4/9/20

TRANSFORM OF INTEGRALS AND INTEGRALS OF TRANSFORM

* Find Inverse Laplace Transform

1. $\dfrac{1}{s^2+\frac{s}{4}}$

W.K.T $L^{-1}\left[\dfrac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]\,dt$

$$L^{-1}\left[\frac{1}{s(s+\frac{1}{4})}\right] = L^{-1}\left[\frac{1/(s+1/4)}{s}\right]$$

$$= \int_0^t L^{-1}\left[\frac{1}{s+\frac{1}{4}}\right]dt$$

$$= \int_0^t e^{-1/4t}\,dt = \left.\frac{e^{-1/4t}}{-1/4}\right]_0^t$$

$$= \boxed{4-4e^{-1/4t}}$$

2. $\dfrac{1}{s^3-9s^2}$

W.K.T $L^{-1}\left[\dfrac{F(s)}{s^2}\right] = \int_0^t\int_0^t L^{-1}[F(s)]\,dt.dt$

$$\therefore L^{-1}\left[\frac{1/s-9}{s^2}\right] = \int_0^t\int_0^t L^{-1}\left[\frac{1}{s-9}\right]dt.dt$$

$$= \int_0^t\int_0^t e^{9t}\,dt.dt = \int_0^t \left.\frac{e^{9t}}{9}\right]_0^t dt$$

$$= \int_0^t \frac{e^{9t}}{9} - \frac{1}{9}\,dt$$

$$= \frac{1}{9}\int_0^t (e^{9t}-1)\,dt = \frac{1}{9}\left[\frac{e^{9t}}{9}-t\right]_0^t$$

$$= \frac{1}{9}\left(\frac{e^{9t}-1-9t}{9}\right)$$

$$= \boxed{\frac{e^{9t}-1-9t}{81}}$$

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3. $\dfrac{s}{s^3-5s}$

$$F(s) = \frac{s}{s^2-5}$$

$$L^{-1}[F(s)] = \sqrt{5}\sinh\sqrt{5}t$$

$$L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]\,dt$$

$$L^{-1}\left[\frac{s/s^2-5}{s}\right] = \int_0^t L^{-1}\left[\frac{s}{s^2-5}\right]dt$$

$$= \int_0^t \sqrt{5}\sinh\sqrt{5}t\cdot dt$$

$$= \frac{\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}}\cosh\sqrt{5}t\Big]_0^t$$

$$= \boxed{\cosh\sqrt{5}t - 1}$$

4. $\dfrac{1}{s^4-4s^2}$

$$F(s) = \frac{1}{s^2-4}$$

$$L^{-1}\left[\frac{F(s)}{s^2}\right] = L^{-1}\left[\frac{1/s^2-4}{s^2}\right] = \int_0^t\int_0^t L^{-1}[F(s)]\,dt\,.dt$$

$$= \int_0^t\int_0^t L^{-1}\left[\frac{1}{s^2-4}\right]dt\,.dt$$

$$= \int_0^t\int_0^t \frac{\sinh 2t}{2}\,dt = \frac{1}{2}\int_0^t \frac{\cosh 2t}{2}\Big]_0^t dt$$

$$= \frac{1}{4}\int_0^t \cosh 2t - 1\,dt$$

$$= \frac{1}{4}\left[\frac{\sinh 2t}{2} - t\right]_0^t = \boxed{\frac{1}{8}\sinh 2t - \frac{t}{4}}$$

$$= \frac{1}{8}\left[\frac{e^{2t}-e^{-2t}}{2} - \frac{t}{4}\right]$$

$$= \boxed{\frac{2e^{2t}-2e^{-2t}-t}{8}}$$

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5) $\dfrac{2}{s^3+9s}$

$F(s) = \dfrac{2}{s^2+9}$; $L^{-1}[F(s)] = \dfrac{2}{3}\sin 3t$

$L^{-1}\left[\dfrac{F(s)}{s}\right] = \dfrac{2}{3}\int_0^t L^{-1}\left[\dfrac{3}{s^2+9}\right] dt$

$= \dfrac{2}{3}\int_0^t \sin 3t\, dt = \dfrac{2}{9}(-\cos 3t)_0^t$

$= \boxed{\dfrac{2}{9}(1-\cos 3t)}$

* Find Laplace Transform

1) $\dfrac{1-e^t}{t}$

W.K.T $L\left\{\dfrac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds$

$L\left\{\dfrac{1-e^t}{t}\right\} = \int_s^\infty L(1-e^t)\, ds$

$= \int_s^\infty \dfrac{1}{s} - \dfrac{1}{s-1}\, ds$

$= [\log s - \log(s-1)]_s^\infty$

$= \log\left(\dfrac{s}{s-1}\right)_s^\infty$

$= \left[\log\left(\dfrac{1}{1-1/s}\right)\right]_s^\infty$

$= \log 1 - \log\left(\dfrac{1}{1-1/s}\right)$

$= -\log\left(\dfrac{s}{s-1}\right)$

$= \boxed{\log\left(\dfrac{s-1}{s}\right)}$

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2) $\dfrac{2-2\cosh t}{t}$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$$

$$L\left\{\frac{2-2\cosh t}{t}\right\} = \int_s^\infty L(2-2\cosh t)\,ds$$

$$= \int_s^\infty \frac{2}{s} - \frac{2s}{s^2-1}\,ds$$

$$= \left[2\log s - \log(s^2-1)\right]_s^\infty$$

$$\left(\because \frac{2s}{s^2-1} = \frac{dt}{t}\right)$$

$$= \log\left(\frac{s^2}{s^2-1}\right)\Bigg]_s^\infty$$

$$= \log\left(\frac{1}{1-1/s^2}\right)_s^\infty$$

$$= \log\left(\frac{1}{1-0}\right) - \log\left(\frac{1}{1-1/s^2}\right)$$

$$= -\log\left(\frac{s^2}{s^2-1}\right) = \boxed{\log\left(\frac{s^2-1}{s^2}\right)}$$

3) $2\left(\dfrac{1-\cos t}{t}\right)$

$$L\left\{\frac{f(t)}{t}\right\} = L\left\{\frac{2(1-\cos t)}{t}\right\} = \int_s^\infty 2\left(\frac{1}{s} - \frac{s}{s^2+1}\right)ds$$

$$= 2\log s - \log(s^2+1)\Big]_s^\infty$$

$$= \log\left[\frac{s^2}{s^2+1}\right]_s^\infty = \log\left(\frac{1}{1+1/s^2}\right)_s^\infty$$

$$= \log 1 - \log\left(\frac{s^2}{s^2+1}\right)$$

$$= \boxed{\log\left(\frac{s^2+1}{s^2}\right)}$$

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4) $\frac{e^t}{t}$

Here, $\lim_{t \to 0} \frac{e^t}{t} = \frac{e^0}{0} = \frac{1}{0} = \infty$

$\therefore$ limit does not exist

$\Rightarrow$ Laplace transform does not exist.

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