Computing the Odds of Winning in Tennis and Similar Games

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1 Abstract:

We follow the methodology formulated by Newton and Keller [1] that computes the probability of winning a professional tennis match based on the comparative strength of the players in singles matches (both three and five set matches). The theoretical probability of winning a game, set and match are computed by exhaustively modeling each case, using Binomial probabilities. The probability of a player winning a point on their serve is extracted for the ATP and WTA players [1] and is held constant throughout. The theoretical probabilities of winning a game are found to closely resemble the empirical career probabilities for each player.

2 Introduction:

The calculation of the odds of winning a game for two players by evaluating all possible future scenarios of games using binomial probabilities is first discussed in Laplace and Pascal[4], in the so-called Problem of Points. The present discussion presumes that a rally in tennis is a Bernoulli trial, and the player serving the rally has a constant probability of winning a point on own serve. Further, successive rallies are treated as independent and the outcomes are iid Bernoulli random variables. Thus, score is distributed as Binomial random variables.

The method requires the construction all possible game scenarios, within the tournament rules[2]. We reconstruct the formulae in Newton and Keller[1] for the calculation of these probabilities and frame additional formulas to ease lengthy computation. We expect that this method, wherein two players contend for a single point, as in Tennis tournaments, can be extended to other types of 'game' situations also, making the present discussion more widely applicable.

We calculate the probability of a player winning a game, set and match using the probability that the player wins a rally on their serve. Here, the two outcomes of a rally served

[1] As reported in February-March 2022

[2] See Appendix for a list of tennis nomenclature

by a given player are that they win or lose the rally. Since winning a game is winning a series of rallies, the score behaves like a Binomial random variable under assumptions of iid of the individual rallies.

We calculate the probability that a player ranked in the top 50 of Association of Tennis Professionals (ATP) and a player ranked in the top 50 of Women's Tennis Association (WTA) will win a match against another player ranked in the top 50 of their respective tours[3]. We use the formulas to calculate the probability of a player winning a game, set and match. We seed our formulas with the career-long empirical probability of winning a point on serve. Our results compare well with the career proportion of games won by the players.

3 Calculating the probability of a player winning a game on their serve

The probability that a player A will win a point on their serve is denoted by p_A^R following Newton and Keller. $q_A^R = 1 - p_A^R$ is the probability that the player will lose a point on their serve. To win a game, the player must be the first to reach 4 points and/or must have a two point difference. The possible ways this can happen is (4, 0), (4, 1), (4, 2) and (3, 3) + deuce.
In the following section, we explore each of these situations.

(4, 0):
Player A wins all 4 points. The probability of this happening is

$$(p_A^R)^4$$

.
(4, 1):
Player A wins 4 of the five points and player B wins one of the five point. Player B cannot win the fifth point as the the game would have been won by player A at (4, 0). So, among the first four games, B won one point. The probability of this happening is $(p_A^R)^4 *^4 C_1(q_A^R)$. This is the same as

$$4 * (p_A^R)^4(q_A^R)$$

.
(4, 2):
Player A wins 4 of the six points and player B wins 2 of the six points. Player B cannot have won the sixth points as the game would have been won by player A at (4, 1). Player B can win the fifth point if the score at that stage is (3, 1). Player A would have won 3 out of the first four games and the sixth game. This probability of this happening is ${}^4C_3 * (p_A^R)^3(q_A^R)^2(p_A^R)$. This is the same as $4 * (p_A^R)^4(q_A^R)^2$. Else, B would have won two of the first four points. The probability of this happening is ${}^4C_2 * (q_A^R)^2(p_A)^4$. This is the same as $6 * (q_A^R)^2(p_A)^4$. So, the probability that the score reaches (4, 2) is

$$4 * (p_A^R)^4(q_A^R)^2 + 6 * (q_A^R)^2(p_A)^4 = 10 * (p_A^R)^4(q_A^R)^2$$

[3]Rankings as of Feb-Mar 2022

.

(3, 3):
Player A wins 3 out of six games and player B wins 3 out of six games. The probability of this happening is ${}^6C_3 * (p_A^R)^3(q_A^R)^3$. This is the same as

$$20 * (p_A^R)^3(q_A^R)^3$$

.

Once deuce is reached, player A must reach a two point lead in order to win the game. This means that for all $n \geq 0$, player A must reach a score of $n+2$ and player B must remain at a score of n.
Once both players reach n points, player A wins two consecutive points to win the game. If both the players reach n points, the points played would be of the following nature:
For every point A makes, A loses a point
For every point B makes, B loses a point
The probability of A winning the first point and losing the next point j times is

$$((p_A^R)(q_A^R))^j$$

.
The probability of A losing the first point and winning the next point $n-j$ times is

$$((p_A^R)(q_A^R))^{(n-j)}$$

.
By doing the above j and $n-j$ times respectively, each player will reach a score of n. The points that A first wins and then loses the consecutive point can be chosen in ${}^nC_j$ ways.
So, the probability that a score of n will be reached is ${}^nC_j((p_A^R)(q_A^R))^j((p_A^R)(q_A^R))^{(n-j)}$. This can be written as ${}^nC_j((p_A^R)(q_A^R))^n$.
The probability that A will win the next 2 points is $(p_A^R)^2$. So, the probability that each player will reach a score of n and A will win two consecutive points is

$${}^nC_j((p_A^R)(q_A^R))^n * (p_A^R)^2$$

.
So, for all n, the probability that each player will reach a score of n and A will win two consecutive points is $\sum_{j=0}^{\infty} {}^nC_j((p_A^R)(q_A^R))^n * (p_A^R)^2$.
$\sum_{j=0}^{\infty} {}^nC_j$ is equal to 2^n (The binomial expansion of $(1+1)^n$). Hence, the probability that each player will reach a score of n and A will win two consecutive points to win the game is

$$(p_A^R)^2(p_A^R q_A^R)^n * 2^n$$

.

The probability that the player A will win the tie-break and considers all possible values of n is $\sum_{n=0}^{\infty}(p_A^R)^2(2p_A^R q_A^R)^n$.

The summation of $\sum_{n=0}^{\infty}(2p_A^R q_A^R)^n$ would be the sum up to infinite terms of the geometric progression with the first term equal to 1 and the common ratio equal to $2p_A^R q_A^R$. Hence, the sum up to infinity is $\frac{1}{1-r} = \frac{1}{1-2p_A^R q_A^R}$. The probability that player A will win a game given that their probability of winning a point on their serve is p_A^R is

$$(p_A^R)^4 + 4*(p_A^R)^4(q_A^R) + 10*(p_A^R)^4(q_A^R)^2 + (20*(p_A^R)^3(q_A^R)^3)(\frac{1}{1-2p_A^R q_A^R})$$

.

$$= (p_A^R)^4(1+4(q_A^R)+10(q_A^R)^2) + (20*(p_A^R)^3(q_A^R)^3)((p_A^R)^2(p_A^R q_A^R)^n * 2^n)$$

.

As is clear from the formula, the probability of player A winning a game in their serve is purely based on the probability that A wins a point on their serve. Hence, this is also a measure of the skill of the player and is independent of the opponent.

4 Calculating the probability of a player winning a set on their serve

We have calculated the probability that a player will win a game on their serve. Now, we calculate the probability that player A will win a set if they serve the first game.
To win a set, the player must be the first to reach a score of 6 and have a two point difference.Hence, the possible ways that player A can win the set is by reaching a score of (6, 0), (6, 1), (6, 2), (6, 3), (6, 4), (7, 5) and (6, 6) + tie-break.
In the following section, we explore each of these situations. Similar combinatorics to the calculation of probability of winning a game are used.

(6, 0):
Player A has won all six games. Since player A started to serve, the order of service through the set will be as follows:

$$ABABAB$$

Hence, A must have won all three games on their serve and B must have lost all three games on their serve. The probability of this happening is

$$(p_A^G)^3(q_A^G)^3$$

(6, 1):
Player A wins 6 out of the seven games played and player B wins 1 out of the seven games played. B cannot win the seventh game as the set would have already been won by player A at (6, 0). This means that player B must have won their game in the first

six games. Since player A started to serve, the order of service through the set will be as follows:

$$ABABABA$$

Case 1:
B won the game on their serve.
The probability of this happening is

$$3(p_B^G)(q_B^G)^2(p_A^G)^4$$

.

Case 2:
B win the game on $A's$ serve.
The probability of this happening is

$$3(p_A^G)^3(q_A^G)(q_B^G)^3$$

.

The probability that the score reaches (6, 1) is

$$3(p_B^G)(q_B^G)^2(p_A^G)^4 + 3(p_A^G)^3(q_A^G)(q_B^G)^3$$

.
(6, 2):
Player A wins 6 out of the eight games played and player B win 2 out of the eight games played. B cannot win the eighth game as the set would have been won by A at (6, 1). This means that B must have won both the games in the first seven games. Since player A started to serve, the order of service through the set will be as follows:

$$ABABABAB$$

Case 1:
B wins the seventh game and 1 game in the first six games. This game could have been won either on $A's$ serve or on $B's$ serve. The probability of this is

$$3(p_B^G)(q_B^G)^3(p_A^G)^3(q_A^G) + 3(p_A^G)^2(q_A^G)^2(q_B^G)^4$$

.

Case 2:
B wins both the games in the first six games and A wins the seventh and eighth games.

Case I:
B wins both the games on $A's$ serve. The probability of this happening is

$$3(p_A^G)^2(q_A^G)^2(q_B^G)^4$$

B wins both the games on $B's$ serve. The probability of this happening is

$$3(p_B^G)^2(q_B^G)^2(p_A^G)^4$$

B wins one game on $A's$ serve and one game on $B's$ serve. The probability of this happening is

$$9(q_A^G)(p_B^G)(p_A^G)^3(q_B^G)^3$$

The probability that the score reaches (6, 2) is

$$3(p_B^G)(q_B^G)^3(p_A^G)^3(q_A^G)+3(p_A^G)^2(q_A^G)^2(q_B^G)^4+3(p_A^G)^2(q_A^G)^2(q_B^G)^4+3(p_B^G)^2(q_B^G)^2(p_A^G)^4+9(q_A^G)(p_B^G)(p_A^G)^3(q_B^G)^3$$
$$= 12(q_A^G)(p_B^G)(p_A^G)^3(q_B^G)^3 + 6(p_A^G)^2(q_A^G)^2(q_B^G)^4 + 3(p_B^G)^2(q_B^G)^2(p_A^G)^4$$

.

(6, 3):
Player A wins 6 out of the nine games played and player B wins 3 out of the nine games played. B cannot win the ninth game as the set would have been won by A at $(6, 2)$. This means that B must have won all the three games in the first eight games. Since player A started to serve the order of service through the set will be as follows:

$$ABABABABA$$

Case 1:
B wins the eighth game and 2 games in the first seven games. These games could have been won on $A's$ serve or $B's$ serve or in both. The probability of this is

$$12(p_A^G)^4(q_A^G)(p_B^G)^2(q_B^G)^2 + 6(p_A^G)^3(q_A^G)^2(p_B)(q_B^G)^3 + 3(p_A^G)^5(p_B^G)^3(q_B^G)$$

Case 2:
B wins all three games in the first seven games.
Case I:
B wins all the three games on $A's$ serve. The probability of this happening is

$$4(p_A^G)^2(q_A^G)^3(q_B^G)^4$$

Case II:

B wins all the three games on $B's$ serve. The probability of this happening is

$$(p_A^G)^5(p_B^G)^3(q_B^G)$$

Case III:
B wins two games in $A's$ serve and one game on $B's$ serve. The probability of this happening is

$$18(p_A^G)^3(q_A^G)^2(p_B^G)(q_B^G)^3$$

Case IV:
B wins one game on $B's$ serve and two games on $A's$ serve. The probability of this happening is

$$12(p_A^G)^4(q_A^G)(p_B^G)^2(q_B^G)^2$$

The probability that the score reaches $(6,3)$ is

$$12(p_A^G)^4(q_A^G)(p_B^G)^2(q_B^G)^2+6(p_A^G)^3(q_A^G)^2(p_B)(q_B^G)^3+3(p_A^G)^5(p_B^G)^3(q_B^G)+$$

$$4(p_A^G)^2(q_A^G)^3(q_B^G)^4+(p_A^G)^5(p_B^G)^3(q_B^G)+18(p_A^G)^3(q_A^G)^2(p_B^G)(q_B^G)^3+12(p_A^G)^4(q_A^G)(p_B^G)^2(q_B^G)^2$$

$$=24(p_A^G)^4(q_A^G)(p_B^G)^2(q_B^G)^2+24(p_A^G)^3(q_A^G)^2(p_B)(q_B^G)^3+4(p_A^G)^5(p_B^G)^3(q_B^G)+4(p_A^G)^2(q_A^G)^3(q_B^G)^4$$

$(6,4)$:
Player A wins 6 of the ten games played and player B wins 4 of the ten games played. B cannot win the tenth game as the set would have been won by A at $(6,3)$. This means that B must have won all the four games in the first nine games. Since A started to serve, the order of service through the set will be as follows.

$$ABABABABAB$$

Case 1:
B wins the ninth game and three games in the first eight games. These games could have been won on $A's$ serve or on $B's$ serve or on both. The probability of this is

$$24(p_A^G)^3(q_A^G)^2(p_B^G)^2(q_B^G)^3+24(p_A^G)^2(q_A^G)^3(p_B)(q_B^G)^4+4(p_A^G)^4(q_A)(p_B^G)^3(q_B^G)^2+4(p_A^G)(q_A^G)^4(q_B^G)^5$$

Case 2:
B wins all four games in the first eight games.
Case I:

B wins all four games on $A's$ serve. The probability of this happening is

$$(p_A^G)(q_A^G)^4(q_B^G)^5$$

Case II:
B wins three games on $A's$ serve and one game on $B's$ serve. The probability of this is

$$16(p_A^G)^2(q_A^G)^3(p_B^G)(q_B^G)^4$$

Case III:
B wins two games on $A's$ serve and 2 games on $B's$ serve. The probability of this happening is

$$36(p_A^G)^3(q_A^G)^2(p_B^G)^2(q_B^G)^3$$

Case IV:
B wins one game on $A's$ serve and three games on $B's$ serve. The probability of this is

$$16(p_A^G)^4(q_A^G)(p_B^G)^3(q_B^G)^2$$

Case V:
B wins all four games on $B's$ serve. The probability of this is

$$(p_A^G)^5(p_B^G)^4(q_B^G)$$

The probability that the score reaches $(6,4)$ is

$$24(p_A^G)^3(q_A^G)^2(p_B^G)^2(q_B^G)^3+24(p_A^G)^2(q_A^G)^3(p_B)(q_B^G)^4+4(p_A^G)^4(q_A)(p_B^G)^3(q_B^G)^2+4(p_A^G)(q_A^G)^4(q_B^G)^5$$

$$+(p_A^G)(q_A^G)^4(q_B^G)^5+16(p_A^G)^2(q_A^G)^3(p_B^G)(q_B^G)^4+36(p_A^G)^3(q_A^G)^2(p_B^G)^2(q_B^G)^3+16(p_A^G)^4(q_A^G)(p_B^G)^3(q_B^G)^2$$

$$+(p_A^G)^5(p_B^G)^4(q_B^G)$$

$$=60(p_A^G)^3(q_A^G)^2(p_B^G)^2(q_B^G)^3+40(p_A^G)^2(q_A^G)^3(p_B)(q_B^G)^4+20(p_A^G)^4(q_A)(p_B^G)^3(q_B^G)^2+5(p_A^G)(q_A^G)^4(q_B^G)^5$$

$$+(p_A^G)^5(p_B^G)^4(q_B^G)$$

$(7,5)$:

Player A wins 7 of the twelve games played and player B wins 5 of the twelve games played. B cannot have won the twelfth game as the set would have reached a tie at $(6,6)$. Also, B could not have won the eleventh game. If we assume that B won their fifth game in the eleventh game played, the score would be $(6,5)$. But again, for this to happen, the previous step must have been $(6,4)$and the set would have been won by A. This means that B must have won the five games in the first ten games. Since A started to serve, the order of service though the set would be as follows.

$$ABABABABABAB$$

Case 1:
B wins all five games on $A's$ serve. The probability of this is

$$(p_A^G)(q_A^G)^5(q_B^G)^6$$

Case 2:
B wins four games on $A's$ serve and one game on $B's$ serve. The probability of this is

$$25(p_A^G)^2(q_A^G)^4(p_B^G)(q_B^G)^5$$

Case 3:
B wins three games on $A's$ serve and two games on $B's$ serve. The probability of this is

$$100(p_A^G)^3(q_A^G)^3(p_B^G)^2(q_B^G)^4$$

Case 4:
B wins two games on $A's$ serve and three games on $B's$ serve. The probability of this is

$$100(p_A^G)^4(q_A^G)^2(p_B^G)^3(q_B^G)^3$$

Case 5:
B wins one game on $A's$ serve and four games on $B's$ serve. The probability of this is

$$25(p_A^G)^5(q_A^G)(p_B^G)^4(q_B^G)^2$$

Case 6:
B wins all five games on $B's$ serve. The probability of this is

$$(p_A^G)^6(p_B^G)^5(q_B^G)$$

The probability that the score reaches $(7,5)$ is

$$(p_A^G)(q_A^G)^5(q_B^G)^6+25(p_A^G)^2(q_A^G)^4(p_B^G)(q_B^G)^5+100(p_A^G)^3(q_A^G)^3(p_B^G)^2(q_B^G)^4+100(p_A^G)^4(q_A^G)^2(p_B^G)^3(q_B^G)^3$$

$$25(p_A^G)^5(q_A^G)(p_B^G)^4(q_B^G)^2 + (p_A^G)^6(p_B^G)^5(q_B^G)$$

$(6,6)$:
Player A wins 6 out of the twelve games played and player B wins 6 out of the twelve games played.
Case 1:
The last game was won by A. In this case, the penultimate game must have been won by B. This is because, if the penultimate game is won by A, then the score would have reached $(5,6)$ at this stage. But this would mean that the score after the tenth game must have been $(4,6)$. This would mean that B would have won the set. So, the score at the eleventh game must be $(5,6)$ with B winning the game. This would mean that the score after the tenth game would have been $(5,5)$. Hence, each of the two players must win 5 games each. Therefore, the score of $(5,5)$ leading to $(6,6)$ with A winning the final game could have been reached in the following way.
Case I:
B wins all five games on $B's$ serve in the first ten games and the eleventh game on $A's$ serve as a consequence of service pattern. The probability if this is

$$(p_A^G)^5(q_A^G)(p_B^G)^5(q_B^G)$$

Case II:
B wins four games on $B's$ serve and one game on $A's$ serve in the first ten games and the eleventh game on $A's$ serve. The probability of this is

$$25(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2$$

Case III:
B wins three games on $B's$ serve and two games on $A's$ serve in the first ten games and the eleventh game on $A's$ serve. The probability of this is

$$100(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3$$

Case IV:
B wins two games on $B's$ serve and three games on $A's$ serve in the first ten games and the eleventh game on $A's$ serve. The probability of this is

$$100(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4$$

Case V:
B wins one game on $B's$ serve and four games on $A's$ serve in the first ten games and the eleventh game on $A's$ serve. The probability of this is

$$25(p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5$$

Case VI:
B wins all five games on $A's$ serve in the first ten games and the eleventh game on $A's$ serve. The probability of this is

$$(q_A^G)^6(q_B^G)^6$$

The probability that the score reaches $(6,6)$ with A winning the last point is

$$(p_A^G)^6(p_B^G)^6+25(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2+100(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3+100(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4$$

$$+25(p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5+(q_A^G)^6(q_B^G)^6$$

Case 2:
The last game was won by B. In this case, the penultimate game must have been won by A. This is because, if the penultimate game is won by B, then the score would have reached $(6,5)$ at this stage. But this would mean that the score after the tenth game must have been $(6,4)$. This would mean that A would have won the set. So, the score at the eleventh game must be $(6,5)$ with A winning the game. This would mean that the score after the tenth game would have been $(5,5)$. Hence, each of the two players must win 5 games each. Therefore, the score of $(5,5)$ leading to $(6,6)$ with B winning the final game could have been reached in the following way.

Case I:
B wins all five games on $B's$ serve in the first ten games and the twelfth game also on $B's$ serve as a consequence of service pattern. The probability of this is

$$(p_A^G)^6(p_B^G)^6$$

Case II:
B wins four games on $B's$ serve and one game on $A's$ serve in the first ten games and the twelfth game on $B's$ serve. The probability of this is

$$25(p_A^G)^5(p_B^G)^5(q_A^G)(q_B^G)$$

Case III:
B wins three games on $B's$ serve and two games on $A's$ serve in the first ten games and the twelfth game on $B's$ serve. The probability of this is

$$100(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2$$

Case IV:
B wins two games on $B's$ serve and three games on $A's$ serve in the first ten games and

the twelfth game on $B's$ serve. The probability of this is

$$100(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3$$

Case V:
B wins one game on $B's$ serve and four games on $A's$ serve in the first ten games and the twelfth game on $B's$ serve. The probability of this is

$$25(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4$$

Case VI:
B wins all five games on $A's$ serve in the first ten games and the twelfth games on $B's$ serve. The probability of this is

$$(p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5$$

The probability that the score reaching $(6,6)$ with B serving the last point is

$$(p_A^G)^6(p_B^G)^6 + 25(p_A^G)^5(q_A^G)(p_B^G)^5(q_B^G) + 100(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2 + 100(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3$$

$$+25(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4 + (p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5$$

The probability the score reaches $(6,6)$ is

$$(p_A^G)^6(p_B^G)^6+25(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2+100(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3+100(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4$$

$$+25(p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5+(q_A^G)^6(q_B^G)^6+(p_A^G)^6(p_B^G)^6+25(p_A^G)^5(q_A^G)(p_B^G)^5(q_B^G)+100(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2$$

$$+100(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3 + 25(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4 + (p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5$$

$$= 2(p_A^G)^6(p_B^G)^6 + 125(p_A^G)^4(q_A^G)^2(p_B^G)^4(q_B^G)^2 + 200(p_A^G)^3(q_A^G)^3(p_B^G)^3(q_B^G)^3$$

$$125(p_A^G)^2(q_A^G)^4(p_B^G)^2(q_B^G)^4 + 26(p_A^G)(q_A^G)^5(p_B^G)(q_B^G)^5 + (q_A^G)^6(q_B^G)^6 + 25(p_A^G)^5(q_A^G)(p_B^G)^5(q_B^G)$$

4.1 Probability of winning a seven point tie-break:

The tie-break is played in case the set reaches a score of $(6, 6)$. The player who serves the first ball of the tie-break serves once and the next ball will be served by the opponent. Starting from the second serve, each player will serve two consecutive rallies before switching serves. The first person to reach a score of 7 wins the tie-break. If the score reaches $(6, 6)$ again, the tie-break will be won by the person who reaches a two-point lead.

$(7, 0)$:
Player A wins all 7 points. Since A begins to serve, the order of service through the tie-break will be as follows:

$$ABBAABB$$

All seven points have been won by player A. The probability of this happening is

$$(p_A^R)^3(q_B^R)^4$$

$(7, 1)$:
Player A wins 7 of the eight points played and player B wins 1 of the eight points played. B cannot have won the last point as the tie-break would have been won by A at (7, 0). This means that B must have won the point on the first seven points. Since A begins to serve, the order of service through the tie-break will be as follows:

$$ABBAABBA$$

Case 1:
B won the point on $A's$ serve. The probability of this is

$$3(p_A^R)^2(q_A^R)(q_B^R)^4$$

Case 2:
B won the serve on $B's$ serve. The probability of this is

$$4(p_A^R)^4(p_B^R)(q_B^R)^3$$

The probability of the score reaching $(7, 1)$ is

$$3(p_A)^3(q_A)(q_B)^4 + 4(p_A^R)^4(p_B^R)(q_B^R)^3$$

$(7, 2)$:
Player A wins 7 of the nine points played and player B wins 2 of the nine points played. B cannot have won the last point as the tie-break would have been won by A at $(7, 1)$.

This means that B must have won both the points in the first eight points. Since A begins to serve, the order of service though the tie-break will be as follows:

$$ABBAABBAA$$

Case 1:
B won both the points on $A's$ serve. The probability of this is

$$6(p_A^R)^5(p_B^R)^2(q_B^R)^2$$

Case 2:
B wins one point on $A's$ serve and one point on $B's$ serve. The probability of this is

$$16(p_A^R)^4(q_A^R)(p_B^R)(q_B^R)^3$$

Case 3:
B wins both the points on $A's$ serve. The probability of this is

$$6(p_A^R)^3(q_A^R)^2(q_B^R)^4$$

The probability of the score reaching $(7,2)$ is

$$6(p_A^R)^5(p_B^R)^2(q_B^R)^2 + 16(p_A^R)^4(q_A^R)(p_B^R)(q_B^R)^3 + 6(p_A^R)^3(q_A^R)^2(q_B^R)^4$$

.
$(7,3)$:
Player A wins 7 of the ten points played and player B wins 3 of the ten points played. B cannot have won the last point as the tie-break would have been won by A at $(7,2)$. This means that B must have won all three points in the first nine points. Since A begins to serve, the order of service through the tie-break will be as follows:

$$ABBAABBAAB$$

Case 1:
B wins all three points on $B's$ serve. The probability of this is

$$4(p_A^R)^5(p_B^R)^3(q_B^R)^2$$

Case 2:
B wins two points on $B's$ serve and one point on $A's$ serve. The probability of this is

$$30(p_A^R)^4(q_A^R)(p_B^R)^2(q_B^R)^3$$

Case 3:

B wins one point on $B's$ serve and two points on $B's$ serve. The probability of this is

$$40(p_A^R)^3(q_A^R)^2(p_B^R)(q_B^R)^4$$

Case 4:
B wins all three points on $A's$ serve. The probability of this is

$$10(p_A^R)^2(q_A^R)^3(q_B^R)^5$$

The probability of the score reaching $(7,3)$ is

$$4(p_A^R)^5(p_B^R)^3(q_B^R)^2 + 30(p_A^R)^4(q_A^R)(p_B^R)^2(q_B^R)^3 + 40(p_A^R)^3(q_A^R)^2(p_B^R)(q_B^R)^4 + 10(p_A^R)^2(q_A^R)^3(q_B^R)^5$$

$(7,4)$:
Player A wins 7 of the eleven points played and player B wins 4 of the eleven points played. B cannot have won the last point as the tie-break would have been won by A at $(7,3)$. This means that B must have won all the four points in the first ten points. Since A begins to serve, the order of service through the tie-break will be as follows:

$$ABBAABBAABB$$

Case 1:
B wins all four points on $B's$ serve. The probability of this is

$$5(p_A^R)^5(p_B^R)^4(q_B^R)^2$$

Case 2:
B wins three points on $B's$ serve and one point on $A's$ serve. The probability of this is

$$50(p_A^R)^4(q_A^R)(p_B^R)^3(q_B^R)^3$$

Case 3:
B wins two points on $B's$ serve and two points on $A's$ serve. The probability of this is

$$100(p_A^R)^3(q_A^R)^2(p_B^R)^2(q_B^R)^4$$

Case 4:
B wins one point on $B's$ serve and three points on $A's$ serve. The probability of this is

$$50(p_A^R)^2(q_A^R)^3(p_B^R)(q_B^R)^5$$

Case 5:

B wins all four points on $A's$ serve. The probability of this is

$$5(p_A^R)(q_A^R)^4(q_B^R)^6$$

The probability that the score reaches $(7,4)$ is

$$5(p_A^R)^5(p_B^R)^4(q_B^R)^2 + 50(p_A^R)^4(q_A^R)(p_B^R)^3(q_B^R)^3 + 100(p_A^R)^3(q_A^R)^2(p_B^R)^2(q_B^R)^4$$

$$50(p_A^R)^2(q_A^R)^3(p_B^R)(q_B^R)^5 + 5(p_A^R)(q_A^R)^4(q_B^R)^6$$

.

$(7,5)$:

Player A wins 7 of the twelve points played and player B wins 5 of the twelve games played. B cannot have won the last point by A at the tie-break at $(7,4)$. This means that B must have won all the five points in the first eleven points. Since A begins to serve, the order of service will be as follows:

$$ABBAABBAABBA$$

Case 1:

B wins all five points on $B's$ serve. The probability of this is

$$6(p_A^R)^6(p_B^R)^5(q_B^R)$$

Case 2:

B wins four points on $B's$ serve and one point on $A's$ serve. The probability of this is

$$75(p_A^R)^5(q_A^R)(p_B^R)^4(q_B^R)^2$$

Case 3:

B wins three points on $B's$ serve and two points on $A's$ serve. The probability of this is

$$200(p_A^R)^4(q_A^R)^2(p_B^R)^3(q_B^R)^3$$

Case 4:

B wins two points on $B's$ serve and three points on $A's$ serve. The probability of this is

$$150(p_A^R)^3(q_A^R)^3(p_B^R)^2(q_B^R)^4$$

Case 5:

B wins one point on $B's$ serve and four points on $A's$ serve. The probability of this is

$$30(p_A^R)^2(q_A^R)^4(p_B^R)(q_B^R)^5$$

Case 6:
B wins all five points on $A's$ serve. The probability of this is

$$(p_A^R)(q_A^R)^5(q_B^R)^6$$

The probability that the score reaches $(7,5)$ is

$$6(p_A^R)^6(p_B^R)^5(q_B^R) + 75(p_A^R)^5(q_A^R)(p_B^R)^4(q_B^R)^2 + 200(p_A^R)^4(q_A^R)^2(p_B^R)^3(q_B^R)^3$$

$$150(p_A^R)^3(q_A^R)^3(p_B^R)^2(q_B^R)^4 + 30(p_A^R)^2(q_A^R)^4(p_B^R)(q_B^R)^5 + (p_A^R)(q_A^R)^5(q_B^R)^6$$

$(6,6)$:
Player A wins 6 of the twelve points played and B wins 6 of the twelve points played. In this case, it doesn't matter who wins the last game unlike in the case of sets as the player must reach 7 points to win the tie-break. So, we must calculate the probability that each player wins 6 points. Since A begins to serve, the order of service though the tie-break will be as follows:

$$ABBAABBAABBA$$

Case 1:
B wins all six points on $B's$ serve. The probability of this is

$$(p_A^R)^6(p_B^R)^6$$

Case 2:
B wins five points on $B's$ serve and one point on $A's$ serve. The probability of this is

$$36(p_A^R)^5(q_A^R)(p_B^R)^5(q_B^R)$$

Case 3:
B wins four points on $B's$ serve and two points on $A's$ serve. The probability of this is

$$225(p_A^R)^4(q_A^R)^2(p_B^R)^4(q_B^R)^2$$

Case 4:
B wins three points on $B's$ serve and three points on $A's$ serve. The probability of this is

$$400(p_A^R)^3(q_A^R)^3(p_B^R)^3(q_B^R)^3$$

Case 5:
B wins two points on $B's$ serve and four points on $A's$ serve. The probability of this is

$$225(p_A^R)^2(q_A^R)^4(p_B^R)^2(q_B^R)^4$$

Case 6:
B wins one point on $B's$ serve and five points on $A's$ serve. The probability is

$$36(p_A^R)(q_A^R)^5(p_B^R)(q_B^R)^5$$

Case 7:
B wins all six points on $A's$ serve. The probability of this is

$$(q_A^R)^6(q_B^R)^6$$

The probability that the score reaches $(6,6)$ is

$$(p_A^R)^6(p_B^R)^6+36(p_A^R)^5(q_A^R)(p_B^R)^5(q_B^R)+225(p_A^R)^4(q_A^R)^2(p_B^R)^4(q_B^R)^2+400(p_A^R)^3(q_A^R)^3(p_B^R)^3(q_B^R)^3$$

$$225(p_A^R)^2(q_A^R)^4(p_B^R)^2(q_B^R)^4+36(p_A^R)(q_A^R)^5(p_B^R)(q_B^R)^5+(q_A^R)^6(q_B^R)^6$$

Two point lead after reaching $(6,6)$:
The service order ends with a BBA as the score reaches $(6,6)$. In this case, the order of service will progress as follows:

$$ABBAABBAA\ldots$$

To reach a score of $(n+2,n)$ each player must reach a score of (n,n) and player A must win two consecutive point. This means that player A wins a point on their serve followed by player B winning a point on their serve or vice-versa j times or A losing a point on their serve followed by B losing a point on their serve or vice-versa $n-j$ times. These j points can be chosen in ${}^nC_j$ ways. Finally, player A must win a point on each of their serve consecutively to achieve a lead of two points.
This can be mathematically depicted as follows:

$$\sum_{j=0}^{n} {}^nC_j(p_A^Rp_B^R)^j(q_A^Rq_B^R)^{n-j}p_A^Rq_B^R$$

$$=p_A^Rq_B^R(p_A^Rp_B^R+q_A^Rq_B^R)^n$$

Further, the sum up to infinity of the aforementioned term must be taken as, theoretically, n can range between 0 and infinity. Hence,

$$\sum_{n=0}^{\infty} p_A^R q_B^R (p_A^R p_B^R + q_A^R q_B^R)^n$$

The term $p_A^R q_B^R$ can be taken outside the summation as it is constant. The series $(p_A^R p_B^R + q_A^R q_B^R)^n$ is a geometric progression with first term equal to 1 and the common ration equal to $p_A^R p_B^R + q_A^R q_B^R$. Hence, the sum up to infinity is equal to

$$\frac{1}{1-(p_A^R p_B^R + q_A^R q_B^R)}$$

$$= [1 - p_A^R p_B^R - q_A^R q_B^R]^{-1}$$

Hence, the probability of winning a tie-break is

$$\sum i = 0^5 p_A^T(7,i) + p_A^T(6,6) p_A^R q_B^R [1 - p_A^R p_B^R - q_A^R q_B^R]^{-1}$$

where for all $0 \leq i \leq 5$, $(7,i)$ has been calculated.

The probability that player A wins the set is equal to

$$\sum_{i=0}^{4} p_A^S(6,i) + p_A^S(7,5) + p_A^S(6,6)(\sum i = 0^5 p_A^T(7,i) + p_A^T(6,6) p_A^R q_B^R [1 - p_A^R p_B^R - q_A^R q_B^R]^{-1})$$

where for all $0 \leq 4$, $(6,i)$ and $(7,5)$ and $(6,6)$ + deuce have been calculated.

5 Calculating the probability of a player winning a match if they serve the first game:

The probability that player A wins the set

1. where A served the first and last game
2. where A served the first game and B served the last game

are used to formulate the probability that player A wins the match. We split the probability that player A wins a set into these two conditions due to the rule that if A served the last game of the first set then player B serves the first game of the second set and vice-versa.

Since the players serve alternate games, the possibilities that A serves the first and the last game are as follows:
$(6,1), (6,3)$ and $(6,6)$ + tie-break.
The possibilities that A serves the first game and B serves the last game are as follows:

$(6,0),(6,2),(6,4)$ and $(7,5)$

Hence, $p^S_{AA} + p^S_{AB} = p^S_A$. Similarly, we can also say that $p^S_{BB} + p^S_{BA} = p^S_B$.

The probability that A wins will be the sum of the probability of the two aforementioned cases. We know that the sum of the probability that A wins and the probability that A loses must be 1. Hence, we can say that

$$(p^S_{AA})(q^S_{AA})(p^S_{AB})(q^S_{AB}) = 1$$

Similarly, if B serves the first game of the set,

$$(p^S_{BB})(q^S_{BB})(p^S_{BA})(q^S_{BA}) = 1$$

We use recursive methods to calculate the probability that player A wins the match.

We say that the score of $p^M_{AA}(i,j)$ could have been reached in four ways, namely, $p^M_{AA}(i-1,j)q^S_{BA}$ or $p^M_{AB}(i-1,j)p^S_{AA}$ or $p^M_{AB}(i,j-1)q^S_{AA}$ or $p^M_{AA}(i,j-1)p^S_{BA}$. Hence,

$$p^M_{AA}(i,j) = p^M_{AA}(i-1,j)q^S_{BA} + p^M_{AB}(i-1,j)p^S_{AA} + p^M_{AB}(i,j-1)q^S_{AA} + p^M_{AA}(i,j-1)p^S_{BA}$$

- (1)

Similarly, $p^M_{AB}(i,j)$ could have been reached in four ways, namely, $p^M_{AB}(i-1,j)p^S_{AB}$ or $p^M_{AA}(i-1,j)q^S_{BB}$ or $p^M_{AB}(i,j-1)q^S_{AB}$ or $p^M_{AA}(i,j-1)p^S_{BB}$.

Initialising the scores, we have, The probability if the score reaching $p^M_{AA}(1,0) = p^S_{AA}$ and $p^M_{AB}(1,0) = p^S_{AB}$. Similarly, $p^M_{AA}(0,1) = q^S_{AA}$ and $p^M_{AB}(0,1) = q^S_{AB}$. Best two out of three:
The score must reach $(2,i)$ where $i=0$ or $i=1$. The probability of the score reaching $p^M_{AA}(1,0) = p^S_{AA}$ and $p^M_{AB}(1,0) = p^S_{AB}$.
From the aforementioned recursion formulae (1) (the last set must be won by A), we get

$$p^M_{AA}(2,0) = p^M_{AA}(1,0)q^S_{BA} + p^M_{AB}(1,0)p_{AA})^S$$

$$= p^S_{AA}q^S_{BA} + p^S_{AB}p^S_{AA}$$

$$p^M_{AB}(2,0) = p_A AB^M(1,0)p^S_{AB} + p_A AA^M(1,0)q^S_{BB}$$

$$= p^{p}_{AB}{}^S_{AB} + p^S_{AA}q^S_{BB}$$

Adding the two, we will get $p_A^M(2,0) = p_{AA}^S q_{BA}^S + p_{AB}^S p_{AA}^S + p_{AB}^S p_{AB}^S + p_{AA}^S q_{BB}^S$

$= p_{AA}^S(q_{BA}^S + q_{BB}^S) + p_{AB}^S(p_{AA}^S + p_{AB}^S)$

$= p_{AA}^S q_B^S + p_{AB}^S p_A^S$

For the previously mentioned recursion (1) formulae(the last set must be won by A), we get

$$p_{AA}(2,1) = p_{AA}^M(1,1)q_{BA}^S + p_{AB}^M(1,1)p_{AA}$$

$$p_{AA}^M(1,1) = p_{AB}^M(0,1)p_{AA}^S + p_{AA}^M(0,1)q_{BA} + p_{AB}^M(1,0)q_{AA}^S + p_{AA}(1,0)p_{BA}^S$$

$$p_{AA}^M(1,1) = q_{AB}^S p_{AA}^S + q_{AA}^S q_{BA} + p_{AB}^S q_{AA}^S + p_{AA}^S p_{BA}^S$$

$$p_{AB}^M(1,1) = p_{AB}^M(0,1)p_{AB}^S + p_{AA}^M(0,1)q_{BB}^S + p_{AB}^M(1,0)q_{AB}^S + p_{AA}(1,0)p_{BB}^S$$

$$= q_{AB}^S p_{AB}^S + q_{AA}^S q_{BB}^S + p_{AB}^S q_{AB}^S + p_{AA}^S p_{BB}^S$$

Hence, $p_{AA}^M(2,1) = p_{AA}^M(1,1)q_{BA}^S + p_{AB}^M(1,1)p_{AA}$
$= q_{AB}^S p_{AA}^S q_{BA}^S + q_{AA}^S q_{BA} q_{BA}^S + p_{AB}^S q_{AA}^S q_{BA}^S + p_{AA}^S p_{BA}^S q_{BA}^S + q_{AB}^S p_{AB}^S p_{AA} + q_{AA}^S q_{BB}^S p_{AA} + p_{AB}^S q_{AB}^S p_{AA} + p_{AA}^S p_{BB}^S p_{AA}$

$$p_{AB}^M(2,1) = p_{AB}^M(1,1)p_{AB}^S + p_{AA}^M(1,1)q_{BB}^S$$

$$p_{AA}^M(1,1) = q_{AB}^S p_{AA}^S + q_{AA}^S q_{BA} + p_{AB}^S q_{AA}^S + p_{AA}^S p_{BA}^S$$

and

$$p_{AB}^M(1,1) = q_{AB}^S p_{AB}^S + q_{AA}^S q_{BB}^S + p_{AB}^S q_{AB}^S + p_{AA}^S p_{BB}^S$$

Hence, $p_{AB}^M(2,1) = q_{AB}^S p_{AA}^S q_{BB}^S + q_{AA}^S q_{BA} q_{BB}^S + p_{AB}^S q_{AA}^S q_{BB}^S + p_{AA}^S p_{BA}^S q_{BB}^S + q_{AB}^S p_{AB}^S p_{AA} + q_{AA}^S q_{BB}^S p_{AA} + p_{AB}^S q_{AB}^S p_{AA} + p_{AA}^S p_{BB}^S p_{AA}$

Probability that the score reaches $(2, 1)$ is Hence, the probability of the score reaching $(2, i)$ where $i = 0$ or $i = 1$ is
$p^S_{AA}q^S_B + p^S_{AB}p^S_A + q^S_{AB}p^S_{AA}q^S_{BB} + q^S_{AA}q_{BA}q^S_{BB} + p^S_{AB}q^S_{AA}q^S_{BB} + p^S_{AA}p^S_{BA}q^S_{BB} + q^S_{AB}p^S_{AB}p_{AA} + q^S_{AA}q^S_{BB}p_{AA} + p^S_{AB}q^S_{AB}p_{AA} + p^S_{AA}p^S_{BB}p_{AA}$

Newton and Keller [1] conclusively prove that the decision of who serves the first game in the match does not affect the probability of winning a game, set or match. Using this result, we say that the above result for $(2, i)$ can be written as $p^M_A = (p^S_A)^2 + 2(p^S_A)^2 p^S_B$

Best three out of five:
The score must reach $(3, i)$ where $i = 0, 1, 2$.
From the aforementioned recursion formulae (1), (the last set must be won by A), we get

$$p^M_{AA}(3,0) = p^M_{AA}(2,0)q^S_{BA} + p^M_{AB}(2,0)p^S_{AA}$$

$$= (p^S_{AA}q^S_{BA} + p^S_{AB}p^S_{AA})q^S_{BA} + (p^p_{AB}{}^S_{AB} + p^S_{AA}q^S_{BB})p^S_{AA}$$

$$= p^S_{AA}q^S_{BA}q^S_{BA} + p^S_{AB}p^S_{AA}q^S_{BA} + p^p_{AB}{}^S_{AB}p^S_{AA} + p^S_{AA}q^S_{BB}p^S_{AA}$$

$$p^M_{AB}(3,0) = p^M_{AB}(2,0)p^S_{AB} + p_{AA}(2,0)q^S_{BB}$$

$$= (p^p_{AB}{}^S_{AB} + p^S_{AA}q^S_{BB})p^S_{AB} + (p^S_{AA}q^S_{BA} + p^S_{AB}p^S_{AA})q^S_{BB}$$

$$= p^p_{AB}{}^S_{AB}p^S_{AB} + p^S_{AA}q^S_{BB}p^S_{AB} + p^S_{AA}q^S_{BA}q^S_{BB} + p^S_{AB}p^S_{AA}q^S_{BB}$$

Probability of the score reaching $(3, 0)$ is

$p^S_{AA}q^S_{BA}q^S_{BA} + p^S_{AB}p^S_{AA}q^S_{BA} + p^p_{AB}{}^S_{AB}p^S_{AA} + p^S_{AA}q^S_{BB}p^S_{AA} + p^p_{AB}{}^S_{AB}p^S_{AB} + p^S_{AA}q^S_{BB}p^S_{AB} + p^S_{AA}q^S_{BA}q^S_{BB} + p^S_{AB}p^S_{AA}q^S_{BB}$

Similarly, the previously mentioned recursion formulae (1) can be used to calculate the probability of reaching a score of (3, 1) and (3, 2). Finally, we get that the probability that the score reaches $(3, i)$ where $i = 0, 1, 2$ is

$$p^M_A = (p^S_A)^3 + 3(p^S_A)^3(p^S_B) + 6(p^S_A)^3(p^S_B)^2$$

6 Results

The ATP and WTA websites provide p_A^R, that is, the probability that a player wins a rally on their serve and p_A^G, that is, the probability that a player wins a game on their serve. These probabilities are empirical values, computed from the life-time data of games played by the player. Using the empirical value for p_A^R, we ran the code on the theoretical probabilities listed in the above discussion. The resulting theoretical value of p_A^G agree closely with the corresponding career probabilities reported by ATP and WTA. The tables containing the same have been attached to the end of this paper.

We note that extensions of the work of Newton and Keller, which we have reproduced and checked above, have been used to compute the duration of professional Tennis matches [3]. Further, these methods may be also adapted to other 'game scenarios'.

7 Conclusion

We have calculated the probability that a player would win a game, set and match in a tennis tournament given the empirical probability that the player wins a rally on their serve in Tennis. We did so by reconstructing the formulas described by Newton and Keller (2005). We extracted the probability of winning a rally on their serve for the top fifty players of Association of Tennis Professionals (ATP) and Women's Tennis Association (WTA). We used the empirical probability provided by the two websites in place of the empirical probability for a specific tournament used in the paper. We computed the theoretical probability of the players winning a game on their serve. We observed that the theoretical value calculated by us closely resembled the published empirical data.

8 Appendix

Grand Slam event This includes the four main Tennis tournaments namely, Australian Open, Roland-Garros (French Open), Wimbledon and the US Open.

Scoring System In a regular tennis tournament a game, set and match mean as follows:

Game: The first player to reach 4 points and/or have a lead of two points wins the game.

Set: The first player to reach 6 games and/or have a lead of two games wins the set.
Match:

1. Three set match:
   The player who wins two out of three sets of the match wins the match.

2. Five set match:
   The player who wins three out of five sets of the match wins the match.

Deuce This is reached if the score of a game reaches (3, 3). From this point on, the first player to achieve a lead of two points wins the game.
Tie-break This is reached if the score of a set reaches (6, 6). The first player to reach a score of 7 points and/or have a two point lead wins the tie-break.

References

[1] Newton, P.K. and Keller, J.B., Probability of Winning at Tennis I. Theory and Data (2005), Massachusetts Institute of Technology.

[2] Newton, P.K., Aslam, K., Monte Carlo Tennis (2006), SIAM REVIEW, Vol.48,No.4,pp.722–742.

[3] Kovalchik, S.A., Ingram, M., Estimating the duration of professional tennis matches for varying formats (2018), J. Quant. Anal. Sports 2018; 14(1): 13–23.

[4] https://sites.math.rutgers.edu/courses/436/436-s00/Papers2000/cheng.html (This paper discusses the Problem of Points.)

Ranking	Name	prob_point	prob_game	Computed_prob_game
1	Novak Djokovic	0.67	0.86	0.860918892983435
2	Daniil Medvedev	0.66	0.83	0.845749974534688
3	Alexander Zverev	0.66	0.82	0.845749974534688
4	Stefanos Tsitsipas	0.68	0.86	0.875146382431728
5	Rafael Nadal	0.67	0.86	0.860918892983435
6	Matteo Berrettini	0.69	0.87	0.8884338858106955
7	Andrey Rublev	0.65	0.81	0.8296446444954129
8	Casper Ruud	0.65	0.83	0.8296446444954129
9	Felix Auger-Aliassime	0.66	0.82	0.845749974534688
10	Jannik Sinner	0.64	0.81	0.8126146626848665
11	Hubert Hurkacz	0.65	0.83	0.8296446444954129
12	Cameron Norrie	0.63	0.79	0.7946787458198578
13	Diego Schwartzman	0.6	0.73	0.7357292307692308
14	Denis Shapovalov	0.66	0.84	0.845749974534688
15	Roberto Bautista Agut	0.64	0.8	0.8126146626848665
16	Taylor Fritz	0.65	0.82	0.8296446444954129
17	Pablo Carreno Busta	0.63	0.78	0.7946787458198578
18	Reilly Opelka	0.71	0.9	0.9122249128131928
19	Nikoloz Basilashvili	0.6	0.73	0.7357292307692308
20	Carlos Alcaraz Garfia			
21	Lorenzo Sonego	0.65	0.82	0.8296446444954129
22	Aslan Karatsev	0.63	0.79	0.7946787458198578
23	John Isner	0.72	0.92	0.9227603166369973
24	Marin Cilic	0.66	0.84	0.845749974534688
25	Gael Monfils	0.64	0.81	0.8126146626848665
26	Karen Khachanov	0.65	0.82	0.8296446444954129
27	Cristian Garin	0.62	0.76	0.77586270623177
28	Dan Evans			
29	Roger Federer	0.7	0.89	0.9007889655172414
30	Alexander Bublik	0.63	0.79	0.7946787458198578
31	Albert Ramos-Vinolas	0.62	0.76	0.77586270623177
32	Alex de Minaur	0.64	0.79	0.8126146626848665
33	Grigor Dimitrov	0.66	0.82	0.845749974534688
34	Frances Tiafoe	0.64	0.8	0.8126146626848665
35	Llyod Harris			
36	Marton Fucsovics	0.62	0.76	0.77586270623177
37	Federico Delbonis	0.62	0.76	0.77586270623177
38	Fabio Fognini	0.6	0.72	0.7357292307692308
39	Tommy Paul	0.63	0.78	0.7946787458198578
40	Sebastian Korda	0.64	0.8	0.8126146626848665
41	Ugo Humbert	0.66	0.83	0.845749974534688
42	Kei Nishikori	0.64	0.8	0.8126146626848665
43	Alejandro Davidovich Fokina	0.6	0.71	0.7357292307692308
44	Filip Krajinović	0.62	0.76	0.77586270623177
45	Ilya Ivashka	0.64	0.78	0.8126146626848665
46	Dušan Lajović			
47	Jenson Brooksby	0.64	0.8	0.8126146626848665
48	David Goffin	0.63	0.77	0.7946787458198578
49	Benoît Paire			
50	Botic van de Zandschulp	0.65	0.81	0.8296446444954129

Ranking	Name	prob_point	prob_game	Computed_prob_game
1	Ashleigh Barty	0.706	0.941	0.907759753495446
2	Aryna Sabalenka	0.55	0.624	0.6231485024752477
3	Barbora Krejikova	0.617	0.759	0.7700510115119765
4	Paula Badosa	0.565	0.658	0.6584672644452293
5	Karolina Pliskova	0.527	0.6	0.5672058421793514
6	Maria Sakkari	0.612	0.763	0.760198097822708
7	Anett Kontaveit			
8	Iga Swiatek	0.595	0.718	0.7252058803303989
9	Garbine Muguruza	0.593	0.699	0.7209447575843081
10	Ons Jabeur	0.574	0.676	0.679086479439849
11	Danielle Collins	0.632	0.783	0.7983372678871344
12	Emma Raducanu	0.535	0.658	0.5868608461743171
13	Jelena Ostapenko	0.578	0.696	0.6880956578122669
14	Anastasia Pavlyuchenkova	0.631	0.786	0.7965124249352128
15	Elina Svitolina	0.556	0.641	0.6374070697914348
16	Victoria Azarenka	0.611	0.766	0.7582028625696868
17	Jessica Pegula	0.581	0.691	0.6947862472489357
18	Angelique Kerber	0.506	0.534	0.5149967605908864
19	Leylah Fernanez	0.631	0.798	0.7965124249352128
20	Elena Rybakina	0.669	0.855	0.8594443343176296
21	Petra Kvitova	0.596	0.705	0.7273254518583449
22	Belinda Bencic	0.603	0.739	0.7419523894547994
23	Cori gauff			
24	Tamara Zidansek	0.57	0.66	0.669980148522087
25	Veronika Kudermetova	0.612	0.775	0.760198097822708
26	Elise Mertens	0.574	0.663	0.679086479439849
27	Simona Halep	0.615	0.769	0.7661346624733508
28	Daria Kasatkina			
29	Madison Keys	0.644	0.796	0.8195365735736578
30	Camila Giorgi	0.586	0.678	0.7058058707578017
31	Sorana Cirstea			
32	Sara Sorribes Tormo	0.549	0.624	0.620756518152082
33	Clara Tauson			
34	Liudmila Samsonova	0.639	0.832	0.8108614681043153
35	Maketa Vondrousova	0.583	0.725	0.6992141114192044
36	Viktorija Golubic	0.532	0.592	0.579511019186568
37	Alize Cornet	0.559	0.641	0.6444728638146056
38	Ajla Tomljanovic	0.516	0.541	0.5399386396059646
39	Yulia Putintseva	0.52	0.565	0.5498802427910544
40	Amanda Anisimova	0.616	0.73	0.7680969997965892
41	Jil Teichmann	0.572	0.675	0.674545189150088
42	Tereza Martincova	0.575	0.652	0.6813480490716686
43	Bianca Andreescu			
44	Jasmine Paolini	0.557	0.655	0.6397672160547052
45	Camila Osorio	0.584	0.664	0.7014180973856177
46	Shelby Rogers			
47	Nuria Parrizas Diaz	0.536	0.603	0.58930473048217
48	Katerina Siniakova	0.527	0.575	0.5672058421793514
49	Anhelina Kalinina	0.525	0.578	0.5622663652392458
50	Ekaterina Alexandrova	0.565	0.678	0.6584672644452293