WORKSHEET-1

1.)

a. $5n^6 + n^3 + n^2\log n - n = O(n^6)$

correct, because 6 is the largest power of n, dominates the remaining combinations.

b. $10n^3 + 5n\log n = O(n^3)$

correct because $n^3 > \log n \times n$

c. $\log(n^3) = O(\log n)$

correct because $\log(n^3) = 3\log n$
$= O(\log n)$

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2) (a) $T(n) = 2T(n/2) + n^3$

$T(n/2) = 2T(n/2^2) + \frac{n^3}{8}$

$\therefore T(n) = 2[2T(n/4) + n^3/8] + n^3$

$= 4T(n/4) + n^3(1 + 1/4)$

$T(n/4) = 2T(n/8) + n^3/4^3$

$T(n) = 4\left[2T\left(\frac{n}{8}\right) + \frac{n^3}{4^3}\right] + n^3\left[1 + \frac{1}{4}\right]$

$\Rightarrow T(n) = 8T\left(\frac{n}{8}\right) + n^3\left(\frac{1}{4} + \frac{1}{4^2} + 1\right)$

$\vdots$

$T(n) = 2^kT\left(\frac{n}{2^k}\right) + n^3\left[1 + \frac{1}{4} + \frac{1}{4^2} + \dots \frac{1}{4^k}\right]$

$= 2^kT\left(\frac{n}{2^k}\right) + n^3\left[\frac{1 - (1/4)^{k+1}}{1 - 1/4}\right]$

$\frac{n}{2^k} = 1 \quad 2^k = n \quad k = \log_2 n$

$= n\,T(1) + n^3 \times \frac{4}{3}\left[1 - \left(\frac{1}{4}\right)^{\log_2 n + 1}\right]$

$= n + \frac{4n^3}{3}\left(1 + \frac{1}{4n^2}\right)$

$\boxed{= O(n^3)}$

(b) $T(n) = T(9n/10) + n$

$T[9n/10] = T\left(\frac{9^2}{10^2} n\right) + \frac{9}{10} n$

$T(n) = T\left(\frac{9^2}{10^2} n\right) + n\left(1 + \frac{9}{10}\right)$

$\vdots$

$T(n) = T\left(\left(\frac{9}{10}\right)^k n\right) + n\left[1 + \frac{9}{10} + \left(\frac{9}{10}\right)^2 + \dots\right]$

$\left(\frac{9}{10}\right)^k n = 1 \rightarrow n = \left(\frac{10}{9}\right)^k \quad k = \log_{10/9} n$

$T(n) = 1 + n\left[\frac{1 - (9/10)^{k+1}}{1 - 9/10}\right]$

$= 1 + 10n\left[1 - \left(\frac{9}{10}\right)^{\log_{\frac{10}{9}} n + 1}\right]$

$= 1 + 10n\left(1 - \frac{1}{\frac{10}{9} n}\right) = 1 + 10n\left(1 - \frac{9}{10n}\right)$

$\boxed{= O(n)}$

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3) $T(n) = T(n/4) + 1$

$T(n/4) = T(n/4^2) + 1$

$\vdots$

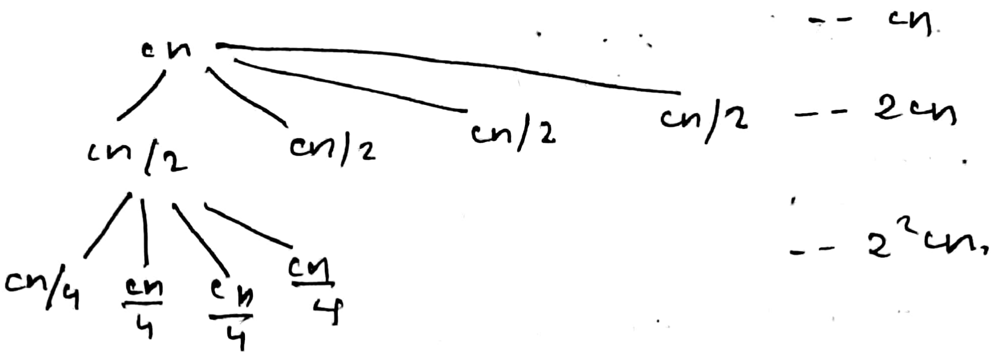
$T(n) = T(n/4^2) + 2 \Rightarrow T(n) = T(n/4^k) + k.$

$n/4^k = 1 \quad n = 4^k \quad k = \log_4 n$

$T(n) = T(1) + \log_4 n$

$\Rightarrow \boxed{O(\log_4 n)}$

4) $T(n) = 4T(n/2) + cn$.



$\Rightarrow cn, \frac{cn}{2}, \frac{cn}{2^2}, \ldots \frac{cn}{2^i} = 1$

$2^i = cn \rightarrow i = \log_2 cn$

$\text{sum} = cn[1 + 2 + 2^2 + \ldots]$

$= cn\left[\frac{2^{i+1} - 1}{2-1}\right] = cn\left[2^{\log_2 cn + 1} - 1\right]$

$= cn(2cn - 1) \Rightarrow \boxed{O(n^2)}$

$T(n) = 4T(n/2) + cn$

$T(n/2) = 4T(n/2^2) + cn/2$

$T(n) = 4[4T(n/2^2) + cn/2] + cn$

$= 4^2 T(n/2^2) + cn(1 + 2)$

$\vdots$

$T(n) = 4^k T(n/2^k) + cn(1 + 2 + 2^2 + \ldots 2^k)$

$= 4^k T\left(\frac{n}{2^k}\right) + cn\left(\frac{2^{k+1} - 1}{2-1}\right)$

$\frac{n}{2^k} = 1 \rightarrow k = \log_2 n$

$= 4^{\log_2 n} + cn[2^{\log_2 n + 1} - 1]$

$= n^2 + cn[2n - 1]$

$\boxed{= O(n^2)}$

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6)

a) for (i=0 ; i<n ; i++)

m += i $\rightarrow n$

This loop iterates n times. $\therefore$ $\boxed{O(n)}$

b) for (i=0 ; i<n ; i++) — n times

for (j=0 ; j<n ; j++) — n times

sum[i] += array[i][j] $\} \rightarrow \boxed{O(n^2)}$

c) for (i=0 ; i<n ; i++)

for (j=0 ; j<i ; j++)

m += j

$$\left.\begin{array}{l} i = 0 \text{ to } n-1 \\ j = 0, 1, 2 \ldots n-2 \end{array}\right\} \quad \sum_{i=1}^{n-1} i = \frac{(n-1)n}{2} = \frac{n^2-n}{2}$$

$$= \boxed{O(n^2)}$$

d) i = 1 ;

while (i<n) { tot += i ;

i = i * 2 ;

}

$2^k = n \quad k = \log_2 n \Rightarrow \boxed{O(\log n)}$

e) i = n ;

while (i > 0)

{ tot += i ;

i = i / 2 ;

}

$\frac{1}{2^k} = n$

$k = \log_2 n$

$\boxed{O(\log n)}$

f) for(i=0; i<n; i++) — n times

for(j=0; j<n; j++) — n times

for(k=0; k<n; k++) — n times

sum[i][j] += entry[i][j][k];

$\therefore$ $n \times n \times n = \boxed{O(n^3)}$

g) for(i=0; i<n; i++) — n times

for(j=0; j<n; j++) — n times

sum[i] += entry[i][j] — $n \times n$ times

$\boxed{= O(n^2)}$

h) for(i=0; i<n; i++)

for(j=0; j<$\sqrt{n}$; j++) — $O(n \cdot \sqrt{n})$

m += j; $\Rightarrow$ $\boxed{O(n^{3/2})}$

i) for i=0 to n-2

do {

for j=i+1 to n-1

do { If A[i, j] != A[j, i]

return false

}

return true

}

i = 0 to n-2

j = n-2 to 1

| |

n-1-(1) n-1-(n-2)

$$\sum_{j=1}^{n-2} 1 = \frac{n-2(n-1)}{2}$$

$\boxed{= O(n^2)}$

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b) $T(n) = 2T(n/2+17)+n$

$T(n/2) = 2T(n/2^2+17)+n/2$

$T(n) = 2[2T(n/2^2+17)+n/2]+n$

$T(n) = 2^2 T(n/2^2+17)+2n$

$T(n/2^2) = 2T(n/2^3+17)+n/2^2$

$T(n) = 2^3 T(n/2^3+17)+3n$

⋮

$T(n) = 2^k T(\frac{n}{2^k}+17)+kn$

$\frac{n}{2^k}+17 = 18$, $\frac{n}{2^k} = 1$, $k = \log_2 n$

$T(n) = n + n\log n \Rightarrow \boxed{O(n \log n)}$

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2) a) $3x^2$, x^2+5

$$\lim_{x\to\infty} \frac{3x^2}{x^2+5} = \lim_{x\to\infty} \frac{6x}{2x} = 3$$

Hence order of growth is same

b) x and $\ln x$.

$$\lim_{x\to\infty} \frac{x}{\ln x} = \lim_{x\to\infty} \frac{1}{1/x} = \infty$$

Hence order growth of $x > \ln x$.

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3) show that $5n^2 - 6n = \Theta(n^2)$

$c_1 n^2 \le 5n^2 - 6n \le c_2 n^2$

for several values of c_1 & c_2 including

For eg. $c_1 = 1$, $c_2 = 10$, this is possible

hence $5n^2 - 6n = \boxed{\Theta(n^2)}$.

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9)

1. $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2, c = 2$

$c < \log_b a \Rightarrow \boxed{T(n) = O(n^2)}$

2. $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2, c = 2$

$c = \log_b a \Rightarrow \boxed{T(n) = \Theta(n^2 \log n)}$

3. $T(n) = T(n/2) + 2^n$

conditions does not imply (not n^a format)

$\Rightarrow$ Masters theorem not applicable.

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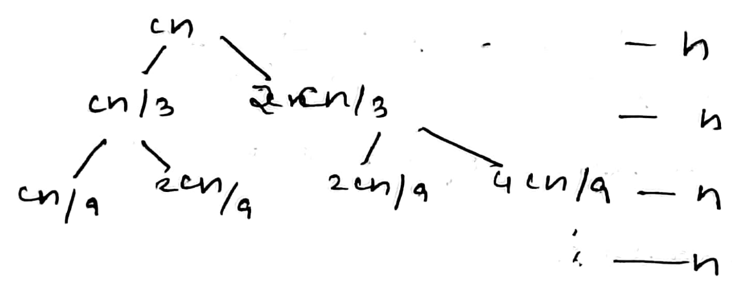
10) $T(n) = T(n/3) + T(2n/3) + cn$



compared to $\frac{cn}{3}$, $\frac{2cn}{3}$ has a longer branch

so for max no: of levels,

$$\frac{cn}{\left(\frac{3}{2}\right)^i} = 1 \Rightarrow (3/2)^i = cn$$

$$i = \log_{3/2} cn$$

Total time $= n + n + \ldots + n$ i times

$= i \cdot n = \log_{3/2} cn \cdot n$

$\Rightarrow \boxed{O(n \log n)}$