Post optimality analysis

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Note :

(i) In equal constraint,

Change it to less than or equal to and greater than or equal to.

(ex) $x_1 + x_2 = 5$

$\Downarrow$

$$\left.\begin{array}{l} x_1 + x_2 \leq 5 \\ x_1 + x_2 \geq 5 \end{array}\right\} \Rightarrow \begin{array}{l} x_1 + x_2 \leq 5 \\ -x_1 - x_2 \leq -5 \end{array}$$

2) While checking dual optimality condition for entering variable in IBFS, if $\alpha_j \geq 0$ (for all non basic x_j) the problem has no feasible solution.

3) Dual - Primal Relation

P \ D	I	O	U
I	✓	✗	✓
O	✗	✓	✗
U	✓	✗	✗

✓ Possible

✗ Impossible

I - infeasible

O - optimal

U - unbounded

Sensitivity Analysis

Post optimality Analysis

(1) Changes affecting feasibility

Toy Co Problem (Introduction to operation Research - Taha)

$$\text{Max } Z = 3x_1 + 2x_2 + 5x_3$$

Cars
trucks
trains

s.t.

$$x_1 + 2x_2 + x_3 \le 430 \quad \text{(operation 1)} \rightarrow \text{minutes}$$
$$3x_1 + 2x_3 \le 460 \quad \text{(operation 2)}$$
$$x_1 + 4x_2 \le 420 \quad \text{(oper 3)}$$
$$x_1, x_2, x_3 \ge 0$$

420 machine 3

430 1 minute operation 1\$/min

Opt. solution.

$Z = 1350$, $x_1 = 0$, $x_2 = 100$, $x_3 = 230$

Dual

Min $Z = 430y_1 + 460y_2 + 420y_3$

s.t.

$$y_1 + 3y_2 + y_3 \ge 3$$
$$2y_1 + 4y_3 \ge 2$$
$$y_1 + 2y_2 \ge 5$$
$$y_1, y_2, y_3 \ge 0$$

Soln
$y_1 = 1$
$y_2 = 2$ — 2\$/m
$y_3 = 0$ — no worth
$Z = 1350$

Primal optimal table

	x_1	x_2	x_3	x_4	x_5	x_6	sol
Z	4	0	0	1	2	0	1350
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100
x_3	$3/2$	0	1	0	$1/2$	0	230
x_6	2	0	0	-2	1	1	$\boxed{20}$

$Ax = B$
$x = A^{-1}B$

$x_1 = 0$
$x_2 = 100$
$x_3 = 230$
$\left.\begin{matrix} x_4 \\ x_5 \end{matrix}\right\} = 0$
$x_6 = 20$
slack

(1) If daily capacity of operation 1, 2 & 3 is increased to 600, 640, 590, what is the effect in total revenue?

New RHS in iteration i = inverse in iteration i × New RHS constraint

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 600 \\ 640 \\ 590 \end{pmatrix}$$

$$= \begin{pmatrix} 140 \\ 320 \\ 30 \end{pmatrix} \quad \text{feasible}$$

New Soln is

$x_2 = 140$, $x_3 = 320$

$x_6 = 30$

New $\boxed{Z = 1880}$ Increase in profit $530

Total increase in revenue $\Big\} =$ New Z − old Z

$= 1880 - 1350$

$= \boxed{530}$

Using Dual

$\boxed{y_1 = 1}$, $y_2 = 2$, $\boxed{y_3 = 0}$

Resource worth operation 1 is 1/unit

Resource worth operation 2 is 2/unit

" " 3 is 0/unit

Operation 1 → charges in unit

600 − 430

= 170

Total increase w.r.t. Oper 1 = 170

Operation 2 → Charges in unit

= 640 − 460

= 180

Operation 3 → Charges in unit

= 590 − 420 = 170

Total Resource Value

= (No. of units for oper 1 × oper 1 worth)

+ (No. of units for oper 2 × oper 2 worth)

+ (No. of units for oper 3 × oper 3 worth)

feasibility range

limits is

Sens. analysis..

$$= (170 \times 1) + (180 \times 2) + (170 \times 0)$$

$$= 170 + 360$$

$$= 530$$ increase in revenue

(Using Dual price, we got same ans)

2) Shift the slack capacity of operation 3 ($x_6 = 20$) to the capacity of operation 1.

How would this change impact the optimum solution?

New RHS in iteration i = Optimal Inverse in itr. 'i' × New RHS constraints

$$= \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 450 \\ 460 \\ 400 \end{pmatrix}$$

430 + 20

420 − 20

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 110 \\ 230 \\ -40 \end{pmatrix}$$

$x_2 = 110$, $x_3 = 230$, $x_6 = -40$

'Infeasible'

Solve by dual simplex method.

$x_2 = 110$ $x_3 = 230$ $x_6 = -40$

$$z = 3x_1 + 2x_2 + 5x_3$$

$$= (3 \times 0) + 2(110) + 5 \times 230$$

$$= 1370$$

Rewrite the optimal table as follows

Z 1370

x_2 110

x_3 230

x_6 . . . . -40

Starting table for dual Simplex

Soln
$Z = 1350$
$x_2 = 100$, $x_3 = 230$, $x_4 = 20$

The optimal Solution remains same. Shifting slack $x_6 = 20$ to $x_4 = 20$ is not advantageous

(iii) Shifting $x_6 = 20$ to operation 2.
what is the impact?

$$\text{New RHS} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 430 \\ 480 \\ 400 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 95 \\ 240 \\ 20 \end{pmatrix}$$

$x_1 = 0$, $x_2 = 95$, $x_3 = 240$, $x_6 = 20$

$3x_1 + 2x_2 + 5x_3$

$Z = (3 \times 0) + (2 \times 95) + (5 \times 240)$

$= 1390$ $\boxed{\$40}$

Increase in optimal soln.

$\boxed{\text{Selection of operation 2 over operation 1 is preferred}}$

2) Addition of new constraint

✓ 3 constraints in Toyco

4^{th} constraint → operation 4

$$\boxed{4^{th} \rightarrow 3x_1 + x_2 + x_3 \leq 500}$$

Current optimal solution.

$x_1 = 0 \quad x_2 = 100 \quad x_3 = 230$

$Z = 1350$

if the constraint is Redundant
'no change in optimum'

$(3 \times 0) + (100) + (230)$
$= 330 \leq 500$ ✓

True

Redundant constraint

No change in optimal value.

eg: 4^{th} constraint

$3x_1 + 3x_2 + x_3 \leq 500$

$530 \leq 500$ ✗ (not true)

Change in optimal value;
we have to add constraint
& we have to solve
dual simplex method