Graphical method example1

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Example: (Product Mix)

Consider a small manufacturer making two products A and B. Two resources R1 & R2 are used. Each unit of Product A requires 1 unit of R1 and 3 units of R2. Each unit of Product B requires 1 unit of R1 and 2 units of R2. The manufacturer has 5 units of R1 and 12 units of R2 available. The manufacturer makes a profit of Rs 6/unit for product A sold and Rs 5/unit of Product B sold. Find the optimum production of products A and B to maximize profit.

Ans:

Step 1 : Decision Variables

x_1 – The number of units of Product A to be produced

x_2 – The number of units of Product B to be produced

Step 2: Objective function

Profit per unit for Product A = Rs 6 ✓

" " " B = Rs 5 ✓

$\therefore$ Maximize $Z = 6x_1 + 5x_2$

Step 3 : Constraints

Based on Resource R1

Max. available = 5
Required for Product A = 1
Required for Product B = 1

$\therefore x_1 + x_2 \le 5$

Based on Resource R2

Max. available = 12
Required for Product A = 3
Required for Product B = 2

$\therefore 3x_1 + 2x_2 \le 12$

Step 4 : Non Negativity Constraints

$x_1, x_2 \geq 0$

$\therefore$ Model is

$$\boxed{\begin{aligned} \text{Max } Z &= 6x_1 + 5x_2 \\ \text{s.t.} \quad x_1 + x_2 &\le 5 \\ 3x_1 + 2x_2 &\le 12 \\ x_1, x_2 &\geq 0 \end{aligned}}$$

Solution by Graphical Method

Step 1 : Determination of feasible Solution

Replacing each inequality Constraints as equations and finding coordinates.

(i) $x_1 + x_2 = 5$

Put $x_1 = 0 \implies x_2 = 5 \therefore$ Pt is $(0,5)$
Put $x_2 = 0 \implies x_1 = 5 \therefore$ Pt is $(5,0)$

$\boxed{\text{Coordinates } (5,0) \text{ \& } (0,5)}$

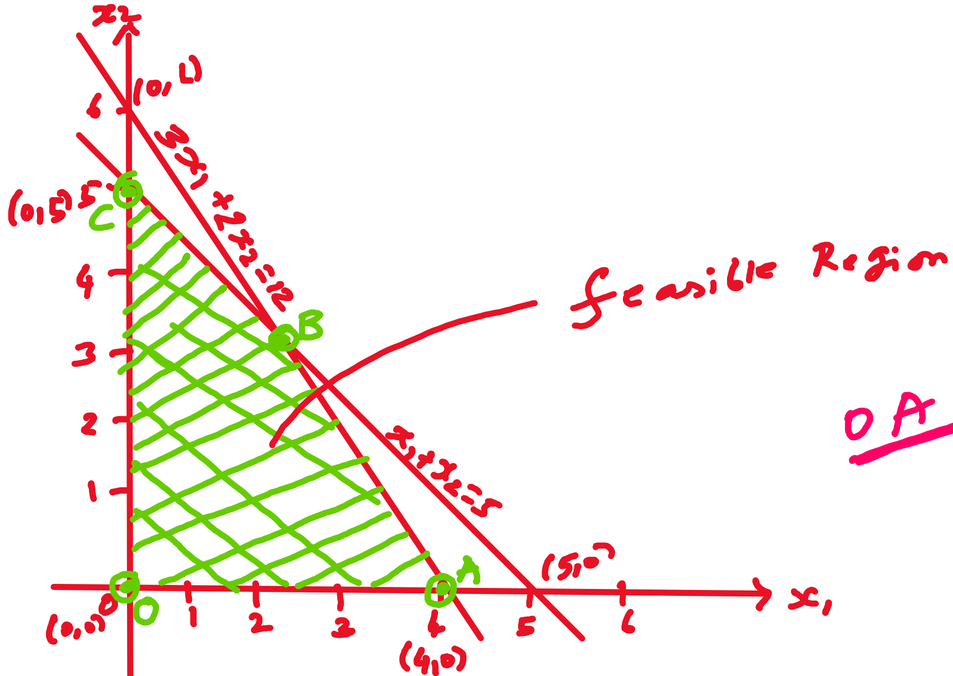
(ii) $3x_1 + 2x_2 = 12$

Put $x_1 = 0 \Rightarrow x_2 = 6$ ∴ pt is $(0, 6)$ ✓
Put $x_2 = 0 \Rightarrow x_1 = 4$ ∴ pt is $(4, 0)$ ✓

$\boxed{\text{Coordinates } (4,0) \text{ \& } (0,6)}$

(iii) Non negativity constraints $x_1, x_2 \geq 0$ implies first quadrant.

Plotting the st. lines in the graph.



OABC

The feasible solution space is OABC. There are infinite number of solutions in feasible region.

Step 2 : To find optimum solution

The optimum solution is identified with corner points. OABC.

(ie) O → (0,0) ✓
A → (4,0) ✓

B → To find B,

we have to solve the equations

$$x_1 + x_2 = 5 \quad \text{—(1)}$$
$$3x_1 + 2x_2 = 12 \quad \text{—(2)}$$

(1) $\times 2 \rightarrow \quad 2x_1 + 2x_2 = 10$

(2) $\rightarrow \quad 3x_1 + 2x_2 = 12$

$- \quad -x_1 = -2 \Rightarrow x_1 = 2$

Put $x_1 = 2$ in (1).

$x_2 = 3$.

$\therefore \boxed{B \text{ is } (2,3)}$

we have $C \rightarrow (0,5)$

Corresponding values of objective fn is,

$Z_0(0,0) \rightarrow 6\times 0 + 5\times 0 = 0$

$Z_A(4,0) \rightarrow 6\times 4 + 5\times 0 = 24$

$Z_B(2,3) \rightarrow 6\times 2 + 5\times 3 = 27$

$Z_C(0,5) \rightarrow 6\times 0 + 5\times 5 = 25$

$\therefore$ Maximum Value of Z is 27 at $(2,3)$.

$\therefore$ we have to produce 2 units of Product A and 3 units of product B to get a max profit of Rs. 27/-.

(ie) $\boxed{\text{Max } Z = 27 \quad x_1 = 2, \; x_2 = 3}$