FFT → algorithm to compute DFT

FFT: decimation in Time (DIT), DIF

$$\boxed{\begin{array}{l} DFT - N^2 \\ FFT - N\log_2 N \end{array}}$$

DIT Algorithm

$$W_N = e^{\frac{-2\pi i}{N}}$$

PHASE FACTORS:

$W_2 = -1$

$W_4^{1} = -i$

$W_4^{-1} = i$

$W_8^{1} = 1 - i/\sqrt{2}$

$W_8^{2} = -i$

$W_8^{3} = -1 - i/\sqrt{2}$

$W_8^{-1} = 1 + i/\sqrt{2}$

$W_8^{-2} = i$

$W_8^{-3} = -1 + i/\sqrt{2}$

FFT → requires $N\log_2 N$ no: of operations

DFT → N^2

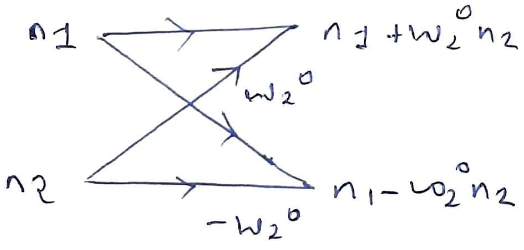
$2i + \sqrt{2}(1-i)$

$2i - \sqrt{2} + \sqrt{2}i$

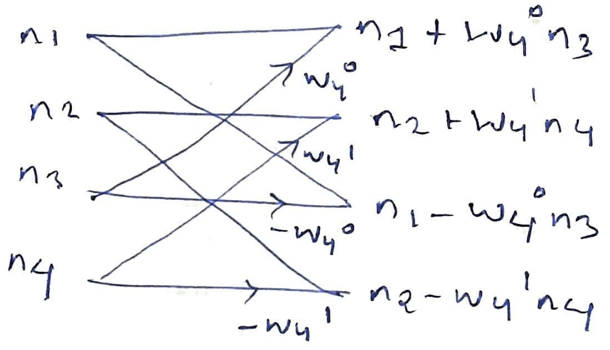
$-\sqrt{2} + i(2+\sqrt{2})$

* DIT Algorithm

i) 2 point formula



3) 4 - point formula



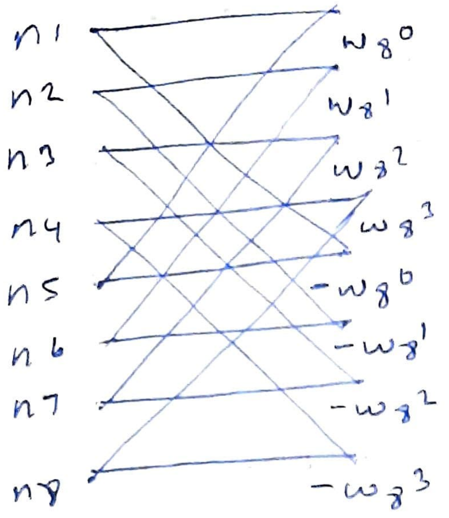
$n_1 + n_3$

$n_2 - i n_4$

$n_1 - n_3$

$n_2 + i n_4$

3) 8-point:



$n1 + n5$

$n2 + W_8^1 n6$

$n3 + W_8^2 n7$

$n4 + W_8^3 n8$

$n1 - n5$

$n2 - W_8^1 n6$

$n3 - W_8^2 n7$

$n4 - W_8^3 n8$

$n1 + n5$

$n2 + \frac{n6}{\sqrt{2}} - \frac{i n6}{\sqrt{2}}$

$n3 - i n7$

$n4 - \frac{n8}{\sqrt{2}} - \frac{i n8}{\sqrt{2}}$

$n1 - n5$

$n2 - \left(\frac{1-i}{\sqrt{2}}\right) n6$

$n3 + i n7$

$n4 + \left(\frac{1+i}{\sqrt{2}}\right) n8.$

* Inverse: → -ve power phase factor
→ Final ans divide by N

2 point: $n1$ → $n1 + W_2^0 n2$

$n2$ → $n1 - W_2^0 n2$

4-point: $n1$	$n1 + W_4^0 n3$	$n1 + n3$
$n2$	$n2 + W_4^{-1} n4$	$n2 + i n4$
$n3$	$n1 - W_4^0 n3$	$n1 - n3$
$n4$	$n2 - W_4^{-1} n4$	$n2 - i n4$

8-point:

$n1$	$n1 + W_8^0 n5$	$n1 + n5$
$n2$	$n2 + W_8^{-1} n6$	$n2 + \frac{(1+i)}{\sqrt{2}} n6$
$n3$	$n3 + W_8^{-2} n7$	$n3 + i n7$
$n4$	$n4 + W_8^{-3} n8$	$n4 + \frac{(-1+i)}{\sqrt{2}} n8$
$n5$	$n1 - W_8^0 n5$	$n1 - n5$
$n6$	$n2 - W_8^{-1} n6$	$n2 - \frac{(1+i)}{\sqrt{2}} n6$
$n7$	$n3 - W_8^{-2} n7$	$n3 - i n7$
$n8$	$n4 - W_8^{-3} n8$	$n4 - \frac{(-1+i)}{\sqrt{2}} n8$

* DIF Algorithm

2 point:

$n1$	1	$n1+n2$
$n2$	-1	$n1-n2$

4-point

$n1$	$n1+n3$	
$n2$	$n2+n4$	
$n3$	n_1-n3	$\longrightarrow (n1-n3)W_4^0$
$n4$	n_2-n4	$\longrightarrow (n2-n4)W_4^1$
		$(n2-n4)-i$

8-point:

$n1$	$n1+n5$	
$n2$	$n2+n6$	
$n3$	$n3+n7$	
$n4$	$n4+n8$	
$n5$	$(n_1-n5)W_8^0$	(n_1-n5)
$n6$	$(n_2-n6)W_8^1$	$(n2-n6)1-i/\sqrt{2}$
$n7$	$(n3-n7)W_8^2$	$(n3-n7)-i$
$n8$	$(n4-n8)W_8^3$	$(n4-n8)-1-i/\sqrt{2}$

* DIF Inverse

2 point:

$n1$	1	$n1+n2$
$n2$	-1	n_1-n2

4-point:

$n1$	$n1+n3$
$n2$	$n2+n4$
$n3$	$(n_1-n3)w_4^0$
$n4$	$(n_1-n_4)w_4^{-1} = (n_1-n4)i$

8-point: n_1 n_1+n_5
n_2 n_2+n_6
n_3 n_3+n_7
n_4 n_4+n_8
n_5 $(n_1-n_5)w_8^0$
n_6 $(n_2-n_6)w_8^{-1} = (n_2-n_6)\,1+i/\sqrt{2}$
n_7 $(n_3-n_7)w_8^{-2} = (n_3-n_7)\,i$
n_8 $(n_4-n_8)w_8^{-3} = (n_4-n_8)\frac{-1+i}{\sqrt{2}}$

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* DISCRETE CONVOLUTION

w.k.t convolution of cts fn is

$$f(t) * g(t) = \int_0^t f(u)\,g(t-u)\,du.$$

suppose we need convolution for discrete functions,

$x(n)$ & $y(n)$ are 2 finite sequences then their convolution

$$x(n) * y(n) = \sum_{r=0}^{n} x(r)\,y(n-r)$$

$$z(n) = x(n) * y(n)$$

duration of sequence $x(n) = N1$