PAGE NO.:
DATE: / /

3/10/22

Unit 1 :- Set theory & Logic

Collection of distinct objects

Representation of Set - A, B, - - Z

— 1 — Member - a, b, c, - - - z

$a \in A$ a is element of set A

$a \notin A$ a is not — " " —

$\sim\in$

- A is a set of the integers -> A = {1, 2, 3, - - - }
- Set A is planets in solar system
  -> A = { Earth, Mars, Saturn, Jupiter, - - - }
- Set of all the integers less than 10
  -> A = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Representation of sets.
1) Roaster or Tabular form [Eg { , , }]
2) Descriptive form [Eg - A = A set of first five natural no.]
3) Set builder notation [Eg - $A = \{ x \in N \mid x \leq 5 \}$
   ($\in$ - is a member, $\mid$ - such that)

- A set of the even integer less than or equal to 50
  A = { 2, 4, . . . 48 }
  $A = \{ x \in N \mid x \leq 50$ & x is even $\}$

set of

Natural no	N
integer	Z
+ve integer	$Z+$
rational nos.	$Q = \{ p/q \mid p \in Z, q \in Z \text{ \& } q \neq 0 \}$
+ve — " —	$Q+$
Real no.	R
+ve — " —	$R+$
Complex nos.	C

★ Type of sets

1) Equal sets :- 2 sets are equal if and only if they have the same elements

Ex 1) $A = \{1, 5, 7\}$, $B = \{7, 5, 1\}$

2) $A = \{1, 2, 3, 6\}$, $A = \{$ set of +ve divisor of 6 $\}$

$A = \{3, 1, 6, 2\}$, $A = \{1, 2, 2, 6, 6, 3\}$
repeation is allowed.

~~A = B~~

2) Equivalent sets :- 2 finite sets A & B are said to equivalent if they have the same no. of elements

$A \cong B$

Ex - $A = \{p, q, r, z\}$ · $B = \{a, b, c, d\}$

Equal sets are always equivalents , equivalent set may not be equal.

3) Empty / Null set :-

ϕ or $\{ \}$

PAGE NO.:
DATE: / /

4) Singleton / unit set :- only 1 element.

5) universal set :- It is set of all elements under
U consideration.

☆ Five operation

1) Complement - The complement of a set A is the
$\overline{A}$, $\vec{A}$ set of all those element of the universal set U which are not in A.

Ex - $U = \{1, 2, 3, 4\}$
$A = \{1, 2\}$
$\overline{A} = \{3, 4\}$

Complement of A using set builder notification
$\overline{A} = \{x \mid x \in U \,\&\, x \notin A\}$

2) Union of a set
3) Intersation of a set
4) Different of a set
5) Symmetric of a set.

PAGE NO. :
DATE : / /

Q1) If U is the universal set containing 50 students of class X of a co educational school & A be the set of all girls & it contains 25 girls. Find the no. of elements of the complement of set of girls.

→ ~~25~~ $A + \bar{A} = U$

$25 + \bar{A} = \cancel{50}$ 50 $\quad \bar{A} = 25$

~~Q2)~~

Q 2) If $B = \{p \mid p$ is a multiple of $3, p \in N\}$
Find B'

→ $B = \{3, 6, 9, \ldots\}$

$B' = \{1, 2, 4, 5, 7, 8, \ldots\}$

$B' = \{p \mid p$ is not multiple of $3, p \in N\}$

2) Union of set :- The union of 2 set A & B, is set of all those elements which are either in A or B

$A \cup B \quad \{x \mid x \in A$ or $x \in B\}$

Eg - $A = \{2, 5, 9, 15\}$

$B = \{1, 2, 4, 5, 6\}$

$A \cup B = \{1, 2, 5, 4, 6, 9, 15\}$

Q1) ST $(A \cup B)' = A' \cap B'$

where $U = \{11, 12, 13, 14, 15, 16\}$

$A = \{12, 13\}$

$B = \{13, 14, 15\}$

PAGE NO.:
DATE: / /

→ $A \cup B = \{12, 13, 14, 15\}$
$(A \cup B)' = \{11, 16\}$ — ①
$A' = \{11, 14, 15, 16\}$
$B' = \{11, 12, 16\}$
$A' \cap B' = \{11, 16\}$ — ②

from ① & ②
$(A \cup B)' = A' \cap B'$

3) Intersection of sets
$A \cap B = \{x \mid x \in A \text{ \& } x \in B\}$
$A \cap A = A$ $A \cap \phi = \phi$
$A \cap B = B$ if $B \subseteq A$ (B is subset of A)
$A \cap U = A$
$A \cap \bar{A} = \phi$
~~Diff~~

4) Difference of sets
$A - B = \{x \mid x \in A \text{ \& } x \notin B\}$
$A - B \neq B - A$

5) Symmetric Diff^n
$A \oplus B = \{x \mid x \in A - B \text{ or } x \in B - A\}$
$= (A - B) \cup (B - A)$

6) Power set
$A = \{a, b, c, d\}$
Power set of A = {a, b, c, d, ab, ac, ad,
bc, bd, cd,
abc, abd, bcd, abcd
Φ, φ}
$2^4 = 16$

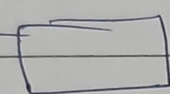
7) Cartesian Product

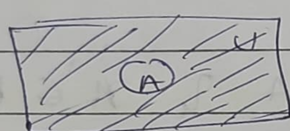
$A \times B = B \times A$

8

* Venn Dig.

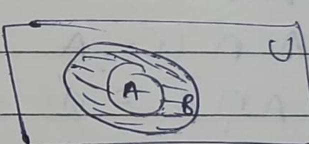
Set — 

U set — 

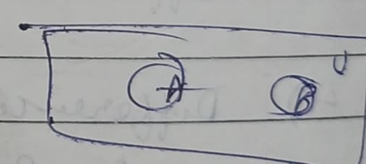
$\bar{A}$ → 

union of set

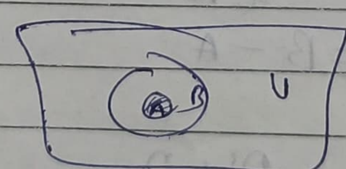
$A \cup B = B$ if $A \subseteq B$



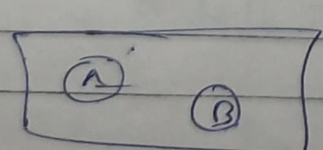
$A \cup B$ when neither $A \subseteq B$ or $B \subseteq A$



$A \cap B$ when $A \subseteq B$



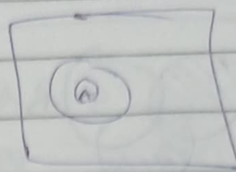
$A \cap B$ when A & B are disjoint sets



$\Rightarrow \phi$

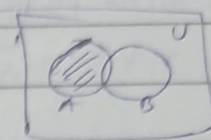
Diffⁿ

$A - B$ if $A \subseteq B$



$\Rightarrow \phi$

$A - B$ when neither $A \subset B$ nor $B \subseteq A$



☆ Laws

$A \cup B = B \cup A$

$A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

Indempotent $A \cup A = A$

$A \cap A = A$

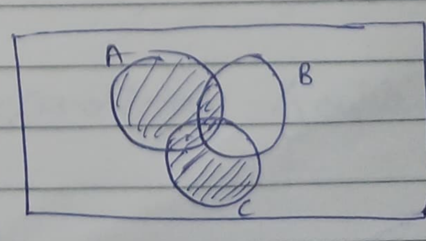
$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

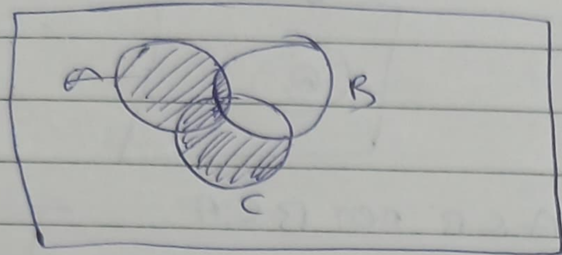
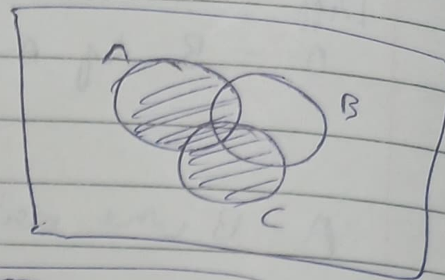
Q1) ST $A \cup (\overline{B} \cap C) = (A \cup \overline{B}) \cap (A \cup C)$

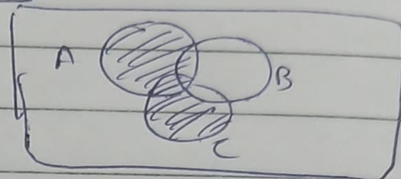
→ ~~LHS = $(A \cup \overline{B}) \cap (A \cup C)$~~

LHS =



$RHS = (A \cup \bar{B}) \cap (A \cup C)$

$=$  $\cap$ 

$=$ 

H.P.

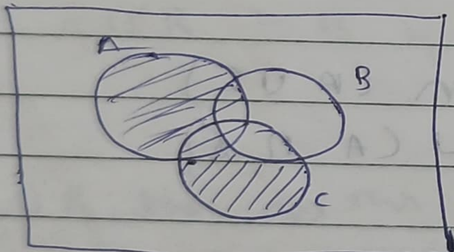
H.W

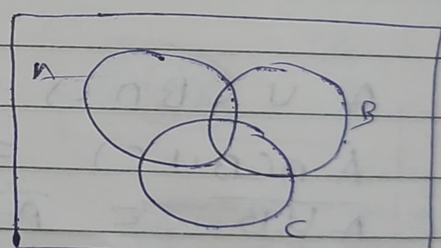
Q) Prove the expressions by using vinn diagram.

1) $A \cup (\bar{B} \cap C) = (A \cup B) \cap (A \cup C)$

2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3) $A \cap \overline{\{B} = \bar{A} \cup \bar{B}$

1) $\Rightarrow$ LHS $=$ 

RHS $=$ 

PAGE NO.:
DATE: / /

✲ Cardinality $|A|$

For any 2 disjoint set A & B

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C|$
$- |B \cap C| + |A \cap B \cap C|$

Q) A computer company must hire 20 programmers to handle system programming jobs & 30 programmers for application prog. job, out of those hire 5 are expected to perform jobs of both types. How many programmers must be hired

→ $|A \cap B| = 5$ $\quad n(A \cap B) = 50$

$|A| = 20$

$|B| = 30$

$|A \cup B| = 20 + 30 - 5$
$= 45$

Q) Out of 250 candidates who failed in an examination, it was revealed that 128 failed in maths, 87 failed in physics & 134 in chem. 31 failed in maths & phy, 54 failed in chem & maths, 30 failed in chem & phy.

Find out how many of candidate failed in

a) all three subjects
b) In math but not physics.

PAGE NO.:
DATE: / /

A - Math
B - Phy
C -

$\rightarrow$ $|M \cup P \cup C| = 250$

$|M| = 128$

$|P| = 87$

$|C| = 134$

$|M \cap P| = 31$

$|M \cap C| = 54$

$|P \cap C| = \cancel{82}\ 30$

$-|M \cap P \cap C| = 128 + 87 + 134 - 31 - 54 - 30 - 250$

$|M \cap P \cap C| = \cancel{174} \cancel{-(-70)} \quad -(-16)$

$= 16$

$\cancel{|M - P| = |M|}$

$|M \cap P| = |M| - |M \cap P|$

$= 128 - 31$

$= 97$

Q) Consider a set of integers from 1 to 250. find out

1) how many of these nos are divisible by 3 or 5

2) —— " —— by 5 or 7

3) —— " —— by 3 or 7

4) —— by 3 or 5 or 7

5) —— 3 or 5 by not by 7.

& $\cap$ or $\cup$

PAGE NO.:
DATE: / 11

6) 5 or 7 but not by 3.
7) 3 or 7 but not by 5
8) betn 1 to 250 not divisible by 3, 5 & 7

→

$3 \to 3, 6, 9, \ldots, 249$ $3\overline{)250}$ = 83

$|A_3| = 83 = \left[\frac{250}{3}\right]$ $|A_3 \cap A_5| = 16$

$|A_5| = 50$ $|A_3 \cap A_7| = 11$

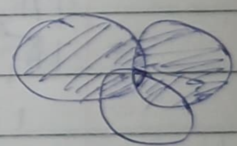
$|A_7| = 35$ $|A_5 \cap A_7| = 7$

$15\overline{)250}$ = 16

$|A_3 \cap A_5 \cap A_7| = 2$

1) $|A_3 \cup A_5| = 83 + 50 - 16 = 117$
2) $|A_5 \cup A_7| = 50 + 35 - 7 = 78$
3) $|A_3 \cup A_7| = 83 + 35 - 11 = 107$
4) $|A_3 \cup A_5 \cup A_7| = 83 + 50 + 35 - 16 - 11 - 7 + 2$
$= 136$

5) $|A_3 \cup A_5| - |A_3 \cap A_5 \cap A_7| = 117 - 2$
$= 115$



6) $|A_5 \cup A_7| - |A_3 \cap A_5 \cap A_7| = 78 - 2$
$= 76$

7) $|A_3 \cup A_7| - |A_3 \cap A_5 \cap A_7| = 107 - 2$
$= 105$

8) $250 - 136$
$= 114$

a) Among 200 student in a class 104 students got A grade in 1st examination & 84 students got A grade in 2nd examination if 68 student didn't get A grade in either of examination. Find out

1) How many students got A grade in both examination

2) If no. of students who got A grade in 1st examination = those who got A grade in 2nd examination. If total students who got A in exactly 1 exam is 160. If 16 students students doesn't get A in both exam. Determine the no. of students who got A in 1st exam, those who got A in 2nd exam & no. of students who got in both exam.

Sol →

$U = 200$

$|A_1| = 104$

$|A_2| = 84$

$|\overline{A_1 \cup A_2}| = 68$

$|A_1 \cap A_2| = ?$

$|A_1 \cup A_2| = 200 - 68$

$= 132$

$|A_1 \cap A_2| = 104 + 84 - 132$

$= 188 - 132$

$= 56$

$|A_1| = |A_2|$

$\cancel{|A_1 \cap A_2|}$ $|A_1 \cup A_2| - |A_1 \cap A_2| = 160$

$|A_1 \cap A_2| = 200 - 16$

$= 184$

$|A_1| = ?$

$|A_2| = ?$

$|A_1 \cap A_2| = ?$

$2|A_1| - 184 - 184 = 160$

$2|A_1| = 160 + 368$

$|A_1| = \frac{528}{2}$

$|A_1| = |A_2| = 264$

Mathematical ~~induct~~ Induction:

Proof: It is a technique which is used to prove a statement / formulae or a theorm is true for every natural no.

- It is the process to establish the validity of an ordinary result involving natural nos.

Step 1) Basis $P(n_0)$ is true

2) $P(n)$ is true

3) $P(n+1)$ must be true

Q1) Prove the following using principal of MI for all $n \in N$

1) $1^3 + 2^3 + 3^3 + \dots n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\Rightarrow$ $P(1)$

$LHS = 1$ $\quad$ $RHS = \left(\frac{1.(1+1)}{2}\right)^2 = 1$

$P(K) = $ true

$P(K+1)$

$LHS = 1^3 + 2^3 + 3^3 + \dots K^3 + (K+1)^3$

$= \left[\frac{K(K+1)}{2}\right]^2 + (K+1)^3$

$= (K+1)^2 \left[\frac{K^2 + K + 1}{4}\right]$

$= \cancel{(K+1)^2 \left[\frac{3K+2}{2}\right]}$

$= (K+1)^2 \left[\frac{K^2 + 4K + 4}{4}\right]$

$RHS = \left[\frac{(K+1)(K+2)}{2}\right]^2$

$= (K+1)^2 \frac{(K^2 + 4K + 4)}{4}$

$LHS = RHS.$

2) $\frac{1}{2\times 5} + \frac{1}{5\times 8} + \frac{1}{8\times 11} + \dots + \frac{1}{(3K-1)(3K+2)}$

$= \frac{K}{6K+4}$

$\to P(1)$

$LHS = \frac{1}{2\times 5} = \frac{1}{10}$ $\qquad RHS = \frac{1}{6+4} = \frac{1}{10}$

for $p(k) = $ true

$P(k+1)$

$$LHS = \frac{1}{2\times 5} + \frac{1}{5\times 8} + \cdots + \frac{1}{[3(k+1)-1][3(k+1)+2]}$$

$$= \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{1}{3k+2}\left[\frac{3k+5+3k-1}{(3k+5)(3k-1)}\right]$$

$$= \frac{6k+4}{(3k+2)(3k+5)(3k-1)}$$

$$= \frac{2}{(3k+5)(3k-1)}$$

$$RHS = \frac{k+1}{6k+10}$$

$$= \frac{k}{2(3k+5)} + \frac{1}{2(3k+5)} = \frac{k+1}{2(3k+5)}$$

$$= \frac{1}{2}\Big[$$

$$\frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{1}{3k+2}\left[\frac{k}{2} + \frac{1}{3k+5}\right] = \frac{1}{3k+2}\left[\frac{3k^2+5k+2}{2(3k+5)}\right]$$

$$\frac{1}{3K+2}\left[\frac{(3K+2)(K+1)}{2(3K+5)}\right] = \frac{(K+1)}{2(3K+5)}$$

$$RHS = \frac{K+1}{6K+10} = \frac{K+1}{2(3K+5)}$$

LHS = RHS.

Q3) For all integers ST $8^n - 3^n$ is divisible by 5 by MI for $n \geq 1$

→ $P(n) = 8^n - 3^n$

$P(1) = 8 - 3 = 5$ divisible by 5

$P(1) = $ true

~~P(n) = true for n = K~~

Let us assum $P(n) = 8^n - 3^n$ is divisible by 5 for all $n = K$

$$P(K+1) = 8^{K+1} - 3^{K+1}$$
$$= 8^K \cdot 8 - 3^K \cdot 3$$
$$= 8^K \cdot 8 - 3^K(8 - 5)$$
$$= 8^K \cdot 8 - 3^K \cdot 8 + 5 \cdot 3^K$$
$$= 8(8^K - 3^K) + 5 \cdot 3^K$$

divisible by 5

∴ Their addition will also be divisible by 5.

PAGE NO.:
DATE: / /

Q3) S.T. the sum of the cubes of 3 consecutive natural no. is divisible by 9.

→ $P(n) = n^3 + (n+1)^3 + (n+2)^3$
$P(1) = 1 + 2^3 + 3^3$
$= 1 + 8 + 27 = 36$ divisible by 9.

$P(1) =$ true

$P(k+1) = (k+1)^3 + (k+2)^3 + (k+3)^3$
$= \underbrace{(k+1)^3 + (k+2)^3 + k^3}_{\text{divisible by 9}} + \underbrace{27 + 9k^2 + 27k}_{9(3+k^2+3k)\text{ divisible by 9}}$

Q4) Prove the foll using principal of MI where $n \in N$
~~P(n)~~ $x^{2n} - y^{2n}$ is divisible by $(x+y)$

→ $P(n) = x^{2n} - y^{2n}$
$P(1) = x^2 - y^2 = (x-y)(x+y)$
which is divisible by $x+y$.

∴ $P(1) =$ true
∴ $P(k) = x^{2k} - y^{2k}$ is true.
$P(k+1) = x^{2(k+1)} - y^{2(k+1)}$
$= x^{2k} \cdot x^2 - y^{2k} \cdot y^2$
$= x^{2k} \cdot x^2 - y^{2k} y^2 + x^2 y^{2k} - x^2 y^{2k}$
$= \cancel{x^2[x^{2k} - y^{2k}] + y^2[x^{2k} - y^{2k}]}$ ~~divisible by~~ $(x+y)$

$= x^2[\underbrace{x^{2k} - y^{2k}}_{\text{divisible by } x+y}] + y^{2k}[x^2 - y^2]$

Q 5) ST for any integer n, $11^{n+2} + 12^{2n+1}$ is divisible by 133

Q 6) For all $n \geq 1$, PT. $1^2 + 2^2 + 3^2 + \cdots n^2 = \left[\frac{n(n+1)(2n+1)}{6}\right]$

Q 7) PT $(ab)^n = a^n b^n$ is true for every natural no n.

5) $\Rightarrow$ $P(n) = 11^{n+2} + 12^{2n+1}$

$P(1) = 11^3 + 12^3 = 1331 + 1728 = 3059$

$3059 \div 133 = 23$

$\therefore$ It is divisible by 133.

$\therefore$ $P(K) = 11^{K+2} + 12^{2K+1}$ is true for all $K \in N$

$P(K+1) = 11^{K+3} + 12^{2K+3}$

$= \cancel{11^K \cdot 11^3 + 12^{2K} \cdot (12^3 + 11^3 - 11^3)}$

$= 11^K \cdot 11^3 + 12^{2K} \cdot 12^3 + 11^3 \cdot 12^{2K} - 11^3 \cdot 12^{2K}$

$-11(+1)$

$= 12^{2K}(12^3 + 11^3) + 11^3(11^K - 12^{2K})$

$= 11^{K+2} \cdot 11 + 12^{2K+1} \cdot 12^2$

$= 11 \cdot 11^{K+2} + 12^{2K+1}(12^2 - 11 + 11)$

$\cancel{= 11^{K+2}(11 + 12^2}$

$= \underbrace{11(11^{K+2} + 12^{2K+1})}_{\text{divisible}} + \underbrace{12^{2K+1}\underbrace{(12^2 - 11)}_{133}}_{\text{by } 133}$

divisible by 133.

$$\frac{(r+1)(k+2)(2k+3)}{6}$$

PAGE NO.:
DATE: / /

6) → $P(n) = 1^2 + 2^2 + \dots n^2 = \frac{n(n+1)(2n+1)}{6}$

LHS
$P(1) = 1$ RHS $= \frac{1(2).(3)}{6} = 1$

∴ ~~For~~ $P(K) = \frac{K(K+1)(2K+1)}{6}$ for all $K \in N$

$$\text{LHS} = P(K+1) = 1^2 + 2^2 + \dots k^2 + (k+1)^2$$
$$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$$
$$= \frac{K+1}{6}\left[K(2K+1) + 6K+6\right]$$
$$= \frac{(K+1)(2K^2+7K+6)}{6}$$

$$\text{RHS} = \frac{(K+1)(K+2)(2K+3)}{6}$$
$$= \frac{(K+1)(2K^2+7K+6)}{6}$$

LHS = RHS.

7) For $n=1$

LHS = ab RHS = $a.b$

∴ For for all $K \in N$
$(ab)^K = a^K b^K$ is true

$n = k+1$

LHS = $(ab)^{K+1}$ RHS = $a^{K+1} . b^{K+1}$
$= a^K . a . b^K . b$ $= a^K . a . b^K . b$

∴ LHS = RHS

* Proposition (statement)

A proposition is a declarative statement that is either true or false but not both.

	symbol	
Conjunction (AND) (join)	$\wedge$	$p \wedge p$
negation (NOT)	$\sim$ / $\neg$	$\sim p$
disconjunction (OR)	$\vee$	$p \vee q$
Conditional (~~if & only if~~) (if then)	$\longrightarrow$	
Biconditional (if & only if	$\longleftrightarrow$	

If you get 100% in end term exam then you will get A grade

both $p \to q$, $q \to p$ should be same.

p	q	$p \to q$	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$
T	T	T	T	T	T
T	F	F	F	T	f
F	T	T	F	T	f
F	F	T	T	f	f

Q) Translate the foll english statement into english statement.

1) You can access the internet from campus only if you are a CS student or your not a fresher.

→ p = you can access internet from campus.
q = you are CS student.
r = you are fresher

PAGE NO.:

DATE: / /

1/11/22

$p \rightarrow (q \vee (\sim r))$

Q) Express foll statement in propositional form.

1) There are many clouds in the sky but it did not rain.

2) I will get 1^{st} class if & only if I study well & score above 80 in mathematics.

3) If I finish my submission before 5 in the evening & it is not very hot, I will go & play a game of hockey.

→ 1) p → There are many clouds in the sky

q → It did rain.

$p \wedge \sim q$

2) p → I will get 1^{st} class

q → I study well

r → I ~~will~~ score above 80 in math

$p \leftrightarrow (q \wedge r)$

3) P → I finish my submission before 5 in the evening

q → it is very hot

r → I will go & play a game of hockey

$(p \wedge \sim q) \rightarrow r$

Q) $p \rightarrow$ I will ~~will~~ study ~~DM~~ discreate strum.
$q \rightarrow$ I will go to movie.
$r \rightarrow$ I am in good mood.

Write the eng statement that correspond to follo.

1) $\sim r \rightarrow q$
2) $\sim q \wedge p$
3) $q \rightarrow \sim p$
4) $\sim p \rightarrow \sim r$

→ 1) I am not in a good mood ~~then~~, I will go to movie.
2) I will not go to movie and I will study discreate strum
3) If I will go to movie ~~then~~, I will not study Dm.
4) If I ~~don't~~ will not stay Dm, I am not in good mood.

Quantifiers

1) Universal → $\forall$
2) Existential → $\exists$

Let p(x) be the statement "$x + 1 > x$"
& let us assume that Domain set of all +ve int.

$P(1) = 1+1>1$ T

$P(2) = 2+1>2$ T $\quad \forall x \quad P(x)$

Q) What is the truth value of :- $\exists x P(x)$ is the statement $x^2 > 10$ & the universe of dispos consist of the +ve integer not exceeding 4.

PAGE NO.:

DATE: /

$\Rightarrow \{1, 2, 3, 4\}$

$1^2 > 10$ F

$2^2 > 10$ F

$3^2 > 10$ F

$4^2 > 10$ T