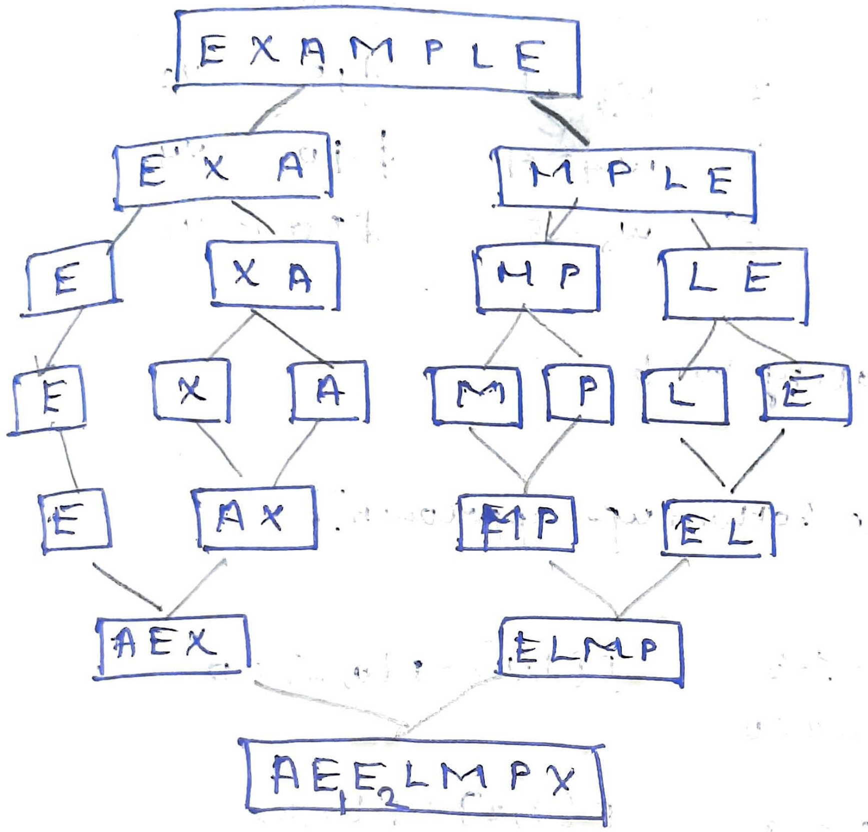
DIVIDE AND CONQUER; GREEDY APPROACH

1) Apply merge sort to sort the list E, X, A, M, P, L, E in alphabetical order. Check if Algorithm is stable.



Here, the Algorithm is stable because 1st E in EXAMPLE is before 2nd E.

Stable algorithm - if 2 objects with equal keys appear in same order in sorted o/p as they appear in i/p array.

2) Apply Quick sort to list A, B, C, D, E in alphabetical order. check no: of comparisons by choosing

i) First element as pivot

A B C D E | comparisons

p = A i = 0 j = 4

i = 1	B > A go to j	1
j = 4	E > A	2
j = 3	D > A	3
j = 2	C > A	4
j = 1	B > A	5
j = 0	A > A (false)	6

since i > j, swap A[j] and p

B C D E

p = B i = 0 j = 3

i = 1	C > B go to j	7
j = 3	E > B	8
j = 2	D > B	9
j = 1	C > B	10
j = 0	B ≤ B	11

since i > j, swap A[j], p

C D E

P = C i = 0 j = 0

i = 1	D > C go to j	12
j = 2	E > C	13
j = 1	D > C	14

j = 0 C ≤ C 15

since i > j swap A[j] & p

D E

p = D i = 0 j = 1

i = 1 E > D go to j 16

j = 1 E > D 17

j = 0 D ≤ D 18

swap A[j] and p

E - No comparison [∵ 1 element]

∴ No: of comparisons = $\boxed{18}$

ii)

A B C D E

p = E i = 0 j = 4

		comparisons
i = 0	A ≤ E	1
i = 1	B ≤ E	2
i = 2	C ≤ E	3
i = 3	D ≤ E	4
i = 4	E ≤ E	5

i out of bounds
swap A[j] and p

A B C D

p = D i = 0 j = 3

i = 0	A ≤ D	6
i = 1	B ≤ D	7
i = 2	C ≤ D	8
i = 3	D ≤ D	9

i out of bounds, swap A[j], P

A B C

$p = C$ $i = 0$ $j = 2$

$i = 0$ $A \leq C$ 10

$i = 1$ $B \leq C$ 11

$i = 2$ $C \leq C$ 12

i out of bounds, swap A[j] & P

A B

$p = B$ $i = 0$ $j = 1$

$i = 0$ $A \leq B$ 13

$i = 1$ $B \leq B$ 14

i out of bounds, swap A[j], P

A - single element (No comparison)

$\therefore$ No: of comparisons = $\boxed{14}$

3) Apply strassen's algorithm to compute

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 4 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 5 & 0 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 4 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & 5 & 0 \end{bmatrix}$$

A :

$a_{00} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ $a_{01} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ $a_{10} = \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix}$

$a_{11} = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

$b_{00} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ $b_{01} = \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix}$ $b_{10} = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}$

$b_{11} = \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$

$M1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$

$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix}$

Thus for M1 ;

$m1 = (a_{00} + a_{11}) * (b_{00} + b_{11}) = 6 * 2 = 12$

$m2 = (a_{10} + a_{11}) * b_{00} = 8$

$m3 = a_{00} * (b_{01} - b_{11}) = 4$

$m4 = a_{11} * (b_{10} - b_{00}) = 12$

$m5 = (a_{00} + a_{01}) * b_{11} = 4 * 1 = 4$

$m6 = (a_{10} - a_{00}) * (b_{00} + b_{01}) = 2 * 3 = 6$

$m7 = (a_{01} - a_{11}) * (b_{10} + b_{11}) = -2 * 8 = -16$

$m_1 + m_4 - m_5 + m_7 = 12 + 12 - 4 - 16$

$\boxed{= 4}$

$m_3 + m_5 = \boxed{8}$

$m_2 + m_4 = \boxed{20}$

$m_1 + m_3 - m_2 + m_6 = \boxed{14}$

Thus, $M1 = \begin{bmatrix} 4 & 8 \\ 20 & 14 \end{bmatrix}$

$m_2 = (a_{10} + a_{11}) * b_{00}$

$$= \left(\begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \right) * \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 7 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

For M_2 :

$m_1 = 4 * 1 = 4 = (a_{00} + a_{11})(b_{00} + b_{11})$

$m_2 = (a_{10} + a_{11}) * b_{00} = 0$

$m_3 = a_{00} * (b_{01} - b_{11}) = 0$

$m_4 = a_{11} * (b_{10} - b_{00}) = 2$

$m_5 = (a_{00} + a_{01}) * b_{11} = 4$

$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{11}) = 4$

$m_7 = (a_{10} - a_{11}) * (b_{10} + b_{11}) = 0$

$m_1 + m_4 - m_5 + m_7 = 4 + 2 - 4 + 0 = 2$

$m_3 + m_5 = 4$

$m_2 + m_4 = 0 + 2 = 2$

$m_1 + m_3 + m_6 - m_2 = 4 + 0 - 0 + 4$

$= 8$

$$M_2 = \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix}$$

$M_3 = a_{00} * (b_{01} - b_{11})$

$$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 \\ -5 & 4 \end{bmatrix}$$

For M_3 :

$m_1 = 2 \times 3 = 6$

$m_2 = 5 * -1 = -5$

$m_3 = -1 * -4 = -4$

$m_4 = 1 * -4 = -4.$

$m_6 = 3 * -1 = -3$

$m_5 = 1 * 4 = 4$

$m_7 = -1 * -1 = 1$

$m_1 + m_4 - m_5 + m_7 = 6 + -4 - 4 + 1 = -1$

$m_3 + m_5 = 0$

$m_2 + m_4 = -9$

$m_1 + m_3 - m_2 + m_6 = 4$

$$M_3 = \begin{bmatrix} -1 & 0 \\ -9 & 4 \end{bmatrix}$$

$M_4 = a_{00} * (b_{10} - b_{00})$

$$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix}$$

$m_1 = 4 \times 4 = 16$

$m_2 = 3 \times 2 = 6$

$m_3 = 3 \times -3 = -9$

$m_4 = 1 \times -3 = -3$

$m_5 = 3 \times 2 = 6$

$m_6 = -1 * 1 = -1$

$m_7 = -1 \times 1 = -1$

$m_1 + m_4 - m_5 + m_7 = 6$

$m_3 + m_5 = -3$

$m_2 + m_4 = 3$

$m_1 + m_3 - m_2 + m_6 = 0$

$$M4 = \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix}$$

$M5 = (a00 + a01) * L11$

$$= \left(\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \right) * \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$m_1 = 4 \times 1 = 4$

$m_2 = 6 \times 1 = 6$

$m_3 = 3 \times 1 = 3$

$m_4 = 4 \times 1 = 4$

$m_5 = 4 \times 0 = 0$

$m_6 = 2 \times 2 = 4$

$m_7 = 0 \times 5 = 0$

$m_1 + m_4 - m_5 + m_7 = 8$

$m_3 + m_5 = 3$

$m_2 + m_4 = 10$

$m_1 + m_3 - m_2 + m_6 = 5$

$$M5 = \begin{bmatrix} 8 & 3 \\ 10 & 5 \end{bmatrix}$$

$M6 = (a10 - a00) * (b00 + b01)$

$$= \left(\begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \right) * \left(\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 0 & 2 \\ 2 & 5 \end{bmatrix}$$

$m_1 = -2 \times 5 = -10$

$m_2 = 0 \times 0 = 0$

$m_3 = -1 \times -3 = 3$

$m_4 = -1 \times 2 = -2$

$m_5 = 0 \times 5 = 0$

$m_6 = 2 \times 2 = 4$

$m_7 = 2 \times 7 = 14$

$m_1 + m_4 - m_5 + m_7 = 2$

$m_3 + m_5 = 3$

$m_2 + m_4 = -2$

$m_1 + m_3 - m_2 + m_6 = -3$

$$M6 = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$$

$$M_7 = (a_{01} - a_{11}) * (b_{10} + b_{11})$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 6 & 3 \end{bmatrix}$$

$m_1 = -2 \times 6 = -12$

$m_2 = -2 \times 3 = -6$

$m_3 = -1 \times -2 = 2$

$m_4 = -1 \times 3 = -3$

$m_5 = 0 \times 3 = 0$

$m_6 = 0 \times 4 = 0$

$m_7 = 2 \times 9 = 18$

$m_1 + m_4 - m_5 + m_7 = 3$

$m_3 + m_5 = 2$

$m_2 + m_4 = -9$

$m_1 + m_3 - m_2 + m_6 = -4$

$$M_7 = \begin{bmatrix} 3 & 2 \\ -9 & -4 \end{bmatrix}$$

$$\therefore M_1 + M_4 - M_5 + M_7 = \begin{bmatrix} 4 & 8 \\ 20 & 44 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 8 & 3 \\ 20 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ -9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$M_3 + M_5 = \begin{bmatrix} -1 & 0 \\ -9 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 3 \\ 10 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 1 & 9 \end{bmatrix}$$

$$M_2 + M_4 = \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 5 & 8 \end{bmatrix}$$

$M_1 + M_3 - M_2 + M_6 =$

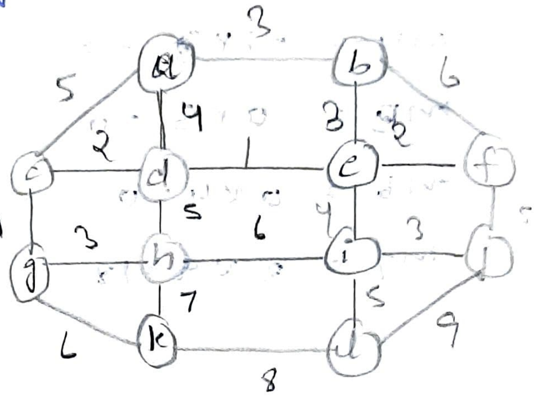
$$\begin{bmatrix} 4 & 8 \\ 20 & 44 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -9 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 7 \\ 7 & 7 \end{bmatrix}$$

Ans : $\begin{bmatrix} 5 & 4 & 7 & 3 \\ 4 & 5 & 2 & 9 \\ 8 & 1 & 3 & 7 \\ 5 & 8 & 7 & 7 \end{bmatrix}$

4 Find minimal spanning tree of graph given below using

i) Prim's Algorithm

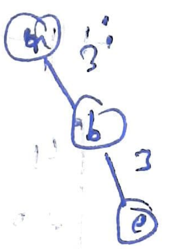


a(–,–) · b(a,3) 4
c(a,5)
d(a,4)
e(–,∞)
f(–,∞)
g(–,∞)
h(–,∞) k(–,∞)
i(–,∞) l(–,∞)
j(–,∞).

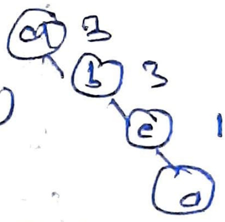
2) b(a,3) · c(a,5) · d(a,4) e(b,3) ·
f(b,6) g(–,∞) h(–,∞)
i(–,∞) j(–,∞) k(–,∞)
l(–,∞)

3) e(b,3) c(a,5) d(e,1) f(e,2)
g(–,∞) h(–,∞) i(e,4)
j(–,∞)

k(–,∞) l(–,∞)

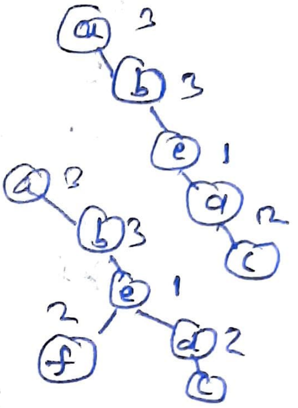


4) d(e,1) = c(d,2) f(e,2) g(-,∞) h(d,5) .
i(e,4) j(-,∞) k(-,∞) l(-,∞)

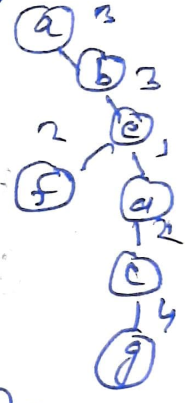


5) c(d,2) = f(e,2) g(c,4) h(d,∞) i(e,4)
j(-,∞) k(-,∞) l(-,∞)

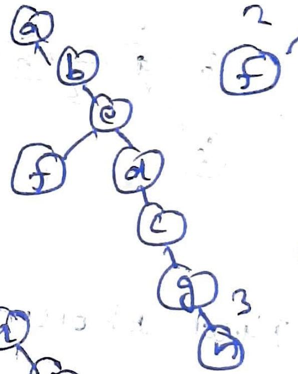
6) f(c,2) = g(c,4) h(d,5) i(e,4)
j(f,5) k(-,∞) l(-,∞)



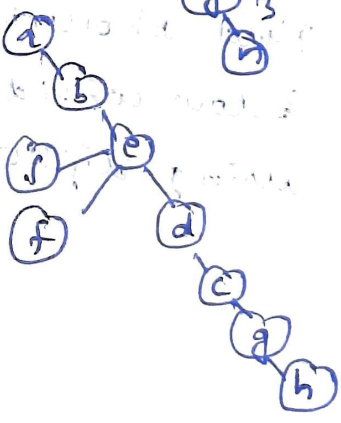
7) g(c,4) h(g,3) i(e,4) j(f,5) k(g,6)
l(-,∞)



8) h(g,3) i(e,4) j(f,5) k(g,6)
l(-,∞)



9) i(e,4) j(i,3) k(g,6)
l(i,5)

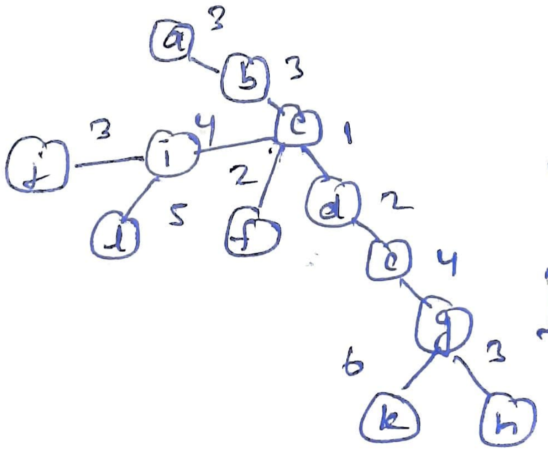


10) j(i,3) k(g,6) l(i,5)

11) l(i,5) k(g,6)

12) k(g,6) - empty

final Tree :



sum of weights

$\boxed{= 36}$

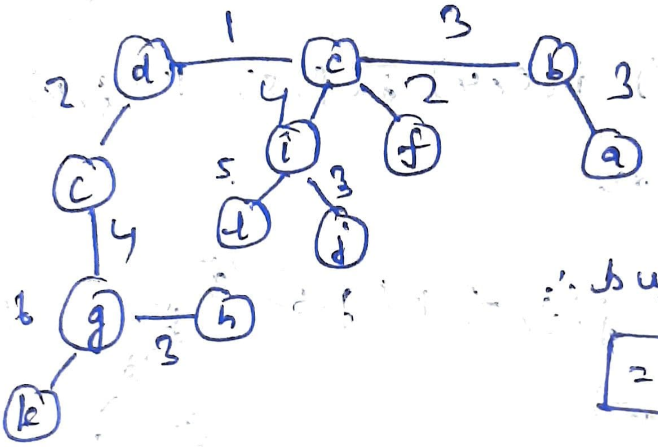
ii) Kruskal's Algorithm

1 de, 2 cd, 2 ef, 3 ab, 3 eb

3 gh, 3 ij, 4 ad, 4 eg, 4 ei

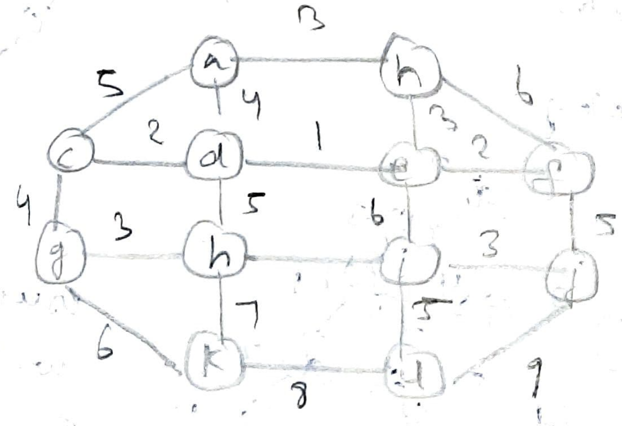
5 ac, 5 dh, 5 fj, 5 li, 6 bf, 6 hi, 6 gk

7 hk, 8 kl, 9 lf



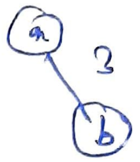
∴ sum of weights $\boxed{= 36}$

5) Find shortest path in the graph given below considering source vertex as 'a' using dijkstra's algorithm

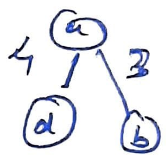


a(-,0)   b(a,3) c(a,5) d(a,4)
e(-,∞) f(-,∞) g(-,∞)
h(-,∞) i(-,∞) j(-,∞)
k(-,∞) l(-,∞)

b(a,3)   c(a,5) d(a,4) e(b,6)
f(b,9) g(-,∞) h(-,∞)
i(-,∞) j(-,∞) k(-,∞)
l(-,∞)

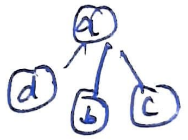


d(a,4)   c(a,5) e(d,5) f(b,9)
g(-,∞) h(d,9) i(-,∞)
j(-,∞) k(-,∞) l(-,∞)



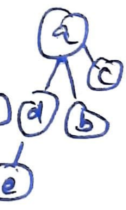
c(a,5)   e(d,5) f(b,9) g(c,9) h(d,9)
i(-,∞) j(-,∞) k(-,∞) l(-,∞)

e(d,5)   f(e,7) g(c,9) h(d,9) i(e,9)
j(-,∞) k(-,∞) l(-,∞)



f(e,7)   g(c,9) h(d,9) i(e,9) j(f,12)
k(-,∞) l(-,∞)

g(c,9)   h(d,9) j(f,12) k(g,15) l(-,∞)



h(d,9)   i(e,9) j(f,12)
k(g,15) l(-,∞)

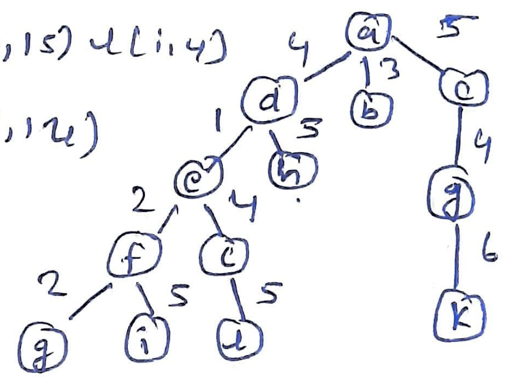
Front

i(e,9)   j(f,12) k(g,15) l(i,14)

j(f,12)   k(g,15) l(i,14)

l(i,14)   k(g,15)

k(g,15)



DYNAMIC PROGRAMMING

1)

item	weight	value	
1	$w_1 = 3$	\$25	v1
2	$w_2 = 2$	\$20	v2
3	$w_3 = 1$	\$15	v3
4	$w_4 = 4$	\$40	v4
5	$w_5 = 5$	\$50	v5

capacity $W = 6$

soln: [use bottom up approach]

$j - w_i = 1-3 = -2 < 0$ $\quad F(1,1) = F(0,1) = 0$

$j - w_i = 2-3 = -1 < 0$ $\quad F(1,2) = F(0,2) = 0$

$j - w_i = 3-3 = 0$ $\quad F(1,3) = \max[F(0,3), F(0,0) + 25]$

$= 25$

$j - w_i = 4-3 = 1 > 0$ $\quad F(1,4) = \max[F(0,4), F(0,1) + 25]$

$= \max[0, 25] = 25$

$j - w_i = 5-3 = 2 > 0$ $\quad F(1,5) = \max[F(0,5), F(0,2) + 25]$

$= 25$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	25	25	25	25
2	0	0	20	25	25	45	45
3	0	15	20	35	40	45	60
4	0	15	20	35	40	55	60
5	0	15	20	35	40	55	$\boxed{65}$

$j-w_i = 6-3 > 0$ $F(1,6) = \max[F(0,6), F(0,3)+25]$

$= 25$

similarly;

$j-w_i = 1-3 < 0$ $F(2,1) = F(0,1) = 0$

$j-w_i = 2-2 = 0$ $F(2,2) = \max[0, 20] = 20$

$j-w_i = 3-2 = 1 > 0$ $F(2,3) = \max[25, 20] = 25$

$j-w_i = 4-2 = 2 > 0$ $F(2,4) = \max[25, 20] = 25$

$j-w_i = 5-2 = 3 > 0$ $F(2,5) = \max[25, 45] = 45$

$j-w_i = 6-2 = 4 > 0$ $F(2,6) = \max[25, 45] = 45$

$j-w_i = 1-1 = 0$ $F(3,1) = \max[0, 15] = 15$

$j-w_i = 2-1 = 1 > 0$ $F(3,2) = \max[20, 15] = 20$

$j-w_i = 3-1 = 2 > 0$ $F(3,3) = \max[25, 35] = 35$

$j-w_i = 4-1 = 3 > 0$ $F(3,4) = \max[25, 40] = 40$

$j-w_i = 5-1 = 4 > 0$ $F(3,5) = \max[45, 40] = 45$

$j-w_i = 6-1 = 5 > 0$ $F(3,6) = \max[45, 60] = 60$

$j-w_i = 1-4 = -3 \not> 0$ $F(4,1) = F(3,1) = 15$

$j-w_i = 2-4 = -2 < 0$ $F(4,2) = F(3,2) = 20$

$j-w_i = 3-4 = -1 < 0$ $F(4,3) = F(3,3) = 35$

$j-w_i = 4-4 = 0$ $F(4,4) = max[40,40] = 40$

$j-w_i = 5-4 = 1$ $F[4,5] = max[45,55] = 55$

$j-w_i = 6-4 = 2 > 0$ $F[4,6] = max[60,60] = 60$

$j-w_i = -4$ $F[5,1] = F(4,1) = 15$

$j-w_i = -3$ $F[5,2] = F(4,2) = 20$

$j-w_i = -2$ $F[5,3] = F(4,3) = 35$

$j-w_i = -1$ $F[5,4] = F(4,4) = 40$

$j-w_i = 0$ $F[5,5] = max[55,50] = 55$

$j-w_i = 1$ $F[5,6] = max[60,65] = 65$

↳ Now, find what items are included in knapsack,

capacity = 6

i) $F(5,6) > F(4,6) \Rightarrow 65 > 60$

Hence, 5th item included.

ii) capacity = $6-5 = 1$

$F(5,1) = F(4,1) = 15$

$F(4,1) = F(3,1) = 15$

So consider $F(3,1) > F(2,1) \Rightarrow 15 > 0$

Hence, 3rd item is included.

Now capacity is $1-1 = 0$ (knapsack is full)

Therefore, 3rd & 5th items are used to fill knapsack and most optimal solution is F(5, 6) with value of 65.

2) To find Transitive closure (Warshall's Alg)

$$R_0 : A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3) To solve: All pairs shortest path (Floyd's Algorithm)

$$D_0 = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

$$D_0 = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & \infty & 4 & 0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 8 & 4 & 0 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 8 & 4 & 0 \end{bmatrix}$$

$$D_4 = \begin{bmatrix} 0 & 2 & 3 & 1 & 4 \\ 6 & 0 & 3 & 2 & 5 \\ \infty & \infty & 0 & 4 & 7 \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 6 & 4 & 0 \end{bmatrix}$$

$$D_5 = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 2 & 3 & 1 & 4 \\ b & 6 & 0 & 3 & 2 & 5 \\ c & 10 & 12 & 0 & 4 & 7 \\ d & 6 & 3 & 2 & 0 & 3 \\ e & 3 & 5 & 6 & 4 & 6 \end{array}$$