{-j2\pi(x_0 u/M + y_0 v/M)} \quad \text{--- (6)}$$

$f(x-y)$ by the exponential shown shifts the origin of the DFT to

$x = r\cos\theta$, $y = r\sin\theta$

⇒ Periodicity:

In 1D case, 2-D FT & its inv are infinitely perio[dic]

⇒ Unsharp masking, High-boost Filtering and High frequency-emphasis filtering 10-4-21

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

$$f_{LP}(x,y) =$$

→ Homomorphic filtering:

$$f(x,y) = i(x,y)\, r(x,y)$$

$$\mathcal{J}[f(x,y)] \neq \mathcal{J}[i(x,y)]\,.\,\mathcal{J}[r(x,y)]$$

$$z(x,y) = \ln f(x,y)$$

$$= \ln i(x,y) + \ln r(x,y)$$

$$= \mathcal{J}[\ln i(x,y)] + \mathcal{J}[\ln r(x,y)]$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

using $H(u,v)$

$$S(u,v) = H(u,v)\, Z(u,v)$$

$$= H\, F_i(u,v) + H\, F_r(u,v)$$

in spatial domain

$$s(x,y) = T^{-1}[s(u,v)]$$

$$= T^{-1}[H F_i] + T^{-1}[H F_r]$$

$$i'(x,y) = T^{-1}[H F_i]$$

$$r'(x,y) = T^{-1}[H F_r]$$

reverse the process

$$g(x,y) = e^{s(x,y)}$$

$$= e^{i'(x,y)} \, e^{r'(x,y)}$$

$$= i_0(x,y) \, r_0(x,y)$$

$$f(x,y) \rightarrow \boxed{\ln} \rightarrow \boxed{DFT} \rightarrow \boxed{H(u,v)} \rightarrow \boxed{DFT^{-1}} \rightarrow \boxed{\exp} \rightarrow g(x,y)$$