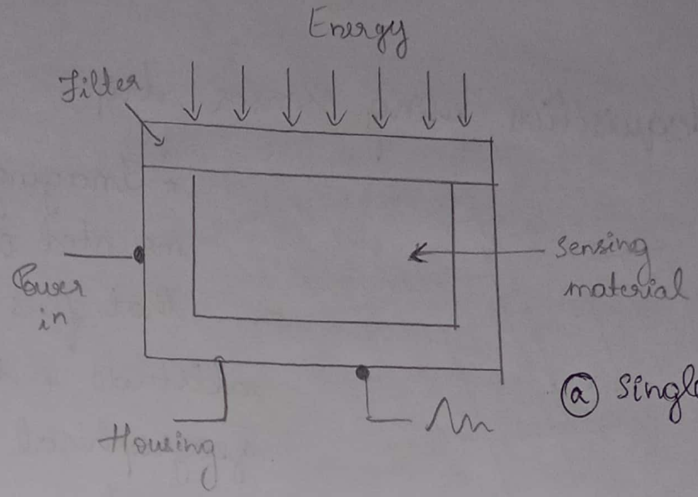
Digital Image Processing.

Text Book: Raful c Gonzaliz & Richard & Woods 4th edition

★ **Image Sensing & Acquisition:**

→ Image Acquisition using single sensor
→ " " " sensor stripe
→ " " " Sensor Arrays.

Any 1 in test. pakka



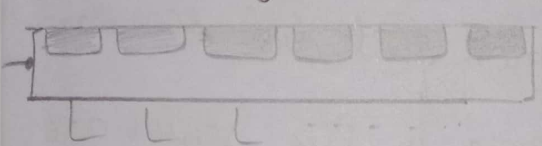
ⓐ Single sensing element.

★ Sensor of this type is the photodiode which is constructed of silicon materials & O/P voltage proportional to light intensity.

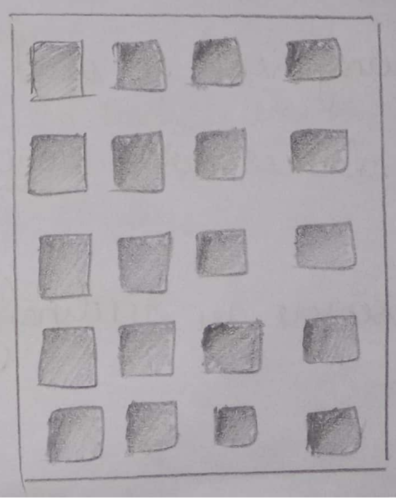
★ Filter in front of a sensor improves the selectivity.

★ Sensor O/P would be stronger for given light.

★ To generate 2-D image using a single scanning element that has to be relative displacements in both the x & y direction b/w sensore & the area to be imaged.



ⓑ line sensor.



ⓒ Array sensor

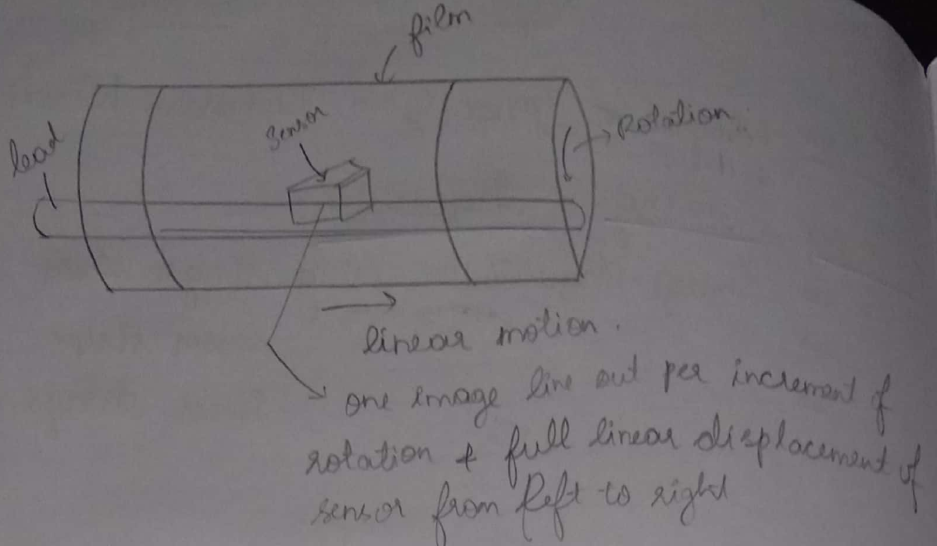
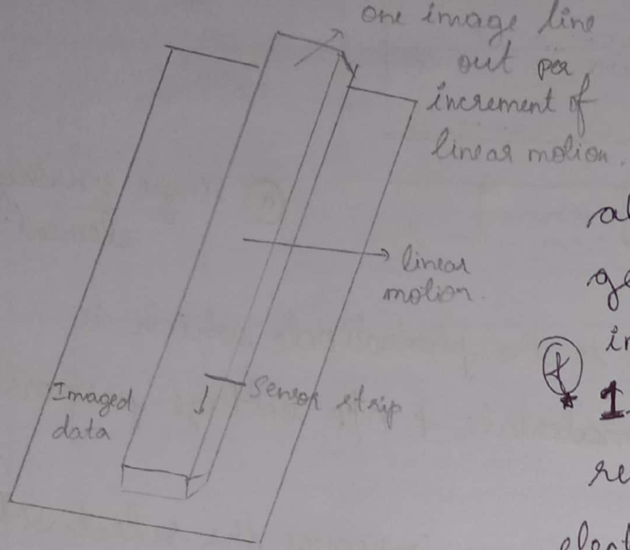


Image Acquisition using sensor strips:



Fig[a]: Image Acquisition using a linear sensor strip

(e) * Imaging system is mounted on the aircraft that flies of a constant altitude & speed over the geographical area to be imaged.

(f) * 1D imaging sensor strip respond to various bands of electromagnetic spectrum are mounted $\perp^r$ to direction of flight.

a.) The strip provides imaging element in one direction motion $\perp^r$ to the strip provides imaging in the other direction.

b.) This arrangement is used in most flat bed scanners.

c.) Sensing devices with 4000 or more in line sensor are possible.

d.) In the sensors are radinely in airbone imaging application

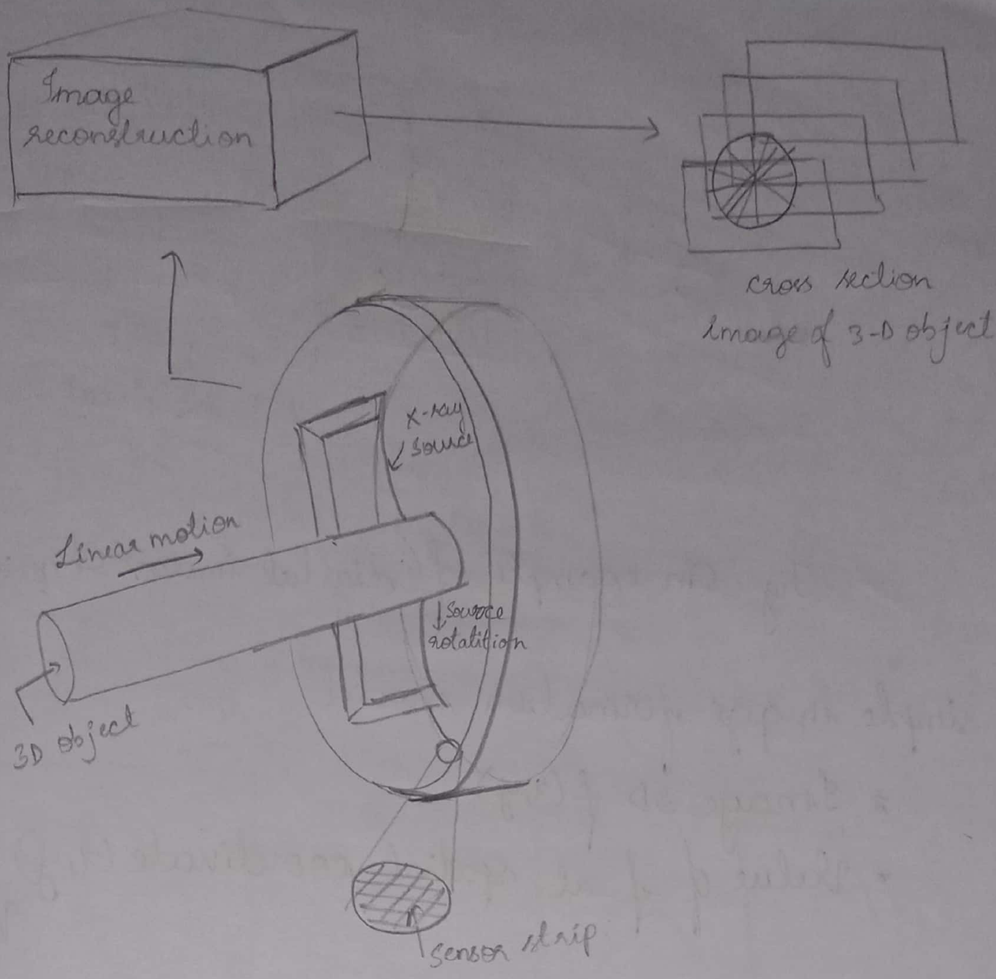


Fig 6: Image acquisition using a circular sensor strip.

* It is used in medical & industrial imaging.
* A rotating X-ray source provides illumination & X-ray sensitive sensor opposite the source collect the energy that passes through the object.
* This is the basic for medical & industrial computerised axial tomography (CAT) imaging.
* CAT includes MRI & PET (Position emission tomography)

Image acquisition using sensor arrays:

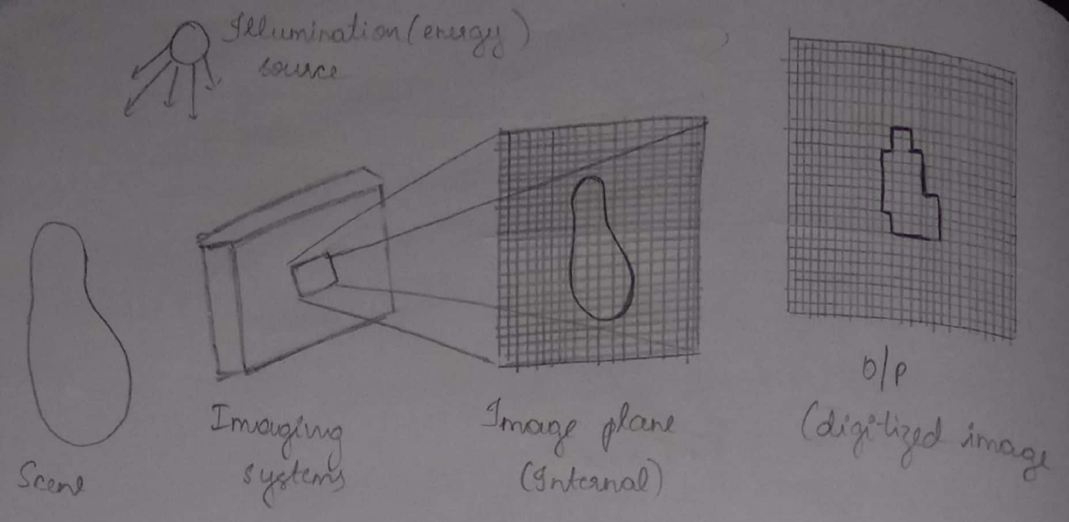


Fig: An example of digital image acquisition.

A simple image formation model:

* Image 2D $f(x,y)$
* Value of f at spatial coordinate $(x,y) \rightarrow$ scalar quantity
* $0 \leq f(x,y) < \infty$ - ①

Function $f(x,y)$ is characterised by 2 components,

a) The amount of source illumination incident on the scene being viewed.

b) The amount of illumination reflected by the objects in the scene.

$i(x,y)$ & $r(x,y)$

$$f(x,y) = i(x,y) \cdot r(x,y) \quad - ②$$

where, $0 \leq i(x,y) < \infty$ - ③

$0 \leq r(x,y) \leq 1$ - ④

* The reflectance is bounded by 0 (Total observation) & 1 (total reflections)

Eg: chest X-ray.

* Let the intensity (gray level) a monochrome image at any coordinate (x,y) is denoted by,

$$l = f(x,y) \quad - (5)$$

From eq^n (2) through (4) it is evident that l lies in the range, $L_{min} \leq l \leq L_{max}$ — (6)

$L_{min} = i_{min} r_{min}$, $L_{max} = i_{max} r_{max}$.

$[L_{min}, L_{max}]$

$[0, 1]$

0 = Black

1 = White

Image sampling & ~~reconstruction~~ Quantisation: (5 marks)

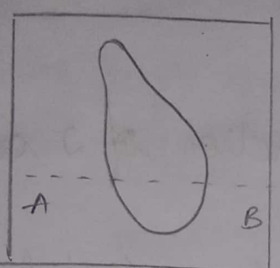


fig (a)

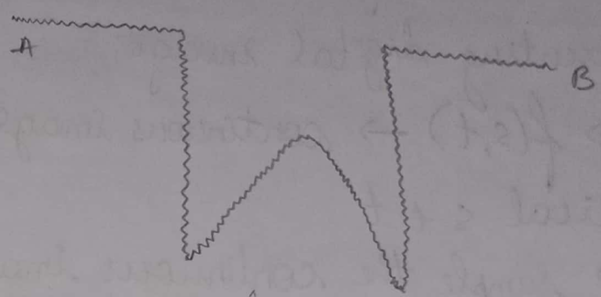
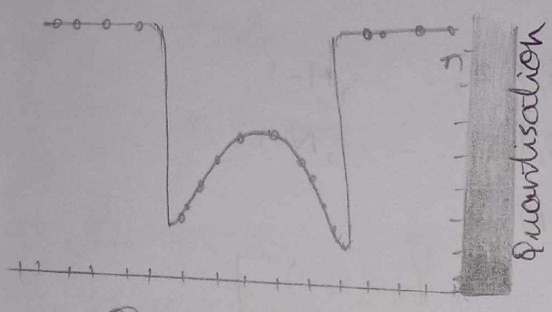
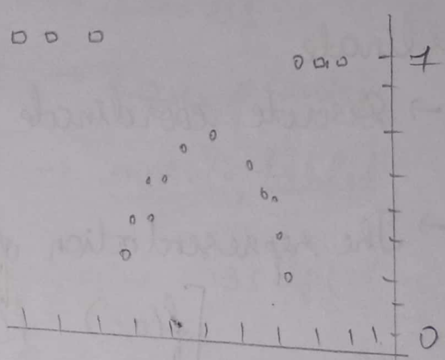


fig (b)



(c) Sampling

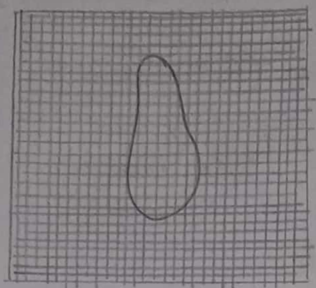


(d)

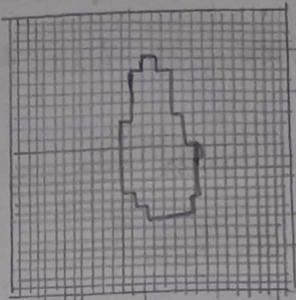
* Digitizing the coordinate values is called sampling. * Digitizing the amplitude value is called quantisation.

* Fig (a) → continous image f that is to be converted into digital ~~frame~~ form.

fig ⓑ. One D function fig is a plot of amplitude (intensity) level of the continuous image along the line segment AB in fig ⓒ



Continuos Image projected on the sensor array



Result of sampling & Quantisation.

Representing digital image:

→ $f(s,t)$ → continuous image function of 2 continuous vertical s & t.

→ Sample the continuous image into a digital image $f(x,y)$.

→ M rows & N columns where (x,y) are discrete coordinate.

→ Discrete coordinate $x: 0, 1, 2, \ldots, M-1$
$y: 0, 1, 2, \ldots, N-1$

→ The representation of $M \times N$

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1),N-1) \end{bmatrix}$$

The representation of digital image in traditional matrix

$$A = \begin{bmatrix} a_{0,0} & \cdots & a_{0,N-1} \\ a_{1,0} & & a_{1,N-1} \\ \vdots & & \vdots \\ a_{M-1,0} & & a_{M-1,N-1} \end{bmatrix}$$

→ The no. of intensity level typically is an integer power of 2. $L = 2^k$ where k is an integer range $[0, L-1]$

→ The no. of bits required to store a digitized image, $b = M \times N \times K$, where $M = N$

$\therefore b = N^2 K$.

Spatial & Intensity Resolution:

→ Spatial resolution is a measure of the smallest discernible detail in image

→ High spatial resolution

Medium " "

Low " "

Measuring Spatial Resolution,

* Dots per inch @ DPI → Monitors.
* Line " LPI → Laser printer
* Pixels PPI → mobile, tablet.

Newspaper - 75 dpi Magazine - 133 dpi

glossy brochure - 175 dpi Book page - 2400 dpi

* Intensity resolution: The smallest discernible change in intensity level. No. of pixels per square inch.

Image interpolation:

→ Tasks → Zooming, Shrinking, rotating, geometric corrections.

→ Image resizing (shrinking & zooming)

Eg: 500 × 500 pixels.

Enlarged 1.5 times, 750×750

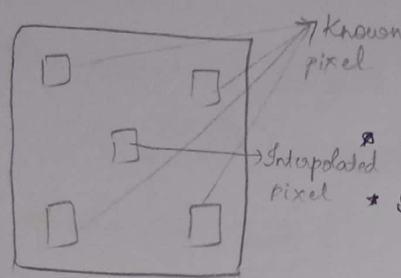
a.) Nearest neighbour interpolation:

Allign to each new location, the intensity of the nearest neighbor in the original image.

b.) Bilinear interpolation:

* 4 nearest neighbor to estimate the intensity at the given locations $I_0(x, y) = ax + by + cxy + d$.
* Better result than nearest neighbor.

c.) Bicubic interpolation: Sixteen N.N of a point.

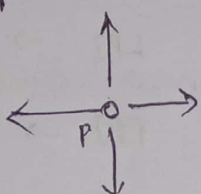


$$v(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

* Sharper images.
* Adobe photoshop, printer devices.

Neighbours of a pixel:

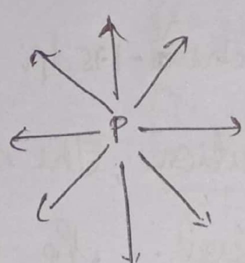
① 4-neighbor of a pixel P are V & H denoted by $N_4(P)$



$N_4(P)$

② 8-neighbour of P.

N_D	N_4	N_D
N_4	P	N_4
N_D	N_4	N_D



$N_8(P)$

N_4 = 4 neighbours

N_8 = 8 neighbours

N_D = Diagonal neighbour

$(N_4 \cup N_D) = N_8$

Scanned with CamScanner

Intensity transformations & spatial filtering.

Two principale categories:

1. Intensity transformation:

   Operates on single pixel of an image for tasks such as contrast manipulation & image thresholding.

2. Spatial filtering:

   Operates on neighborhood of every pixel in an image. Eg: Image smoothening & sharpening

Spatial domain: Operates directly on the pixels of an image.

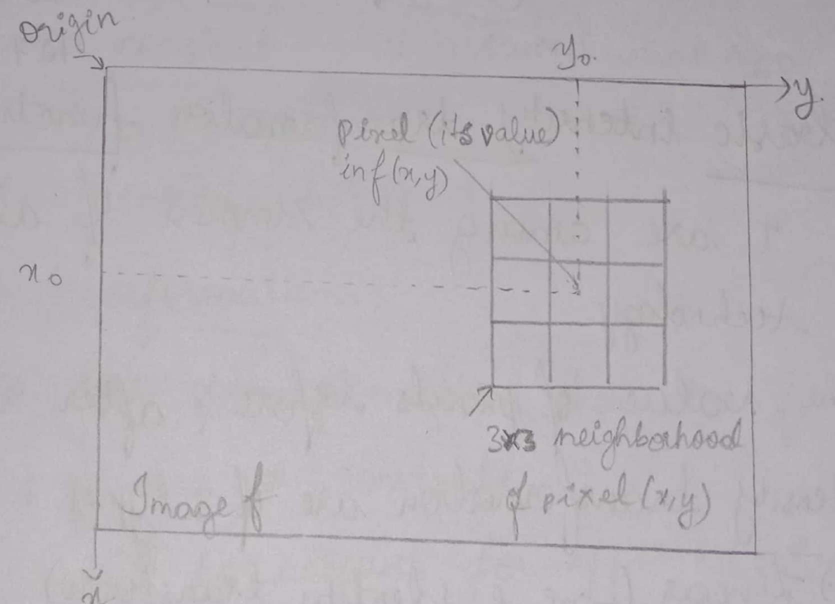
Frequency domain: operations are performed on the fourier transform of an image.

Spatial domain process: $g(x,y) = T[f(x,y)]$ -①.

$f(x,y)$ = I/P image

$g(x,y)$ = o/p image.

T = operator.

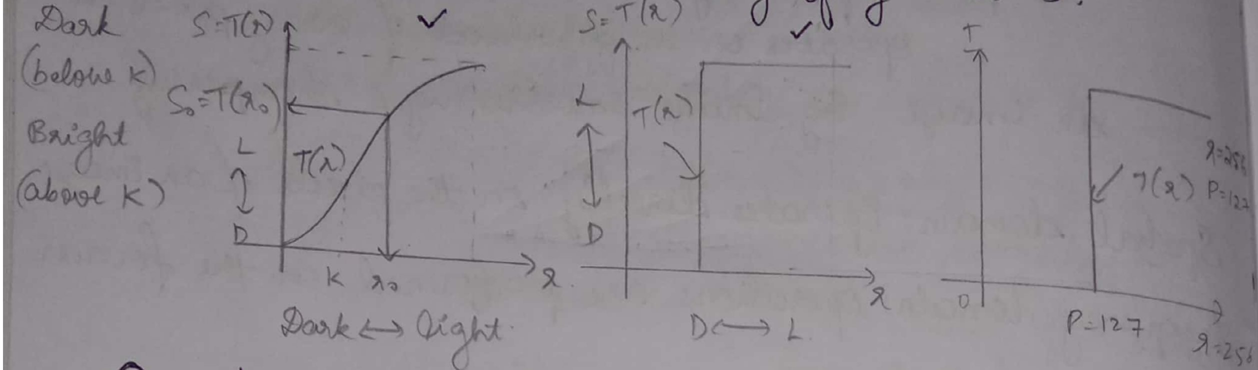


→ operator can be applied to the pixels of a single image as to the pixels of a set of images.

→ orbiting location in an image (100,150) o/p g(100,150) is sum of f(100,150) & its 8 neighborhood divided by 9

→ The process start from left top corner of i/p image
→ The smallest possible neighbourhood is 1x1
→ g depends only on the value of f ~~of~~ a single point (x, y) & T in eq^n ① becomes an intensity transformation function $S = T(r)$ — ②

S & T denote the intensity of g and s.



ⓐ contrast stretching

ⓑ Thresholding function

ⓒ Produce 2 level image.
→ 2 level of intensity 0 & 1
→ Below 127 = Black
→ Above 127 = white
→ exact point of 127

Mathematically the transfer fun^c;

$$g(x,y) = \begin{cases} 0, & f(x,y) < 127 \\ 1, & f(x,y) > 127 \end{cases}$$

Some basic intensity transformation functions:

→ I T are among the simplest of all image Process technology.
→ The value of pixels before & after r & s
→ Intensity transformation are of 3 types.

i) Linear (-ve & identity transform)
ii) Logarithmic (log & inverse log transform)
iii) Power law (n^{th} power & n^{th} root transform)

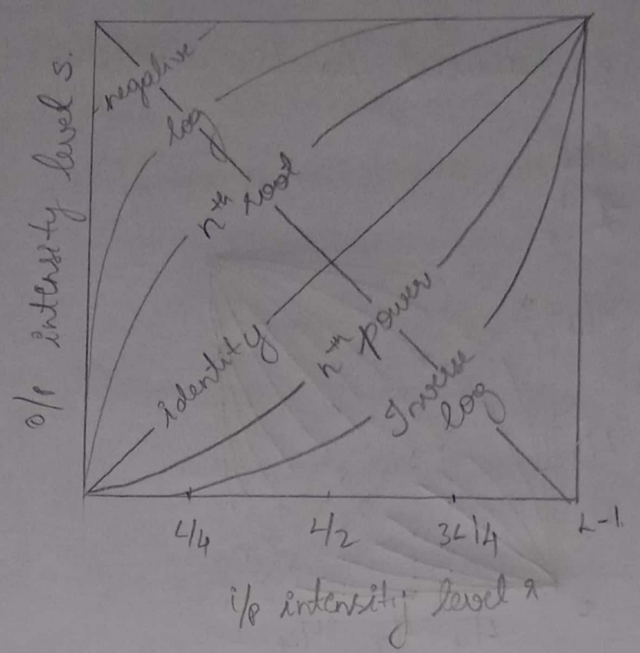


Image negative: $[0, L-1]$

$S = L-1-r$.

S = o/p intensity level / o/p gray level.

L = intensity level

r = o/p intensity level / i/p gray level

log tranformation: $S = c \log(1+r)$

c = a constant & it is assumed that $r \geq 0$.

r = i/p intensity level

S = o/p intensity level.

Power low tranformations:

Power low transformations have the form $S = c r^{\gamma}$

when c & γ are +ve constants.

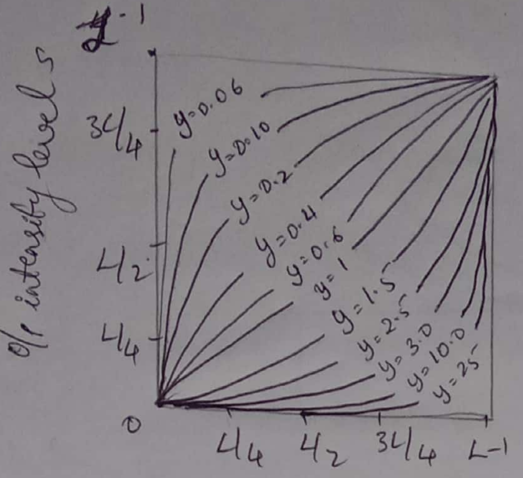
$S = c(r+\epsilon)^{\gamma}$ to account for offsets (i.e measurable

→ Power low curve with fractional value of γ mapp. a narrow range of dark i/p value into a wide range of o/p value with o/p being true for higher value.

→ value of $\gamma > 1$ have exactly o/p effect as those generated with value of $\gamma < 1$

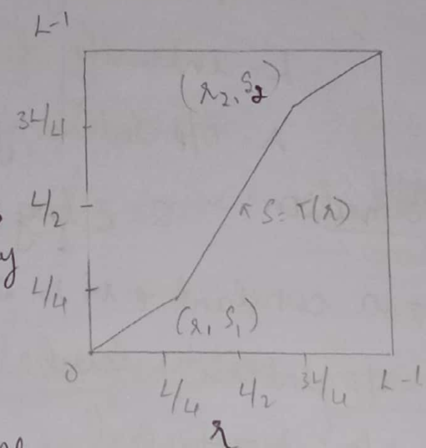
→ when $c = \gamma = 1$ is reduced to the identity transformation

→ gamma correction.



Piece wise linear transformation function:

Contract stretching



→ If $r_1 = s_1$, $r_2 = s_2$

No changes in intensity

→ If $r_1 = r_2$, $s_1 = 0$ & $s_2 = L-1$
the transformation become
a thresholding function that create a binary image.

→ Intermediate value of (r_1, s_1) & (r_2, s_2) produce various degree of spread in the intensity level of o/p image.

Intensity level slicing:

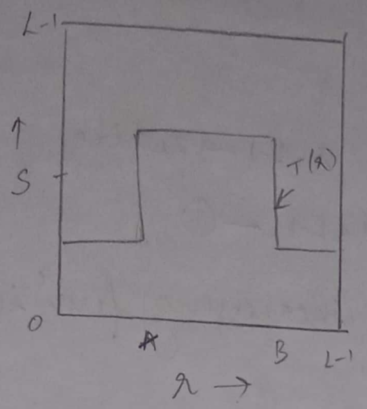
→ The applications in which it is intrest to highligh a specific range of intensities in an image.

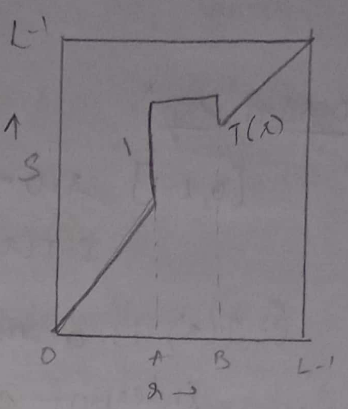
→ Sattelite imaging such as mass of water & enhancing flow in X-ray image.

2 basic themes:

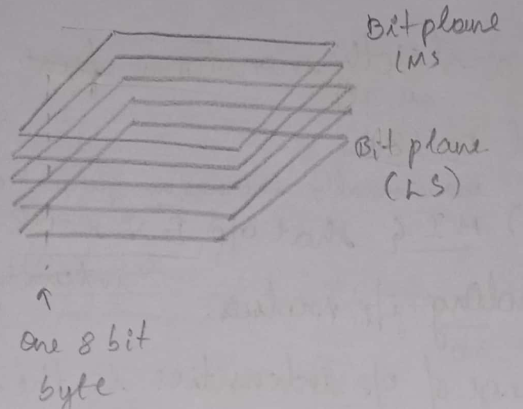
1) One approach is to display in one value (white) all the values in the range of interest & another (black) all other intensities.

2.) Second approach based on the transformation.





Bit plane slicing:



→ Plane 8 is lowest order bit all pixel in the image

→ Plane 8 is highest order bit.

fig: Bit plane of 8 bit image.

Histogram processing:

Let r_k $k = 0, 1 \ldots L-1$ - $f(x,y)$

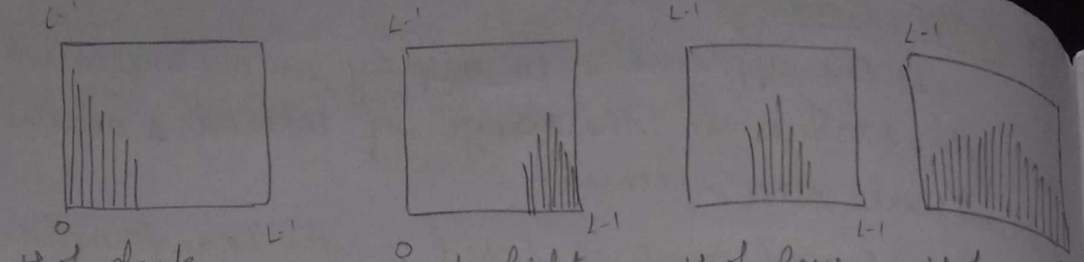
unnormalised histogram f is defined as

$\boxed{h(r_k) = n_k}$ for $k = 0, 1 \ldots N-1$

n_k = no. of pixel in f with intensity r_k.

Illy normalised histogram of f is defined as

$P(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$ M & N are rows & columns.



H of dark image — H of light image — H of low contrast img — H of High Contrast img

Histogram eqn:

$[0, L-1]$ $r=0 \to$ Black $r=L-1 \to$ white

$s = T(r) \quad 0 \le r \le L-1$ — ①

ⓐ $T(r)$ is a monotonic increasing func in the interval $0 \le r \le L-1$

ⓑ $0 \le T(r) \le L-1$ for $0 \le r \le L-1$

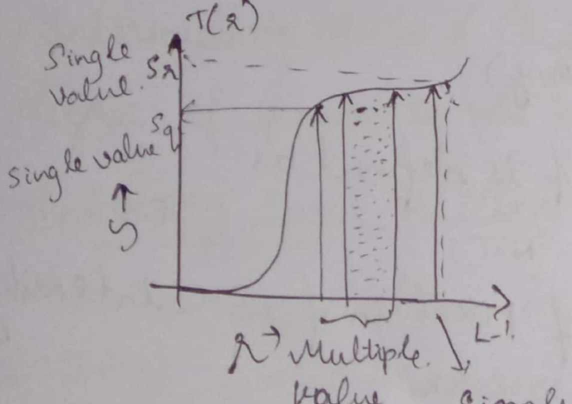
Inverse transformation.

(a') $T(r)$ is a strictly monotonic ~~func~~ increasing func in the interval $0 \le r \le L-1$

→ condition ⓐ $T(r)$ MIG (monotonically increasing gurantee) that o/p IV (intensity values) will never be less than corresponding i/p values.

→ condition (b) Range of o/p intensities is the same as i/p

→ (a') ~~generates~~ guarentees that the mapping from s back to r will be one to one



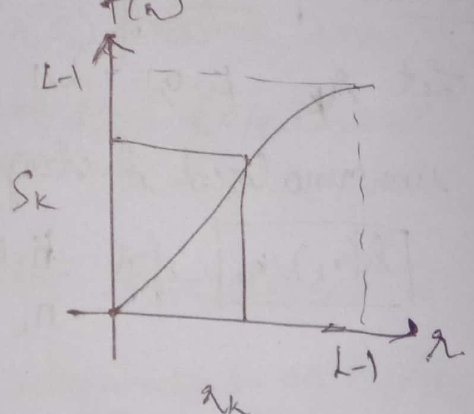


Fig ⓐ MIF showing how multiple value can map to single value.

ⓑ Strictly monotonic

→ Let $P_r(r)$ & $P_s(s)$ denote PDF of intensity value r & s in 2 d/f image.

→ P indicate P_r & P_s as d/f func.

→ PDF of the tranformed variable s can be.

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right| \quad - (3).$$

→ A transformation func of particular importance in image processing.

$$s = T(r) = (L-1)\int_0^r P_r(\omega)\,d\omega \quad - (4)$$

ω → dummy variable of integration.

→ The integration ~~the~~ right ~~sig~~ side is commutative distribution func (CDF) of random variable.

→ Eqn (3) to find $P_s(s)$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r P_r(\omega)\,d\omega\right] \quad - (5)$$

$$\frac{ds}{ds} = (L-1)P_r(r)$$

Substituting the result for $\frac{dr}{ds}$ in eqn (3)

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

$$= P_r(r)\left|\frac{1}{(L-1)\,P_r(r)}\right| \quad - (6)$$

$$P_s(s) = \frac{1}{L-1} \qquad 0 \le s \le L-1$$

$P_s(s)$ = uniform probability func

For <u>discrete values</u>,

→ For probability of occurance of intensity level r_k in a digital image in $P_r(r_k) = \frac{n_K}{MN}$ − (7)

MN = Rows & colums $\boxed{r_K \in (0, L-1)}$

n_K = no. of pixel in i/p image with intensity r_K

→ The discrete form of the transformation in eqn ⑦

$$s_k = T(r_k) = (L-1)\sum_{i=0}^{k} P_r(r_i) \quad k = 0, \dots L-1$$

L = intensity level of img. — ⑧

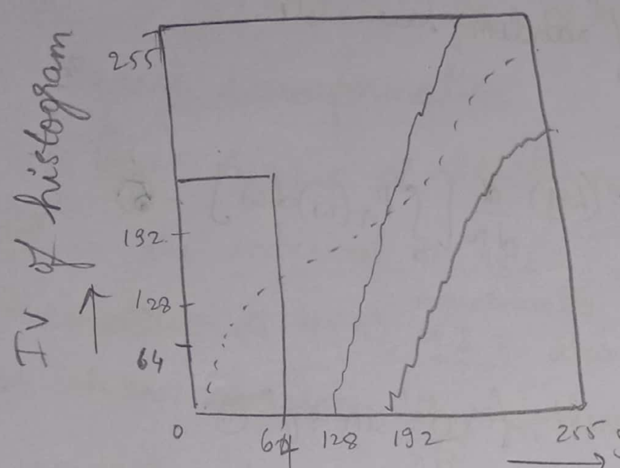
The processed image obtained from eqn ⑧ to map each pixel in the i/p image with intensity r_k into a ~~compan~~ corresponding pixel level s_k in the o/p image.

→ The inverse transformation from s to r

$$r_k = T^{-1}(s_k) \text{ — } ⑨$$

Inverse then satisfies (a') & (b)

Histogram matching (specification).



→ r & z PDF, $P_r(r)$ & $P_z(z)$

→ r & z i/p & o/p

→ $P_r(r)$ → i/p , $P_z(z)$ – o/p image.

$$s = T(r) = (L-1)\int_0^r P_r(w)\,dw \text{ — } ⑨$$

w – dummy variable

$$G(z) = (L-1)\int_0^z P_z(v)\,dv \text{ — } 10$$

v – dummy variable.

$$G(z) = s = T(r)$$

$$z = G^{-1}(s) = G^{-1}[T(r)] \quad (11)$$

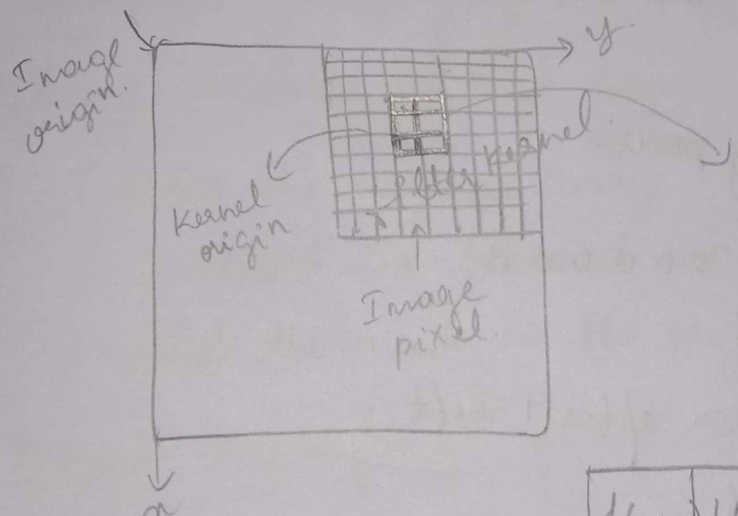
Eq^n (9) to (11)

1. Obtain $P_r(r)$ from the i/p image to use eq^n (9)
2. Use the specified PDF $P_z(z)$ in eq^n (10) to obtain the func $G(z)$
3. Compute inverse transformation $z = G^{-1}(s)$. Build mapping for s to z.
4. Obtain the o/p image by 1st equalising the i/p image using eq^n (9). the pixel values in this image are the s value. For each pixel with value s in the equalised image perform the inverse mapping $z = G^{-1}(s)$ to obtain the corresponding pixels in the o/p image.

Fundamentals of Spatial filters:

→ Filtering refers to passing, modifying or rejecting specified frequency components of an image.

→ Filter that passes low frequencies is called lowpass filter



$w(-1,1)$	$w(1,0)$	$w(1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(-1,-1)$	$w(-1,0)$	$w(1,-1)$

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

fixed value index kernel

$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots$$
$$\dots + w(0,0)f(x,y) + \dots + w(1,1)f(x+1,y+1) \quad \text{--- (1)}$$

Spatial correlation & convolution:

→ S C consists of moving the center of kernel over an image computing the sum of the products of each location.

→ Spatial convolution are some except that to convolution kernel rotated by 180°

$$g(x) = \sum_{s=-a}^{a} w(s) f(x+s)$$

Correlation

Origin

0 0 0 1 0 0 0 0 f   w 1 2 4 2 8

0 0 0 1 0 0 0 0

1 2 4 2 8

Starting position

Zero padding

0 0 0 0 0 1 0 0 0 0 0 0 0

1 2 4 2 8

Starting position

0 0 0 0 0 1 0 0 0 0 0 0

1 2 4 2 8

Position after 1 shift

correlation result

0 8 2 4 2 1 0 0

full convolution result

0 0 0 8 2 4 2 1 0 0 0 0

Smoothing (Lowpass) spatial filter:

→ Used to reduce the sharp transitions in intensity

Low pass filter ─┬─ Box filter kernel
                 └─ Gaussian kernel.

Box kernel: It is suitable for quick experimentation

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

→ $m \times n$ box filter is an $m \times n$ array.

Gaussian filter:

$$w(s,t) = G(s,t) = K e^{\frac{s^2+r^2}{2\sigma^2}}$$

$$r = (s^2 + r^2)^{1/2}$$

$$G(r) = K e^{\frac{-r^2}{2\sigma^2}}$$

$\sqrt{2}$	1	$\sqrt{2}$
1	0	1
$\sqrt{2}$	1	$\sqrt{2}$

Sharpening (Higpass) filter:

→ Sharpening is often referred to as highpass filter.

→ electronic printing & medical imaging, industrial inspection & autonomous guidence in military system.

1st order derivative: $\frac{\delta f}{\delta x} = f(x+1) - f(x)$

2nd order derivative: $\frac{\delta^2 f}{\delta x^2} = f(x+1) + f(x-1) - 2f(x)$

Unsharp masking & high boast filtering:

1) Blur the original image.
2) Subtract the blurred images from original image
3) Add the mask to the original

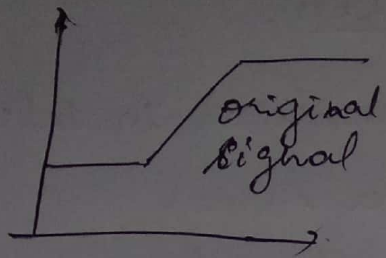
$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

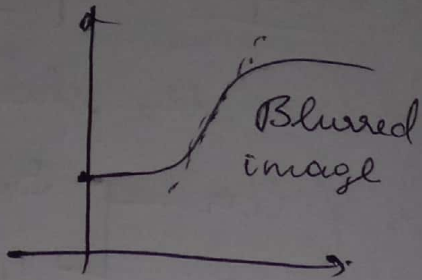
$$g(x,y) = f(x,y) + K \cdot g_{mask}(x,y)$$

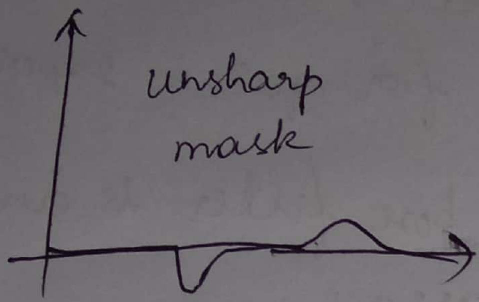
$K = 1$ unsharp masking

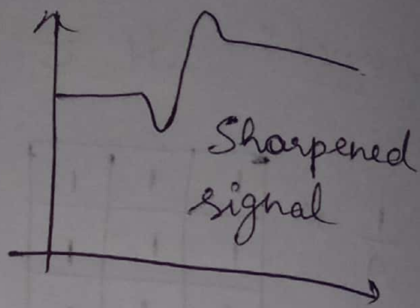
$K > 1$ highboost filtering

$K < 1$ reduce the ~~contri~~ contribution of unsharpened image


original signal


Blurred image


unsharp mask


Sharpened signal