Linear programming Problem(LPP)

15 January 2021 18:48

- To Solve Problems with linear objective and Constraints.

  (Objective and Constraints are Linear equations)
- General mathematical model of LPP is of the form

Maximize/Minimize
$$Z = C_1 x_1 + C_2 x_2 + \cdots + C_n x_n$$

$c_1, c_2, \ldots c_n$ – Cost/profit Coefficients

$x_1, x_2, \ldots x_n$ – decision Variables
x_i's are non negative

s.t.

$$\text{Constraints}\begin{cases} a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \ (\le\ =\ \ge)\ b_1 \\ a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \ (\le\ =\ \ge)\ b_2 \\ \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \ (\le\ =\ \ge)\ b_m \end{cases}$$

b_i – available resources
$(i = 1, 2, \ldots m)$

a_{ij} – Requirement
$\begin{pmatrix} i = 1, 2, \ldots m \\ j = 1, 2, \ldots n \end{pmatrix}$

decision Variable

eg. Maximize $Z = \underbrace{2x_1 + 3x_2}$ } objective fn.
s.t.

$$\left.\begin{array}{r} x_1 + x_2 \le 5 \\ 2x_1 - x_2 \le 3 \end{array}\right\} \text{Constraint}$$

$$x_1, x_2 \ge 0 \Big\} \text{Non negativity Constraint}$$

Feasible Solution

- The values which satisfies the Constraints are called <u>feasible Solution</u>.

Optimal Solution

Feasible Solution which maximizes or minimizes the objective function is called <u>Optimal Solution</u>.

Steps in LPP

1) Formulation of LPP
2) Solution methods of LPP model.

Formulation of LPP

Example 1 : (Reddy Mikks Product mix Problem)

Reddy Mikks Produces both interior and exterior paints from two raw materials M_1 & M_2. The following table provides the basic data of the problem.

		Tons Ext. Paint	Int. Paint	Max daily availability
Raw	M_1	[illegible]	[illegible]	[illegible]

Material ...	6	4	24
Raw Material M2	1	2	6
Profit per ton (in $1000's)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than one ton. Also the maximum daily demand for interior paint is 2 tons.

Reddy Mikks wants to determine the optimum product mix of interior and exterior paints that maximizes the total daily profit.

LPP Model Formulation

Step 1 : Decision Variables

x_1 : Tons Produced of exterior paint daily.

x_2 : Tons Produced of Interior Paint daily.

Step 2 : Objective function

Profit Per ton (in $1000) for exterior paint is 5 and interior Paint is 4. We want to Maximize Profit.

Maximize $Z = 5x_1 + 4x_2$

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Step 3 : Constraints

Two resources : M1 & M2

For M1 :

Total Max availability/day = 24

Requirement : 6 tons of exterior
4 tons of Interior

(ie) $6x_1 + 4x_2 \leq 24$

For M2 : $x_1 + 2x_2 \leq 6$

Based on Daily Demand

(i) $x_2 - x_1 \leq 1$

(ie) $-x_1 + x_2 \leq 1$

(ii) $x_2 \leq 2$

Step 4 : Non negativity Constraints

$$x_1, x_2 \geq 0$$

∴ LPP model is

$$\boxed{\begin{array}{l} \text{Max } Z = 5x_1 + 4x_2 \\ \text{s.t.} \\ 6x_1 + 4x_2 \leq 24 \\ x_1 + 2x_2 \leq 6 \\ -x_1 + x_2 \leq 1 \end{array}}$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$