ASSIGNMENT-1

Q1) 2 websites - Medical parts supplier,
Fitness e-zine (magazine)

	Medical parts	Fitness
Average hits per day per page	1,200,000	2,000,000
cost to advertise	\$ 1,100	\$ 1,600

The director would like atleast 15 ads, is able to allocate upto \$ 50,000 for advertising. At least 3 ads should be placed on each website. How many adds should be placed on each website to maximise the potential no: of readers?

SOLUTION : Step-1 : Decision variables

x_1 - Number of ads in med-supplier website

x_2 - Number of ads in e-zine website.

Z = max reach for readers,

Step-2 : objective function

$$\text{Max } Z = 1,200,000x_1 + 2,000,000x_2$$

Step-3 : constraints

$x_1 \geq 3,\ x_2 \geq 3$ (least ads per website)

- cost of advertising in med website } = \$ 1100

cost of advertising in fitness ezine } = \$ 1600.

so, $1100x_1 + 1600x_2 \leq 50000$ (total cost constraint)

$\Rightarrow x_1 + x_2 \geq 15.$

Also, $x_1, x_2 \geq 0$ (non negative constraints)

Graphical Method :

$x_1 \geq 3$ $\quad x_2 \geq 3$

$x_1, x_2 \geq 0 \Rightarrow$ 1st quadrant

$x_1 + x_2 \geq 15$

when $x_1 = 0, x_2 = 15$

when $x_2 = 0, x_1 = 15$

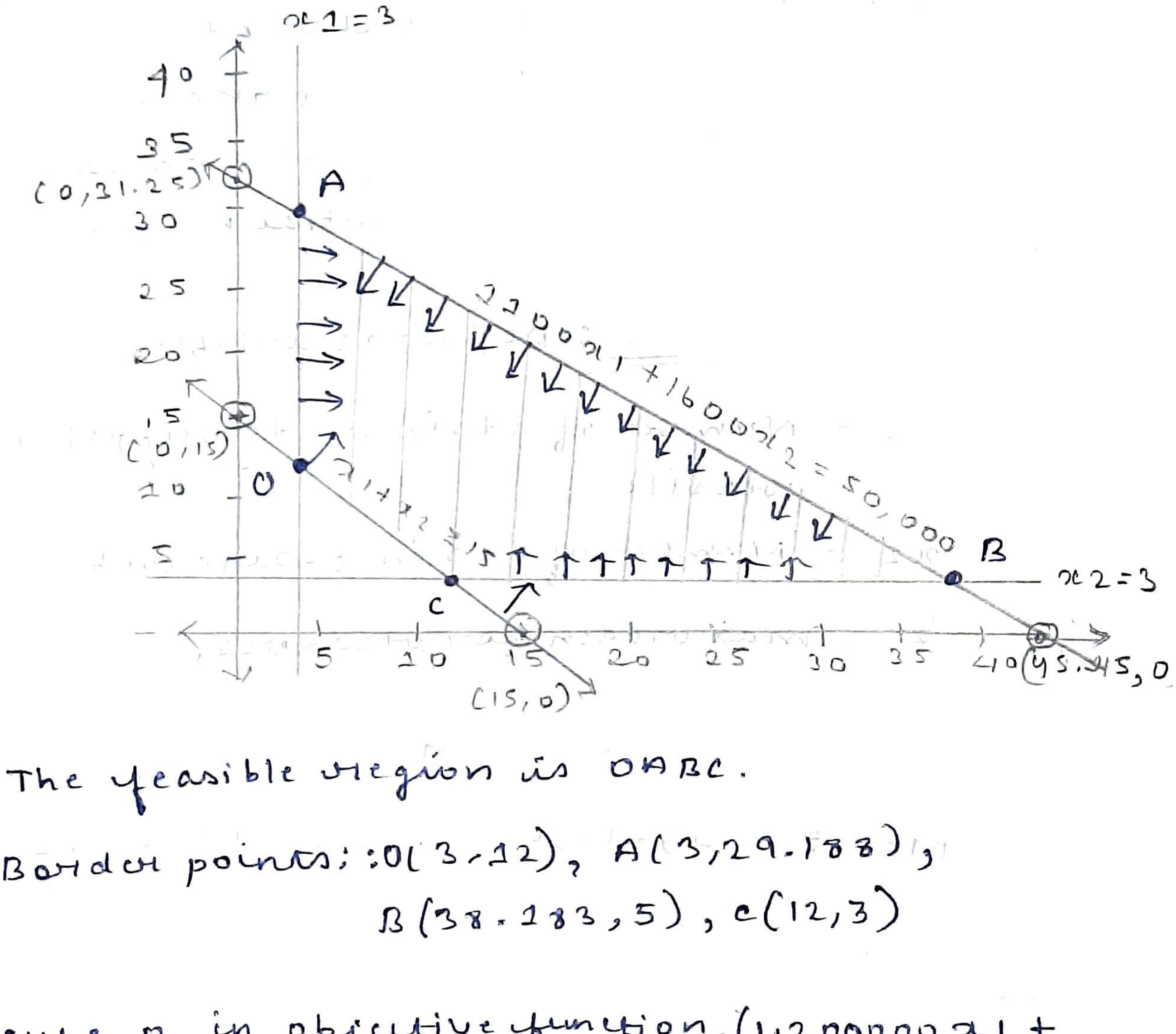
$(15, 0)$, $(0, 15)$

$1100x_1 + 1600x_2 \leq 50,000$

when $x_1 = 0$, $x_2 = 31.25$

when $x_2 = 0$, $x_1 = 45.45$

points : $(0, 31.25)$, $(45.45$



The feasible region is OABC.

Border points: O(3, 12), A(3, 29.188),
B(38.183, 5), C(12, 3)

subs O in objective function, $(1,200000x_1 + 2,000,000x_2)$

$Z_O = 1,200,000 \times 3 + 2,000,000 \times 12$

$Z_O = 27,600,000$

$Z_A = 1200000 \times 3 + 2000000 \times 29.188$

$= 61976000$

$Z_B = 55819600$

$Z_C = 20400000$

So, A(3, 29.188) is the optimal solution

Thus, for LPP

$$Z = 1,200,000x_1 + 2,00,000x_2$$

S.T $\quad x_1 \geq 3,\ x_2 \geq 3$

$1100x_1 + 1600x_2 \leq 50,000$

$x_1 + x_2 \geq 15$

$x_1, x_2 \geq 0$

To maximise potential no: of readers, the no: of adds in medical suppliers website must be '3' and must be '29' in fitness website (floor of 29.188 is taken to round off value to integer)

Q2) 2 Different budgets - Public expenditure, Grade school initiatives

Board willing to pay at least half of initiatives budget from public expenditure budget.

Government mandates atleast $2000 for local initiatives. both budgets partially funded by federal emergency funding.

Public expenditure - 55%.

Grade - school - 23%.

In order to use emergency funding wise, district likes to minimize federal dollars. How much must come from each budget?

SOLUTION :

Step - 1 - Decision variables

x_1 : Money from public expenditure budget

x_2 : Money from grade school initiatives budget.

Step - 2 - objective function

$$\text{Min } Z = 0.55x_1 + 0.23x_2$$

Step-3 : constraints

$x_1 \geq \frac{1}{2} x_2$ (public budget atleast half greater than grade-initiatives.

$2x_1 - x_2 \geq 0$

$x_2 \geq 2000$ (Min \$2000 for school)

$x_1, x_2 \geq 0$ (Non -ve constraints)

LPP Model :

$Z_{min} = 0.55x_1 + 0.23x_2$

S.T, $2x_1 - x_2 \geq 0$

$x_2 \geq 2000$

$x_1, x_2 \geq 0$

Graphical Method

$2x_1 - x_2 = 0$

when $x_1 = 0$, $x_2 = 0$

when $x_1 = 1000$, $x_2 = 2000$

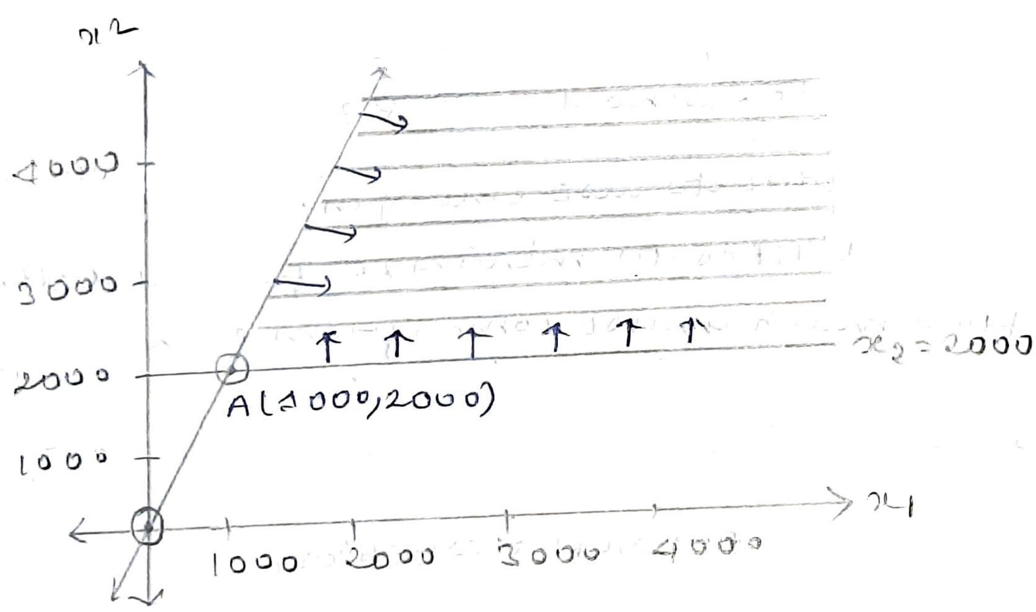
points : (0,0) (1000,2000)

$x_2 = 2000$

$x_1, x_2 \geq 0$

$\Downarrow$

1st quadrant.



$Z_A = 0.55(1000) + 0.23(2000)$

$= 1010$ (Minimum)

Thus, \$1000 and \$2000 from public expenditures, grade school initiatives must be given to minimise use of federal dollars to \$1010.

3) increase in starting salaries for :
New administrative secretaries, faculty.

	Administrative Secretary	Faculty
Start Salary	\$ 28,000	\$ 40,000
hire for next year	8	7

The college has at most \$ 5000 to put towards raises. What increase 4 each group?

SOLUTION :

step - 1 : Decision variables

x_1 - percentage increase for secretaries
x_2 - percentage increase for faculty.

step - 2 : objective function

$$Z_{min} = (28,000 \times 8) x_1 + (40,000 \times 7) x_2$$
$$= 2,24,000 \, x_1 + 2,80,000 \, x_2$$

step - 3 : constraints

$$2,24,000 \, x_1 + 2,80,000 \, x_2 \leq 5000$$
$$x_1, x_2 \geq 0.$$

LPP Model :

$$Z_{min} = 224000 x_1 + 280000 x_2$$

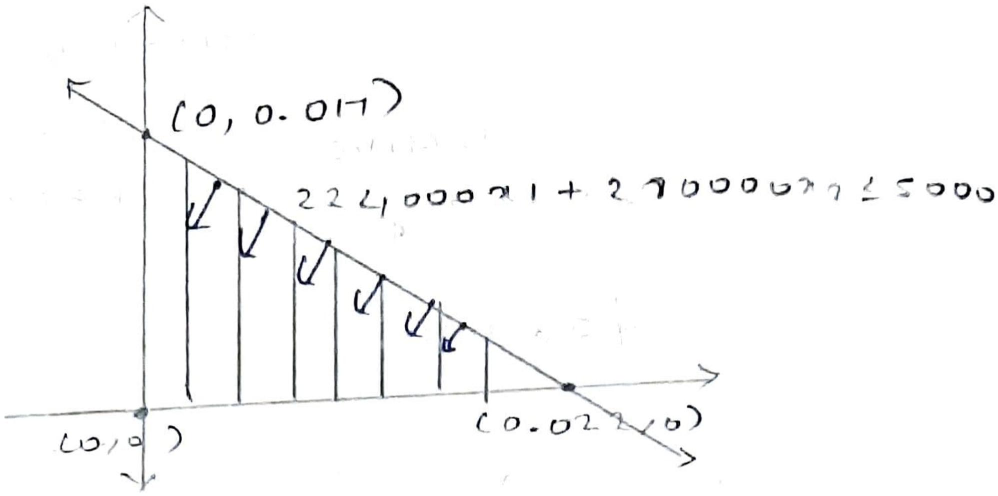
S.T $224000 x_1 + 280000 x_2 \leq 5000$

$x_1, x_2 \geq 0.$

Graphical Method :

$$224000 x_1 + 280000 x_2 = 5000$$

$x_1 = 0 \Rightarrow x_2 = 0.017$
$x_2 = 0 \Rightarrow x_1 = 0.022$ $\Bigg\} \Rightarrow$ pts are : $(0.022, 0)$ $(0, 0.017)$



Border points : (0,0) (0, 0.017) (0.022, 0)
1 2 3

$Z_1 = 0$

$Z_2 = 0 + 280000 \times 0.017 = 5000$

$Z_3 = 0.022 \times 224000 = 5000.$

Thus, given constraint are not sufficient to find the optimal solution as raise has to be given for both secretary + faculty.

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4) Box - 10 fruit bars

Potassium per serving is :

Dried apricots - 407 mg

Dried dates - 271 mg

Cost :

Dried apricots - \$9.99/lb (3 serving)

Dates - \$7.99/lb (4 servings)

The company would like the box to have atleast 4700 mg potassium intake. In order to minimize cost, how many servings of each dry fruit should go inside box of the bars?

SOLUTION:

Step-1 - Decision variables.

x_1: No of dried apricots serving

x_2: No of dried dates serving

Step-2 - objective function

$$Z_{min} = \frac{9.99}{3} x_1 + \frac{7.99}{4} x_2$$

$$= 3.33 x_1 + 1.99 x_2$$

Step-3 - constraints

$$407 x_1 + 271 x_2 \geq 4700$$

$$407 x_1 + 271 x_2 \leq 9400$$

LPP is, $Z = 3.33 x_1 + 1.99 x_2$

S.T $407 x_1 + 271 x_2 \geq 4700$

$407 x_1 + 271 x_2 \leq 9400$

Graphical Method

$407 x_1 + 271 x_2 = 4700$

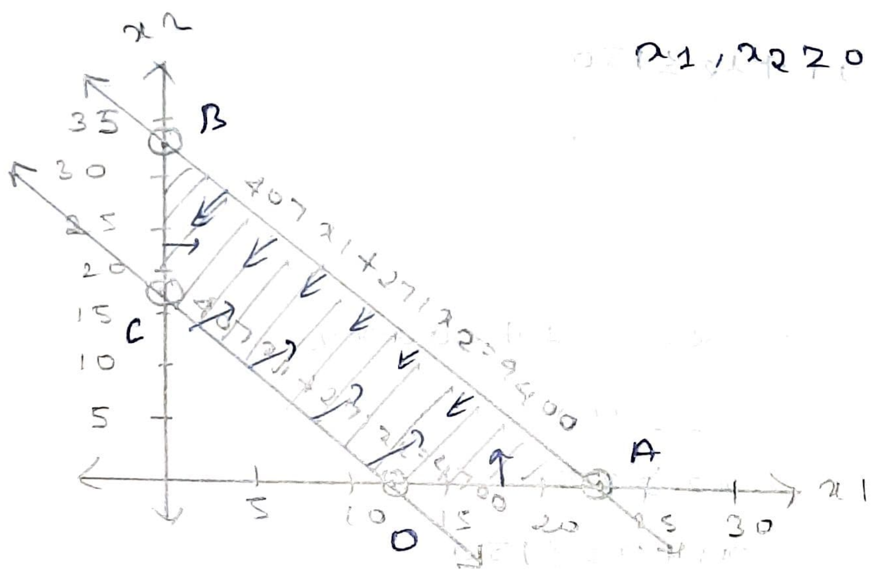
$x_1 = 0 \quad x_2 = 17.3$

$x_2 = 0 \quad x_1 = 11.54$

$407 x_1 + 271 x_2 = 9400$

$x_1 = 0 \quad x_2 = 34.6$

$x_2 = 0 \quad x_1 = 23.09$



$x_1, x_2 \geq 0 \Rightarrow$ 1st quadrant.

Border points: $O(11.58, 0)$, $A(23.09, 0)$, $B(0, 34.686)$, $C(0, 17.343)$

$Z_O = 38.30 \qquad Z_B = 69.40$

$Z_A = 76.92 \qquad Z_C = 34.60$

Thus, $Z_C = 34.60$ which is $(0, 17.343)$ is optimal solution.

5) Airline : coach, first class Tickets

	Coach Ticket	First class Ticket
minimum tickets	40	25
profit per ticket	$225	$200

Atmost plane capacity = 150 travelers.
How many of each must be sold to maximise profit?

SOLUTION:

step-1 - Decision variables

x_1 : No: of coach tickets

x_2 : No: of first class tickets.

step-2 - objective function

$$225x_1 + 200x_2 = Z_{max}$$

step-3 - constraints

$$x_1 \geq 40$$
$$x_2 \geq 25$$
$$x_1 + x_2 \leq 150$$
$$x_1, x_2 \geq 0.$$

LPP Model:

$$Z_{max} = 225x_1 + 200x_2$$

S.T

$$x_1 \geq 40$$
$$x_2 \geq 25$$
$$x_1 + x_2 \leq 150$$
$$x_1, x_2 \geq 0$$

Graphical Method:

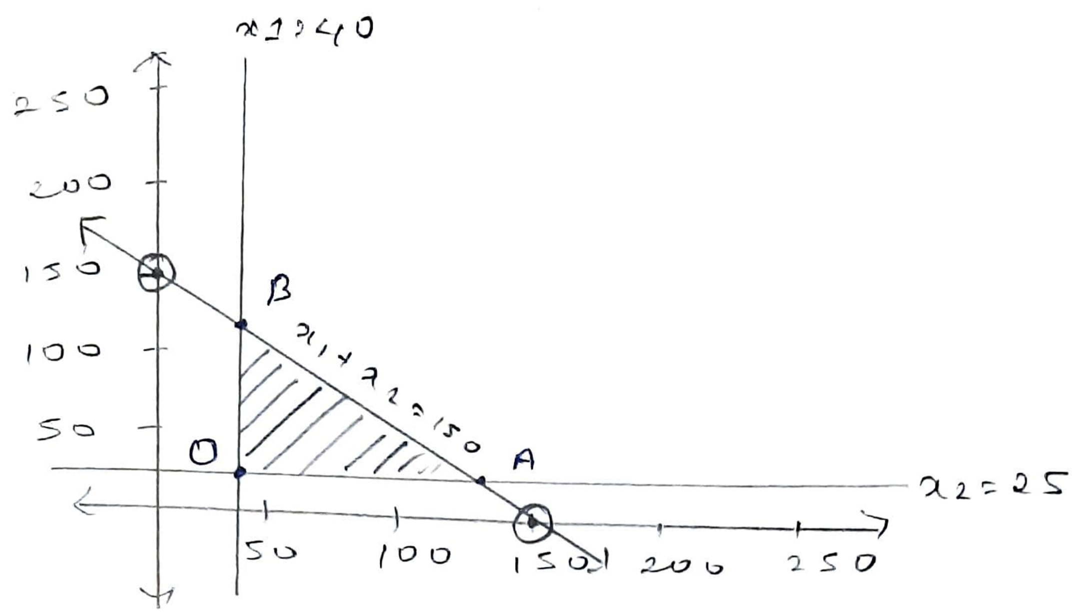
$$x_1 + x_2 = 150$$

$x_1 = 0 \quad x_2 = 150$

$x_2 = 0 \quad x_1 = 150$

points: $(0,150), (150,0)$

$x_1, x_2 \geq 0 \Rightarrow$ 1st quadrant.



Border points: O(40,25), A(125,25), B(40,110)

$Z_O = 225 \times 40 + 25 \times 200 = 14000$

$Z_A = 225 \times 125 + 25 \times 200 = 33125$

$Z_B = 225 \times 40 + 200 \times 110 = 31,000$

Thus $Z_A = 33125$ is the maximum value

i.e, point (125, 25) maximises profit

$\Rightarrow$ 125 coach tickets + 25 first class tickets must be sold for maximum profit.