Assignment Problem

16 March 2021 09:30

- It is a special case of Transportion problem

- Assignment model is "Best Person for the job"

- The objective of assignment model is to determine minimum cost assignment of workers to jobs.

- It is a special case of Transportation Problem.

- It is an LP model defined as

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

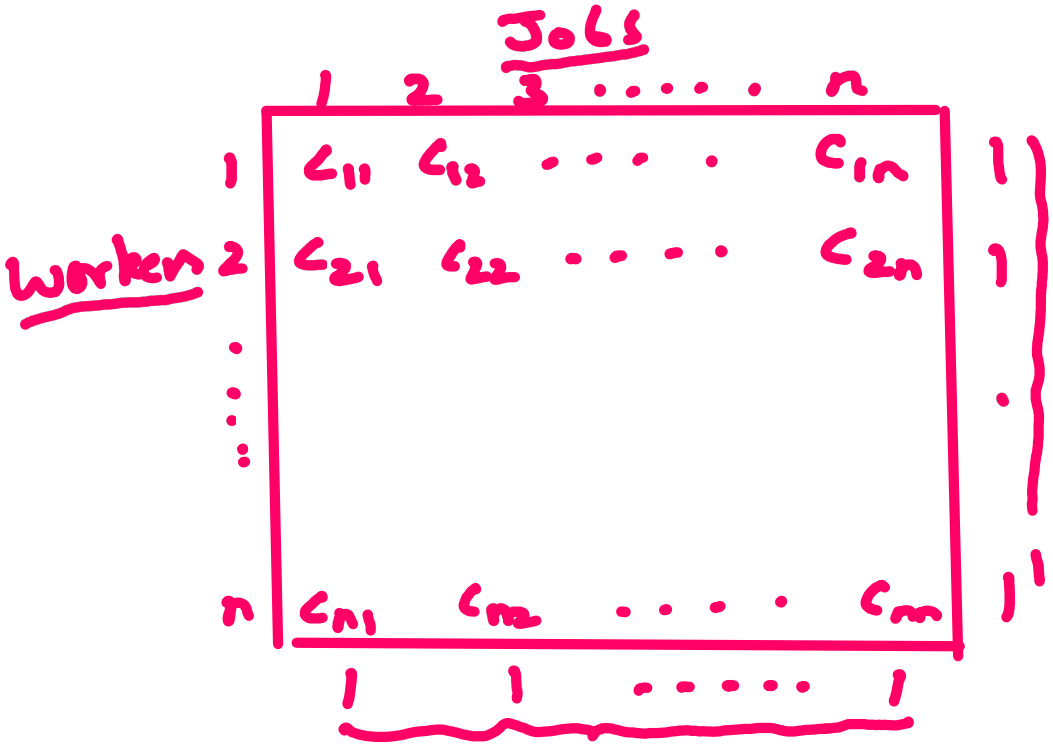
s.t.

$$\sum_{j=1}^{n} x_{ij} = 1 \quad , \quad i = 1, 2, \ldots n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad , \quad j = 1, 2, \ldots n$$

$x_{ij} = 1$ or 0

Assignment model



Example

Klyne's three children John, Karen, Teri want to earn money for personal expenses. Mr. Klyn has chosen three jobs namely mowing, Painting and Washing. He asked his children to submit bids for

these jobs. The bids are given as follows.

	Mowing	Paint	Wash
John	15	10	9
Karen	9	15	10
Teri	10	12	8

Klyne has to assign jobs to his children in such a way that the cost is minimum. Find the optimal assignment.

Ans (Hungarian Algorithm)

Step 1:

Find row minima & subtr from all elements. (for each in cost matrix.

Row min

15	10	9	9
9	15	10	9
10	12	8	8

Subtract Row min from all elements in each row.

15-9
10-9
9-9

6	1	0
0	6	1
2	4	0

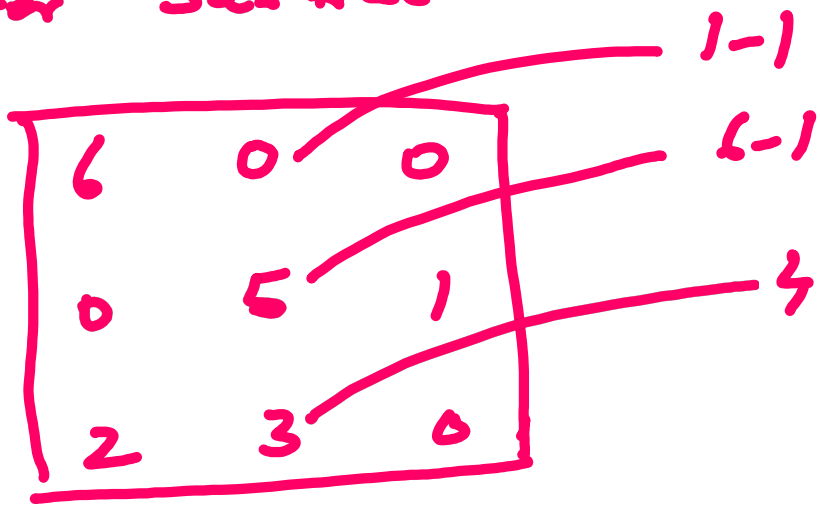
Step 2

For each column find Column min and subtract from other elements.

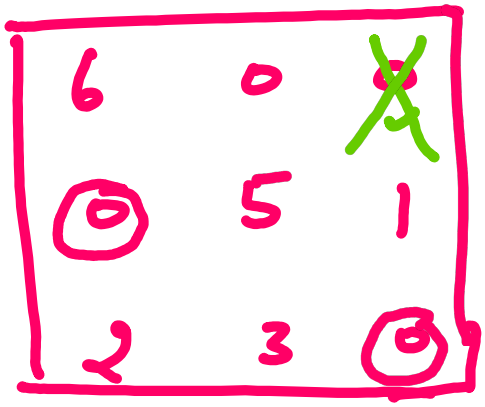
6	1	0
0	6	1
2	4	0

Col min 0 1 0

After Subtraction.

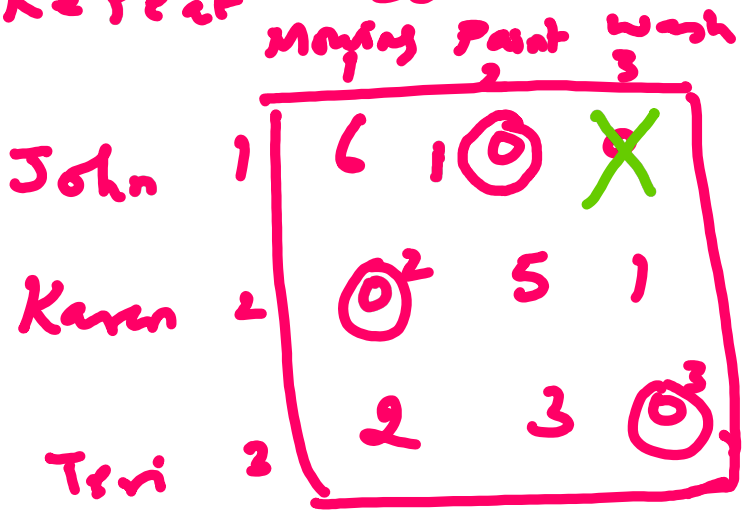


Step 3: Do optimal assignm



(i) Check row wise. Identify row with unique Zero. Circle it. If the are any Zero in that Corresponding Column Cross

(i) Repeat Column wise.



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(iii) If no. of assignment = order of matrix then stop. optimal A is made. else Adjust Computation are needed

No. of assignments = 3
= Order

∴ optimal assignment is

John → Paint

Karen → Mowi

Teri → Wash

Min cost is,

$= 10 + 9 + 8$

$= \$27$ //

Example 2:

The cost matrix of 4 & 4 jobs is given as

	J1	J2	J3	J4
C1	(1)	4	6	2
C2	9	7	(10)	9
C3	4	(5)	11	7
C4	8	7	8	(3)

Do optimal assignment & find min cost.

Final assignment is

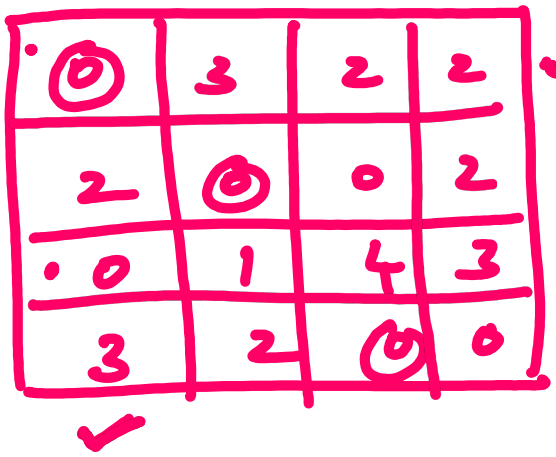
(0)	3	2	2
2	(0)	X	2
X	1	4	3

No. of assignment ≠ order of matrix

3 ≠ 4

Not optimal.

Additional Computation



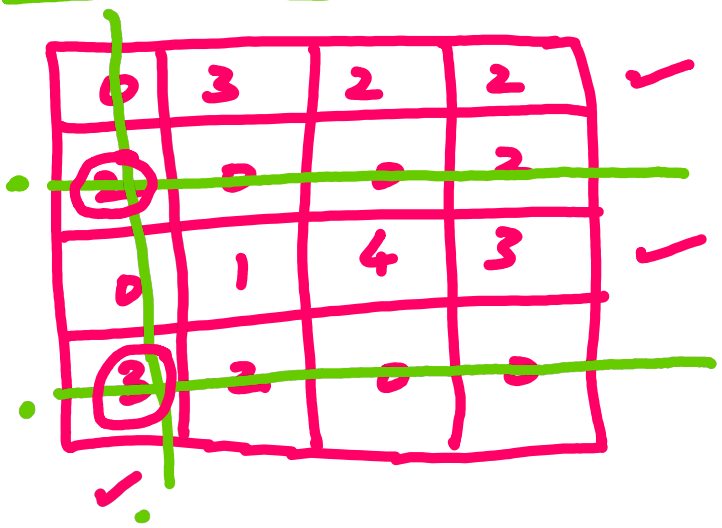
(i) Look for the row in which assignment is not done. Put ✓. (for all rows where assignme is not d

(ii) In the ticked row, look for 0 cell. Put ✓ in Co column.

(iii) Examine ✓ marked colu If any assigned 0 exists in that column, tick Cov

row.

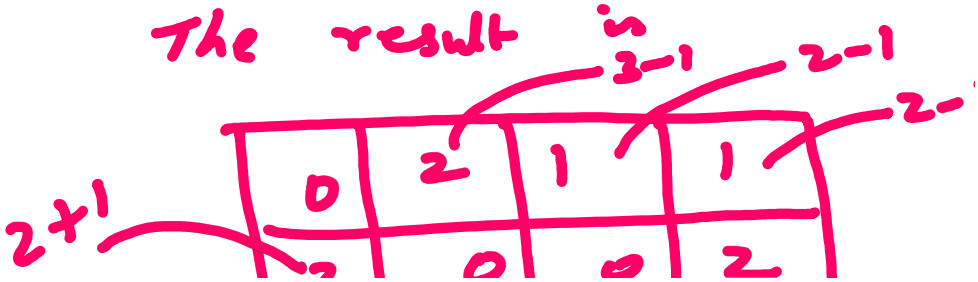
(iv) Draw horizontal & vertical lines in untracked rows & tracked cols.



v) In the uncovered elements Choose smallest element (ie) 1

vi) Subtract the smallest ele 1 from all remaining uncovered elements.

vii) Add : Smallest at the point of intersection of The result is



0	0	3	2
4	2	0	0

3+1

Do assignment

Job

Child	1	2	3	4
1	(0)	2	1	1
2	3	X	(0)	2
3	X	(0)	3	2
4	4	2	X	(0)

Optimal Assignment

Child 1 — Job 1

Child 2 — Job 3

Child 3 — Job 2

Child 4 — Job 4

Min cost = 1 + 10 + 5 + 5

= 21 //

Note: In case of Maximization

Subtract all elements from the highest element. Then apply minimization Procedure.

		Zone A	Zone B	C	D
Sales	P	140	112	98	154
	Q	70	72	63	99
	R	110	88	77	121
	S	80	64	56	88

Largest element = 154
Subtract all elements from highest.

14	42	56	0
64	82	91	55
44	66	77	33
74	90	98	66

Now minimize.

HW

Ans 392