Duality

16 February 2021 09:58

Rules for Constructing dual

Primal Problem Objective	Dual Problem Objective	Dual Problem Constraint type	Dual Problem Variable Sign
Maximization ✓	Minimization ✓	$\geq$ ✓	Unrestricted
Minimization	Maximization	$\leq$	Unrestricted

Eg 1:

$$\text{Max} \quad Z = 5x_1 + 12x_2 + 4x_3$$

s.t.

$$x_1 + 2x_2 + x_3 \leq 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3 \geq 0$$

Convert the above LPP to Dual.

Soln

Writing the given Primal in standard form,

$$\text{Max} \quad Z = 5x_1 + 12x_2 + 4x_3 + 0x_4$$

s.t.

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

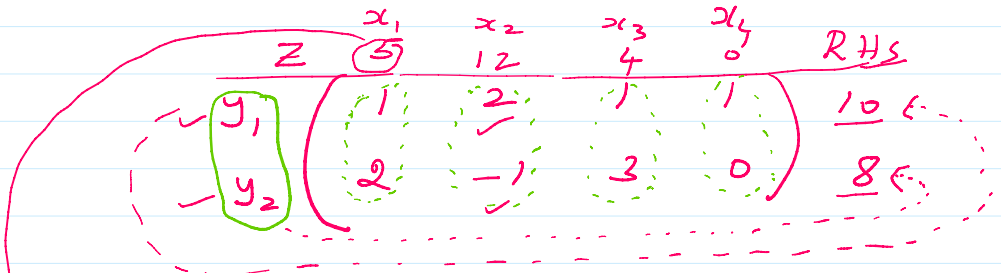
$$2x_1 - x_2 + 3x_3 = 8 \quad \text{(No. Artificial Variables)}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

In matrix form,

x_1 x_2 x_3 x_4 RHS

In matrix form,



Dual is,

Minimize $Z = 10\,y_1 + 8\,y_2$

s.t.

$$y_1 + 2y_2 \geq 5$$

$$2y_1 - y_2 \geq 12$$

$$y_1 + 3y_2 \geq 4$$

$$y_1 \geq 0$$

$y_1^{\times}$, y_2 are unrestricted

$$y_1 \geq 0$$

y_2 unrestricted

Dual

Min $Z = 10\,y_1 + 8\,y_2$

s.t.

$$y_1 + 2y_2 \geq 5$$

$$2y_1 - y_2 \geq 12$$

$$y_1 + 3y_2 \geq 4$$

$$y_1 \geq 0$$

y_2 unrestricted

2) Min $Z = 15\,x_1 + 12\,x_2$

s.t.

s.t.

$$x_1 + 2x_2 \geq 3$$

$$2x_1 - 4x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

Write dual.

Ans

Primal → Standard

$$\text{Min } Z = 15x_1 + 12x_2 + 0x_3 + 0x_4$$

s.t.

$$x_1 + 2x_2 - x_3 = 3$$

$$2x_1 - 4x_2 + x_4 = 5$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Matrix

	x_1	x_2	x_3	x_4	RHS
Z	15	12	0	0	
y_1	1	2	−1	0	3
y_2	2	−4	0	1	5

Dual is,

$$\text{Max } Z = 3y_1 + 5y_2$$

s.t.

$$y_1 + 2y_2 \leq 15$$

$$2y_1 - 4y_2 \leq 12$$

$-y_1 \le 0 \longrightarrow$ $y_1 \ge 0$ ✓

$y_2 \le 0 \longrightarrow$ $y_2 \le 0$ ✓

$\times$ y_1, y_2 are unrestricted

Dual is

Max $Z = 3y_1 + 5y_2$

s.t.

$y_1 + 2y_2 \le 15$

$2y_1 - 4y_2 \le 12$

$y_1 \ge 0$

$y_2 \le 0$

3) Max $Z = 5x_1 + 6x_2$

s.t.

$x_1 + 2x_2 = 5$

$-x_1 + 5x_2 \ge 3$

$4x_1 + 7x_2 \le 8$

x_1 unrestricted

$x_2 \ge 0$

Ans:

Since x_1 is unrestricted,

we can write $x_1 = x_1' - x_1''$, $x_1', x_1'' \ge 0$

Rewriting Primal in standard form

Max $Z = 5x_1' - 5x_1'' + 6x_2 + 0x_3 + 0x_4$

s.t.

$x_1' - x_1'' + 2x_2 = 5$

$-x_1' + x_1'' + 5x_2 - x_3 = 3$

$4x_1' - 4x_1'' + 7x_2 + x_4 = 8$

$$4x_1' - 4x_1'' + 7x_2 + x_4 = 8$$

$$x_1', x_1'', x_2, x_3, x_4 \geq 0$$

	Z	x_1'	x_1''	x_2	x_3	x_4	RHS
		5	−5	6	0	0	
y_1		1	−1	2	0	0	5
y_2		−1	1	5	−1	0	3
y_3		4	−4	7	0	1	8

Dual is,

$$\text{Min} \quad Z = 5y_1 + 3y_2 + 8y_3$$

s.t.

$$y_1 - y_2 + 4y_3 \geq 5$$
$$-y_1 + y_2 - 4y_3 \geq -5$$
$$2y_1 + 5y_2 + 7y_3 \geq 6$$
$$-y_2 \geq 0$$
$$y_3 \geq 0$$

y_1, y_2, y_3 unrestricted

$y_1 - y_2 + 4y_3 \geq 5$ ✓
$y_1 - y_2 - 4y_3 \leq 5$ ✓

$$\text{Min} \quad Z = 5y_1 + 3y_2 + 8y_3$$

s.t

$$y_1 - y_2 + 4y_3 = 5$$
$$2y_1 + 5y_2 + 7y_3 \geq 6$$
$$y_2 \leq 0 \checkmark$$
$$y_3 \geq 0 \checkmark$$
$$y_1 \text{ unrestricted}$$

Dual.