Identical vectors : (or) Equal vectors :

Two vectors are said to be identical, if there difference is zero.

$\vec{A}$ and $\vec{B}$ are identical, if $\vec{A} - \vec{B} = 0$ (ie) $\vec{A} = \vec{B}$

→ coordinate systems :

In mathematics, there are various coordinate systems, here we consider only three.

(1) cartesian (or) rectangular coordinate system

(2) cylindrical coordinate system.

(3) spherical coordinate system.

(1) cartesian (or) rectangular coordinate system :

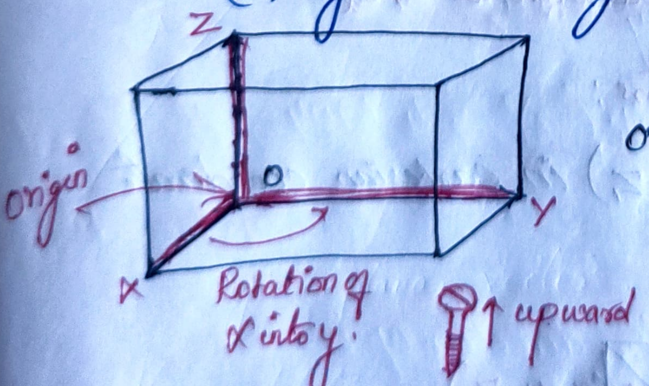
* Three coordinate axes – x, y and z – which are mutually at right angles to each other.

* origin – Three axes intersect at a common point.

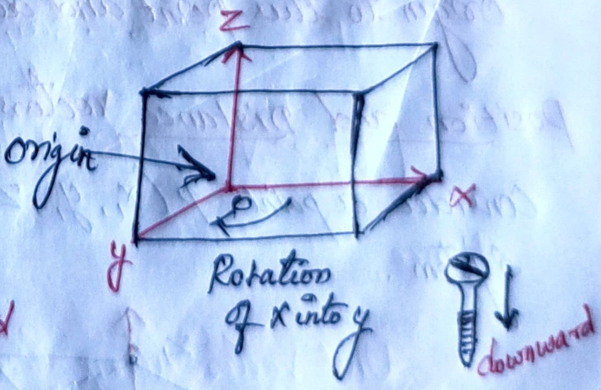
* There are two types of system

(i) Right handed system

(2) left handed system.



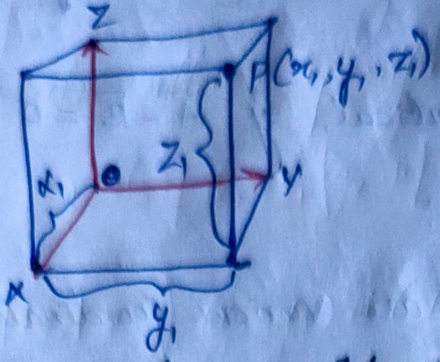
(a) Right handed system.



(b) left handed system.

In cartesian coordinate system x=0 plane indicates two dimensional y-z plane, y=0 plane indicates two dimensional x-z plane. similarly z=0 plane indicates x-y plane.

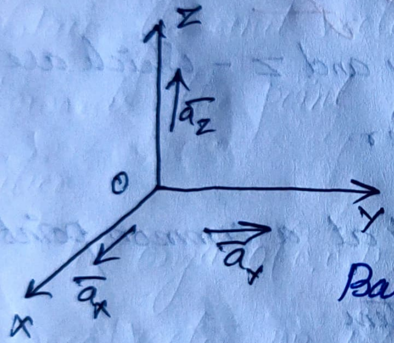
* Representing a point in rectangular coordinate system:



Similarly, $Q(3,-1,2)$ point can be represented.

Another method to define a point, consider three surfaces namely x = constant, y = constant and z = constant planes. The common intersection point of these three surfaces is the point to be defined.

Base vectors:



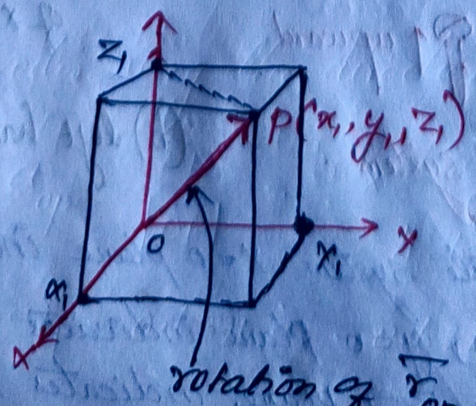
Base vectors are the unit vectors which are strictly oriented along the directions of the coordinate axes.

Base vectors are $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$.

* Example $(x_1, 0, 0)$ can be represented by a vector joining origin to this point and denoted as $x_1 \bar{a}_x$.

Position and Distance vectors:

Consider a point $P(x_1, y_1, z_1)$ in cartesian coordinate system.



rotation of $\bar{r}_{op}$ vector.

* position vector of point P is represented by the distance of point P from the origin.

* This is also called radius vector.

* Thus the position vector of point P can be

$$\overrightarrow{r_{op}} = x_1 \overline{a_x} + y_1 \overline{a_y} + z_1 \overline{a_z}$$

Magnitude of this vector

$$|\overline{r_{op}}| = \sqrt{(x_1)^2 + (y_1)^2 + (z_1)^2}$$

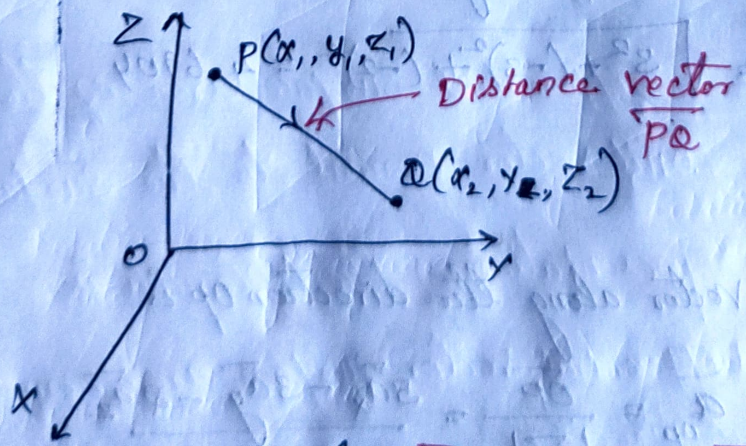
If point P has coordinates (1, 2, 3), then its position vector

$$|\overline{r_{op}}| = \sqrt{(1)^2 + (2)^2 + (3)^2} \Rightarrow \overline{r_{op}} = 1\overline{a_x} + 2\overline{a_y} + 3\overline{a_z}$$

$$= \sqrt{14} = 3.7416$$

* Mostly, position vector is denoted by the vector representing that point itself.
  - for point P, the position vector is $\overline{P}$
  - for point Q, the position vector is $\overline{Q}$.

* Consider two points in a cartesian coordinates. P and Q with the coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2)



Individual position vectors $\overline{P} = x_1 \overline{a_x} + y_1 \overline{a_y} + z_1 \overline{a_z}$

$$\overline{Q} = x_2 \overline{a_x} + y_2 \overline{a_y} + \overline{z_2} \overline{a_z}$$

Distance vector $\overline{PQ} = \overline{Q} - \overline{P} = [x_2 - x_1]\bar{a}_x + (y_2 - y_1)\bar{a}_y + (z_2 - z_1)\bar{a}_z$

This is also called seperation vector.

$\overline{PQ} = \overline{Q} - \overline{P} = [x_2 - x_1]\bar{a}_x + [y_2 - y_1]\bar{a}_y + [z_2 - z_1]\bar{a}_z$.

Magnitude $|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Unit vector along the direction PQ is

$$\bar{a}_{PQ} = \text{Unit vector along } PQ = \frac{(\overline{PQ})}{|\overline{PQ}|}$$

problems:

(i) obtain the unit vector in the direction from the origin towards the point P(3, -3, -2).

Solution:

The origin O(0,0,0) while P(3, -3, -2) hence the distance vector $\overline{OP}$ is,

$$\overline{OP} = (3 - 0)\bar{a}_x + (-3 - 0)\bar{a}_y + (-2 - 0)\bar{a}_z$$

$$\overline{OP} = 3\bar{a}_x - 3\bar{a}_y - 2\bar{a}_z$$

$$|\overline{OP}| = \sqrt{3^2 + (-3)^2 + (-2)^2} = 4.6904$$

Hence,

Unit vector along the direction OP is

$$\bar{a}_{OP} = \frac{\overline{OP}}{|\overline{OP}|} = \frac{3\bar{a}_x - 3\bar{a}_y - 2\bar{a}_z}{4.6904}$$

$$\bar{a}_{OP} = 0.6396\,\bar{a}_x - 0.6396\,\bar{a}_y - 0.4264\,\bar{a}_z$$

Example: 2 Two points A(2, 2, 1) and B(3, −4, +2) are given in cartesian system. Obtain the vector from A to B and a unit vector directed from A to B.

Soln Starting point is A and terminating point is B

$\bar{A} = 2\bar{a}_x + 2\bar{a}_y + \bar{a}_z \qquad \bar{B} = 3\bar{a}_x - 4\bar{a}_y + 2\bar{a}_z$

$\therefore \bar{AB} = \bar{B} - \bar{A} = (3-2)\,\bar{a}_x + (-4-2)\bar{a}_y + (2-1)\,\bar{a}_z$

$\bar{AB} = \bar{a}_x - 6\bar{a}_y + \bar{a}_z$

Magnitude $|\bar{AB}| = \sqrt{(1)^2 + (-6)^2 + (1)^2} = 6.1644$

Unit vector directed from A to B $\quad \bar{a}_{AB} = \dfrac{\bar{AB}}{|\bar{AB}|} = \dfrac{\bar{a}_x - 6\bar{a}_y + \bar{a}_z}{6.1644}$

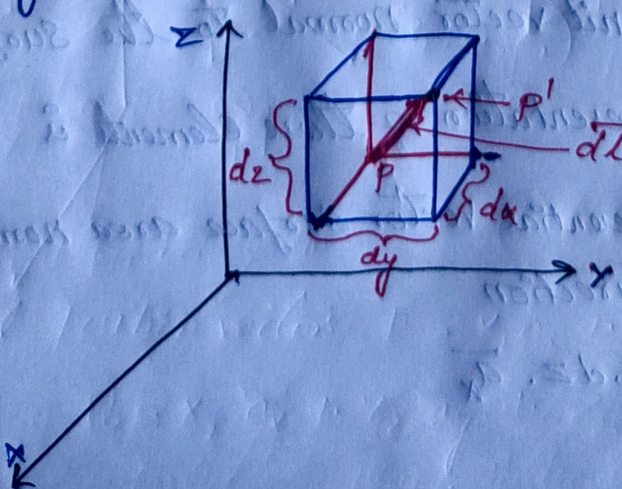
$\bar{a}_{AB} = 0.1622\,\bar{a}_x - 0.9733\,\bar{a}_y + 0.1622\,\bar{a}_z$

Cross verify $|\bar{a}_{AB}| = 1.$

⇒ Differential elements in cartesian coordinate system:

Consider a point $p(x, y, z)$ in the rectangular coordinate system. Let us increase each coordinate by a differential amount.

A new point p' will be obtained having coordinates $(x + dx,\ y + dy,\ z + dz)$



Thus dx = Differential length in x direction

dy = Differential length in y direction

dz = Differential length in z direction.

Hence, differential vector length also called elementary vector length

$$\overline{dL} = dx\,\overline{a_x} + dy\,\overline{a_y} + dz\,\overline{a_z}$$

The distance of P' from P is given by magnitude of the differential vector length

$$|\overline{dL}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

Hence, differential volume of the rectangular parallelopiped is given by

$$dv = dx \cdot dy \cdot dz.$$

Note: $\overline{dL}$ is a vector, dv is a scalar.

Let us define, differential surface area. The differential surface element $\overline{ds}$ is represented as

$$d\overline{S} = dS \cdot \overline{a_n}$$

Where, ds = Differential surface area of the element

$\overline{a_n}$ = Unit vector normal to the surface ds.

The vector representation of these elements is

$d\overline{S}_x$ = Differential vector surface area normal to x direction

$$d\overline{S}_x = dy \cdot dz \cdot \overline{a_x}$$

$d\bar{s}_y$ = Differential vector surface area normal to y direction

$= dx.dz.\bar{a}_y$

$d\bar{s}_z$ = Differential vector surface area normal to z direction

$= dx.dy.\bar{a}_z$