

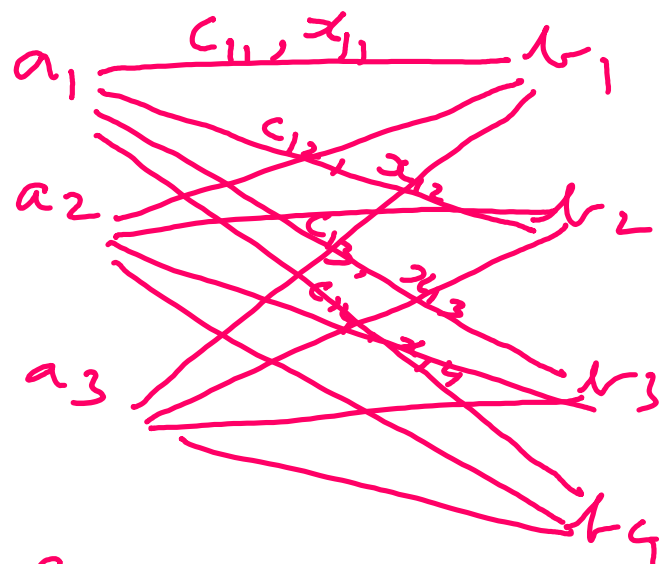
Transportation Problem

02 March 2021 10:01

Transportation Model

Transporting a single item from a given set of supply points to a given set of destination points.

(eg)



a_i
(Supply points
or source)

$i = 1, 2, 3$

In general.

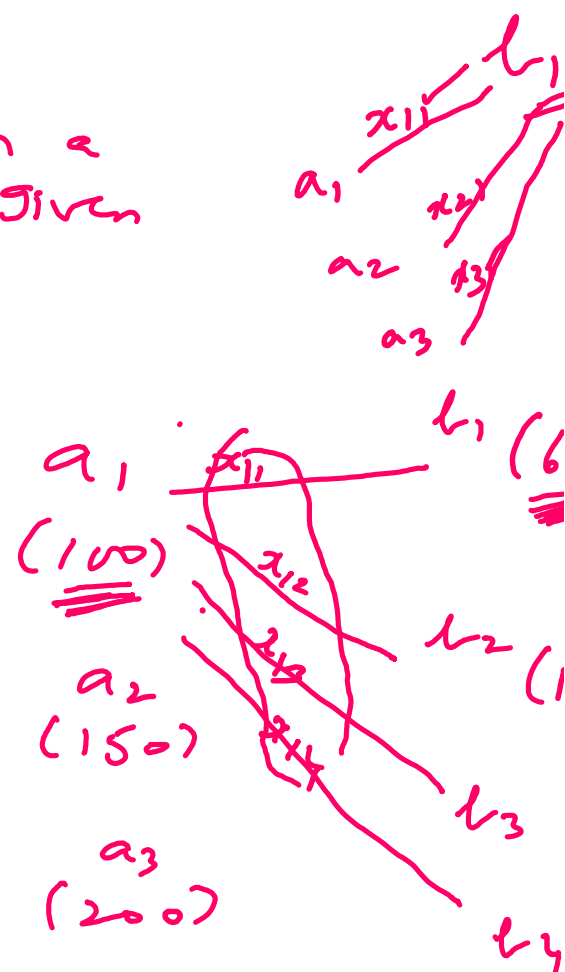
$i = 1, 2, \dots, m$

b_j
(demand points)
or destination

$j = 1, 2, 3, 4$

In general

$j = 1, 2, \dots, n$



x_{11}

x_{12}

x_{13}

x_{14}

C_{ij} — cost of transporting one unit from supply point 'i' to demand point 'j'.

x_{ij} — The quantity shipped from source 'i' to dest 'j'.

Objective

Minimize total transportation cost satisfying supply and demand restriction

Transportation represented as LPP as follows.

$$\text{Min. } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

s.t.

$$\sum_{j=1}^n x_{ij} = a_i \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, 2, \dots, n$$

prev example

$$x_{11} + x_{12} + x_{13} + x_{14} = a_1 \checkmark$$

$$\underline{x_{11}} + \underline{x_{21}} + \underline{x_{31}} = \underline{b_1}$$

$$x_{ij} \geq 0 \quad i=1, 2, \dots, m \\ j=1, 2, \dots, n$$

$$\boxed{\text{Total Supply} = \text{Total Demand}}$$

is called Balanced Transportation Problem

$$\boxed{\sum_i a_i = \sum_j b_j}$$

Solution to Transportation Problem

Step 1 : Initial Basic Feasible Solution

Method I : North West Corner rule.

eg:

Sunray Transport Company ships
.....
(Refer Taha)

		Mill				
		1	2	3	4	Supply
Silo	1	10 x_{11}	2 x_{12}	20 x_{13}	11 x_{14}	15
	2	12 x_{21}	7 x_{22}	9 x_{23}	20 x_{24}	25
	3	4 x_{31}	14 x_{32}	16 x_{33}	18 x_{34}	10
Demand		5	15	15	15	50

100's
of dollars

m Sources

n destination

m + n - 1 constraint eqns

$m+n$
 $m+n-1$ basic variables.

$$m=3, n=4$$

$$m+n=7 \text{ eqns}$$

No. of basic Variables $m+n-1$
 $= 7-1=6$ variables.

		Mill.				
		1	2	3	4	Supply
Silo	1	5	10	20	1	15
	2	12	7	9	2	25
	3	4	4	11	18	10
	Demand	5	15	15	10	

1st cell North
West

$$x_{11} = 5, \quad x_{12} = 10$$

$$x_{22} = 5, \quad x_{23} = 15, \quad x_{24} = 5$$

$$x_{34} = 10$$

The cost is,

$$\begin{aligned}
 &= (5 \times 10) + (2 \times 10) + (7 \times 5) \\
 &\quad + (9 \times 15) + (20 \times 5) \\
 &\quad + (18 \times 10)
 \end{aligned}$$

$$= 50 + 20 + 35 + 135 + 100 + 180$$

$$= 520$$

Cost is \$ 520

IBFS, $Z = 520$

$$x_{11} = 5, \quad x_{12} = 10, \quad x_{22} = 5$$

$$x_{23} = 15, \quad x_{24} = 5, \quad x_{34} = 10$$