Primal Dual optimal solution

19 February 2021 11:58

eg :

$$\text{Max } Z = 5x_1 + 12x_2 + 4x_3$$

s.t.

$$x_1 + 2x_2 + x_3 \leq 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3 \geq 0$$

Big 'M'

Solving Primal

$$\text{Max } Z = 5x_1 + 12x_2 + 4x_3 + 0x_4 - MA_1$$

s.t.

$$\left.\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 10 \\ 2x_1 - x_2 + 3x_3 + A_1 &= 8 \end{aligned}\right\}$$

$$x_1, x_2, x_3, x_4, A_1 \geq 0$$

The optimal Primal table (Big 'M' method)

Basic	x_1	x_2	x_3	x_4	A_1	Soln RHS
Z row	0	0	$3/5$	$29/5$	$-2/5 + M$	$\frac{274}{5}$
x_2	0	1	$-1/5$	$2/5$	$-1/5$	$12/5$
x_1	1	0	$7/5$	$1/5$	$2/5$	$26/5$

To find optimal dual

Method I :

		x_4	A_1
1)	Starting Primal basic variables	x_4	A_1
2)	Z-row Coefficients	$29/5$	$-2/5 + M$
3)	original obj Coefficient fn.	0	$-M$
4)	Dual Variables	y_1	y_2
5)	Optimal dual Values	$\frac{29}{5} + 0 = 29/5$	$-2/5 + \cancel{M} - \cancel{M} = -2/5$

$$y_1 = 29/5 , \quad y_2 = -2/5$$

objective fn of dual

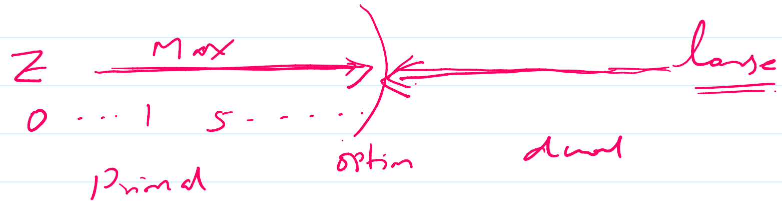
$$\underline{\text{Min}}\ Z = 10 y_1 + 8 y_2$$

$$= 10 \left(29/5\right) + 8 \times \left(-2/5\right)$$

$$= \frac{290 - 16}{5} = \underline{\underline{\frac{274}{5}}}$$

$$\boxed{\left.\begin{array}{l}\text{dual}\\ \text{soln}\end{array}\right\} \longrightarrow \underline{y_1 = 27/5} \quad \underline{y_2 = -2/5} \quad \text{Min } Z = \frac{274}{5}}$$

$\boxed{\text{Max Primal} = \text{Min dual}}$



Method 2

The optimal inverse matrix in the table under starting basic variables x_4 & A_1

$$\text{optimal Inverse} = \begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix}$$

Order of Primal basic Variables in optimal table is (x_2, x_1)

original objective coefficient is $(12, 5)$

optimal dual values = original objective fn coeff $\times$ optimal Inverse

$$(y_1, y_2) = (12, 5) \times \begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix}$$

$$(y_1, y_2) = \left(\tfrac{29}{5}, -\tfrac{2}{5}\right)$$

$$\boxed{\begin{array}{l} \therefore \text{dual Soln } \; y_1 = 29/5,\; y_2 = -2/5 \\ \& \; Z = 274/5 \end{array}}$$

Economic Interpretation of dual variable

Reddy Mikk's Problem

$$\text{Max } Z = 5x_1 + 4x_2$$

s.t.

$$6x_1 + 4x_2 \le 24 \text{ (Resource M1)}$$

$$x_1 + 2x_2 \le 6 \text{ (Resource M2)}$$

$$-x_1 + x_2 \le 1 \text{ (Resource Market)}$$

$$x_2 \le 2 \text{ (Resource demand)}$$

$$x_1, x_2 \ge 0$$

Opt. Soln.

$x_1 = 3$, $x_2 = 1.5$, $Z = 21$

$$\boxed{\begin{array}{l} \text{Slack } x_5 = 5/2 \\ x_6 = 1/2 \\ x_3 = x_4 = 0 \end{array}}$$

Dual (Reddy mikks)

$$\text{Min } W = 24y_1 + 6y_2 + y_3 + 2y_4$$

s.t.

$$6y_1 + y_2 - y_3 \ge 5$$

$$4y_1 + 2y_2 + y_3 + y_4 \geq 4$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Optimal soln

$y_1 = 0.75$, $y_2 = 0.5$

$y_3 = 0$, $y_4 = 0$

$Z = 21$

Meaning

dual Price (Shadow Price)

(or) worth per unit of resource

dual Price of Resource M1 is 0.75 ✓

" " M2 is 0.5 ✓

M3 is 0 } not critical
M4 is 0 } in determining optimal soln.

slack	
x_3	y_1
x_4	y_2
x_5	y_3
x_6	y_4

Primal Slack x_5, x_6 } they are not zero.

$\Downarrow$

Corresponding dual Values are 0.

Primal Slack x_3, x_4 } they are zero

Primal slack
x_3, x_4 } they are zero

⇓

Corresponding dual values
are not 0.

Slack +ve } Resources are
available

Slack 0 } Resources are
not available