Simplex method

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(Eg. Product mix Problem
Graphical method eg. 2.)

Max $Z = 6x_1 + 5x_2$

s.t.

$x_1 + x_2 \leq 5$

$3x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$

Solution

Converting the given LPP into equation form

$$Z = \underline{6}\,\underline{x_1} + \underline{5}\,\underline{x_2} + \underline{0}\,x_3 + \underline{0}\,x_4$$

s.t.

$x_1 + x_2 + x_3 = 5$

$3x_1 + 2x_2 + x_4 = 12$

$x_1, x_2, \underline{x_3}, \underline{x_4} \geq 0$

slack variables added for $\leq$ constraints.

Rewriting objective function

$$\underline{Z - 6x_1}\ \underline{-5x_2}\ \underline{+0x_3 + 0x_4}\ \underline{=0}$$

Initial Simplex table

non basic

Basic variable	x_1 (c_r)	x_2	x_3	x_4	Solution RHS (b_r)	Ratio (b_r/c_r)
Z row	-6	-5	0	0	0	–
R_1 x_3	1	1	1	0	5	$5/1 = 5$
R_2 x_4	3	2	0	1	12	$12/3 = 4$

[Ratio is taken for all $c_r > 0$]

Initial Basic Feasible Solution is (IBFS)

$$\boxed{\begin{array}{ll} Z = 0 & \\ x_3 = 5 & x_1 = 0 \\ x_4 = 4 & x_2 = 0 \end{array}}$$

Basic variables x_3, x_4 (appearing under basic variable column)

Non Basic variables x_1, x_2

Basis = {Set of basic variables}

$= \{x_3, x_4\}$

Entering Variable

x_1 enters the basis, since its coefficient is most negative in Z-row.

Leaving variable

x_4 leaves the basis, since the ratio b_r/c_r is minimum.

Pivot element

Intersection of entering variable column and leaving variable row is called pivot.

Here pivot element is 3.
We have to make pivot element as 1 and all other elements in that column as 0.
This is shown in next iteration.

Iteration 1

	Basic variable	x_1	x_2 (c_r)	x_3	x_4	Solution (RHS) b_r	b_r/c_r
	Z row	0	−1	0	2	24	−
R_1	x_3	0	1/3	1	−1/3	1	$\frac{1}{1/3} = 3$
R_2'	x_1	1	2/3	0	1/3	4	$\frac{4}{2/3} = 6$

[Ratio should be taken for all $c_r > 0$]

$\boxed{R_2' = R_2/3}$ Pivot row

[Rough work :

i) Z row : −6 −5 0 0 0
Pivot row $R_2' \times (-6)$: −6 −4 0 −2 −24
(−) 0 −1 0 2 24

x_3 row
ii) R_1 : 1 1 1 0 5
Pivot row $R_2' \times 1$: 1 2/3 0 1/3 4
(−) 0 1/3 1 −1/3 1

Entering Variable

The most negative coefficient in Z row is -1 and corresponds to variable x_2.

$\therefore x_2$ enters the basis

__Leaving variable__

The min-ratio is 3 and corresponds to variable x_3.

$\therefore x_3$ leaves the basis.

Pivot element is $1/3$.

Making Pivot element as 1 and other elements in that column as Zero.,

__Iteration: 2__

	Basic variable	x_1	x_2	x_3	x_4	RHS soln (b_r)	b_r/c_r	
	Z row	0	0	3	1	27		
R_1'	x_2	0	1	3	-1	3		$R_1' = R_1 \times 3$
R_2	x_1	1	0	-2	1	2		

[__Rough work:__

Z-row:	0	-1	0	2	24
Pivot row $R_1' \times -1$:	0	-1	-3	1	-3
(−)	0	0	3	1	27

R_2-Row:	1	$2/3$	0	$1/3$	4
Pivot row $R_1 \times 2/3$	0	$2/2$	2	$-2/3$	2
(−)	1	0	-2	1	2

$\frac{1}{3} + \frac{2}{3}$]

All coefficients in Z-row is non negative (≥ 0).

$\therefore$ optimality condition is reached ...

∴ Optimality condition is reached.
Stop iteration

Optimal Soln is
$Z = 27$ ✓
$x_1 = 2$ ✓
$x_2 = 3$ ✓

non basic
$x_3 = 0$
$x_4 = 0$ } Slack Variables

1) Optimality Condition

All Z- row coefficient are non negative, then optimum is reached.

2) Feasibility Condition

— The leaving variable is the basic variable associated with smallest non negative ratio with strictly positive denominator. ($c_r > 0$)