Digital Image Processing

Morphological Image Processing

Introduction

- *Morphology* – a branch of biology concerned with the form and structure of plants and animals
- *Mathematical morphology* – a tool for extracting image components useful in the representation and description of image shape including:
  - Boundaries
  - Skeletons
  - Convex hull
- We will also look at morphological techniques for
  - Filtering
  - Thinning
  - Pruning

The language of mathematical morphology is set theory

Sets in mathematical morphology represent objects in an image

For example, the set of all black pixels in a binary image is a complete morphological description of the image

Mathematic Morphology

- used to extract image components that are useful in the representation and description of region shape, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

- Mathematical morphology as a tools for extracting image compounds that are useful in the representation and description of region shape such as boundaries, skeletons and the convex hull.
- A collection of **non-linear operations related to the shape or morphology of features in an image** is known as Morphological Operation in Image Processing

- **Binary images** (consists of pixels that can have one of exactly two colors, usually black and white) may contain numerous imperfections.

- **Morphological Operations** in Image Processing pursues the goals of removing these imperfections by accounting for the form and structure of the image.

Mathematic Morphology

mathematical framework used for:

- pre-processing
  - noise filtering, shape simplification
- enhancing object structure
  - skeletonization, convex hull...
- Segmentation
  - watershed,...
- quantitative description
  - area, perimeter, ...

- In binary images, the sets are members of the 2-D integer space Z^2, where each element of a set is a tuple (2-D vector) whose coordinates are the (x, y) coordinates of a black (or white, depending on convention) pixel in the image
- Gray-scale digital images can be represented as sets whose components are in Z^3

Some Basic Concepts from Set Theory

- Let A be a set in Z^2
- If a = (a1, a2) is an element of A, then we write

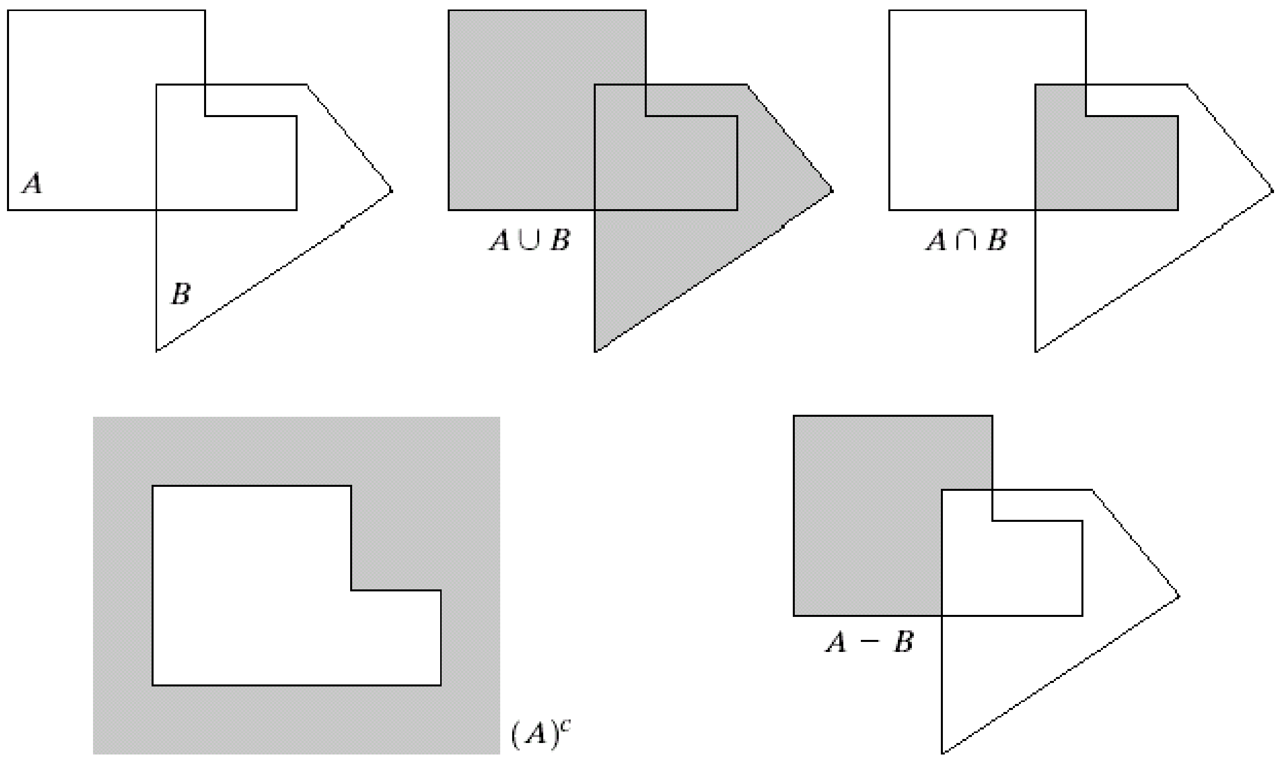
$$a \in A$$

- Similarly, if a is not an element of A, we write

$$a \overline{\in} A$$

- The set with no elements is called the null or empty set and is denoted by the symbol ϕ

Basic Set Theory



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Logic Operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Example

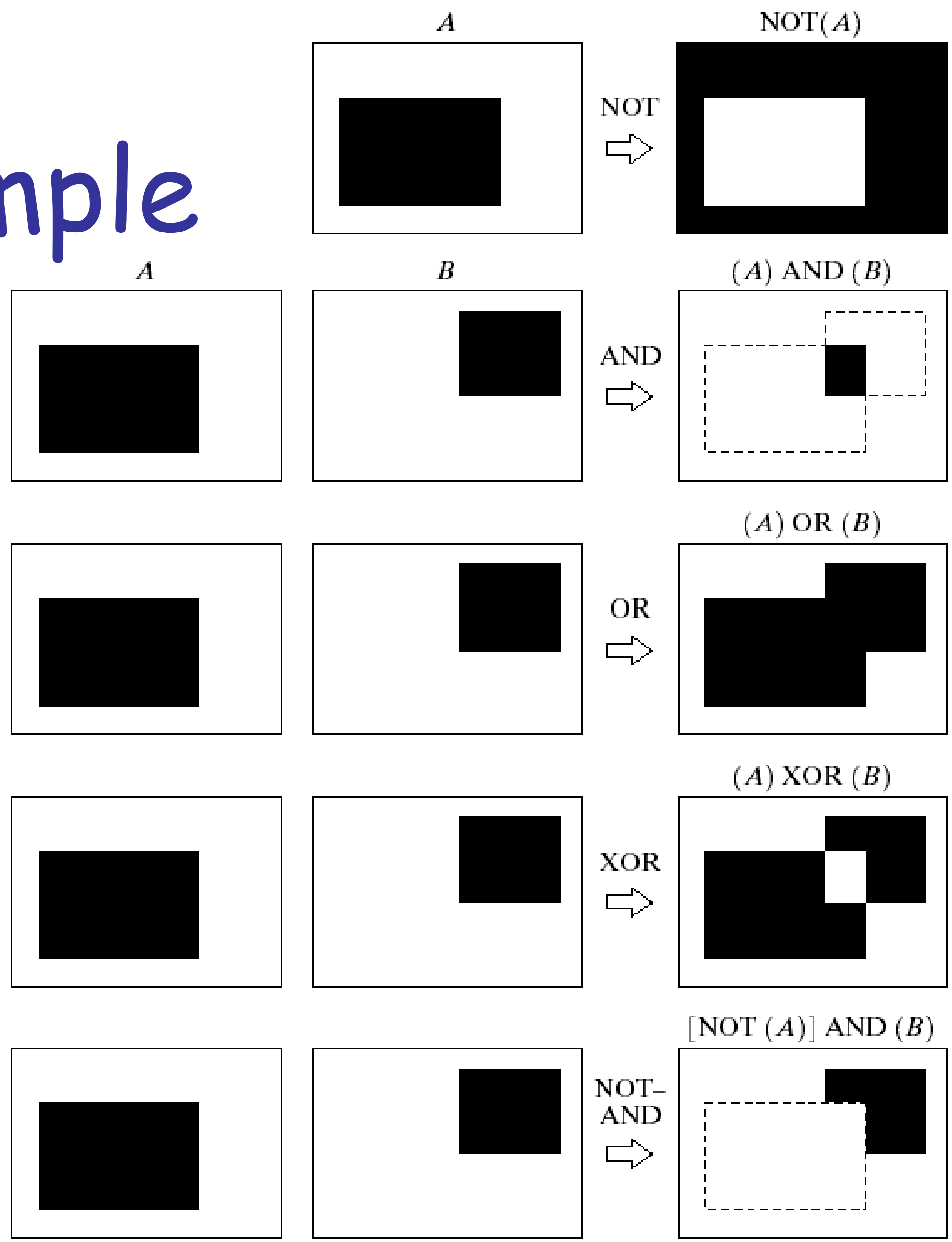


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

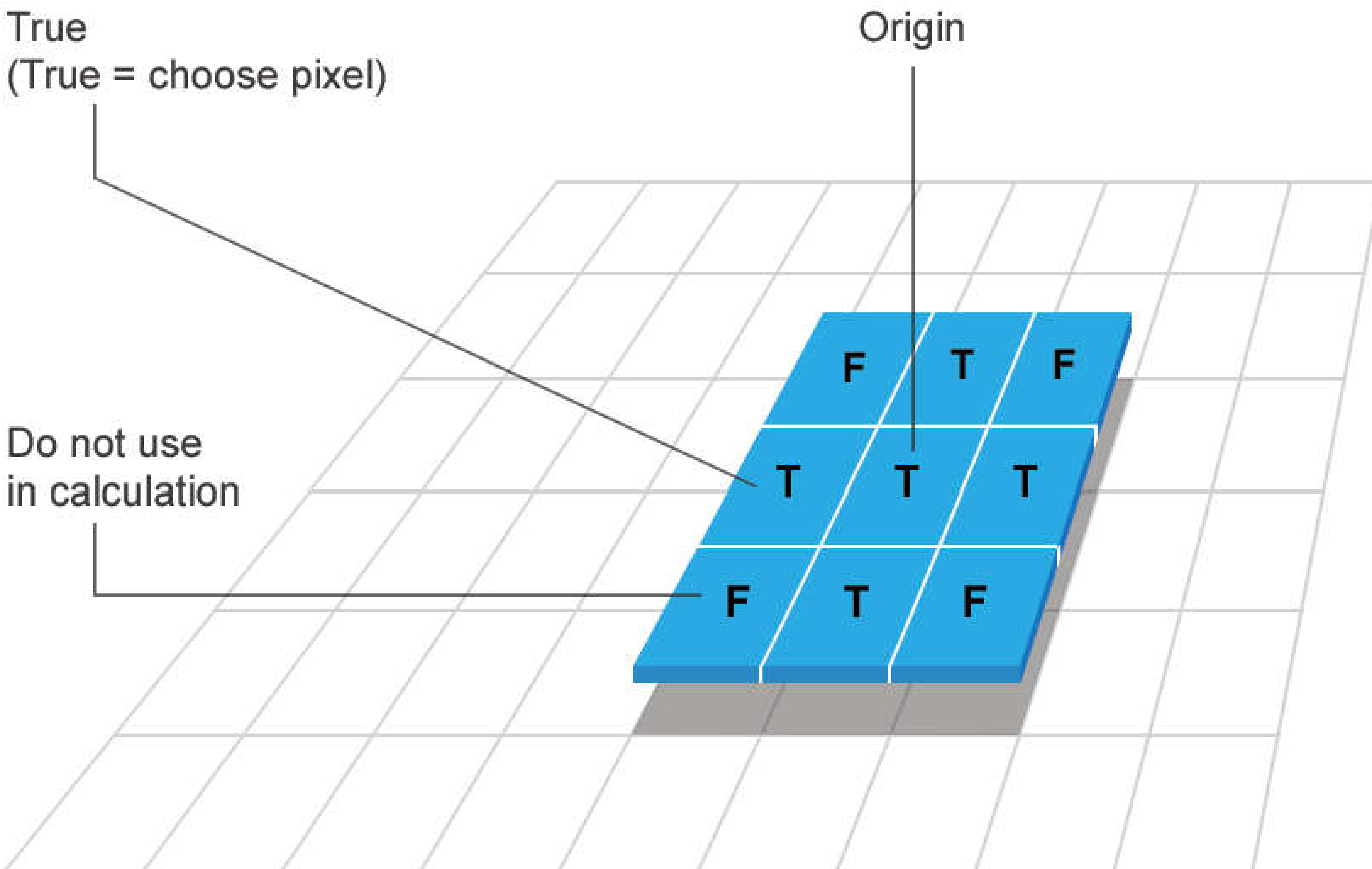
Structuring element (SE)

An essential part of the morphological dilation and erosion operations is the structuring element used to probe the input image.

A structuring element is a matrix that identifies the pixel in the image being processed and defines the neighborhood used in the processing of each pixel.

You typically choose a structuring element the same size and shape as the objects you want to process in the input image.

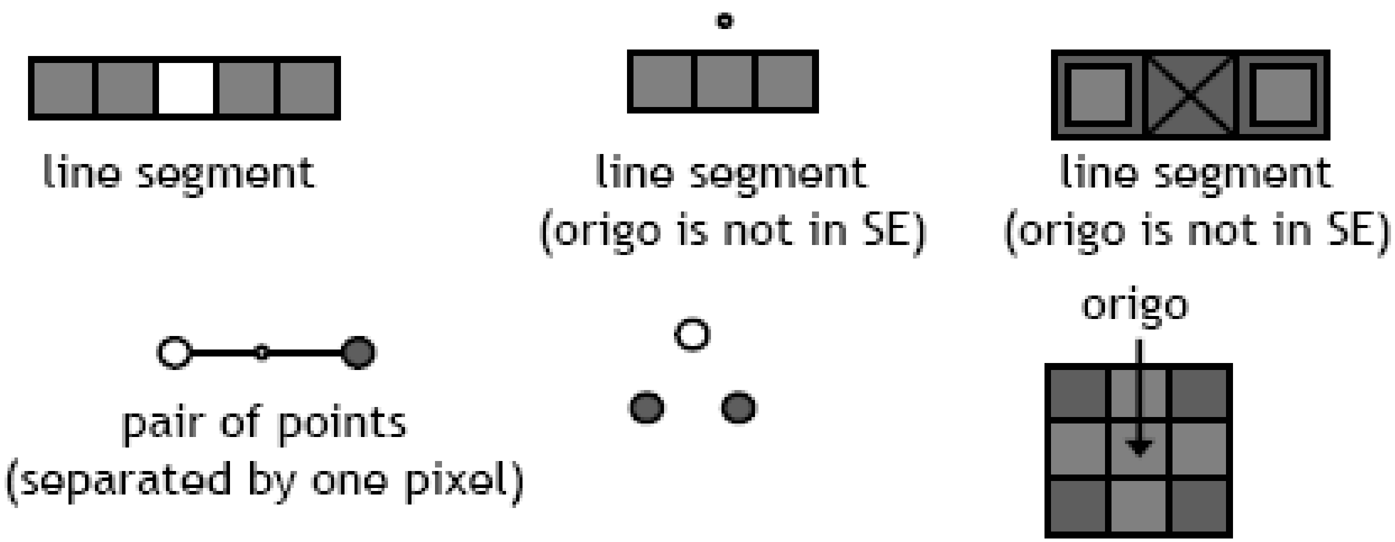
For example, to find lines in an image, create a linear structuring element.



FLAT (binary valued)

How to describe SE

- many different ways!
- information needed:
  - position of origo for SE
  - positions of elements belonging to SE



Basic morphological operations

- Erosion shrink
- Dilation grow
- combine to
  - Opening → object
  - Closening → background

keep general shape but smooth with respect to

Erosion

- Erosion is one of the two basic operators in the area of mathematical morphology, the other being dilation.
- It is typically applied to binary images, but there are versions that work on grayscale images.
- The basic effect of the operator on a binary image is to erode away the boundaries of regions of foreground pixels (*i.e.* white pixels, typically).
- Thus areas of foreground pixels shrink in size, and holes within those areas become larger.

Erosion

- Erosion shrink-ens the image pixels i.e. it is used for shrinking of element A by using element B.
- Erosion removes pixels on object boundaries.:
- The value of the output pixel is the minimum value of all the pixels in the neighborhood. A pixel is set to 0 if any of the neighboring pixels have the value 0.

Erosion

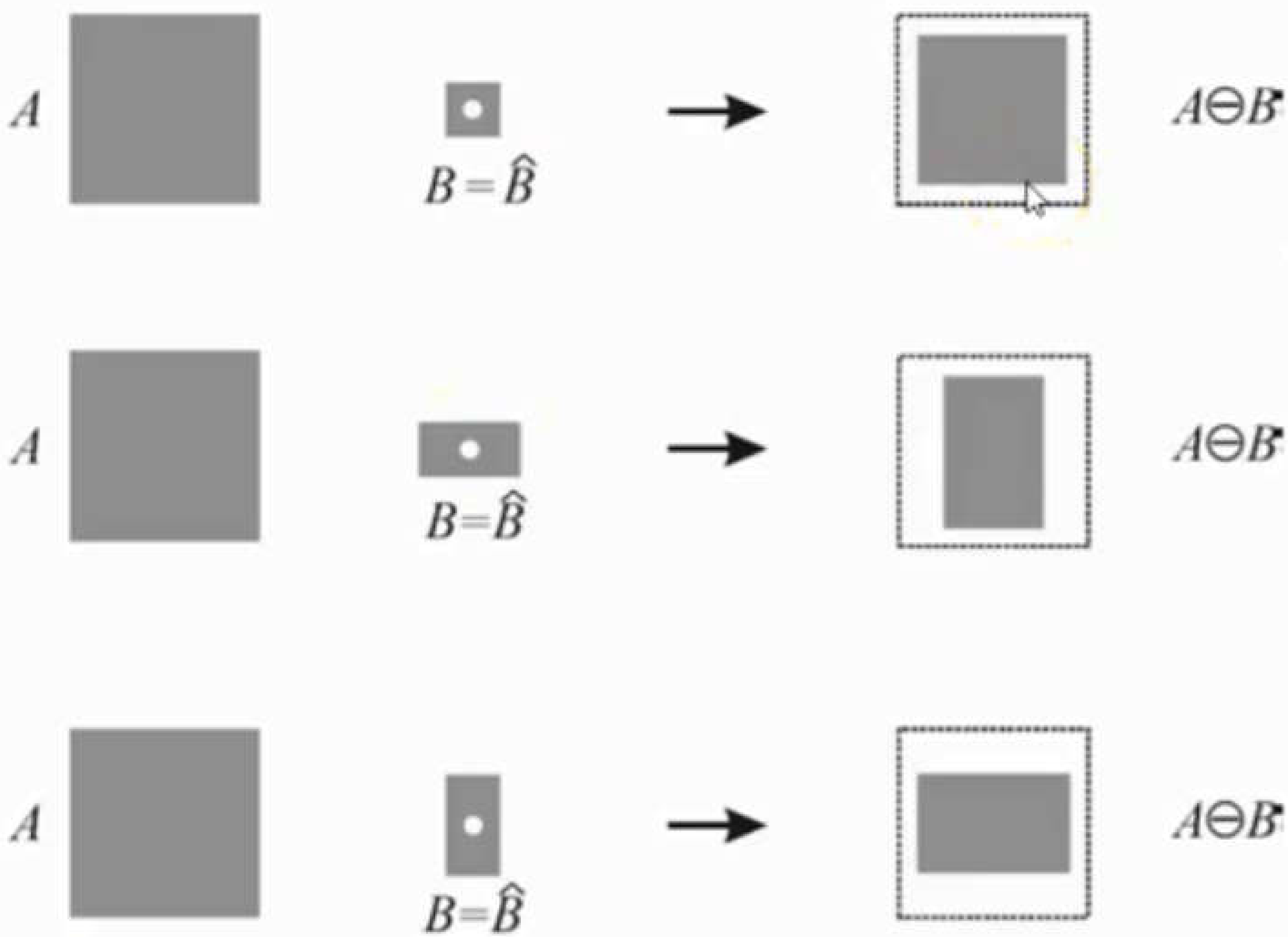
Erosion of a set A by structuring element B: all z in A such that B is in A when origin of B=z

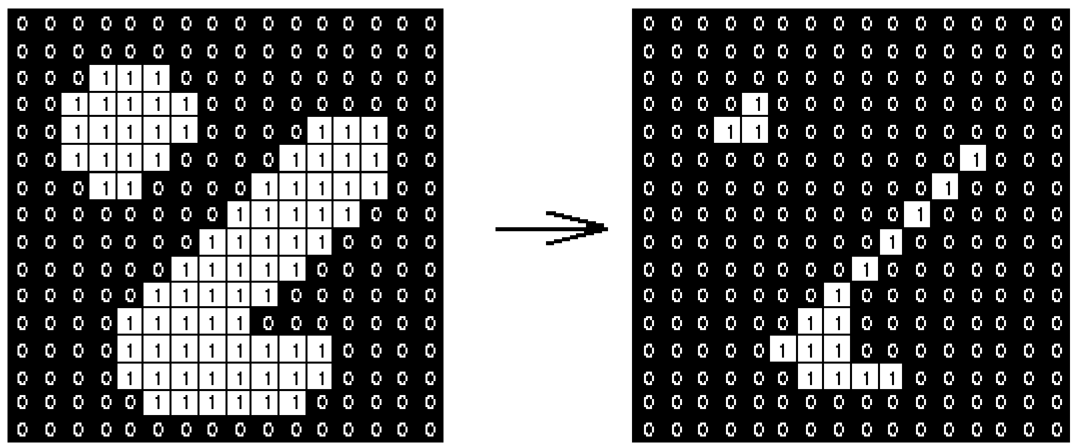
$$A \ominus B = \{ z | (B)_z \subseteq A \}$$

shrink the object

where B_z is the translation of B by the vector z

Example of erosion using three different rectangular structuring elements





Erosion: a 3×3 square structuring element

Dilation

Dilation adds pixels to the boundaries of objects in an **image**

Dilation –
Dilation expands the image pixels i.e. it is used for expanding an element A by using structuring element B. Dilation adds pixels to object boundaries.
The value of the output pixel is the maximum value of all the pixels in the neighborhood. A pixel is set to 1 if any of the neighboring pixels have the value 1

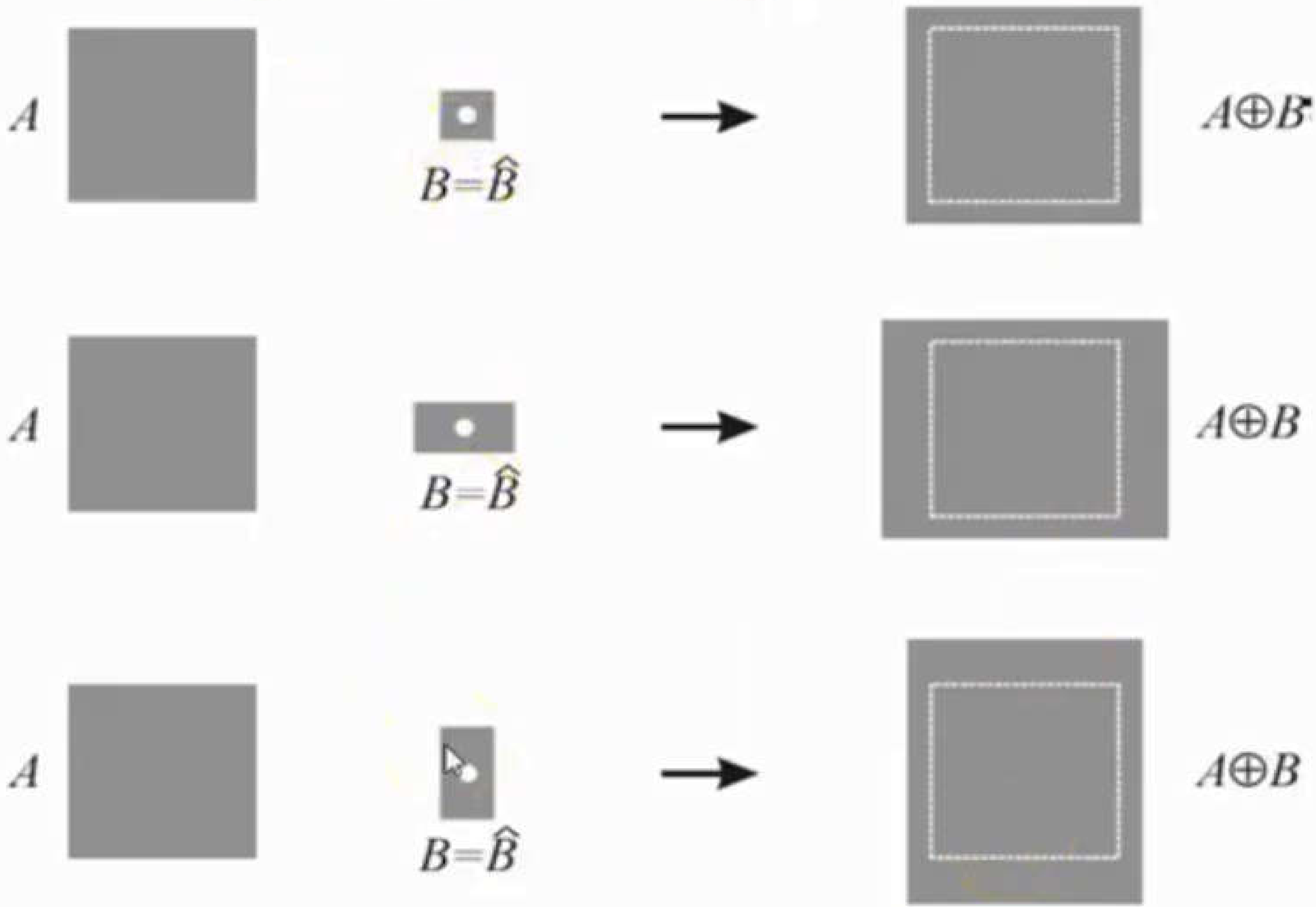
Dilation

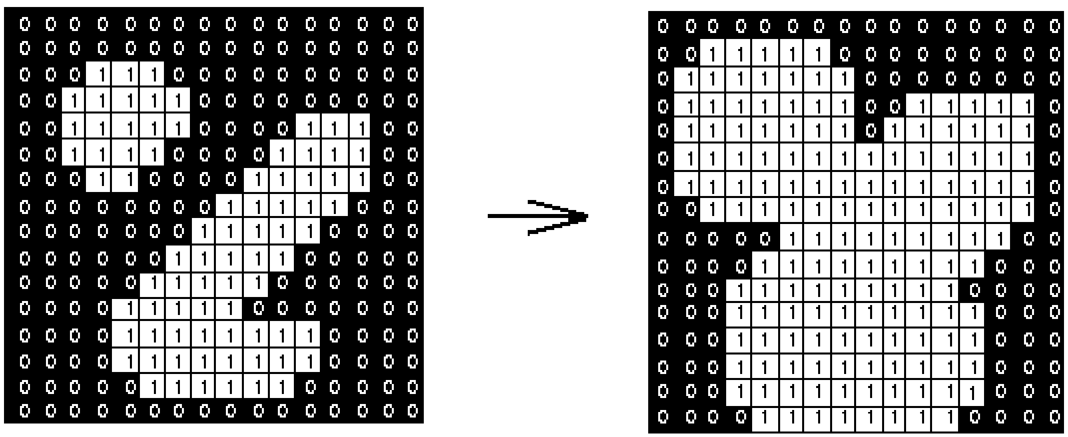
- Dilation of a set A by structuring element B: all z in A such that B hits A when origin of B=z

$$A \oplus B = \{ z | (\hat{B})_z \cap A \neq \Phi \}$$

- **grow the object**

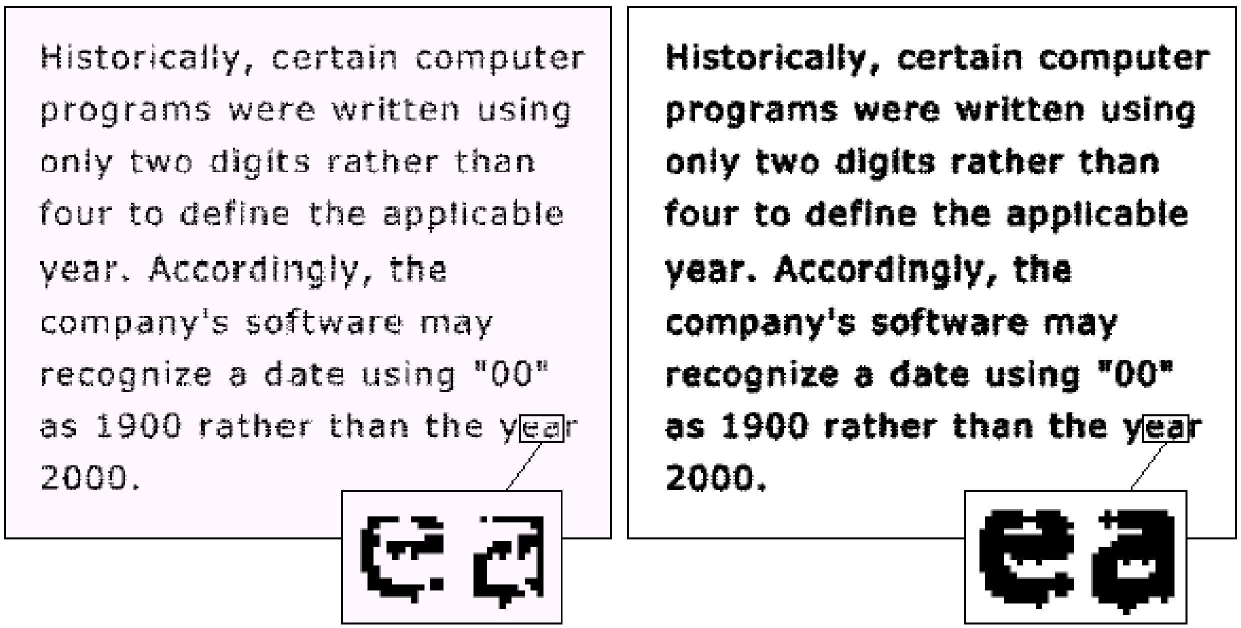
Example of dilation using three different rectangular structuring elements





Dilation: a 3×3 square structuring element

Dilation : Bridging gaps



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

useful

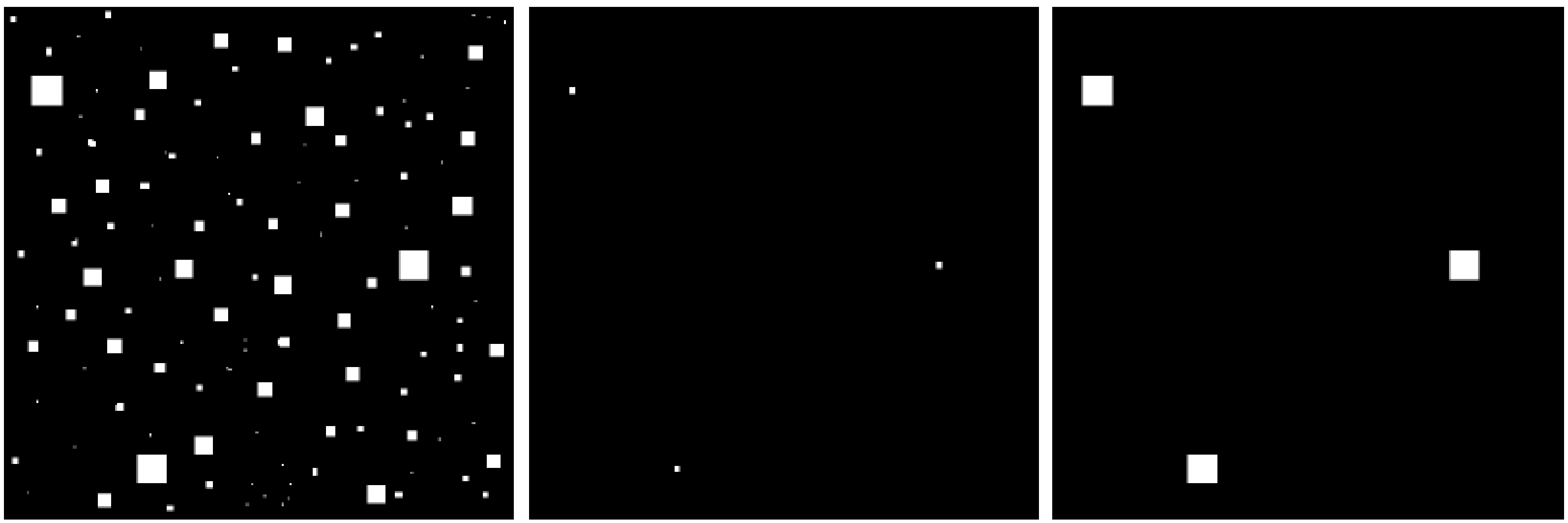
- Erosion
  - removal of structures of certain shape and size, given by SE
- Dilation
  - filling of holes of certain shape and size, given by SE

Combining erosion and dilation

- WANTED:
  - remove structures / fill holes
  - without affecting remaining parts

- SOLUTION:
- combine erosion and dilation
- (using same SE)

Erosion : eliminating irrelevant detail



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of gray level 1

Opening

In mathematical morphology, **opening** is the dilation of the erosion of a set
A by a structuring element B:

$$A \circ B = (A \ominus B) \oplus B,$$

where and denote erosion and dilation, respectively.

Opening

Opening removes small objects from the foreground (usually taken as the bright pixels) of an image, placing them in the background, while closing removes small holes in the foreground, changing small islands of background into foreground.

These techniques can also be used to find specific shapes in an image.

Opening can be used to find things into which a specific structuring element can fit (edges, corners, ...).

Opening

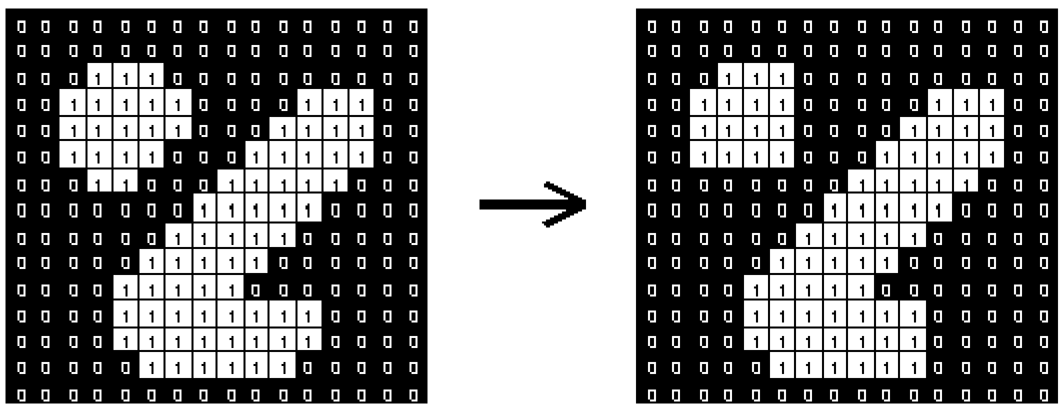


Figure 1 Effect of opening using a 3×3 square structuring element

Closing

In mathematical morphology, the **closing** of a set (binary image) *A* by a structuring element *B* is the erosion of the dilation of that set,

$$A \bullet B = (A \oplus B) - B$$

In image processing, closing is, together with opening, the basic workhorse of morphological noise removal.

Opening removes small objects, while closing removes small holes, fill gaps in the contour, smooth contour..

Closing

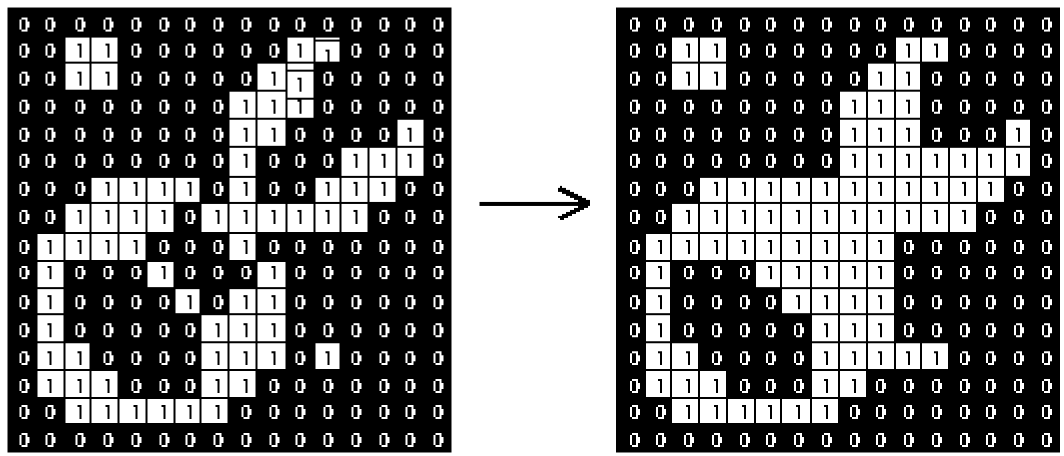
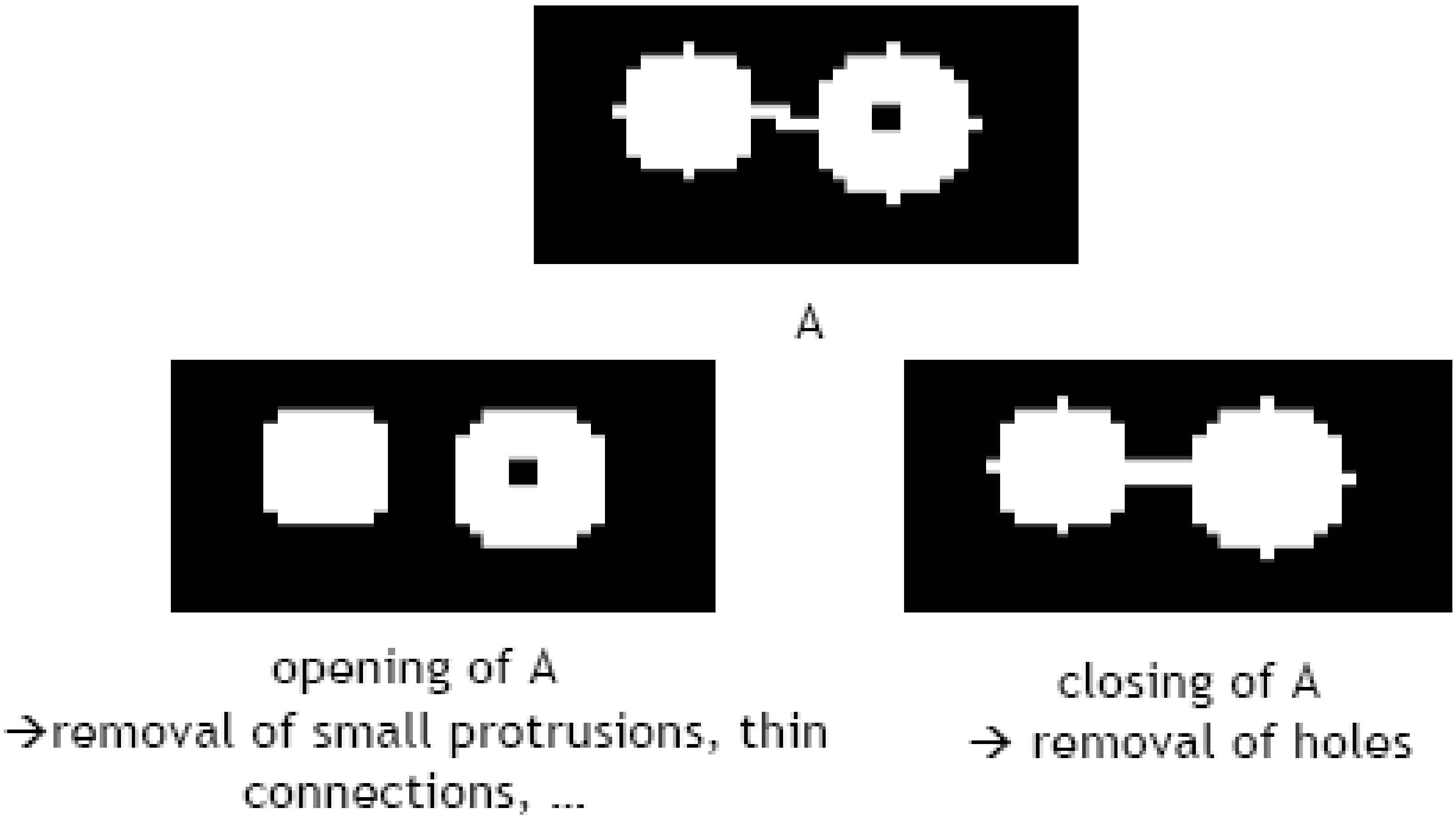
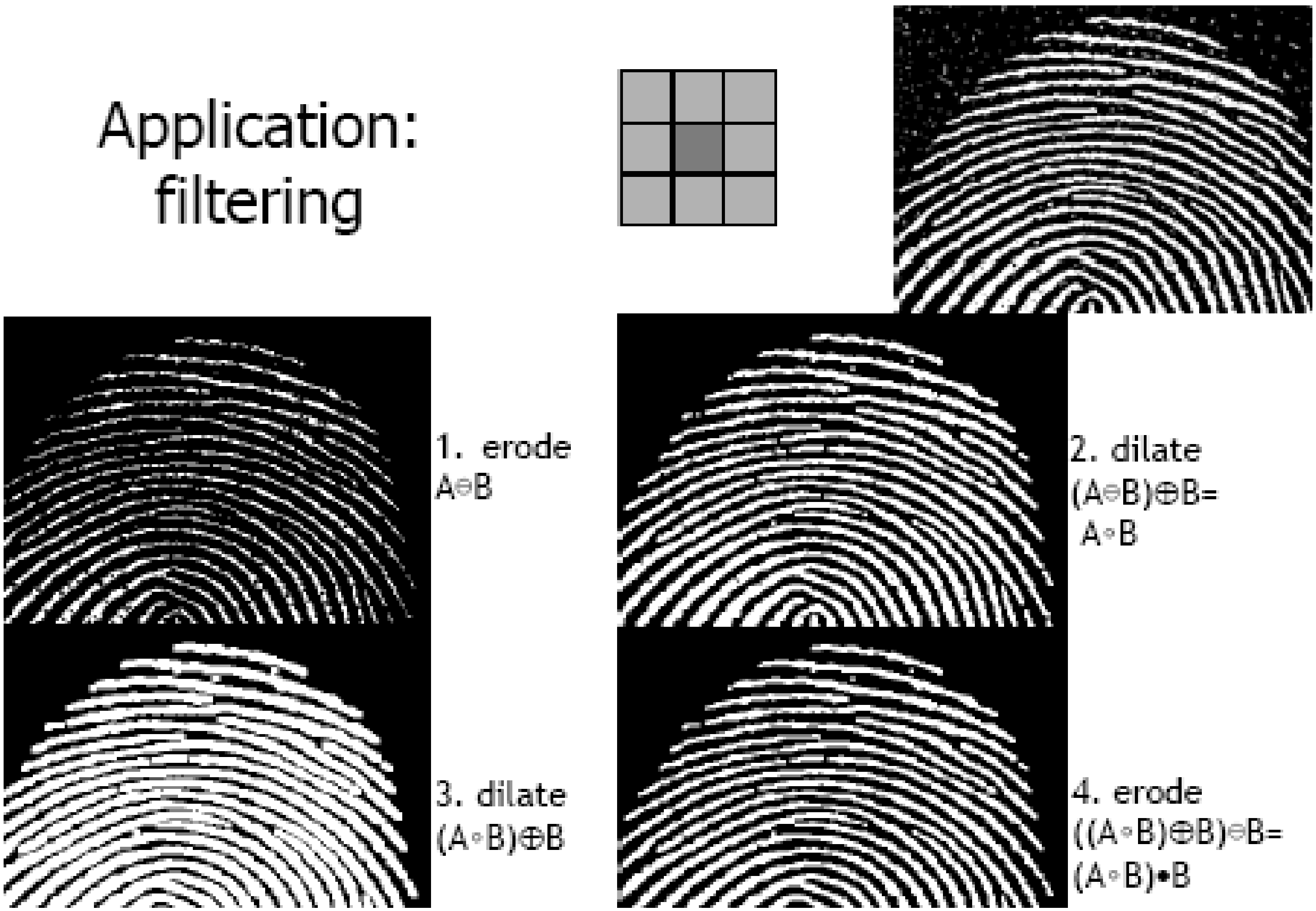


Figure 1 Effect of closing using a 3×3 square structuring element

Useful: open & close



Application: filtering



Hit-or-Miss Transformation

The hit-and-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.

As with other binary morphological operators it takes as input a binary image and a structuring element, and produces another binary image as output.

Hit-or-Miss Transformation

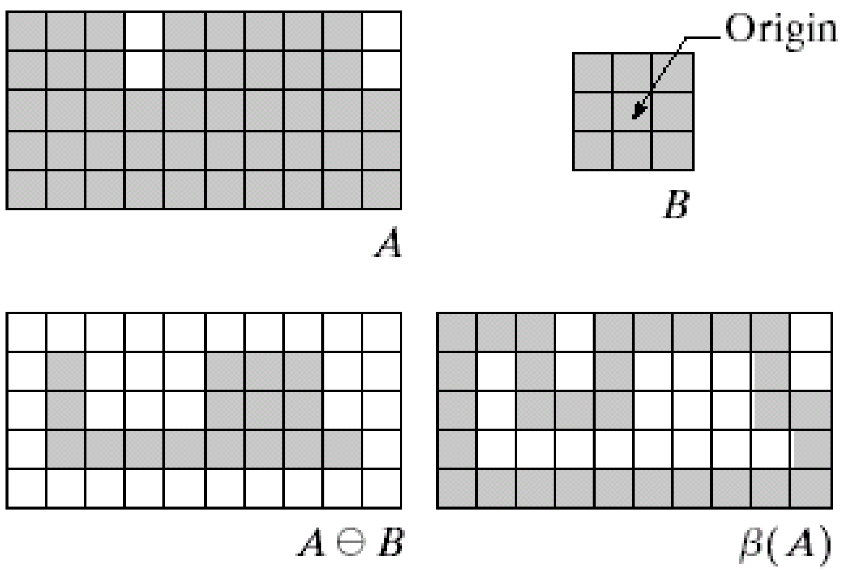
Hit-or-miss transform is an operation that detects a given configuration (or pattern) in a binary image, using the morphological erosion operator and a pair of disjoint structuring elements.

The result of the hit-or-miss transform is the set of positions where the first structuring element fits in the foreground of the input image, and the second structuring element misses it completely.

Boundary Extraction

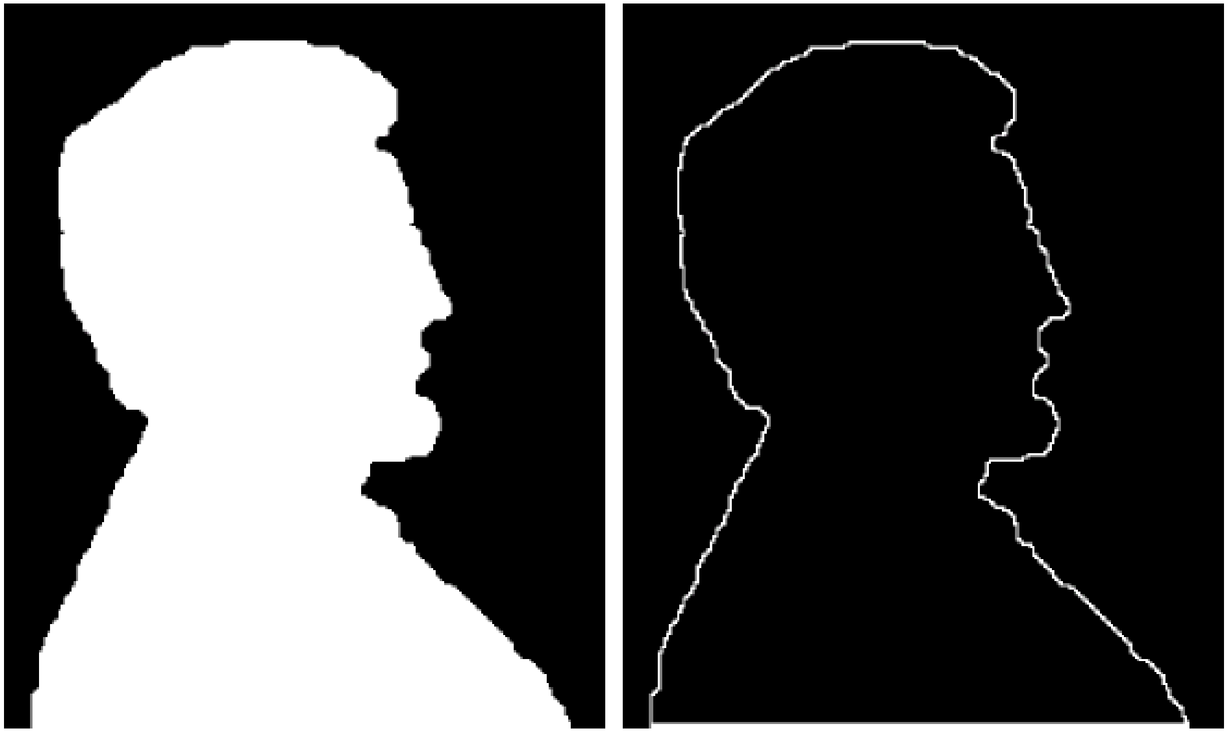
a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



$$\beta(A) = A - (A \ominus B)$$

Example



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Thank You