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UNIT STEP FUNCTION

1. $f(t) = \begin{cases} \sin t & 0 \le t \le \pi/4 \\ \sin t + \cos(t - \pi/4) & \pi/4 \le t \end{cases}$

$f(t) = \sin t + \cos(t-\pi/4)\, u(t-\pi/4)$

$L[f(t)] = \dfrac{1}{s^2+1} + e^{-\frac{\pi s}{4}} \dfrac{s}{s^2+1}$

$= \boxed{\dfrac{1}{s^2+1}\left(1 + e^{-\frac{\pi s}{4}}\right)}$

2. $f(t) = \begin{cases} 0 & 0 \le t < 3 \\ e^t & 3 \le t < 4 \\ 0 & 4 \le t \end{cases}$

$f(t) = e^t \cdot u(t-3) - e^t u(t-4)$

$L[f(t)] = e^{-3s} L(e^{t+3}) - e^{-4s} L(e^{t+4})$

$= \boxed{\dfrac{e^{-3s+3}}{s-1} - \dfrac{e^{-4s+4}}{s-1}}$

3. $f(t) = \begin{cases} 0 & 0 \le t < 2 \\ t+3 & 2 \le t \end{cases}$

$f(t) = (t+3)u(t-2) = t u(t-2) + 3u(t-2)$

$L[f(t)] = e^{-2s} L(t+2) + \dfrac{3e^{-2s}}{s}$

$= \dfrac{e^{-2s}}{s^2} + \dfrac{2e^{-2s}}{s} + \dfrac{3e^{-2s}}{s}$

$$= \boxed{\frac{e^{-2s}}{s^2} + \frac{5e^{-2s}}{s}}$$

4. $g(t) = \begin{cases} 1 & 0 \le t < 2 \\ t^2 & 2 \le t \end{cases}$

$$g(t) = t^2 u(t-2) + u(t) - u(t-2)$$

$$L[g(t)] = e^{-2s} L[(t+2)^2] + L(1) - L[u(t-2)]$$

$$= e^{-2s} L(t^2 + 4t + 4) + \frac{1}{s} + (-) \frac{e^{-2s}}{s}$$

$$\therefore L[g(t)] = \boxed{e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{3}{s}\right) + \frac{1}{s}}$$

5. $g(t) = \begin{cases} 0 & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$

$$g(t) = u(t-1)$$

$$L[g(t)] = \boxed{\frac{e^{-s}}{s}}$$

6. $g(t) = \begin{cases} 0 & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ 0 & 2\pi \le t \end{cases}$

$$g(t) = u(t-\pi) - u(t-2\pi)$$

$$L[g(t)] = \boxed{\frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}}$$

7. $g(t) = \begin{cases} 0 & 0 \le t < 4 \\ e^t & 4 \le t < 5 \\ 0 & 5 \le t \end{cases}$

$g(t) = e^t u(t-4) - e^t u(t-5)$

$L[g(t)] = \boxed{\dfrac{e^{-4s+4}}{s-1} - \dfrac{e^{-5s+5}}{s-1}}$

8. $g(t) = \begin{cases} 0 & 0 \le t < 5 \\ (t-5)/5 & 5 \le t < 10 \\ 1 & 10 \le t \end{cases}$

$g(t) = \dfrac{t-5}{5} u(t-5) - \dfrac{t-5}{5} u(t-10) + u(t-10)$

$L[g(t)] = \dfrac{e^{-5s}}{5} \times \dfrac{1}{s^2} + \dfrac{e^{-10s}}{s} - \dfrac{1}{5} (t-10+10-5) \times u(t+0)$

$= \dfrac{e^{-5s}}{5s^2} + \dfrac{e^{-10s}}{s} - \dfrac{1}{5}\left\{L[(t-10)u(t-10)]\right\}$

$- \dfrac{5}{5}\left\{L[u(t-10)]\right\}$

$= \dfrac{e^{-5s}}{5s^2} + \dfrac{e^{-10s}}{s} - \dfrac{1}{5}\dfrac{e^{-10s}}{s^2} - \dfrac{e^{-10s}}{s}$

$= \boxed{\dfrac{1}{5s^2}(e^{-5s} - e^{-10s})}$

9) $f(t) = \begin{cases} t & 0 \le t < 2 \\ t^2 & 2 \le t < 6 \\ t^3 & 6 \le t \end{cases}$

$f(t) = t + (t^2 - t) u(t-2) + t^3 - t^2 u(t-6)$

$$L[f(t)] = \frac{1}{s^2} + e^{-2s} L\{(t+2)^2 - (t+2)\} +$$

$$e^{-6s} L\{(t+2)^3 - (t+2)^2\}$$

$$= \frac{1}{s^2} + e^{-2s} L\{t^2 + 3t + 2\}$$

$$+ e^{-6s} L\{t^3 + 5t^2 + 8t + 4\}$$

$$= \frac{1}{s^2} + \frac{e^{-2s}}{s}\left[\frac{2}{s^2} + \frac{3}{s} + 2\right] + e^{-6s}\left(\frac{6}{s^4} + \frac{10}{s^3}\right.$$

$$\left. + \frac{8}{s^2} + \frac{4}{s}\right)$$