SOLVING DIFFERENCE EQUATION

* Solve using z-Transform

1) $y_{n+1} + y_n = 1, \; y_0 = 0$

$$y_{n+1} + y_n - 1 = 0$$

$$Z\{y_{n+1}\} + Z\{y_n\} - Z\{1\} = 0$$

$$zZ\{y_n\} + Z\{y_n\} - z y_0 - \frac{z}{z-1} = 0$$

$$Z\{y_n\}[z+1] = \frac{z}{z-1} \Rightarrow Z\{y_n\} = \frac{z}{(z-1)(z+1)}$$

$$x(z) = \frac{z}{(z-1)(z+1)} \qquad \frac{x(z)}{z} = \frac{1}{(z-1)(z+1)}$$

$$\frac{x(z)}{z} = \frac{1}{2}\left[\frac{z+1-(z-1)}{(z-1)(z+1)}\right] = \frac{1}{2}\left[\frac{1}{z-1} - \frac{1}{z+1}\right]$$

$$x(z) = \frac{1}{2}\left[\frac{z}{z-1} - \frac{z}{z+1}\right]$$

$$Z^{-1}[x(z)] = Z^{-1}\left[\frac{1}{2}\frac{z}{z-1}\right] - Z^{-1}\left[\frac{1}{2}\frac{z}{z+1}\right]$$

$$y_n = \frac{1}{2}\left[1 - (-1)^n\right]$$

2) $y_{n+2} + 6y_{n+1} + 9y_n = 2^n, \; y_0 = 0, \; y_1 = 0$

$$Z\{y_{n+2}\} + 6Z\{y_{n+1}\} + 9Z\{y_n\} - Z\{2^n\} = 0$$

$$\left.\begin{array}{l} z^2 Z\{y_n\} - z^2 y_0 - z y_1 + 6[zZy(n) - z y_0] \\ \quad + 9Z\{y_n\} - \frac{z}{z-2} \end{array}\right] = 0$$

Madhumitha

$$(z^2+6z+9)\,Z\{y(n)\} = \frac{z}{z-2}$$

$$Z\{y(n)\} = \frac{z}{(z-2)(z+3)^2}$$

$$y(n) = Z^{-1}\left[\frac{z}{(z-2)(z+3)^2}\right] = x(z)$$

thus, $\dfrac{x(z)}{z} = \dfrac{1}{(z-2)(z+3)^2}$

$$= \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$1 = A(z+3)^2 + B(z+3)(z-2) + C(z-2)$$

$$1 = A(z^2+6z+9) + B(z^2-z-6) + C(z-2)$$

$$z^2(A+B) + z(6A+B+C) + 9A - 6B - 2C = \cancel{0}\,1$$

$A+B=0$	$6A+B+C=0$	$9A-6B-2C=1$
$A=-B$	$-6B+B+C=0$	$-9B-6B-2C=1$
	$C-5B=0$	$-15B-2C=1$
	$C=5B$	$-15B-10B=1$
$\boxed{A=1/25}$	$\boxed{C=-1/5}$	$\boxed{B=-1/25}$

$$\frac{x(z)}{z} = \frac{1}{25(z-2)} - \frac{1}{25(z+3)} - \frac{1}{5(z+3)^2}$$

$$x(z) = \frac{z}{25(z-2)} - \frac{z}{25(z+3)} - \frac{z}{5(z+3)^2}$$

$$Z^{-1}[x(z)] = \frac{1}{25}Z^{-1}\left[\frac{z}{z-2}\right] - \frac{1}{25}Z^{-1}\left[\frac{z}{z+3}\right]$$
$$-\frac{1}{5}Z^{-1}\left[\frac{z}{(z+3)^2}\right]$$

$$y(n) = \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n - \frac{1}{5}(-3)^{n-1} \times n$$

3) $y_{n+2} + y_n = 5 \cdot 2^n, \ y_0 = 1, \ y_1 = 0$

$$z\{y_{n+2}\} + z\{y_n\} = z\{5 \cdot 2^n\}$$

$$z^2 z\{y_n\} - z^2 y_0 - z y_1 + z\{y_n\} = \frac{5 \cdot z}{z-2}$$

$$z^2 z\{y_n\} - z^2 + z\{y_n\} = \frac{5z}{z-2}$$

$$z\{y_n\}[z^2+1] = \frac{5z}{z-2} + z^2$$

$$z\{y_n\} = \frac{5z + z^3 - 2z^2}{(z^2+1)(z-2)}$$

$$\frac{x(z)}{z} = \frac{5 + z^2 - 2z}{(z^2+1)(z-2)}$$

$$\frac{x(z)}{z} = \frac{A}{z-2} + \frac{Bz+C}{z^2+1} = \frac{5+z^2-2z}{(z^2+1)(z-2)}$$

$$A(z^2+1) + Bz+C(z-2) = 5+z^2-2z$$

$z=2$; $A(5) = 5+4-4 \Rightarrow \boxed{A=1}$

$z=0$; $A(1) + C(-2) = 5 \Rightarrow \boxed{C=-2}$

$z=1$; $A(2) + (B-2)(1-2) = 5+1-2$

$\cancel{2} - B + \cancel{2} = \cancel{2} + \cancel{2} \Rightarrow \boxed{B=0}$

$$\frac{x(z)}{z} = \frac{1}{z-2} - \frac{2}{z^2+1}$$

$$x(z) = \frac{z}{z-2} - \frac{2z}{z^2+1}$$

$$z^{-1}[x(z)] = z^{-1}\left[\frac{z}{z-2}\right] - 2z^{-1}\left[\frac{z}{z^2+1}\right]$$

$$z^{-1}[x(z)] = \boxed{2^n - 2\sin\frac{n\pi}{2}}$$

Madhumitha

4) $y_{n+2} - 7y_{n+1} + 12y_n = 2^n, \quad y_0 = 0, y_1 = 0$

$$Z\{y_{n+2}\} - 7Z\{y_{n+1}\} + 12Z\{y_n\} = Z\{2^n\}$$

$$\left.\begin{array}{l} z^2 Z\{y_n\} - z^2 y_0 - z y_1 - 7\{z Z\{y_n\} \\ \quad + z y_0\} + 12 Z\{y_n\} - \dfrac{z}{z-2} \end{array}\right] = 0$$

$$(z^2 - 7z + 12) Z\{y_n\} = \frac{z}{z-2}$$

$$Z\{y_n\} = \frac{z}{(z-2)(z^2-7z+12)}$$

$$y_n = Z^{-1}\left[\frac{z}{(z-2)(z^2-7z+12)}\right]$$

$$z^2 - 7z + 12 = (z-a)(z-b)$$

$$x(z) = \frac{z}{(z-2)(z-a)(z-b)}$$

$$\frac{x(z)}{z} = \frac{1}{(z-2)(z-a)(z-b)} = \frac{A}{z-2} + \frac{B}{z-a} + \frac{C}{z-b}$$

$$1 = A(z-a)(z-b) + B(z-2)(z-b) + C(z-2)(z-a)$$

$z=2$; $\quad 1 = A(2-a)(2-b)$

$z=a$; $\quad 1 = B(a-2)(a-b)$

$z=b$; $\quad 1 = C(b-a)(b-2)$

$$x(z) = \frac{1}{(2-a)(2-b)}\left[\frac{z}{z-2}\right] + \frac{1}{(a-2)(a-b)}\left[\frac{z}{z-a}\right] + \frac{1}{(b-a)(b-2)}\left[\frac{z}{z-b}\right]$$

$$z^{-1}[x(z)] = \frac{1}{(2-a)(2-b)} 2^n + \frac{1}{(a-2)(a-b)} a^n$$

$$+ \frac{1}{(b-a)(b-2)} b^n$$

$z^2 - 76z + 12 \Rightarrow a = 38 + 2\sqrt{358}$

$b = 38 - 2\sqrt{358}$

$$y_n = \frac{1}{(-36+2\sqrt{358})(-36-2\sqrt{358})} \times 2^n +$$

$$\frac{1}{(36+2\sqrt{358})(4\sqrt{358})} \times (38+2\sqrt{358})^n +$$

$$\frac{1}{(-4\sqrt{358})(36-2\sqrt{358})} \times (38-2\sqrt{358})^n$$

Thus,

$$y_n = \frac{-1}{36}(2^n) + \frac{(38+2\sqrt{358})^n}{(36+2\sqrt{358})(4\sqrt{358})}$$

$$+ \frac{(38-2\sqrt{358})^n}{(-4\sqrt{358})(36-2\sqrt{358})}$$

5) $y_{n+2} + 5y_{n+1} + 6y_n = 2^n$, $y_0 = 0$, $y_1 = 0$

$$\left.\begin{array}{l} z\{y_{n+2}\} + 5z\{y_{n+1}\} + \\ 6z\{y_n\} = z/z-2 \end{array}\right\} = 0$$

Madhumitha

$$z^2 Z\{y(n)\} - z^2 y_0 - z y_1 + 5[z Z\{y(n)\} - z y_0] + 6 Z\{y_n\} = \frac{z}{z-2}$$

$$(z^2+5z+6)\, Z\{y_n\} = \frac{z}{z-2}$$

$$Z\{y_n\} = \frac{z}{(z-2)(z+2)(z+3)}$$

$$\frac{x(z)}{z} = \frac{1}{(z-2)(z+2)(z+3)}$$

$$= \frac{A}{z-2} + \frac{B}{z+2} + \frac{C}{z+3}$$

$$1 = A(z+2)(z+3) + B(z-2)(z+3) + C(z-2)(z+2)$$

when $z=2$; $1 = A(4)(5) \Rightarrow \boxed{A = 1/20}$

when $z=-2$; $1 = B(-4)(1) \Rightarrow \boxed{B = -1/4}$

when $z=-3$; $1 = C(-5)(-1) \Rightarrow \boxed{C = 1/5}$

$$x(z) = \frac{1}{20}\frac{z}{z-2} - \frac{1}{4}\frac{z}{z+2} + \frac{1}{5}\frac{z}{z+3}$$

$$Z^{-1}[x(z)] = \frac{1}{20}2^n - \frac{1}{4}(-2)^n + \frac{1}{5}(-3)^n$$

Thus, $y(n) = \dfrac{2^n}{20} - \dfrac{(-2)^n}{4} + \dfrac{(-3)^n}{5}$

Madhumitha