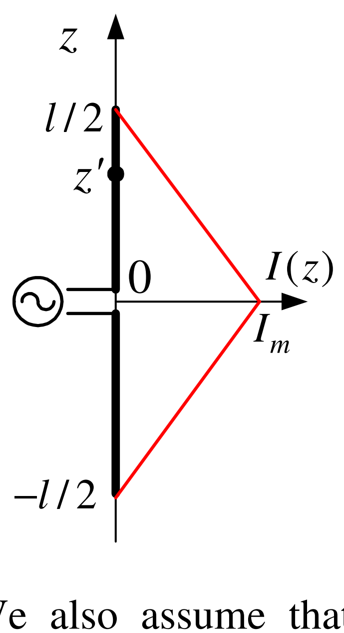
Lecture 9: Linear Wire Antennas – Dipoles and Monopoles

(*Small electric dipole antenna. Finite-length dipoles. Half-wavelength dipole. Method of images – revision. Vertical infinitesimal dipole above a conducting plane. Monopoles. Horizontal infinitesimal dipole above a conducting plane.*)

The dipole and the monopole are arguably the two most widely used antennas across the UHF, VHF and lower-microwave bands. Arrays of dipoles are commonly used as base-station antennas in land-mobile systems. The monopole and its variations are common in portable equipment, such as cellular telephones, cordless telephones, automobiles, trains, etc. It has attractive features such as simple construction, sufficiently broadband characteristics for voice communication, small dimensions at high frequencies. Alternatives to the monopole antenna for hand-held units is the inverted F and L antennas, the microstrip patch antenna, loop and spiral antennas, and others. The printed inverted F antenna (PIFA) is arguably the most common antenna design used in modern handheld phones.

1. Small Dipole



The small dipole features short electrical length:

$$\frac{\lambda}{50} < l \leq \frac{\lambda}{10} \tag{9.1}$$

If we assume that (9.1) holds, the maximum phase error in (βR) that can occur is

$$e_{\max} = \frac{\beta l}{2} = \frac{2\pi}{\lambda}\frac{\lambda}{20} = \frac{\pi}{10} \approx 18°,$$

which error corresponds to an observation direction at $\theta = 0^\circ$. As a reminder, a maximum total phase error less than $\pi/8$ is acceptable for the approximation $e^{-jkR} \approx e^{-jkr}$ to be made in the integral solution for the vector potential **A**.

We also assume that the observation distance fulfills $r \gg l$, so that the approximation $R \approx r$ can be made in the amplitude-decay term $1/R \approx 1/r$.

On such a short dipole, the current distribution is a triangular function of z':

$$I(z') = \begin{cases} I_m \cdot \left(1 - \dfrac{z'}{l/2}\right), & 0 \le z' \le l/2 \\ I_m \cdot \left(1 + \dfrac{z'}{l/2}\right), & -l/2 \le z' \le 0. \end{cases} \tag{9.2}$$

Then, the VP integral is obtained as

$$\mathbf{A} = \hat{\mathbf{z}} \frac{\mu}{4\pi} \left[\int_{-l/2}^{0} I_m \left(1 + \frac{z'}{l/2}\right) \frac{e^{-j\beta R}}{R} dz' + \int_{0}^{l/2} I_m \left(1 - \frac{z'}{l/2}\right) \frac{e^{-j\beta R}}{R} dz' \right]. \tag{9.3}$$

The solution of (9.3) is simple when we assume that $R \approx r$ in both the amplitude-decay and the phase-delay terms:

$$\mathbf{A} \approx \hat{\mathbf{z}} \frac{1}{2} \left[\frac{\mu}{4\pi} I_m l \frac{e^{-j\beta r}}{r} \right]. \tag{9.4}$$

The further away from the antenna the observation point is, the more accurate the expression in (9.4).

Note that *the result in* (9.4) *is exactly one-half of the VP* **A** *of an infinitesimal dipole of the same length* (where the current magnitude $I_0 = I_m$ is constant along the dipole). This is expected because we made the same approximation for R as in the case of the infinitesimal dipole but, this time, we integrated a triangular function along l, whose average is $I_{\text{av}} = I_m / 2$.

Now, we need not repeat all the calculations of the field components, power and antenna parameters; we simply use the infinitesimal-dipole field multiplied by a factor of 0.5:

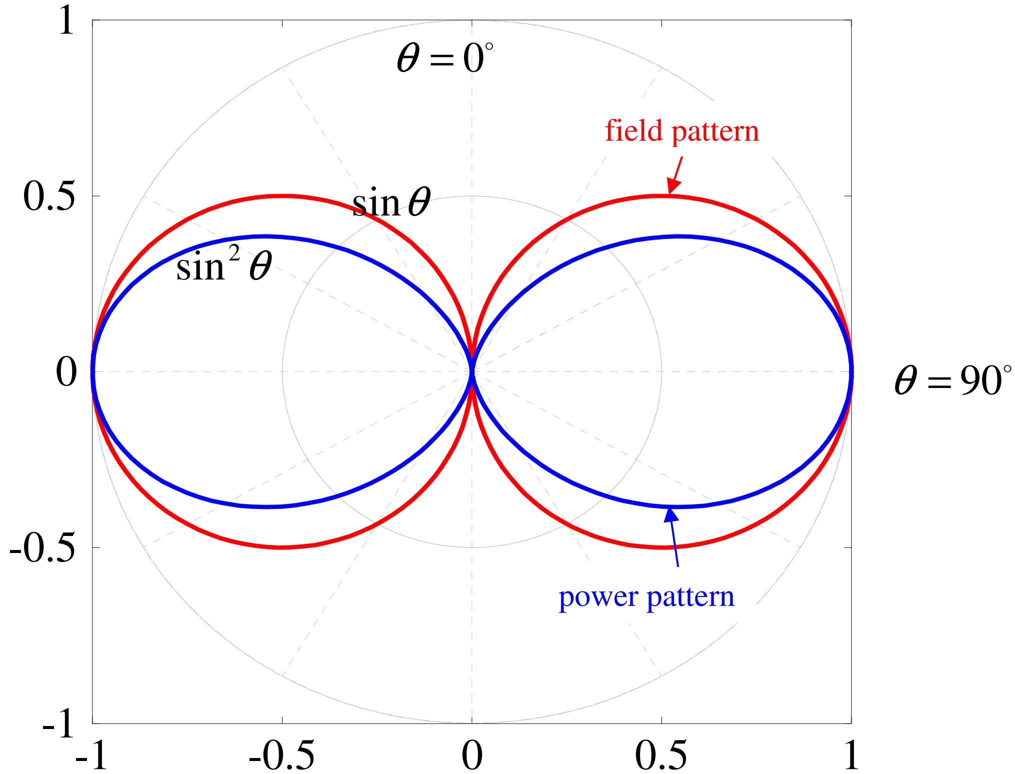
$$\begin{aligned} E_\theta &\approx j\eta \frac{\beta I_m l}{8\pi} \frac{e^{-j\beta r}}{r} \sin\theta \\ H_\varphi &\approx j \frac{\beta I_m l}{8\pi} \frac{e^{-j\beta r}}{r} \sin\theta \ , \ \beta r \gg 1. \\ E_r &= E_\varphi = H_r = H_\theta = 0 \end{aligned} \tag{9.5}$$

The normalized field pattern is the same as that of the infinitesimal dipole:

$$\bar{E}(\theta,\varphi) = \sin\theta \tag{9.6}$$

and the power pattern is

$$\bar{U}(\theta,\varphi) = \sin^2\theta. \tag{9.7}$$



<u>The beam solid angle</u>:

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \cdot \sin\theta d\theta d\varphi,$$

$$\Omega_A = 2\pi \cdot \int_0^{\pi} \sin^3\theta d\theta = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$$

<u>The directivity</u>:

$$D_0 = \frac{4\pi}{\Omega_A} = \frac{3}{2} = 1.5. \tag{9.8}$$

As expected, the directivity, the beam solid angle as well as the effective aperture are the same as those of the infinitesimal dipole (the normalized patterns of both dipoles are the same).

The radiated power is four times less than that of an infinitesimal dipole of the same length and current $I_0 = I_m$ because the far fields are twice smaller in magnitude:

$$\Pi = \frac{1}{4}\cdot\frac{\pi}{3}\eta\left(\frac{I_m l}{\lambda}\right)^2 = \frac{\pi}{12}\eta\left(\frac{I_m l}{\lambda}\right)^2. \tag{9.9}$$

As a result, the radiation resistance is also four times smaller than that of the infinitesimal dipole:

$$R_r = \frac{\pi}{6}\eta\left(\frac{l}{\lambda}\right)^2 = 20\pi^2\left(\frac{l}{\lambda}\right)^2. \tag{9.10}$$

2. Finite-length Infinitesimally Thin Dipole

A good approximation of the current distribution along the dipole's length is the sinusoidal one:

$$I(z') = \begin{cases} I_0 \sin\left[\beta\left(\frac{l}{2} - z'\right)\right], & 0 \le z' \le l/2 \\ I_0 \sin\left[\beta\left(\frac{l}{2} + z'\right)\right], & -l/2 \le z' \le 0. \end{cases} \tag{9.11}$$

It can be shown that the VP integral

$$\mathbf{A} = \hat{\mathbf{z}}\frac{\mu}{4\pi}\int_{-l/2}^{l/2} I(z')\frac{e^{-j\beta R}}{R}dz' \tag{9.12}$$

has an analytical (closed form) solution. Here, however, we follow a standard approach used to calculate the far field for an arbitrary wire antenna. It is based on the solution for the field of the infinitesimal dipole. The finite-length dipole is divided into an infinite number of infinitesimal dipoles of constant-current elements of length dz'. Each such dipole produces the elementary far field given by

$$\begin{aligned} dE_\theta &\approx j\eta\beta I_e(z')\frac{e^{-j\beta R}}{4\pi R}\sin\theta\cdot dz' \\ dH_\varphi &\approx j\beta I_e(z')\frac{e^{-j\beta R}}{4\pi R}\sin\theta\cdot dz' \\ dE_r &\approx dE_\varphi \approx dH_r \approx dH_\theta \approx 0 \end{aligned} \tag{9.13}$$

where $R = [x^2 + y^2 + (z - z')^2]^{1/2}$ and $I_e(z')$ denotes the value of the current element at z'. Using the far-zone approximations,

$$\left|\begin{array}{l}\dfrac{1}{R} \approx \dfrac{1}{r}, \text{ for the amplitude factor} \\ R \approx r - z' \cos\theta, \text{ for the phase factor}\end{array}\right. \tag{9.14}$$

the following approximation of the elementary far field is obtained:

$$dE_\theta \approx j\eta\beta I_e \left(\frac{e^{-j\beta r}}{4\pi r}\right) e^{j\beta z' \cos\theta} \cdot \sin\theta dz'. \tag{9.15}$$

Using the superposition principle, the total far field is obtained as

$$E_\theta = \int_{-l/2}^{l/2} dE_\theta \approx j\eta\beta \left(\frac{e^{-j\beta r}}{4\pi r}\right) \sin\theta \cdot \int_{-l/2}^{l/2} I_e(z') e^{j\beta z' \cos\theta} dz'. \tag{9.16}$$

The *first factor*

$$g(\theta) = j\eta\beta \left(\frac{e^{-j\beta r}}{r}\right) \sin\theta \tag{9.17}$$

is called the ***element factor***. The element factor in this case is the far field produced by an infinitesimal dipole of unit current element $Il = 1$ (A×m). The element factor is the same for any current element, provided the angle θ is always associated with the axis of the current flow. The *second factor*

$$f(\theta) = \int_{-l/2}^{l/2} I_e(z') e^{j\beta z' \cos\theta} dz' \tag{9.18}$$

is the ***space factor (or pattern factor, or, array factor)***. The pattern factor is dependent on the amplitude and phase distribution of the current of the antenna (the source distribution in space).

For the sinusoidal current distribution of (9.11), the pattern factor is

$$f(\theta) = I_0 \left\{ \int_{-l/2}^{0} \sin\left[\beta\left(\frac{l}{2} + z'\right)\right] e^{j\beta z' \cos\theta} dz' + \int_{0}^{l/2} \sin\left[\beta\left(\frac{l}{2} - z'\right)\right] e^{j\beta z' \cos\theta} dz' \right\}. \tag{9.19}$$

The above integrals are solved having in mind that

$$\int \sin(a + b \cdot x) e^{c \cdot x} dx = \frac{e^{cx}}{b^2 + c^2} \left[c \sin(a + bx) - b \cos(a + bx)\right]. \tag{9.20}$$

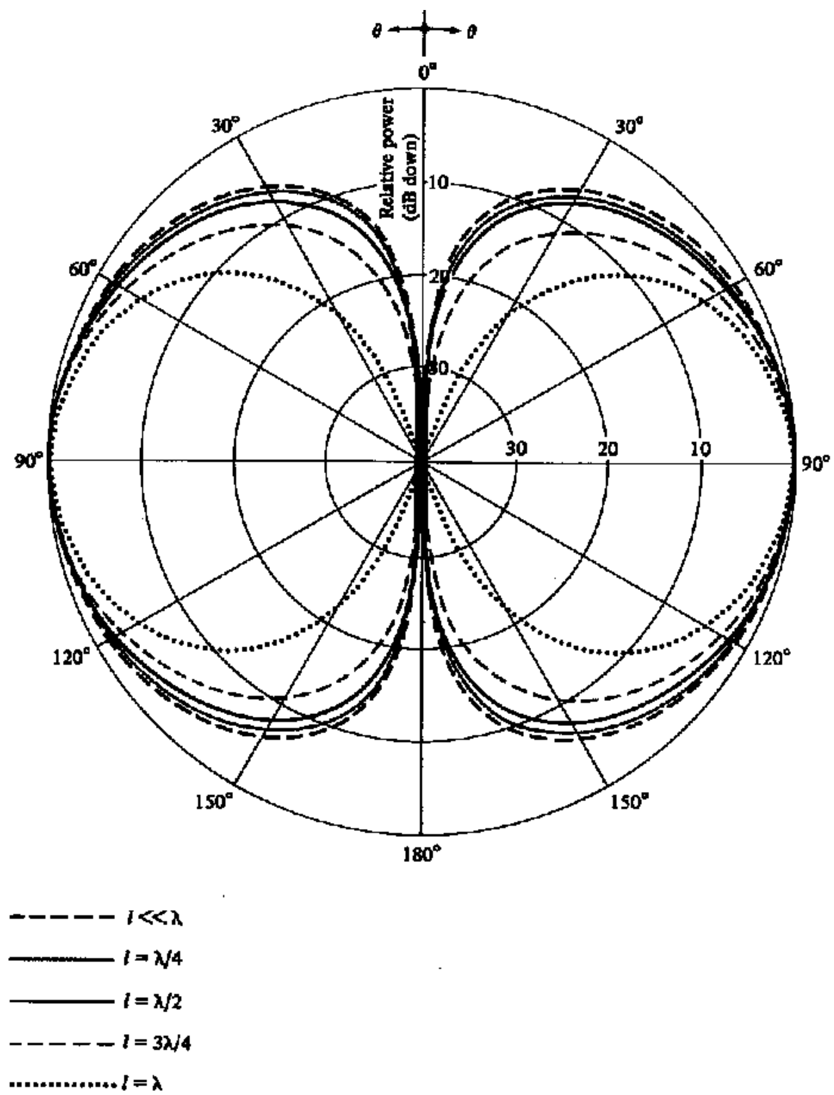
The far field of the finite-length dipole is thus obtained as

$$E_\theta = g(\theta) \cdot f(\theta) = j\eta I_0 \left(\frac{e^{-j\beta r}}{2\pi r} \right) \cdot \frac{\left[\cos\left(\frac{\beta l}{2} \cos\theta \right) - \cos\left(\frac{\beta l}{2} \right) \right]}{\sin\theta}. \quad (9.21)$$

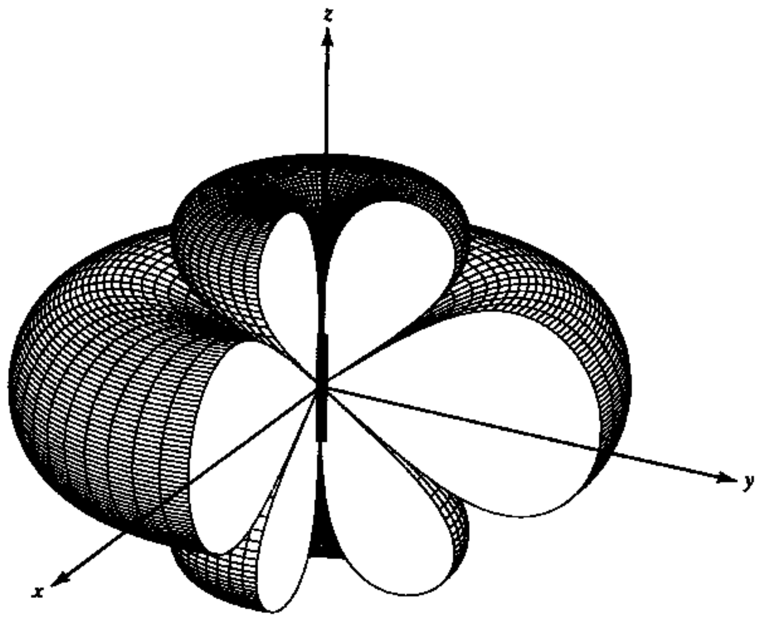
Amplitude pattern:

$$\boxed{\bar{E}(\theta,\varphi) = \frac{\cos\left(\frac{\beta l}{2} \cos\theta \right) - \cos\left(\frac{\beta l}{2} \right)}{\sin\theta}}. \quad (9.22)$$

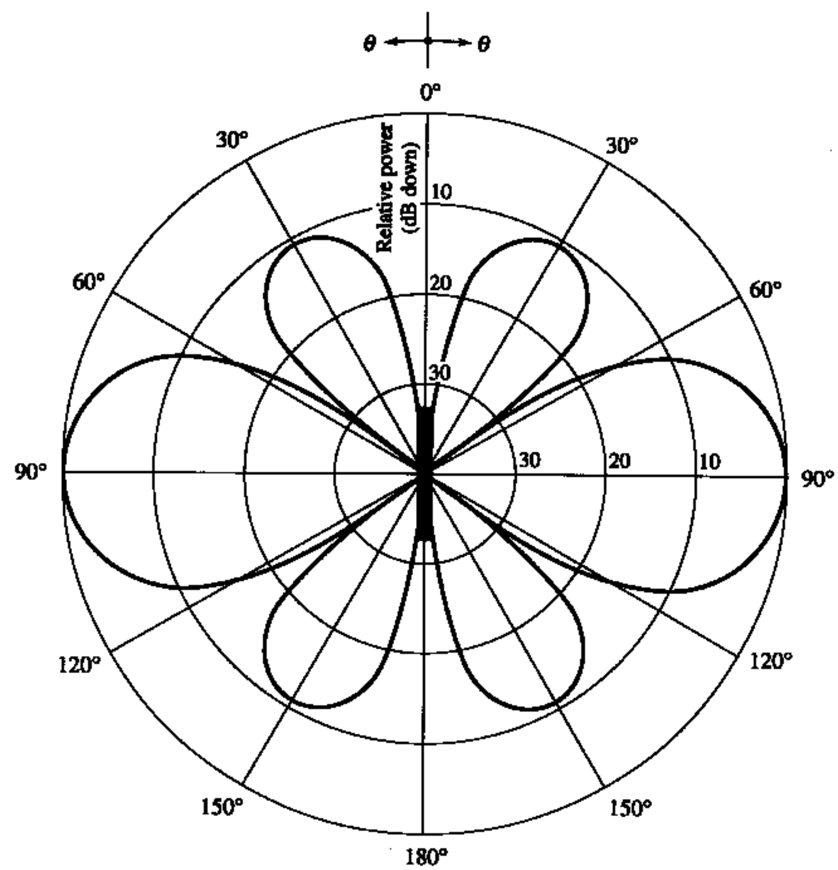
Some elevation patterns (in dB) for dipole lengths $l \le \lambda$ are plotted below [from Balanis]. Notice that the direction of maximum radiation is always in the azimuth plane. Also, as the dipole's length increases, a slight increase in the pattern directivity is observed (beamwidth decreases).



For dipoles of length $l > \lambda$, the patterns develop secondary beams. The 3-D pattern of the dipole $l = 1.25\lambda$ is shown below [from Balanis].



(a) Three-dimensional



(b) Two-dimensional

Power pattern:

$$F(\theta,\varphi)=\frac{\left[\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]^2}{\sin^2\theta}. \qquad (9.23)$$

Note: The maximum of $F(\theta,\varphi)$ above is not necessarily unity, but for $l<1.5\lambda$ the major maximum is always at $\theta=90^\circ$.

Radiated power

First, the far-zone power flux density is calculated as

$$\mathbf{P}=\hat{\mathbf{r}}\frac{1}{2\eta}|E_\theta|^2=\hat{\mathbf{r}}\eta\frac{I_0^2}{8\pi^2r^2}\left[\frac{\cos(0.5\beta l\cos\theta)-\cos(0.5\beta l)}{\sin\theta}\right]^2. \qquad (9.24)$$

The total radiated power is then

$$\Pi=\oiint\mathbf{P}\cdot d\mathbf{s}=\int_0^{2\pi}\int_0^{\pi}P\cdot r^2\sin\theta d\theta d\varphi \qquad (9.25)$$

$$\Pi=\eta\frac{I_0^2}{4\pi}\underbrace{\int_0^{\pi}\frac{\left[\cos(0.5\beta l\cos\theta)-\cos(0.5\beta l)\right]^2}{\sin\theta}d\theta}_{\Im}. \qquad (9.26)$$

$\Im$ is solved in terms of the cosine and sine integrals:

$$\begin{aligned}\Im&=C+\ln(\beta l)-C_i(\beta l)+\frac{1}{2}\sin(\beta l)\left[S_i(2\beta l)-2S_i(\beta l)\right]+\\&+\frac{1}{2}\cos(\beta l)\left[C+\ln(\beta l/2)+C_i(2\beta l)-2C_i(\beta l)\right].\end{aligned} \qquad (9.27)$$

Here,

$C\approx0.5772$ is the Euler's constant,

$C_i(x)=\int_\infty^x\frac{\cos y}{y}dy=-\int_x^\infty\frac{\cos y}{y}dy$ is the cosine integral,

$S_i(x)=\int_0^x\frac{\sin y}{y}dy$ is the sine integral.

Thus, the radiated power can be written as

$$\Pi = \eta \frac{I_0^2}{4\pi} \cdot \Im . \tag{9.28}$$

Radiation resistance

The radiation resistance is defined as

$$\boxed{R_r = \frac{2\Pi}{I_m^2} = \frac{I_0^2}{I_m^2} \cdot \frac{\eta}{2\pi} \cdot \Im} \tag{9.29}$$

where I_m is the maximum current magnitude along the dipole. If the dipole is half-wavelength long or longer ($l \geq \lambda / 2$), $I_m = I_0$, see (9.11). However, if $l < \lambda / 2$, then $I_m < I_0$ as per (9.11). For $l < \lambda / 2$, the maximum current is at the dipole center (the feed point $z' = 0$) and its value is

$$I_m = I_{(z'=0)} = I_0 \sin(\beta l / 2) \tag{9.30}$$

where $\beta l / 2 < \pi / 2$, and, therefore, $\sin(\beta l / 2) < 1$. In summary,

$$\begin{aligned} I_m &= I_0 \sin(\beta l / 2), \text{ if } l \leq \lambda / 2 \\ I_m &= I_0, \qquad\qquad\quad \text{if } l > \lambda / 2. \end{aligned} \tag{9.31}$$

Therefore,

$$\boxed{\begin{aligned} R_r &= \frac{\eta}{2\pi} \cdot \frac{\Im}{\sin^2(\beta l / 2)}, \text{ if } l < \lambda / 2 \\ R_r &= \frac{\eta}{2\pi} \cdot \Im \qquad\qquad\quad , \text{ if } l \geq \lambda / 2. \end{aligned}} \tag{9.32}$$

Directivity

The directivity is obtained as

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = 4\pi \frac{F_{\max}}{\int_0^{\pi} \int_0^{2\pi} F(\theta,\varphi) \sin\theta d\theta d\varphi} \tag{9.33}$$

where

$$F(\theta,\varphi) = \left[\frac{\cos(0.5\beta l \cos\theta) - \cos(0.5\beta l)}{\sin\theta} \right]^2$$

is the power pattern [see (9.23)]. Finally,

$$D_0 = 2F_{\max} / \Im . \tag{9.34}$$

Input resistance of center-fed dipoles

The radiation resistance given in (9.32) is not necessarily equal to the input resistance because the current at the dipole's center I_{in} (if its center is the feed point) is not necessarily equal to I_m. In particular, $I_{in} \neq I_m$ if $l > \lambda / 2$ and $l \neq (2n+1)\lambda / 2$, n is any integer. Note that when $l \geq \lambda / 2$, $I_m = I_0$.

To obtain a general expression for the current magnitude I_{in} at the center of the dipole (assumed to be the feed point), we note that if the dipole is lossless, the input power is equal to the radiated power. Therefore, in the case of a dipole longer than half a wavelength,

$$P_{in} = \frac{|I_{in}|^2}{2} R_{in} = \Pi = \frac{|I_0|^2}{2} R_r \text{ for } l > \lambda / 2, \tag{9.35}$$

and the input and radiation resistances relate as

$$R_{in} = \frac{|I_0|^2}{|I_{in}|^2} R_r \text{ for } l > \lambda / 2. \tag{9.36}$$

Since the current at the center of the dipole ($z' = 0$) is [see (9.11)]

$$I_{in} = I_0 \sin(\beta l / 2), \tag{9.37}$$

then,

$$R_{in} = \frac{R_r}{\sin^2(\beta l / 2)} \quad . \tag{9.38}$$

Using the 2nd expression for R_r in (9.32), we obtain

$$\boxed{R_{in} = \frac{\eta}{2\pi} \cdot \frac{\Im}{\sin^2(\beta l / 2)}}, \; l > \lambda / 2. \tag{9.39}$$

For a short dipole ($l \leq \lambda / 2$), $I_{in} = I_m$. It then follows from

$$P_{in} = \frac{|I_{in}|^2}{2} R_{in} = \frac{|I_m|^2}{2} R_r \text{ and } I_{in} = I_m, \; l \leq l / 2, \tag{9.40}$$

that

$$\boxed{R_{in} = R_r = \frac{\eta}{2\pi} \cdot \frac{\Im}{\sin^2(\beta l / 2)}}, \; l \leq \lambda / 2, \tag{9.41}$$

where we have taken into account the first equation in (9.32).

In summary, the dipole's input resistance, regardless of its length, depends on the integral $\Im$ as in (9.39) or (9.41), as long as the feed point is at the center.

Loss can be easily incorporated in the calculation of R_{in} bearing in mind that the power-balance relation (9.35) can be modified as

$$P_{in} = \frac{|I_{in}|^2}{2} R_{in} = \Pi + P_{\text{loss}} = \frac{|I_m|^2}{2} R_r + P_{\text{loss}}. \quad (9.42)$$

Remember that in Lecture 4, we obtained the expression for the loss of a dipole of length l as:

$$P_{\text{loss}} = \frac{I_0^2 R_{hf}}{4}\left[1 - \frac{\sin(\beta l)}{\beta l}\right]. \quad (9.43)$$

3. Half-wavelength Dipole

This is a classical and widely used thin wire antenna. Substituting $l \approx \lambda/2$ in (9.21) yields:

$$\boxed{\begin{aligned} E_\theta &= j\eta \frac{I_0 e^{-j\beta r}}{2\pi r} \cdot \frac{\cos(0.5\pi\cos\theta)}{\sin\theta} \\ H_\varphi &= E_\theta / \eta \end{aligned}} \quad (9.44)$$

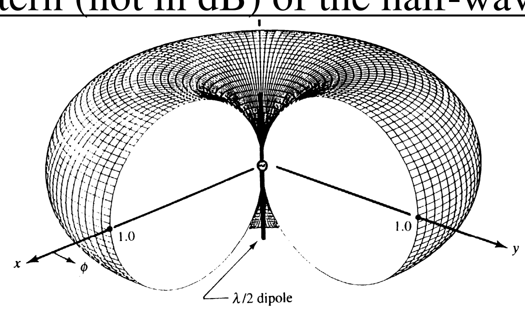
Radiated power flux density:

$$P = \frac{1}{2\eta}|E_\theta|^2 = \eta \frac{|I_0|^2}{8\pi^2 r^2} \underbrace{\left[\frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\right]^2}_{F(\theta)\ -\ \text{normalized power pattern}} \approx \eta \frac{|I_0|^2}{8\pi^2 r^2} \sin^3\theta. \quad (9.45)$$

Radiation intensity:

$$U = r^2 P = \eta \frac{|I_0|^2}{8\pi^2} \underbrace{\left[\frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\right]^2}_{F(\theta)\ -\ \text{normalized power pattern}} \approx \eta \frac{|I_0|^2}{8\pi^2} \sin^3\theta. \quad (9.46)$$

3-D power pattern (not in dB) of the half-wavelength dipole



Radiated power

The radiated power of the half-wavelength dipole is a special case of the integral in (9.26):

$$\Pi = \eta \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\cos^2(0.5\pi\cos\theta)}{\sin\theta} d\theta \tag{9.47}$$

$$\Pi = \eta \frac{|I_0|^2}{8\pi} \int_0^{2\pi} \frac{1-\cos y}{y} dy \tag{9.48}$$

$$\mathcal{J} = 0.5772 + \ln(2\pi) - C_i(2\pi) \approx 2.435 \tag{9.49}$$

$$\Rightarrow \underline{\Pi = 2.435 \frac{\eta}{8\pi} |I_0|^2 = 36.525 |I_0|^2}. \tag{9.50}$$

Radiation resistance:

$$R_r = \frac{2\Pi}{|I_0|^2} \approx 73 \quad \Omega. \tag{9.51}$$

Directivity:

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = 4\pi \frac{U_{/\theta=90^\circ}}{\Pi} = \frac{4}{\Im} = \frac{4}{2.435} = 1.643. \tag{9.52}$$

Maximum effective area:

$$A_e = \frac{\lambda^2}{4\pi} D_0 \approx 0.13\lambda^2. \tag{9.53}$$

Input impedance

Since $l = \lambda / 2$, the input resistance is the same as the radiation resistance:

$$R_{in} = R_r \approx 73 \quad \Omega. \tag{9.54}$$

The imaginary part of the input impedance is approximately $+j42.5 \ \Omega$. To achieve maximum power transfer, this reactance has to be removed by matching (e.g., shortening) the dipole:

- thick dipole $l \approx 0.47\lambda$
- thin dipole $l \approx 0.48\lambda$.

The input reactance of the dipole is very frequency sensitive, i.e., it depends strongly on the ratio l / λ. This is to be expected from a resonant narrow-band structure operating at or near resonance. We should also keep in mind that the input impedance is influenced by the capacitance associated with the physical junction to the transmission line. The structure used to support the antenna, if any, can also influence the input impedance. That is why the curves below describing the antenna impedance are only representative.

Measured input impedance of a dipole vs. its electrical length

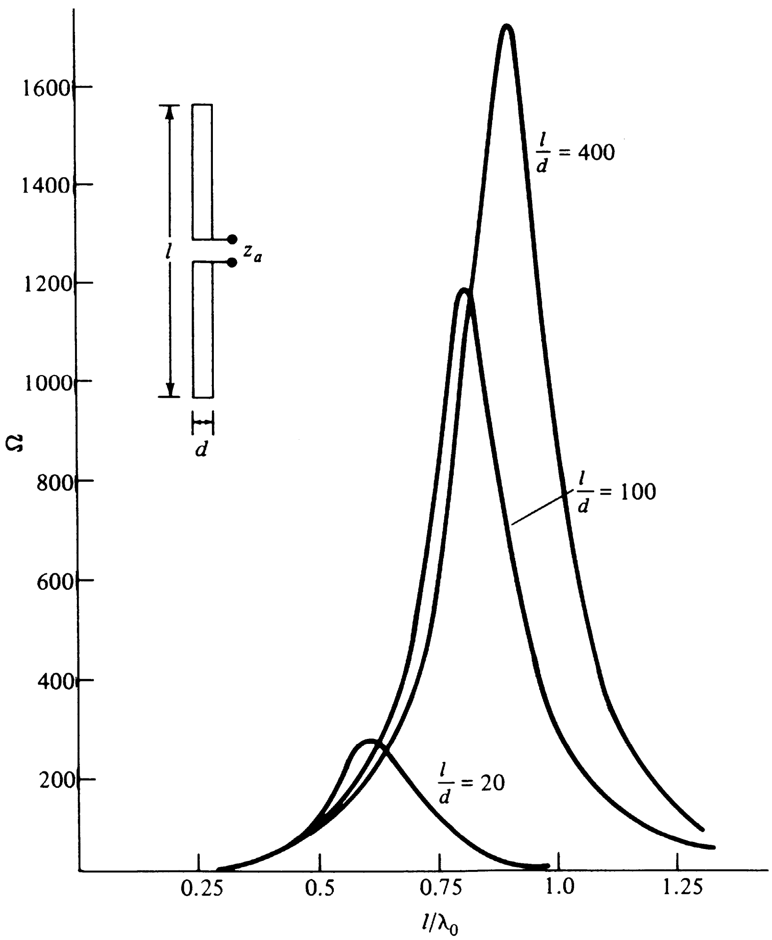
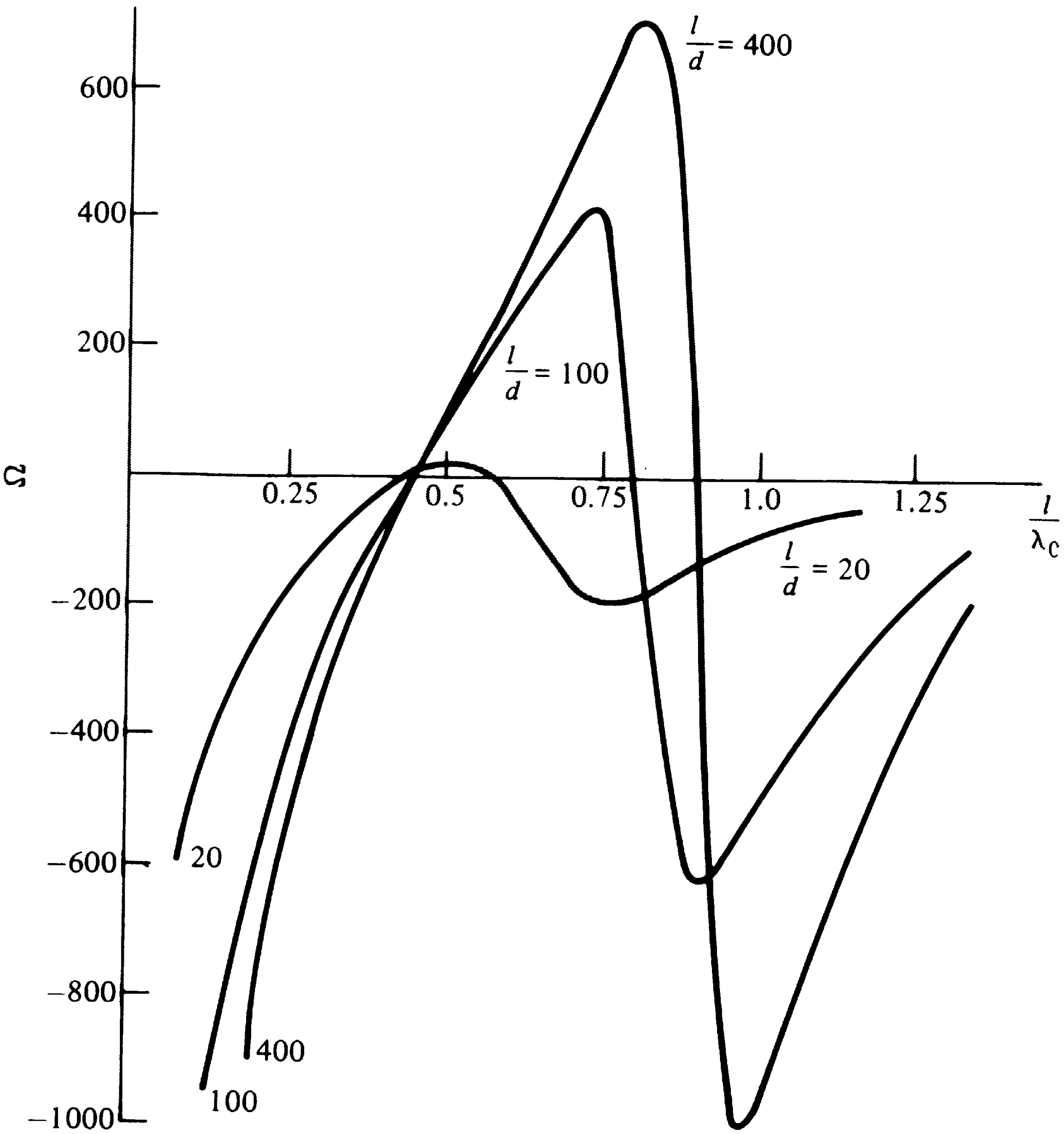


Figure 2.13 Input resistance of dipole antenna.

† G. H. Brown, and O. M. Woodward, Jr., "Experimentally Determined Impedance Characteristics of Cylindrical Antennas," *Proc. IRE*, vol. 33, 1945, pp. 257–262.

(a) input resistance

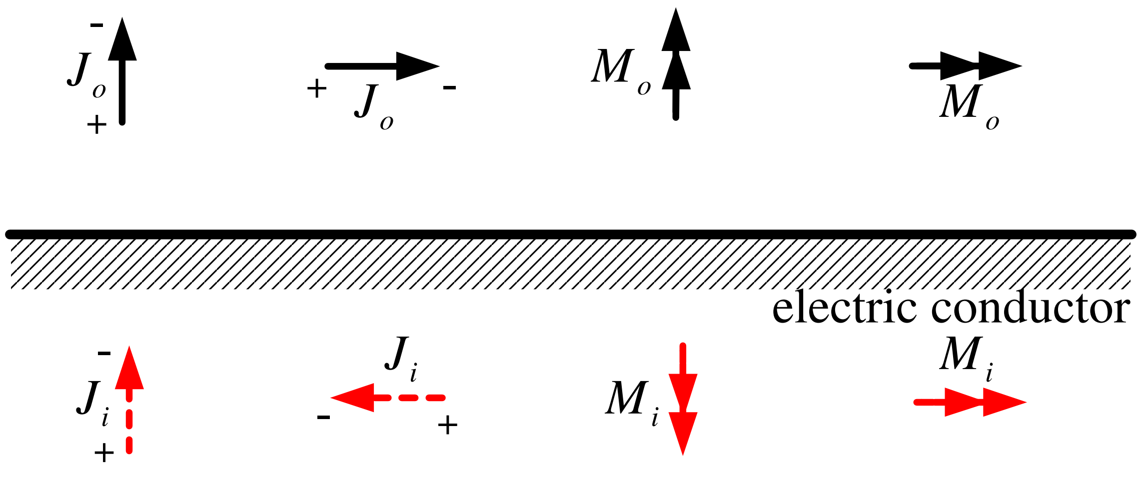
Note the strong influence of the dipole's diameter on the dipole's input resistance at resonance (maximum input resistance). But when $l \approx 0.5\lambda_0$, the impedance is close to about 73 Ω regardless of the dipole's diameter.

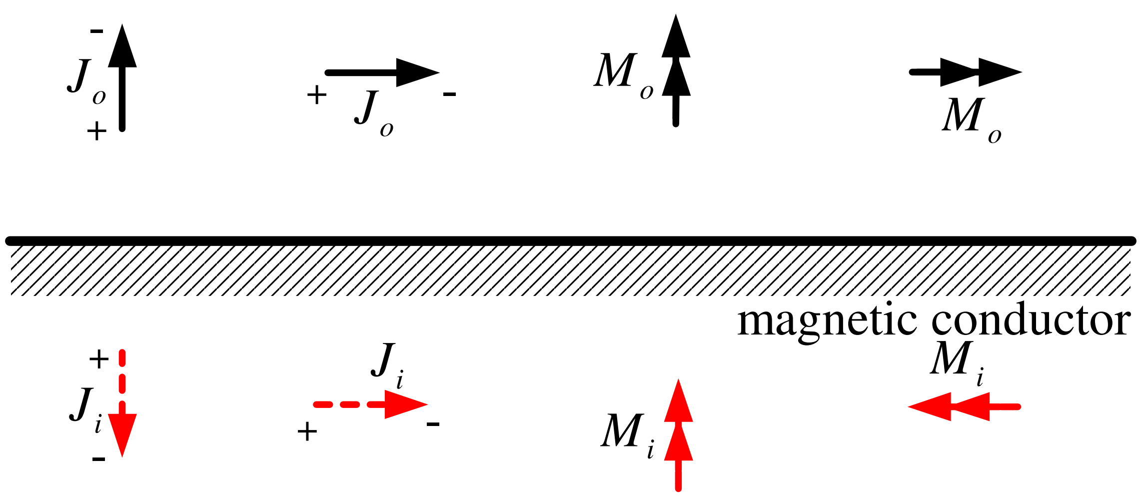


(b) input reactance

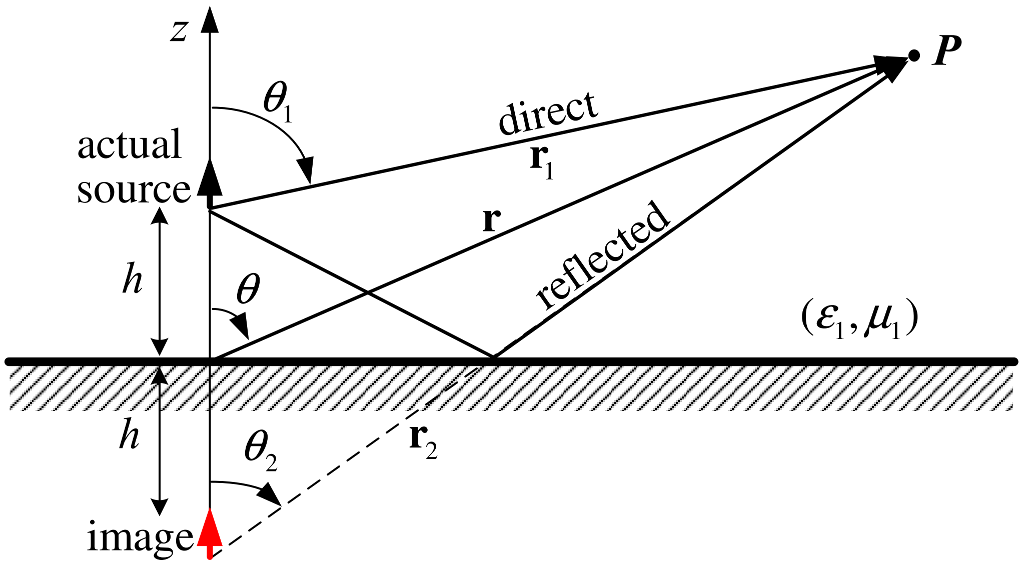
We can calculate the input *resistance* as a function of l/λ using (9.29) and (9.39). These equations, however, are valid only for infinitesimally thin dipoles. Besides, they do not produce the reactance. In practice, dipoles are most often tubular and they have substantial diameter *d*. High-frequency simulators within antenna/microwave CAD packages can calculate accurately the complex antenna input impedance. However, there exists a classical method that produces fairly accurate closed form solutions for the self-impedance and the mutual impedance of straight-wire antennas. This is the ***induced electromotive force (emf) method***, which is discussed later.

4. Method of Images – Revision





5. Vertical Electric Current Element Above Perfect Conductor



The field at the observation point P is a superposition of the fields of the actual source and the image source, both radiating in a homogeneous medium of constitutive parameters (ε_1, μ_1). The actual (or original) source is a current element $I_0 \Delta l$ (infinitesimal dipole). Therefore, the image source is also an infinitesimal dipole. The respective field components are:

$$
\begin{aligned}
E_\theta^d &= j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_1}}{4\pi r_1}\cdot\sin\theta_1, \\
E_\theta^r &= j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_2}}{4\pi r_2}\cdot\sin\theta_2\ .
\end{aligned}
\tag{9.55}
$$

Expressing the distances $r_1 = |\mathbf{r}_1|$ and $r_2 = |\mathbf{r}_2|$ in terms of $r = |\mathbf{r}|$ and h (using the cosine theorem) gives

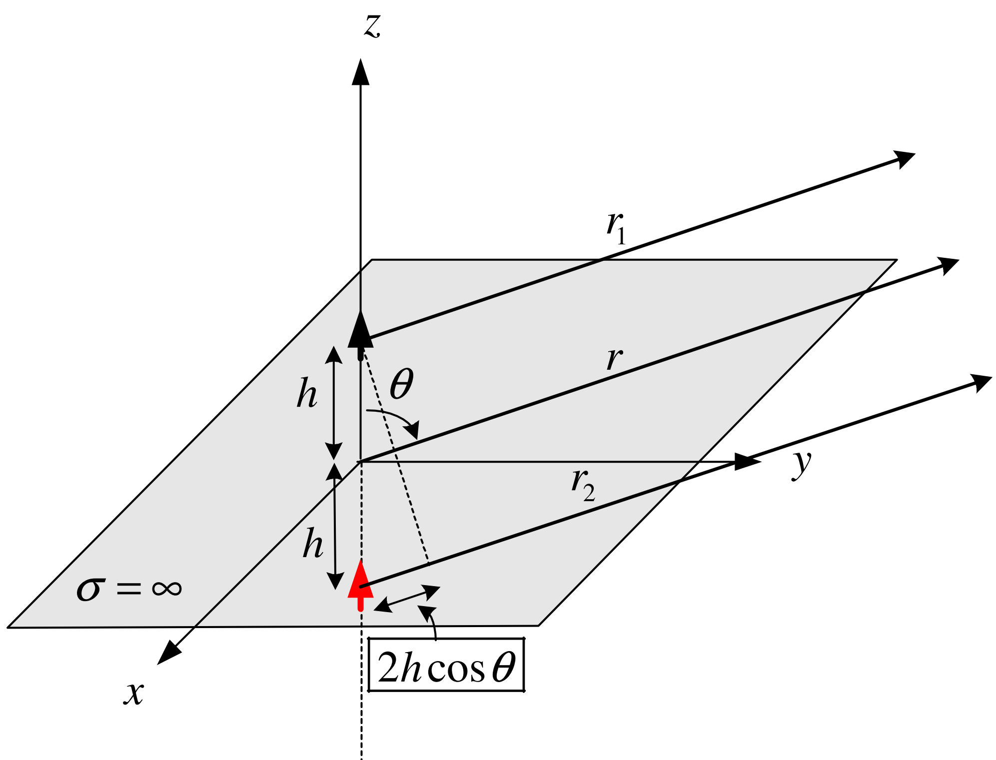
$$
\begin{aligned}
r_1 &= \sqrt{r^2 + h^2 - 2rh\cos\theta}, \\
r_2 &= \sqrt{r^2 + h^2 - 2rh\cos(\pi - \theta)}\ .
\end{aligned}
\tag{9.56}
$$

We make use of the binomial expansions of r_1 and r_2 to approximate the amplitude and the phase terms, which simplify the evaluation of the total far field and the VP integral. For the amplitude term,

$$\frac{1}{r_1} \approx \frac{1}{r_2} \approx \frac{1}{r}. \tag{9.57}$$

For the phase term, we use the second-order approximation (see also the geometrical interpretation below),

$$\begin{aligned} r_1 &\approx r - h\cos\theta \\ r_2 &\approx r + h\cos\theta. \end{aligned} \tag{9.58}$$



The total far field is

$$E_\theta = E_\theta^d + E_\theta^r \tag{9.59}$$

$$E_\theta = j\eta\beta \frac{(I_0 \Delta l)}{4\pi r} \cdot \sin\theta \left[e^{-j\beta(r - h\cos\theta)} + e^{-j\beta(r + h\cos\theta)} \right] \tag{9.60}$$

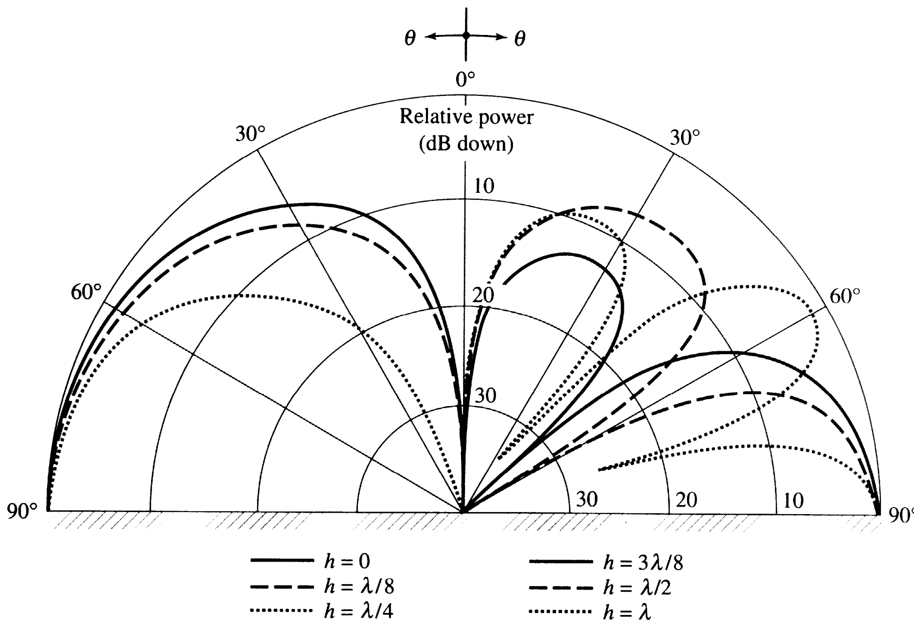
$$\boxed{\begin{aligned} &E_\theta = \underbrace{j\eta\beta (I_0 \Delta l) \frac{e^{-j\beta r}}{4\pi r} \sin\theta}_{\text{element factor } g(\theta)} \cdot \underbrace{\left[2\cos(\beta h \cos\theta) \right]}_{\text{array factor } f(\theta)}, \quad z \ge 0 \\ &E_\theta = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad , \quad z < 0 \end{aligned}} \tag{9.61}$$

Note that the far field can be decomposed into two factors: the field of the elementary source $g(\theta)$ and the pattern factor (also array factor) $f(\theta)$.

The normalized power pattern is

$$F(\theta) = \left[\sin\theta \cdot \cos\left(\beta h \cos\theta\right)\right]^2. \quad (9.62)$$

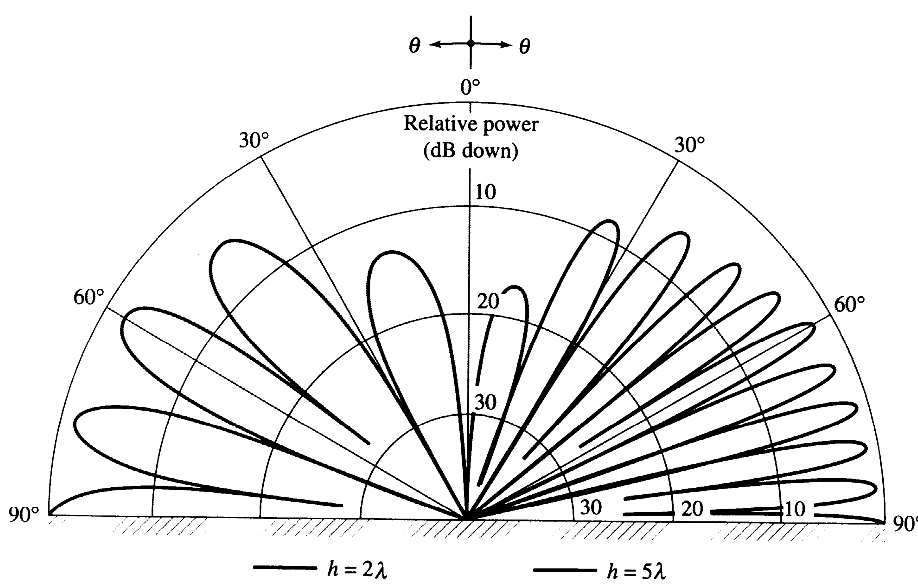
Below, the elevation plane patterns are plotted for vertical infinitesimal electric dipoles of different heights above a perfectly conducting plane:



[Balanis]

As the vertical dipole moves further away from the infinite conducting (ground) plane, more and more lobes are introduced in the power pattern. This effect is called ***scalloping*** of the pattern. The number of lobes is

$$n = \mathrm{nint}\left[(2h/\lambda)+1\right].$$



Total radiated power

$$\Pi = \oiint \mathbf{P} \cdot d\mathbf{s} = \frac{1}{2\eta} \int_0^{2\pi} \int_0^{\pi/2} |E_\theta|^2 \, r^2 \sin\theta d\theta d\varphi,$$

$$\Pi = \frac{\pi}{\eta} \int_0^{\pi/2} |E_\theta|^2 \, r^2 \sin\theta d\theta, \quad (9.63)$$

$$\Pi = \eta\beta^2 (I_0 \Delta l)^2 \int_0^{\pi/2} \sin^2\theta \cdot \cos^2(\beta h \cos\theta) d\theta,$$

$$\boxed{\Pi = \pi\eta \left(\frac{I_0 \Delta l}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right].} \quad (9.64)$$

- As $\beta h \to 0$, the radiated power of the vertical dipole above ground approaches twice the value of the radiated power of a dipole of the same length in free space.
- As $\beta h \to \infty$, the radiated power of the vertical dipole above ground tends toward that of the vertical dipole in open space.

The above asymptotic behavior is explained by the limits:

$$\lim_{h\to 0} \left[-\frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3} \right] = \frac{1}{3}, \quad (9.65)$$

$$\lim_{h\to \infty} \left[-\frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3} \right] = 0. \quad (9.66)$$

Radiation resistance

$$\boxed{R_r = \frac{2\Pi}{|I_0|^2} = 2\pi\eta \left(\frac{\Delta l}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right].} \quad (9.67)$$

- As $\beta h \to 0$, the radiation resistance of the vertical dipole above ground approaches twice the value of the radiation resistance of a dipole of the same length in free space:

$$R_{in}^{vdp} = 2R_{in}^{dp}, \quad \beta h = 0. \quad (9.68)$$

- As $\beta h \to \infty$, the radiation resistances of both dipoles (in free space and above ground) become the same.

Radiation intensity

$$U = r^2 P = r^2 \frac{|E_\theta|^2}{2\eta} = \frac{\eta}{2}\left(\frac{I_0 \Delta l}{\lambda}\right)^2 \sin^2\theta \cos^2(\beta h \cos\theta). \qquad (9.69)$$

The maximum of $U(\theta)$ occurs at $\theta = \pi/2$:

$$U_{\max} = \frac{\eta}{2}\left(\frac{I_0 \Delta l}{\lambda}\right). \qquad (9.70)$$

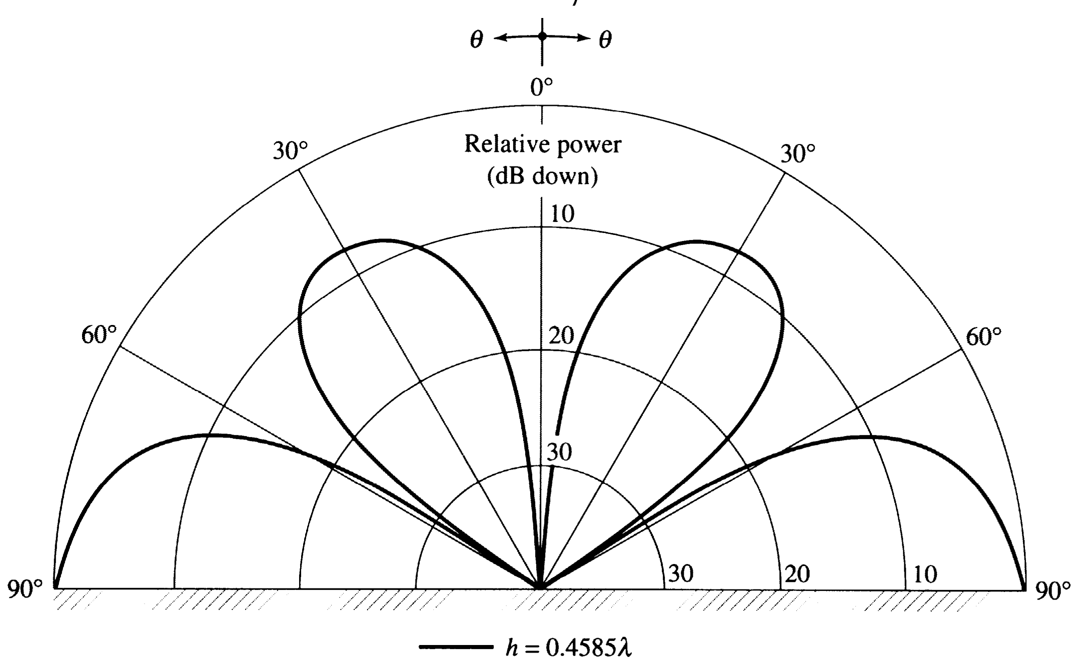
This value is 4 times greater than $U_{\max}$ of a free-space dipole of the same length. Can you provide a physical explanation of this result?

Maximum directivity

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = \frac{2}{\dfrac{1}{3} - \dfrac{\cos(2\beta h)}{(2\beta h)^2} + \dfrac{\sin(2\beta h)}{(2\beta h)^3}}. \qquad (9.71)$$

If $\beta h = 0$, $D_0 = 3$, which is twice the maximum directivity of a free-space current element ($D_0^{id} = 1.5$). Can you explain why that is when in fact the two field patterns are identical in the upper half-space?

The maximum of D_0 as a function of the height h occurs when $\beta h \approx$ 2.881 ($h \approx 0.4585\lambda$). Then, $D_0 \approx 6.566_{/\beta h = 2.881}$.



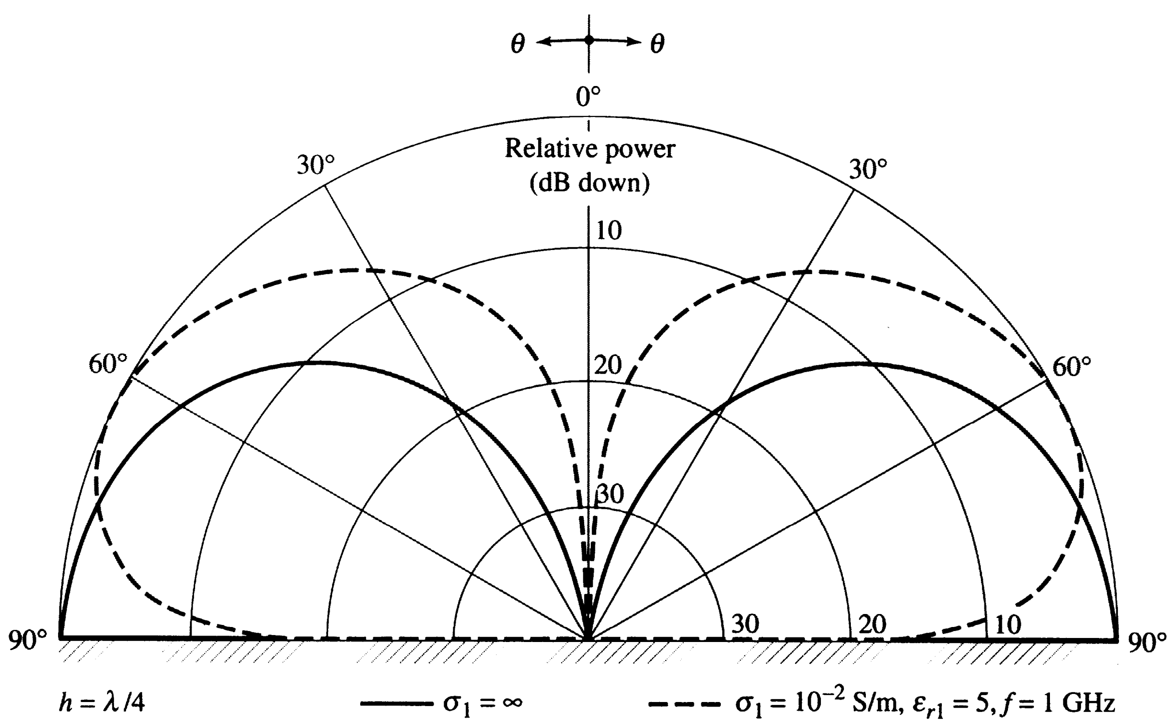
6. Monopoles

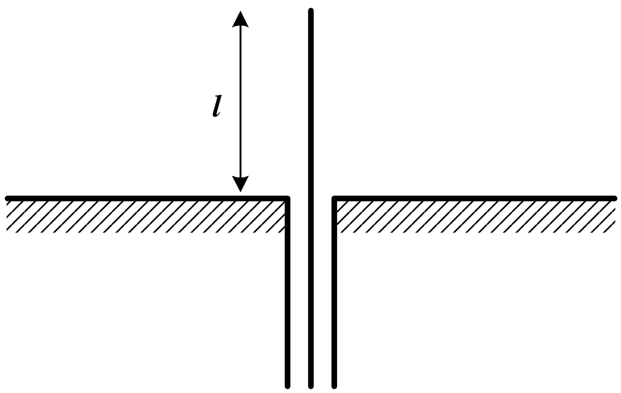
A monopole is a dipole that has been reduced by one-half and is fed against a ground plane. It is normally $\lambda/4$ long (a ***quarter-wavelength monopole***), but it might by shorter if there are space restrictions. In the latter case, the monopole is a ***small monopole*** the counterpart of which is the ***small dipole*** (see Section 1). Its current has linear distribution with its maximum at the feed point and its null at the end.

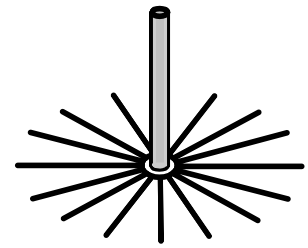
The vertical monopole is a common antenna for AM broadcasting (f = 500 to 1500 kHz, λ = 200 to 600 m), because it is the shortest most efficient antenna at these frequencies. Also, the vertically polarized waves suffer less attenuation at close-to-ground propagation. Vertical monopoles are widely used as base-station antennas in mobile communications, too.

Monopoles at base stations and radio-broadcast stations are supported by towers and guy wires. The guy wires must be separated into short enough ($\leq \lambda/8$) pieces insulated from each other to suppress parasitic currents.

Special care is taken when grounding the monopole. Usually, multiple radial wires or rods, each $0.25-0.35\lambda$ long, are buried at the monopole base in the ground to simulate perfect ground plane, so that the pattern approximates closely the theoretical one, i.e., the pattern of the $\lambda/2$-dipole. Losses in the ground plane cause undesirable deformation of the pattern as shown below (infinitesimal dipole above an imperfect ground plane).







Monopole fed against a large solid ground plane

Practical monopole with radial wires to simulate perfect ground

Several important conclusions follow from the image theory and the discussion in Section 5:

- The field distribution in the upper half-space is the same as that of the respective free-space dipole.
- The currents and charges on a monopole are the same as on the upper half of its dipole counterpart but the terminal voltage is only one-half that of the dipole. <u>The input impedance of a monopole is therefore only half that of the respective dipole</u>:

$$Z_{in}^{mp} = 0.5 Z_{in}^{dp} . \quad (9.72)$$

- The radiation pattern of a monopole is one-half the dipole's pattern since it radiates in half-space and, at the same time, the field normalized distribution in this half-space is the same as that of the dipole. As a result, the beam solid angle of the monopole is half that of the respective dipole and <u>its directivity is twice that of the dipole</u>:

$$D_0^{mp} = \frac{4\pi}{\Omega_A^{mp}} = \frac{4\pi}{0.5\Omega_A^{dp}} = 2D_0^{dp} . \quad (9.73)$$

<u>The quarter-wavelength monopole</u>

This is a straight wire of length $l = \lambda / 4$ mounted over a ground plane. From the discussion above, it follows that the quarter-wavelength monopole is the counterpart of the half-wavelength dipole as far as the radiation in the hemisphere above the ground plane is concerned.

- Its radiation pattern is the same as that of a free-space $\lambda/2$-dipole, but it is non-zero only for $0° \le \theta \le 90°$ (above ground).
- The field expressions are the same as those of the $\lambda/2$-dipole.
- The total radiated power of the $\lambda/4$-monopole is half that of the $\lambda/2$-dipole.
- The radiation resistance of the $\lambda/4$-monopole is half that of the $\lambda/2$-dipole: $Z_{in}^{mp} = 0.5Z_{in}^{dp} \approx 0.5(73 + j42.5) = 36.5 + j21.25, \ \ \Omega$.
- The directivity of the $\lambda/4$-monopole is

$$D_0^{mp} = 2D_0^{dp} \approx 2 \cdot 1.643 = 3.286.$$

<u>Some approximate formulas for rapid calculations of the input resistance of a dipole and the respective monopole</u>:

Let
$$\left| \begin{array}{l} G = \dfrac{\beta l}{2} = \pi \dfrac{l}{\lambda}, \ \text{ for } \ \text{dipole} \\ \\ G = \beta l = 2\pi \dfrac{l}{\lambda}, \ \text{ for } \ \text{monopole.} \end{array} \right.$$

Approximate formulas:

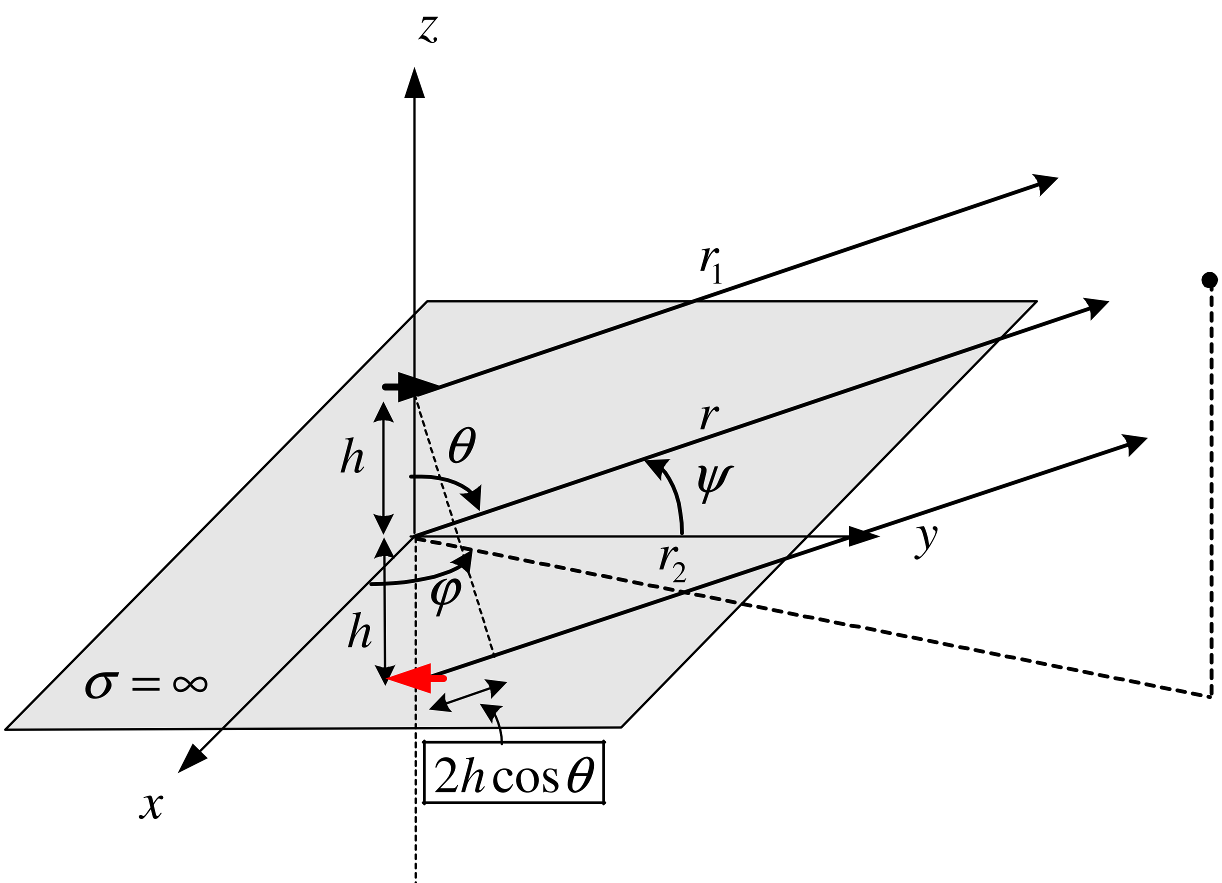
- If $0 < G < \dfrac{\pi}{4}$, then $\left| \begin{array}{l} R_{in} = 20G^2 \quad \text{,dipole} \\ R_{in} = 10G^2 \quad \text{,monopole} \end{array} \right.$

- If $\dfrac{\pi}{4} < G < \dfrac{\pi}{2}$, then $\left| \begin{array}{l} R_{in} = 24.7G^{2.5} \quad \text{,dipole} \\ R_{in} = 12.35G^{2.5} \quad \text{,monopole} \end{array} \right.$

- If $\dfrac{\pi}{2} < G < 2$, then $\left| \begin{array}{l} R_{in} = 11.14G^{4.17} \quad \text{,dipole} \\ R_{in} = 5.57G^{4.17} \quad \text{,monopole} \end{array} \right.$

7. Horizontal Current Element Above a Perfectly Conducting Plane

The analysis is analogous to that of a vertical current element above a ground plane. The difference arises in the element factor $g(\theta)$ because of the horizontal orientation of the current element. Let us assume that the current element is oriented along the *y*-axis, and the angle between $\mathbf{r}$ and the dipole's axis (*y*-axis) is ψ.



$$\mathbf{E}(P) = \mathbf{E}^d(P) + \mathbf{E}^r(P), \tag{9.74}$$

$$E_\psi^d = j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_1}}{4\pi r_1}\sin\psi, \tag{9.75}$$

$$E_\psi^r = -j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_2}}{4\pi r_2}\sin\psi. \tag{9.76}$$

We can express the angle ψ in terms of (θ,φ):

$$\cos\psi = \hat{\mathbf{y}}\cdot\hat{\mathbf{r}} = \hat{\mathbf{y}}\cdot(\hat{\mathbf{x}}\sin\theta\cos\varphi + \hat{\mathbf{y}}\sin\theta\sin\varphi + \hat{\mathbf{z}}\cos\theta)$$

$$\Rightarrow \cos\psi = \sin\theta\sin\varphi$$
$$\Rightarrow \sin\psi = \sqrt{1-\sin^2\theta\sin^2\varphi}. \tag{9.77}$$

The far-field approximations are:

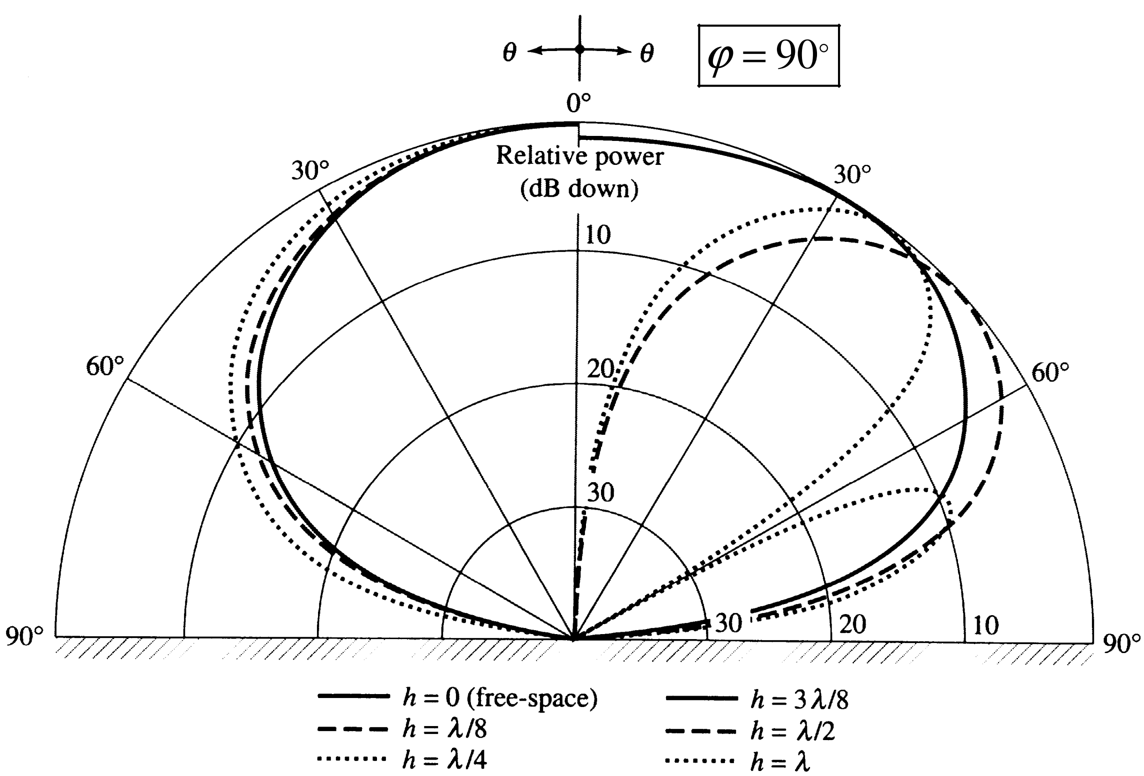
$$\left|\begin{array}{l} \dfrac{1}{r_1} = \dfrac{1}{r_2} = \dfrac{1}{r}, \quad \text{for the amplitude term} \\ \left.\begin{array}{l} r_1 \approx r - h\cos\theta \\ r_2 \approx r + h\cos\theta \end{array}\right\} \quad \text{for the phase term.} \end{array}\right.$$

The substitution of the far-field approximations and equations (9.75), (9.76), (9.77) into the total field expression (9.74) yields

$$\boxed{E_\psi(\theta,\varphi) = \underbrace{j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r}}{4\pi r}\sqrt{1-\sin^2\theta\sin^2\varphi}}_{\text{element factor } g(\theta,\varphi)} \cdot \underbrace{\left[2j\sin\left(\beta h\cos\theta\right)\right]}_{\text{array factor } f(\theta,\varphi)}.} \tag{9.78}$$

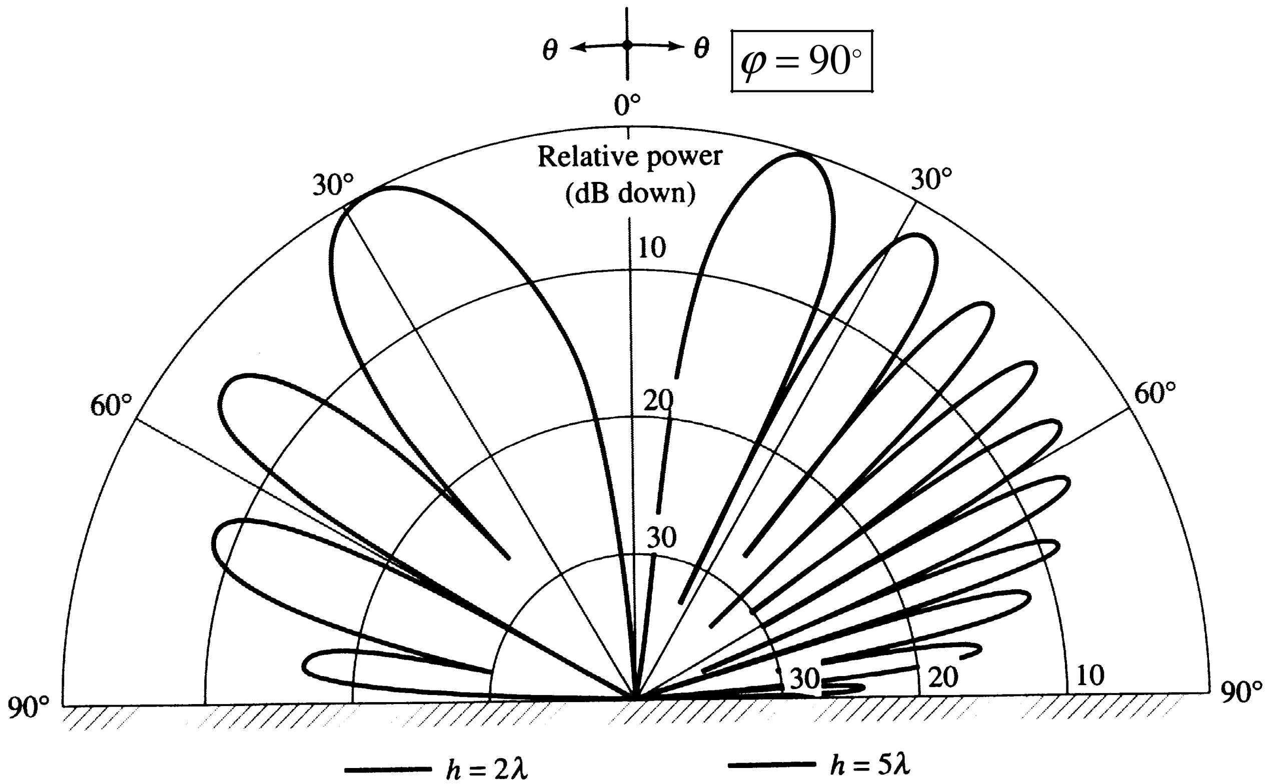
<u>The normalized power pattern</u>

$$F(\theta,\varphi) = \left(1-\sin^2\theta\cdot\sin^2\varphi\right)\cdot\sin^2\left(\beta h\cos\theta\right) \tag{9.79}$$



As the height increases beyond a wavelength ($h > \lambda$), scalloping appears with the number of lobes being

$$n = \mathrm{nint}\left(2\frac{h}{\lambda}\right). \tag{9.80}$$



Following a procedure similar to that of the vertical dipole, the radiated power and the radiation resistance of the horizontal dipole can be found:

$$\Pi = \frac{\pi}{2}\eta\left(\frac{I_0\Delta l}{\lambda}\right)^2 \underbrace{\left[\frac{2}{3} - \frac{\sin(2\beta h)}{2\beta h} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]}_{R(\beta h)} \tag{9.81}$$

$$R_r = \pi\eta\left(\frac{\Delta l}{\lambda}\right)^2 \cdot R(\beta h). \tag{9.82}$$

By expanding the sine and the cosine functions into series, it can be shown that for small values of (βh) the following approximation holds:

$$R_{/\beta h \to 0} \approx \frac{32\pi^2}{15}\left(\frac{h}{\lambda}\right)^2. \tag{9.83}$$

It is also obvious that if $h=0$, then $R_r=0$ and $\Pi=0$. This is to be expected because the dipole is short-circuited by the ground plane.

Radiation intensity

$$U=\frac{r^2}{2\eta}|\mathbf{E}_\psi|^2=\frac{\eta}{2}\left(\frac{I_0\Delta l}{\lambda}\right)^2\left(1-\sin^2\theta\cdot\sin^2\varphi\right)\cdot\sin^2\left(\beta h\cos\theta\right) \qquad (9.84)$$

The maximum value of (9.84) depends on whether (βh) is less than $\pi/2$ or greater:

- If $\beta h\le\frac{\pi}{2}$ $\left(h\le\frac{\lambda}{4}\right)$

$$U_{\max}=\frac{\eta}{2}\left(\frac{I_0\Delta l}{\lambda}\right)^2\sin^2\left(\beta h\right)_{/\theta=0^\circ}. \qquad (9.85)$$

- If $\beta h>\frac{\pi}{2}$ $\left(h>\frac{\lambda}{4}\right)$

$$U_{\max}=\frac{\eta}{2}\left(\frac{I_0\Delta l}{\lambda}\right)^2_{/\theta=\arccos\left(\frac{\pi}{2\beta h}\right),\varphi=0^\circ}. \qquad (9.86)$$

Maximum directivity

- If $h\le\frac{\lambda}{4}$, then $U_{\max}$ is obtained from (9.85) and the directivity is

$$D_0=4\pi\frac{U_{\max}}{\Pi}=\frac{4\sin^2(\beta h)}{R(\beta h)}. \qquad (9.87)$$

- If $h>\frac{\lambda}{4}$, then $U_{\max}$ is obtained from (9.86) and the directivity is

$$D_0=4\pi\frac{U_{\max}}{\Pi}=\frac{4}{R(\beta h)}. \qquad (9.88)$$

For very small βh, the approximation $D_0\approx 7.5\left(\frac{\sin\left(\beta h\right)}{\beta h}\right)^2$ is often used.