1. Find Laplace Transform

i) $L(4te^t)$

$L\{t f(t)\} = -F'(s)$

$4L\{te^t\} = 4 \times -\frac{d}{ds}[L(e^t)]$

$= -4\frac{d}{ds}\left(\frac{1}{s-1}\right) = \boxed{\frac{4}{(s-1)^2}}$

ii) $L(t\sin\omega t)$

$= -\frac{d}{ds}\{L(\sin\omega t)\} = -\frac{d}{ds}\left\{\frac{\omega}{\omega^2+s^2}\right\}$

$= \boxed{\frac{2s\omega}{(s^2+\omega^2)^2}}$

iii) $L(-t\cosh 2t)$

$= \frac{d}{ds}[L(\cosh 2t)] = \frac{d}{ds}\left(\frac{s}{s^2-4}\right)$

$= \frac{s^2-4-2s^2}{s^2-4} = \boxed{\frac{-(s^2+4)}{(s^2-4)^2}}$

iv) $t\cos(t+k)$

$= t[\cos t\cos k - \sin t\sin k]$

$= t\cos t\cos k - t\sin t\sin k.$

$= \cos k\left(-\frac{d}{ds}[L(\cos t)]\right) - \sin k\left[-\frac{d}{ds}(L[\sin t])\right]$

$= -\cos k\frac{d}{ds}\left(\frac{s}{s^2+1}\right) - \sin k\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$

$$\boxed{= \frac{s^2-1}{(s^2+1)^2} \cos k - \sin k \times \frac{2s}{(s^2+1)^2}}$$

v) $L(te^{-2t}\sin t)$

$$= \frac{-d}{ds}\left\{L(e^{-2t}\sin t)\right\} = \frac{-d}{ds}\left(\frac{1}{s^2+1}\right)_{s\to s+2}$$

$$= \frac{-d}{ds}\left(\frac{1}{(s+2)^2+1}\right) = \boxed{\frac{2(s+2)}{(s+2)^2+1}}$$

vi) $L(t^2\sinh 4t)$

$$= \frac{d^2}{ds^2}\left[L(\sinh 4t)\right] = \frac{d^2}{ds^2}\left(\frac{4}{s^2-16}\right)$$

$$= \frac{d}{ds}\left(\frac{-8s}{(s^2-16)^2}\right) = \frac{d}{ds}\left[\frac{-8s}{(s^2-16)^2}\right]$$

$$= \frac{-8(s^2-16)^2 - 2(s^2-16)2s \times -8s}{(s^2-16)^4}$$

$$= s^2-16\left[\frac{-8(s^2-16)+32s^2}{(s^2-16)^4}\right]$$

$$= \frac{-8s^2+128-32s^2}{(s^2-16)^3} = \frac{128-24s^2}{(s^2-16)^3}$$

$$= \boxed{\frac{8(16-3s^2)}{(s^2-16)^3}}$$

vii) $L(t^2 \sin \omega t)$

$$= \frac{d^2}{ds^2}\left[L(\sin\omega t)\right] = \frac{d^2}{ds^2}\left(\frac{\omega}{\omega^2+s^2}\right)$$

$$= \frac{d}{ds}\left[\frac{-2\omega s}{(\omega^2+s^2)^2}\right] = \frac{-2\omega(\omega^2+s^2)^2 + 2(\omega^2+s^2)2s \times 2\omega s}{(\omega^2+s^2)^4}$$

$$= \frac{8\omega s^2 - 2(\omega^2+s^2)\omega}{(\omega^2+s^2)^3} = \frac{8\omega s^2 - 2\omega^3 - 2\omega s^2}{(\omega^2+s^2)^3}$$

$$= \boxed{\frac{6\omega s^2 - 2\omega^3}{(\omega^2+s^2)^3}}$$

viii) $L[\sin(t+k) * t]$

$$= t\sin t\cos k + t\cos t\sin k$$

$$= \cos k\, L\{t\sin t\} + \sin k\, L(t\cos t)$$

$$= \boxed{\cos k \times \frac{2s}{(s^2+1)^2} + \sin k \times \frac{(s^2-1)}{(s^2+1)^2}}$$

ix) $L(t\cos\omega t)$

$$= \frac{-d}{ds}\left(\frac{s}{s^2+\omega^2}\right) = -\left[\frac{(s^2+\omega^2) - 2s^2}{(s^2+\omega^2)^2}\right]$$

$$= \frac{2s^2 - s^2 - \omega^2}{(s^2+\omega^2)^2} = \boxed{\frac{s^2-\omega^2}{(s^2+\omega^2)^2}}$$

x) $L\{te^{-kt}\sin t\}$

$$L[t\sin t] = \frac{-d}{ds}\left(\frac{1}{s^2+1}\right) = \frac{2s}{(s^2+1)^2}$$

$$L(te^{-kt}\sin t) = \boxed{\frac{2(s+k)}{[(s+k)^2+1]^2}}$$

II. Find Inverse Laplace Transform

i) $\ln\left(1-\frac{a^2}{s^2}\right)$

$$= \ln\left(\frac{s^2-a^2}{s^2}\right) = \ln(s^2-a^2) - \ln s^2$$

$$F'(s) = \frac{2s}{s^2-a^2} - \frac{2}{s}$$

$$L^{-1}[F'(s)] = 2\cosh at - 2$$

$$L^{-1}[F(s)] = \frac{-2}{t}(\cosh at - 1) = \boxed{\frac{2-2\cosh at}{t}}$$

ii) $\ln\left(\frac{s}{s-1}\right) = \ln s - \ln(s-1)$

$$F'(s) = \frac{1}{s} - \frac{1}{s-1}$$

$$L^{-1}[F'(s)] = 1 - e^{-t}$$

$$L^{-1}[F(s)] = \boxed{\frac{-(1-e^{t})}{t}}$$

iii) $\ln\left(\frac{s+a}{s+b}\right) = \ln(s+a) - \ln(s+b)$

$$F'(s) = \frac{1}{s+a} - \frac{1}{s+b}$$

$$L^{-1}[F'(s)] = e^{-at} - e^{-bt}$$

$$L^{-1}[F(s)] = \boxed{\frac{e^{-bt} - e^{-at}}{t}}$$

iv) $\operatorname{arc}\cot \frac{s}{\omega}$

$$f(s) = \cot^{-1}\left[\frac{s}{\omega}\right]$$

$$F'(s) = \frac{-1}{1+s^2/\omega^2} = \frac{-\omega^2}{\omega^2+s^2}$$

$$= -\omega L^{-1}\left[\frac{\omega}{\omega^2+s^2}\right] = -\omega \sin\omega t$$

$$L^{-1}[F(s)] = \boxed{\frac{\omega}{t}\sin\omega t}$$