$\frac{12}{9}/20$

S- SHIFTING THEOREM

a) $L\{f(t)\} = \frac{s+7}{s^2+4s+8}$. for $f(t)$

$$\frac{s+7}{s^2+4s+8} = \frac{s+7}{s^2+4s+4+4} = \frac{s+7}{(s+2)^2+2^2}$$

$$F(s+2) = \frac{s+2}{(s+2)^2+2^2} + \frac{5}{(s+2)^2+2^2}$$

$$\therefore F(s) = \frac{s}{s^2+2^2} + \frac{5}{s^2+2^2}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{s}{s^2+2^2}\right] + L^{-1}\left[\frac{5}{s^2+2^2}\right]$$

$$= \cos 2t + \frac{5}{2}\sin 2t$$

$$\therefore L^{-1}[F(s+2)] = \boxed{e^{-2t}\left(\cos 2t + \frac{5}{2}\sin 2t\right)}$$

b) $L\{f(t)\} = \frac{3s-137}{s^2+2s+401}$

$$F(s+1) = \frac{3s-137}{s^2+2s+1+400} = \frac{3s-137}{(s+1)^2+20^2}$$

$$= \frac{3(s+1)-140}{(s+1)^2+20^2} = \frac{3(s+1)}{(s+1)^2+20^2} - \frac{20\times 7}{(s+1)^2+20^2}$$

$$F(s) = \frac{3s}{s^2+20^2} - \frac{20\times 7}{s^2+20^2}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{3s}{s^2+20^2}\right] - 7L^{-1}\left[\frac{20}{s^2+20^2}\right]$$

$$= 3\cos 20t - 7\sin 20t$$

$$\therefore L^{-1}[F(s+1)] = \boxed{e^{-t}(3\cos 20t - 7\sin 20t)}$$

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* Find Inverse Laplace Transform.

1. $\dfrac{7}{(s-1)^3}$

$$F(s-1) = \frac{7}{(s-1)^3}$$

Thus, $F(s) = \dfrac{7}{s^3}$; $L^{-1}[F(s)] = \dfrac{7t^2}{2}$

$$L^{-1}[F(s-1)] = e^t \times \frac{7t^2}{2}$$

$$= \boxed{\frac{7e^t t^2}{2}}$$

2. $\dfrac{\sqrt{8}}{(s+\sqrt{2})^3}$

$F(s+\sqrt{2}) = \dfrac{\sqrt{8}}{(s+\sqrt{2})^3}$ $\Rightarrow$ $F(s) = \dfrac{\sqrt{8}}{s^3}$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{\sqrt{8}}{s^3}\right] = \frac{\sqrt{8}\,t^2}{2}$$

$$L^{-1}[F(s+\sqrt{2})] = e^{-\sqrt{2}t} \times \frac{\sqrt{8}\,t^2}{2}$$

$$= \boxed{\sqrt{2}\,(t^2 e^{-\sqrt{2}t})}$$

3. $\dfrac{15}{s^2+4s+29}$

$$= \frac{15}{s^2+4s+4+25} = \frac{15}{(s+2)^2+5^2}$$

$$\therefore F(s+2) = \frac{15}{(s+2)^2+5^2} = 3\left[\frac{5}{(s+2)^2+5^2}\right]$$

$$F(s) = \frac{3 \times 5}{s^2+5^2} \quad ; \quad L^{-1}[F(s)] = 3\sin 5t$$

$$\therefore L^{-1}[F(s+2)] = e^{-2t} \times 3\sin 5t$$

$$= \boxed{3e^{-2t}\sin 5t}$$

4. $\dfrac{\pi}{(s+\pi)^2}$

$$F(s+\pi) = \frac{\pi}{(s+\pi)^2} \Rightarrow F(s) = \frac{\pi}{s^2}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{\pi}{s^2}\right] = \pi t$$

$$\therefore L^{-1}[F(s+\pi)] = \boxed{e^{-\pi t}(\pi t)}$$

5. $\dfrac{s-6}{(s-1)^2+4}$

$$F(s-1) = \frac{(s-1)-5}{(s-1)^2+4} \Rightarrow F(s) = \frac{s-5}{s^2+4}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{s}{s^2+2^2}\right] - L^{-1}\left[\frac{5}{s^2+2^2}\right]$$

$$= \cos 2t - \frac{5}{2}\sin 2t$$

$$L^{-1}[F(s-1)] = \boxed{e^{-t}(\cos 2t - 5/2 \sin 2t)}$$

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6. $\dfrac{4s-2}{s^2-6s+18}$

$$= \frac{4s-2}{s^2-6s+9+3^2} = \frac{4s-2}{(s+3)^2+3^2}$$

$$F(s-3) = \frac{4(s-3)+10}{(s-3)^2+32}$$

$$\therefore F(s) = \frac{4s+10}{s^2+3^2}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{4s}{s^2+32}\right] + 10\, L^{-1}\left[\frac{1}{s^2+32}\right]$$

$$= 4\cos 3t - \frac{10}{3}\sin 3t$$

$$L^{-1}[F(s-3)] = \boxed{e^{3t}(4\cos 3t - 10/3 \sin 3t)}$$

7. $\dfrac{2s-56}{s^2-4s-12}$

$$= \frac{2s-56}{s^2-4s+4-16} = \frac{2s-56}{(s-2)^2-4^2}$$

$$F(s-2) = \frac{2(s-2)-52}{(s-2)^2-4^2} \Rightarrow F(s) = \frac{2s-52}{s^2-4^2}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{2s}{s^2-4^2}\right] - 13\, L^{-1}\left[\frac{4}{s^2-4^2}\right]$$

$$= 2\cosh 4t - 13\sinh 4t$$

$$\therefore L^{-1}[F(s-2)] = \boxed{e^{2t}(2\cosh 4t - 13\sinh 4t)}$$

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