

CS 480

Introduction to Artificial Intelligence

November 10, 2022

Announcements / Reminders

- Follow Week 12 TO DO List
- Written Assignment #03 due on ~~Sunday (11/06/22)~~ **Thursday (11/10)** at 11:00 PM CST
- Programming Assignment #02 due on **Sunday (11/20/22)** at 11:00 PM CST
- Quiz #03 posted due on **Sunday (11/13/22)** at 11:00 PM CST
- Grading TA assignment:
https://docs.google.com/spreadsheets/d/1ExS0bKnGt_fdf4LHa3YS1qRA7-lq4xqXVjfSAPMaGVk/edit?usp=sharing
- UPDATED Final Exam date:
 - **December 1st, 2022 (last week of classes!)**
 - **Ignore the date provided by the Registrar**

Plan for Today

- **Decision Networks**

Decision Theory

- **Decisions**: every plan (**actions**) leads to an outcome (state)
- Agents have preferences (**preferred outcomes**)
- Preferences → outcome **utilities**
- Agents have **degrees of belief** (**probabilities**) for actions

Decision theory = **probability theory** + **utility theory**

Decision Theory

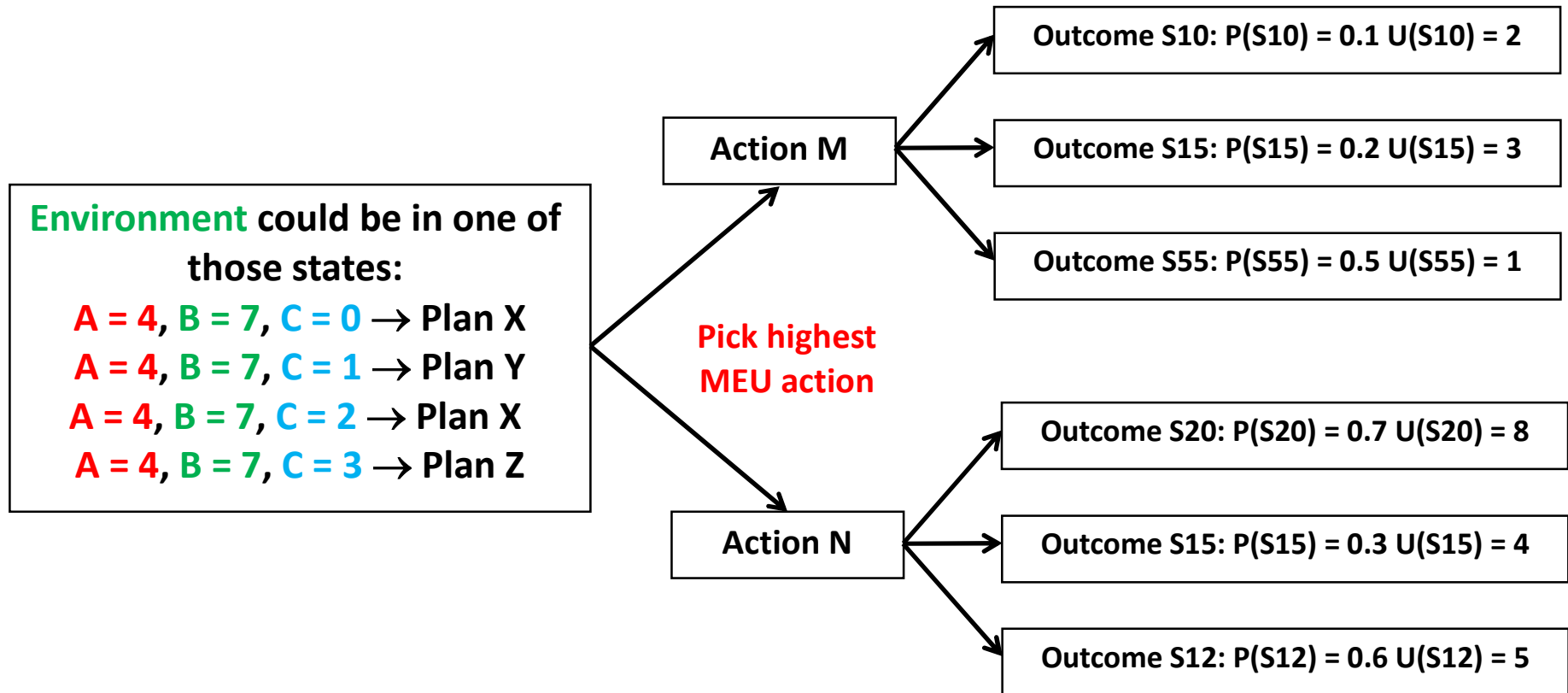
- **Decisions**: every plan (**actions**) leads to an outcome (state)
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Decision theory = **probability theory** + **utility theory**

BELIEFS **DESIRES**

Maximum Expected (Average) Utility

$$MEU(M) = P(S10) * U(S10) + P(S15) * U(S15) + P(S55) * U(S55)$$



$$MEU(N) = P(S20) * U(S20) + P(S15) * U(S15) + P(S12) * U(S12)$$

Agents Decisions

Recall that agent **ACTIONS** change the state:

- if we are in state **s**
- action **a** is expected to
- lead to another state **s'** (outcome)

Given uncertainty about the current state **s** and action outcome **s'** we need to define the following:

- probability (belief) of being in state **s**: $P(\mathbf{s})$
- probability (belief) of action **a** leading to outcome **s'**: $P(\mathbf{s}' \mid \mathbf{s}, \mathbf{a})$

Now:

$$P(\mathbf{s}' \mid \mathbf{s}, \mathbf{a}) = P(\text{RESULT}(\mathbf{a}) = \mathbf{s}') = \sum_{\mathbf{s}} P(\mathbf{s}) * P(\mathbf{s}' \mid \mathbf{s}, \mathbf{a})$$

State Utility Function

Agent's **preferences (desires)** are captured by the **Utility function** $U(s)$.

Utility function assigns a value to each state s to express how desirable this state is to the agent.

Expected Action Utility

The **expected utility** of an action **a** given the evidence is the **average utility value** of all **possible outcomes s'** of action **a**, **weighted by their probability (belief) of occurrence**:

$$EU(a) = \sum_{s'} \sum_s P(s) * P(s' | s, a) * U(s') = \sum_{s'} P(Result(a) = s') * U(s')$$

Rational agent should choose an action that **maximizes the expected utility**:

$$\text{chosen action} = \underset{a}{\operatorname{argmax}} EU(a)$$

How Did We Get Here?

Let's start with relationships (and related notation) between agent's preferences:

- agent **prefers** A over B:

$$A \sqsupset B$$

- agent is **indifferent** between A and B:

$$A \sim B$$

- agent prefers A over B or is indifferent between A and B (**weak preference**):

$$A \sqsupseteq B$$

The Concept of Lottery

Let's assume the following:

- an **action** a is a lottery ticket
- the **set of outcomes (resulting states)** is a lottery

A lottery L with possible outcomes S_1, \dots, S_n that occur with probabilities p_1, \dots, p_n is written as:

$$L = [p_1, S_1; p_2, S_2; \dots ; p_n, S_n]$$

Lottery outcome S_i : atomic state or another lottery.

Lottery Constraints: Orderability

Given two lotteries A and B , a rational agent must either prefer one or else rate them as equally preferable:

Exactly one of $(A \sqsubset B)$, $(B \sqsubset A)$, or $(A \sim B)$ holds

Lottery Constraints: Transitivity

Given three lotteries A, B, and C, if an agent prefers A to B AND prefers B to C, then the agent must prefer A to C:

$$(A \square B) \wedge (B \square C) \Rightarrow (A \square C)$$

Lottery Constraints: Continuity

If some lottery B is between A and C in preference, then there is some probability p for which the rational agent will be indifferent between getting B for sure or some other lottery that yields A with probability p and C with probability $1 - p$:

$$(A \square B \square C) \Rightarrow \exists p [p, A; 1-p, C] \sim B$$

Lottery Constraints: Substitutability

If an agent is indifferent between two lotteries A and B, then the agent is indifferent between two more complex lotteries that are the same, except that B is substituted for A in one of them:

$$(A \sim B) \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C]$$

Lottery Constraints: Monotonicity

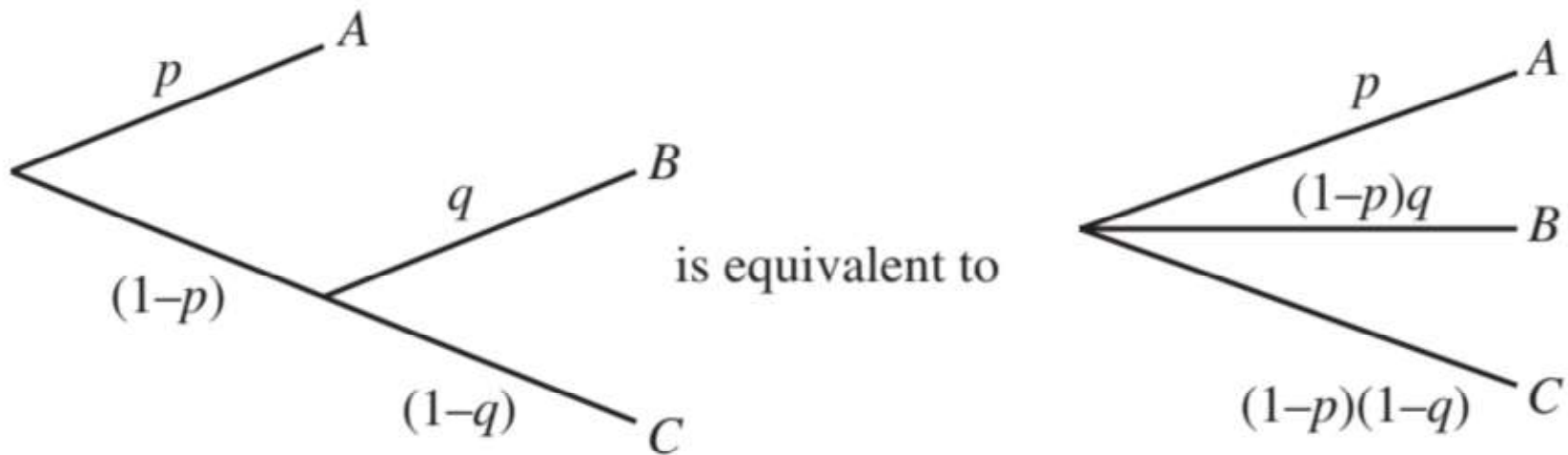
Suppose two lotteries have the same two possible outcomes, A and B. If an agent prefers A to B, then the agent must prefer the lottery that has a higher probability for A:

$$(A \succ B) \Rightarrow (p > q \Leftrightarrow [p, A; 1-p, B] \succ [q, A; 1-q, B])$$

Lottery Constraints: Decomposability

Compound lotteries can be reduced to smaller ones using the laws of probability:

$$[p, A; 1-p, [q, B; 1-q, C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C]$$



Preferences and Utility Function

An agent whose preferences between lotteries follow the set of axioms (**of utility theory**) below:

- Orderability
- Transitivity
- Continuity
- Substitutability
- Monotonicity
- Decomposability

can be described as possessing a utility function and maximize it.

Preferences and Utility Function

If an agent's preferences obey the axioms of utility theory, then there exist a function U such that:

$$U(A) = U(B) \text{ if and only if } (A \sim B)$$

and

$$U(A) > U(B) \text{ if and only if } (A \succ B)$$

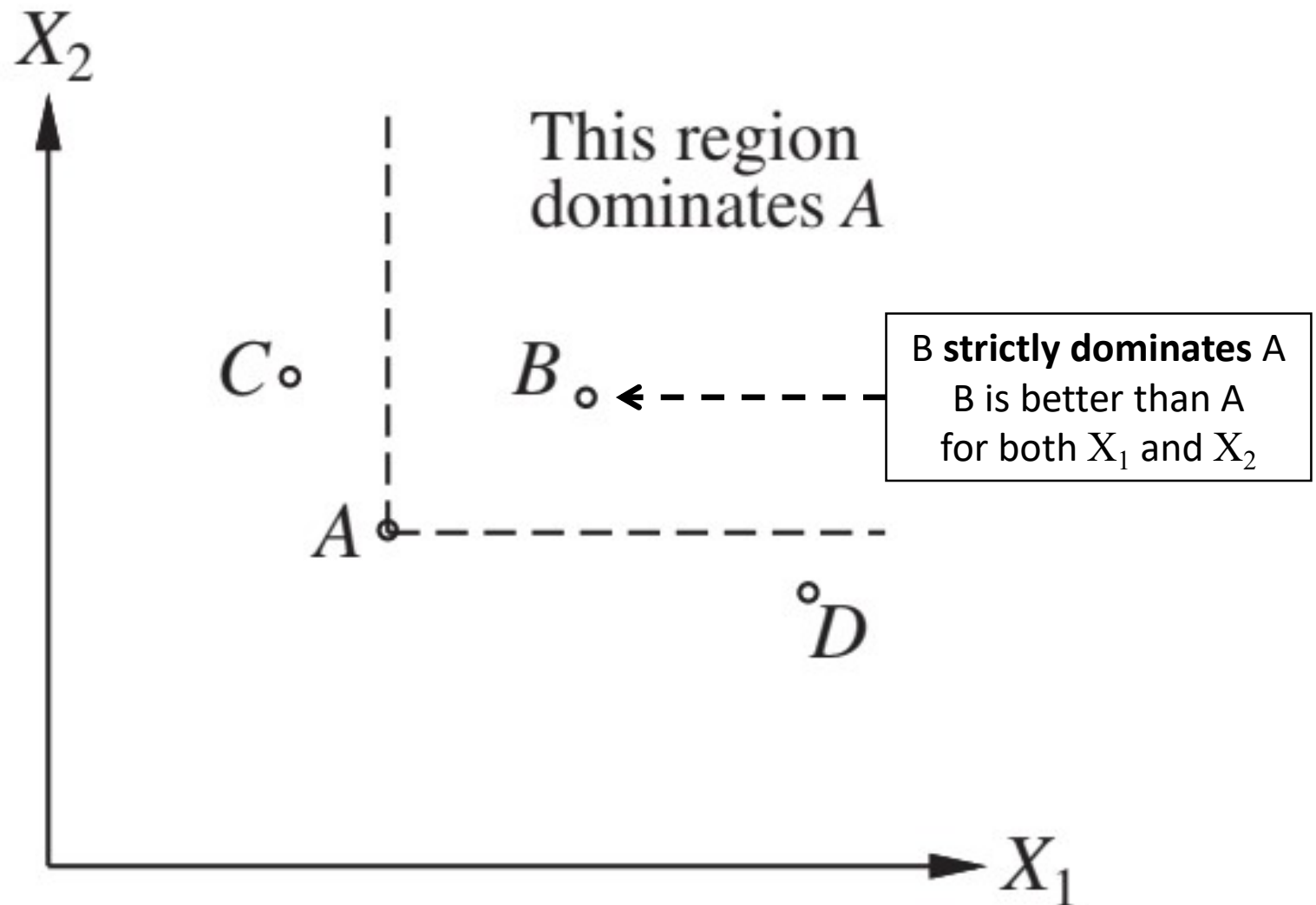
Multiattribute Outcomes

Outcomes can be characterized by more than one attribute. Decisions in such cases are handled by Multiattribute Utility Theory.

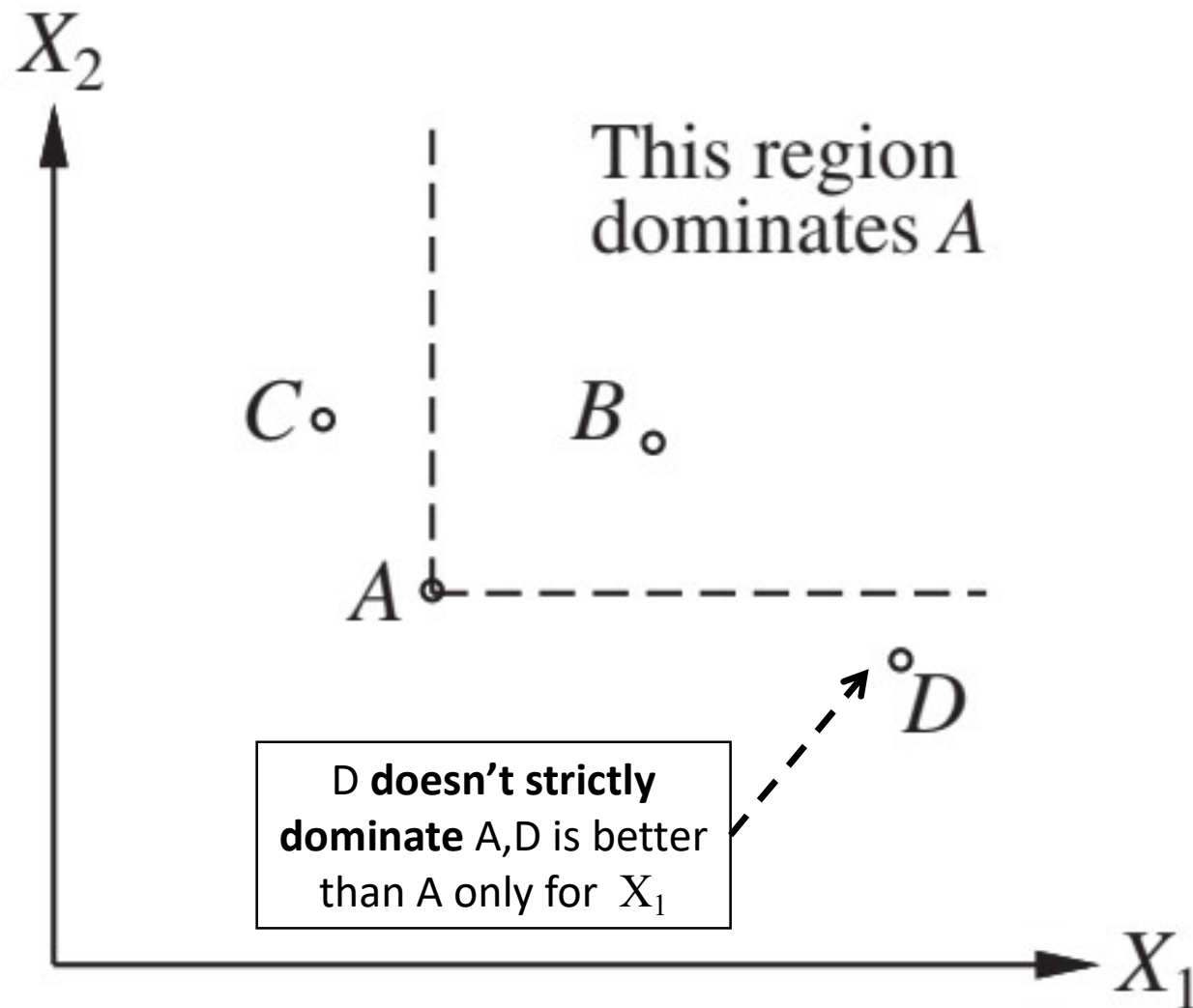
Attributes: $X = X_1, \dots, X_n$

Assigned values: $x = \langle x_1, \dots, x_n \rangle$

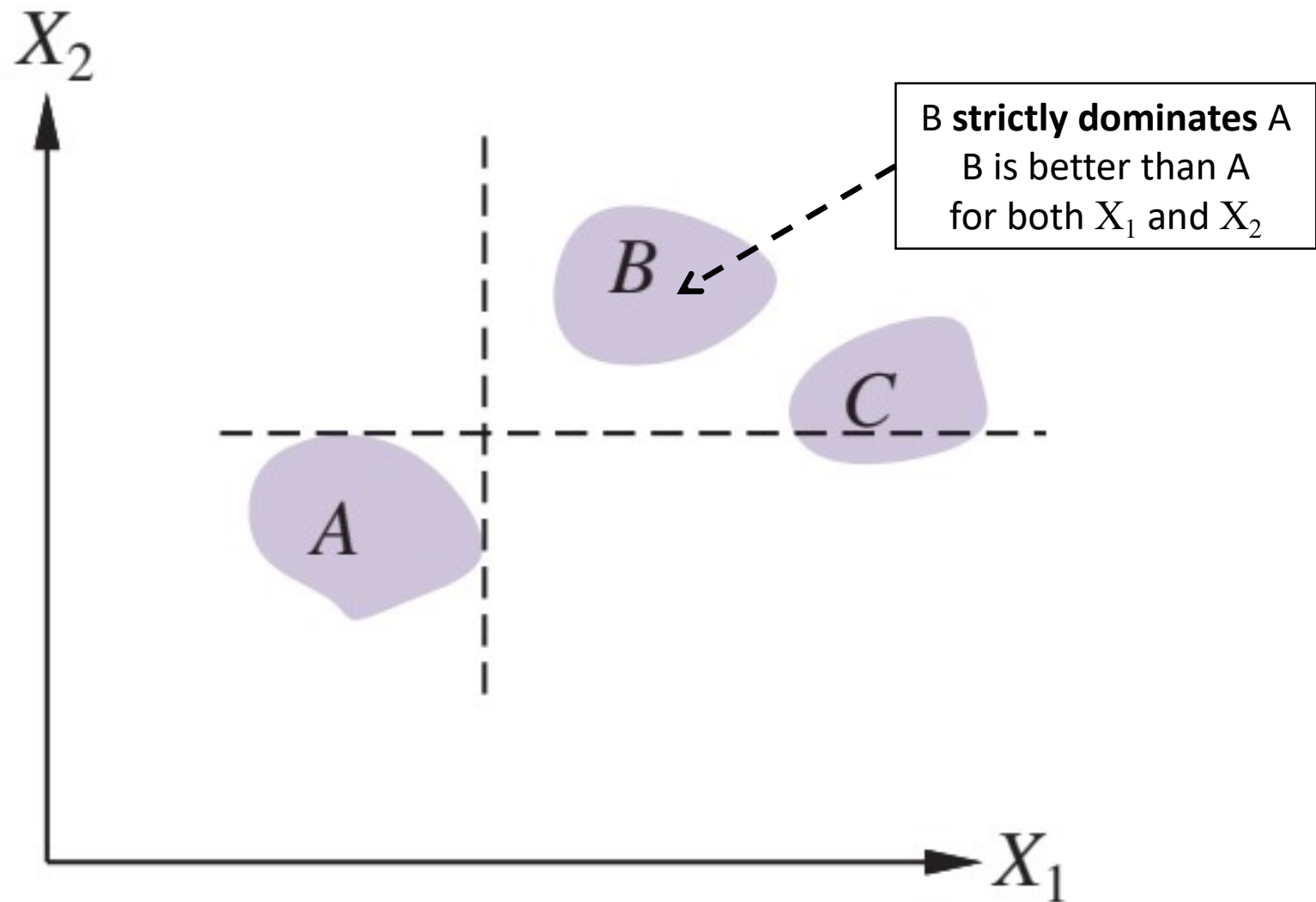
Strict Dominance: Deterministic



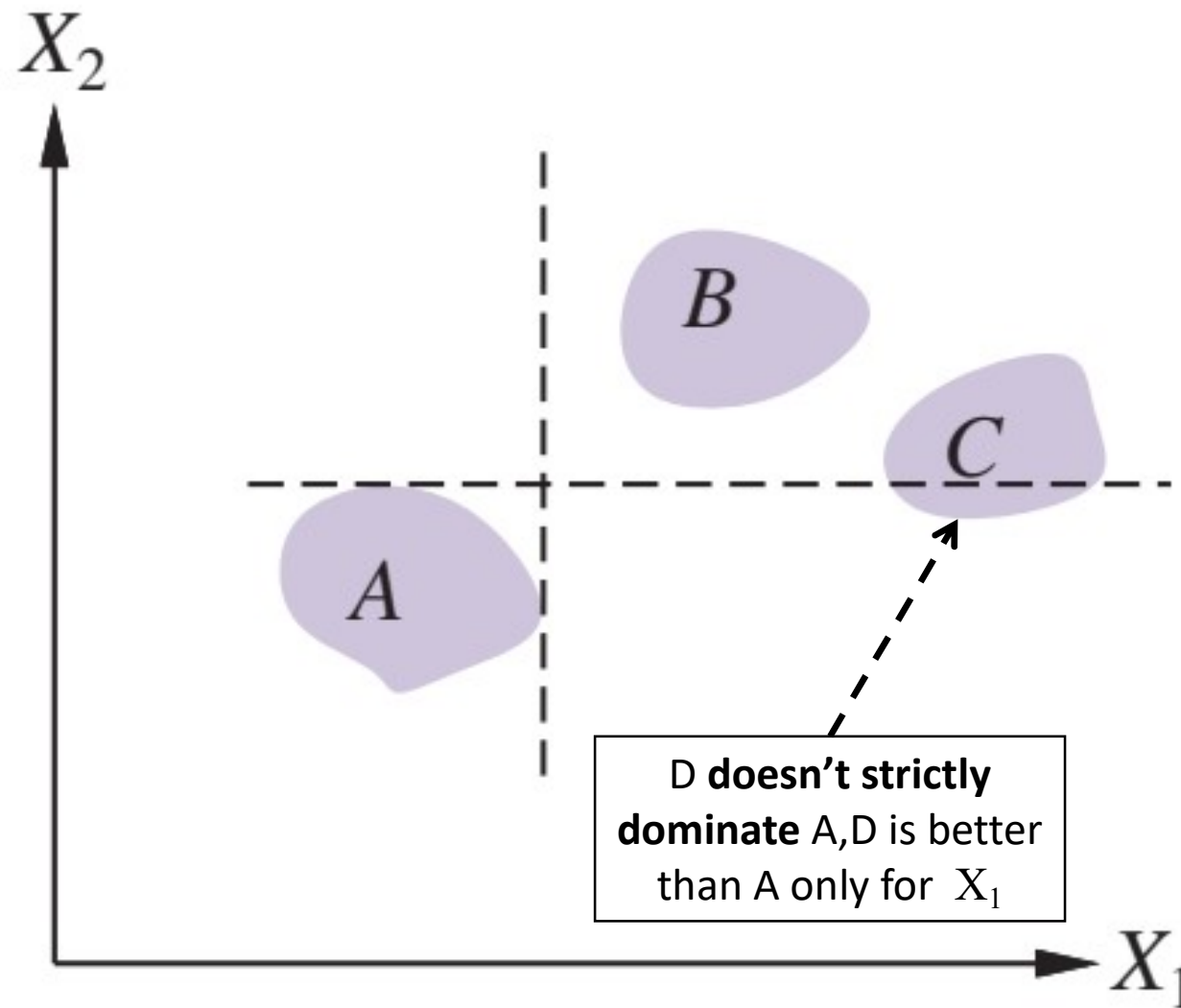
Strict Dominance: Deterministic



Strict Dominance: Uncertain



Strict Dominance: Uncertain



Decision Network (Influence Diagram)

Decision networks (also called influence diagrams) are structures / mechanisms for making rational decisions.

Decision networks are based on Bayesian networks, but include additional nodes that represent **actions** and **utilities**.

Decision Networks

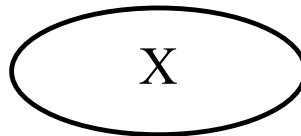
The most basic decision network needs to include:

- information about current state s
- possible actions
- resulting state s' (after applying chosen action a)
- utility of the resulting state $U(s')$

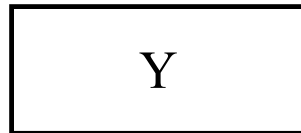
Decision Network Nodes

Decision networks are built using the following nodes:

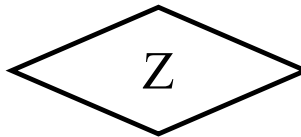
- chance nodes:



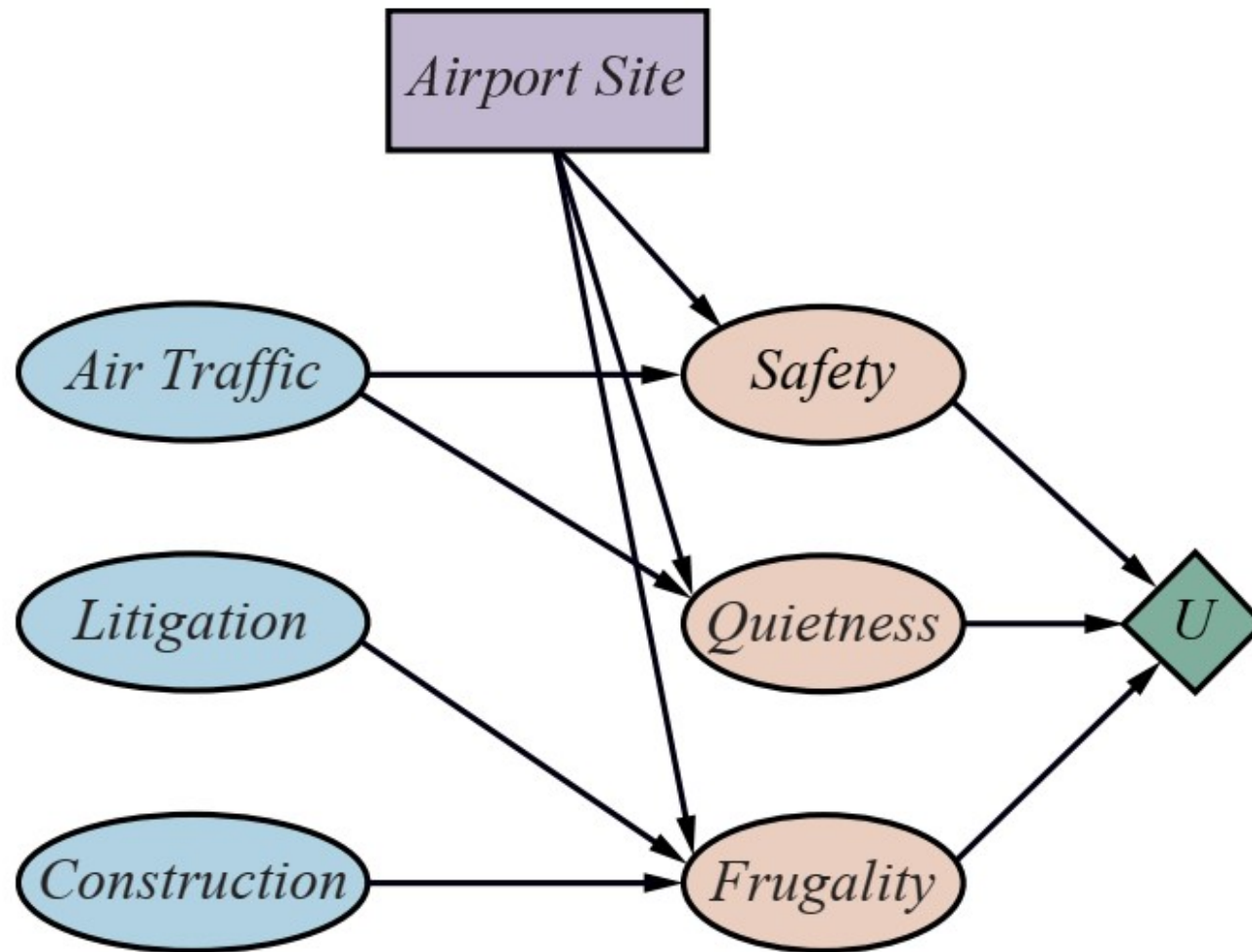
- decision nodes:



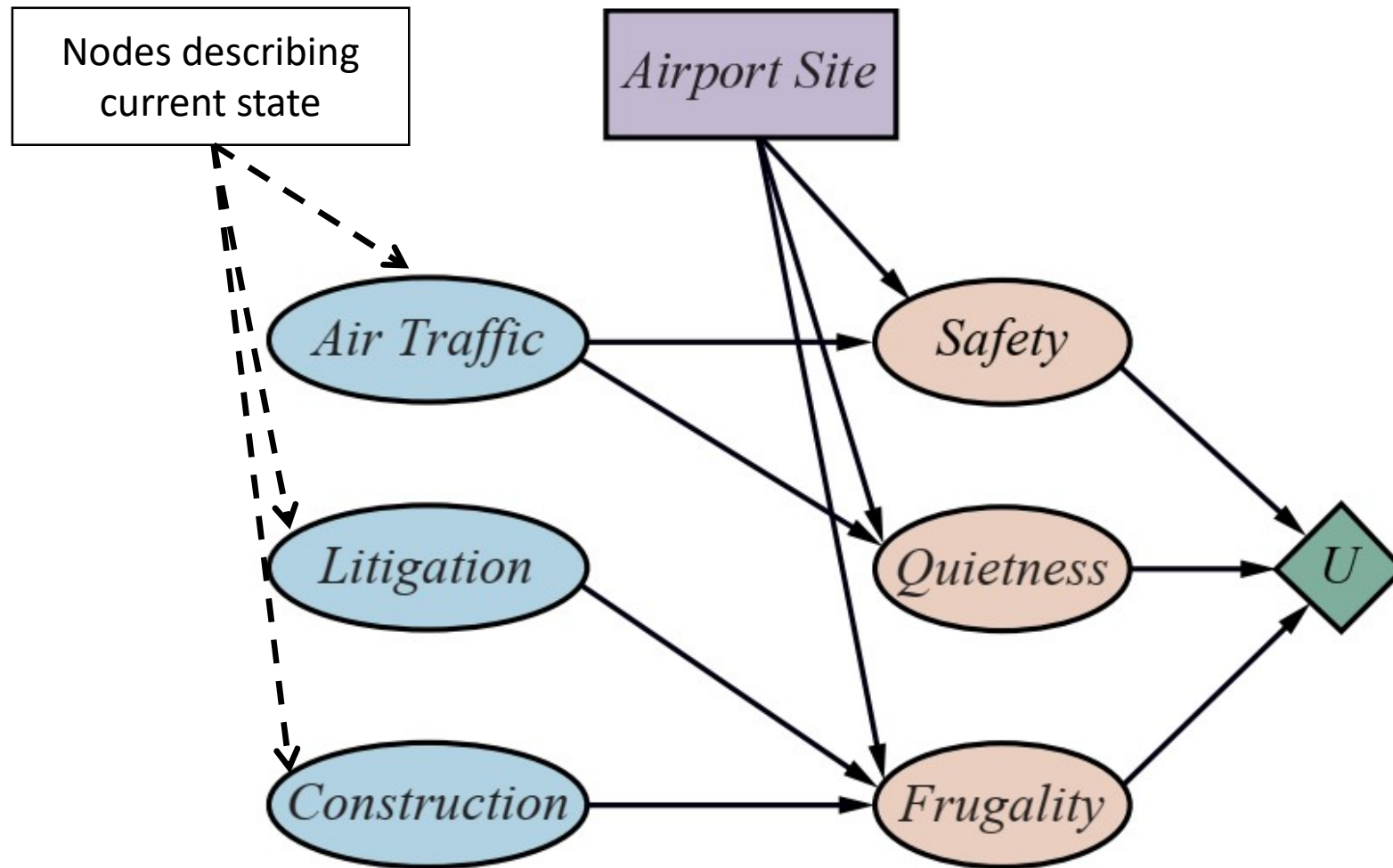
- utility (or value) nodes



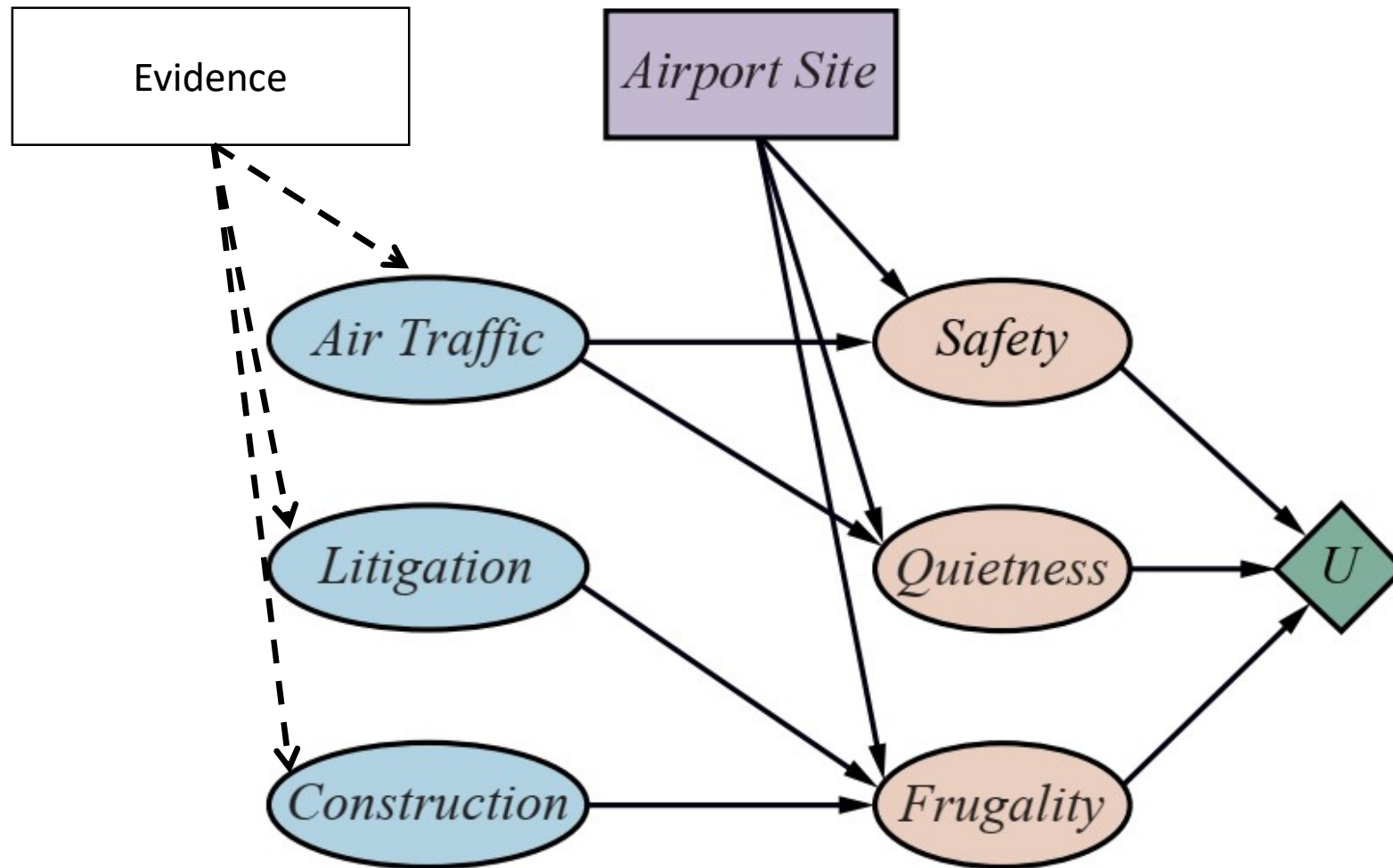
Decision Network: Example



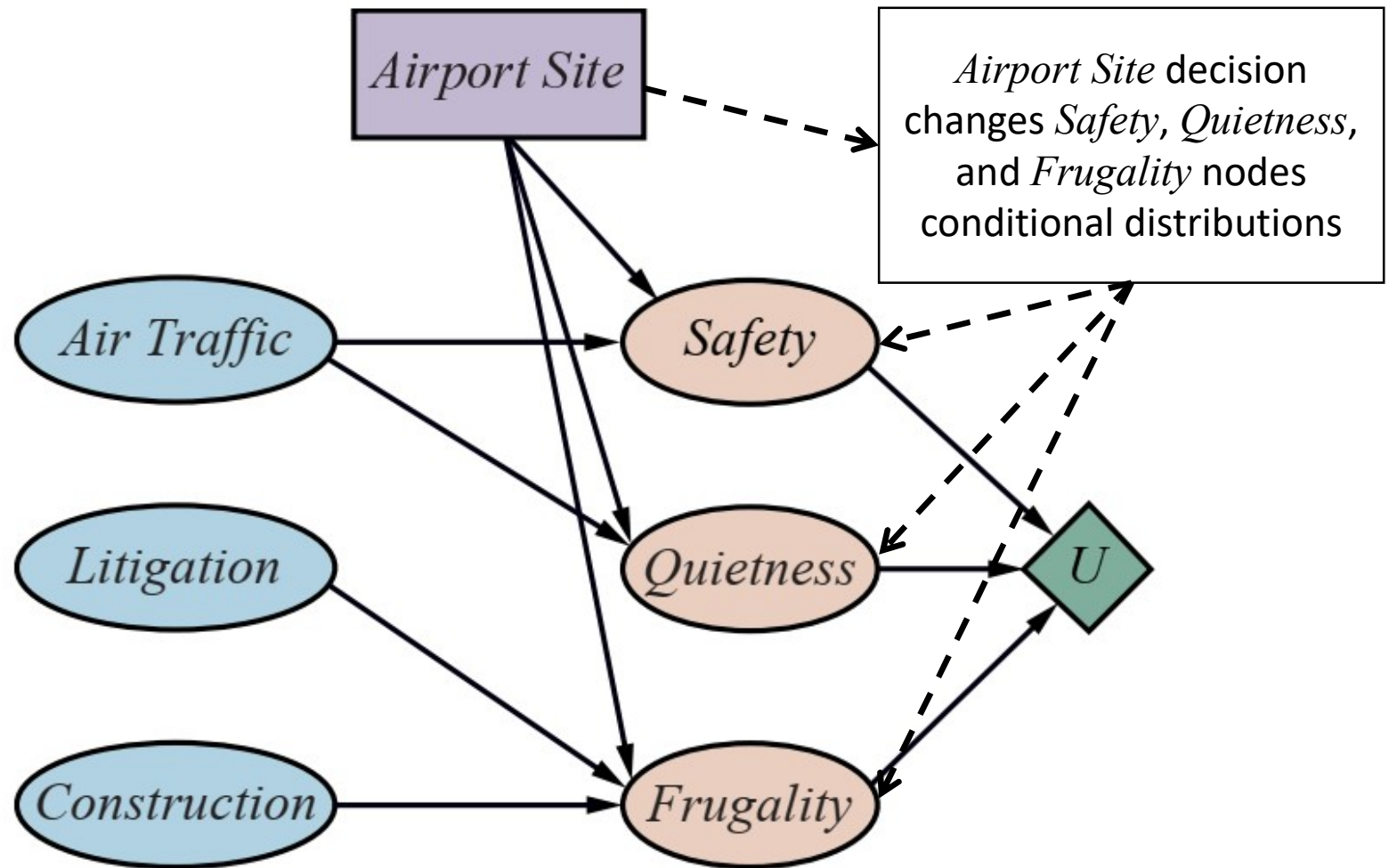
Decision Network: Example



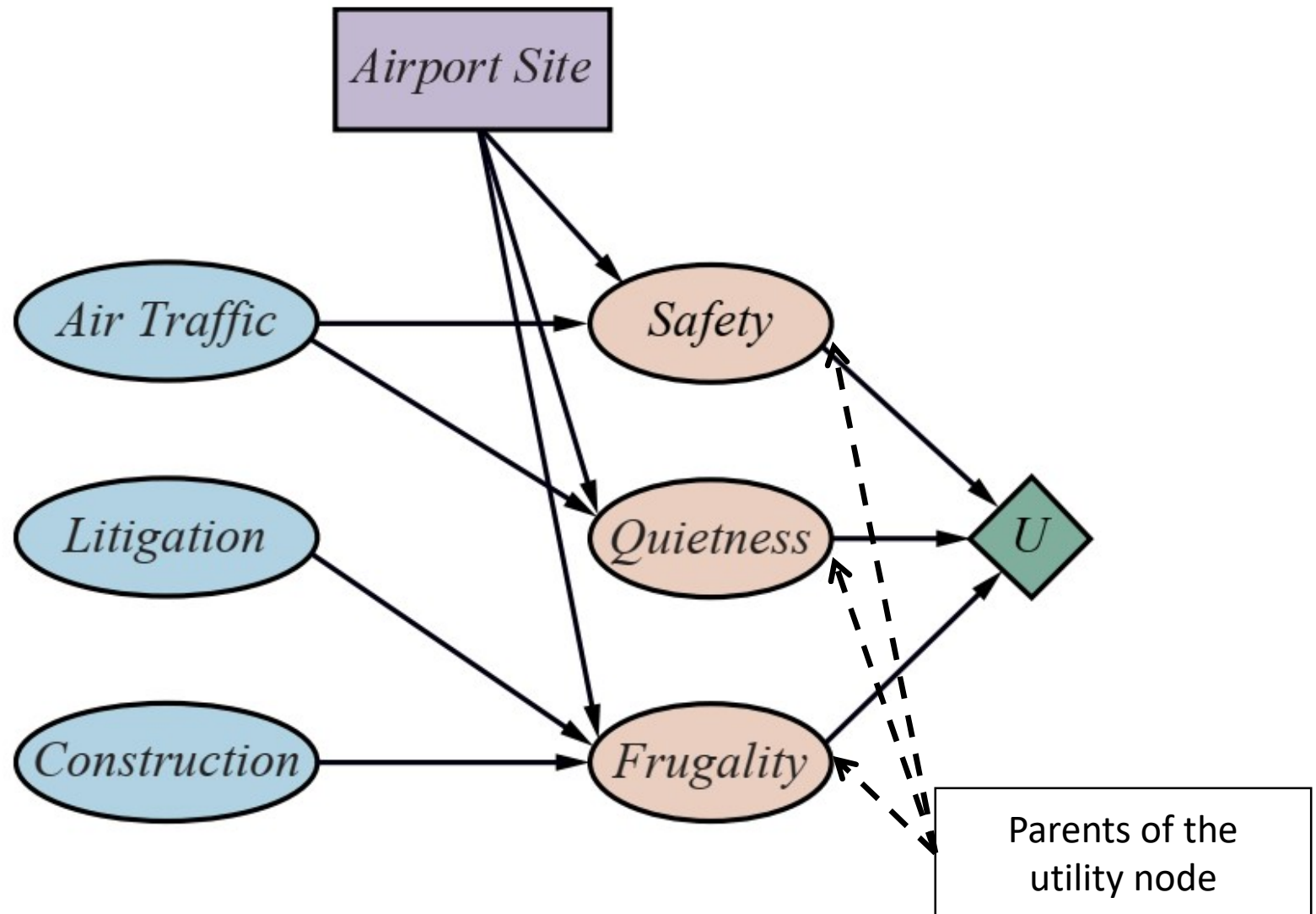
Decision Network: Example



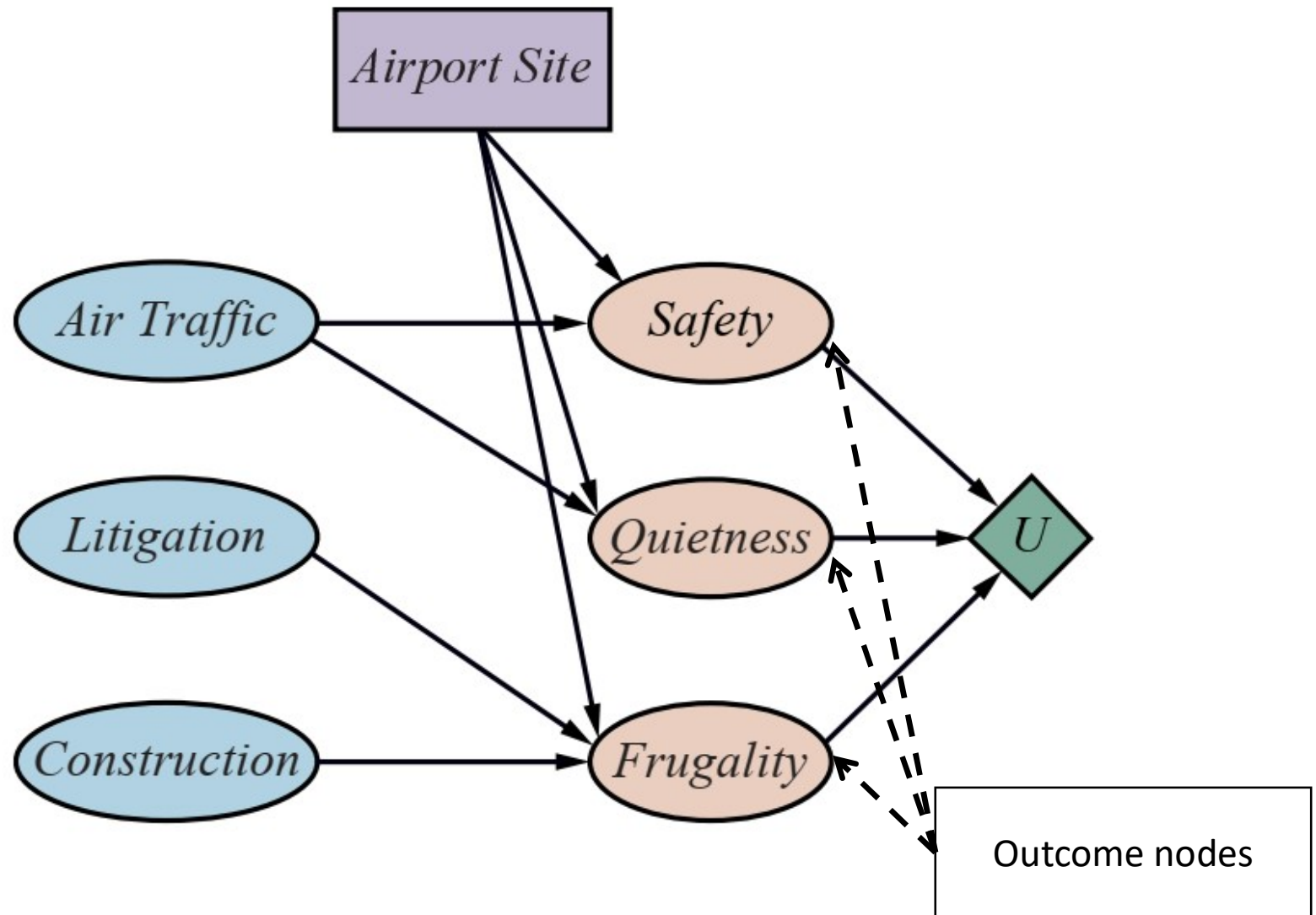
Decision Network: Example



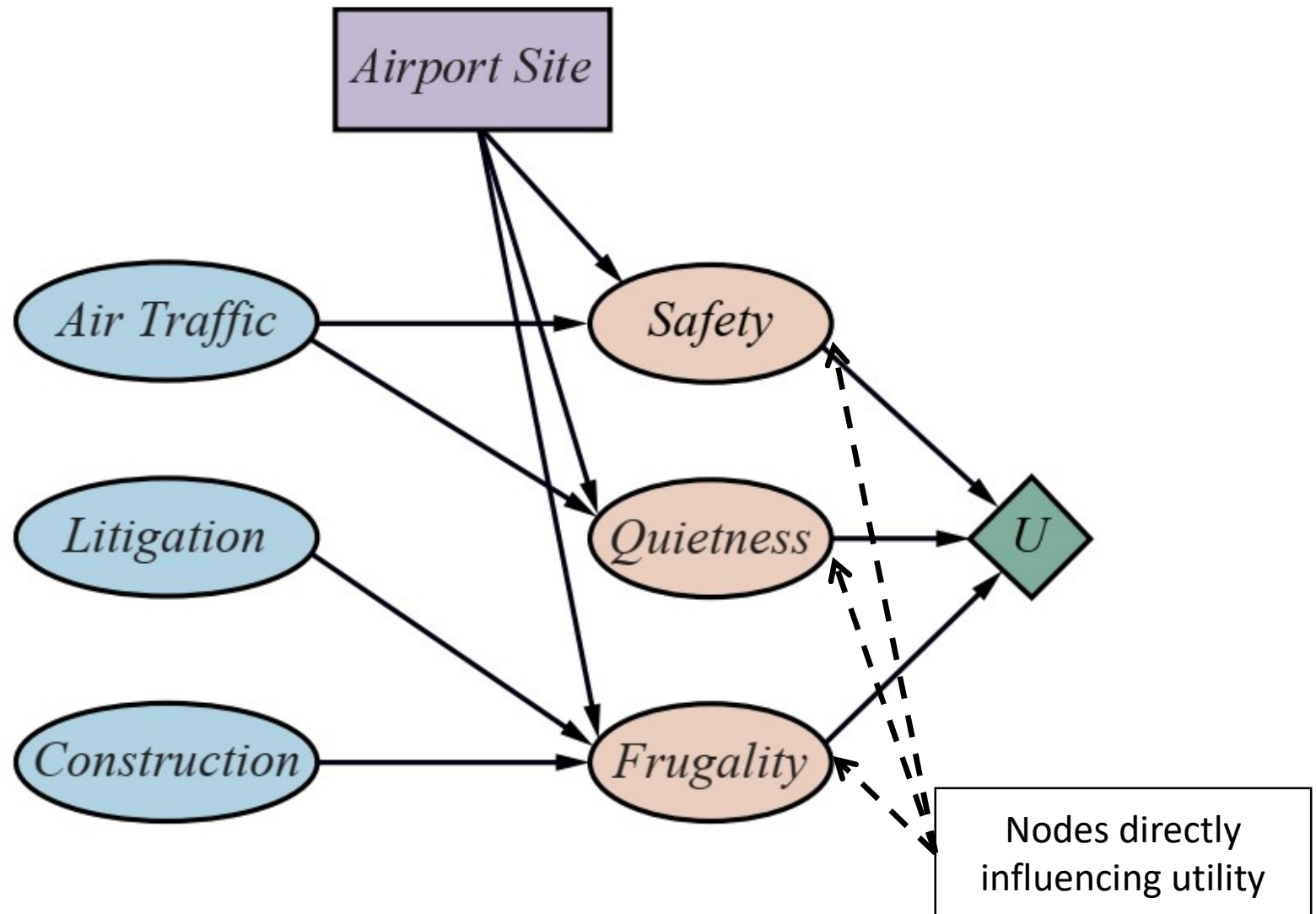
Decision Network: Example



Decision Network: Example



Decision Network: Example

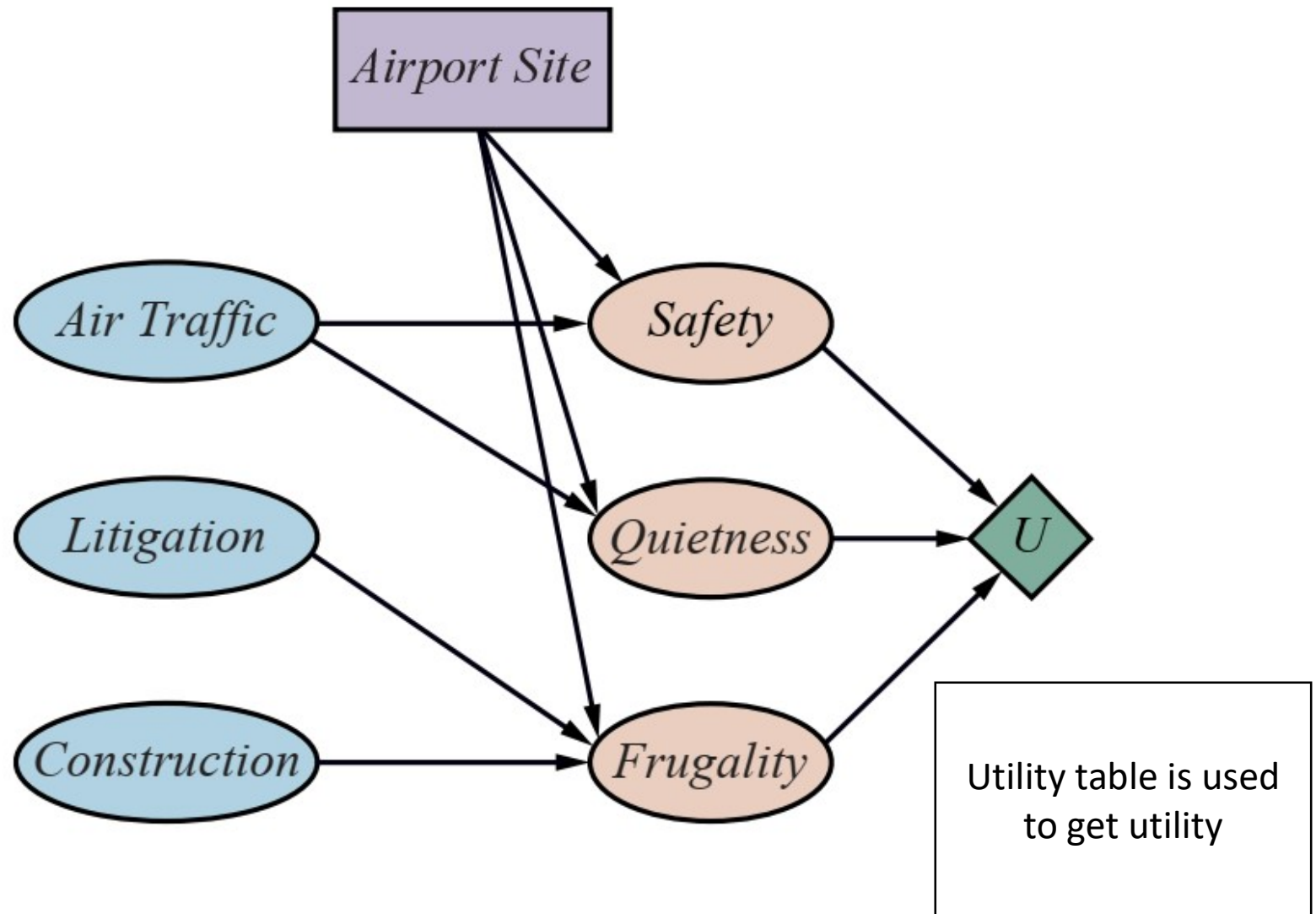


Decision Network: Evaluation

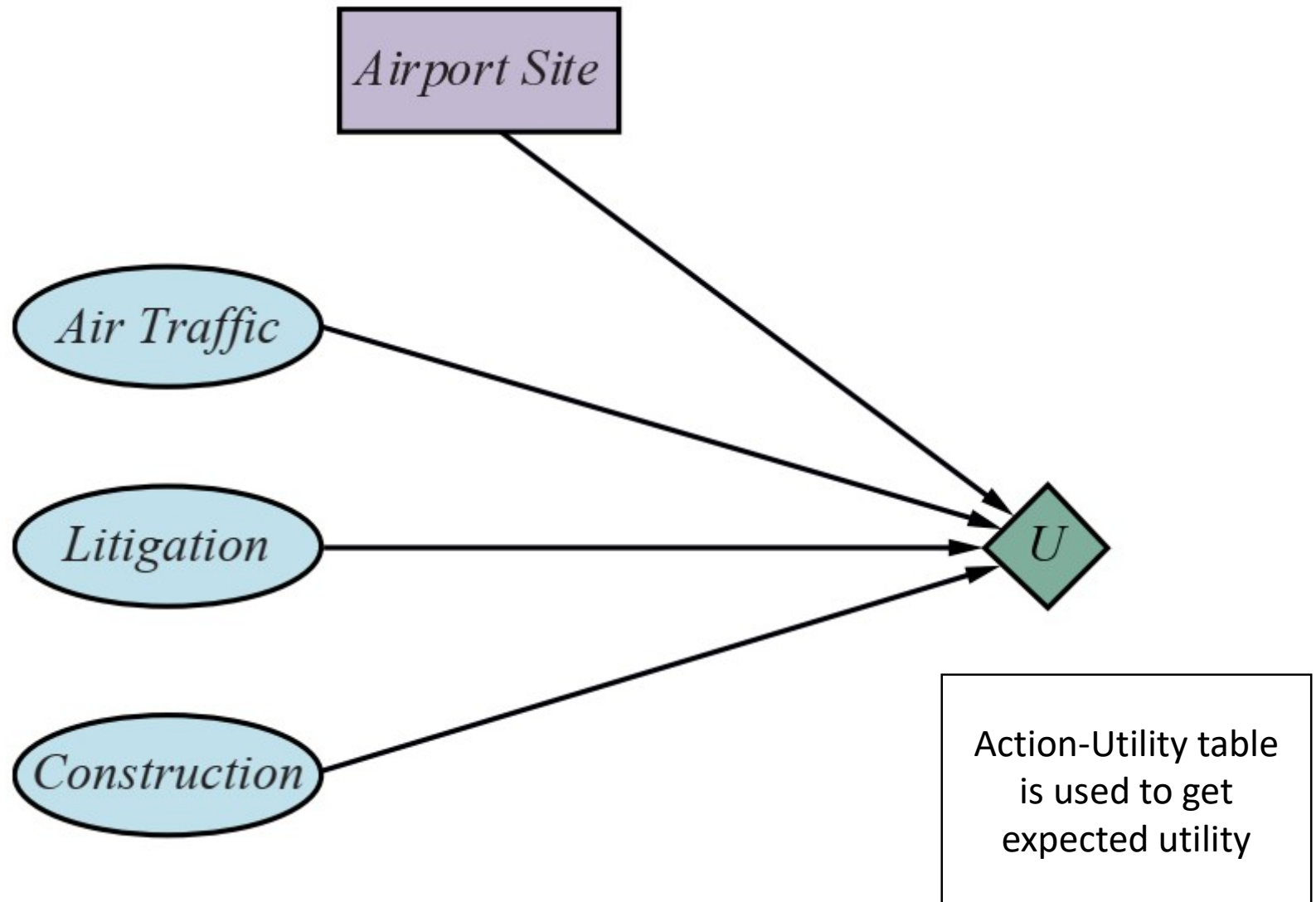
The algorithm for decision network evaluation is as follows:

1. Set the evidence variables for the current state
2. For each possible value a of decision node:
 - a. Set the decision node to that value
 - b. Calculate the posterior probabilities for the parent nodes of the utility node
 - c. Calculate the utility for the action / value a
3. Return the action with highest utility

Decision Network: Example

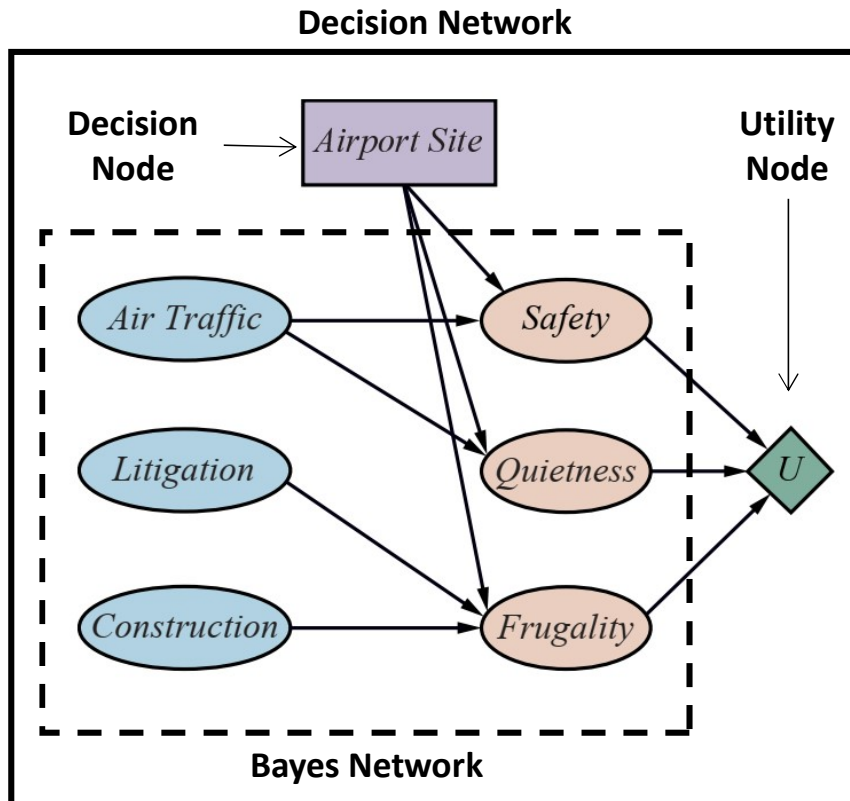


Decision Network: Simplified Form

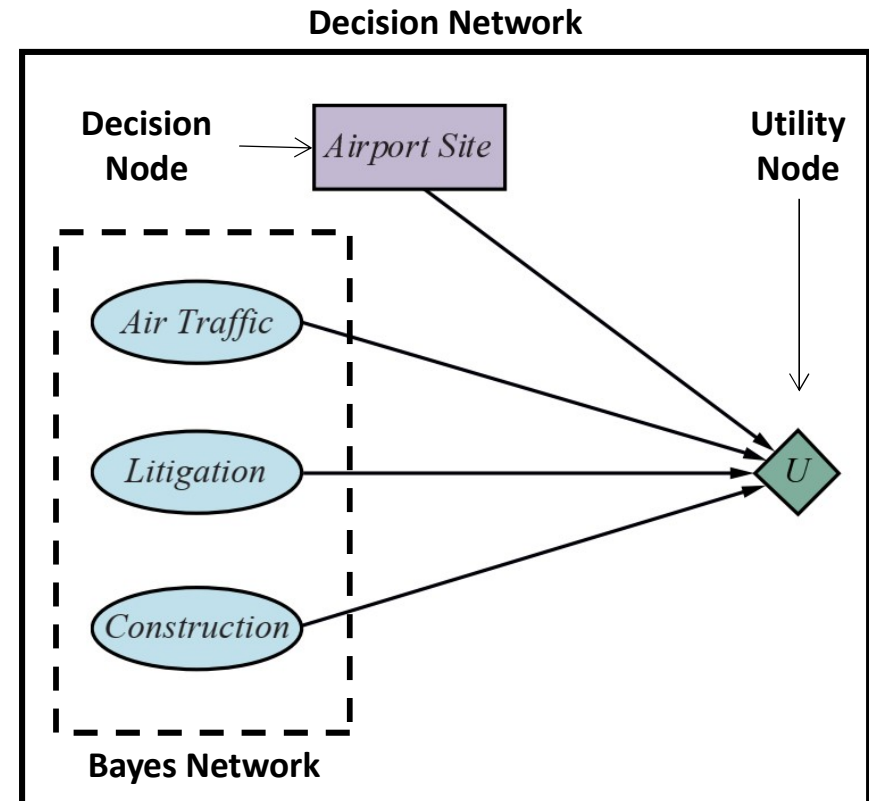


(Single-Stage) Decision Networks

General Structure

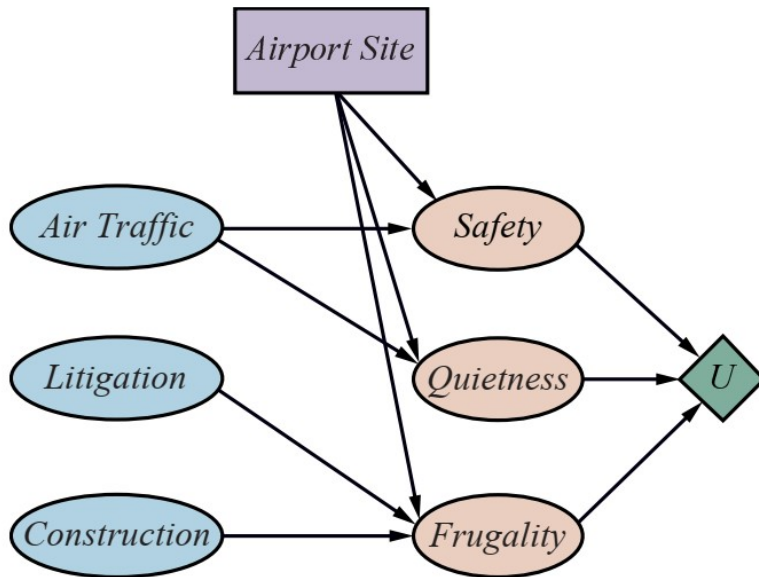


Simplified Structure



(Single-Stage) Decision Networks

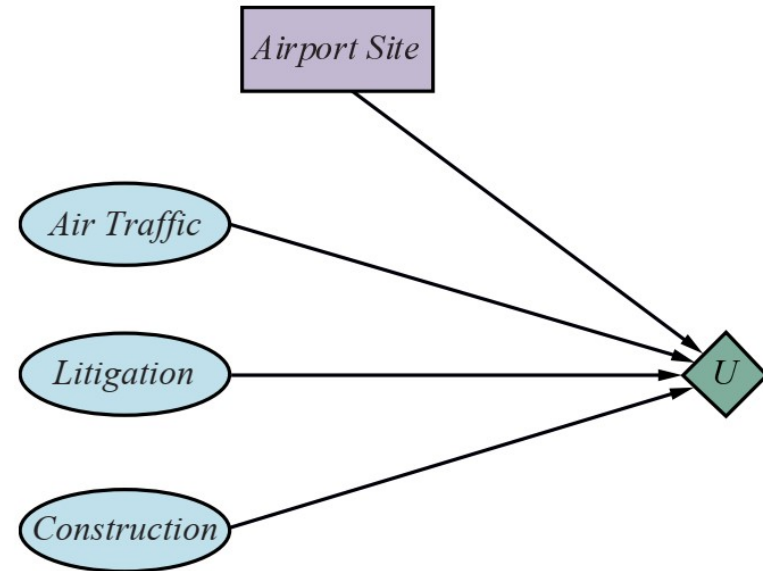
General Structure



Utility Table

S	low	low	low	low	high	high	high	high
Q	low	low	high	high	low	low	high	high
F	low	high	low	high	low	high	low	high
U	10	20	5	50	70	150	100	200

Simplified Structure



Action-Utility Table (not all columns shown)

AT	low	low	low	---	---	high	high	high
L	low	low	high	---	---	low	high	high
C	low	high	low	---	---	high	low	high
AS	A	A	A	---	---	B	B	B
U	10	20	5	---	---	150	100	200

Decision Network: Evaluation

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The **expected utility** of an action **a** given the evidence is the **average utility value** of all **possible outcomes s'** of action **a**, **weighted by their probability (belief) of occurrence**:

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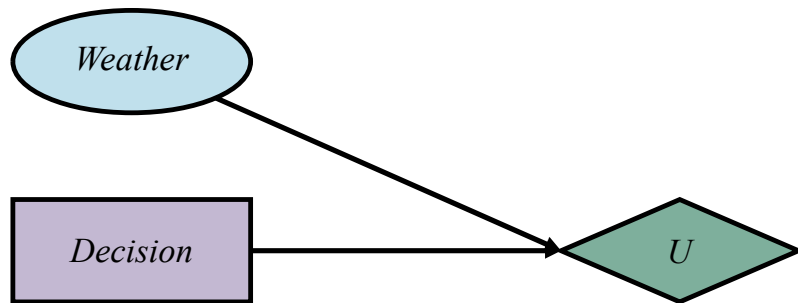
Rational agent should choose an action that **maximizes the expected utility**:

$$\text{chosen action} = \underset{a}{\operatorname{argmax}} EU(a)$$

Decision Networks: Example

Decision: **take** umbrella

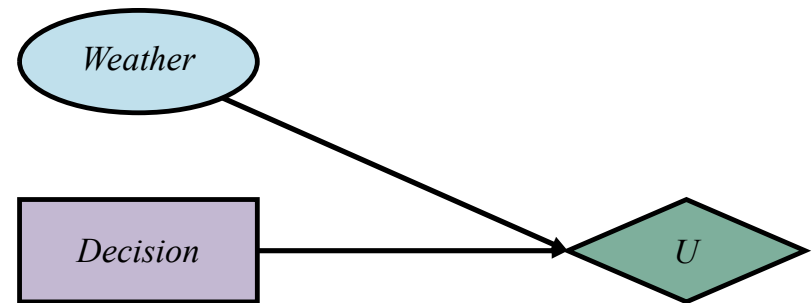
$P(W=\text{rain})$	$P(W=\text{sun})$
0.30	0.70



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decision: **leave** umbrella

$P(W=\text{rain})$	$P(W=\text{sun})$
0.30	0.70



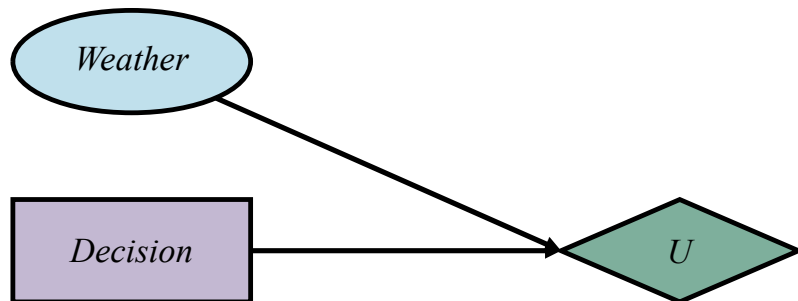
D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decision Networks: Example

Decision: **take** umbrella

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s') * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70



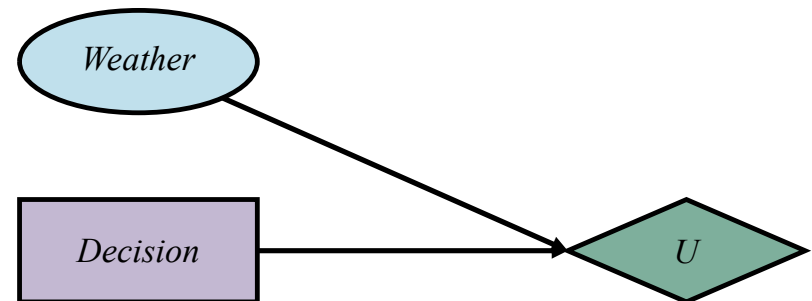
D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$EU(\text{take}) = ???$$

Decision: **leave** umbrella

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s') * U(s')$$

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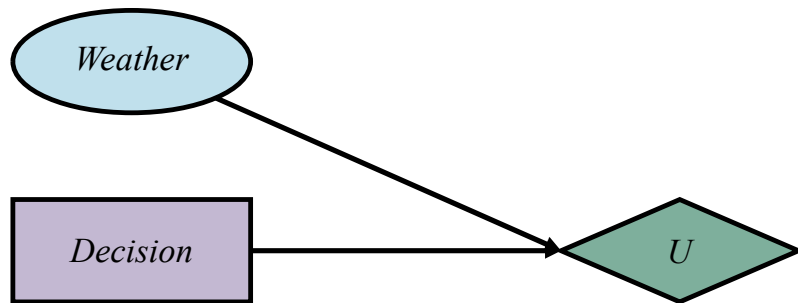
$$EU(\text{leave}) = ???$$

Decision Networks: Example

Decision: **take** umbrella

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s') * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70



S_1' : D = take, W = sun

S_2' : D = take, W = rain

$EU(\text{take}) =$

$P(\text{Result}(\text{take}) = S_1') * U(S_1') +$

$P(\text{Result}(\text{take}) = S_2') * U(S_2') =$

$0.70 * 20 + 0.30 * 70 = 35$

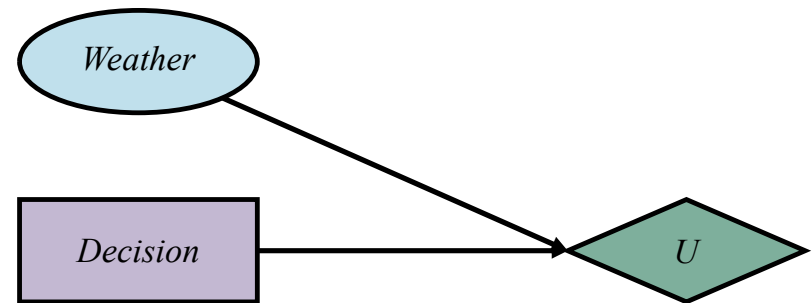
D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$EU(\text{take}) = 35$

Decision: **leave** umbrella

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s') * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70



S_3' : D = leave, W = sun

S_4' : D = leave, W = rain

$EU(\text{leave}) =$

$P(\text{Result}(\text{leave}) = S_3') * U(S_3') +$

$P(\text{Result}(\text{leave}) = S_4') * U(S_4') =$

$0.70 * 100 + 0.30 * 0 = 70$

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

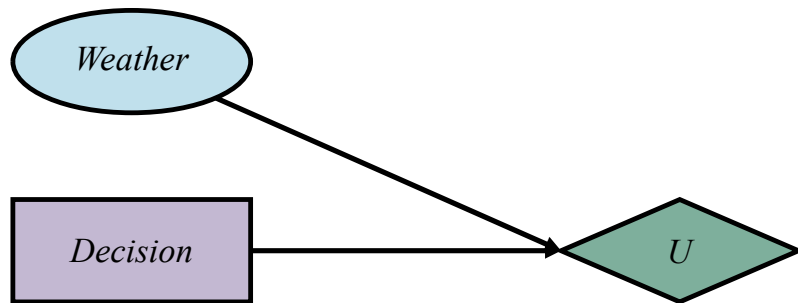
$EU(\text{leave}) = 70$

Decision Networks: Example

Which action to choose: **take** or **leave** Umbrella?

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s') * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70



S_1' : D = take, W = sun

S_2' : D = take, W = rain

$EU(\text{take}) =$

$P(\text{Result}(\text{take}) = S_1') * U(S_1') +$

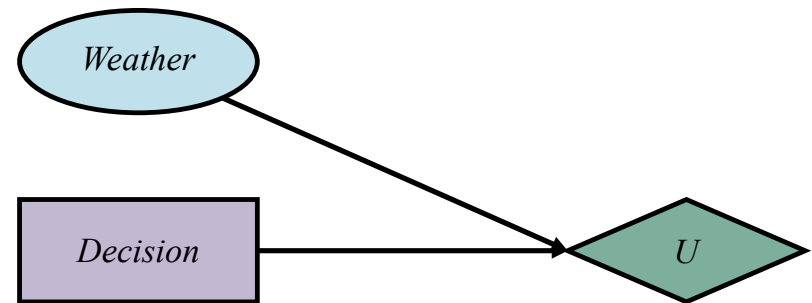
$P(\text{Result}(\text{take}) = S_2') * U(S_2') =$

$0.70 * 20 + 0.30 * 70 = 35$

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$EU(a) = \sum_{s'} P(\text{Result}(a) = s') * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70



S_3' : D = leave, W = sun

S_4' : D = leave, W = rain

$EU(\text{leave}) =$

$P(\text{Result}(\text{leave}) = S_3') * U(S_3') +$

$P(\text{Result}(\text{leave}) = S_4') * U(S_4') =$

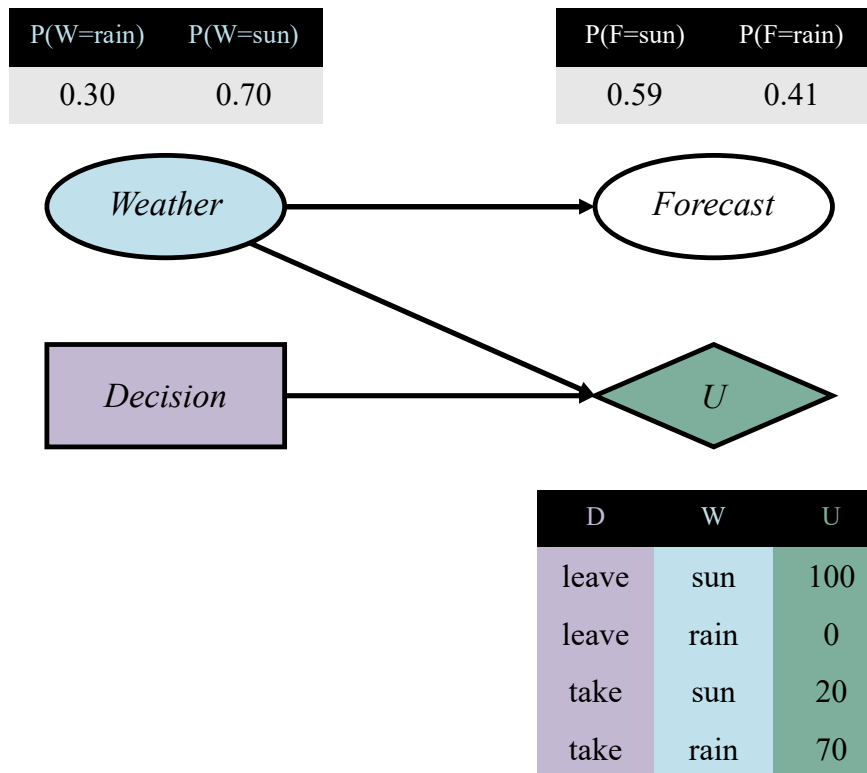
$0.70 * 100 + 0.30 * 0 = 70$

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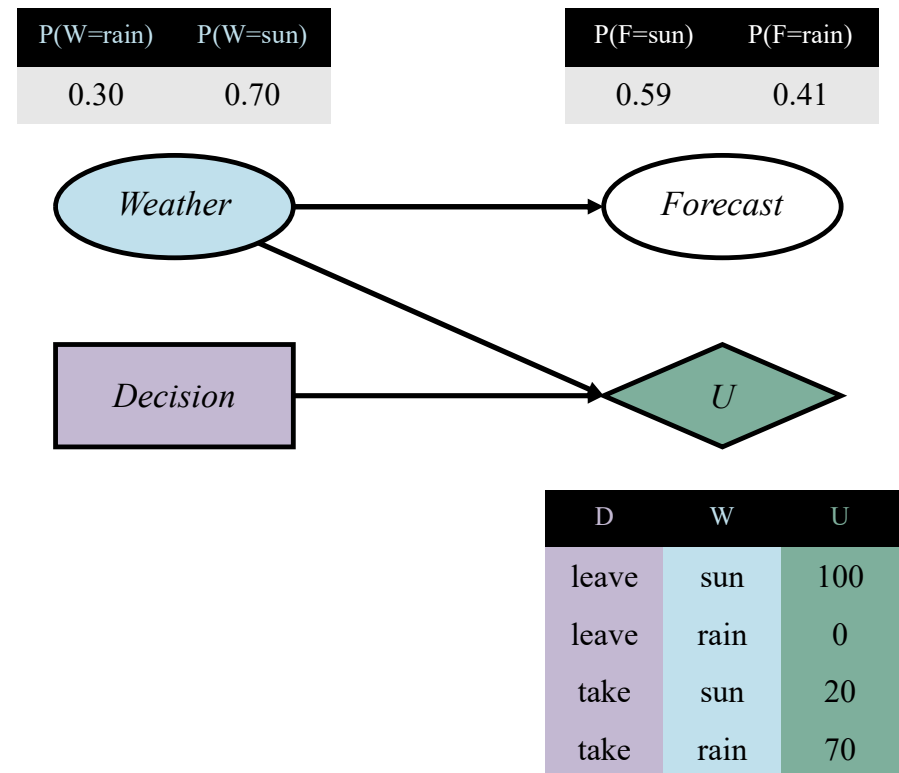
action = $\underset{a}{\operatorname{argmax}} EU(a) \mid \max(EU(\text{take}), EU(\text{leave})) = \max(35, 70) \rightarrow \text{leave}$

Decision Networks: Example

Decision: **take** umbrella



Decision: **leave** umbrella



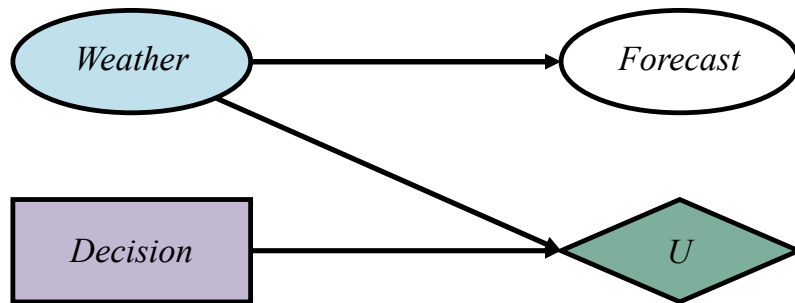
Decision Networks: Example

Decision: **take** umbrella given **e**

$$EU(a \mid e) = \sum_{s'} P(\text{Result}(a) = s' \mid e) * U(s')$$

P(rain F)	P(sun F)
???	???

P(F=sun)	P(F=rain)
0.59	0.41



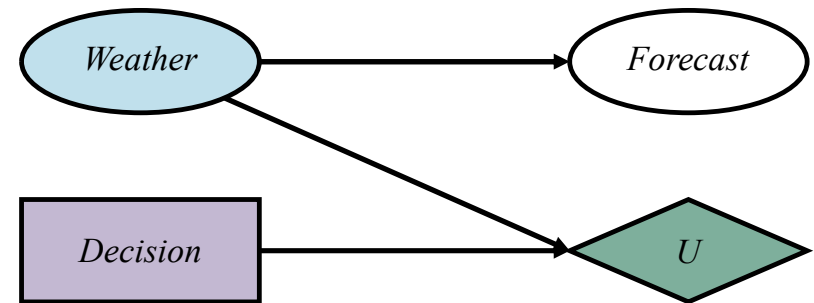
D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decision: **leave** umbrella given **e**

$$EU(a \mid e) = \sum_{s'} P(\text{Result}(a) = s' \mid e) * U(s')$$

P(rain F)	P(sun F)
???	???

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

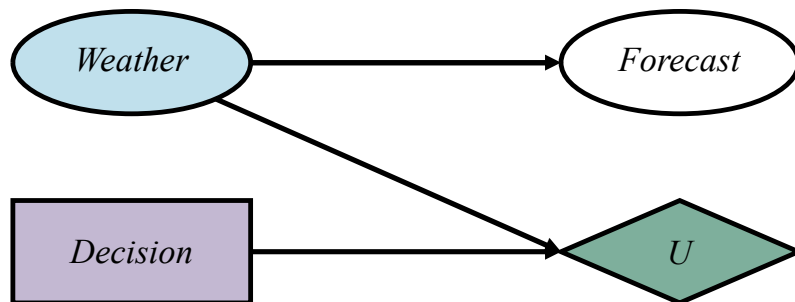
Decision Networks: Example

Decision: **take** umbrella given **e**

$$EU(a | e) = \sum_{s'} P(Result(a) = s' | e) * U(s')$$

P(W=rain)	P(W=sun)
0.30	0.70

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Conditional probabilities
Assume that we are given:

F	W	P(F W)
sun	sun	0.80
rain	sun	0.20
sun	rain	0.10
rain	rain	0.90

By Bayes' Theorem:

$$P(W = \text{sun} | F = \text{sun}) = \frac{P(F = \text{sun} | W = \text{sun}) * P(W = \text{sun})}{P(F = \text{sun})} = \frac{0.80 * 0.70}{0.59} = 0.95$$

$$P(W = \text{sun} | F = \text{rain}) = \frac{P(F = \text{rain} | W = \text{sun}) * P(W = \text{sun})}{P(F = \text{rain})} = \frac{0.20 * 0.70}{0.41} = 0.34$$

$$P(W = \text{rain} | F = \text{sun}) = \frac{P(F = \text{sun} | W = \text{rain}) * P(W = \text{rain})}{P(F = \text{sun})} = \frac{0.10 * 0.30}{0.59} = 0.05$$

$$P(W = \text{rain} | F = \text{rain}) = \frac{P(F = \text{rain} | W = \text{rain}) * P(W = \text{rain})}{P(F = \text{rain})} = \frac{0.90 * 0.30}{0.41} = 0.66$$

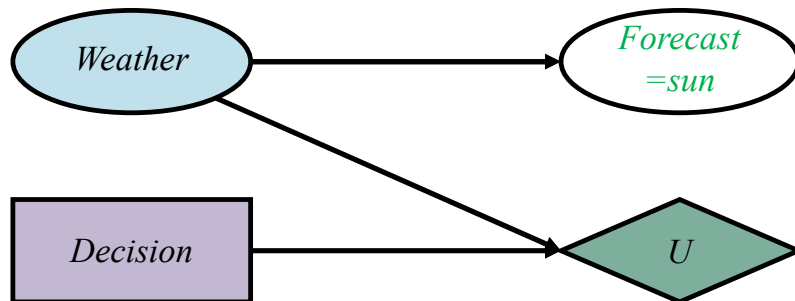
Decision Networks: Example

Decision: **take** umbrella given **sun**

$$EU(a \mid e) = \sum_{s'} P(\text{Result}(a) = s' \mid e) * U(s')$$

P(rain F)	P(sun F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

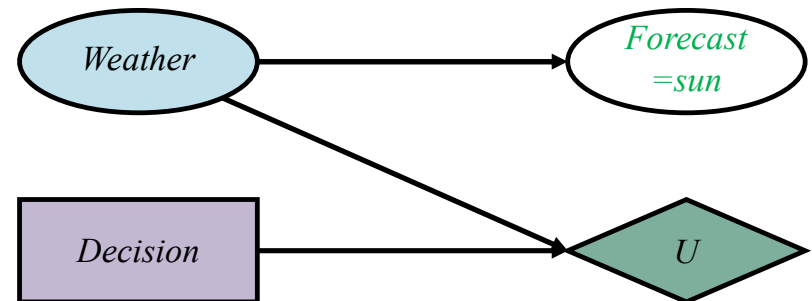
$$EU(\text{take given sun forecast}) = ???$$

Decision: **leave** umbrella given **sun**

$$EU(a \mid e) = \sum_{s'} P(\text{Result}(a) = s' \mid e) * U(s')$$

P(rain F)	P(sun F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$EU(\text{leave given sun forecast}) = ???$$

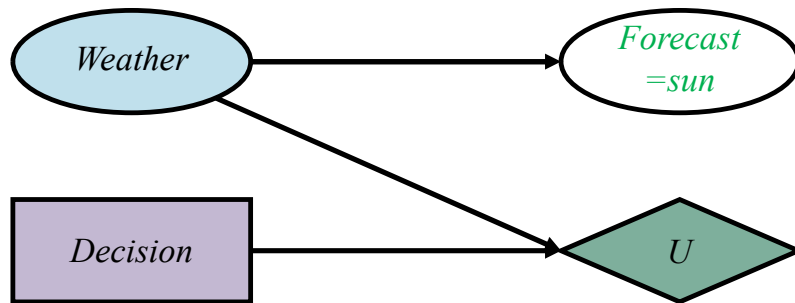
Decision Networks: Example

Decision: **take** umbrella given **sun**

$$EU(a | e) = \sum_{s'} P(\text{Result}(a) = s' | e) * U(s')$$

P(rain F)	P(sun F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



S_1' : D = take, W = sun

S_2' : D = take, W = rain

$EU(\text{take}) =$

$$P(\text{Result}(\text{take})=S_1'|e)*U(S_1') + P(\text{Result}(\text{take})=S_2'|e)*U(S_2') = 0.95 * 20 + 0.05 * 70 = 22.5$$

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

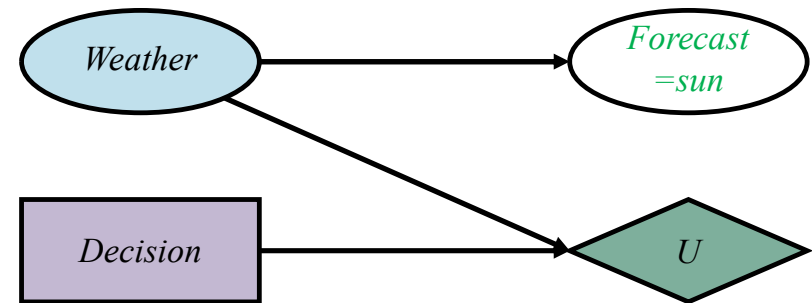
$$EU(\text{take given sun forecast}) = 22.5$$

Decision: **leave** umbrella given **sun**

$$EU(a | e) = \sum_{s'} P(\text{Result}(a) = s' | e) * U(s')$$

P(rain F)	P(sun F)
0.05	0.95

P(F=sun)	P(F=rain)
0.59	0.41



S_3' : D = leave, W = sun

S_4' : D = leave, W = rain

$EU(\text{leave}) =$

$$P(\text{Result}(\text{leave})=S_3'|e)*U(S_3') + P(\text{Result}(\text{leave})=S_4'|e)*U(S_4') = 0.95 * 100 + 0.05 * 0 = 95$$

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$EU(\text{leave given sun forecast}) = 95$$

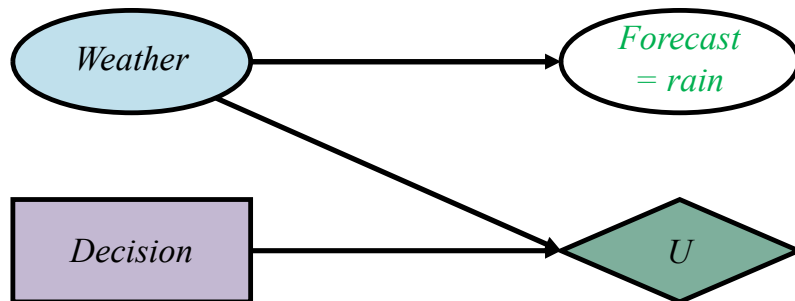
Decision Networks: Example

Decision: **take** umbrella given **rain**

$$EU(a \mid e) = \sum_{s'} P(\text{Result}(a) = s' \mid e) * U(s')$$

P(rain F)	P(sun F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

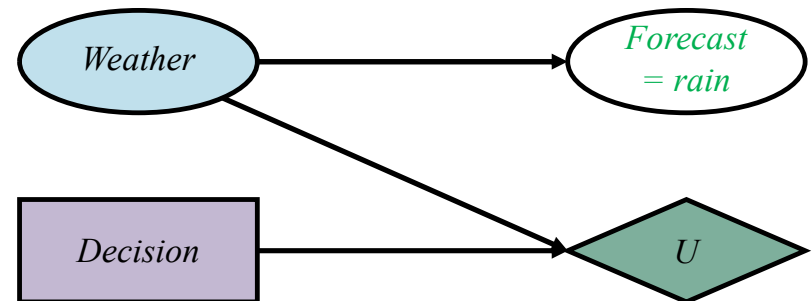
$$EU(\text{take given rain forecast}) = ???$$

Decision: **leave** umbrella given **rain**

$$EU(a \mid e) = \sum_{s'} P(\text{Result}(a) = s' \mid e) * U(s')$$

P(rain F)	P(sun F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	0.41



D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$EU(\text{leave given rain forecast}) = ???$$

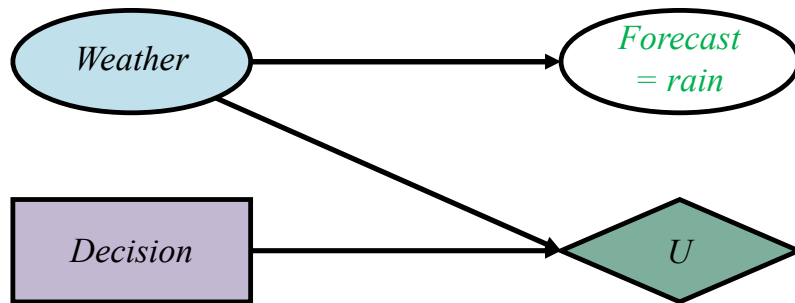
Decision Networks: Example

Decision: **take** umbrella given **rain**

$$EU(a | e) = \sum_{s'} P(\text{Result}(a) = s' | e) * U(s')$$

P(rain F)	P(sun F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	0.41



S_1' : D = take, W = sun

S_2' : D = take, W = rain

$EU(\text{take}) =$

$$P(\text{Result}(\text{take})=S_1'|e)*U(S_1') + P(\text{Result}(\text{take})=S_2'|e)*U(S_2') = 0.34 * 20 + 0.66 * 70 = 53$$

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

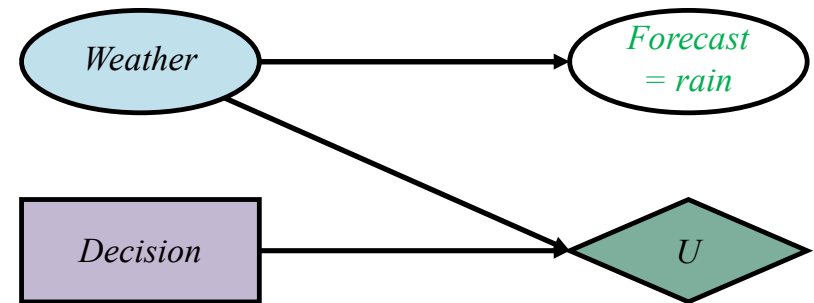
$EU(\text{take given rain forecast}) = 53$

Decision: **leave** umbrella given **rain**

$$EU(a | e) = \sum_{s'} P(\text{Result}(a) = s' | e) * U(s')$$

P(rain F)	P(sun F)
0.66	0.34

P(F=sun)	P(F=rain)
0.59	0.41



S_3' : D = leave, W = sun

S_4' : D = leave, W = rain

$EU(\text{leave}) =$

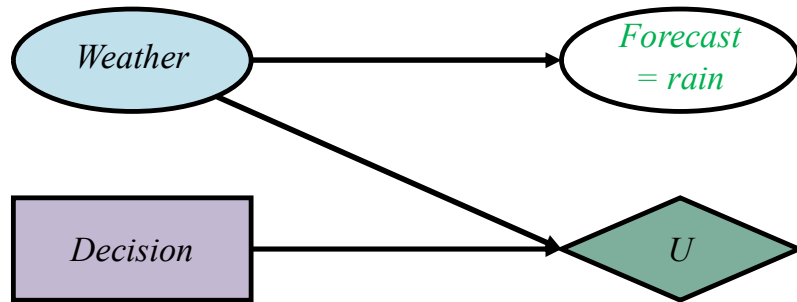
$$P(\text{Result}(\text{leave})=S_3'|e)*U(S_3') + P(\text{Result}(\text{leave})=S_4'|e)*U(S_4') = 0.34 * 100 + 0.66 * 0 = 34$$

D	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$EU(\text{leave given rain forecast}) = 34$

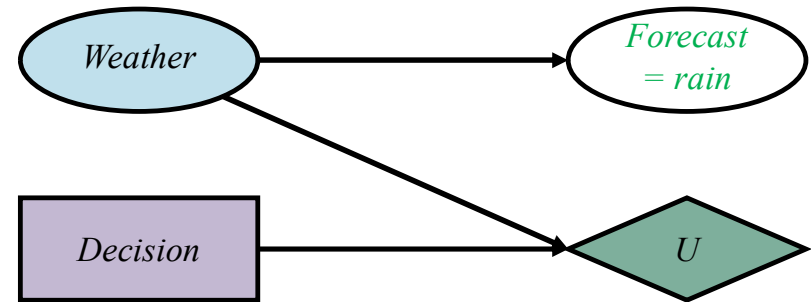
Decision Networks: Example

Decision: **take** umbrella given **rain**



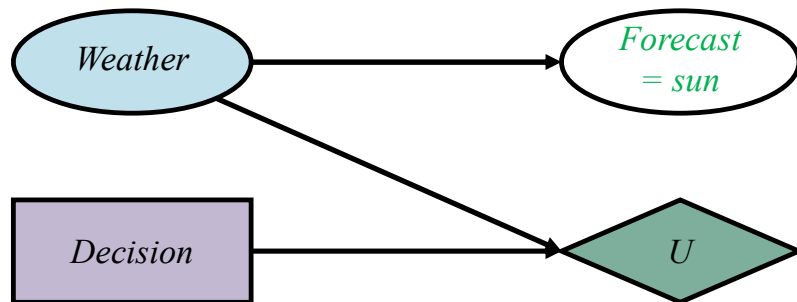
$$EU(\text{take given rain forecast}) = 53$$

Decision: **leave** umbrella given **rain**



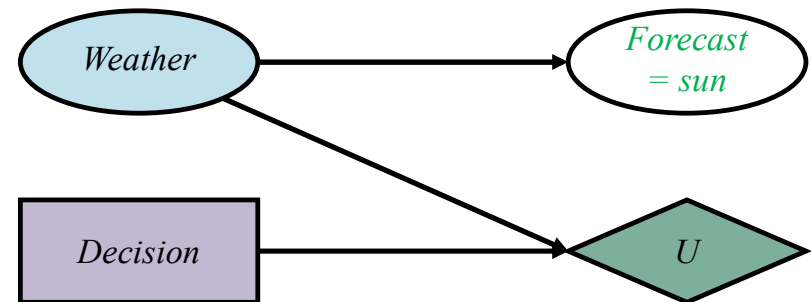
$$EU(\text{leave given rain forecast}) = 34$$

Decision: **take** umbrella given **sun**



$$EU(\text{take given sun forecast}) = 22.5$$

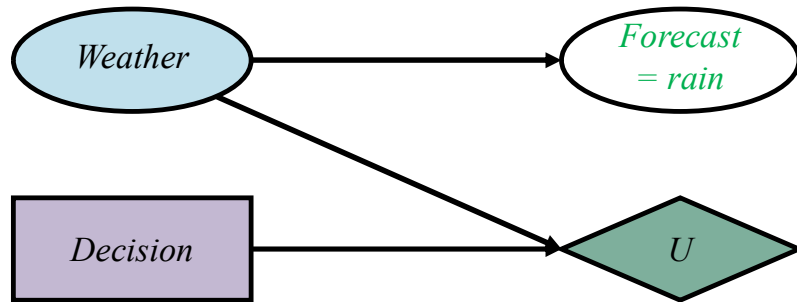
Decision: **leave** umbrella given **sun**



$$EU(\text{leave given sun forecast}) = 95$$

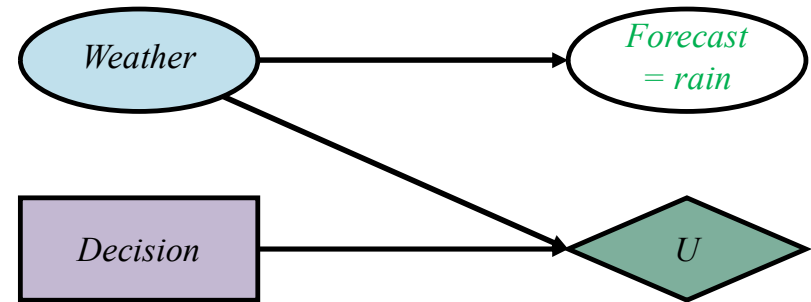
Decision Networks: Example

Decision:take umbrella given rain



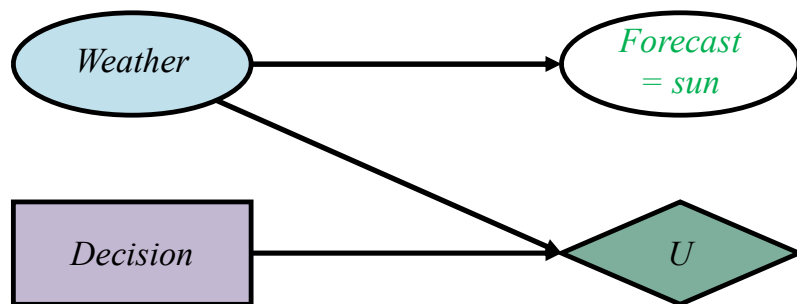
$$EU(\text{take given rain forecast}) = 53$$

Decision:leave umbrella given rain



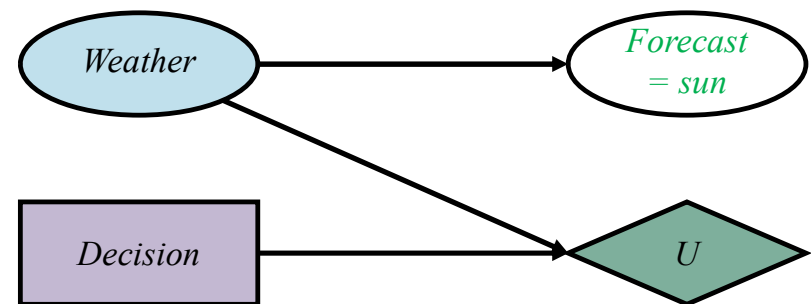
$$EU(\text{leave given rain forecast}) = 34$$

Decision:take umbrella given sun



$$EU(\text{take given sun forecast}) = 22.5$$

Decision:leave umbrella given sun



$$EU(\text{leave given sun forecast}) = 95$$

Value of Perfect Information

The value/utility of best action α without additional evidence (information) is :

$$MEU(\alpha) = \max_a \sum_{s'} P(Result(a) = s') * U(s')$$

If we include new evidence/information ($E_j = e_j$) given by some variable E_j , value/utility of best action α becomes:

$$MEU(a_{e_j} | e_j) = \max_a \sum_{s'} P(Result(a) = s' | e_j) * U(s')$$

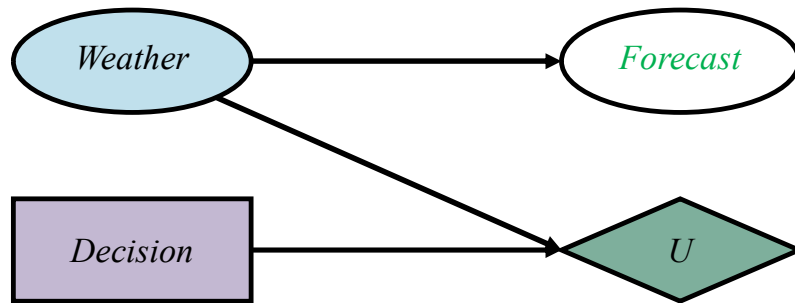
The value of additional evidence/information from E_j is:

$$VPI(E_j) = \left(\sum_{e_j} P(E_j = e_j) * MEU(a_{e_j} | E_j = e_j) \right) - MEU(a)$$

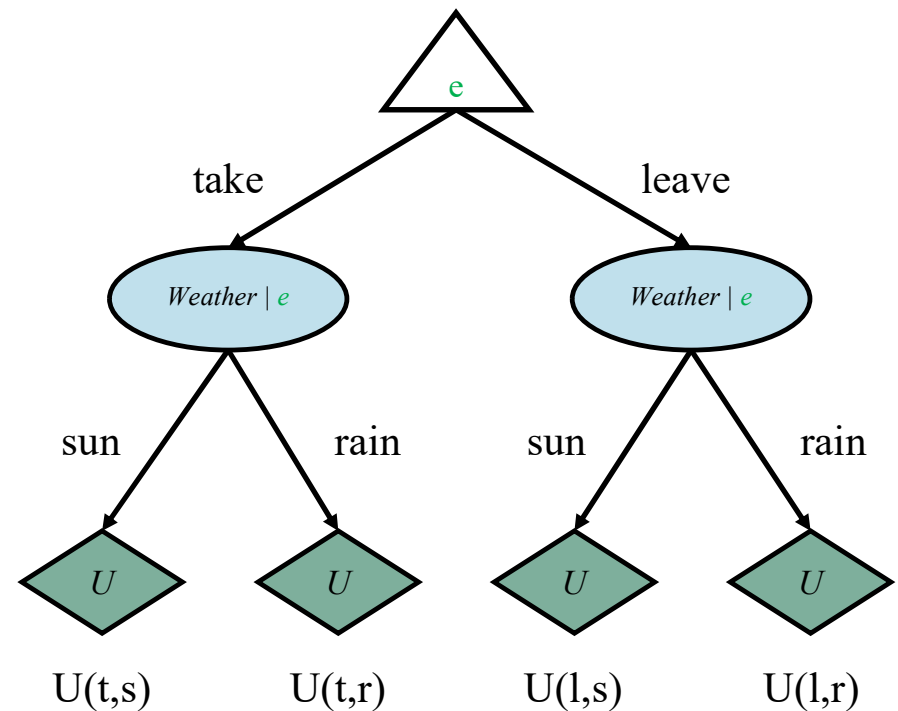
using our current **beliefs** about the world.

Decision Network: Example

Decision network



Outcome tree



The value of best action α without additional evidence

$$MEU(\alpha) = MEU(\text{leave}) = 70$$

With evidence information ($E_j = e_j$) given by Forecast:

$$MEU(a_{e_1} | e_1) = MEU(\text{take} | F = \text{rain}) = 53$$

$$MEU(a_{e_2} | e_2) = MEU(\text{leave} | F = \text{sun}) = 95$$

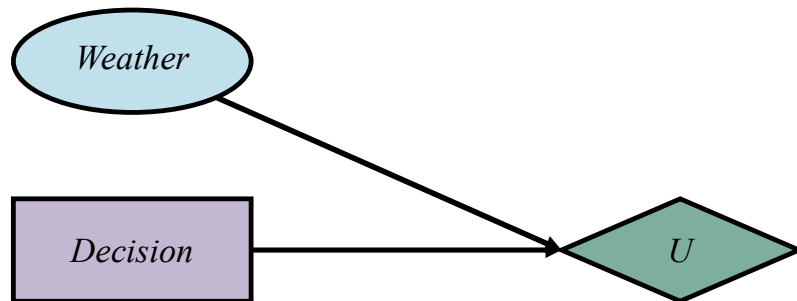
The value of additional evidence / information from F is:

$$VPI(E_j) = \left(\sum_{e_j} P(E_j = e_j) * MEU(a_{e_j} | E_j = e_j) \right) - MEU(\alpha)$$

$$\begin{aligned} VPI(F) &= (P(F = \text{rain}) * MEU(\text{take} | F = \text{rain}) + P(F = \text{sun}) * \\ &\quad MEU(\text{leave} | F = \text{sun})) - MEU(\text{leave}) = \\ &\quad (0.41 * 53 + 0.59 * 95) - 70 = 7.78 \end{aligned}$$

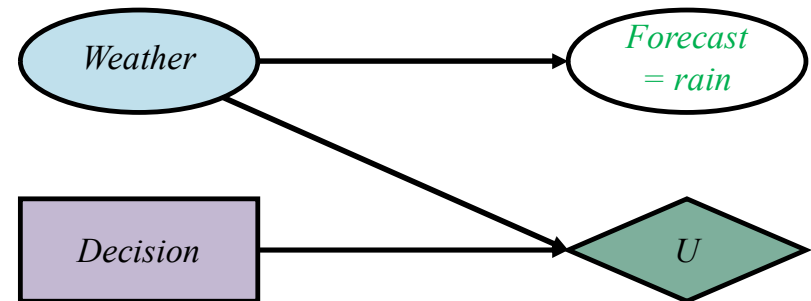
Decision Networks: Example

Decision: **leave** umbrella



$$EU(\text{leave}) = 70$$

Decision: **take** umbrella given **rain**



$$EU(\text{take given rain forecast}) = 53$$

The value of best action α without additional evidence

$$MEU(\alpha) = MEU(\text{leave}) = 70$$

With evidence information ($E_j = e_j$) given by Forecast:

$$MEU(a_{e_1} | e_1) = MEU(\text{take} | F = \text{rain}) = 53$$

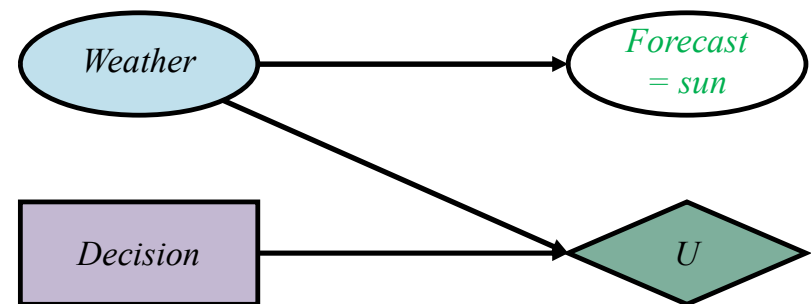
$$MEU(a_{e_2} | e_2) = MEU(\text{leave} | F = \text{sun}) = 95$$

The value of additional evidence / information from F is:

$$VPI(E_j) = \left(\sum_j P(E_j = e_j) * MEU(a_{e_j} | E_j = e_j) \right) - MEU(\alpha)$$

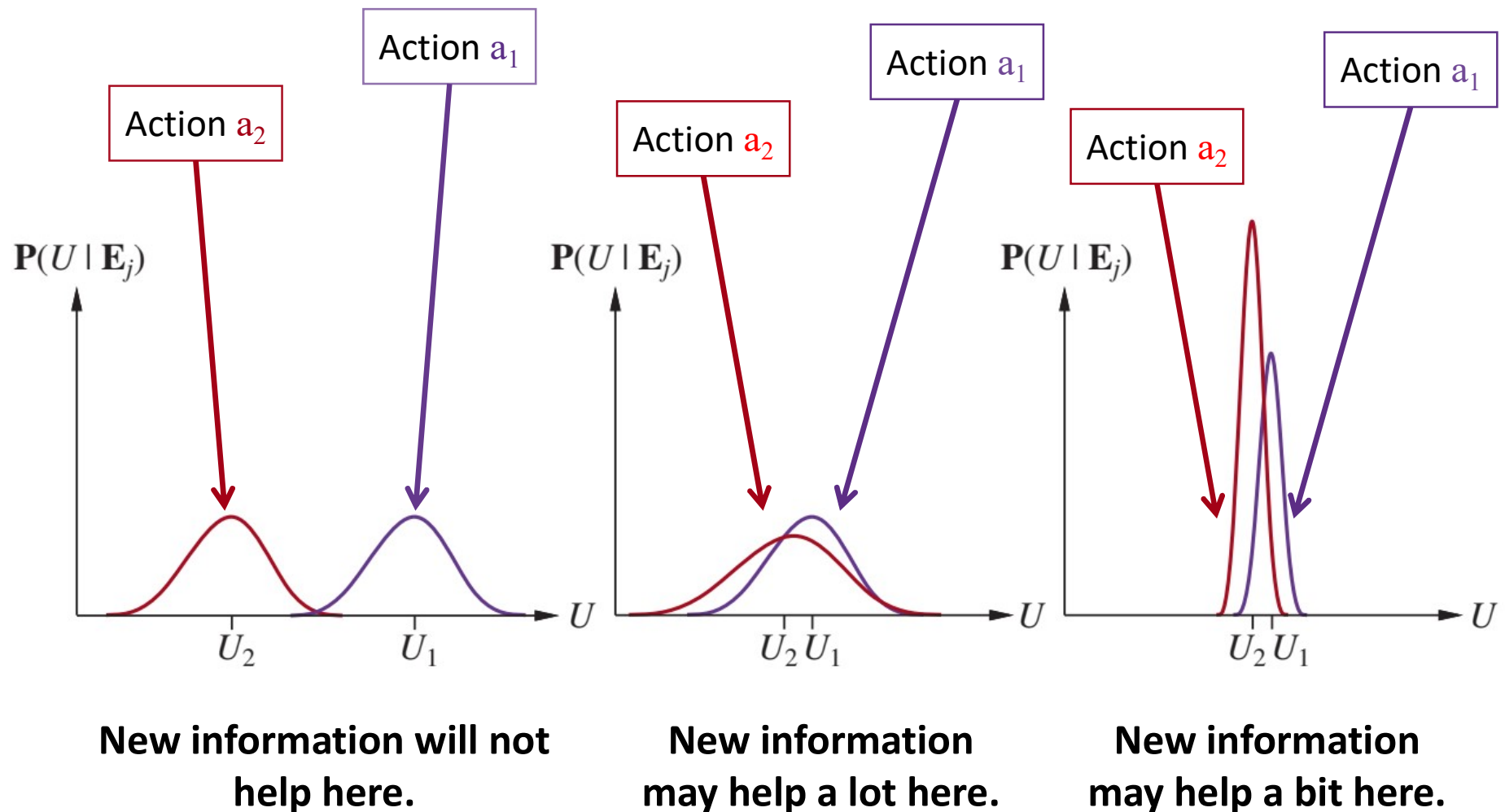
$$\begin{aligned} VPI(F) &= (P(F = \text{rain}) * MEU(\text{take} | F = \text{rain}) + P(F = \text{sun}) * \\ &\quad MEU(\text{leave} | F = \text{sun})) - MEU(\text{leave}) = \\ &\quad (0.41 * 53 + 0.59 * 95) - 70 = 7.78 \end{aligned}$$

Decision: **leave** umbrella given **sun**



$$EU(\text{leave given sun forecast}) = 95$$

Utility & Value of Perfect Information



VPI Properties

Given a decision network with possible observations E_j (sources of new information / evidence):

- The expected value of information is nonnegative:

$$\forall_j \text{VPI}(E_j) \geq 0$$

- VPI is not additive:

$$\text{VPI}(E_j, E_k) \neq \text{VPI}(E_j) + \text{VPI}(E_k)$$

- VPI is order-independent:

$$\text{VPI}(E_j, E_k) = \text{VPI}(E_j) + \text{VPI}(E_k | E_j) = \text{VPI}(E_k) + \text{VPI}(E_j | E_k) = \text{VPI}(E_k, E_j)$$

Information Gathering Agent

function INFORMATION-GATHERING-AGENT(*percept*) **returns** an *action*
persistent: D , a decision network

integrate *percept* into D

$j \leftarrow$ the value that maximizes $VPI(E_j) / C(E_j)$

if $VPI(E_j) > C(E_j)$

then return $Request(E_j)$

else return the best action from D