CS 480

Introduction to Artificial Intelligence

November 15, 2022

Announcements / Reminders

- **Follow Week 13 TO DO List**
- **Programming Assignment #02 due on Sunday (11/20/22) at 11:00 PM CST**
- **Grading TA assignment:**

  https://docs.google.com/spreadsheets/d/1ExS0bKnGt_fdf4LHa3YS1qRA7-Iq4xqXVjfSAPMaGVk/edit?usp=sharing
- **UPDATED Final Exam date:**
  - **December 1st, 2022 (last week of classes!)**
    - **Ignore the date provided by the Registrar**

Plan for Today

- **Decision Networks**
- **Markov models [BONUS MATERIAL]**
- **Fuzzy logic [BONUS MATERIAL]**
- **Bio-Inspired AI [BONUS MATERIAL]**

Decision Network (Influence Diagram)

Decision networks (also called influence diagrams) are structures / mechanisms for making rational decisions.

Decision networks are based on Bayesian networks, but include <u>additional nodes</u> that represent actions and utilities.

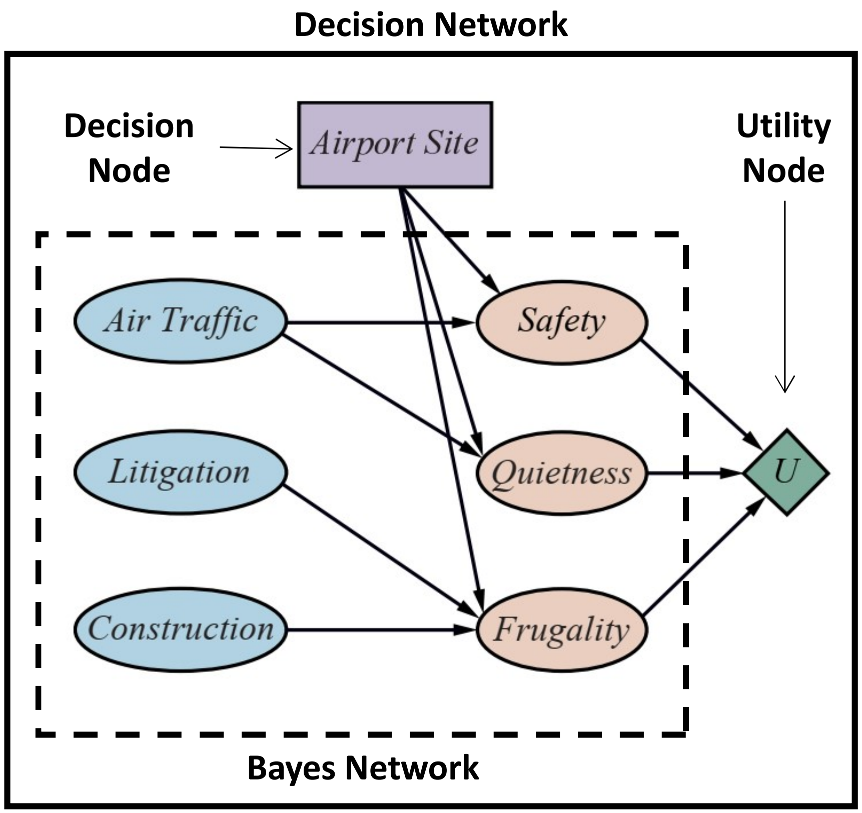
Decision Networks

The most basic decision network needs to include:

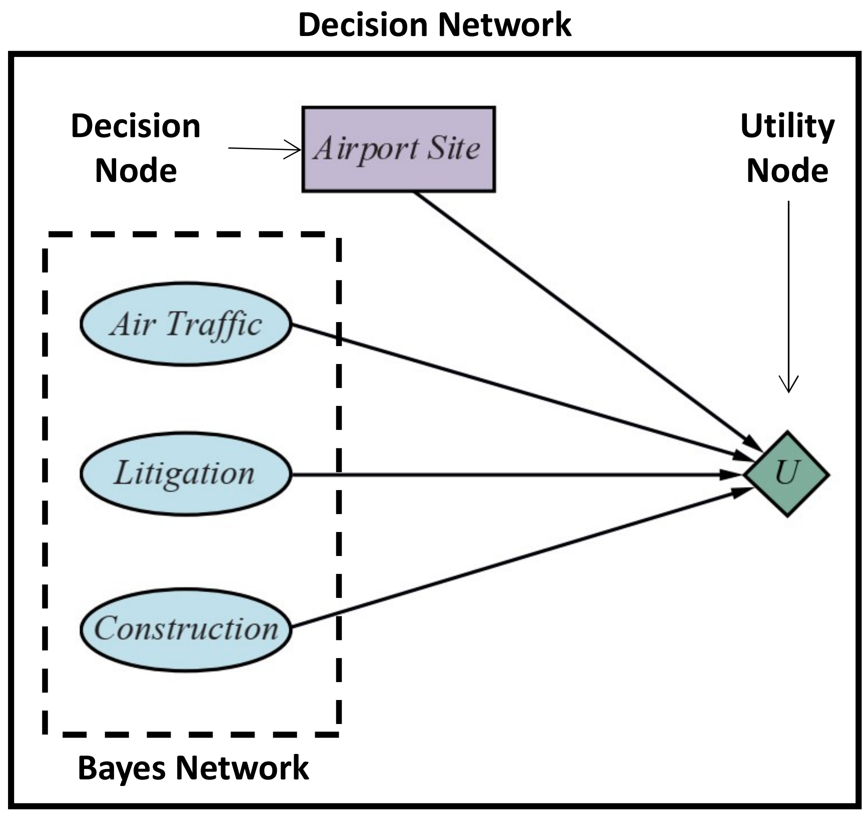
- **information about current state** s
- **possible actions**
- **resulting state** s' **(after applying chosen action** a**)**
- **utility of the resulting state** $U(s')$

(Single-Stage) Decision Networks

General Structure

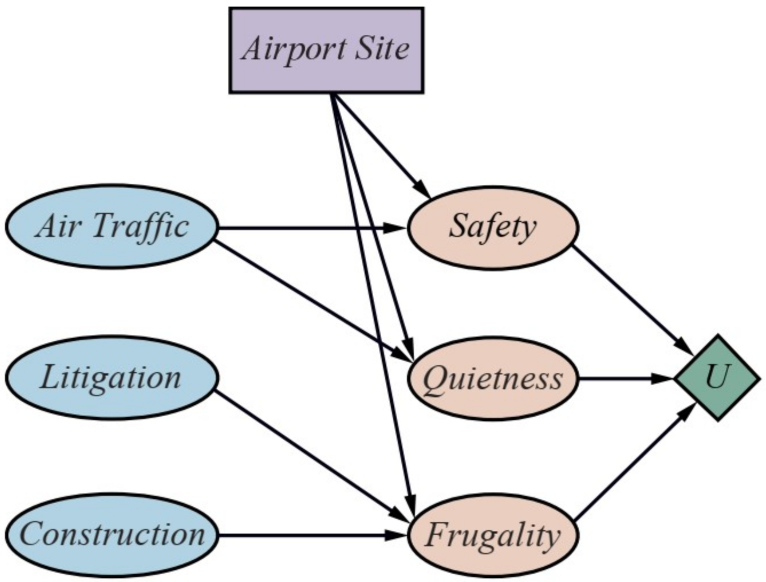


Simplified Structure



(Single-Stage) Decision Networks

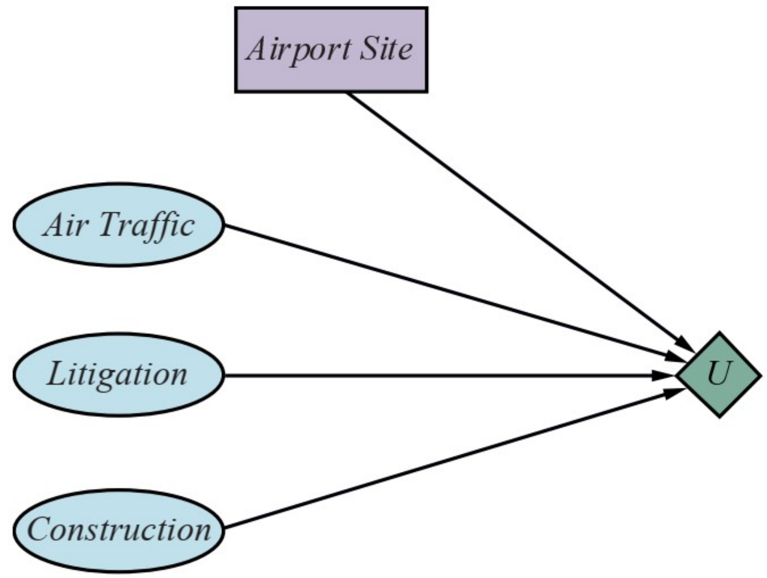
General Structure



Utility Table

S	low	low	low	low	high	high	high	high
Q	low	low	high	high	low	low	high	high
F	low	high	low	high	low	high	low	high
U	10	20	5	50	70	150	100	200

Simplified Structure



Action-Utility Table (not all columns shown)

AT	low	low	low	---	---	high	high	high
L	low	low	high	---	---	low	high	high
C	low	high	low	---	---	high	low	high
AS	A	A	A	---	---	B	B	B
U	10	20	5	---	---	150	100	200

Decision Network: Evaluation

The algorithm for decision network evaluation is as follows:

1. **Set the evidence variables for the current state**
2. **For each possible value a of decision node:**
   a. **Set the decision node to that value**
   b. **Calculate the posterior probabilities for the parent nodes of the utility node**
   c. **Calculate the utility for the action / value a**
3. **Return the action with highest utility**

Agent's Decisions

Recall that agent ACTIONS change the state:

- **if we are in state** s
- **action** a **is expected to**
- **lead to another state** s' **(outcome)**

Given uncertainty about the current state s **and action outcome** s' **we need to define the following:**

- **probability (belief) of being in state** s**:** $P(s)$
- **probability (belief) of action** a **leading to outcome** s'**:** $P(s' \mid s, a)$

Now:

$$P(s' \mid s, a) = P(\text{RESULT}(a) = s') = \sum_s P(s) * P(s' \mid s, a)$$

Expected Action Utility

The expected utility of an action a given the evidence is the average utility value of all possible outcomes s' of action a, weighted by their probability (belief) of occurence:

$$\mathrm{EU}(a) = \sum_{s'} \sum_{s} \mathrm{P}(s) * \mathrm{P}(s' \mid s, a) * \mathrm{U}(s') = \sum_{s'} \mathrm{P}(Result(a) = s') * \mathrm{U}(s')$$

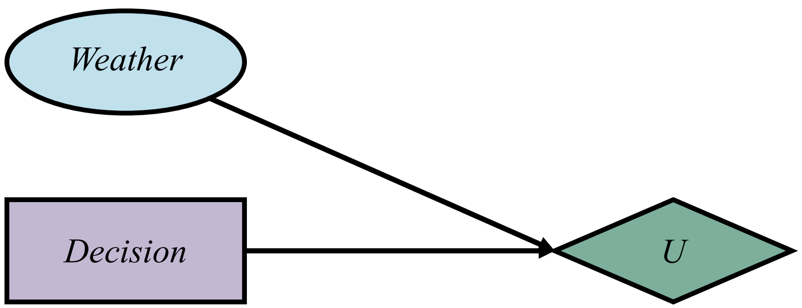
Rational agent should choose an action that maximizes the expected utility:

$$\text{chosen action} = \underset{a}{\text{argmax}}\ \mathrm{EU}(a)$$

Decision Networks: Example

Decision: take umbrella

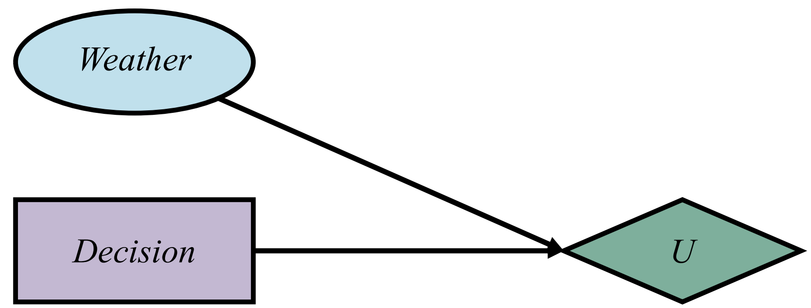
P(W=rain)	P(W=sun)
0.30	0.70



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decision: leave umbrella

P(W=rain)	P(W=sun)
0.30	0.70



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take	rain	70

Decision Networks: Example

Decision: take umbrella

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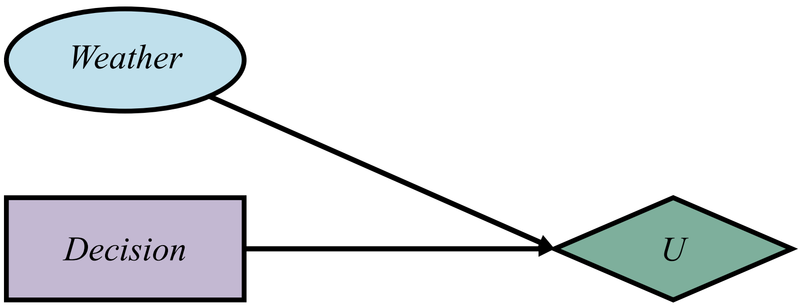

D	W	U
leave	sun	100
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take	rain	70

$\text{EU}(\text{take}) = ???$

Decision: leave umbrella

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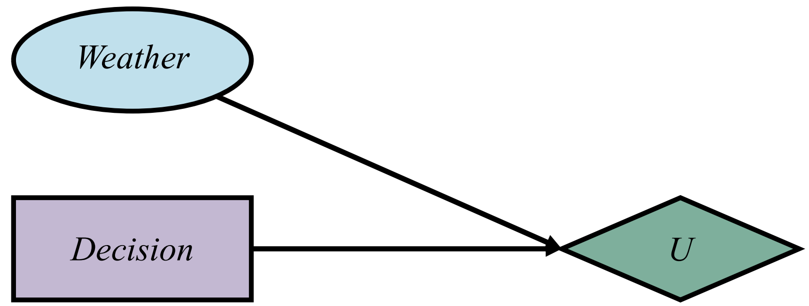

D	W	U
leave	sun	100
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$\text{EU}(\text{leave}) = ???$

Decision Networks: Example 1

Decision: take umbrella

$$\mathrm{EU}(a) = \sum_{s'} \mathrm{P}(Result(a) = s') * \mathrm{U}(s')$$

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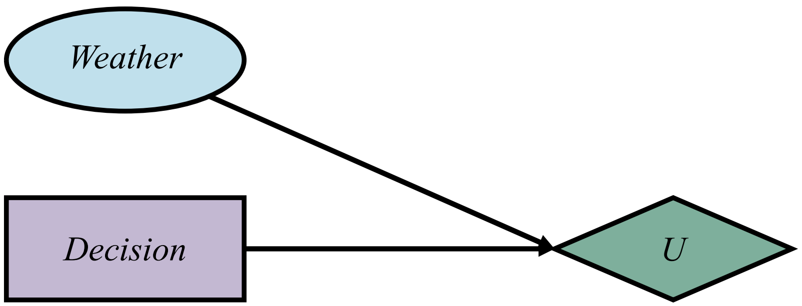

S_1': D = take, W = sun
S_2': D = take, W = rain
EU(take) =
P(Result(take) = S_1')*U(S_1') +
P(Result(take) = S_2')*U(S_2') =
0.70 * 20 + 0.30 * 70 = 35

D	W	U
leave	sun	100
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$$\mathrm{EU}(\text{take}) = 35$$

Decision: leave umbrella

$$\mathrm{EU}(a) = \sum_{s'} \mathrm{P}(Result(a) = s') * \mathrm{U}(s')$$

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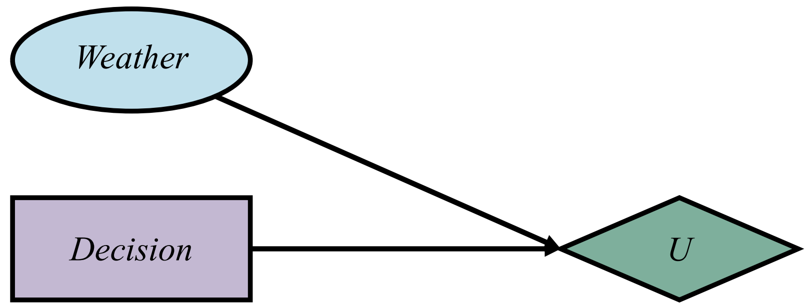

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P(Result(leave) = S_4')*U(S_4') =
0.70 * 100 + 0.30 * 0 = 70

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

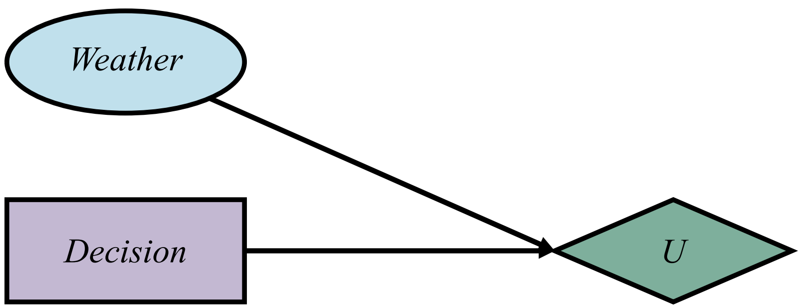
$$\mathrm{EU}(\text{leave}) = 70$$

Decision Networks: Example 1

Which action to choose: take or leave Umbrella?

$EU(a) = \sum_{s'} P(Result(a) = s') * U(s')$

P(W=rain)	P(W=sun)
0.30	0.70



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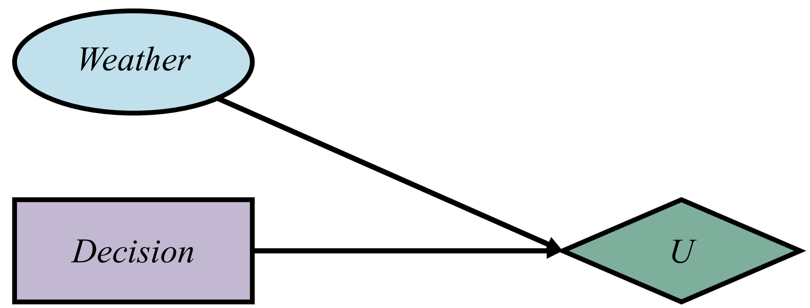

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0.70 * 100 + 0.30 * 0 = 70

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

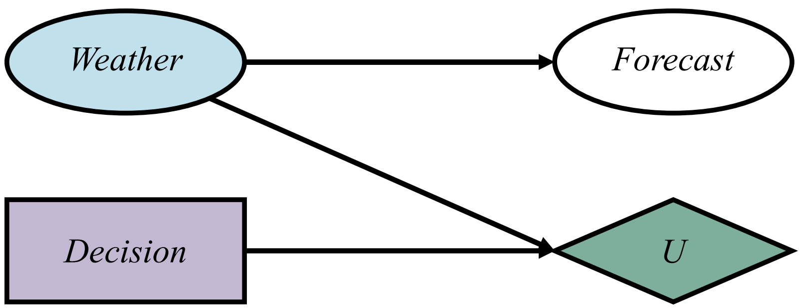
$\text{action} = \underset{a}{\text{argmax}}\ EU(a) \mid \max(EU(\text{take}), \underline{EU(\text{leave})}) = \max(35, \mathbf{70}) \rightarrow \textbf{leave}$

Decision Networks: Example 2a

Decision: take umbrella

P(W=rain)	P(W=sun)
0.30	0.70

P(F=sun)	P(F=rain)
0.59	0.41



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Decision: leave umbrella

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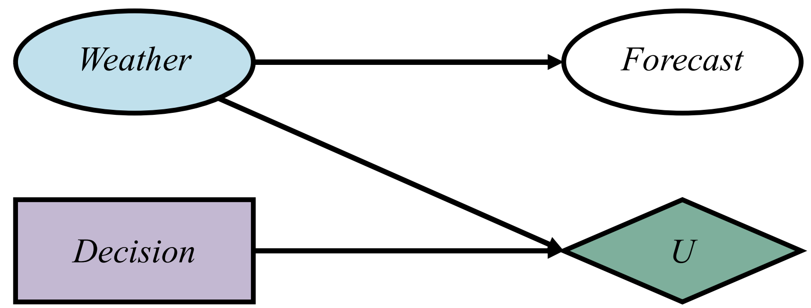

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Decision Networks: Example 2a

Decision: take umbrella

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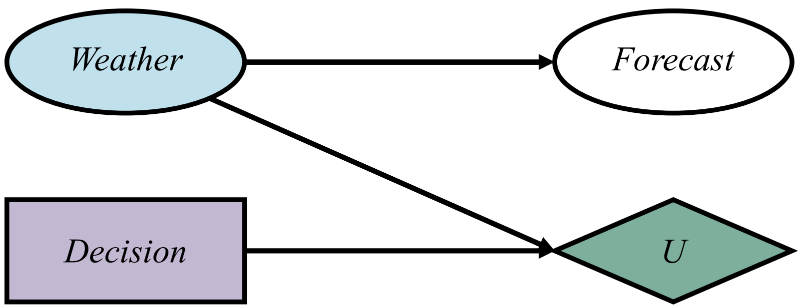

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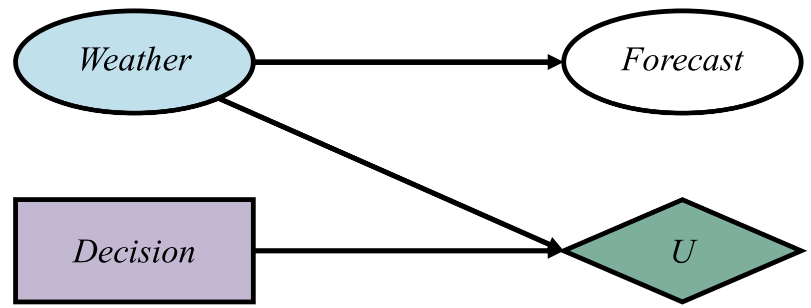

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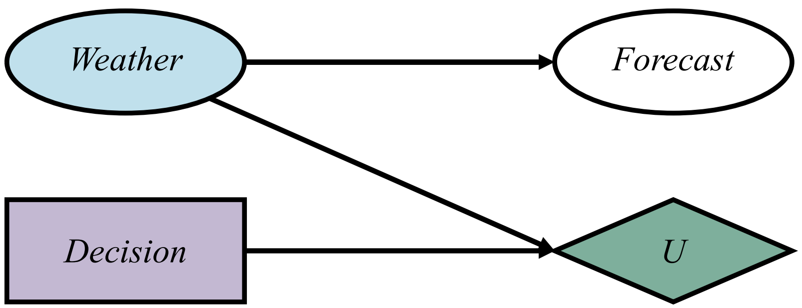

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$$EU(take) = 35$$

Decision: leave umbrella

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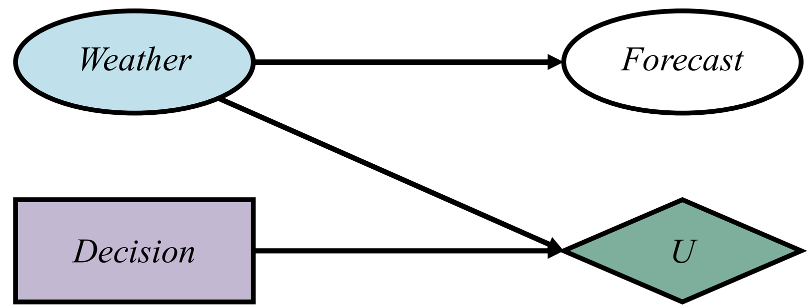

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0.70 * 100 + 0.30 * 0 = 70

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$EU(leave) = 70$$

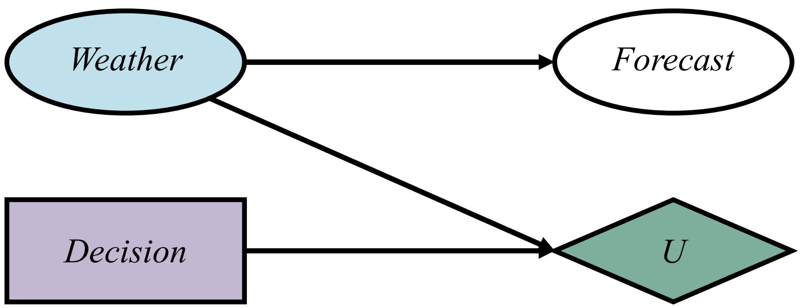
Decision Networks: Example 2a

Which action to choose: take or leave Umbrella?

$$EU(a) = \sum_{s'} P(Result(a) = s') * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70

P(F=sun)	P(F=rain)
0.59	0.41



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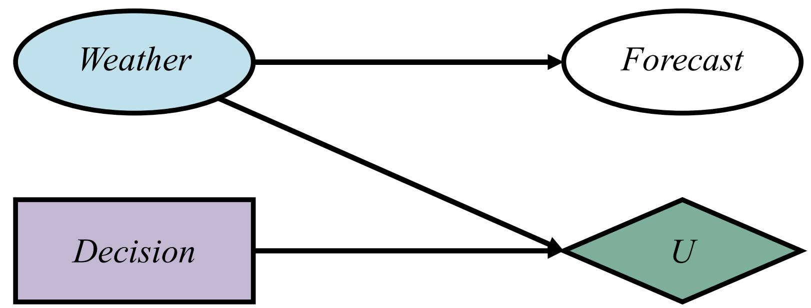

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D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

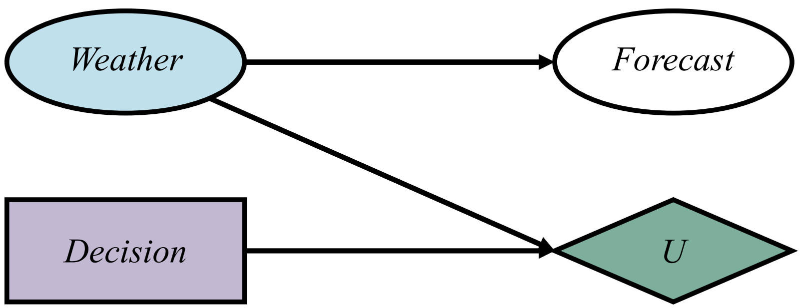
$$action = \underset{a}{\operatorname{argmax}} \; EU(a) \mid \max(EU(take), \underline{EU(leave)}) = \max(35, \mathbf{70}) \rightarrow \mathbf{leave}$$

Decision Networks: Example 2a

Decision: take umbrella

P(W=rain)	P(W=sun)
0.30	0.70

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
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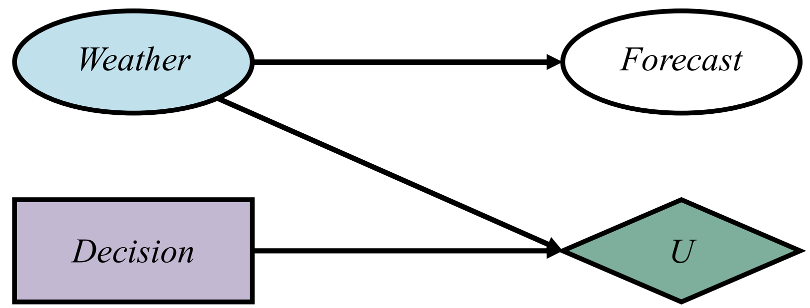
take	rain	70

Decision: leave umbrella

P(W=rain)	P(W=sun)
0.30	0.70

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

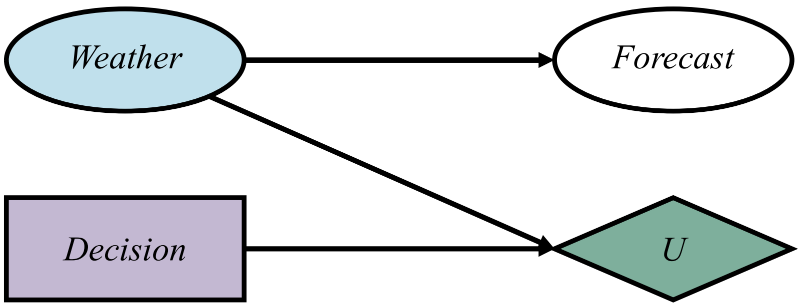
Decision Networks: Example 2b

Decision: take umbrella given e

$\text{EU}(a \mid e) = \sum_{s'} \text{P}(Result(a) = s' \mid e) * \text{U}(s')$

P(rain | F)	P(sun | F)
???	???

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decision: leave umbrella given e

$\text{EU}(a \mid e) = \sum_{s'} \text{P}(Result(a) = s' \mid e) * \text{U}(s')$

P(rain | F)	P(sun | F)
???	???

P(F=sun)	P(F=rain)
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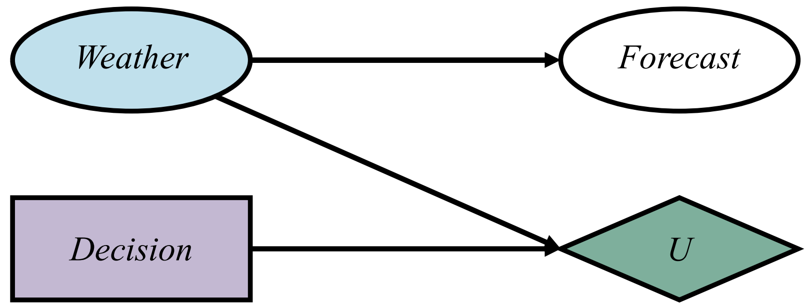


D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

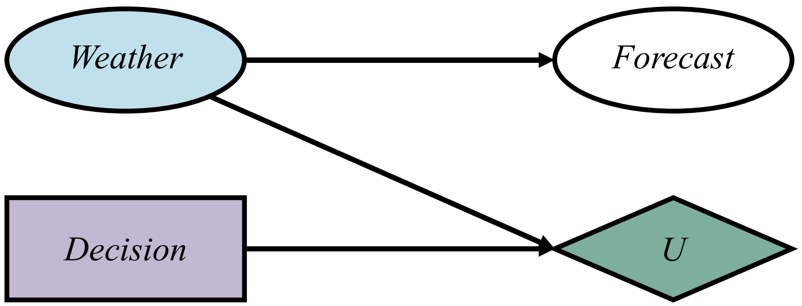
Decision Networks: Example 2b

Decision: take umbrella given e

$$EU(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Conditional probabilities

Assume that we are given:

F	W	P(F|W)
sun	sun	0.80
rain	sun	0.20
sun	rain	0.10
rain	rain	0.90

By Bayes' Theorem:

$$P(W = sun \mid F = sun) = \frac{P(F = sun \mid W = sun) * P(W = sun)}{P(F = sun)} = \frac{0.80 * 0.70}{0.59} = 0.95$$

$$P(W = sun \mid F = rain) = \frac{P(F = rain \mid W = sun) * P(W = sun)}{P(F = rain)} = \frac{0.20 * 0.70}{0.41} = 0.34$$

$$P(W = rain \mid F = sun) = \frac{P(F = sun \mid W = rain) * P(W = rain)}{P(F = sun)} = \frac{0.10 * 0.30}{0.59} = 0.05$$

$$P(W = rain \mid F = rain) = \frac{P(F = rain \mid W = rain) * P(W = rain)}{P(F = rain)} = \frac{0.90 * 0.30}{0.41} = 0.66$$

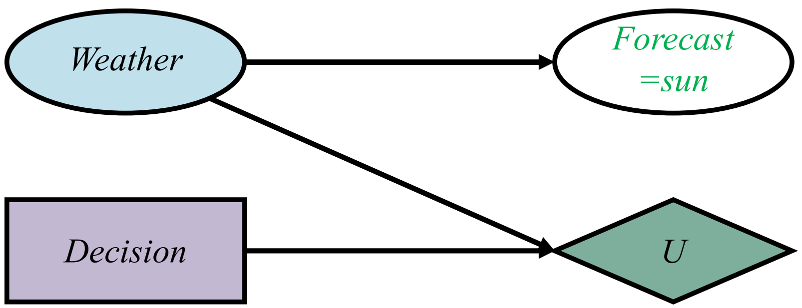
Decision Networks: Example 2b

Decision: take umbrella given sun

$\text{EU}(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$

P(rain | F)	P(sun | F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

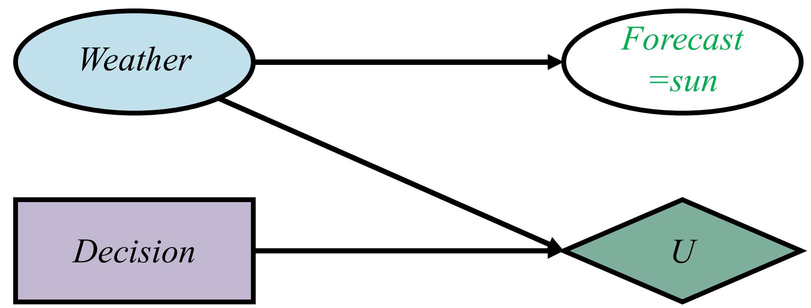
EU(take given sun forecast) = ???

Decision: leave umbrella given sun

$\text{EU}(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$

P(rain | F)	P(sun | F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
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take	rain	70

EU(leave given sun forecast) = ???

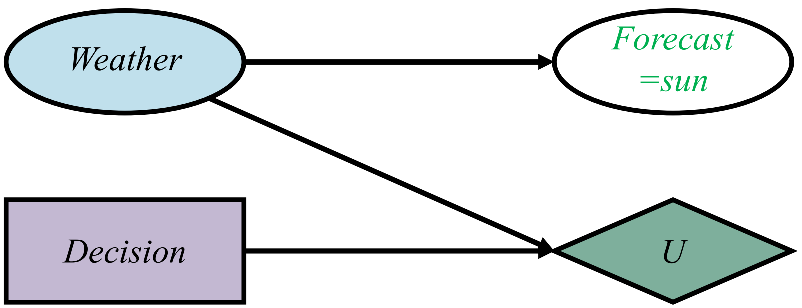
Decision Networks: Example 2b

Decision: take umbrella given sun

$EU(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$

P(rain | F)	P(sun | F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



S_1': D = take, W = sun
S_2': D = take, W = rain
EU(take) =
P(Result(take)=S_1'|e)*U(S_1') +
P(Result(take)=S_2'|e)*U(S_2') =
0.95 * 20 + 0.05 * 70 = 22.5

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

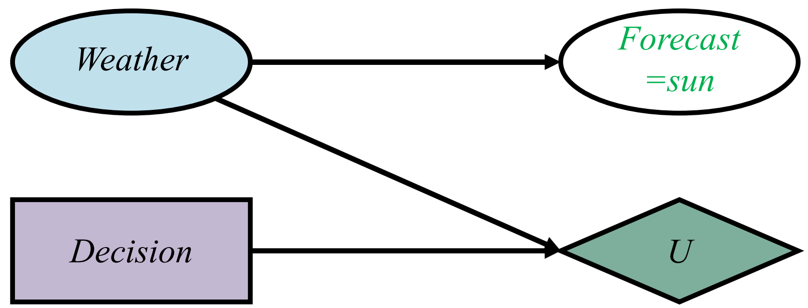
EU(take given sun forecast) = 22.5

Decision: leave umbrella given sun

$EU(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$

P(rain | F)	P(sun | F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



S_3': D = leave, W = sun
S_4': D = leave, W = rain
EU(leave) =
P(Result(leave)=S_3'|e)*U(S_3') +
P(Result(leave)=S_4'|e)*U(S_4') =
0.95 * 100 + 0.05 * 0 = 95

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

EU(leave given sun forecast) = 95

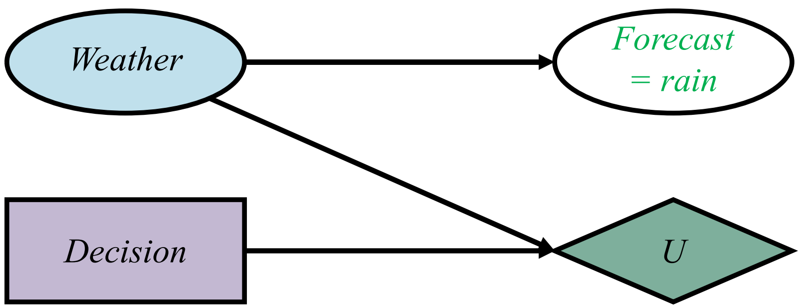
Decision Networks: Example 2b

Decision: take umbrella given rain

$\mathrm{EU}(a \mid e) = \sum_{s'} \mathrm{P}(Result(a) = s' \mid e) * \mathrm{U}(s')$

P(rain | F)	P(sun | F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	**0.41**



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

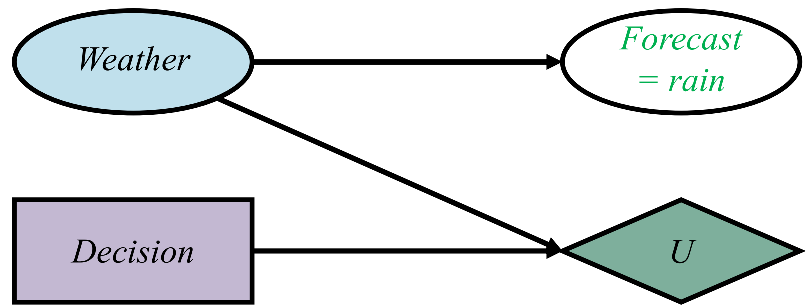
EU(take given rain forecast) = ? ? ?

Decision: leave umbrella given rain

$\mathrm{EU}(a \mid e) = \sum_{s'} \mathrm{P}(Result(a) = s' \mid e) * \mathrm{U}(s')$

P(rain | F)	P(sun | F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	**0.41**



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

EU(leave given rain forecast) = ? ? ?

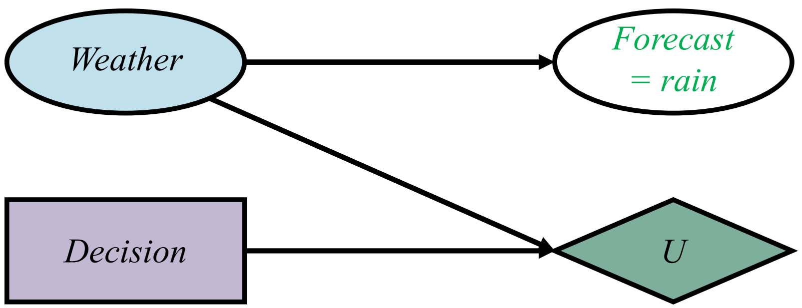
Decision Networks: Example 2b

Decision: take umbrella given rain

$EU(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$

P(rain | F)	P(sun | F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	**0.41**



S_1': D = take, W = sun
S_2': D = take, W = rain
EU(take) =
P(Result(take)=S_1'|e)*U(S_1') +
P(Result(take)=S_2'|e)*U(S_2') =
0.34 * 20 + 0.66 * 70 = 53

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

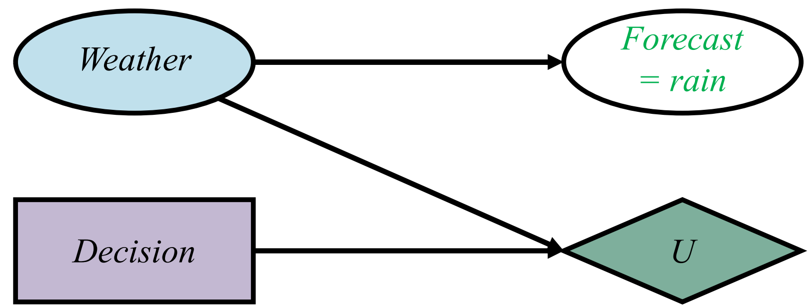
EU(take given rain forecast) = 53

Decision: leave umbrella given rain

$EU(a \mid e) = \sum_{s'} P(Result(a) = s' \mid e) * U(s')$

P(rain | F)	P(sun | F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	**0.41**



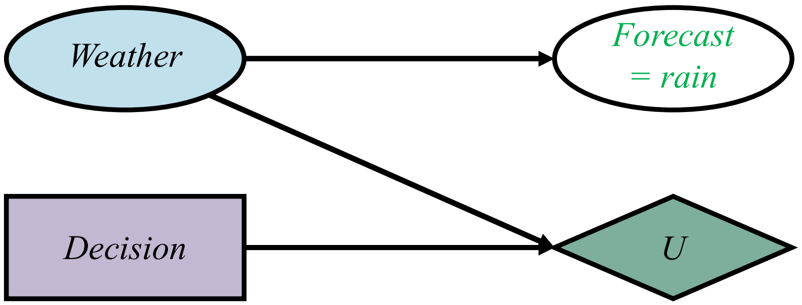
S_3': D = leave, W = sun
S_4': D = leave, W = rain
EU(leave) =
P(Result(leave)=S_3'|e)*U(S_3') +
P(Result(leave)=S_4'|e)*U(S_4') =
0.34 * 100 + 0.66 * 0 = 34

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

EU(leave given rain forecast) = 34

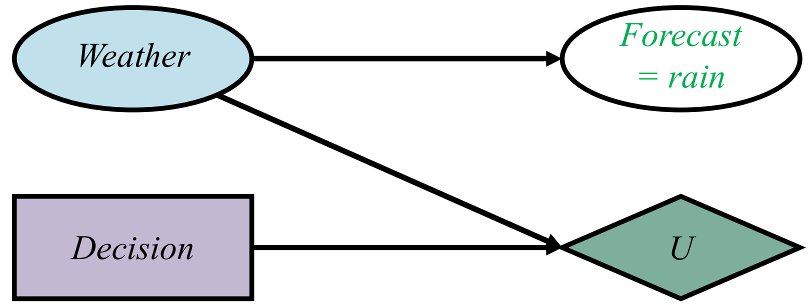
Decision Networks: Example 2b

Decision:take umbrella given rain



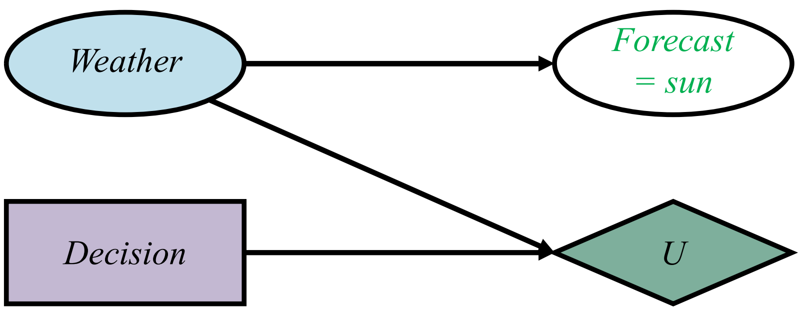
EU(take given rain forecast) = 53

Decision:leave umbrella given rain



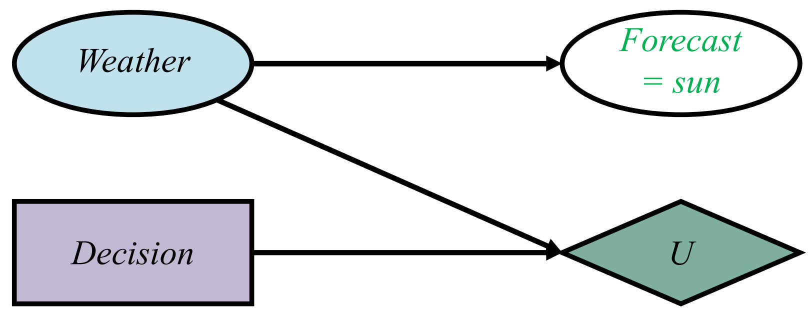
EU(leave given rain forecast) = 34

Decision:take umbrella given sun



EU(take given sun forecast) = 22.5

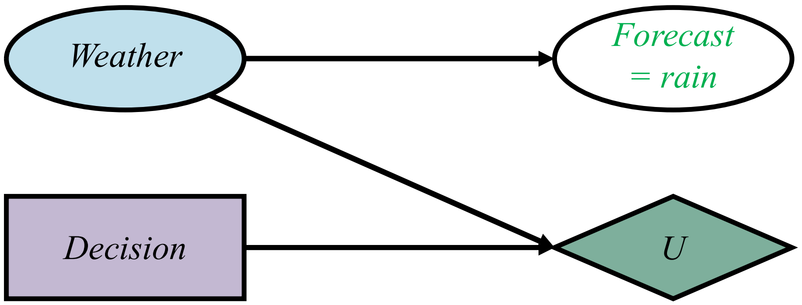
Decision:leave umbrella given sun



EU(leave given sun forecast) = 95

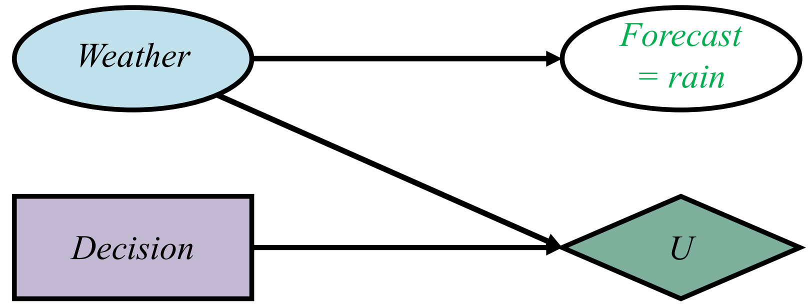
Decision Networks: Example 2b

Decision:take umbrella given rain



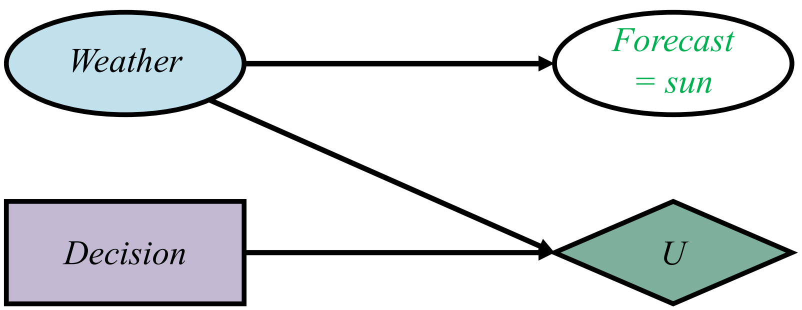
EU(take given rain forecast) = 53

Decision:leave umbrella given rain



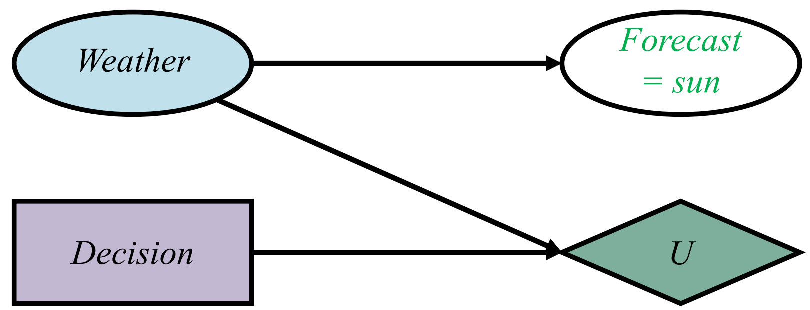
EU(leave given rain forecast) = 34

Decision:take umbrella given sun



EU(take given sun forecast) = 22.5

Decision:leave umbrella given sun



EU(leave given sun forecast) = 95

Value of Perfect Information

The value/utility of best action α without additional evidence (information) is :

$$\mathrm{MEU}(\alpha|\varepsilon) = \max_{a} \sum_{s'} \mathrm{P}(Result(a) = s') * \mathrm{U}(s')$$

If we include new evidence/information ($\mathrm{E_j} = \mathrm{e_j}$) given by some variable Ej, value/utility of best action α becomes:

$$\mathrm{MEU}(a_{e_j} \mid e_{j.sss}) = \max_{a} \sum_{s'} \mathrm{P}(Result(a) = s' \mid e_j) * \mathrm{U}(s')$$

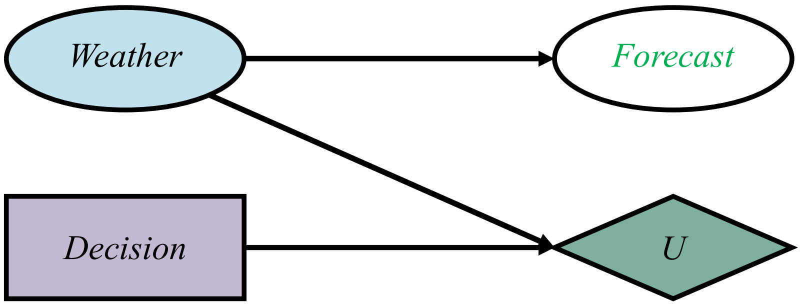
The value of additional evidence/information from Ej is:

$$\mathrm{VPI}(E_j) = \left(\sum_{e_j} \mathrm{P}(E_j = e_j) * \mathrm{MEU}(a_{e_j} \mid E_j = e_j)\right) - MEU(a)$$

using our current beliefs about the world.

Decision Network: Example

Decision network



The value of best action α without additional evidence

$$\text{MEU}(\alpha) = \text{MEU}(\text{leave}) = 70$$

With evidence information ($\mathrm{E_j} = \mathrm{e_j}$) given by Forecast:

$$\text{MEU}(a_{e_1} \mid e_1) = \text{MEU}(\text{take} \mid F = rain) = 53$$

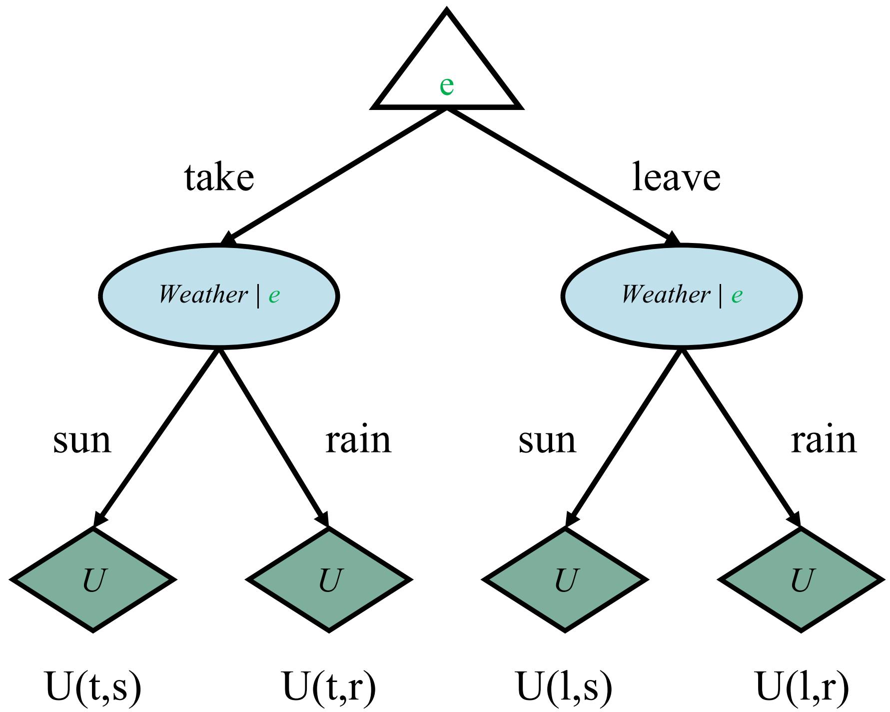
$$\text{MEU}(a_{e_2} \mid e_2) = \text{MEU}(\text{leave} \mid F = sun) = 95$$

The value of additional evidence / information from F is:

$$\text{VPI}(E_j) = \left(\sum_{e_j} \text{P}(E_j = e_j) * \text{MEU}(a_{e_j} \mid E_j = e_j)\right) - MEU(a)$$

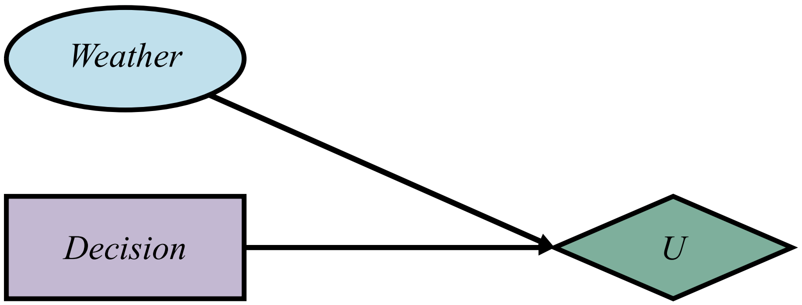
$$\text{VPI}(F) = (\text{P}(F = rain) * \text{MEU}(take \mid F = rain) + \text{P}(F = sun) * \text{MEU}(\text{leave} \mid F = sun)) - MEU(leave) = (0.41 * 53 + 0.59 * 95) - 70 = 7.78$$

Outcome tree



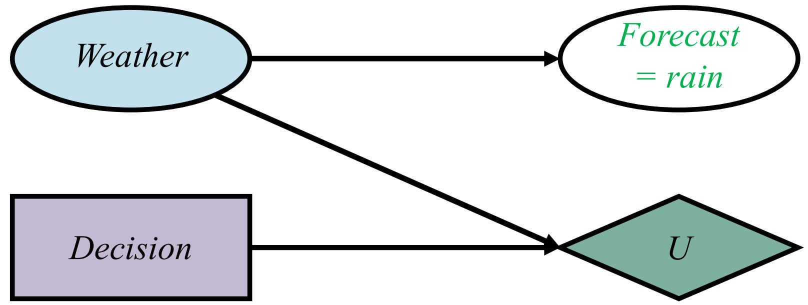
Decision Networks: Example

Decision: leave umbrella



$$\mathrm{EU}(\text{leave}) = 70$$

Decision: take umbrella given rain



$$\mathrm{EU}(\text{take given rain forecast}) = 53$$

The value of best action α without additional evidence

$$\mathrm{MEU}(\alpha) = \mathrm{MEU}(\text{leave}) = 70$$

With evidence information ($\mathrm{E_j} = \mathrm{e_j}$) given by Forecast:

$$\mathrm{MEU}(a_{e_1} \mid e_1) = \mathrm{MEU}(\text{take} \mid F = rain) = 53$$

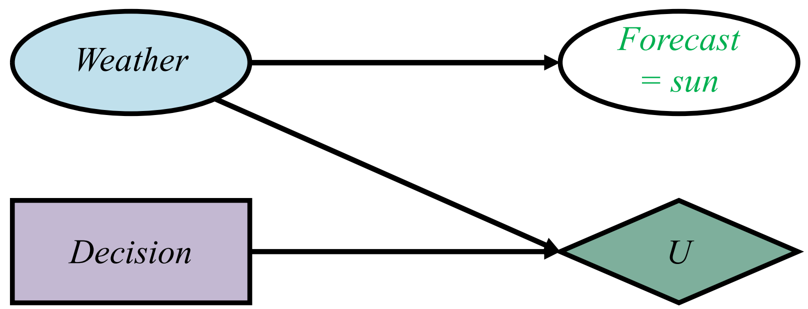
$$\mathrm{MEU}(a_{e_2} \mid e_2) = \mathrm{MEU}(\text{leave} \mid F = sun) = 95$$

The value of additional evidence / information from F is:

$$\mathrm{VPI}(E_j) = \left(\sum_{e_j} \mathrm{P}(E_j = e_j) * \mathrm{MEU}(a_{e_j} \mid E_j = e_j)\right) - MEU(a)$$

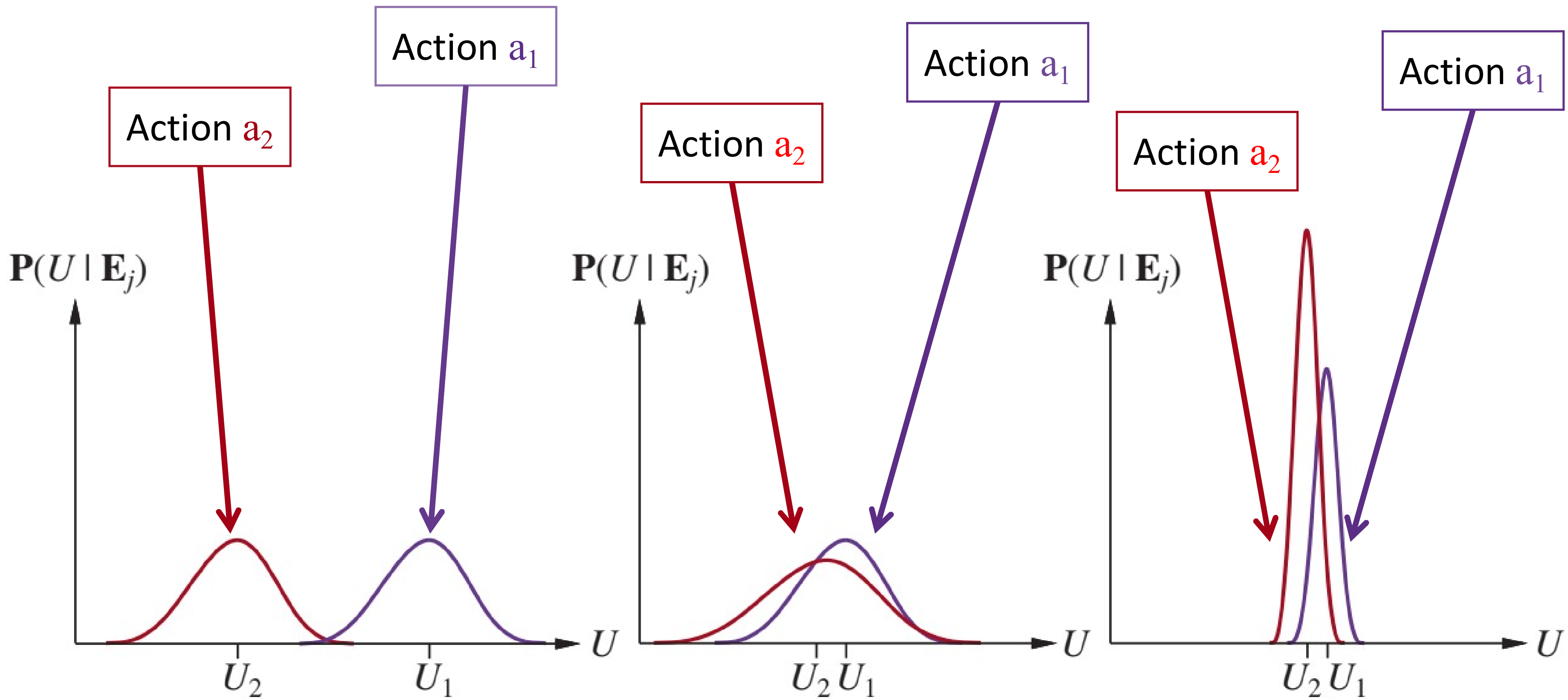
$$\mathrm{VPI}(F) = (\mathrm{P}(F = rain) * \mathrm{MEU}(take \mid F = rain) + \mathrm{P}(F = sun) * \mathrm{MEU}(\text{leave} \mid F = sun)) - MEU(leave) = (0.41 * 53 + 0.59 * 95) - 70 = 7.78$$

Decision: leave umbrella given sun



$$\mathrm{EU}(\text{leave given sun forecast}) = 95$$

Utility & Value of Perfect Information



New information will not help here.

New information may help a lot here.

New information may help a bit here.

VPI Properties

Given a decision network with possible observations $\mathrm{E_j}$ (sources of new information / evidence):

- **The expected value of information is nonnegative:**

$$\forall_{\mathrm{j}}\,\mathrm{VPI}(E_j) \geq 0$$

- **VPI is not additive:**

$$\mathrm{VPI}(E_j, E_k) \neq \mathrm{VPI}(E_j) + \mathrm{VPI}(E_k)$$

- **VPI is order-independent:**

$$\mathrm{VPI}(E_j,\ E_k) = VPI(E_j) + VPI(\ E_k \mid E_j) = VPI(E_k) + VPI(\ E_j \mid E_k) = VPI(E_k\ ,\ E_j)$$

Information Gathering Agent

function INFORMATION-GATHERING-AGENT($percept$) **returns** an $action$
persistent: D, a decision network

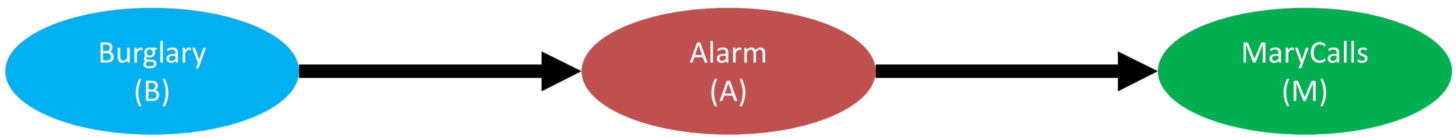
integrate $percept$ into D
$j \leftarrow$ the value that maximizes $VPI(E_j) \ / \ C(E_j)$
if $VPI(E_j) \ > \ C(E_j)$
then return $Request(E_j)$
else return the best action from D

BONUS MATERIAL

(NOT ON EXAMS!)

Conditional Independence

Causal Chain:



$$P(M \mid A, B) = \frac{P(A, B, M)}{P(A, B)} = \frac{P(B) * P(A \mid B) * P(M \mid A)}{P(B) * P(A \mid B)} = P(M \mid A)$$

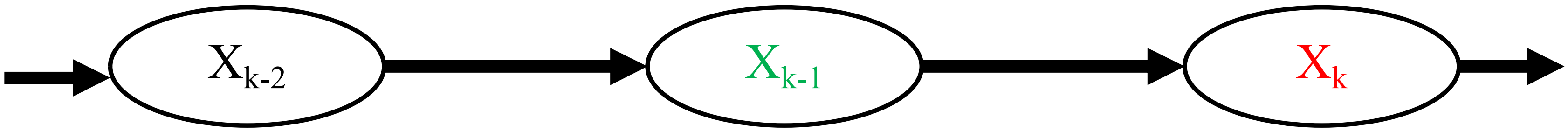
Burglary and MaryCalls are CONDITIONALLY independent given Alarm.

If Alarm is given, what "happened before" does not directly influence MaryCalls.

Markov Chains / Markov Property

A sequence of random variables $\{X_i\}$ is called a Markov chain if it has the Markov property (memoryless property):

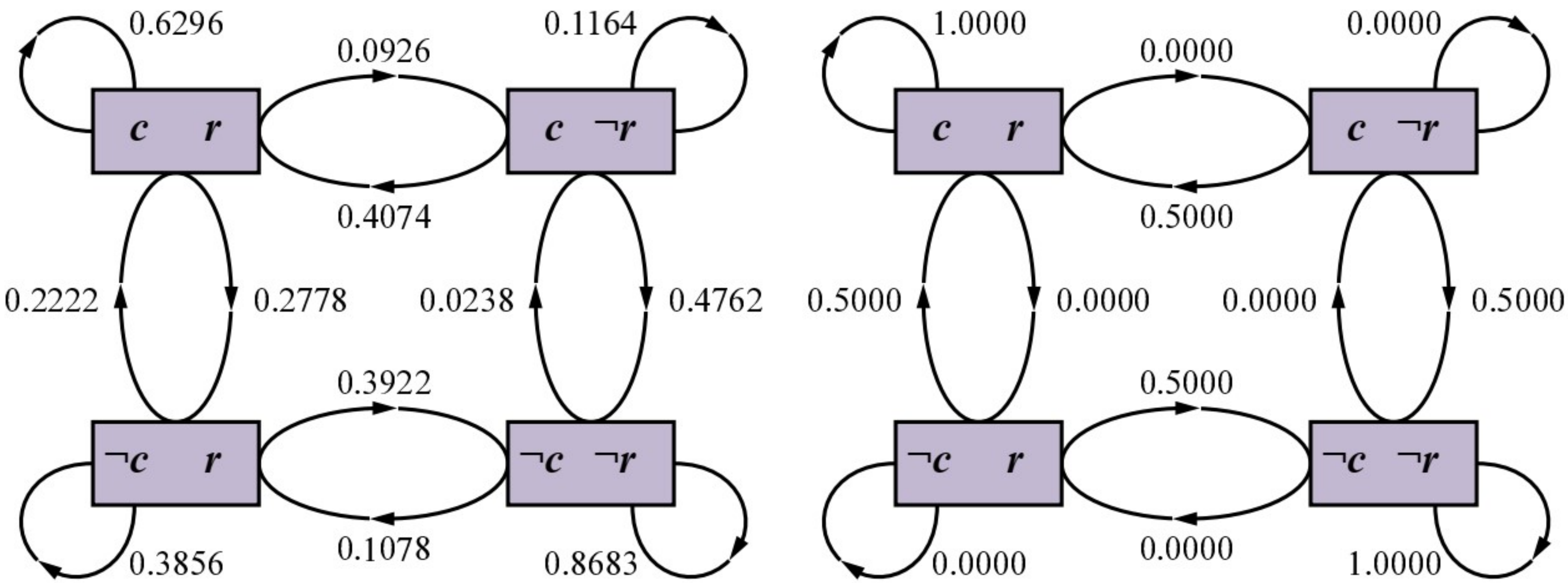
$$P(X_k = a \mid X_{k-1} = b, X_{k-2} = c, \ldots, X_1 = z) = P(X_k = a \mid X_{k-1} = b)$$



Markov Model

A Markov model is a stochastic model used to model (pseudo-) randomly changing systems.

Its key future is the assumption that future states depend only on the current state, not on the events that occurred before it (it assumes the Markov property).



Check out this demo: https://setosa.io/ev/markov-chains/

Fuzzy Logic: the Idea

- Boolean ("crisp") logic

true false

- Fuzzy (many valued) logic

true false

Fuzzy Logic: the Idea

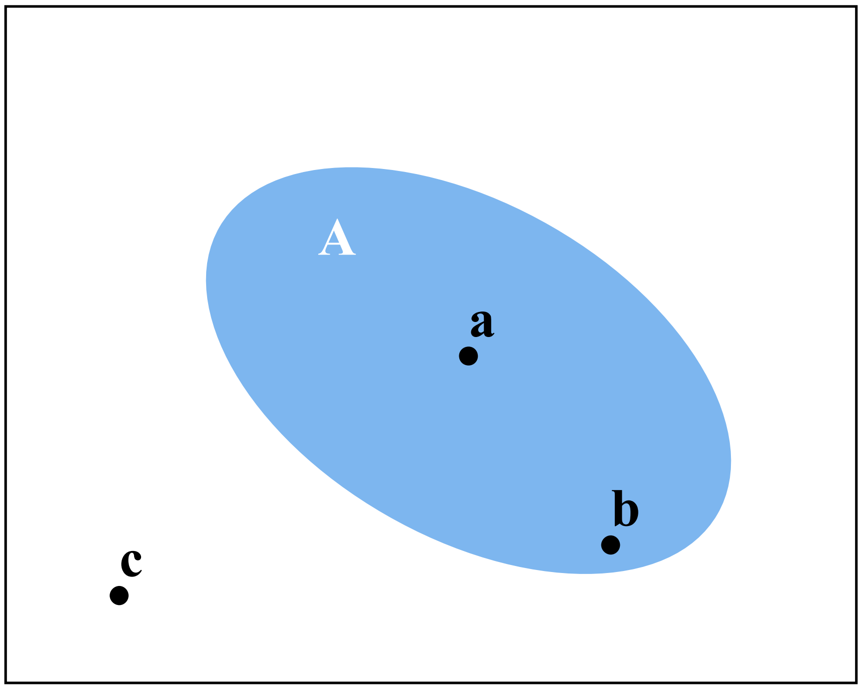
- Boolean ("crisp") logic

- Fuzzy (many valued) logic

Fuzzy Logic: Fuzzy Sets

"Crisp" Set A

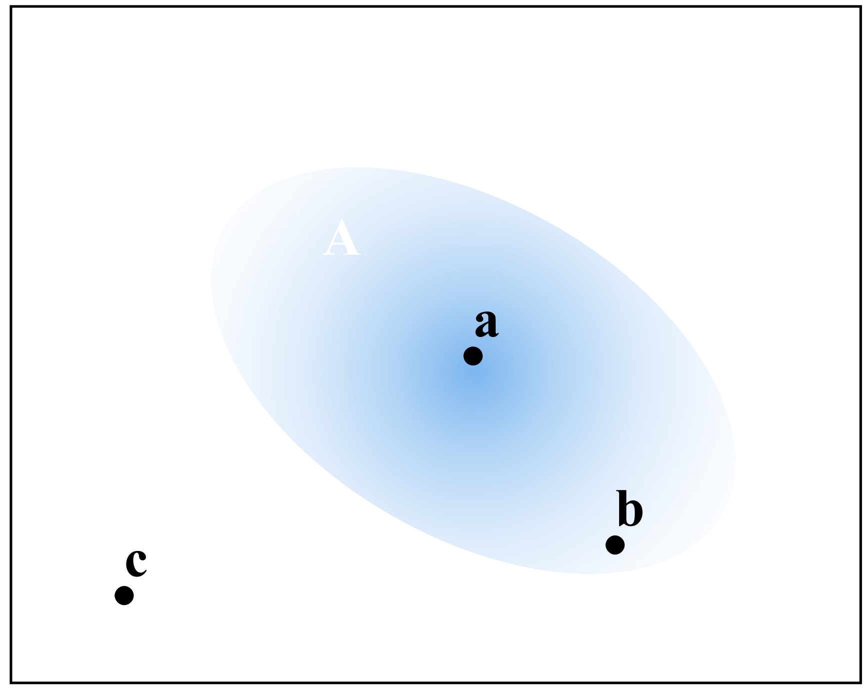
an element is a set member or not



$a \in A$
$b \in A$
$c \notin A$

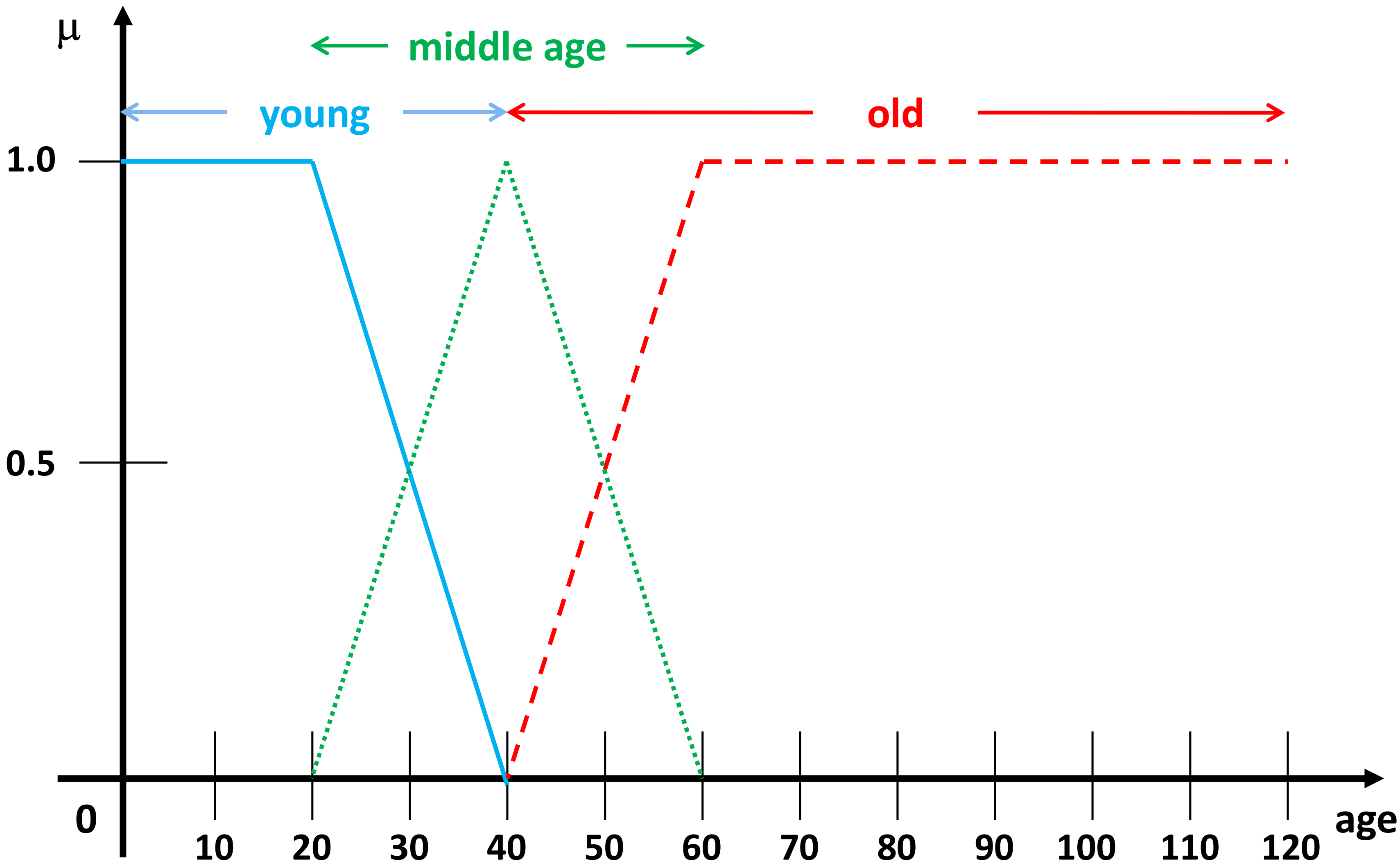
Fuzzy Set A:

an element is a set member
with some membership degree μ

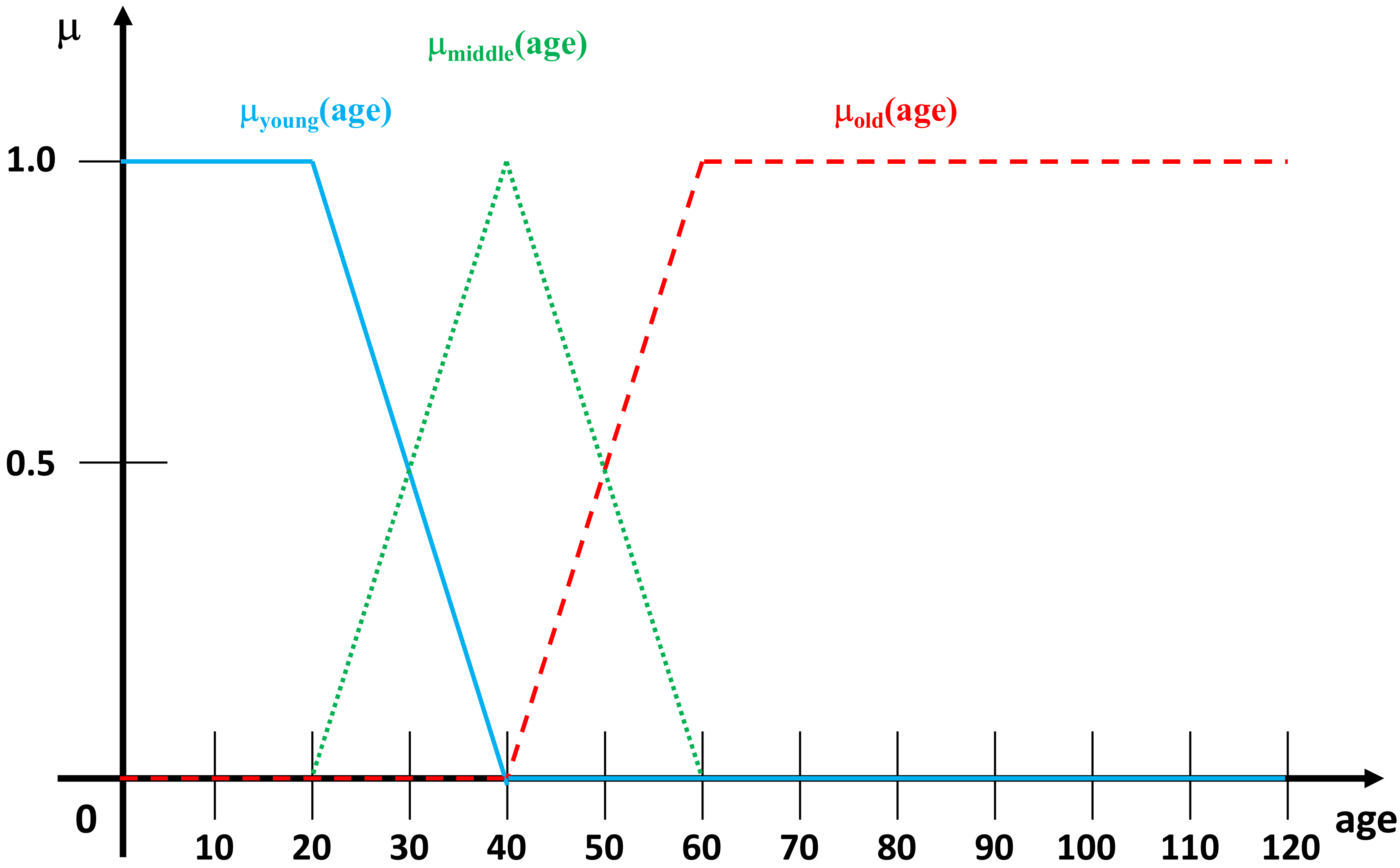


$\mu(a) = 1.0$
$\mu(b) = 0.1$
$\mu(c) = 0.0$

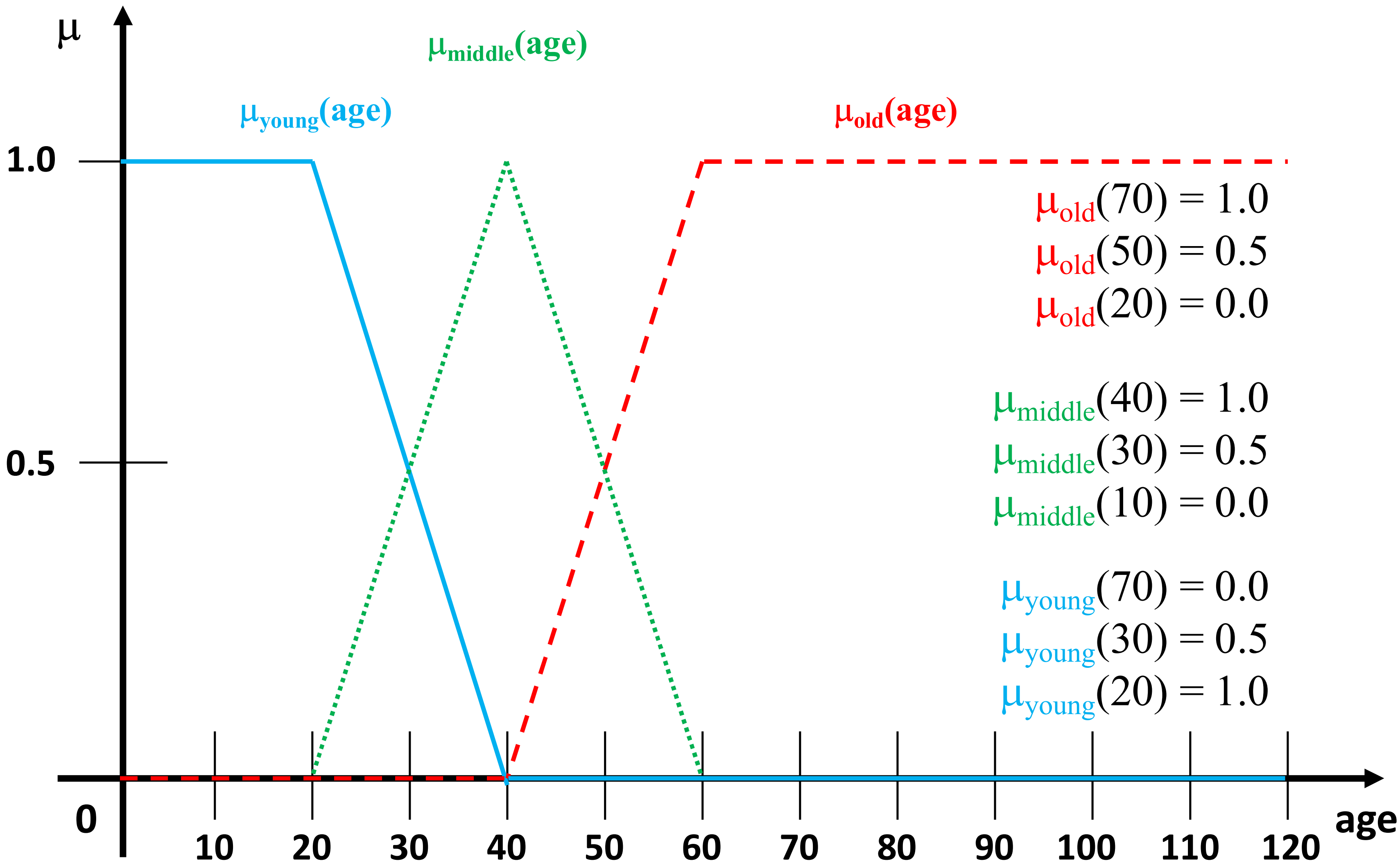
Fuzzy Logic: Fuzzy Sets



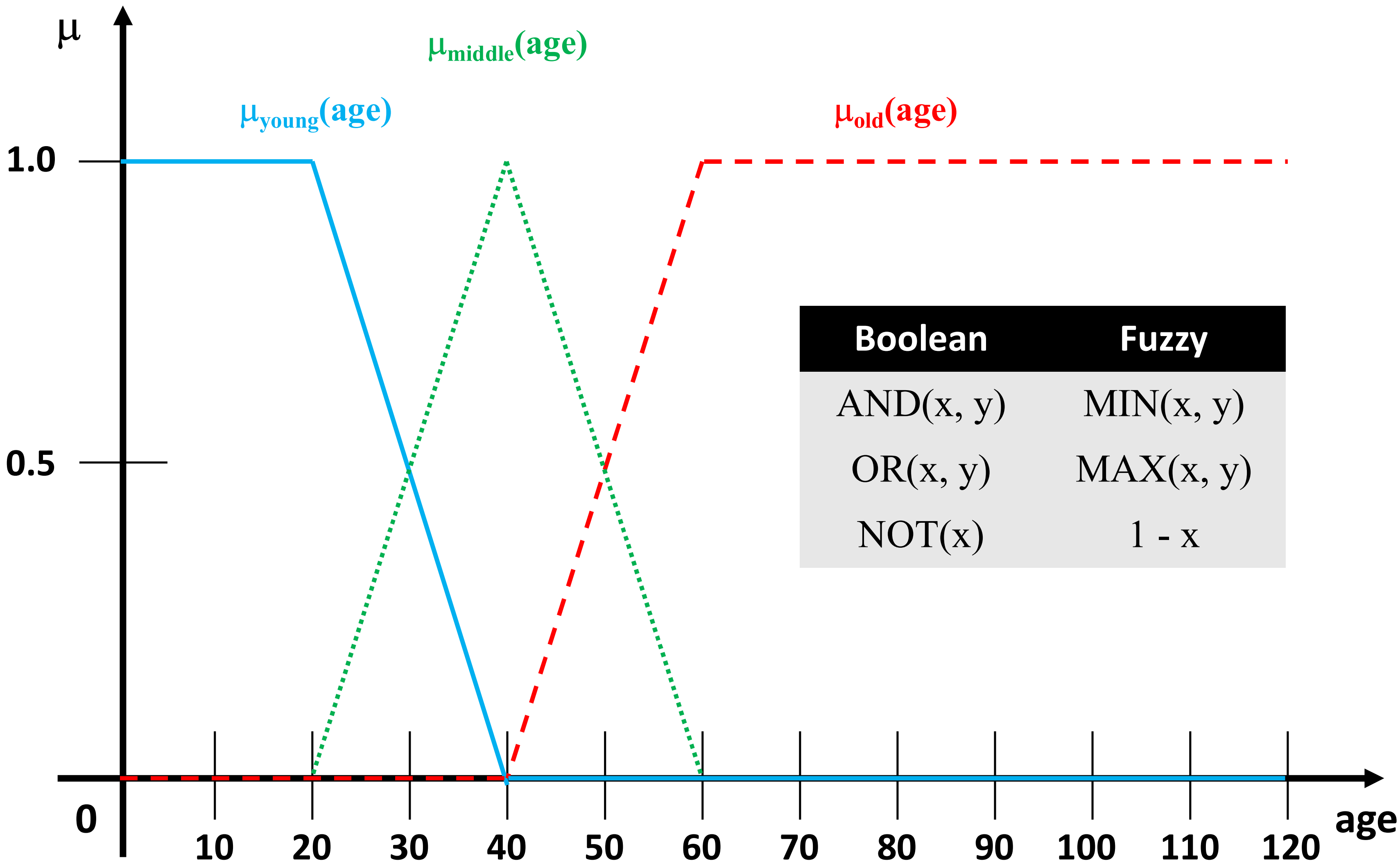
Fuzzy Logic: Membership Functions



Fuzzy Logic: Membership Functions

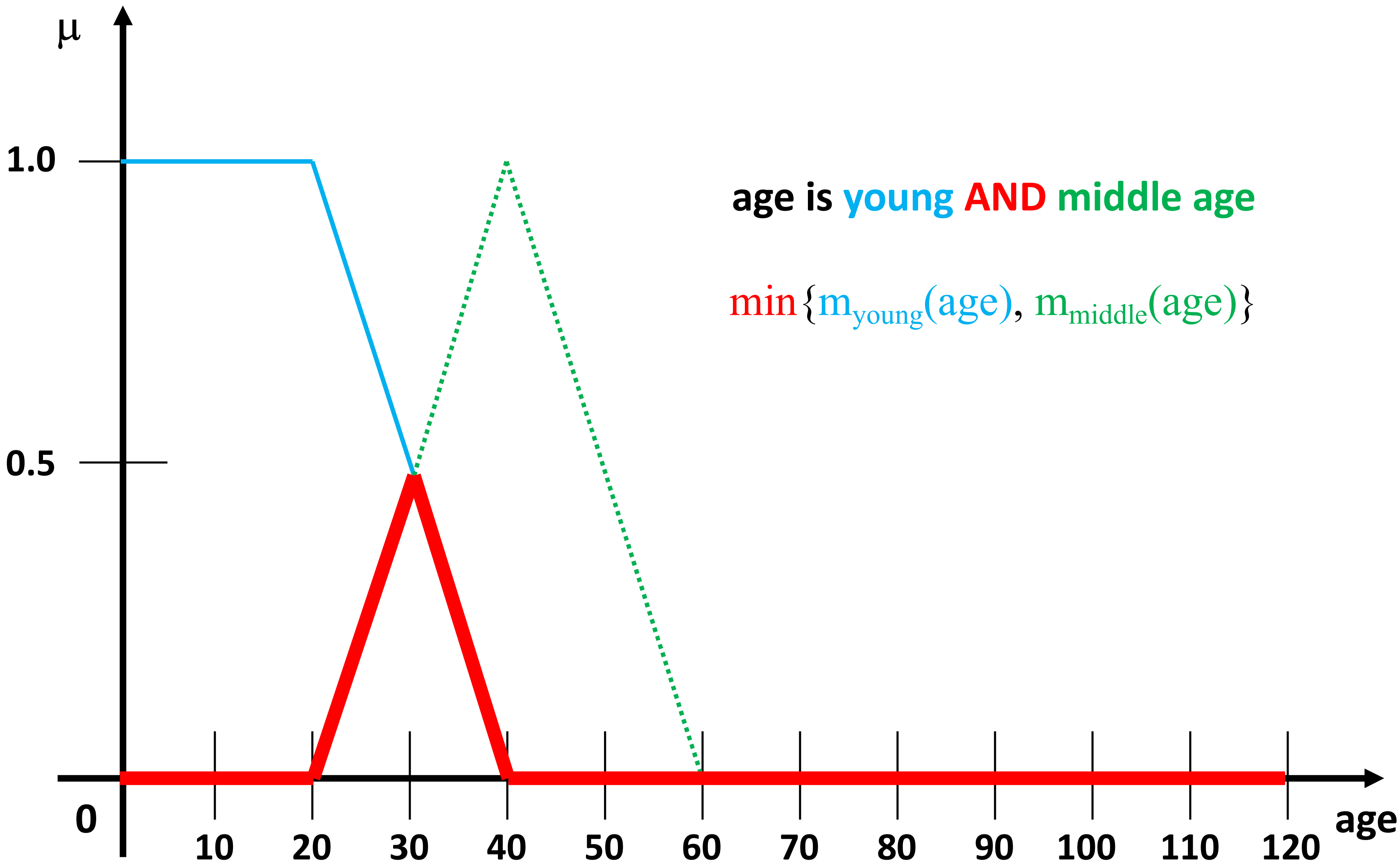


Fuzzy Logic: Logic Operators



Boolean	Fuzzy
AND(x, y)	MIN(x, y)
OR(x, y)	MAX(x, y)
NOT(x)	1 - x

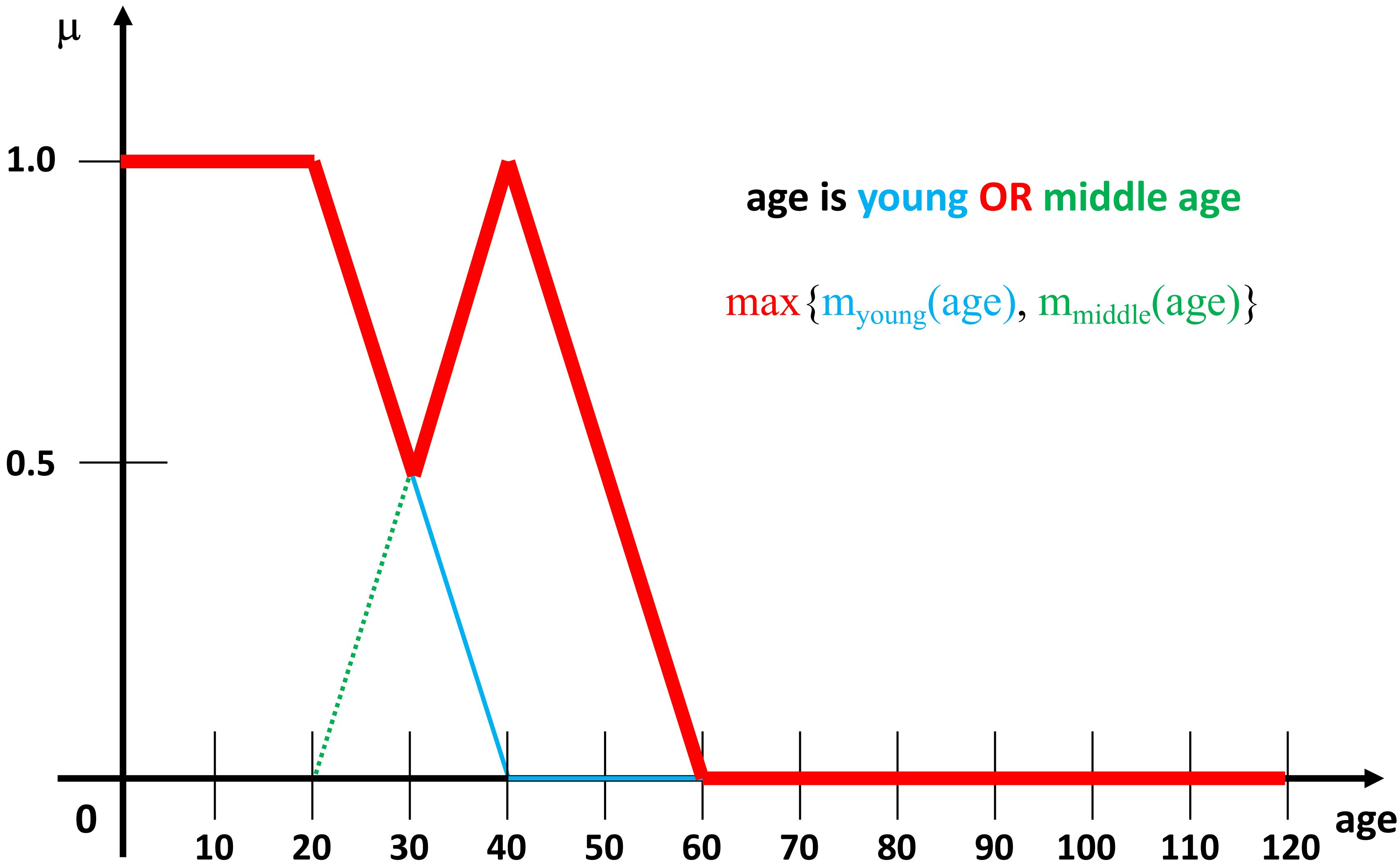
Fuzzy Logic: the AND Operator



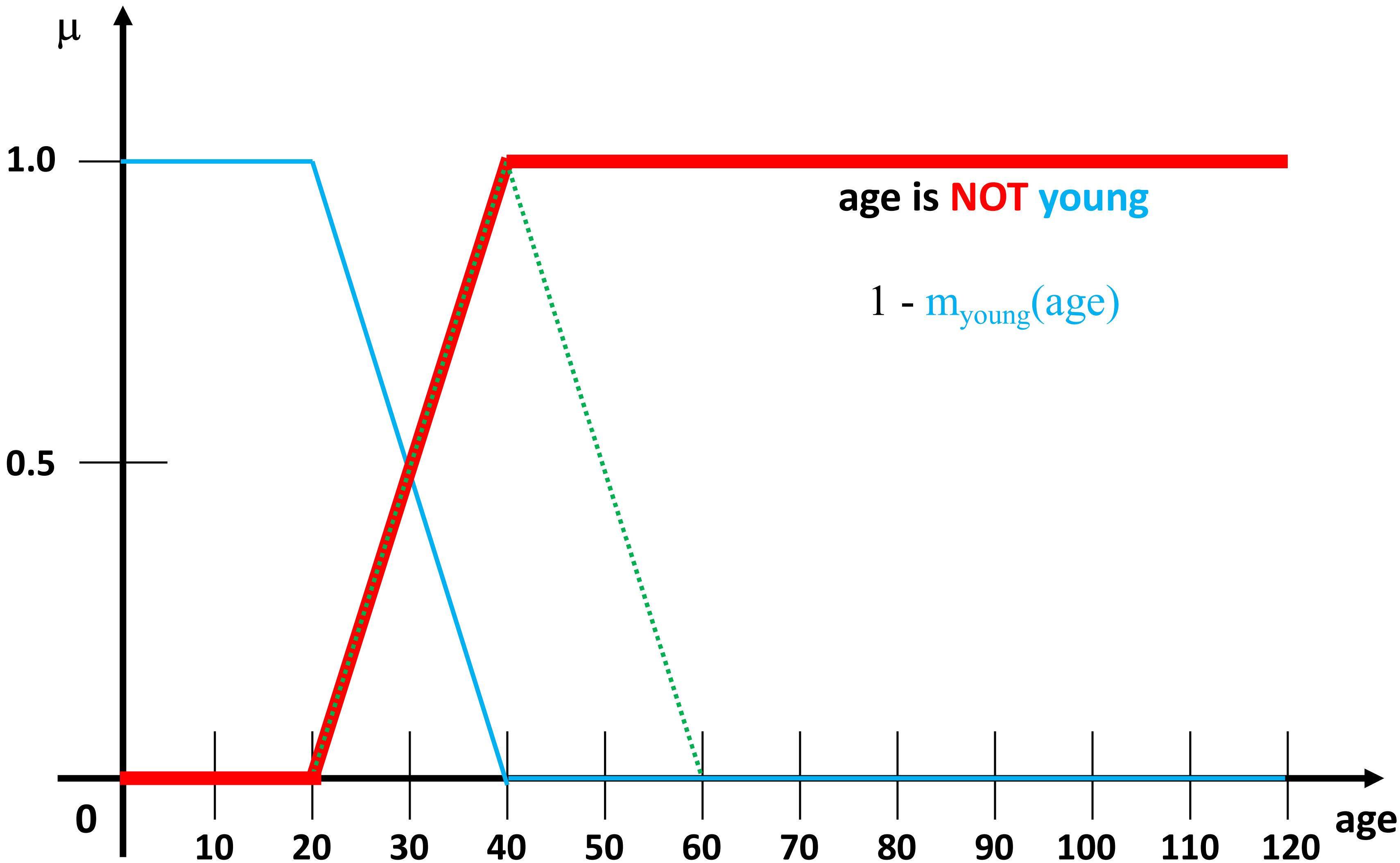
age is young AND middle age

$\min\{m_{young}(age), m_{middle}(age)\}$

Fuzzy Logic: the OR Operator

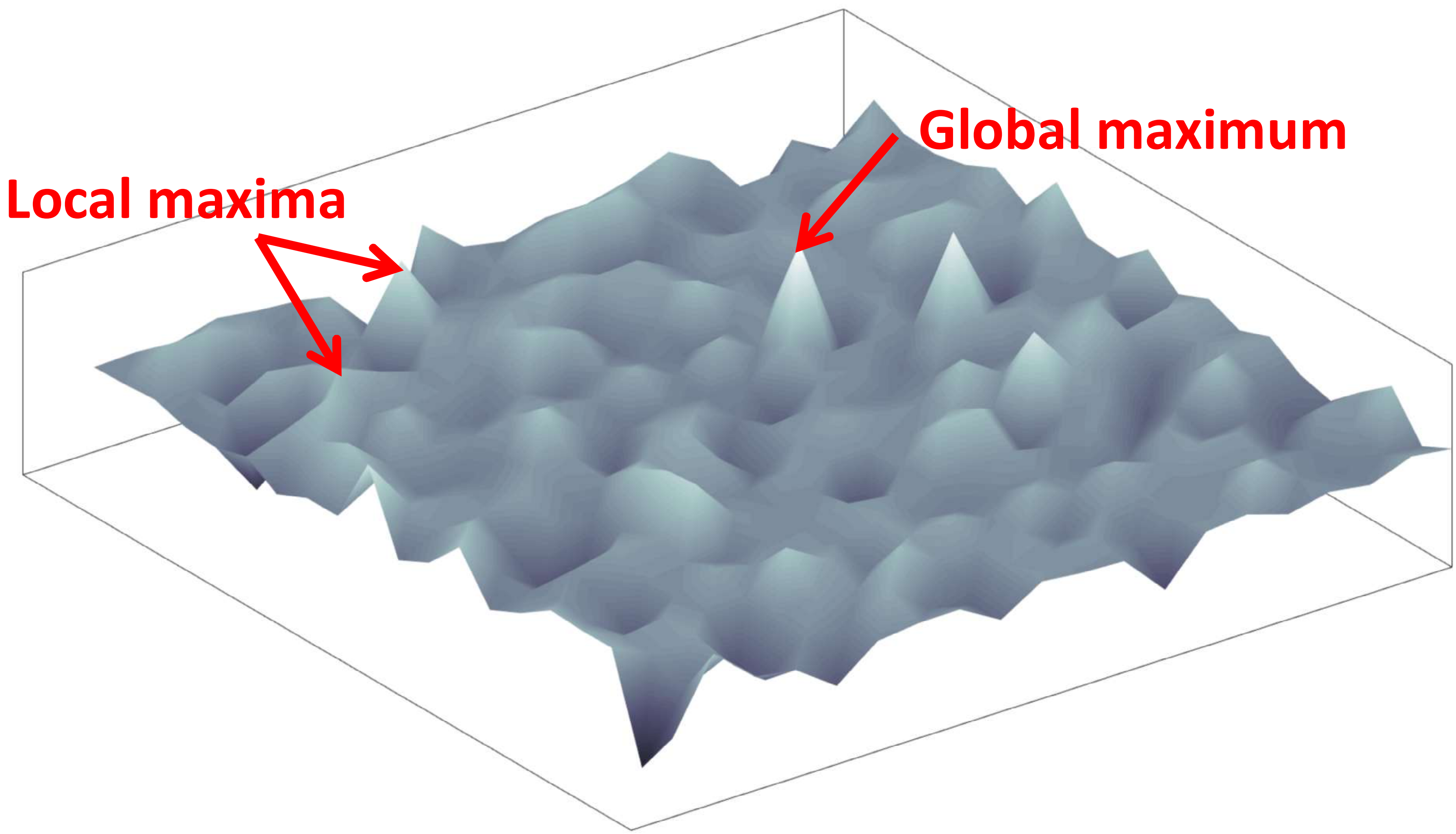


Fuzzy Logic: the NOT Operator

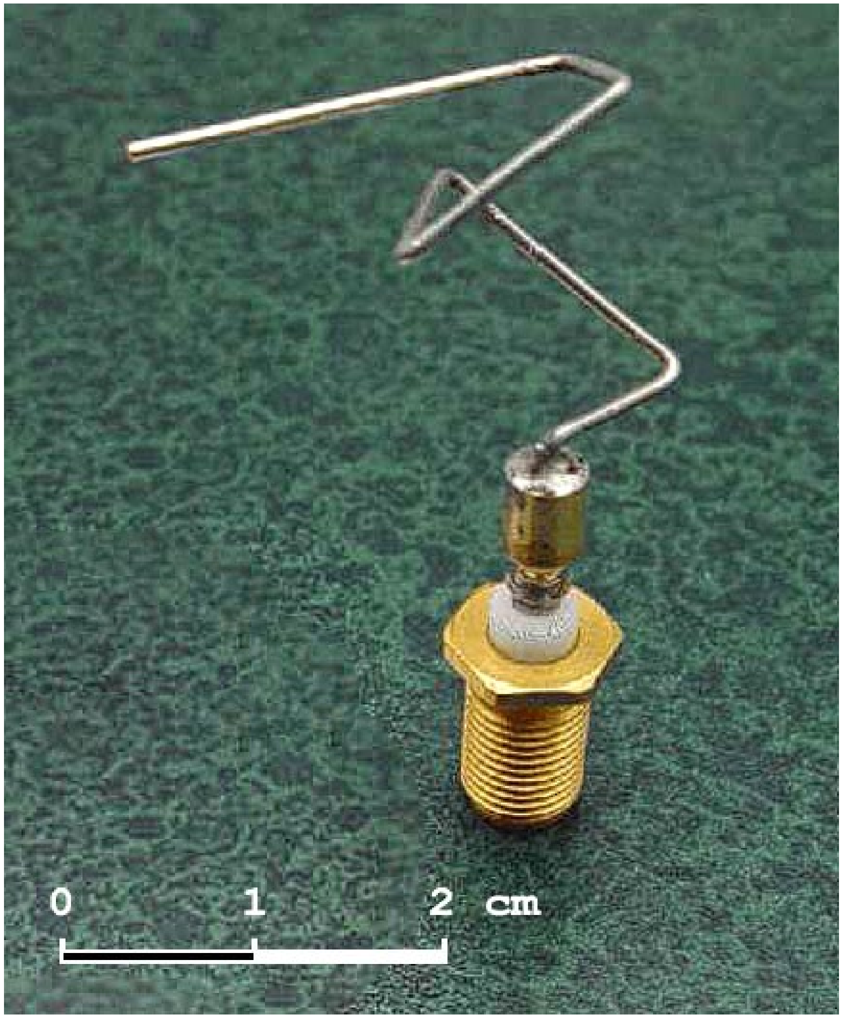


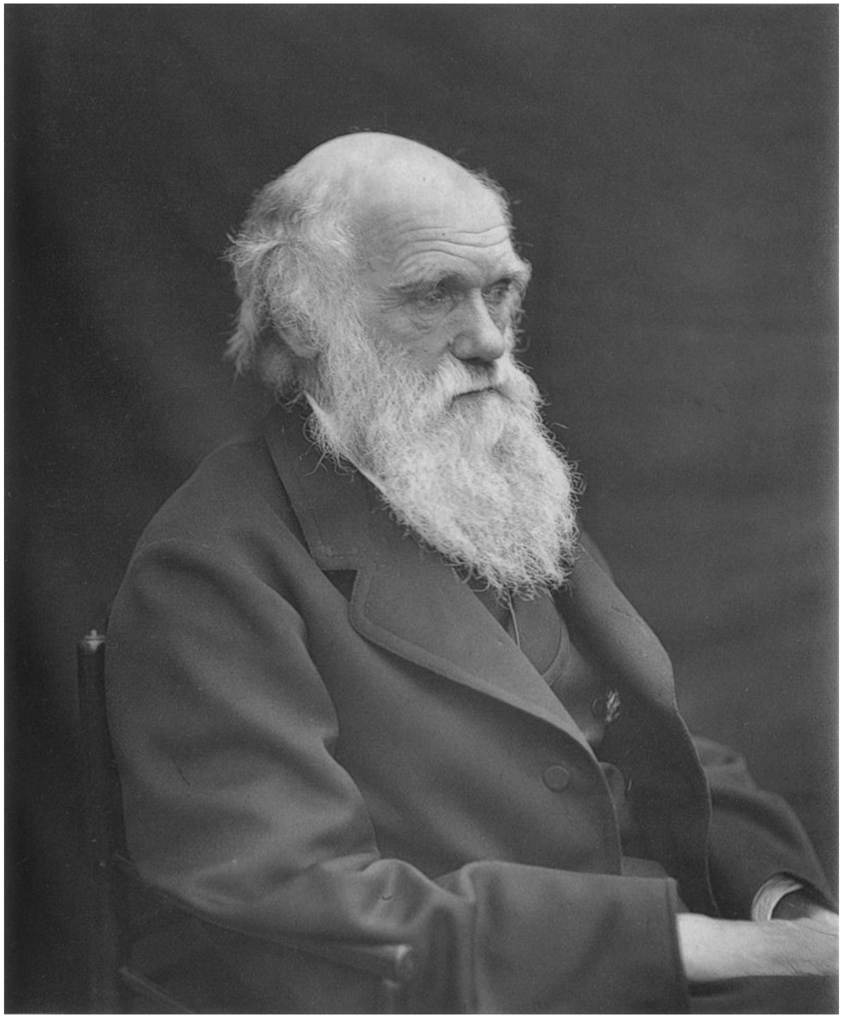
MORE Bonus Material
Chapter 4 - related
(NOT ON EXAMS!)
Search in Complex Environments

Complex Environments



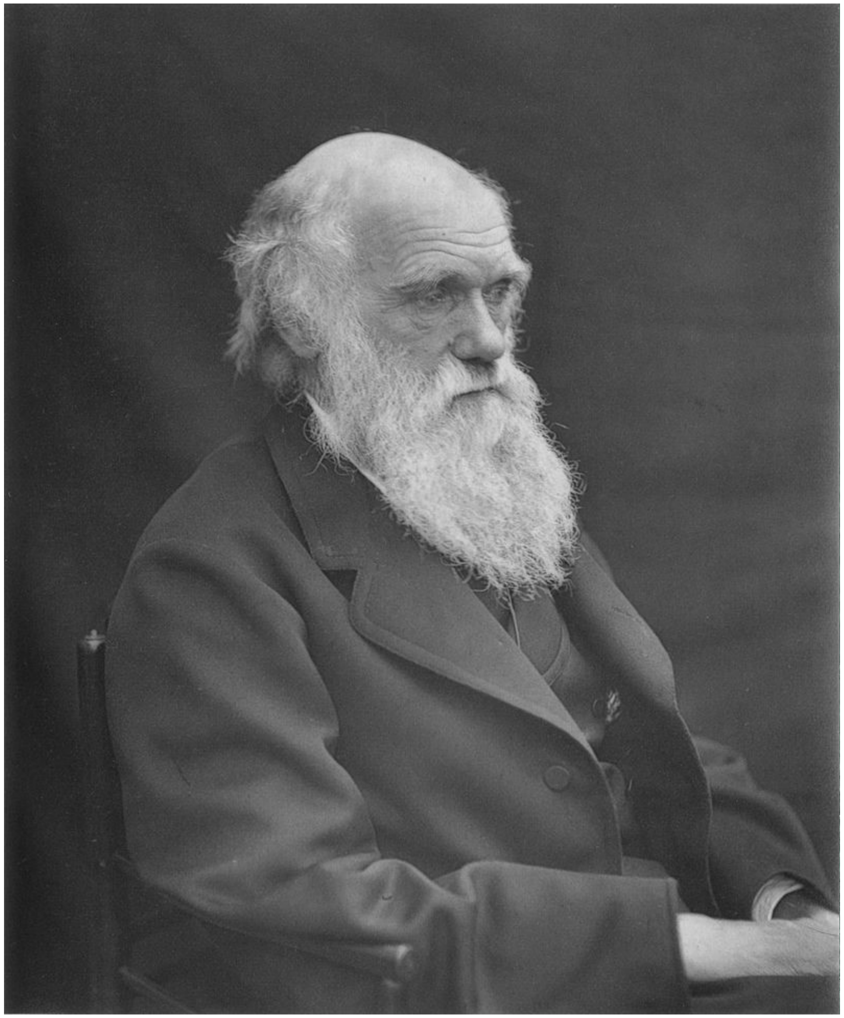
What's the Connection Here?





Source: https://wikipedia.org/

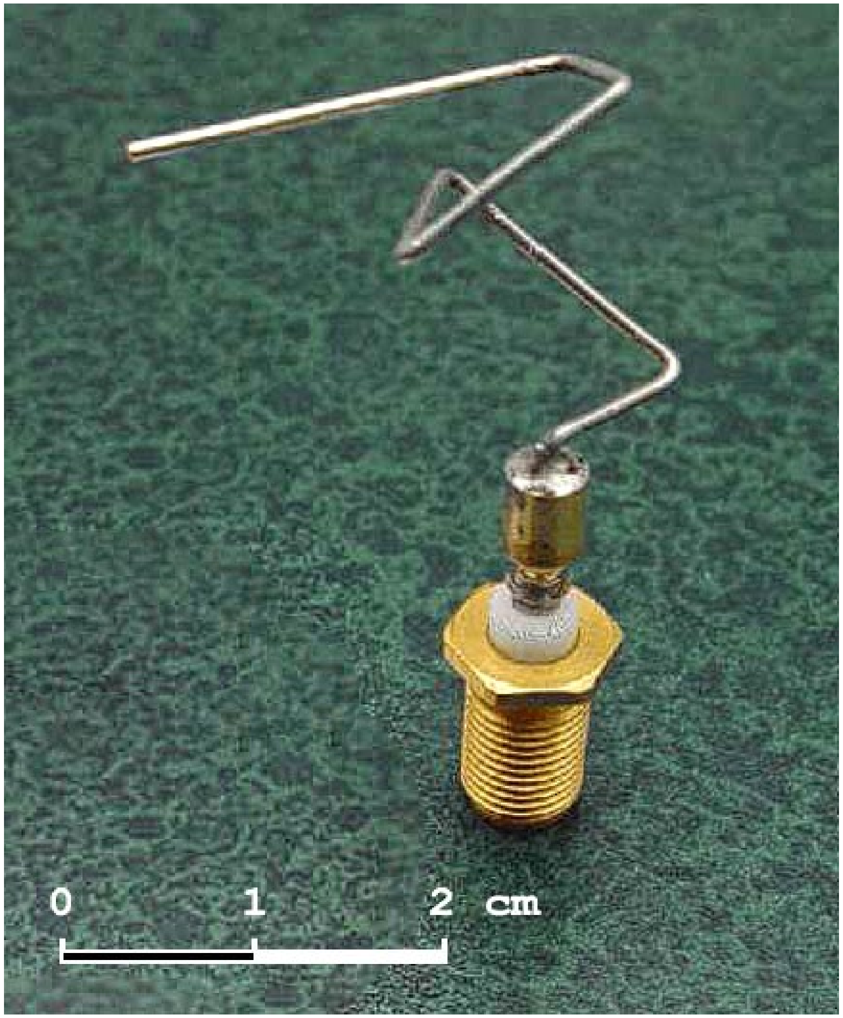
Charles Darwin



Source: https://wikipedia.org/

Charles Robert Darwin was an English naturalist, geologist and biologist, best known for his contributions to the science of evolution. His proposition that all species of life have descended over time from common ancestors is now widely accepted, and considered a foundational concept in science.

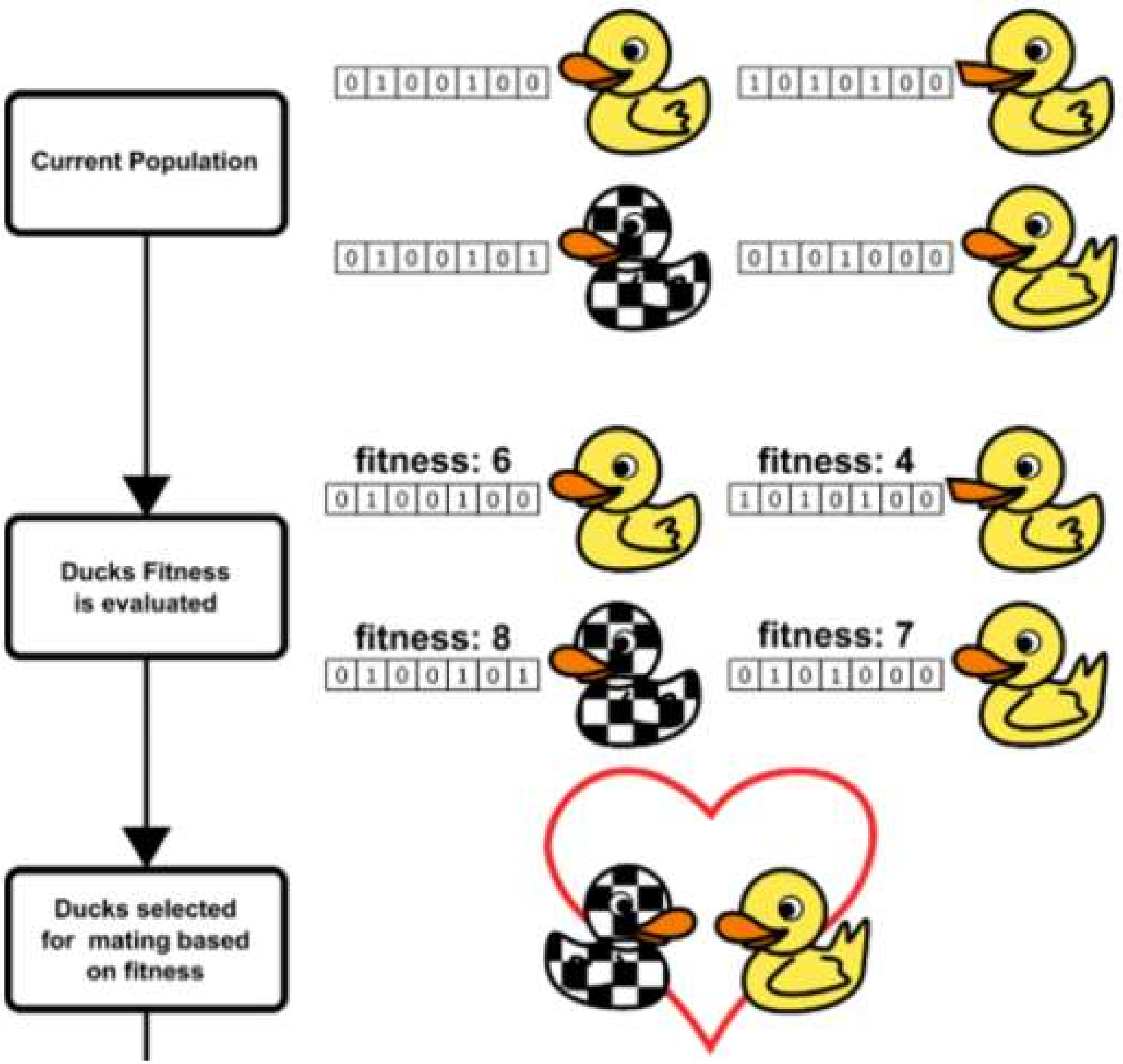
Evolved Antenna



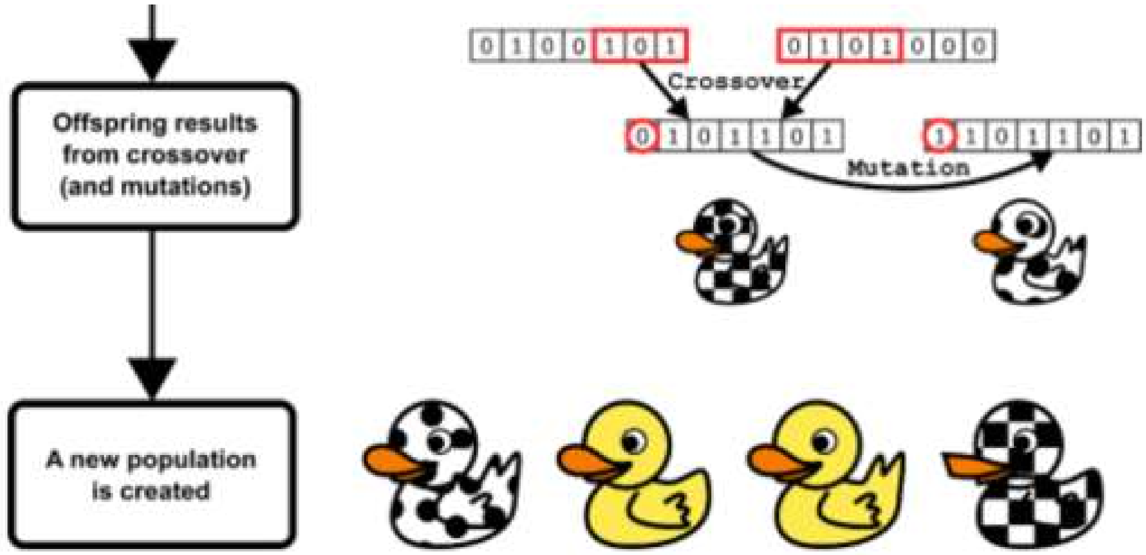
Source: https://wikipedia.org/

An evolved antenna is an antenna designed fully or substantially by an automatic computer design program that uses an evolutionary algorithm that mimics Darwinian evolution.

Genetic Algorithm: The Idea



Genetic Algorithm: The Idea



Source: https://livebook.manning.com/book/algorithms-and-data-structures-in-action/chapter-18/v-14/102

Genetic Algorithm: Example

Population of points (solutions)

x y

0	1	0	1	0	1

x y

0	1	1	1	0	1

x y

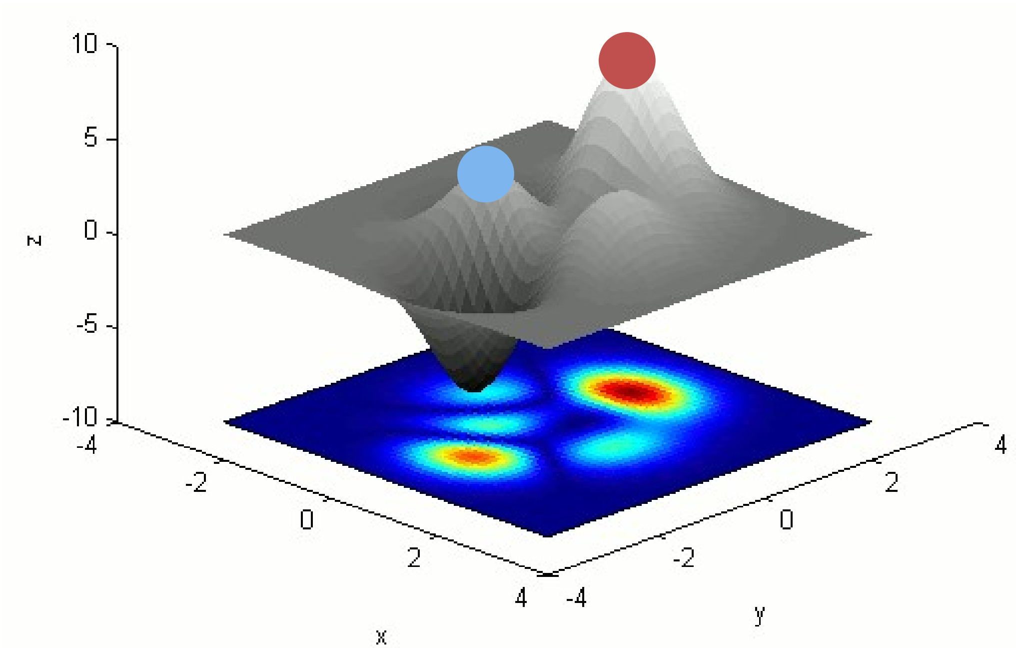
0	0	0	1	0	1

x y

0	1	0	1	0	0

x y

0	0	1	0	0	0

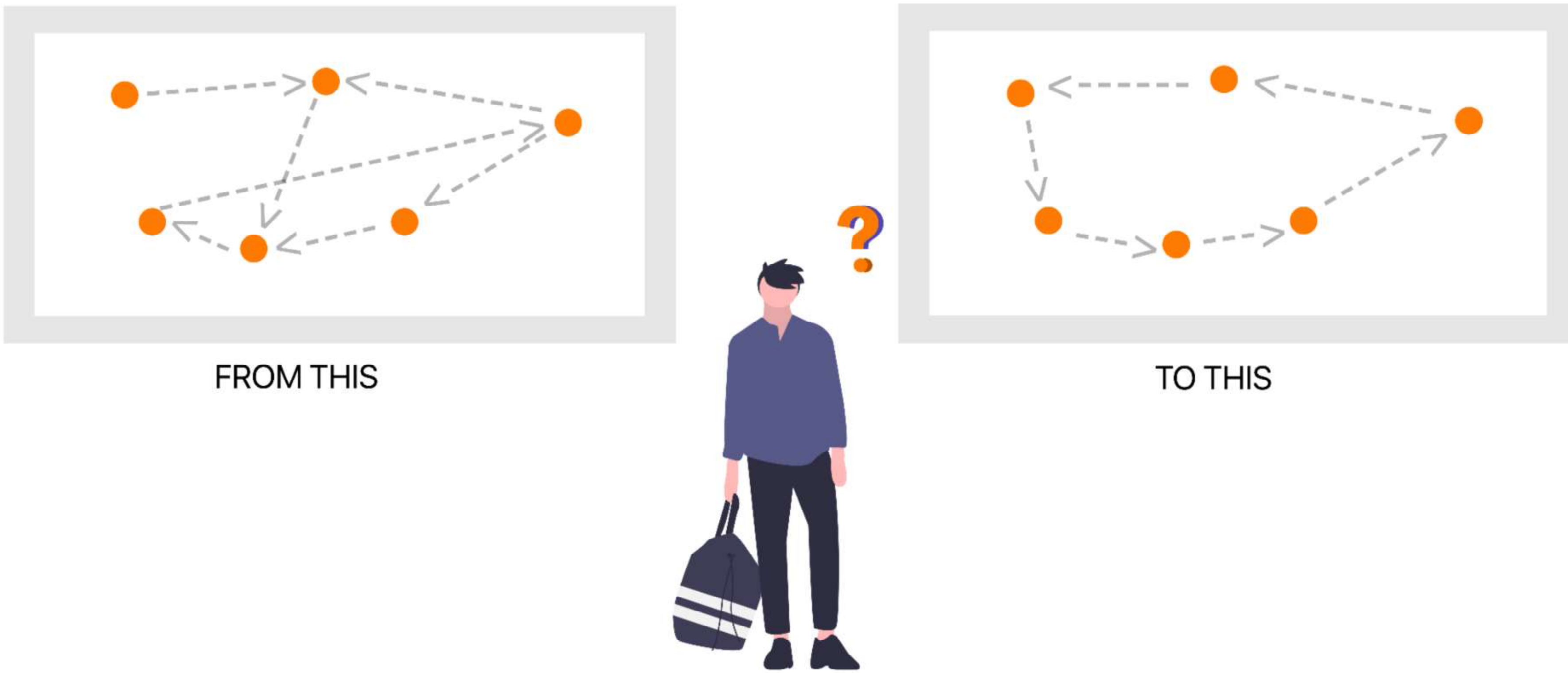


y = f(x,y) - fitness function

“Good enough” / local maximum

Best / global maximum

Traveling Salesman Problem



A traveler needs to visit all the cities from a list, where distances between all the cities are known and each city should be visited just once. What is the shortest possible route that he visits each city exactly once and returns to the origin city?

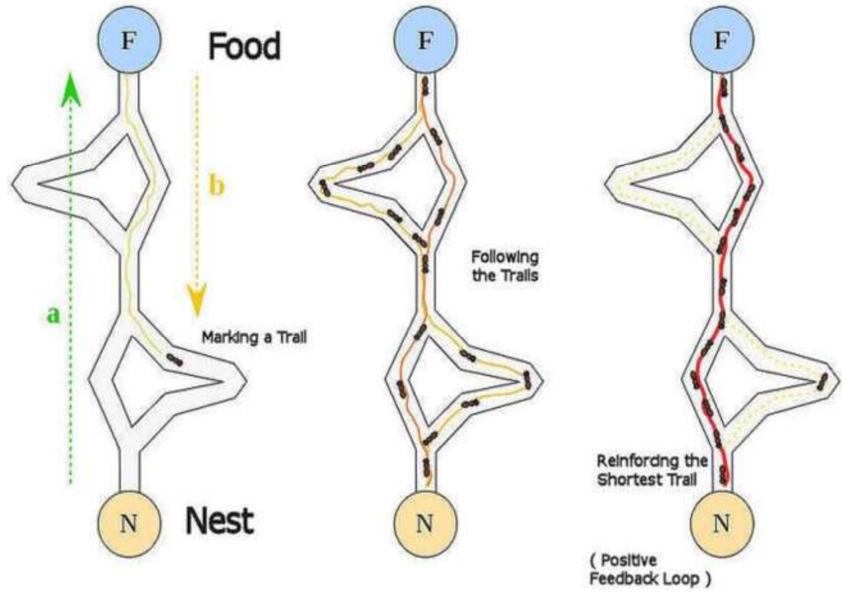
N cities → (N-1)!/2 paths | 15 cities → 43589145600 paths

Source: https://medium.com/ivymobility-developers/traveling-salesman-problem-9ab623c88fab

Example: Genetic Algorithm

http://ostap0207.github.io/web-ga-tsp/

Ant Colony Optimization: The Idea

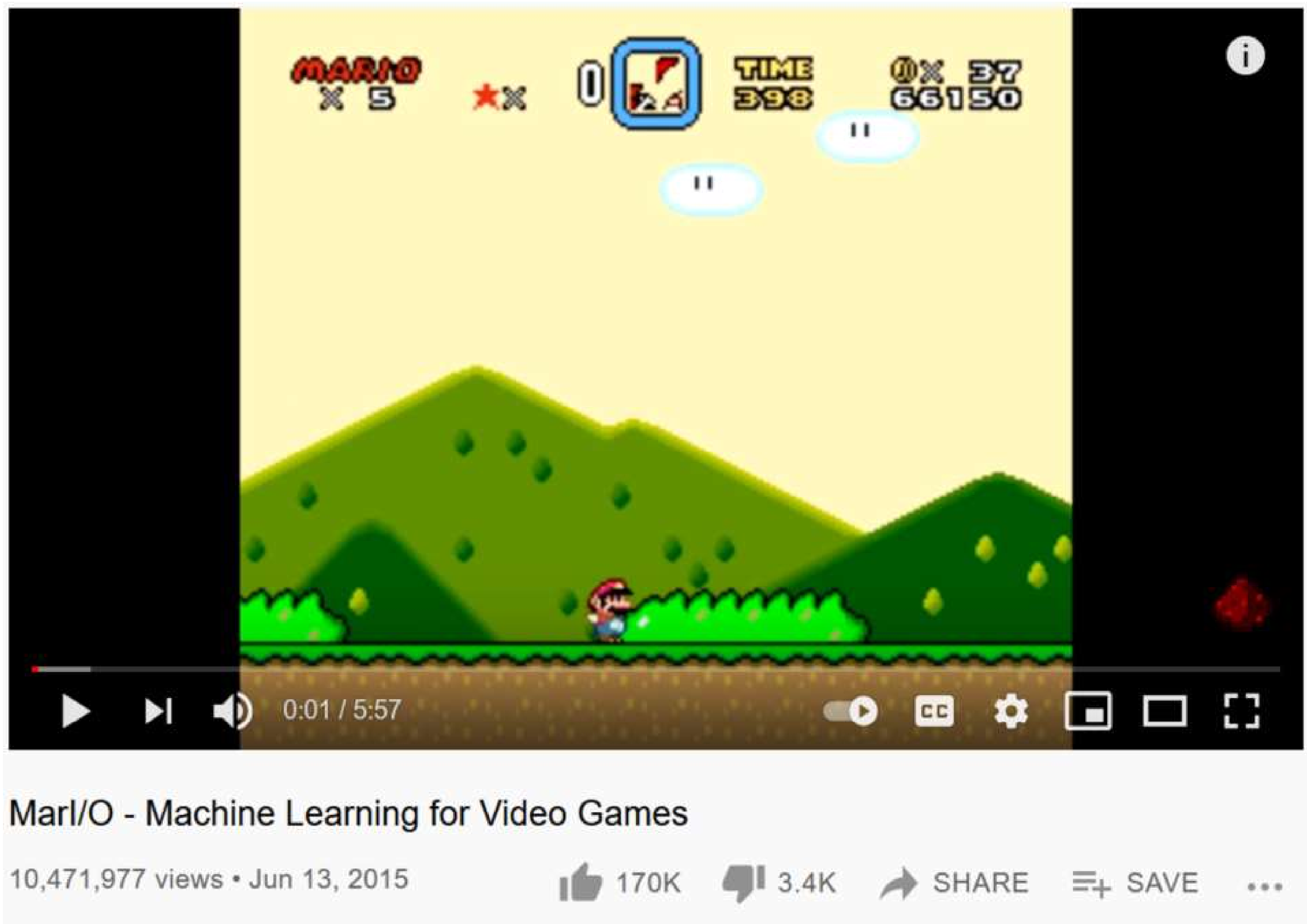


Source: https://wikipedia.org/

Example: Ant Colony Optimization

https://courses.cs.ut.ee/demos/visual-aco/

Genetic Algorithm in Action



Source: https://www.youtube.com/watch?v=qv6UVOQ0F44

Bonus DEFINITELY OPTIONAL Material