Sensitivity Analysis

- In LP, the parameters (input data) of the model can change within certain limits without causing the optimum solution to change. This is referred to as sensitivity analysis.

Graphical Sensitivity Analysis:

- Two cases will be considered:
  - Sensitivity of the optimum solution to changes in the availability of the resources (right-hand side of the constraints)
  - Sensitivity of the optimum solution to changes in unit profit or unit cost (coefficients of the objective function)

Changes in the Right-Hand Side

- JOBCO produces two products on two machines. A unit of product 1 requires 2 hours on machine 1 and 1 hour on machine 2. For product 2, a unit requires 1 hour on machine 1 and 3 hours on machine 2. The revenues per unit of products 1 and 2 are $30 and $20, respectively. The total daily processing time available for each machine is 8 hours.

- $x1$ = the daily number of units of products 1
- $x2$ = the daily number of units of products 2
- The LP model is given as

Maximize $z = 30x1 + 20x2$

Subject to

$2x1 + x2 \leq 8$ $x1$ (Machine 1)

$+ 3x2 \leq 8$ (Machine 2)

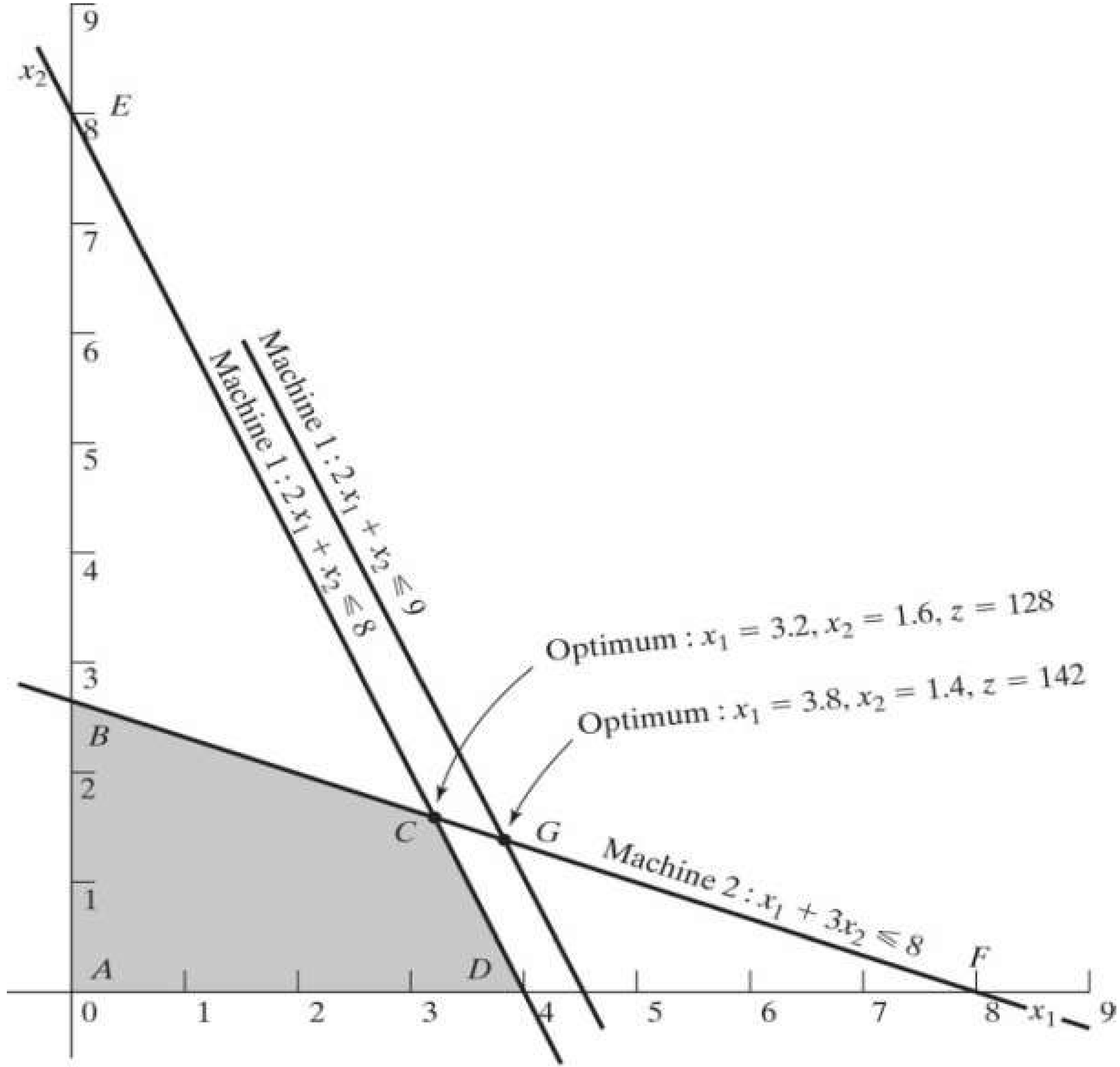
$x1, x2 \geq 0$

First: change in resources

- If the daily capacity is increased from 8 hours to 9 hours, the new optimum will occur at point G. The rate of change in optimum z resulting from changing machine 1 capacity from 8 hours to 9 hours can be computed as follows:

Rate of revenue change (point C to point G)

$$= \frac{zG - zC}{Capacity\ change} = \frac{142 - 128}{9 - 8} = \$\ 14.00\ /hr$$



Machine 1 : $2x_1 + x_2 \leqslant 9$
Machine 1 : $2x_1 + x_2 \leqslant 8$
Optimum : $x_1 = 3.2, x_2 = 1.6, z = 128$
Optimum : $x_1 = 3.8, x_2 = 1.4, z = 142$
Machine 2 : $x_1 + 3x_2 \leqslant 8$
x_2
x_1
A
B
C
D
E
F
G
0
1
2
3
4
5
6
7
8
9

Shadow Price

- Dual or Shadow price/unit worth of resource: is the change in the optimal objective value per unit change in the availability of the resource.
- e.g. a unit increase (decrease) in machine I capacity will increase (decrease) revenue by $14.00

Feasibility Range

- We can see that the dual price of \$14.00/hr remains valid for changes (increases or decreases) in machine 1 capacity that move its constraint parallel to itself to any point on the line segment BF.
- Minimum machine 1 capacity [at B = (0, 2.67)] =
  $2\times0+1\times2.67 = 2.67$ hr
- Maximum machine 1 capacity [at F = (8,0)] =
  $2\times8+1\times0=16$ hr
- The dual price of \$14.001hr will remain valid for the range
  2.67 hr ≤ Machine 1 capacity ≤ 16 hr

Changes outside this range will produce a different dual price.

- You can verify that the dual price for machine 2 capacity is \$2.00/hr and it remains valid for changes (increases or decreases) that move its Constraint parallel to itself to any point on the line segment DE.
- Minimum machine 2 capacity [at D = (4, 0)] =

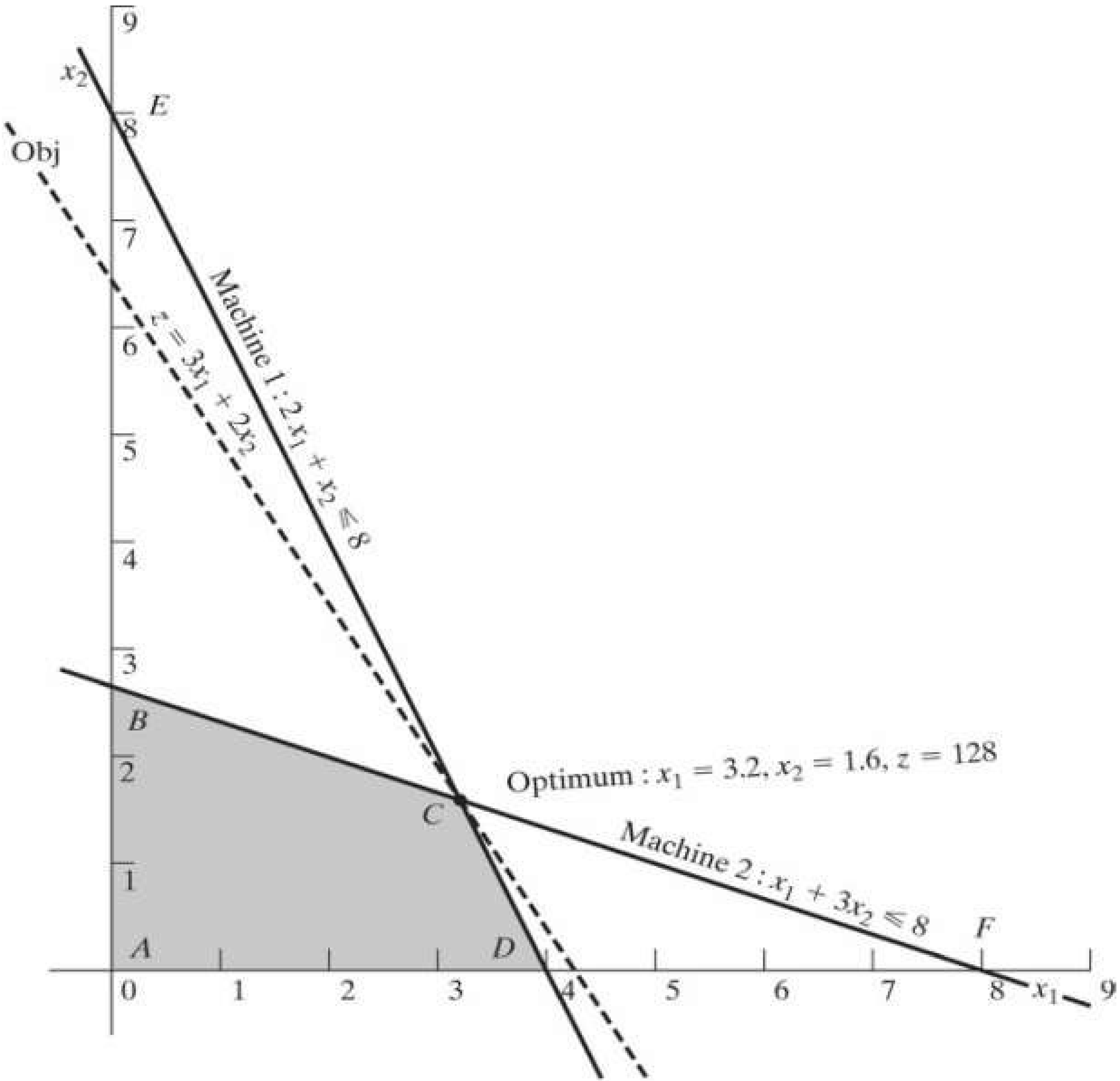
  $$1\times4+3\times0 = 4 \text{ hr}$$
- Maximum machine 2 capacity [at E = (0,8)] =

  $$1\times0+3\times8=24 \text{ hr}$$
- The dual price of \$2.00/hr for machine 2 will remain applicable for the range

  $$4 \text{ hr} \le \text{Machine 2 capacity} \le 24 \text{ hr}$$

Second: Changes in the Objective Coefficients

- Changes in revenue units (i.e., objective-function coefficients) will change the slope of z.
- The optimum solution will remain at point C so long as the objective function lies between lines BF and DE, the two constraints that define the optimum point.



x_2
9
E
8
Obj
7
6
5
4
3
B
2
1
A
0
1
2
3
4
5
6
7
8
9
x_1
C
D
F
Machine 1 : $2x_1 + x_2 \leqslant 8$
$z = 3x_1 + 2x_2$
Optimum : $x_1 = 3.2, x_2 = 1.6, z = 128$
Machine 2 : $x_1 + 3x_2 \leqslant 8$

- We can write the objective function in the general format
  Maximize $z = c1x1 + c2x2$
- The optimum solution will remain at point C so long as $z=c1x1 + c2x2$ lies between the two lines
  - $x1+3x2 = 8$
  - $2x1+x2 = 8$
- This means that the ratio $c1/c2$ can vary between 1/3 and 2/1, which yields the following condition:

$$(1/3) \le (c1/c2) \le (2/1) \text{ or}$$

$$.333 \le (c1/c2) \le 2$$

Optimality Range

- The unit revenue of product 2 is fixed at its current value of $c2 = \$20.00$. The associated range for c1:

  [<u>remember:</u> $(1/3) \leq (c1/c2) \leq (2/1)$]

  $(1/3) \times 20 \leq c1 \leq 2 \times 20$ or $6.67 \leq c1 \leq 40$
- Similarly, if we fix the value of c1 at \$30.00 $c2 \leq 30 \times 3$ and $c2 \geq (30/2)$ or $15 \leq c2 \leq 90$

- Suppose that the unit revenues for products 1 and 2 are changed to \$35 and \$25, respectively.

  Will the optimum remain the same?

- The new objective function is Maximize $z = 35x_1 + 25x_2$

- The solution will remain optimal :

$c_1/c_2 = 35/25 = 1.4 \rightarrow$ within the optimality range $(.333, 2)$