Special cases

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1) Infeasible solution

If there exists no solution that satisfy all constraint then it is said to be infeasible solution or no feasible solution exists.

E1

Min $Z = 200x_1 + 300x_2$

S.t.

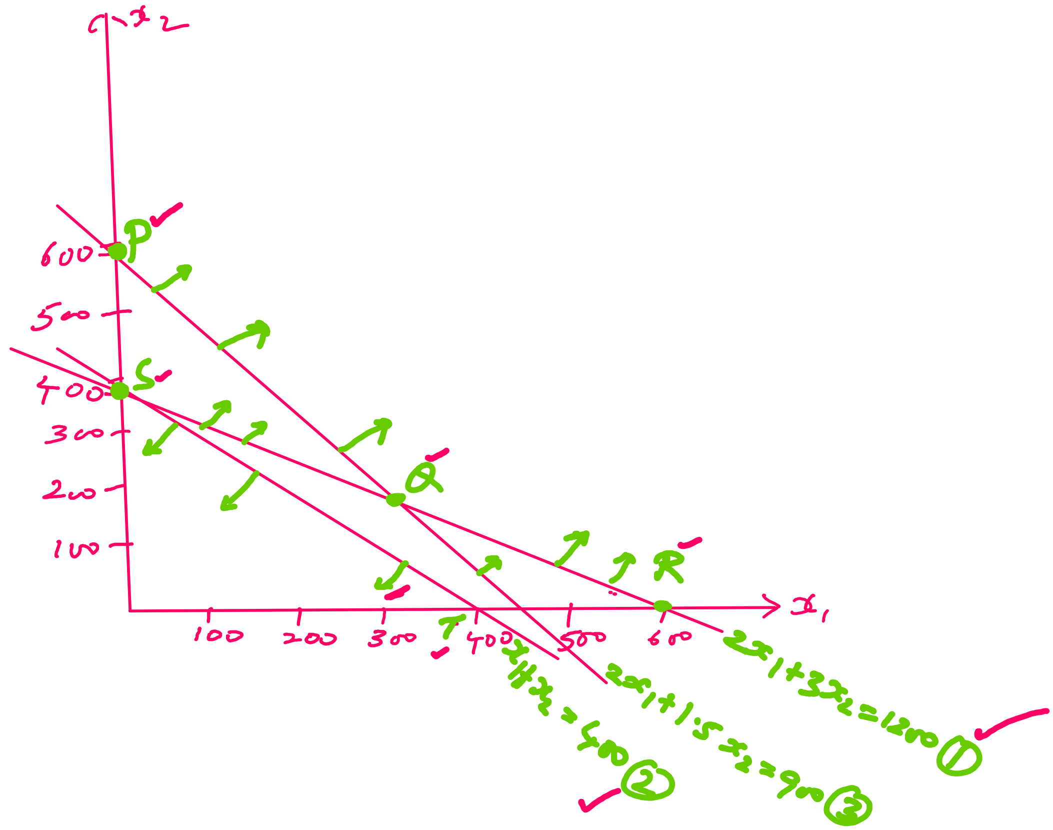
$2x_1 + 3x_2 \geq 1200$

$x_1 + x_2 \leq 400$

$2x_1 + 1.5x_2 \geq 900$

$x_1, x_2 \geq 0$

Plotting in the graph.



Region above PQR satisfies Constraint (1) & (3).

Region below ST satisfies Constraint 2

No Common Region which satisfies all constraints.

$\therefore$ Infeasible Solution

2) Multiple optimal solution
(Alternative optima)

When the line representing objective function is parallel to a line satisfied by an equation at optimal solution, then multiple optimal solution exists.

eg.

$$\text{Max } Z = 4x_1 + 3x_2$$

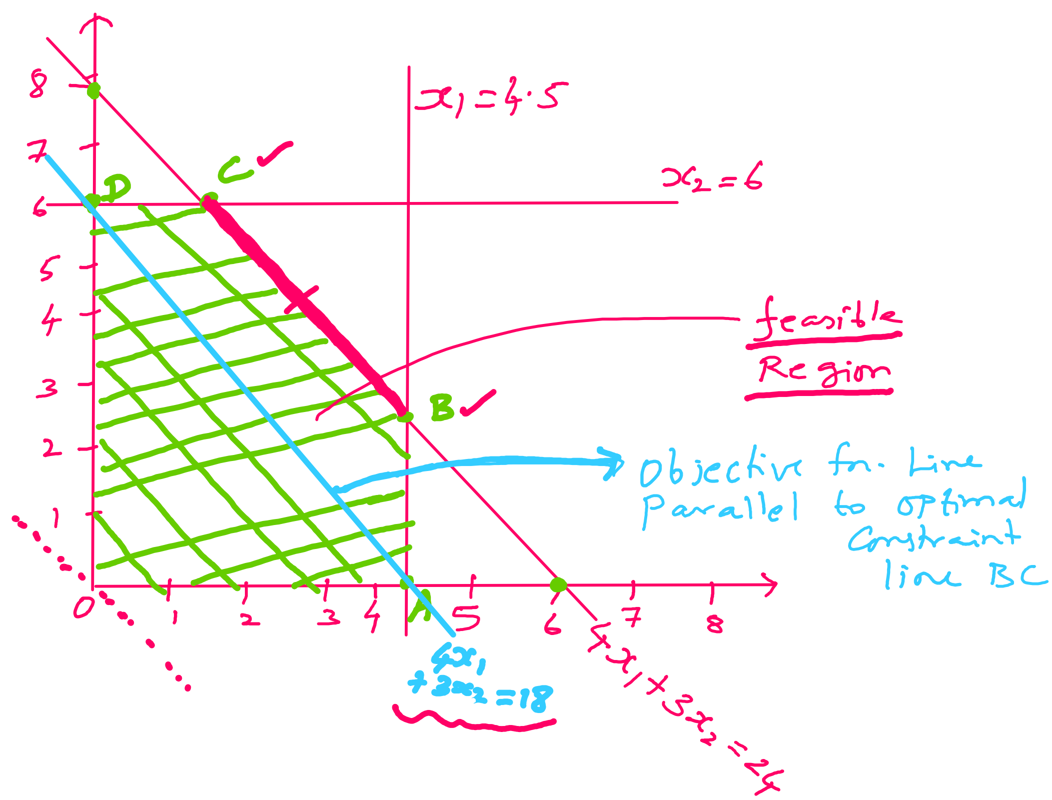
s.t.

$$4x_1 + 3x_2 \leq 24$$

$$x_1 \leq 4.5$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$



Feasible Region OABCD

Corner point	Obj fn
A (4.5, 0)	18
B (4.5, 2)	24 ✓
C (1.5, 6)	24 ✓
D (0, 6)	18

Maximum occurs at B and C.
Also the maximum occurs at all points in line segment BC.
Infinite number of optimal solutions exists.
However from the corner points the optimal soln is

$$\text{Alternative optima}\begin{cases} x_1 = 4.5, \quad x_2 = 2, \quad \text{Max } Z = 24 \\ \quad\quad (\text{or}) \\ x_1 = 1.5 \quad x_2 = 6, \quad \text{Max } Z = 24 \end{cases}$$

Alternative optima can be identified by plotting objective function Z.

Here $Z = 4x_1 + 3x_2$

Let us consider some arbitrary value for Z as 18.

(ie) $4x_1 + 3x_2 = 18$

Plotting in the graph, we find that the objective function equation is parallel to constraint equation which has optimal points B & C.

3) Unbounded Solution

If feasible region is

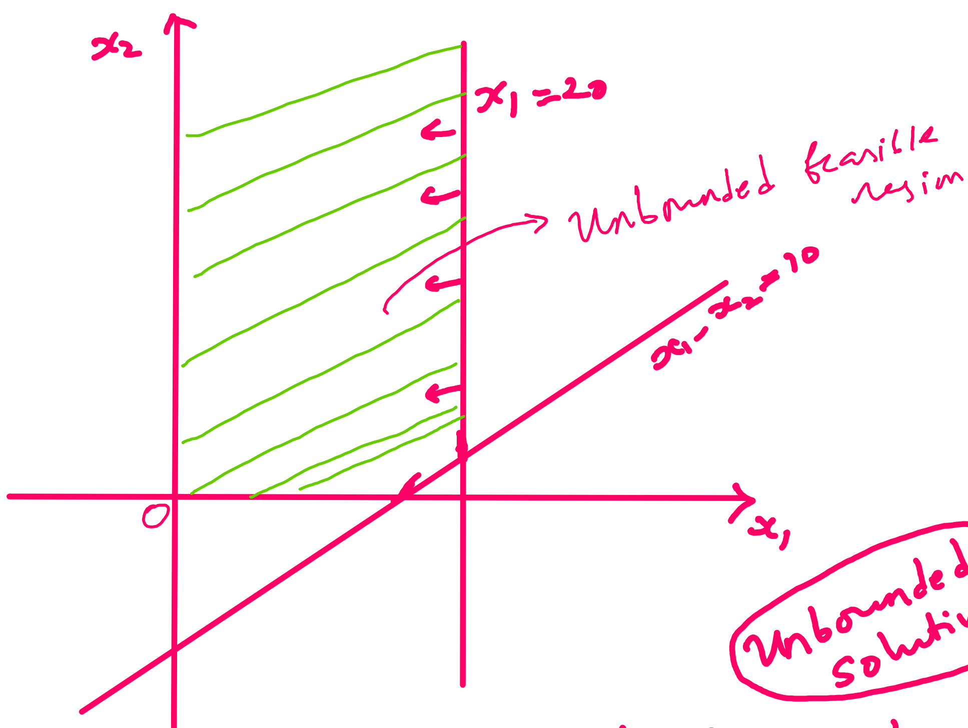
If feasible region is Unbounded, then in case of maximization Problem one or more decision Variables will increase indefinitely without violation of constraint. In this case solution is unbounded.

(For minimization problem, the solution exists at corner points, if the coefficients are positive in Z equation)

eg Max $Z = 2x_1 + 7x_2$

s.t.

$$x_1 - x_2 \leq 10$$
$$2x_1 \leq 40$$
$$x_1, x_2 \geq 0$$



Unbounded solution

x_2 can be increased indefinitely

But x_1 has to take values ≤ 20

4) Redundancy

A Redundant Constraint is

one that does not affect the feasible solution region.

$$\text{Max } Z = 3x_1 + 4x_2$$

s.t.

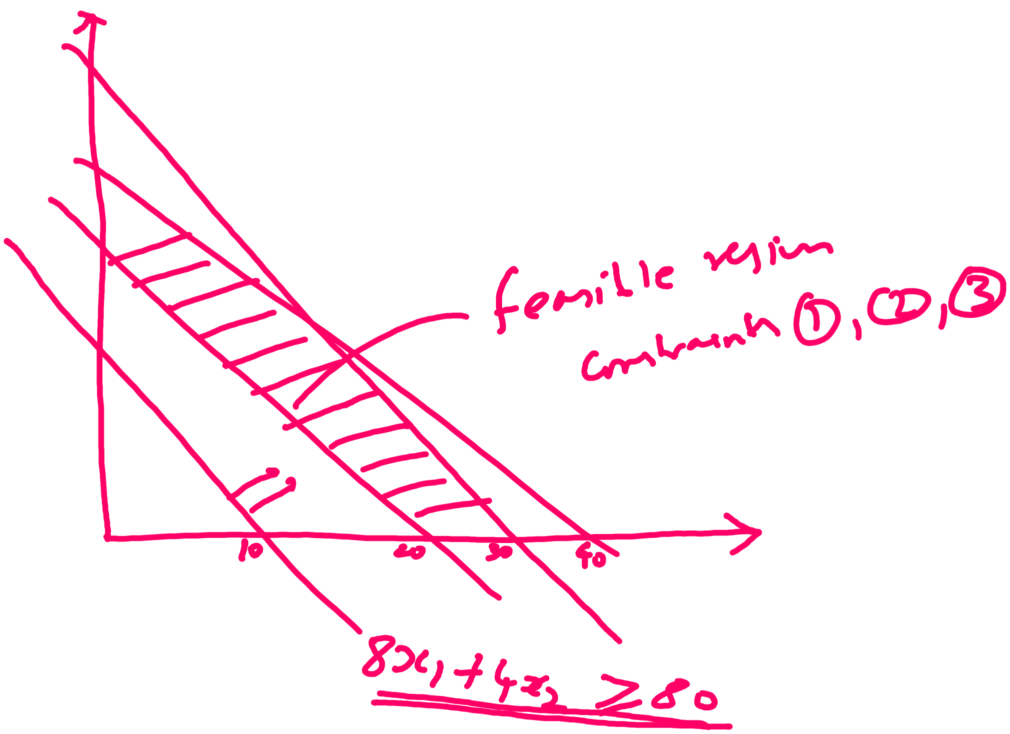
$5x_1 + 4x_2 \le 200$ (0,50), (40,0)

$3x_1 + 5x_2 \le 150$ (0,30), (50,0)

$5x_1 + 4x_2 \ge 100$ (20,0) (0,25)

$8x_1 + 4x_2 \ge 80$ (10,0) (0,20)

$x_1, x_2 \ge 0$



Constraint $8x_1 + 4x_2 \ge 80$ doesn't affect the feasible region and hence it is called redundant constraint.

Note:

1) Graphical Method limitation: two variables.

2) The set of feasible solution of a LPP is a convex set.

A set D is a Convex set if the line segment joining any pair of Points in D is contained in D.

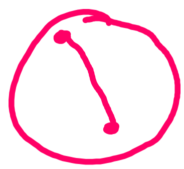
(Mathematically,

Let x_1, x_2 be any two points in the region S. Then,

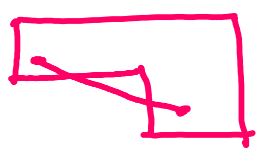
$\lambda x_1 + (1-\lambda) x_2 \in S$,

where $\lambda \in [0, 1]$, then region S is convex.

Eg.



Convex



Non Convex
(There are some points not contained in given region)