PSG COLLEGE OF TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS AND COMPUTATIONAL SCIENCES

M.Sc (SS) – Design and Analysis of Algorithms

Assignment -I

1. Which of the following conjectures are correct and which are incorrect? Give a short justification for each answer.
a) $5n^6 + n^3 + n^2 \log n - n = O(n^6)$
b) $10n^3 + 5n \log n = O(n^3)$
c) $\log(n^3) = O(\log n)$

2. Determine the solutions of the following recurrences in O- notation. You may assume that n is a power of some suitable constant (for example n is a power of 2 in the first recurrence). In all cases, $T(1) = 1$.
a) $T(n) = 2T(n/2) + n^3$
b) $T(n) = T(9n/10) + n$

3. Determine the exact solution of the recurrence $T(n) = T(n/4) + 1$ by using substitution. Consider $T(1) = 1$ as the base criteria and you may assume that n is a power of some suitable constant.

4. Draw the recursion tree for $T(n) = 4T(n/2) + cn$, where c is a constant, and provide a tight asymptotic bound on its solution. Verify the bound by substitution method.

5. Analyse the time complexity of the following non recursive algorithms
   a. for (i=0 ; i<n ; i++)
           m += i;

   b. for (i=0 ; i<n ; i++)
        for(j=0 ; j<n ; j++)
           sum[i] += entry[i][j];

   c. for (i=0 ; i<n ; i++)
        for(j=0 ; j<i ; j++)
           m += j;

   d. i = 1;
   while (i< n) {
           tot += i;
           i = i * 2;
   }

   e. i = n;
   while (i> 0) {
           tot += i;
           i = i / 2;
   }

   f. for (i=0 ; i<n ; i++)
     for(j=0 ; j<n ; j++)
        for(k=0 ; k<n ; k++)
           sum[i][j] += entry[i][j][k];

g. for (i=0 ; i<n ; i++)
    for(j=0 ; j<n ; j++)
        sum[i] += entry[i][j][0];

h. for (i=0 ; i<n ; i++)
    for(j=0 ; j<sqrt(n) ; j++)
        m += j;

i. Enigma(A[0..n − 1, 0..n − 1])
        //Input: A matrix A[0..n − 1, 0..n − 1] of real numbers
        fori←0 **to** n − 2 **do**
          forj ←i+ 1 **to** n − 1 **do**
                ifA[i, j] != A[j, i]
                    return false
        return true

6. Solve the recurrence $T(n) = 2T(n/2 + 17) + n$ using substitution method.

7. Use limits to compare the order of growth of

   (a) $3x^2$ and x^2+5
   (b) x and ln x

8. Show that $5n^2-6n=\Theta(n^2)$.
9. For each of the following recurrences, give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.
   1. $T(n) = 3T(n/2)+ n^2$
   2. $T(n) = 4T(n/2)+ n^2$
   3. $T(n) = T(n/2) + 2^n$
10. Use recursion tree to solve $T(n) = T(n/3)+T(2n/3)+cn$