Student Name : Madhushalini Murali ; A Id: A20513784

CS 480 Fall 2022 Written Assignment #02

Due: **Sunday, October 16th, 11:00 PM CST**

Points: **90**

Instructions:

1. Use this document template to report your answers. Name the complete document as follows:

`LastName_FirstName_CS480_Written02.doc`

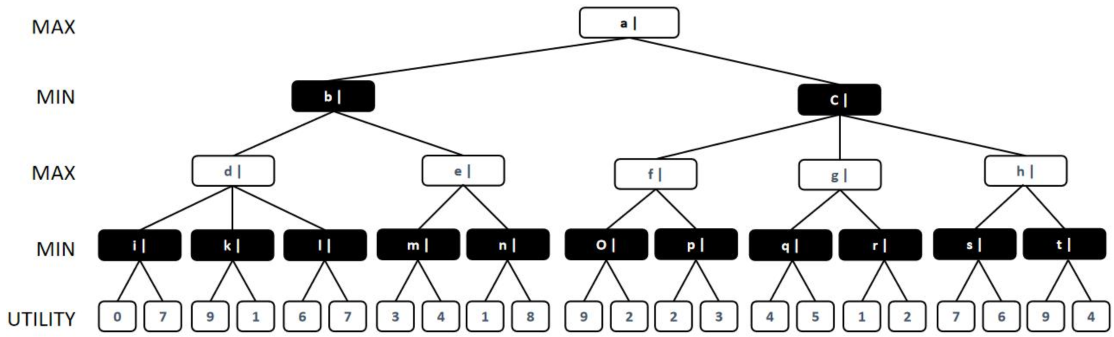
2. Submit the final document to Blackboard Assignments section before the due date. No late submissions will be accepted.

Objectives:

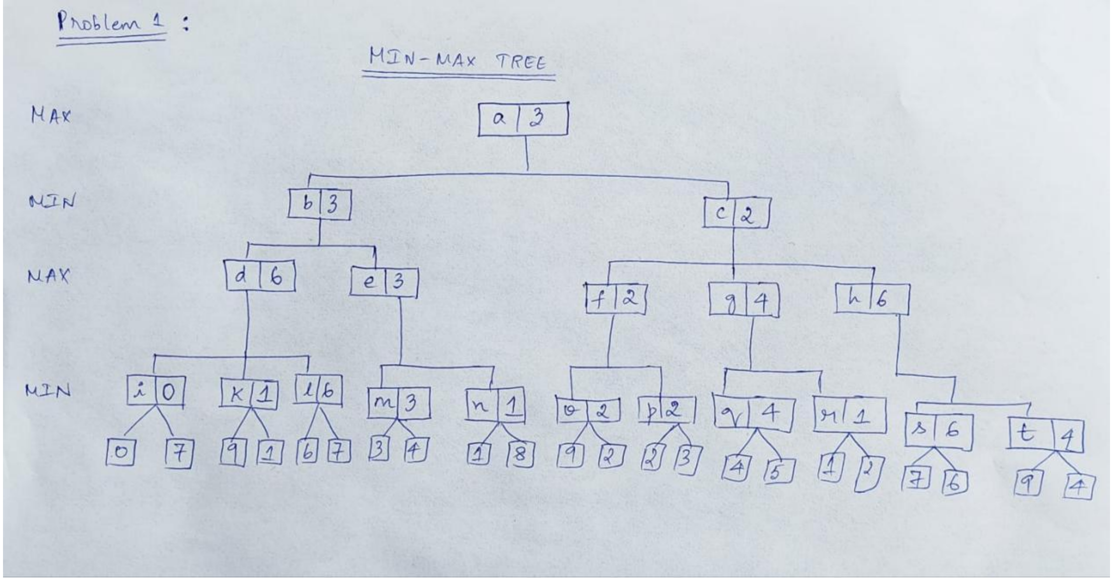
1. (20 points) Demonstrate your understanding of MinMax games and α–β pruning algorithm.
2. (50 points) Demonstrate your understanding of propositional logic, its syntax, equivalence laws, and CNF form
3. (20 points) Demonstrate your understanding of proof by resolution.

Problem 1 [20 pts]:

Consider the following MinMax game tree



Evaluate MinMax values for all nodes (you can paste in an edited version of this tree below) **[10 pts]**:

Your solution:


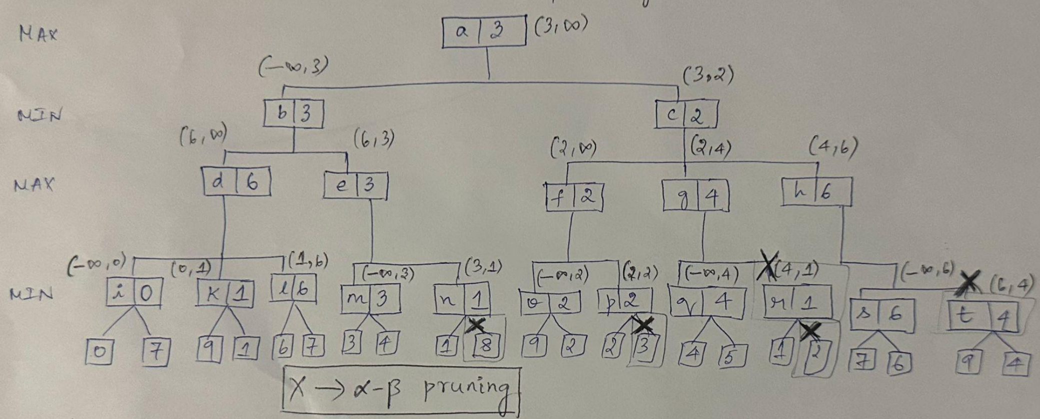
Now, apply **alpha-beta (α–β) pruning** to prune some of the tree branches. Show (you can paste in an edited version of this tree below) which sections of the tree will be pruned and **justify your answer [10 pts]**:

Your solution:

Problem 1 :

Madhushalini Murali
A20513784

MIN-MAX TREE [α-β Pruning]



Explanation : In this tree, 5 prunings are done.

We know that the condition for α-β pruning is $\boxed{\alpha >= \beta}$

- In case 1, $\alpha = 3$ and $\beta = 1$ which satisfies the condition
- In case 2, $\alpha = 2$ and $\beta = 2$ which satisfies the condition
- In case 3, $\alpha = 4$ and $\beta = 1$ which satisfies the condition
- In case 4, $\alpha = 6$ and $\beta = 4$ which satisfie the condition
- In case 5, $\alpha = 4$ and $\beta = 1$ which satisfies the condition

Since all the 5 case satisfies the condition, the corresponding path is pruned and hence not visited

Problem 2 [10 pts]:

Use **truth tables** to show that the following sentences are **tautologies [5 pts]**:

1. $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ **[5 pts]**

   Place your truth table here.

p	q	¬p	¬q	p∧q	¬(p∧q)	¬p∨¬q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

From the last two rows of the truth table, we can infer that $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
Hence it is a **Tautology.**

2. $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ **[5 pts]**

   Place your truth table here.

p	q	¬p	¬q	(p⇒q)	(¬q⇒¬p)
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

From the last two rows of the truth table,we can infer that $(p \Rightarrow q) \Leftrightarrow(\neg q \Rightarrow \neg p)$
Hence it is a **Tautology.**

Problem 3 [15 pts]:

Use **deduction** to show (**prove**) that the following sentences are **tautologies**:

1. $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
Given	$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$	
1	$(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p) \wedge (\neg q \Rightarrow \neg p) \Rightarrow (p \Rightarrow q)$	Equivalence law
2	(¬p∨q)⇒(¬(¬q)∨¬p) ∧ (¬(¬q)∨¬p)⇒(¬p∨q)	Implication law
3	(¬p∨q)⇒(q∨¬p) ∧ (¬q∨¬p)⇒(¬p∨q)	Double negation law
4	¬(¬p∨q) ∨ (q∨¬p) ∧ ¬(¬q∨¬p) ∨ (¬p∨q)	Implication law
5	¬(¬p∨q) ∨ ⊥ ∨ (¬p∨q)	Contradiction (Negation law)
6	¬(¬p∨q) ∨ (¬p∨q)	Identity law
7	T	Law of excluded middle
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴		

2. $((p \Rightarrow q) \wedge (q \Rightarrow p)) \Leftrightarrow (p \Leftrightarrow q)$ **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
Given	$((p \Rightarrow q) \wedge (q \Rightarrow p)) \Leftrightarrow (p \Leftrightarrow q)$	
1	$((p \Rightarrow q) \wedge (q \Rightarrow p)) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$	Equivalence law
2	$[(p \Rightarrow q) \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)] \wedge$ $[(p \Rightarrow q) \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)]$	Equivalence law
3	$[(\neg p \vee q) \wedge (\neg q \vee p) \Rightarrow (\neg p \vee q) \wedge (\neg q \vee p)] \wedge$ $[(\neg p \vee q) \wedge (\neg q \vee p) \Rightarrow (\neg p \vee q) \wedge (\neg q \vee p)]$	Implication law
4	$\neg[(\neg p \vee q) \wedge (\neg q \vee p)] \vee [(\neg p \vee q) \wedge (\neg q \vee p)] \wedge$ $\neg[(\neg p \vee q) \wedge (\neg q \vee p)] \vee [(\neg p \vee q) \wedge (\neg q \vee p)]$	Implication law
5	$\neg[(\neg p \vee q) \wedge (\neg q \vee p)] \vee [(\neg p \vee q) \wedge (\neg q \vee p)] \wedge T$	Law of Excluded middle
6	$\neg[(\neg p \vee q) \wedge (\neg q \vee p)] \vee [(\neg p \vee q) \wedge (\neg q \vee p)]$	Identity law
7	T	Law of Excluded middle
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴		

3. $(p \vee q) \wedge (\neg q \vee r) \Rightarrow (p \vee r)$ **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
Given	$(p \vee q) \wedge (\neg q \vee r) \Rightarrow (p \vee r)$	
1	$\neg((p \vee q) \wedge (\neg q \vee r)) \vee (p \vee r)$	Implication Law
2	$\neg (\neg q \vee r) \vee \neg(p \vee q) \vee (p \vee r)$	DeMorgan's law
3	$\neg r \wedge q \vee \neg q \wedge \neg p \vee p \vee r$	DeMorgan's law
4	$\neg r \wedge T \wedge \neg p \vee p \vee r$	Law of Excluded middle
5	$\neg r \wedge \neg p \vee p \vee r$	Identity law
6	$\neg p \vee p \wedge T$	Law of Excluded middle
7	$\neg p \vee p$	Identity law
8	T	Law of Excluded middle
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴		

Problem 4 [15 pts]:

Convert the following sentences into **conjunctive normal form** (**CNF**):

a) $p \wedge q \Leftrightarrow p \vee q$ **[5 pts]**

Your conversion steps:		
Step	**Resulting sentence**	**Applied law / rule**
Given	$p \wedge q \Leftrightarrow p \vee q$	
1	$((p \wedge q) \Rightarrow (p \vee q)) \wedge ((p \vee q) \Rightarrow (p \wedge q))$	Equivalence law
2	$(\neg (p \wedge q) \vee (p \vee q)) \wedge (\neg (p \vee q) \vee (p \wedge q))$	Implication law
3	$((\neg q \vee \neg p) \vee(p \vee q)) \wedge ((\neg q \wedge \neg p) \vee (p \wedge q))$	DeMorgan's law
4	$((\neg q \vee \neg p) \vee(p \vee q)) \wedge (((\neg q \wedge \neg p) \vee p) \wedge (\neg q \wedge \neg p) \vee q))$	Distributive law
5	$(\neg q \vee \neg p \vee p \vee q) \wedge (\neg q\vee p) \wedge (\neg p \vee p) \wedge (\neg q \vee q)\wedge (\neg p \vee q)$	Distributive law and removing parentheses
	The above sentence is in CNF.	
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴		

b) $(p \wedge (p \Rightarrow q)) \Rightarrow q$ **[10 pts]**

Your conversion steps:		
Step	**Resulting sentence**	**Applied law / rule**
Given	$(p \wedge (p \Rightarrow q)) \Rightarrow q$	
1	$\neg (p \wedge (p \Rightarrow q)) \vee q$	Implication law
2	$\neg (p \wedge (\neg p \vee q)) \vee q$	Implication law
3	$(\neg (\neg p \vee q) \vee \neg p)\vee q$	DeMorgan's law
4	$((\neg q \wedge p) \vee \neg p) \vee q$	DeMorgan's law and Double Negation
5	$((\neg q \vee \neg p) \wedge (p\vee \neg p)) \vee q$	Distributive law
6	$((\neg q \vee \neg p\vee q) \wedge (p\vee \neg p\vee q))$	Distributive law
7	$(\neg q \vee \neg p\vee q) \wedge (p\vee \neg p\vee q)$	Removing exterior parentheses
	The above sentence is in CNF.	
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴		

Problem 5 [20 pts]:

Use **proof by resolution** to show that this claim (sentence) below is true (a tautology):

$$\neg((p \Leftrightarrow \neg q) \wedge (q \Leftrightarrow \neg r) \wedge (r \Leftrightarrow \neg p))$$

Show all the necessary steps.

<u>Answer:</u>

Let $S \equiv \neg((p \Leftrightarrow \neg q) \wedge (q \Leftrightarrow \neg r) \wedge (r \Leftrightarrow \neg p))$

Then, $\neg S \equiv ((p \Leftrightarrow \neg q) \wedge (q \Leftrightarrow \neg r) \wedge (r \Leftrightarrow \neg p))$

Converting to CNF,

$$((p \Leftrightarrow \neg q) \wedge (q \Leftrightarrow \neg r) \wedge (r \Leftrightarrow \neg p))$$

By using Equivalence law,

$(p \Rightarrow \neg q) \wedge (\neg q \Rightarrow p) \wedge (q \Rightarrow \neg r) \wedge (\neg r \Rightarrow q) \wedge (r \Rightarrow \neg p) \wedge (\neg p \Rightarrow r)$

By using Implication law,

$(\neg p \vee \neg q) \wedge (q \vee p) \wedge (\neg q \vee \neg r) \wedge (r \vee q) \wedge (\neg r \vee \neg p) \wedge (p \vee r)$

Which is now in CNF form.

Clause 1: $(\neg p \vee \neg q)$; Clause 2: $(q \vee p)$; Clause 3: $(\neg q \vee \neg r)$; Clause 4: $(r \vee q)$;
Clause 5: $(\neg r \vee \neg p)$; Clause 6: $(p \vee r)$

Choosing Clause 1 and Clause 2:

$(\neg p \vee \neg q)$, $(q \vee p)$

By unit resolution , the complimentary pair is deleted and it returns an empty state.

$\neg S = (\ )$ [Empty set]

Choosing Clause 3 and Clause 4:

$(\neg q \vee \neg r)$, $(r \vee q)$

This also returns an empty state by deleting the complimentary pairs.

$\neg S = (\ )$ [Empty set]

Choosing Clause 5 and Clause 6:

$(\neg r \vee \neg p)$, $(p \vee r)$

This also returns an empty state by deleting the complimentary pairs.

$\neg S = (\ )$ [Empty set]

$\therefore$ $\neg S = (\ )$ which is an empty set or a contradiction

Since $\neg S$ is a contradiction or false, S = True or Tautology

Hence the given claim is a Tautology.