Network Problem
CPM & PERT

Introduction

- Any project involves planning, scheduling and controlling a number of interrelated activities with use of limited resources, namely, men, machines, materials, money and time.
    - The projects may be extremely large and complex such as construction of a housing , a highway, a shopping complex etc.
    - introduction of new products and research and development projects.
- It is required that managers must have a dynamic planning and scheduling system to produce the best possible results and also to react immediately to the changing conditions and make necessary changes in the plan and schedule.

A convenient analytical and visual technique of **PERT** and **CPM** prove extremely valuable in assisting the managers in managing the projects.

PERT stands for **Project Evaluation and Review Technique** **developed during 1950's. The technique** was developed and used in conjunction with the planning and designing of the Polaris missile project.

CPM stands for **Critical Path Method** **which was developed by DuPont** Company and applied first to the construction projects in the chemical industry.

Though both PERT and CPM techniques have similarity in terms of concepts, the basic difference is; CPM has single time estimate and PERT has three time estimates for activities and uses probability theory to find the chance of reaching the scheduled time.

Project management generally consists of three phases.

<u>Planning:</u>

Planning involves setting the objectives of the project. Identifying various activities to be performed and determining the requirement of resources such as men, materials, machines, etc.

The cost and time for all the activities are estimated, and a network diagram is developed showing sequential interrelationships (predecessor and successor) between various activities during the planning stage.

<u>Scheduling:</u>

Based on the time estimates, the start and finish times for each activity are worked out by applying forward and backward pass techniques, critical path is identified, along with the slack and float for the non-critical paths.

<u>Controlling:</u>

Controlling refers to analyzing and evaluating the actual progress against the plan. Reallocation of resources, crashing and review of projects with periodical reports are carried out.

COMPONENTS of PERT/CPM NETWORK

PERT / CPM networks contain two major components

i. Activities, and

ii. Events

Activity: *An activity represents an action and consumption of resources (time, money,* energy) required to complete a portion of a project. Activity is represented by an arrow, .

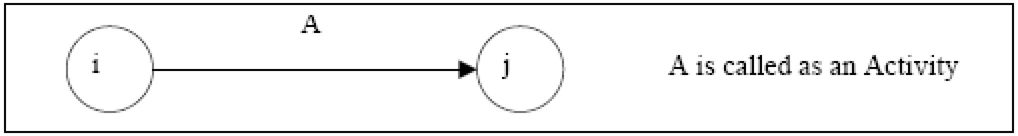


Figure 8.1: An Activity

Event: *An event (or node) will always occur at the beginning and end of an activity. The* event has no resources and is represented by a circle. The ith event and jth event are the tail event and head event respectively.

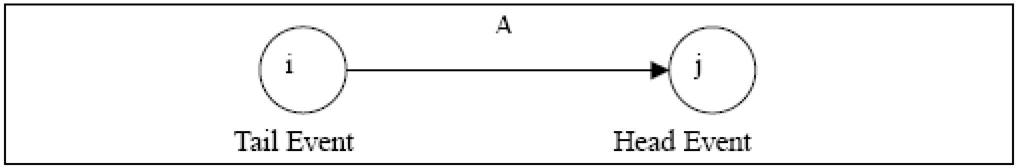


Figure 8.2: An Event

Merge and Burst Events

One or more activities can start and end simultaneously at an event .

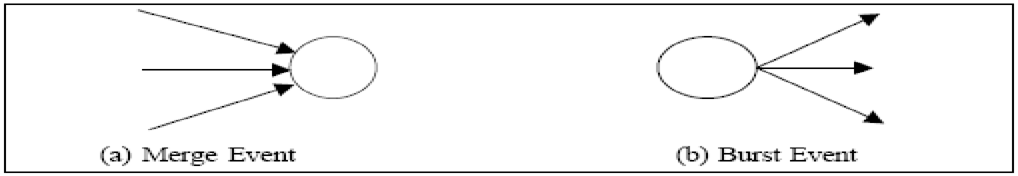


Figure 8.3

Preceding and Succeeding Activities

Activities performed before given events are known as *preceding activities*, and activities performed after a given event are known as *succeeding activities.*

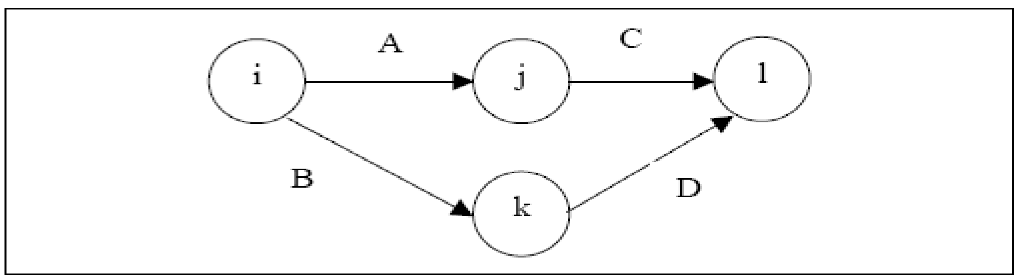


Figure 8.4: Preceding and Succeeding Activities

Activities A and B precede activities C and D respectively.

Dummy Activity

An imaginary activity which does not consume any resource and time is called a ***dummy activity. Dummy activities are simply used to represent a connection between events in*** order to maintain a logic in the network. It is represented by a dotted line in a network.

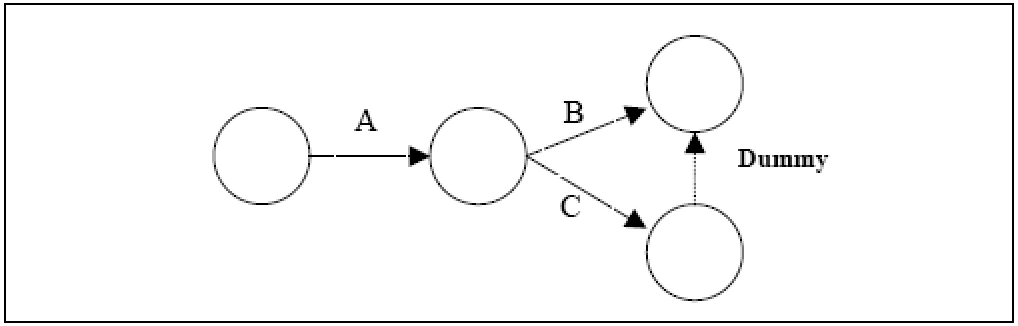


Figure 8.5: Dummy Activity

ERRORS TO BE AVOIDED IN CONSTRUCTING A NETWORK

a. Two activities starting from a tail event must not have a same end event. To ensure this, it is absolutely necessary to introduce a dummy activity, as shown in Figure .

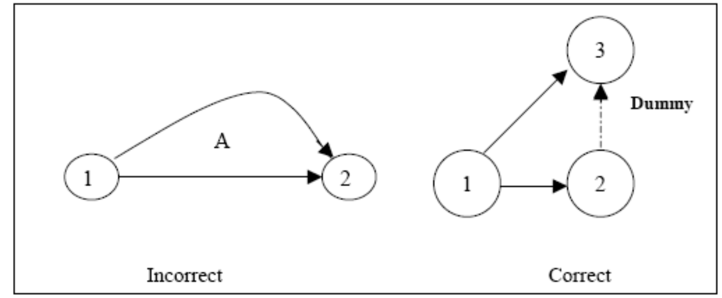


Figure 8.6: Correct and Incorrect Activities

b. Looping error should not be formed in a network, as it represents performance of activities repeatedly in a cyclic manner, as shown below in Figure .

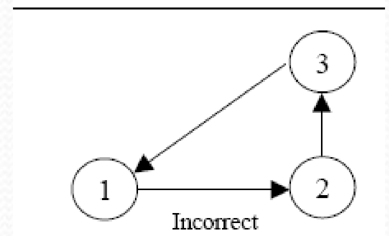


Figure 8.7: Looping Error

c. In a network, there should be only one start event and one ending event as shown below, in Figure .

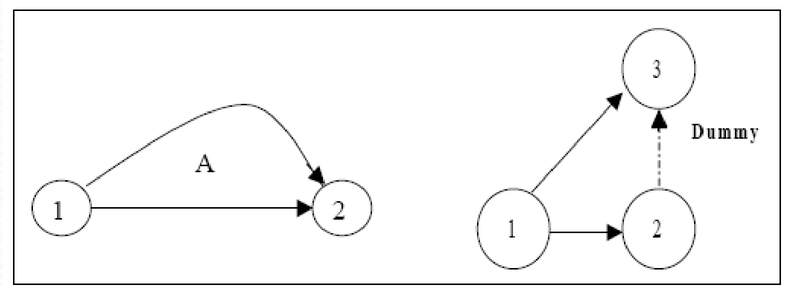


Figure 8.8: Only One Start and End Event

d. The direction of arrows should flow from left to right avoiding mixing of direction as shown in Figure .

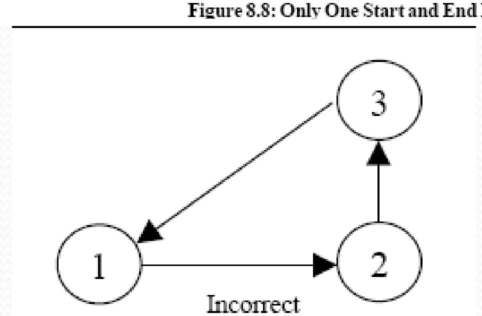


Figure 8.9: Wrong Direction of Arrows

RULES IN CONSTRUCTING A NETWORK

1. No single activity can be represented more than once in a network. The length of an arrow has no significance.

2. The event numbered 1 is the start event and an event with highest number is the end event. Before an activity can be undertaken, all activities preceding it must be completed. That is, the activities must follow a logical sequence (or – interrelationship) between activities.

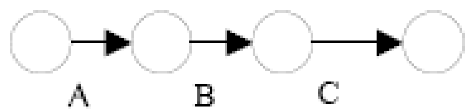
3. In assigning numbers to events, there should not be any duplication of event numbers in a network.

4. Dummy activities must be used only if it is necessary to reduce the complexity of a network.

5. A network should have only one start event and one end event.

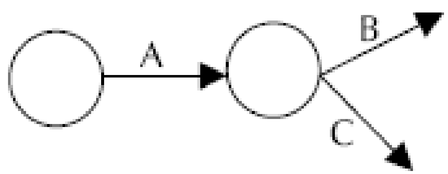
Some conventions of network diagram are shown in Figure (a), (b), (c), (d) below:

(a)



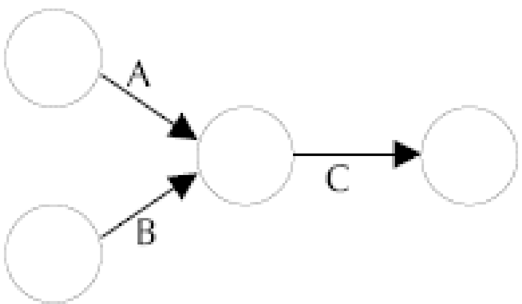
Activity B can be performed only after completing activity A, and activity C can be performed only after completing activity B.

(b)



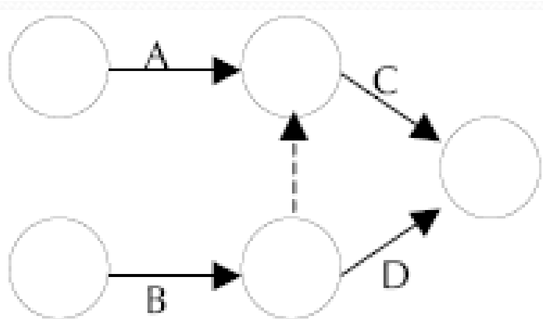
Activities B and C can start simultaneously only after completing A.

(c)



Activities A and B must be completed before start of activity C.

(d)



Activity C must start only after completing activities A and B. But activity D can start after completion of activity B.

PROCEDURE FOR NUMBERING THE EVENTS USING FULKERSON'S RULE

Step1: Number the start or initial event as 1.

Step2: From event 1, strike off all outgoing activities. This would have made one or more events as initial events (event which do not have incoming activities). Number that event as 2.

Step3: Repeat step 2 for event 2, event 3 and till the end event. The end event must have the highest number

Example 1:

Draw a network for a house construction project. The sequence of activities with their predecessors are given in Table , below.

Table 8.1: Sequence of Activities for House Construction Project

Name of the activity	Starting and finishing event	Description of activity	Predecessor	Time duration (days)
A	(1,2)	Prepare the house plan	--	4
B	(2,3)	Construct the house	A	58
C	(3,4)	Fix the door / windows	B	2
D	(3,5)	Wiring the house	B	2
E	(4,6)	Paint the house	C	1
F	(5,6)	Polish the doors / windows	D	1

Solution:

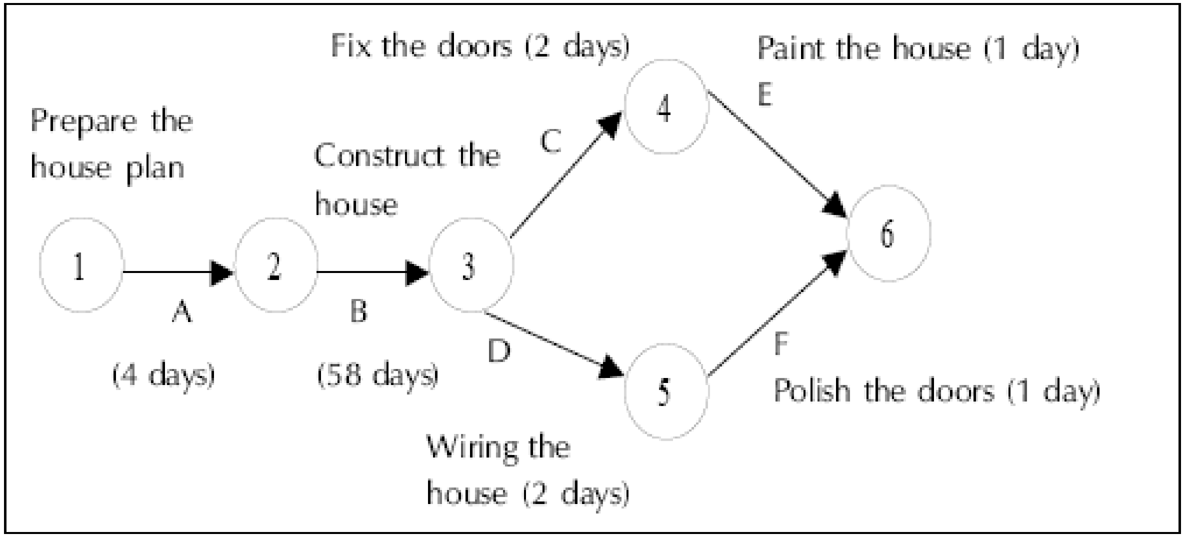


Figure 8.11: Network diagram representing house construction project.

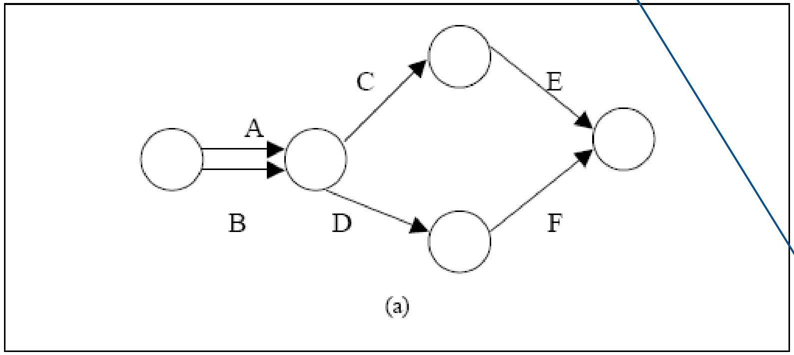
The network diagram in Figure shows the procedure relationship between the activities. **Activity A (preparation of house plan)**, has a start event 1 as well as an ending event 2**. Activity B (Construction of house)** begins at event 2 and ends at event 3. ***The activity B cannot start until activity A has been completed. Activities C and D cannot begin until activity B has been completed,*** but they can be performed simultaneously. Similarly, ***activities E and F can start only after completion of activities C and D respectively.*** Both activities E and F finish at the end of event 6.

Example 2: Consider the project given in Table 8.2 and construct a network diagram. **Table : Sequence of Activities for Building Construction Project**

Activity	Description	Predecessor
A	Purchase of Land	-
B	Preparation of building plan	-
C	Level or clean the land	A
D	Register and get approval	A, B
E	Construct the building	C
F	Paint the building	D

Solution:

The activities C and D have a common predecessor A. The network representation shown in Figure (a), (b) violates the rule that no two activities can begin and end at the same events. It appears as if activity B is a predecessor of activity C, which is not the case. To construct the network in a logical order, it is necessary to introduce a dummy activity as shown in Figure .



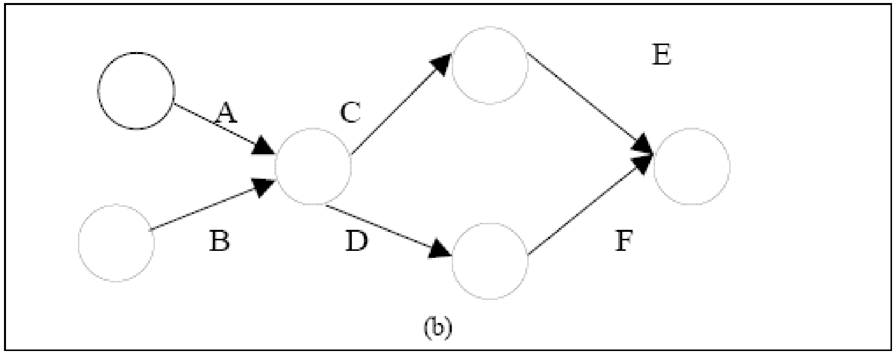


Figure 8.12: Network representing the Error

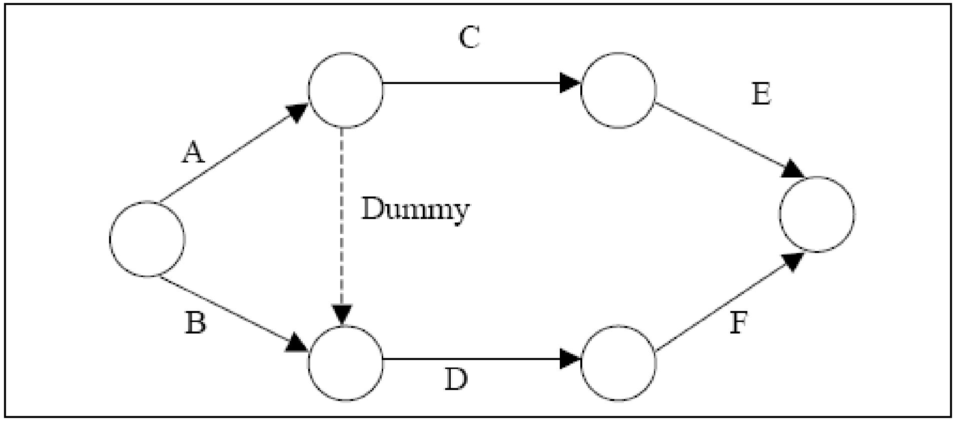


Figure 8.13: Correct representation of Network using Dummy Activity

Example 3:

Construct a network for a project whose activities and their predecessor relationship are given in Table .

Table 8.3: Activity Sequence for a Project

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	-	A	B	B	C	D	E	H, I	F, G

Solution: *The network diagram for the given problem is shown in Figure with* activities A, B and C starting simultaneously.

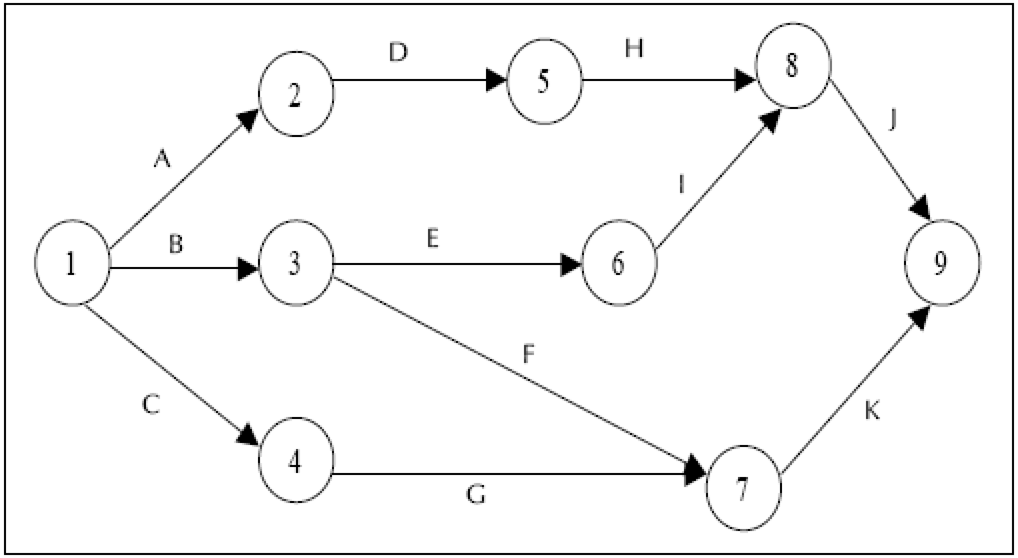


Figure 8.14: Network Diagram

Example 4: Draw a network diagram for a project given in Table .

Table 8.4: Project Activity Sequence

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Immediate Predecessor	-	A	B	A	D	C, E	D	D	H	H	F, H	G, J

***Solution: An activity network diagram describing the project is shown in Figure* , below:**

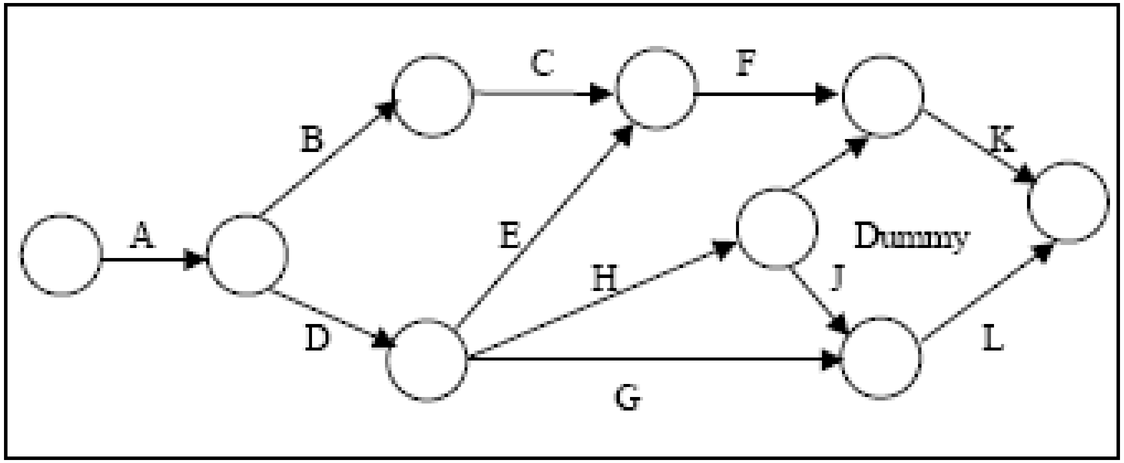


Figure 8.15: Network Diagram

CRITICAL PATH ANALYSIS

The critical path for any network is the longest path through the entire network.

Since all activities must be completed to complete the entire project, the length of the critical path is also the shortest time allowable for completion of the project.

Thus if the project is to be completed in that shortest time, all activities on the critical path must be started as soon as possible.

These activities are called critical activities.

If the project has to be completed ahead of the schedule, then the time required for at least one of the critical activity must be reduced.

Further, any delay in completing the critical activities will increase the project duration.

The activity, which does not lie on the critical path, is called non-critical activity.

These non-critical activities may have some slack time.

The slack is the amount of time by which the start of an activity may be delayed without affecting the overall completion time of the project.

But a critical activity has no slack.

To reduce the overall project time, it would require more resources (at extra cost) to reduce the time taken by the critical activities to complete.

Scheduling of Activities: Earliest Time (TE) and Latest Time(TL)

Before the critical path in a network is determined, it is necessary to find the earliest and latest time of each event to know the earliest expected time (TE) at which the activities originating from the event can be started and to know the latest allowable time (TL) at which activities terminating at the event can be completed.

Forward Pass Computations (to calculate Earliest, Time TE)

Step 1: Begin from the start event and move towards the end event.

Step 2: Put TE = 0 for the start event.

Step 3: *Go to the next event (i.e node 2) if there is an incoming activity for event 2,* add calculate TE of previous event (i.e event 1) and activity time.

Note: If there are more than one incoming activities, calculate TE for all incoming activities and take the maximum value. This value is the TE for event 2.

Step 4: Repeat the same procedure from step 3 till the end event.

Backward Pass Computations (to calculate Latest Time TL)

Procedure :

Step 1: **Begin from end event and move towards the start event. Assume that the direction of arrows is reversed.**

Step 2: **Latest Time TL for the last event is the earliest time. TE of the last event.**

Step 3: **Go to the next event, if there is an incoming activity, subtract the value of TL of previous event from the activity duration time. The arrived value is TL for that event. If there are more than one incoming activities, take the minimum TE value.**

Step 4: **Repeat the same procedure from step 2 till the start event.**

DETERMINATION OF FLOAT AND SLACK TIMES

As discussed earlier, the non – critical activities have some slack or float. The *float of an* activity is the amount of time available by which it is possible to delay its completion time without extending the overall project completion time.

tij = duration of activity
TE = earliest expected time
TL = latest allowable time
ES_{ij} = earliest start time of the activity
EF_{ij} = earliest finish time of the activity
LS_{ij} = latest start time of the activity
LF_{ij} = latest finish time of the activity

Total Float TFij: The total float of an activity is the difference between the latest start time and the earliest start time of that activity.

$TFij = LS_{ij} - ES_{ij}$ …………………(1)
or
$TF_{ij} = (TL - TE) - t_{ij}$ ………….(ii)

Free Float FFij: ***The time by which the completion of an activity can be delayed from*** **its earliest finish time without affecting the earliest start time of the succeeding activity is called free float.**

$$FF_{ij} = (Ej - Ei) - t_{ij} \text{ ....(3)}$$
$$FF_{ij} = \text{Total float} - \text{Head event slack}$$

Independent Float IFij: **The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.**

IF ij = (Ej – Li) – tij ....(4)

IFij = Free float – Tail event slack   Where tail event slack = Li – Ei

The negative value of independent float is considered to be zero.

Critical Path:

After determining the earliest and the latest scheduled times for various activities, the minimum time required to complete the project is calculated. In a network, among various paths, the longest path which determines the total time duration of the project is called the critical path. The following conditions must be satisfied in locating the critical path of a network.

An activity is said to be critical only if both the conditions are satisfied.

1. TL – TE = 0
2. TLj – tij – TEj = 0

Example :

A project schedule has the following characteristics as shown in Table

Table 8.5: Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

i. Construct PERT network.
ii. Compute TE and TL for each activity.
iii. Find the critical path.

Table 8.5: Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

(i) From the data given in the problem, the activity network is constructed as shown in Figure given below

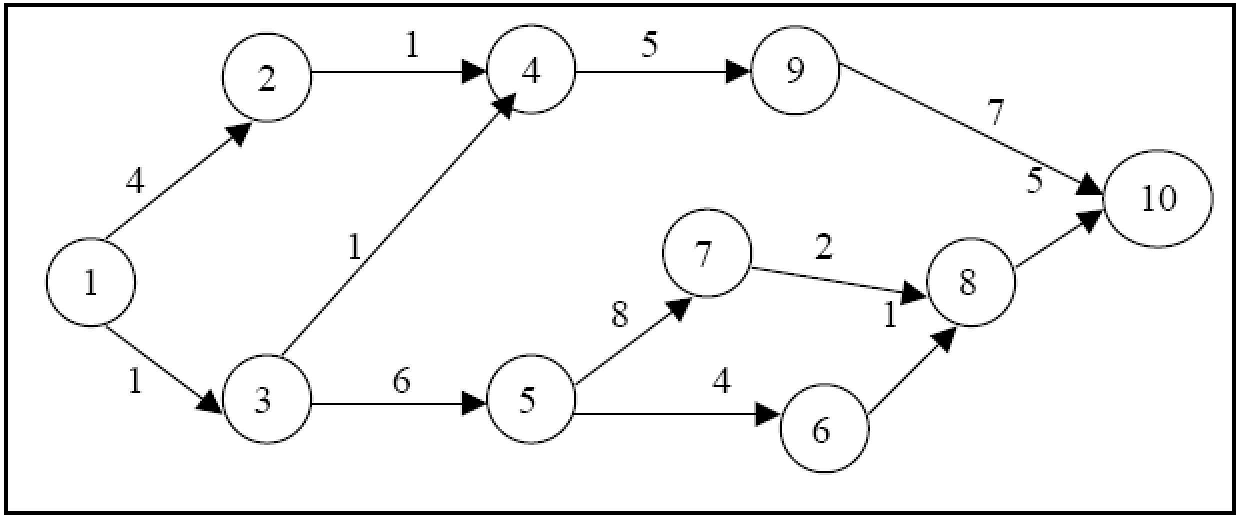


Figure 8.16: Activity Network Diagram

(ii) **To determine the critical path, compute the earliest time TE and latest time TL for each of the activity of the project. The calculations of TE and TL are as follows:,**

To calculate TE for all activities

TE1 = 0
TE2 = TE1 + t1, 2 = 0 + 4 = 4
TE3 = TE1 + t1, 3 = 0 + 1 =1
TE4 = max (TE2 + t2, 4 and TE3 + t3, 4)
= max (4 + 1 and 1 + 1) = max (5, 2)
= 5 days
TE5 = TE3 + t3, 6 = 1 + 6 = 7
TE6 = TE5 + t5, 6 = 7 + 4 = 11
TE7 = TE5 + t5, 7 = 7 + 8 = 15
TE8 = max (TE6 + t6, 8 and TE7 + t7, 8)
= max (11 + 1 and 15 + 2) = max (12, 17)
= 17 days
TE9 = TE4 + t4, 9 = 5 + 5 = 10
TE10 = max (TE9 + t9, 10 and TE8 + t8, 10)
= max (10 + 7 and 17 + 5) = max (17, 22)
= 22 days

To calculate TL for all activities

TL10 = TE10 = 22
TL9 = TE10 – t9,10 = 22 – 7 = 15
TL8 = TE10 – t8, 10 = 22 – 5 = 17
TL7 = TE8 – t7, 8 = 17 – 2 = 15
TL6 = TE8 – t6, 8 = 17 – 1 = 16
TL5 = min (TE6 – t5, 6 and TE7 – t5, 7)
= min (16 – 4 and 15 –8) = min (12, 7)
= 7 days
TL4 = TL9 – t4, 9 = 15 – 5 =10
TL3 = min (TL4 – t3, 4 and TL5 – t3, 5)
= min (10 – 1 and 7 – 6) = min (9, 1)
= 1 day
TL2 = TL4 – t2, 4 = 10 – 1 = 9
TL1 = Min (TL2 – t1, 2 and TL3 – t1, 3)
= Min (9 – 4 and 1 – 1) = 0

Table 8.6: Various Activities and their Floats

Activity	Activity Name	Normal Time (tij)	Earliest Time (TE)		Latest Time (TL)		Total Float
			Start (ES)	Finish (EF)	Start (LS)	Finish (LF)	
1-2	A	4	0	4	5	9	5
1-3	B	1	0	1	0	1	0
2-4	C	1	4	5	9	10	5
3-4	D	1	1	2	9	10	8
3-5	E	6	1	7	1	7	0
4-9	F	5	5	10	10	15	5
5-6	G	4	7	11	12	16	5
5-7	H	8	7	15	7	15	0
6-8	I	1	11	12	16	17	5
7-8	J	2	15	17	15	17	0
8-10	K	5	17	22	17	22	0
9-10	L	7	10	17	15	22	5

(iii) From the Table , we observe that the activities 1 – 3, 3 – 5, 5 – 7,7 – 8 and 8 – 10 are critical activities as their floats are zero.

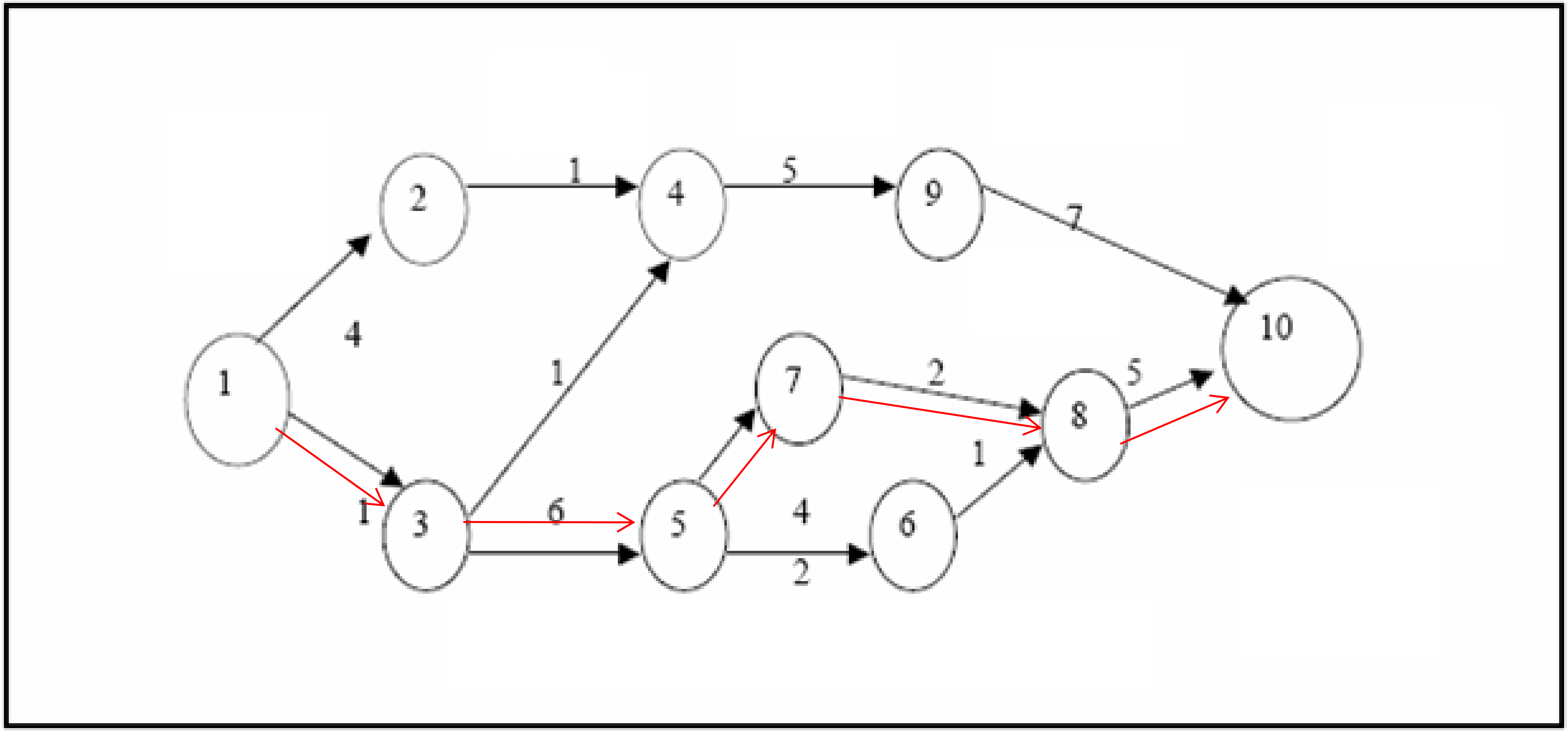


Figure 8.17: Critical Path of the Project

PROJECT EVALUATION REVIEW TECHNIQUE, (PERT)

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities.

Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure 8.22.

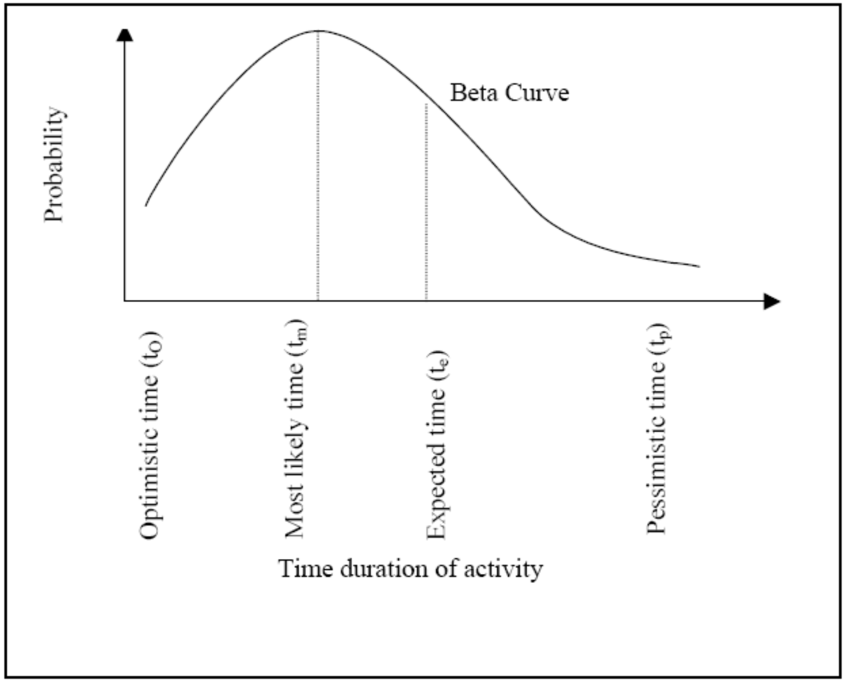


Figure 8.22: PERT Using Probabilistic Method with 3 Time Estimates

Optimistic time to:

It is the shortest time taken to complete the activity. It means that if everything goes well then there is more chance of completing the activity within this time.

Most likely time tm:

It is the normal time taken to complete an activity, if the activity were frequently repeated under the same conditions.

Pessimistic time tp:

It is the longest time that an activity would take to complete. It is the worst time estimate that an activity would take if unexpected problems are faced.

Taking all these time estimates into consideration, the expected time of an activity is arrived at.

The average or mean (ta) value of the activity duration is given by,

$$T_a = \frac{t_0 + 4t_m + t_p}{6} \quad \text{.......................(5)}$$

The variance of the activity time is calculated using the formula,

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6} \right)^2$$

Probability for Project Duration

The probability of completing the project within the scheduled time (Ts) or contracted time may be obtained by using the standard normal deviate where Te is the expected time of project completion.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2} \text{ in critical path}}$$

Probability of completing the project within the scheduled time is,

$$P\,(T \leq T_s) = P\,(\,Z \leq Z_0\,) \quad \text{(from normal tables)} \quad \text{....}$$

Example

An R & D project has a list of tasks to be performed whose time estimates are given in the Table , as follows.

Table 8.11: Time Estimates for R & D Project

Activity i j	Activity Name	T_0	t_m (in days)	t_p
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	9	10

a. Draw the project network.
b. Find the critical path.
c. Find the probability that the project is completed in 19 days. If the probability is less than 20%, find the probability of completing it in 24 days.

Time expected for each activity is calculated using the formula (5): Similarly, the expected time is calculated for all the activities.

$$T_e = \frac{t_0 + 4tm + tp}{6}$$

$$= \frac{4 + 4(6) + 8}{6} = \frac{36}{6} = 6 \text{ days for activity A}$$

The variance of activity time is calculated using the formula (6). Similarly, variances of all the activities are calculated.

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

$$= \left(\frac{8-4}{6}\right)^2 = 0.444$$

Table 8.12: T_e & s^2 Calculated

Activity	T_o	T_m	T_p	T_e	σ^2
1-2	4	6	8	6	0.444
1-3	2	3	10	4	1.777
1-4	6	8	16	9	2.777
2-4	1	2	3	2	0.111
3-4	6	7	8	7	0.111
3-5	6	7	14	8	1.777
4-6	3	5	7	5	0.444
4-7	4	11	12	10	1.777
5-7	2	4	6	4	0.444
6-7	2	9	10	8	1.777

Construct a network diagram:

calculate the time earliest (TE) and time Latest (TL) for all the activities.

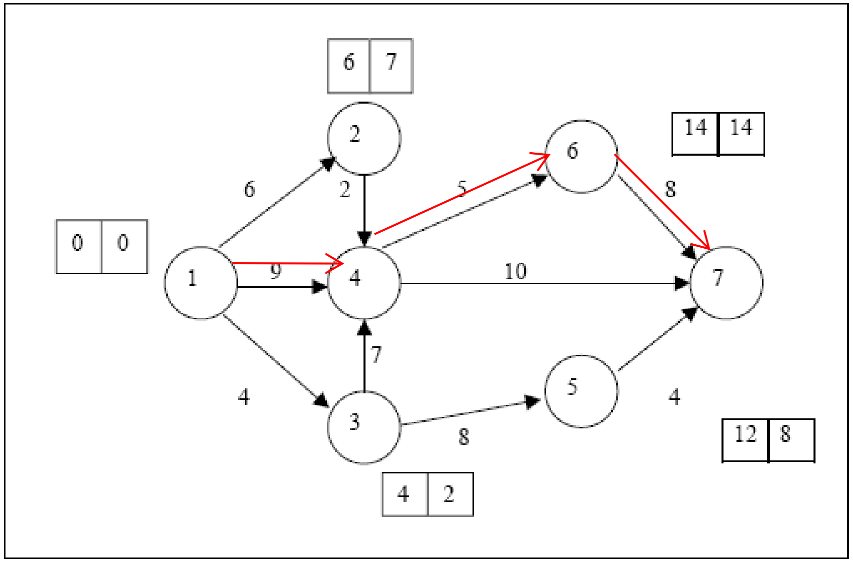


Figure 8.23: Network Diagram

From the network diagram Figure , the critical path is identified as 1-4, 4-6, 6-7, with a project duration of 22 days.

The probability of completing the project within 19 days is given by, P (Z< Zo)

To find Zo ,

$$Z_0 = \left(\frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}} \right)$$

$$= \left(\frac{19 - 22}{\sqrt{2.777 + 0.444 + 1.777}} \right)$$

$$= \left(\frac{-3}{\sqrt{5}} \right) = -1.3416$$

we know, P (Z <Z Network Model 0) = 0.5 – z (1.3416) (from normal tables, z (1.3416) = 0.4099)
= 0.5 – 0.4099
= 0.0901
= 9.01%

Thus, the probability of completing the R & D project in 19 days is 9.01%.

Since the probability of completing the project in 19 days is less than 20% As in question, we find the probability of completing it in 24 days.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}}$$

$$= \left(\frac{24 - 22}{\sqrt{5}} \right) = \left(\frac{2}{\sqrt{5}} \right) = 0.8944 \text{ days}$$

$$P(Z \leq Z_0) = 0.5 - \Upsilon\ (0.8944) \qquad (\text{from normal tables, } \Upsilon\ (0.8944) = 0.3133)$$

$$= 0.5 + 0.3133$$

$$= 0.8133$$

$$= 81.33\%$$

Assignment

1. You are required to prepare a network diagram for constructing a 5 floor apartmer
The major activities of the project are given as follows:

Activity	Description	Immediate Predecessor
A	Selection of site	-
B	Preparation of drawings	-
C	Arranging the for finance	A
D	Selection of contractor	A
E	Getting approval from Govt	A
F	Laying the foundation	E
G	Start construction	D, F
H	Advertise in newspaper	B, C
I	Allocation of tenants	G, H

2. For the problem No.1 the time estimates in days are given. Determine the Time earliest and Time latest, and the critical activities

Activity	A	B	C	D	E	F	G	H	I
Time (days)	3	5	7	2	5	20	60	2	10

Draw a network diagram for the project:

Activity	A	B	C	D	E	F	G	H	I	J
Predecessor	-	A	B	B	B	C	C	F, G	D, E, F	I

. A national conference is planned in a college. The activities are listed down along with their predecessors and time taken. Prepare a network diagram and determine the critical activities.

Activity	Description	Immediate Predecessor	Duration (days)
A	Confirm lead speaker and topic	-	5
B	Prepare brochure	-	1
C	Send letters to other speakers	B	2
D	Get confirmation from speakers	C	5
E	Send letters to participants	C,D	2
F	Obtain travel plans from speakers	D	2
G	Arrange for accommodation for speakers	F	1
H	Get handouts from speakers	F	4
I	Finalize registrations	G,H	10
J	Arrange hall and AV	I	1
K	Conduct of programme	J	1

For the PERT problem find the critical path and project duration. What is the probability that the project will be completed in 25 days?

Activity	Predecessor	Time		
		Optimistic	Most likely	Pessimistic
A	-	2	5	14
B	-	1	10	12
C	A	0	0	6
D	A	1	4	7
E	C	3	10	15
F	D	3	5	7
G	B	1	2	3
H	E,F	5	10	15
I	G	3	6	9

The following table lists the jobs of a network along with their estimates.

Activity	Time (Weeks)		Cost (Rs)	
	Normal	Crash	Normal	Crash
1-2	9	4	1300	2400
1-3	15	13	1000	1380
2-3	7	4	7000	1540
2-4	7	3	1200	1920
2-5	12	6	1700	2240
3-6	12	11	600	700
4-5	6	2	1000	1600
5-6	9	6	900	1200

a. Draw the project network diagram.
b. Calculate the length and variance of the critical path.
c. What is the probability that the jobs on the critical path can be completed in 41 days?