Two phase Simplex Method

22 January 2021 11:48

Phase I:

Convert the LPP into standard form. The objective function is always minimize the sum of artificial variables. If the minimum value is positive then the LPP has no feasible solution. Otherwise (min value is 0) then the LPP has feasible solution and we can proceed to phase II.

Phase II

Use the feasible solution from phase I as a starting basic feasible solution for the original problem.

Eg:

$$\text{Min } Z = 2500x_1 + 3000x_2$$

s.t.

$$x_1 \geq 30$$
$$x_2 \geq 20$$
$$x_1 + x_2 = 60$$
$$x_1, x_2 \geq 0$$

Converting Min to Max Problem

$$\boxed{\text{Min } Z = \text{Max}(-Z)}$$

$$\text{Max } Z = -2500x_1 - 3000x_2$$

s.t.

$$x_1 \geq 30$$
$$x_2 \geq 20$$

$x_1 + x_2 = 60$

$x_1, x_2 \geq 0$

Surplus variable

Convert into standard form

$$\text{Max } Z = -2500x_1 - 3000x_2 + 0x_3 + 0x_4$$

s.t.

Artificial Variable

$$x_1 \boxed{-x_3} + \boxed{A_1} = 30$$

$$x_2 - x_4 + A_2 = 20$$

$$x_1 + x_2 + A_3 = 60$$

$$x_1, x_2, x_3, x_4, A_1, A_2 \geq 0$$

Objective fn is,

$$Z + 2500x_1 + 3000x_2 + 0x_3 + 0x_4 = 0$$

Phase I (Artificial Var elimination process)

The objective function is

$$\text{Min } a = \underline{A_1 + A_2 + A_3}$$

s.t.

$$x_1 - x_3 + A_1 = 30$$

$$x_2 - x_4 + A_2 = 20$$

$$x_1 + x_2 + A_3 = 60$$

Converting Min to Max

$$\text{Max } a = (-a)$$

$$\text{Max } a = \underline{-A_1} \; \underline{-A_2} \; \underline{-A_3}$$

obj. fn $\boxed{a + A_1 + A_2 + A_3 = 0}$

Initial table

Basic var	x_1	x_2	x_3	x_4	A_1	A_2	A_3	Sol (b_r) (RHS)	b_r/c_r	
a row	0	0	0	0	1	1	1	0	–	
A_1	1	0	−1	0	1	0	0	30		R_1
A_2	0	1	0	−1	0	1	0	20		R_2
A_3	1	1	0	0	0	0	1	60		R_3

To make a-row consistent for initial basic feasible solution we do the following operation.

[From the above table,
IBFS is $A_1 = 30$, $A_2 = 20$, $A_3 = 60$
& $a = 0$ — ①
But when we substitute A_1, A_2, A_3
in $a = A_1 + A_2 + A_3$
$= 30 + 20 + 60$
$a = 110$ — ②
From ① & ②, we have inconsistency
Hence to remove inconsistency, we have make the coefficients of A_1, A_2 & A_3 in a-row as zero]

					A_1	A_2	A_3	
a row →	0	0	0	0	$\boxed{1}$	1	1	0
$R_1 \times (-1)$ →	−1	0	1	0	−1	0	0	−30
(+)	−1	0	1	0	0	$\boxed{1}$	1	−30
$R_2 \times (-1)$ →	0	−1	0	1	0	−1	0	−20
(+)	−1	−1	1	1	0	0	$\boxed{1}$	−50
$R_3 \times (-1)$ →	−1	−1	0	0	0	0	−1	−60

(+) -2 -2 1 1 0 0 0 -110

$\underbrace{}_{\text{all } A_1, x_2, A_3 \text{ are } 0}$

$\therefore$ Initial table is

Basic Var	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b_r RHS	b_r/c_r
a row	-2	-2	1	1	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	-110	–
A_1	1	0	-1	0	1	0	0	30	30
A_2	0	1	0	-1	0	1	0	20	–
A_3	1	1	0	0	0	0	1	60	60

Entering Variable

Most negative coef. in a row is -2. Corresponding variable x_1 enters basis. (There is a tie between x_1 & x_2, arbitrarily choose x_1)

Leaving Variable

denominator zero

$$\text{Min ratio } \left(30/1,\ -,\ 60/1\right)$$

$$= (30, -, 60) = 30$$

$\therefore$ Corresponding A_1 leaves.

Pivot element is 1. Making other elements in the column as zero,

Iteration 1:

Basic Var	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b_r RHS	b_r/c_r
a row	0	-2	-1	0	2	0	0	-50	–
x_1	1	0	-1	0	1	0	0	30	–
Pivot row → A_2	0	1	0	-1	0	1	0	20	20
A_3	0	1	1	0	-1	0	1	30	30

Rough work:

Since pivot element is 0, no change in pivot row.

① a row → $\boxed{-2}$ −2 1 1 0 0 0 −110

Pivot row ×(−2) → −2 0 2 0 −2 0 0 −60

(−) 0 −2 −1 0 2 0 0 −50

② A_2 row already zero. no. change

③ A_3 row → 1 1 0 0 0 0 1 60

Pivot row × 1 → 1 0 −1 0 1 0 0 30

(−) 0 1 1 0 −1 0 1 30

Entering Variable:

Most negative coefficient is −2.
∴ Corresponding variable x_2 enters.

Leaving Variable

denominator 0

$$\text{Min ratio} = \left(-, \ 20/1, \ 30/1 \right)$$

$$= 20$$

∴ Corresponding variable A_2 leaves.

Doing Row operations,

Iteration 2:

Basic Var	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS b_i	b_i/c_i
a row	0	0	−1	−1	2	2	0	−10	—
x_1	1	0	−1	0	1	0	0	30	
Pivot → x_2	0	1	0	−1	0	1	0	20	

row	A_3	0	0	1	1	-1	-1	1	10

Rough work

Since pivot element is 1, no change in pivot row.

① a row → 0 -2 -1 1 2 0 0 -50

Pivot row × (-2) → 0 -2 0 2 0 -2 0 -40

(-) 0 0 -1 -1 2 2 0 -10

② x_1 row already zero, no change

③ A_3 row → 0 1 1 0 -1 0 1 30

Pivot row × 1 → 0 1 0 -1 0 1 0 20

0 0 1 1 -1 -1 1 10

Iteration 3.

	x_1	x_2	x_3	x_4	A_1	A_2	A_3	In sol
a	0	0	0	0	1	1	1	0
x_1	1	0	0	1	0	-1	1	40
x_2	0	1	0	-1	0	1	0	20
x_3	0	0	1	1	-1	-1	1	10

a-row all coefficients.

Max (z) = Min (-z) = 0 ✓

Minimum sum of artificial variables is 0.

Phase I produces Basic Feasible sol. We move to phase II.

Eliminate Artificial var col & move to Phase II.

Phase II :

Original Problem is written as

Max $Z = -2500x_1 - 3000x_2 + 0x_3 + 0x_4$

s.t. (look iteration 3 in phase I)

$x_1 + x_4 = 40$

$x_2 - x_4 = 20$

$x_3 + x_4 = 10$

Initial table

Obj fn is $Z + 2500x_1 + 3000x_2 + 0x_3 + 0x_4 = 0$

	Basic Var	x_1	x_2	x_3	x_4	Sol. RHS (b_r)	b_r/c_r
	Z row	2500	3000	0	0	0	–
R_1	x_1	1	0	0	1	40	
R_2	x_2	0	1	0	–1	20	
R_3	x_3	0	0	1	1	10	

Basic Variables : x_1, x_2, x_3

All Basic Variables should have zero coefficient in Z-row.
If not make it zero.
x_1, x_2 non zero coefficients must be substituted out.

$Z \rightarrow$	$\boxed{2500}$	3000	0	0	0
$R_1 \times 2500 \rightarrow$	2500	0	0	2500	100000
(–)	0	3000	0	–2500	–100000

$R_2 \times 3000 \rightarrow$	0	3000	0	-3000	6000
(-)	0	0	0	500	-1,60,000

Initial table

Basic Var	x_1	x_2	x_3	x_4	b or RHS	b/a
Z row	0	0	0	500	-1,60,000	-
x_1	1	0	0	1	40	
x_2	0	1	0	-1	20	
x_3	0	0	1	1	10	

Z-row coefficients are non negative optimum is reached.

$$\text{Max } Z = -1,60,000$$

$$\text{Min } Z = \text{Max}(-Z) = -(-1,60,000) = 1,60,000$$

$x_1 = 40, \quad x_2 = 20, \quad x_3 = 10$

Soln is Min $Z = 1,60,000$
$x_1 = 40$
$x_2 = 20$
$x_3 = 10$