SMAI-M20-L13: PCA(Cont.)

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September 7, 2020

Class Review

Consider the vector $\mathbf{w} = [w_1, w_2]^T$ and the objective function to be minimized as:

$$\min_{\mathbf{w}} (3w_1 + 4w_2 - 12)^2 + \lambda g(\mathbf{w})$$

If $g(\mathbf{w})$ is Lp norm of $\mathbf{w}$, what is the optimal value of $\mathbf{w}$?

- for various p?
- for various λ?

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Recap:

- Problem Space:
  - Learn a function $y = f(\mathbf{W}, \mathbf{x})$ from the data.
  - Dimesnionality Reduction and Representation (Feature Selection, PCA, Neural Embeddings)
  - Matrix Factorization for Data Matrices: (LSI, Matrix Completion, Recommendation Systems)
- Supervised Learning:
  - Notions of Training, Validation and Testing; Loss Function and Optimization
  - Generalization, Overfitting, Occam's razor, Model Complexity, Bias and Variance, Regularization.
  - Performance Metrics, Estimating error using validation set.
- Algorithms:
  - Nearest Neighbour Algorithm
  - Linear Classification; Linear Regression
  - Decide as ω_1 if $P(\omega_1|\mathbf{x}) \geq P(\omega_2|\mathbf{x})$ else ω_2

This Lecture:

1. PCA as Compression
   - Dimensionality reduction that allow minimal loss in the data.
2. Intuitive Intro to Gradient Descent
   - An iterative optimization algorithm for minimizing loss functions.
3. Feature Normalization
   - Dimensions to be brought to some common scale and variation/range; Numerical and practical advantages.

Questions? Comments?

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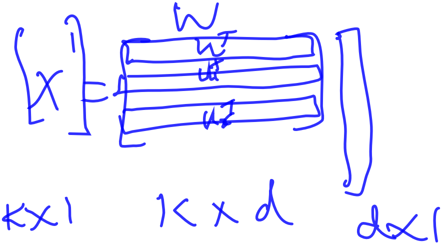
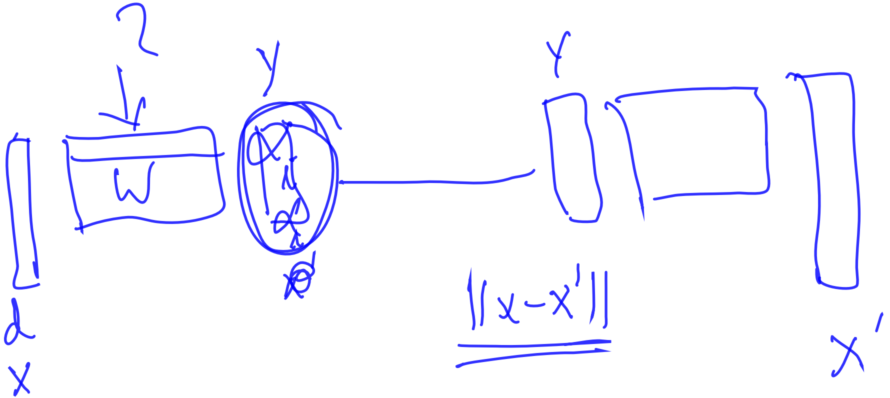


x $d \times 1$

Σ $d \times d$

u $d \times 1$

$a^T b = b^T a$



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Discussions Point - I

- It is common to represent signal/data using some orthogonal basis. Discrete Fourier Transform (DFT) is an example that use complex exponential basis.
- Discrete Cosine Transform(DCT) (a close friend of DFT) is popular in the compression standards such as JPEG. We retain only a small number of coeff; rest as zero; thus obtain compression.
- The optimal linear transform to do this is popularly known as KL Transform (where eigen vectors of the covariance form the basis). Our PCA is a close relative.

Why is PCA/KL-Transform based compression algorithms not yet popular in the standards we all use?

DCT vs KLT

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Comment about the effectiveness of GD for optimizing the loss functions. Initialization is also shown.

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Discussions Point - III

Consider a two dimensional representation with each dimensions as x_1 and x_2. i.e., $\mathbf{x} = [x_1, x_2]^T$.
We know that we need to do a feature normalization to obtain $\mathbf{x}' = [x_1', x_2']^T$

A $x_i' = \frac{x_i}{\alpha_i}$

B $x_i' = x_i - \alpha_i$ (after the class?)

How do we represent

1. $\mathbf{x}' = \mathbf{L}\mathbf{x}$? What is $\mathbf{L}$
2. What should be the matrix $\mathbf{A}$ such that the Euclidean distance in $\mathbf{x}'$ is same as Mahalanobis distance in]$\mathbf{x}$. i.e.,

$$[\mathbf{x} - \mathbf{y}]^T \mathbf{A}^{-1} [\mathbf{x} - \mathbf{y}] = [\mathbf{x}' - \mathbf{y}']^T [\mathbf{x}' - \mathbf{y}']$$

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What Next:? (next three)

1. Application of PCA
2. Perceptron Algorithms
3. Analysis of Gradient Descent Algorithm
4. Implementation of Gradient Descent
5. Analysis of Perceptron Algorithm