# Inverse Z-Transform

* Find Inverse Z-transform:

1) $X(z) = \dfrac{10z}{z^2-3z+2}$

$$\frac{X(z)}{z} = \frac{10}{z^2-3z+2} = \frac{10}{(z-1)(z+2)}$$

$$\frac{10}{(z-1)(z+2)} = \frac{A}{(z-1)} + \frac{B}{(z+2)}$$

$$10 = A(z+2) + B(z-1)$$

when $z=-2$ $\quad B = -10/3$

when $z=1$ $\quad A = 10/3$

Thus, $$\frac{10}{(z-1)(z+2)} = \frac{10}{3(z-1)} + -\frac{10}{3(z+2)}$$

$$z^{-1}[X(z)] = z^{-1}\left[\frac{10}{3} \times \frac{z}{z-1} + -\frac{10}{3} \times \frac{z}{z+2}\right]$$

$$= \frac{10}{3}(1)^n - \frac{10}{3}(-2)^n$$

$$z^{-1}[x(z)] = \boxed{\frac{10}{3}\left[1-(-2)^n\right]}$$

2) $x(z) = \dfrac{z}{z^2+7z+10}$

$$\frac{x(z)}{z} = \frac{1}{(z+2)(z+5)}$$

$$\frac{1}{(z+2)(z+5)} = \frac{A}{z+2} + \frac{B}{z+5}$$

Madhumitha

$$1 = A(z+5) + B(z+2)$$

when $z = -2$ ; $A = 1/3$

$z = -5$ ; $B = -1/3$

Thus $$\frac{1}{(z+2)(z+5)} = \frac{1}{3(z+2)} - \frac{1}{3(z+5)}$$

$$z^{-1}[x(z)] = z^{-1}\left[\frac{1}{3}\frac{z}{z+2} - \frac{1}{3}\frac{z}{z+5}\right]$$

$$= \frac{1}{3}\left[(-2)^n - (-5)^n\right]$$

3) $$X(z) = \frac{8z^2}{(2z-1)(4z-1)}$$

$$\frac{X(z)}{z} = \frac{8z}{(2z-1)(4z-1)} = \frac{z}{(z-1/2)(z-1/4)}$$

$$\frac{z}{(z-1/2)(z-1/4)} = \frac{A}{z-1/2} + \frac{B}{z-1/4}$$

$$z = A\left(z-\frac{1}{4}\right) + B\left(z-\frac{1}{2}\right)$$

when $z = \frac{1}{4}$, $\frac{1}{4} = B\left(\frac{1}{4} - \frac{1}{2}\right)$

$B = \frac{1}{4} \times \frac{4}{-1} \Rightarrow \boxed{B = -1}$

when $z = \frac{1}{2}$, $\frac{1}{2} = A\left(\frac{1}{2} - \frac{1}{4}\right)$

$A = \frac{1}{2} \times \frac{4}{1} \Rightarrow \boxed{A = 2}$

$$\frac{z}{(z-1/2)(z-1/4)} = \frac{2}{z-1/2} - \frac{1}{z-1/4}$$

Hedhumita

$$x(z) = \frac{2z}{z-1/2} + \frac{z}{z-1/4}$$

$$Z^{-1}[x(z)] = 2(1/2)^n - (1/4)^n$$

4) $$x(z) = \frac{z^2+z}{(z-1)(z^2+1)}$$

$$x(z) = \frac{z(z+1)}{(z-1)(z^2+1)}$$

$$\frac{x(z)}{z} = \frac{z+1}{(z-1)(z^2+1)} = \frac{A}{z-1} + \frac{Bz+C}{z^2+1}$$

$$z+1 = A(z^2+1) + (Bz+C)(z-1)$$

$$z+1 = z^2(A+B) - z(B-C) + A - C$$

on comparing LHS, RHS

coefficient of $z^2$ is 0 in LHS

Thus, $A+B=0 \Rightarrow \boxed{A=-B}$

$$-z(B-C) = z(C-B)$$

$C - B = 1 \Rightarrow \boxed{C=B}$ $\quad C = 1+B$ — 1

$A - C = 1 \Rightarrow$ $\quad A = 1 + C$ — 2

From 1 & 2 we get $\left.\begin{array}{l} C = 1+B \\ C = -B-1 \end{array}\right\}$ possible iff $C=0$

$\boxed{C=0}$ $\boxed{A=1}$ $\boxed{B=-1}$

$$\frac{z+1}{(z-1)(z^2+1)} = \frac{1}{z-1} - \frac{z}{z^2+1}$$

$$\therefore Z^{-1}[x(z)] = Z^{-1}\left[\frac{z}{z-1} - \frac{z^2}{z^2+1}\right]$$

$$= \boxed{1 - \cos\frac{n\pi}{2}}$$

Madhumitha

5) $X(z) = \dfrac{z^2}{z^3 - z^2 + z - 1}$

$\dfrac{X(z)}{z} = \dfrac{z}{(z-1)(z^2+1)} = \dfrac{A}{z-1} + \dfrac{Bz+C}{z^2+1}$

$z = A(z^2+1) + (Bz+c)(z-1)$

when $z = 1$; $1 = A(1+1) \Rightarrow \boxed{A = 1}$

when $z = 0$; $0 = A + c(-1)$

$\Rightarrow \boxed{c = -1}$

when $z = -1$; $-1 = A(2) + (-B+c)(-2)$

$\cancel{2} = \cancel{2} + 2B + 2 \Rightarrow \boxed{B = -1}$

Thus, $\dfrac{z}{(z-1)(z^2+1)} = \dfrac{1}{z-1} - \dfrac{z+1}{z^2+1}$

$X(z) = \dfrac{z}{z-1} - \dfrac{z^2}{z^2+1} - \dfrac{z}{z^2+1}$

$z^{-1}[X(z)] = 1 - \cos\left(\frac{n\pi}{2}\right) - \sin\left(\frac{n\pi}{2}\right)$

Madhumitha.