* DISCRETE CONVOLUTION

w.r.t convolution of cts fn is

$$f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du.$$

suppose we need convolution for discrete functions,

$x(n)$ + $y(n)$ are 2 finite sequences then

their convolution

$$x(n) * y(n) = \sum_{r=0}^{n} x(r)\, y(n-r)$$

$$z(n) = x(n) * y(n)$$

duration of sequence $x(n) = N1$

$y(n) = N2$

$z(n) = N1 + N2 - 1$

$x(n) = \{x(0), x(1), \ldots\ldots x(N_1 - 1)\}$

$y(n) = \{y(0), y(1) \ldots\ldots y(N_2 - 1)\}$

$$z(n) = x(0)y(n) + x(1)y(n-1) + \cdots x(n)y(0)$$

$$z(0) = x(0)y(0)$$

$$z(1) = x(0)y(1) + x(1)y(0)$$

$$z(2) = x(0)y(2) + x(1)y(1) + x(2)y(0)$$

$$z(3) = x(0)y(3) + x(1)y(2) + x(2)y(1) + x(3)y(0)$$

$$z(4) = x(0)y(4) + x(1)y(3) + x(2)y(2) + x(3)y(1) + x(4)y(0)$$

$$z(5) = x(0)y(5) + x(1)y(4) + x(2)y(3) + x(3)y(2) + x(4)y(1) + x(5)y(0)$$

$$z(6) = x(0)y(6) + x(1)y(5) + x(2)y(4) + x(3)y(3) + x(4)y(2) + x(5)y(1) + x(6)y(0)$$

* <u>periodic sequence</u>

$x(n)$ is periodic if $x(n+N) = x(n)$ $P = N$

Here, periodicity of $x(n), y(n) = N$

$\Rightarrow P\{z(n)\} = N$

$$z(n) = \sum_{r=0}^{N-1} x(r)\,y(n-r)$$ periodic/ circular/ cyclic convolution

$$z(n) = x(0)y(n) + x(1)y(n-1) + x(2)y(n-2) + \cdots x(N-1)y(n-N+1)$$

$$z(0) = x(0)y(0) + x(1)y(N-1) + x(2)y(N-2) + x(3)y(N-3)$$

* Methods to find circular convolution

1) MMM (Matrix mul method).

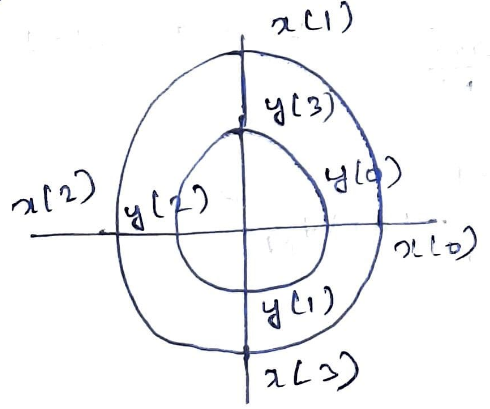
suppose $N=4$, $\{x[n]\} = \{x(0), x(1), x(2), x(3)\}$

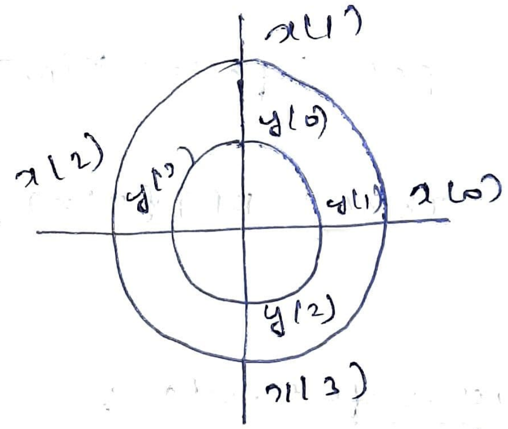
$\{y(n)\} = \{y(0), y(1), y(2), y(3)\}$

$z(n) = x(n) \circledast y(n)$ $\quad \{z(n)\} = z(0), z(1), z(2), z(3)$

$$\begin{pmatrix} z(0) \\ z(1) \\ z(2) \\ z(3) \end{pmatrix} = \begin{pmatrix} x(0) & x(3) & x(2) & x(1) \\ x(1) & x(0) & x(3) & x(2) \\ x(2) & x(1) & x(0) & x(3) \\ x(3) & x(2) & x(1) & x(0) \end{pmatrix} \begin{pmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{pmatrix}$$

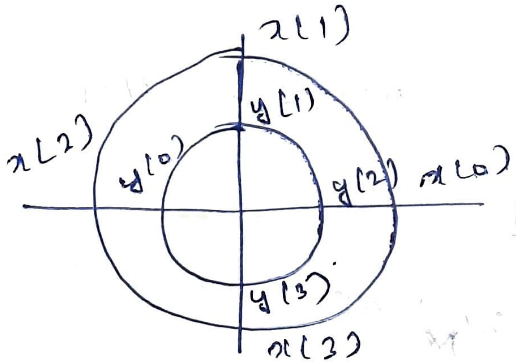
2) circular rep method (CRM)

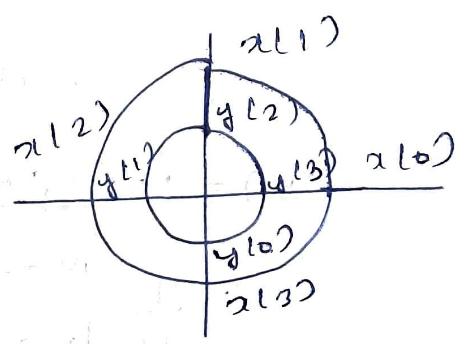




$z(0) = x(0)y(0) + x(1)y(3) + x(2)y(2) + x(3)y(1)$

$z(1) = x(0)y(1) + x(1)y(0) + x(2)y(3) + x(3)y(2)$





$z(2) = x(0)y(2) + x(1)y(1) + x(2)y(0) + x(3)y(3)$

$z(4) = x(0)y(3) + x(1)y(2) + x(2)y(1) + x(3)y(0)$

* circular convolution for $n=4$

$$z(0) = x(0)y(0) + x(1)y(3) + x(2)y(2) + x(3)y(1).$$

$$z(1) = x(0)y(1) + x(1)\cdot y(0) + x(2)y(3) + x(3)y(2)$$

$$z(2) = x(0)y(2) + x(1)y(1) + x(2)y(0) + x(3)y(3)$$

$$z(3) = x(0)y(3) + x(1)y(2) + x(2)y(1) + x(3)y(0)$$

-1

-2 $\quad$ 3

* <u>Discrete fourier Transform</u>

suppose $\{x(n)\}$ is finite sequence of duration N, Then discrete fourier Transform of $x(n)$ is denoted by $DFT\{x(n)\}$

$$DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{2\pi i \times nk}{N}} = X(k)$$

$$k = 0, 1 \ldots N-1$$

$$\text{phase factor } W_N = e^{-\frac{2\pi i}{N}}$$

$$\text{Thus, } DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

<u>Inverse formula</u> :

$$IDFT\{X(k)\} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{2\pi i \times nk}{N}}$$

$$= x(n) \qquad n = 0, 1, \ldots N-1$$

$$x(n) = IDFT\{X(k)\}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-nk}$$

* Properties

$$\frac{2}{\sqrt{3}} \pi^{-\frac{1}{4}} (1-t^2) e^{-\frac{t^2}{2}}$$

1. Linear property:

$$DFT\{c_1 x(n) + c_2 y(n)\} = c_1 DFT\{x(n)\} + c_2 DFT\{y(n)\}$$

$$= c_1 \sum_{n=0}^{N-1} \{x(n)\} W_N^{nk} + c_2 \sum_{n=0}^{N-1} \{y(n)\} W_N^{nk}$$

2. Shifting property:

$m = N-1$ if periodic

$$DFT\{x(n-m)\} = W_N^{mk} \, DFT\{x(n)\}$$ if

$x(n)$ is periodic with period $= N$

$$DFT\{x(n-m)\} = \sum_{n=0}^{N-1} x(n-m) W_N^{nk}.$$

3. Convolution property:

$$DFT\{x(n) * y(n)\} = DFT\{x(n)\} \times DFT\{y(n)\}$$

1, 2, 2, 1

DFT example problem

DFT{1, -1, 1 -1}

Here, $x(n) = 1, -1, 1, -1$

$$DFT\{x(n)\} = \sum_{n=0}^{3} x(n) W_4^{nk}$$ $W_4 = -i$

$$X(k) = x(0) + x(1) W_4^{k} + x(2) W_4^{2k} + x(3) W_4^{3k}$$

$$X(0) = x(0) + x(1) + x(2) + x(3)$$

$$x(1) = x(0) + x(1) W_4^{1} + x(2) W_4^{2} + x(3) W_4^{3}$$

⋮