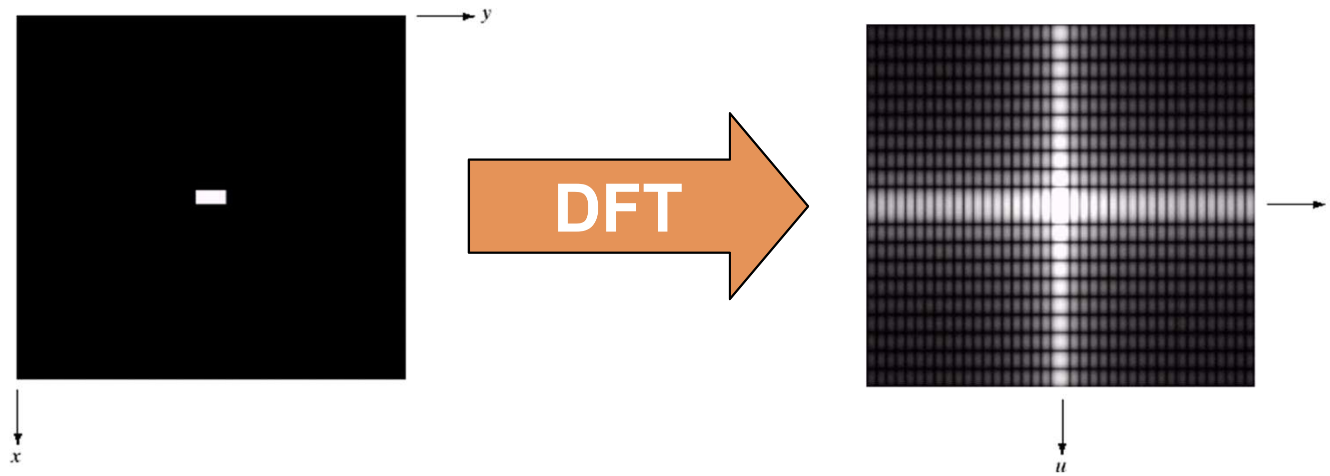
Unit 3

IMAGE ENHANCEMENT: FILTERING IN THE FREQUENCY DOMAIN

Image Enhancement in Frequency Domain

DFT & Images

The DFT of a two dimensional image can be visualised by showing the spectrum of the images component frequencies

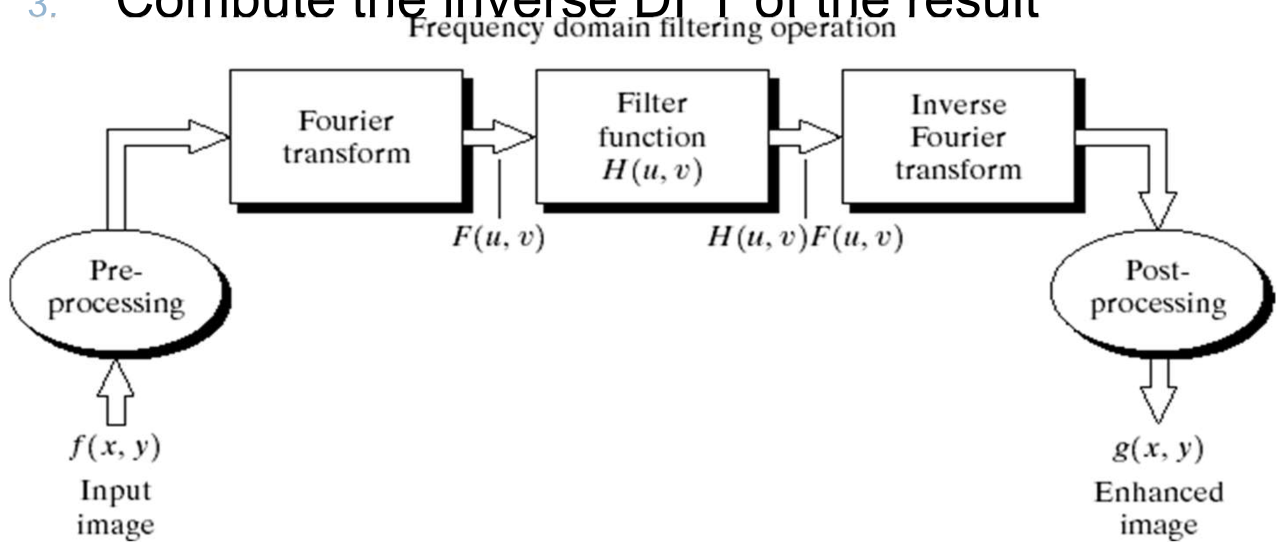


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

The DFT and Image Processing

To filter an image in the frequency domain:

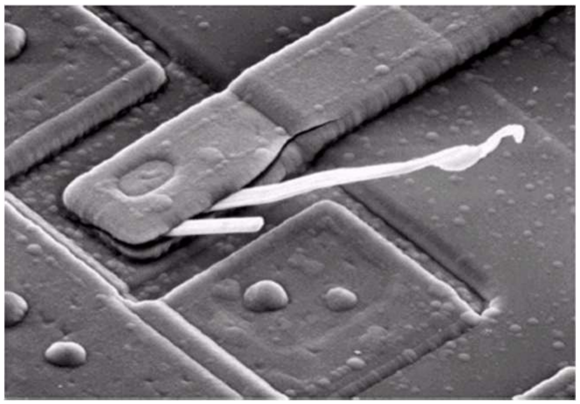
1. Compute *F(u,v)* the DFT of the image
2. Multiply *F(u,v)* by a filter function *H(u,v)*
3. Compute the inverse DFT of the result

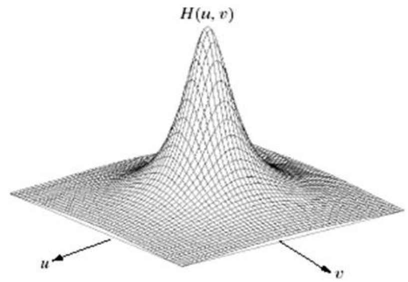


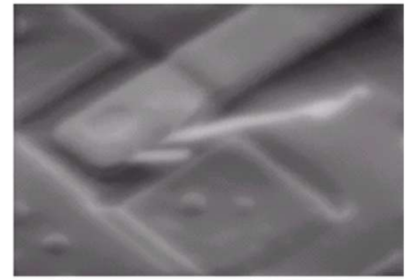
Some Basic Frequency Domain Filters

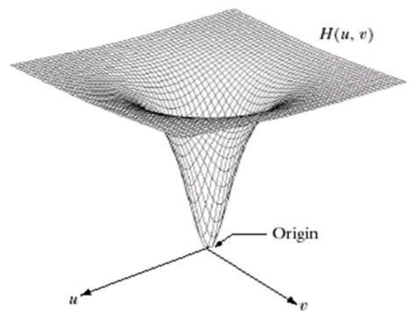
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

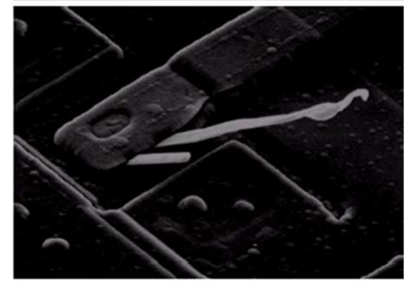
Low Pass Filter











High Pass Filter

Smoothing Frequency Domain Filters

Smoothing is achieved in the frequency domain by dropping out the high frequency components

The basic model for filtering is:

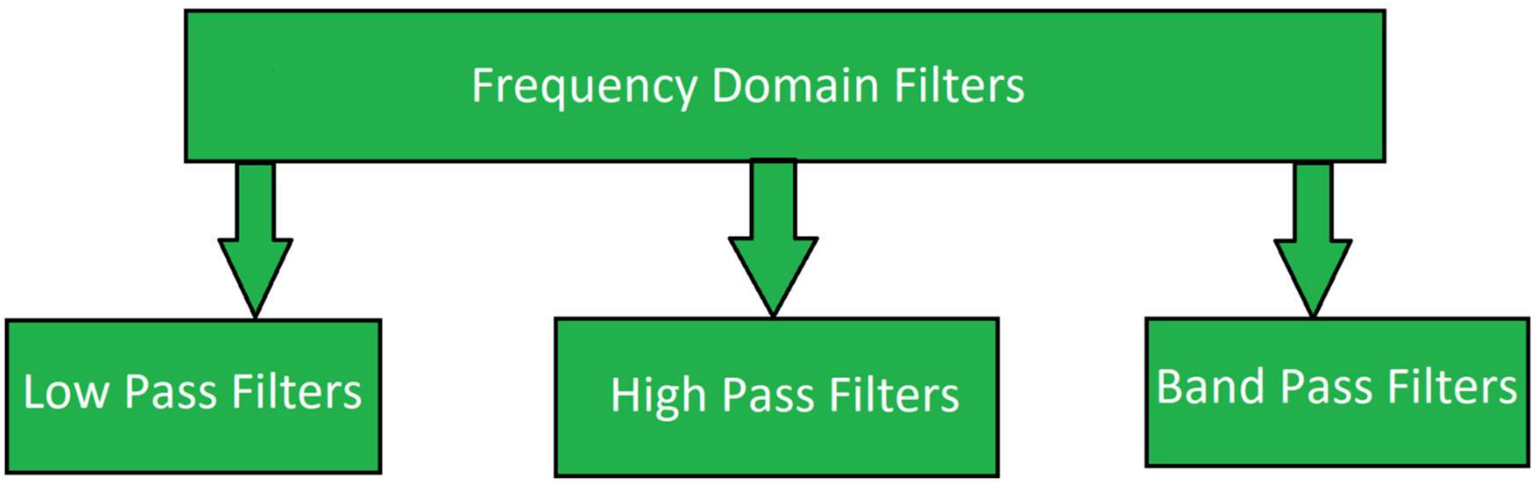
$$G(u,v) = H(u,v)F(u,v)$$

where $F(u,v)$ is the Fourier transform of the image being filtered and $H(u,v)$ is the filter transform function

Low pass filters – only pass the low frequencies, drop the high ones

Smoothing (blurring) is achieved in the frequency domain by high frequency attenuation; that is by low pass filtering

Frequency Domain Filters and its Types



Classification of Frequency Domain Filters

Introduction

Filtering process can also be perform in frequency domain.

- ❑ Frequency Domain Filtering process is based on image transforms.
- ❑ Frequency Domain Filters are used for smoothing and sharpening of image by removal of high or low frequency components.
- ❑ Sometimes it is possible of removal of very high and very low frequency.
- ❑ Frequency domain filters are different from spatial domain filters as it basically focuses on the frequency of the images.

Frequency domain filtering is preferred over spatial domain because it involves less computations

1. **Low pass filter:**
   Low pass filter removes the high frequency components that means it keeps low frequency components. It is used for smoothing the image.

It is used to smoothen the image by attenuating high frequency components and preserving low frequency components. Mechanism of low pass filtering in frequency domain is given by:

```
G(u, v) = H(u, v) . F(u, v) where F(u, v) is the Fourier
Transform of original image and H(u, v) is the Fourier Transform
of filtering mask
```

2. High pass filter:

High pass filter removes the low frequency components that means it keeps high frequency components. It is used for sharpening the image. It is used to sharpen the image by attenuating low frequency components and preserving high frequency components.

Mechanism of high pass filtering in frequency domain is given by:

```
H(u, v) = 1 - H'(u, v) where H(u, v) is the Fourier Transform of
high pass filtering and H'(u, v) is the Fourier Transform of low
pass filtering
```

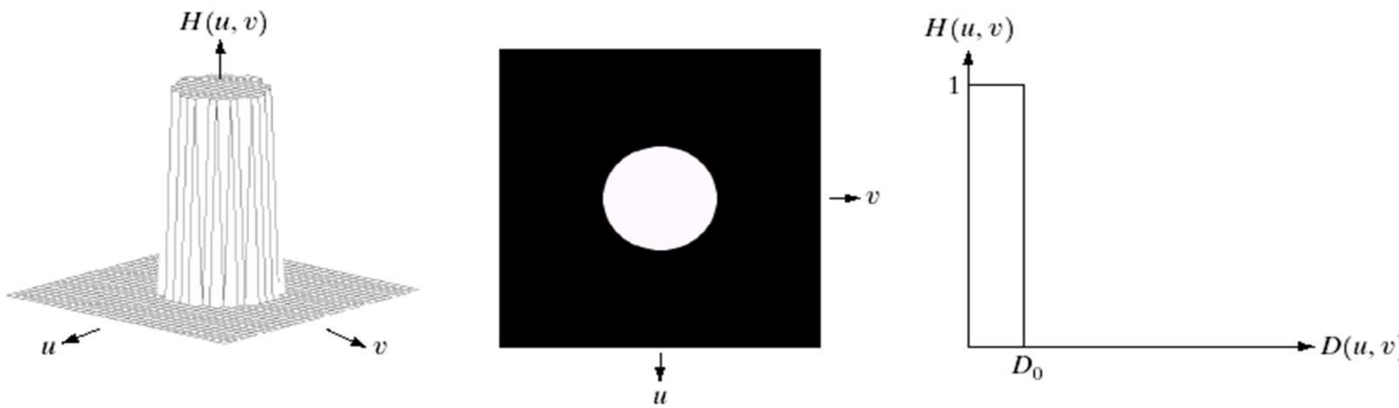
3. Band pass filter:

Band pass filter removes the very low frequency and very high frequency components that means it keeps the moderate range band of frequencies. Band pass filtering is used to enhance edges while reducing the noise at the same time.

Ideal Low Pass Filter

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Simply cut off all high frequency components that are a specified distance D_0 from the origin of the transform



filter

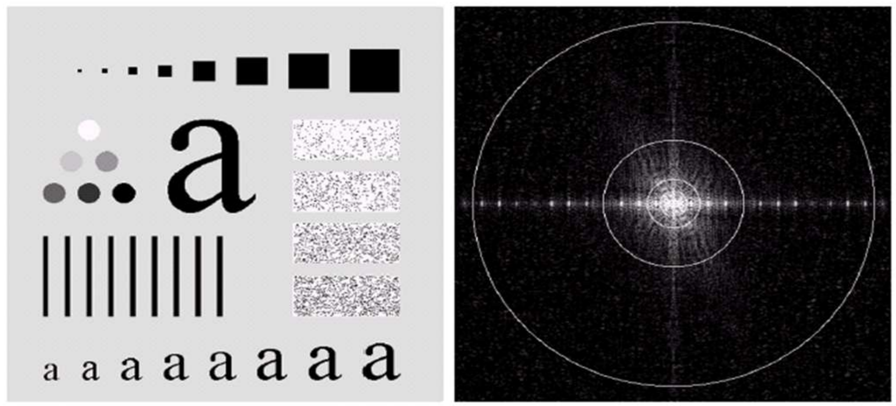
Ideal Low Pass Filter (cont…)

The simplest lowpass filter we can envision is a filter that "cuts off" all high-frequency components of the Fourier transform that are at a distance greater than a specified distance D_0 from the origin of the (centered) transform. Such a filter is called a two-dimensional (2-D) *ideal lowpass filter* (ILPF) and has the transfer function

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases} \tag{4.3-2}$$

Where D0 is the positive quantity

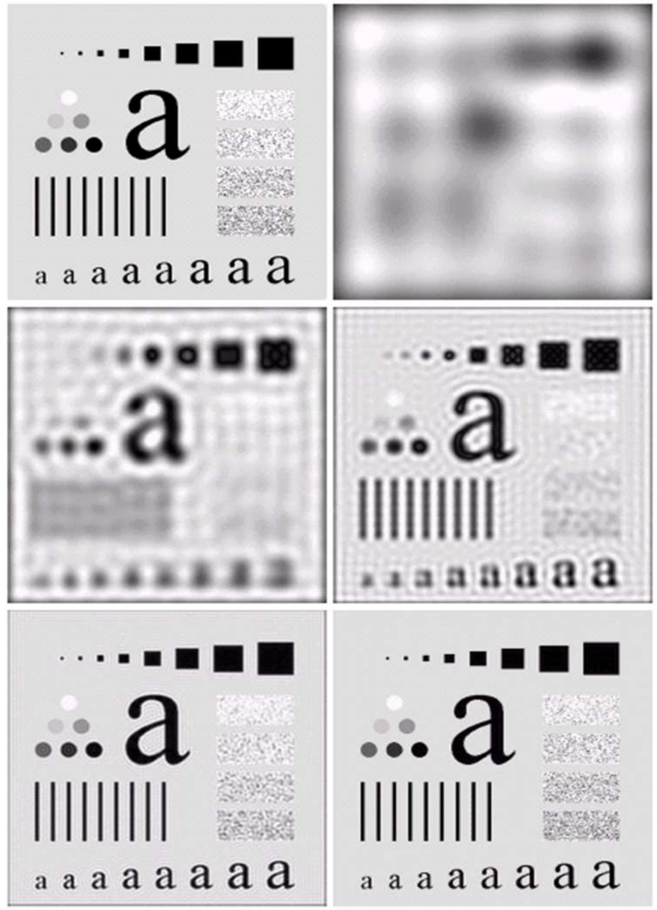
Ideal Low Pass Filter (cont…)



Above we show an image, it’s Fourier spectrum and a series of ideal low pass filters of radius 5, 15, 30, 80 and 230 superimposed on top of it

Ideal Low Pass Filter (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Original image

Result of filtering with ideal low pass filter of radius 5

Result of filtering with ideal low pass filter of radius 15

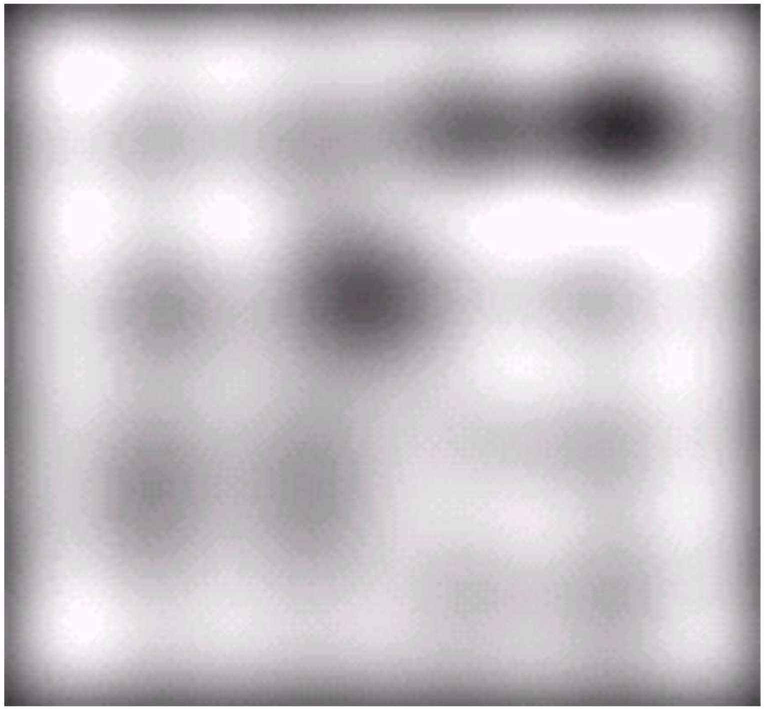
Result of filtering with ideal low pass filter of radius 30

Result of filtering with ideal low pass filter of radius 80

Result of filtering with ideal low pass filter of radius 230

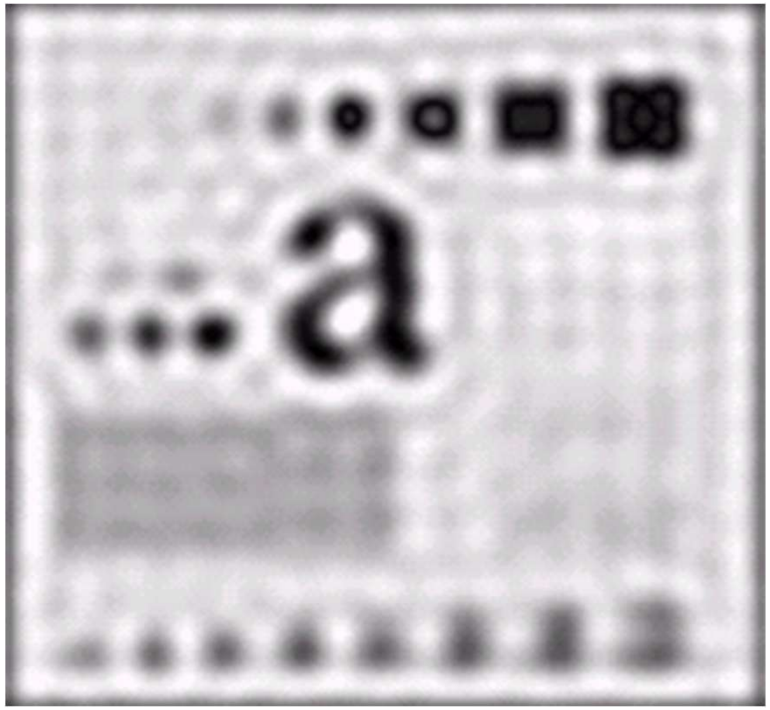
The “ringing” effect can be seen, which becomes finer in texture as the amount of high frequency content removed decreases.

Ideal Low Pass Filter (cont…)



Result of filtering with ideal low pass filter of radius 5

Ideal Low Pass Filter (cont…)



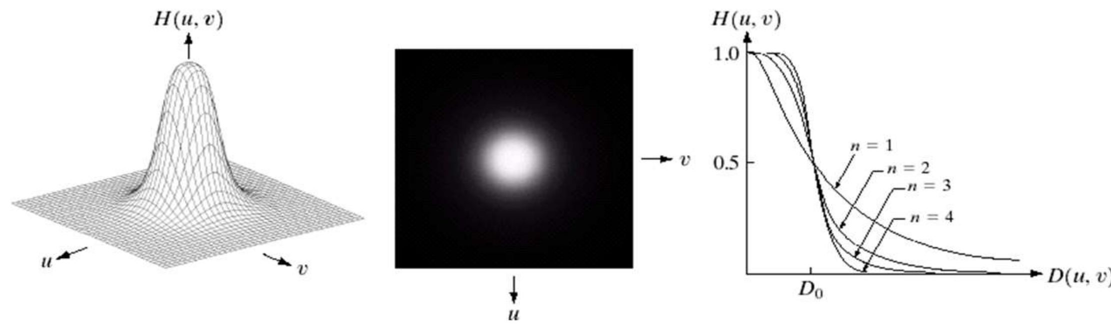
Result of filtering with ideal low pass filter of radius 15

Butterworth Lowpass Filters

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

The transfer function of a Butterworth lowpass filter of order n with cutoff frequency at distance D_0 from the origin is defined as:

$$H(u, v) = \frac{1}{1 + [D(u, v) / D_0]^{2n}}$$

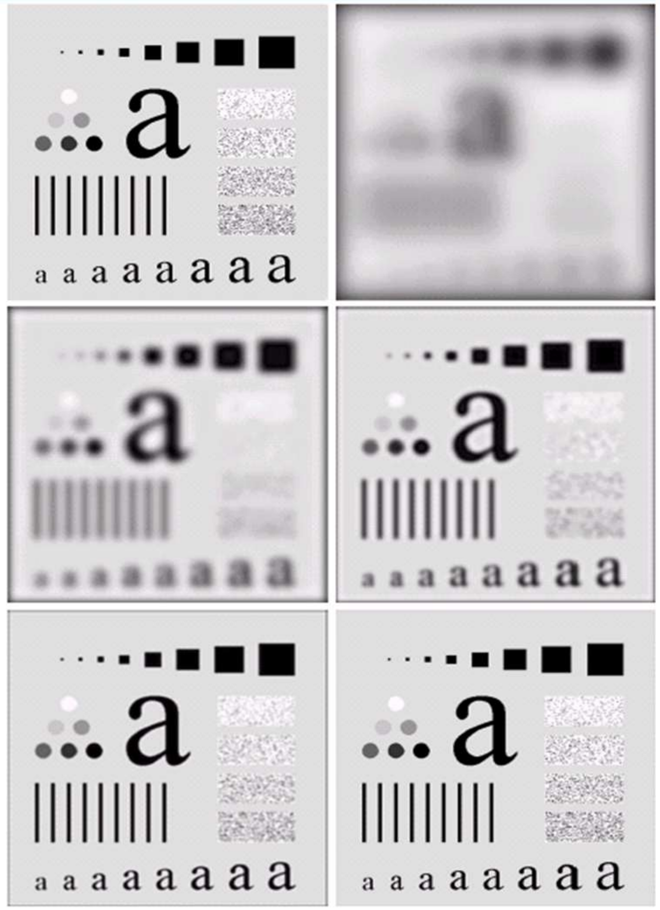


Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuities that gives a clear cut off between passed and filtered frequencies.

Butterworth Lowpass Filter (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Original image



Result of filtering with Butterworth filter of order 2 and cutoff radius 5

Result of filtering with Butterworth filter of order 2 and cutoff radius 15

Result of filtering with Butterworth filter of order 2 and cutoff radius 30

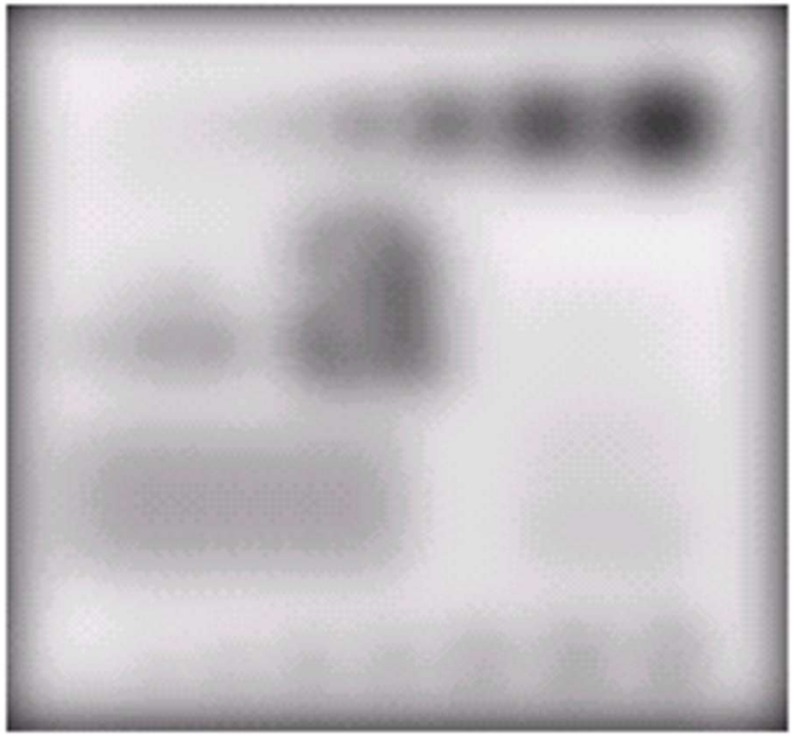
Result of filtering with Butterworth filter of order 2 and cutoff radius 80

Result of filtering with Butterworth filter of order 2 and cutoff radius 230

The BLPFof order 1 has no ringing effect. Order 2 does show mild ringing effect.

Butterworth Lowpass Filter (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

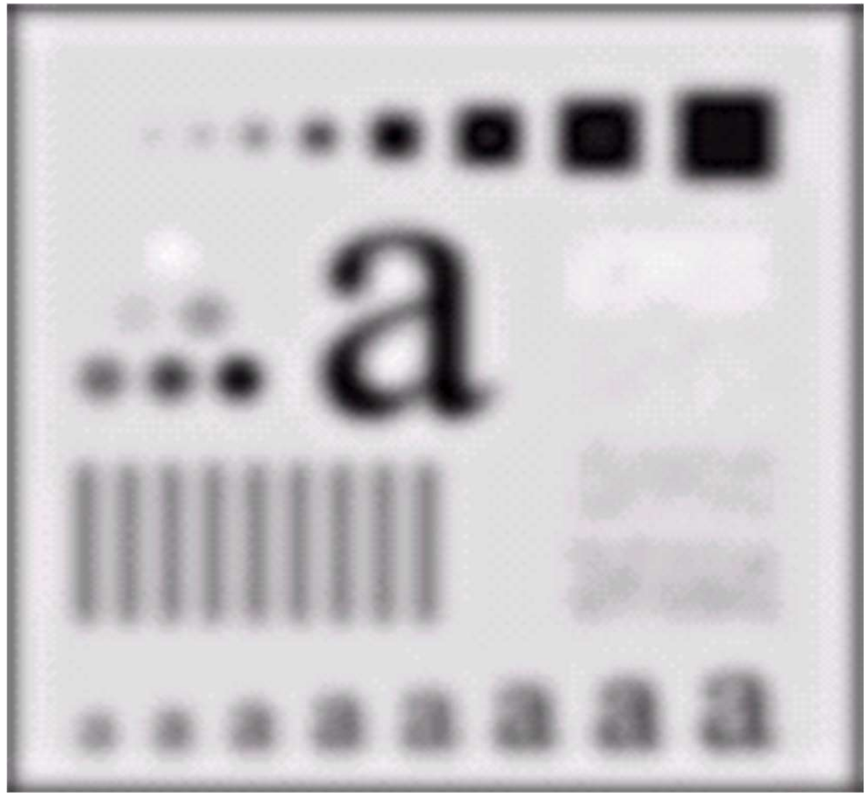


Result of filtering with Butterworth filter of order 2 and cutoff radius 5

Butterworth Lowpass Filter (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

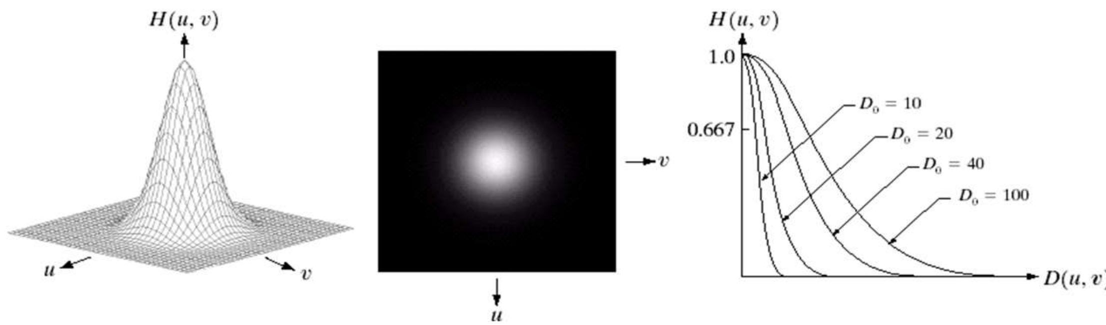
Result of filtering with Butterworth filter of order 2 and cutoff radius 15



Gaussian Lowpass Filters

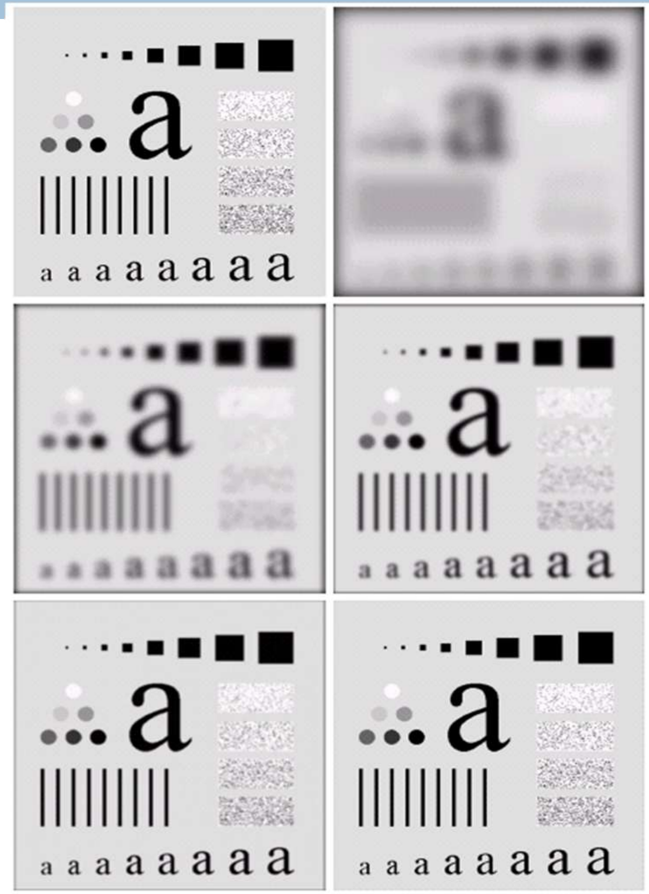
The transfer function of a Gaussian lowpass filter is defined as:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$



Gaussian Lowpass Filters (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Original image

Result of filtering with Gaussian filter with cutoff radius 5

Result of filtering with Gaussian filter with cutoff radius 15

Result of filtering with Gaussian filter with cutoff radius 30

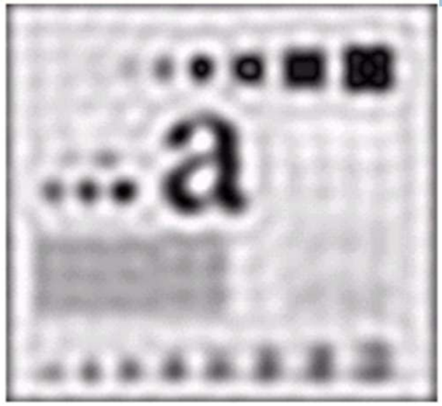
Result of filtering with Gaussian filter with cutoff radius 85

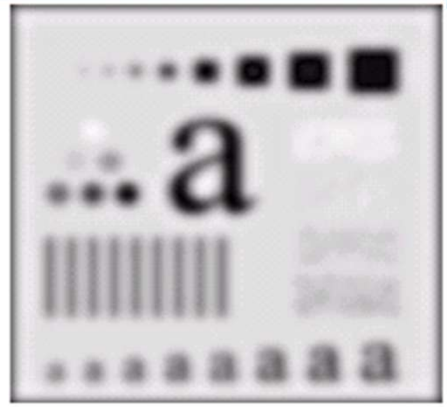
Result of filtering with Gaussian filter with cutoff radius 230

Lowpass Filters Compared

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

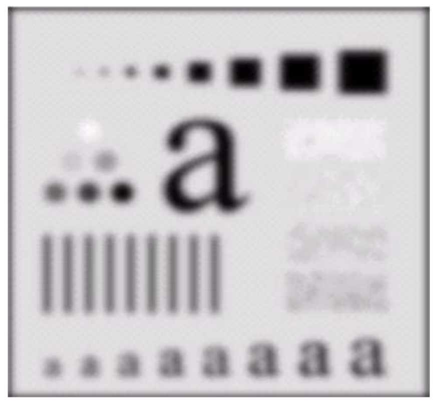
Result of filtering with ideal low pass filter of radius 15





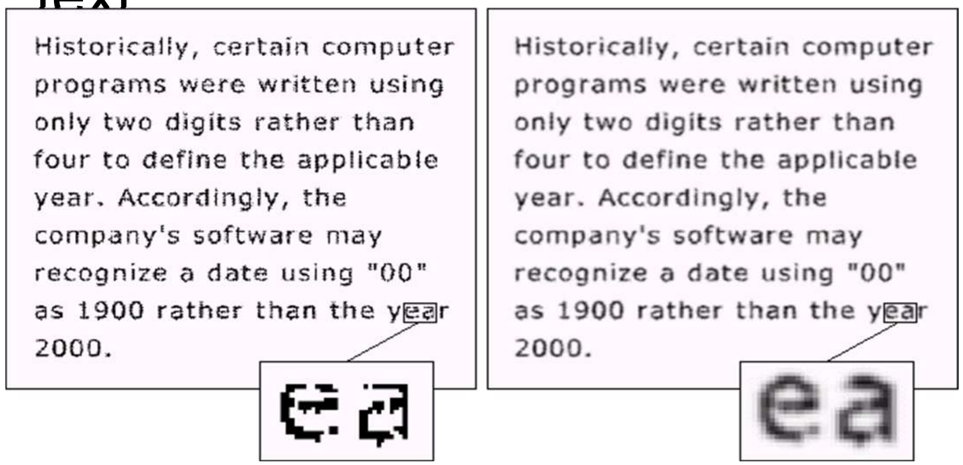
Result of filtering with Butterworth filter of order 2 and cutoff radius 15

Result of filtering with Gaussian filter with cutoff radius 15



Lowpass Filtering Examples

A low pass Gaussian filter is used to connect broken text



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Lowpass Filtering Examples (cont…)

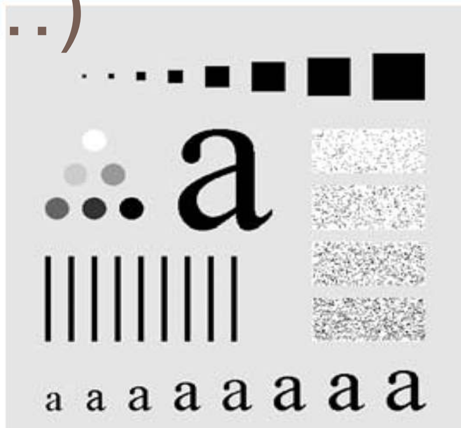
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

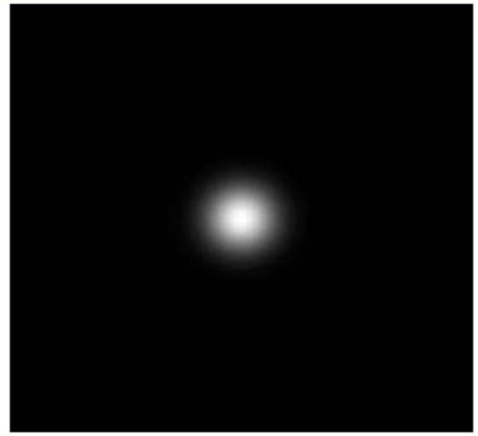
Different lowpass Gaussian filters used to remove blemishes in a photograph

Lowpass Filtering Examples (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

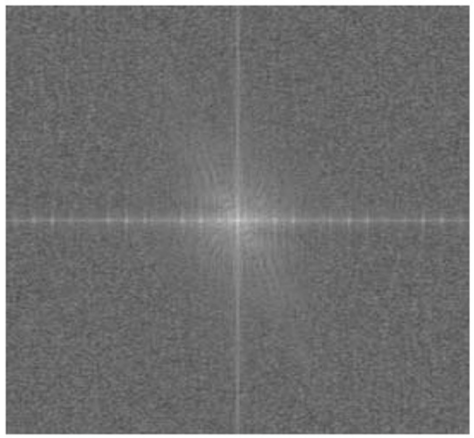
Original image





Gaussian lowpass filter

Spectrum of original image



Processed image

Sharpening in the Frequency Domain

Edges and fine detail in images are associated with high frequency components

High pass filters – only pass the high frequencies, drop the low ones

High pass frequencies are precisely the reverse of low pass filters, so:

$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

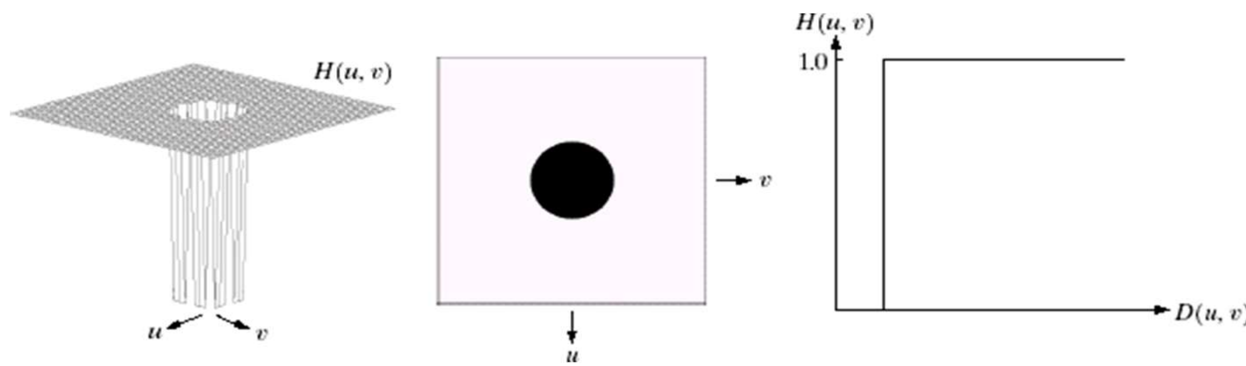
Ideal High Pass Filters

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

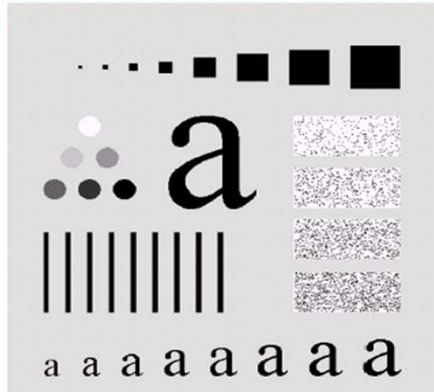
The ideal high pass filter is given as:

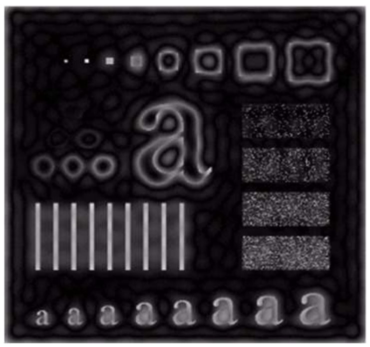
$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

where D_0 is the cut off distance as before

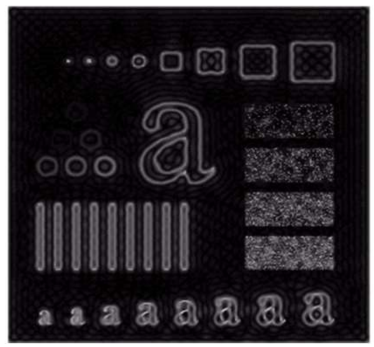


Ideal High Pass Filters (cont…)

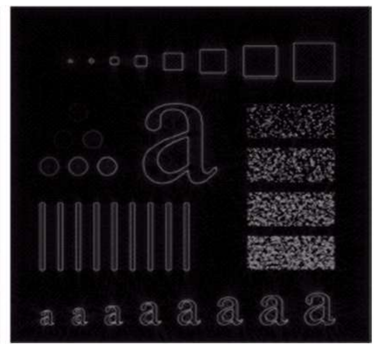




Results of ideal high pass filtering with $D_0 = 15$



Results of ideal high pass filtering with $D_0 = 30$



Results of ideal high pass filtering with $D_0 = 80$

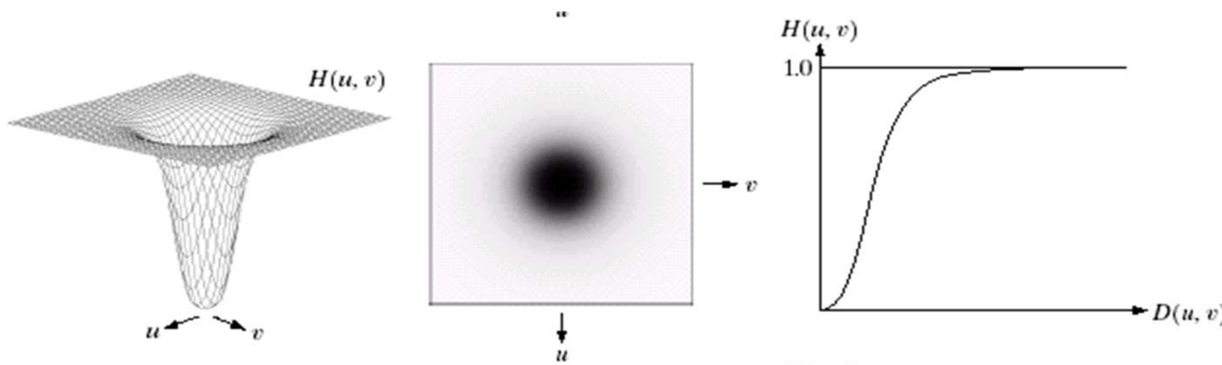
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Butterworth High Pass Filters

The Butterworth high pass filter is given as:

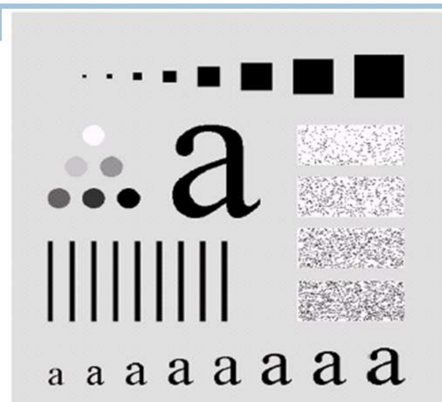
$$H(u, v) = \frac{1}{1 + [D_0 / D(u, v)]^{2n}}$$

where n is the order and D_0 is the cut off distance as before

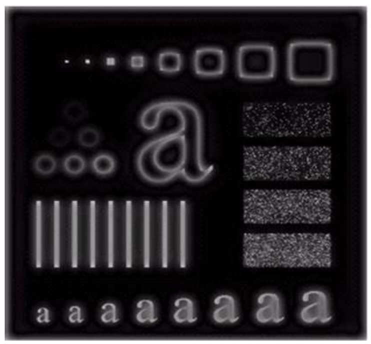


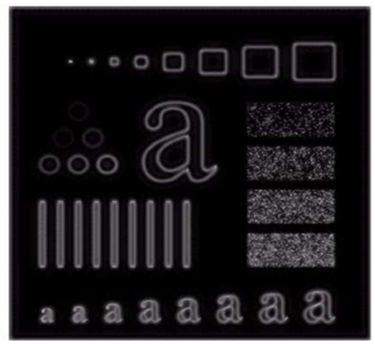
Butterworth High Pass Filters (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

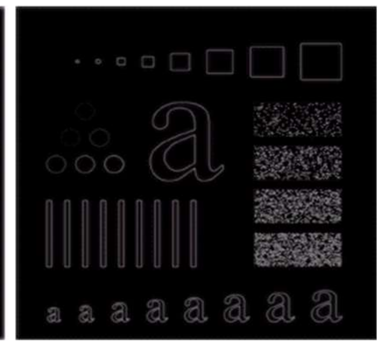


Results of Butterworth high pass filtering of order 2 with $D_0 = 15$





Results of Butterworth high pass filtering of order 2 with $D_0 = 30$



Results of Butterworth high pass filtering of order 2 with $D_0 = 80$

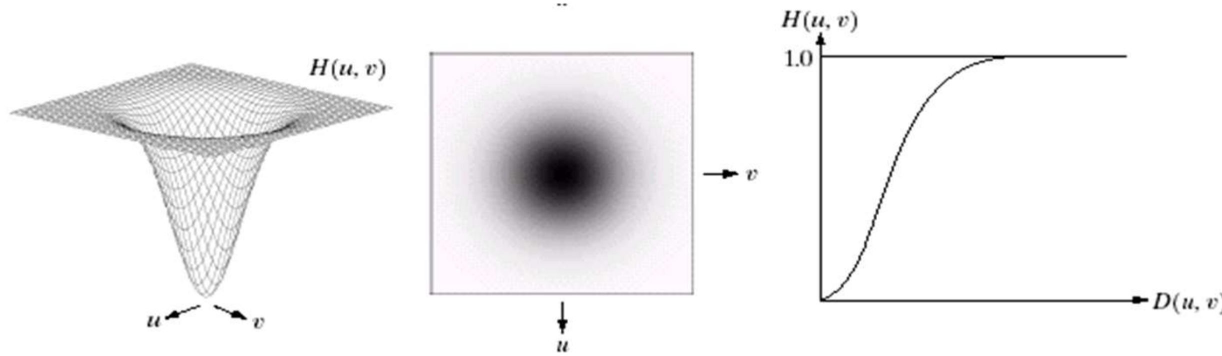
Gaussian High Pass Filters

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

The Gaussian high pass filter is given as:

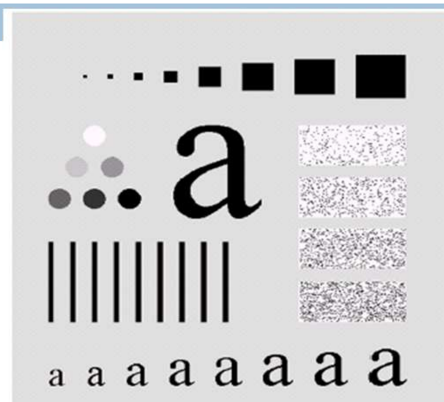
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

where D_0 is the cut off distance as before

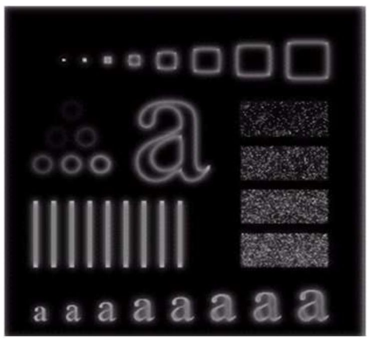


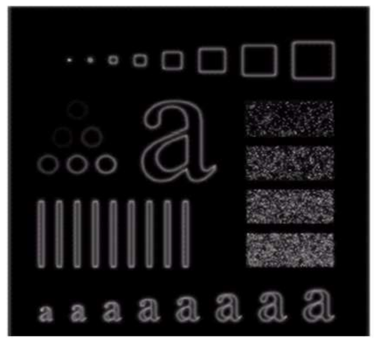
Gaussian High Pass Filters (cont…)

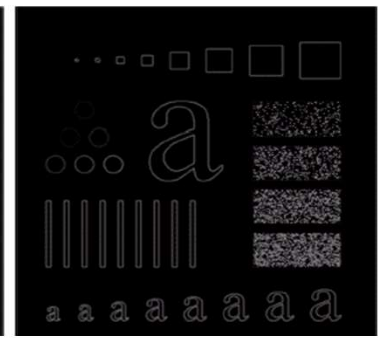
Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Results of Gaussian high pass filtering with $D_0 = 15$





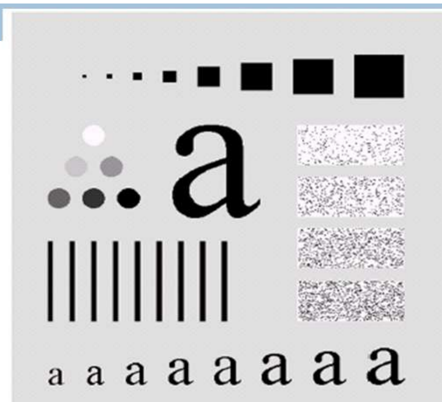


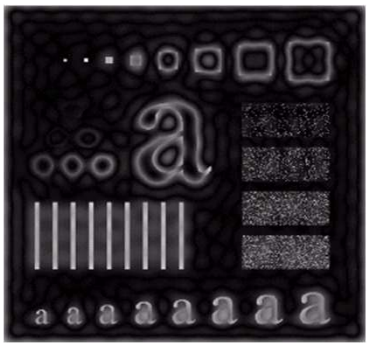
Results of Gaussian high pass filtering with $D_0 = 80$

Results of Gaussian high pass filtering with $D_0 = 30$

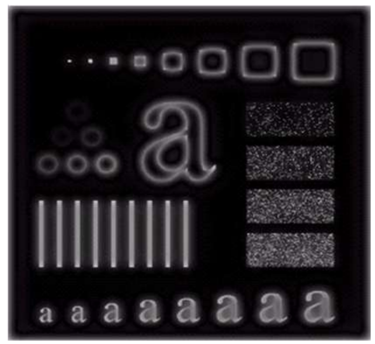
Highpass Filter Comparison

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

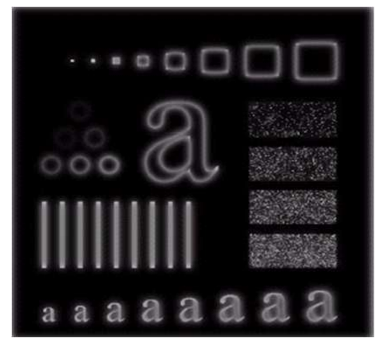




Results of ideal high pass filtering with $D_0 = 15$



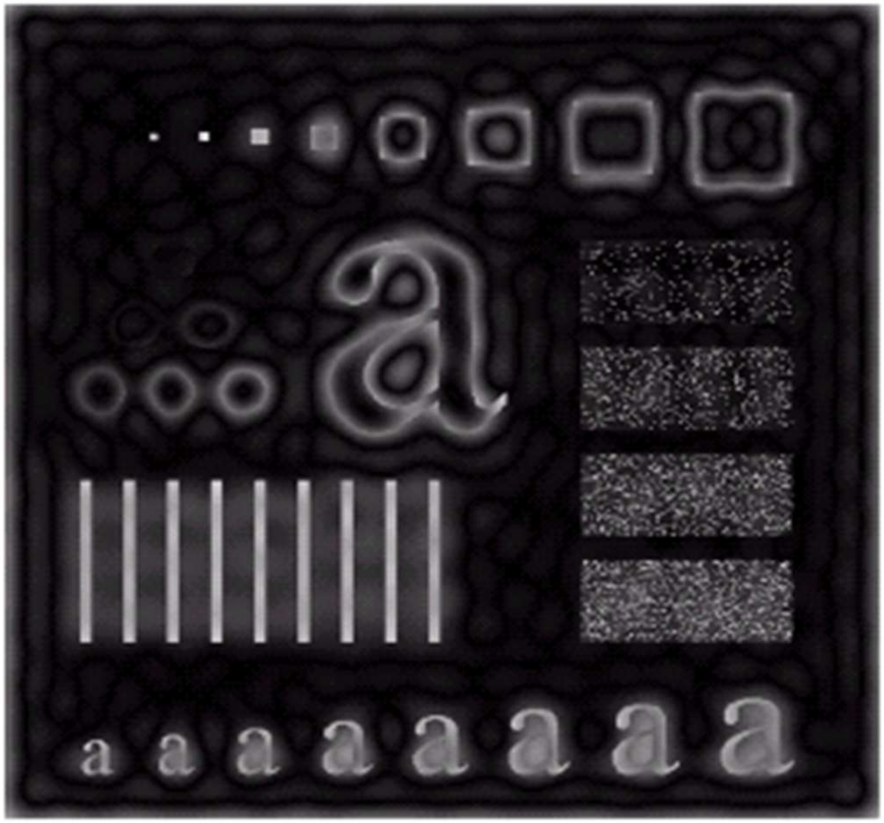
Results of Butterworth high pass filtering of order 2 with $D_0 = 15$



Results of Gaussian high pass filtering with $D_0 = 15$

Highpass Filter Comparison

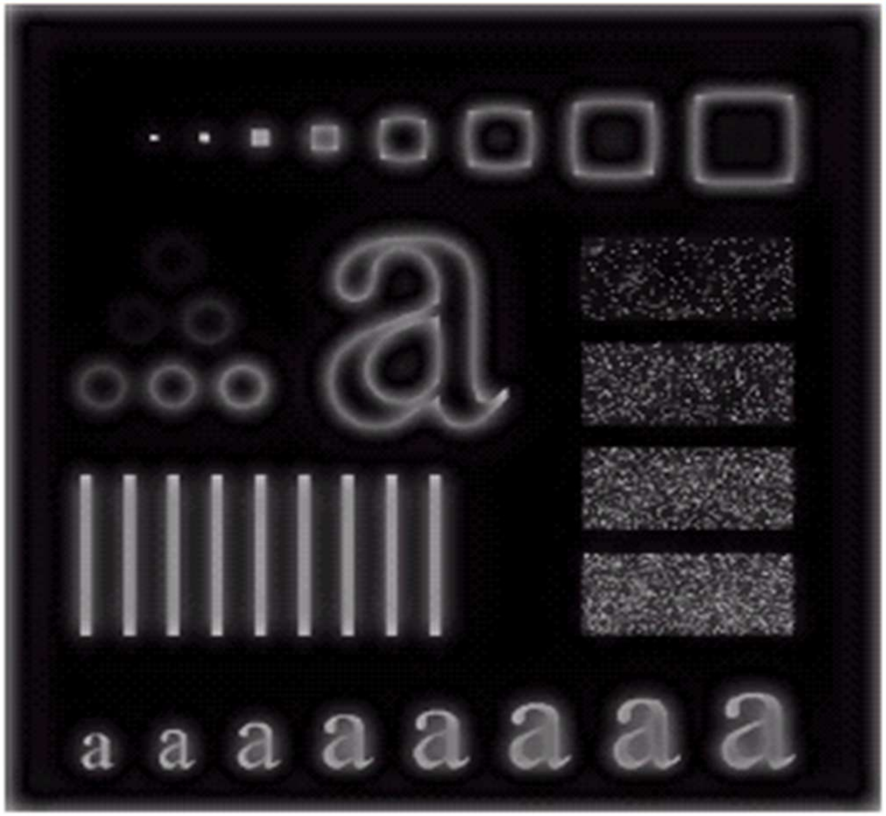
Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Results of ideal high pass filtering with $D_0 = 15$

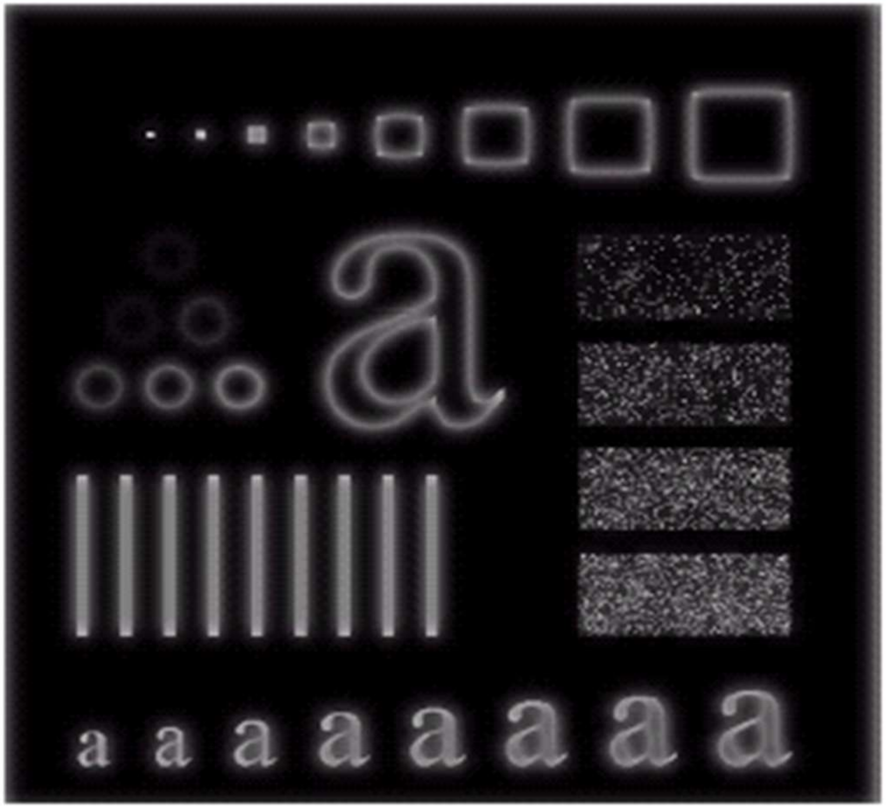
Highpass Filter Comparison

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Results of Butterworth high pass filtering of order 2 with $D_0 = 15$

Highpass Filter Comparison



Results of Gaussian high pass filtering with $D_0 = 15$

Highpass Filtering Example

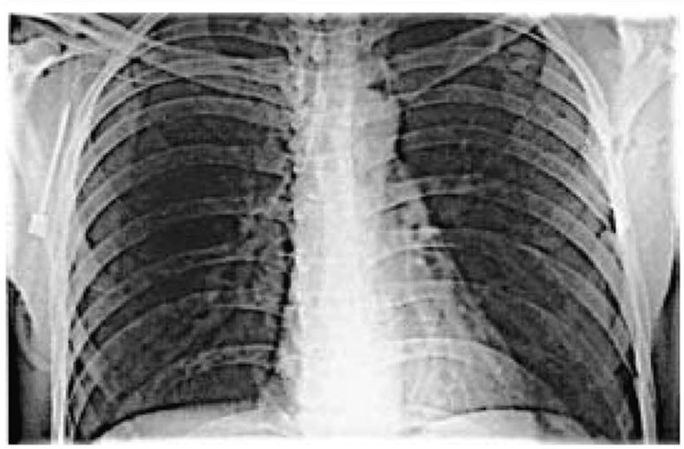
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Original image

Highpass filtering result

High frequency emphasis result

After histogram equalisation



Fast Fourier Transform

The reason that Fourier based techniques have become so popular is the development of the *Fast Fourier Transform (FFT)* algorithm

Allows the Fourier transform to be carried out in a reasonable amount of time

Reduces the amount of time required to perform a Fourier transform by a factor of 100 – 600 times!

Frequency Domain Filtering & Spatial Domain Filtering

Similar jobs can be done in the spatial and frequency domains

Filtering in the spatial domain can be easier to understand

Filtering in the frequency domain can be much faster – especially for large images

- **Homomorphic filtering** is a generalized technique for signal and image processing, involving a nonlinear mapping to a different domain in which linear filter techniques are applied, followed by mapping back to the original domain.

- Homomorphic filtering is sometimes used for image enhancement.

- It simultaneously normalizes the brightness across an image and increases contrast.

- Here homomorphic filtering is used to remove multiplicative noise.

- To make the illumination of an image more even, the high-frequency components are increased and low-frequency components are decreased

- Because the high-frequency components are assumed to represent mostly the reflectance in the scene whereas the low-frequency components are assumed to represent mostly the illumination in the scene.

- That is, high-pass filtering is used to suppress low frequencies and amplify high frequencies

Original Picture

Homomorphic Filter

Thank You