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CS 480 Fall 2022 Written Assignment #04

Due: **Wednesday, November 30, 11:00 PM CST**

Points: **45**

Instructions:

1. Use this document template to report your answers. Name the complete document as follows:

`LastName_FirstName_CS480_Written04.doc`

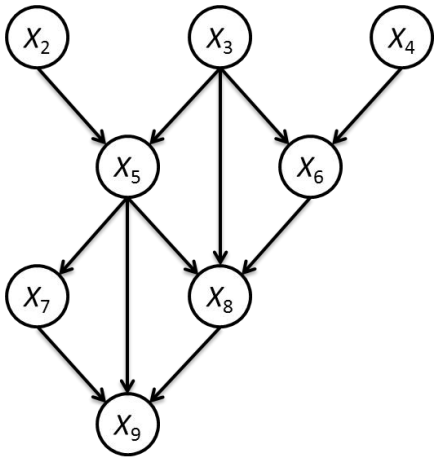
2. Submit the final document to Blackboard Assignments section before the due date. No late submissions will be accepted.

Objectives:

1. (10 points) Demonstrate your understanding of Bayes Networks.
2. (35 points) Demonstrate your understanding of Decision Networks.

Problem 1 [10 pts]:

We are given the following Bayesian network over X_2, X_3, …, X_9. Note that there is no X_1.



What is the Bayesian network factorization of the joint probability $\mathbf{P}(X_2, X_3, \ldots, X_9)$?

Your solution:
The prior probability is for X_2 , X_3, X_4 For X_2 , the prior probability is $P(X_2)$ For X_3 , the prior probability is $P(X_3)$ For X_4 , the prior probability is $P(X_4)$ The posterior probability is for X_5, X_6,X_7,X_8,X_9 For X_5, the posterior probability is $P(X_5\|X_2,X_3)$, X_5 is posterior probability and it depends on X_3 and X_2.

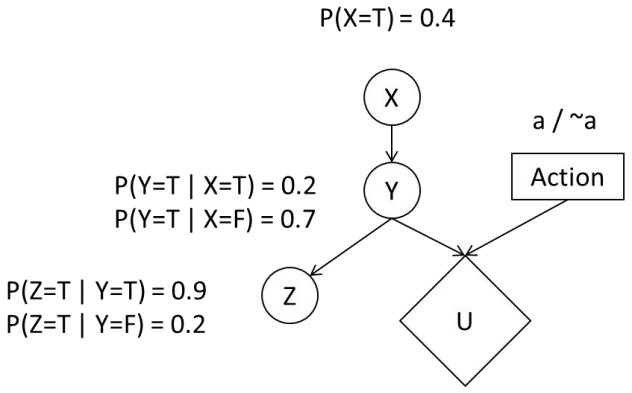
For X_6, the posterior probability is $P(X_6|X_3,X_4)$, X_6 is posterior probability and it depends on X_3 and X_4.
For X_7, the posterior probability is $P(X_7|X_5)$, X_7 is posterior probability and it depends on X_5.
For X_8, the posterior probability is $P(X_8|X_5,X_6,X_3)$, X_8 is posterior probability and it depends on X_5,X_6 and X_3.
For X_9, the posterior probability is $P(X_9|X_7,X_5,X_8)$, X_9 is posterior probability and it depends on X_7,X_5 and X_8.

By chain rule,

$P(X_2) * P(X_3) * P(X_4) * P(X_5|X_2,X_3) * P(X_6|X_3,X_4) * P(X_7|X_5) * P(X_8|X_5,X_6,X_3) * P(X_9|X_7,X_5,X_8)$

Problem 2 [35 pts]:

We are given the following decision network:



Y	Action	U(Y, Action)
T	a	800
T	~a	400
F	a	200
F	~a	1000

a) Which action should be taken? Justify your decision. **[7 pts]**

Your solution:

To find the probability value of P(Y = T)

$P(Y = T) = [P(Y = T|X = F) * P(X = F)] + [P(Y = T|X = T) * P(X = T)]$

$= 0.7 * 0.6 + 0.2 * 0.4$

= 0.50

To find the probability value of P(Y = F)
P(Y = F) = 1- P(Y=T)
=1 - 0.50
= 0.50

To find the utility value - EU(A)
EU(A) = P(Y=T) * U(Y=T,A=T) + P(Y=F) * U(Y=F, A=T)
= 0.50 * 800 + 0.50 *200
= 500

To find the utility value - EU(¬A)
EU(¬A) = P(Y=T) * U(Y=T, ¬A=F) + P(Y=F) * U(Y=F, ¬A=F)
= 0.50 * 400 + 0.50 * 1000
= 700

Choosing the maximum utility value = Max(500, 700) = 700
So the max value is 700 we have to take EU(¬A)

To find the EU (A|Z)
EU (A|Z) = P(Y|Z) * U(Y, A)

To find the EU (¬A|¬Z)
EU (¬A|¬Z) = P(Y|¬Z) * U(Y, ¬A)

To find the EU (A|¬Z)
EU (A|¬Z) = P(Y|¬Z) * U(Y, A)

To find the EU(¬A|Z)
EU(¬A|Z)= P(Y|Z) * U(Y, ¬A)

To find the value for P(Y|Z)

Find the value for P(Z = T)
P(Z = T) = P(Z=T |Y = T) * P(Y = T) + P(Z = T| ¬Y = F) * P(Y = F)
= 0.9 * 0.50 + 0.2 * 0.50
= 0.55

Find the value for P(Z = F)
P(Z = F) = 1- P(Z=T) = 1 – 0.55
= 0.45

To find the value for P(Y|Z)
P(Y,Z) = P(Y|Z)P(Z)
P(Y|Z) = P(Y,Z) / P(Z)

P(Y=T|Z=T) = (P(Z = T|Y = T) * P(Y = T)) / P(Z = T)
= 0.9 * 0.50 / 0.55
= 0.8181

P(Y = T|Z = F) = (P(¬Z = F| Y= T) * P(Y=T)) / P(¬Z = F)
= 0.1 * 0.50 / 0.45
= 0.1111111

P(Y=F|Z=T) = P(Z=T|¬Y=F) * P(Y =F) / P(Z=T)
= 0.2 * 0.5 / 0.55
= 0.1818

P(Y =F|Z=F) = 1 – 0.11111 = 0.8889

To find the expected utility EU(A|Z)

To find the value for EU (A=T|Z = T)

EU (A=T|Z = T) = P(Y=T|Z=T) * U(Y=T, A=T)
= P(Y=T|Z=T)*U(A=T,Y=T) + P(Y=F|Z=T)*U(A=T,Y=F)
= 0.8181 * 800 + 0.1818 * 200
= 690.84

To Find the value for EU (A = F|Z = T)

EU (A = F|Z = T) = P(Y=T|Z=T) * U(Y=T, ¬A=F)
=P(Y=T|Z=T)*U(¬A=F,Y=T) + P(¬Y=F|Z=T)*U(¬A=F, ¬Y=F)
= 0.8181 * 400 + 0.1818 * 1000
= 509.04

To Find the value for EU(A =T|Z = F)

EU(A =T|Z = F) = P(Y=T|¬Z=F) * U(Y=T, A=T)
= P(Y=T|¬Z=F)*U(A=T,Y=T) + P(Y=F|¬Z=F)*U(A=T, Y=F)
= 0.1111 * 800 + 0.8889 * 200
= 266.66

To find the value for EU(A= F| Z = F)

EU(A= F| Z = F) = P(Y=T|¬Z=F) * U(Y=T, ¬ A=F)
= P(Y=T|¬Z=F)*U(¬A=F,Y=T) + P(Y=F|¬Z=F)*U(¬A=F, Y=F)
= 0.1111 * 400 + 0.8889 * 1000
= 933.34

MEU (A=T|Z = T) = 690.84
MEU(A = F| Z = F) = 933.34

b) What is the value of information of Z? Justify your decision. **[8 pts]**

Your solution:

MEU(A) = 700
Use the value from the previous subdivision

MEU(a1|e1) = MEU(A=T|Z=T) = 690.84
MEU(a2|e2) = MEU(¬A=F|¬Z=F) = 933.34

VPI(Z=T) = (P(Z=T)* MEU(A=T|Z=T) +P(¬Z=F) * MEU(¬A=F|¬Z=F)) - MEU(A=T)

= (0.55 * 690.84 + 0.45 * 933.34) - 700
= 800 - 700
= 100

c) What is the value of information of X? Justify your decision. **[10 pts]**

Your solution:

To find P(Y,X)

P(Y,X) = P(Y|X)P(X)
P(Y|X) = P(Y,X) / P(X)
P(Y =T|X=T) = 0.2
P(Y=F|X=T) = 1 – 0.2 = 0.8

P(Y=T|X=F) = 0.7
P(Y=F|X=F) = 1 – 0.7 = 0.3

EU (A=T|X=T) = P(Y=T|X=T) * U(Y=T, A=T)
=P(Y=T|X=T)*U(A=T,Y=T) + P(¬Y=F|X=T)*U(A=T, ¬Y=F)
= 0.2 * 800 + 0.8 * 200
= 320

EU (¬A=F|X=T) = P(Y=T|X=T) * U(Y=T, ¬A=F)
= P(Y=T|X=T) * U(Y=T, ¬A=F) + P(¬Y=F|X=T) * U(¬Y=F, ¬A=F)
= 0.2 * 400 + 0.8 * 1000
= 880

EU (A=T|¬X=F) = P(Y=T|¬X=F) * U(Y=T, A=T)
= P(Y=T|¬X=F) * U(Y=T,A=T) + P(¬Y=F|¬X=F) * U(¬Y=F,A=T)
= 0.7 * 800 + 0.3 * 200
= 620

EU (¬A=F|¬X=F) = P(Y=T|¬X=F) * U(Y=T, ¬A=F)
= P(Y=T|¬X=F) * U(Y=T, ¬A=F) + P(¬Y=F|¬X=F) * U(¬Y=F, ¬A=F)
= 0.7 * 400 + 0.3 * 1000
= 580

To Find MEU value
MEU (A = F|X = T) = 880
MEU (A = T| X = F) = 620

Finding the VPI for X
VPI(X) = (P(X) * MEU(~A|X) + P(~X) * MEU(A|~X)) - MEU (A)
= (0.4 * 880 + 0.6 * 620) - 700
= 24

d) Given Z = T, what is the value of information of X? Justify your decision. **[10 pts]**

Your solution:

Find the P(Y|X,Z)
P(Y|X,Z) = P(X,Y,Z) / P(X,Z)
P(X,Z) = P(Z|X)P(X)

To Find the P(Z|X)
P(Z|X) = P(X,Z)/P(X)
= (P(X,Y,Z) + P(X,~Y,Z)) / P(X)
= (P(Z|X,Y)P(X,Y)*P(Z|X,~Y)P(X,~Y))/P(X)
=(P(Z|Y)*P(Y|X)*P(X) + P(Z|~Y) *P(~Y|X)*P(X))/P(X)

P(Z|X) =P(Z|y)P(Y|X) + P(Z|~Y)P(~Y|X)
P(X,Z) = P(Z|X)P(X)
= (P(Z|y)P(Y|X) + P(Z|~Y)P(~Y|X))*P(X)

P(Y|X,Z) = P(Z|Y)P(Y|X)P(X) / (P(Z|y)P(Y|X) + P(Z|~Y)P(~Y|X))*P(X)
= P(Z|Y)P(Y|X)/P(Z|Y)P(Y|X) + P(Z|~Y)P(~Y|X)
= (0.9 * 0.2) / ((0.9 * 0.2) + (0.2 * 0.8))
= 0.5294

P(Y|~X,Z) = P(Z|Y)P(Y|~X)/P(Z|Y)P(Y|~X) + P(Z|~Y)P(~Y|~X)
= (0.9 * 0.7) / ((0.9 * 0.7) + (0.2 * 0.3))
= 0.9130

P(~Y|X,Z) = P(Z|~Y)P(~Y|X) / P(Z|Y)P(Y|X) + P(Z|~Y)P(~Y|X)
= (0.2 * 0.8) / (((0.9 * 0.2) + (0.2 * 0.8))
=0.4706

P(~Y|~X,Z) = P(Z|~Y)P(~Y|~X)/P(Z|Y)P(Y|~X) + P(Z|~Y)P(~Y|~X)
= (0.2 * 0.3) / ((0.9 * 0.7) + (0.2 * 0.3))
= 0.0869

To find the EU(A|X,Z)
EU (A|X,Z) = P(Y|X,Z) * U(Y, A)
= P(Y|X,Z) * U(Y,A) + P(~Y|X,Z) * U(~Y,A)
= (0.5294 *800) + (0.4706 * 200)
= 517.64

To Find the EU(~A|X,Z)
EU (~A|X,Z) = P(Y|X,Z) * U(Y, ¬A)
= P(Y|X,Z) * U(Y,~A) + P(~Y|X,Z) * U(~Y,~A)
= (0.5294 *400) + (0.4706 * 1000)
= 682.36

To find the EU (~A|~X,Z)
EU (~A|~X,Z) = P(Y|~X,Z) * U(Y, ~A)
=P(Y|~X,Z) * U(Y, ~A) + P(~Y|~X,Z) * U(~Y, ~A)
= (0.9130 * 400) + (0.0869 * 1000)
= 452.1

To find the EU (A|~X,Z)
EU (A|~X,Z) = P(Y|~X,Z) * U(Y, A)
= P(Y|~X,Z) * U(Y, A) + P(~Y|~X,Z) * U(~Y, A)
= (0.9130 * 800) + (0.0869 * 200)
= 747.78

Find MEU value
MEU(A|~X,Z) = 747.78
MEU(~A|X,Z) = 682.36

Find VPI for X

VPI(X) = (P(~X) * MEU(A|~X,Z) + P(X) * MEU(A|~X,Z)) - MEU(A)
VPI(X) = (0.6 * 747.78 + 0.4 * 682.36) - 700
= 721.612 - 700
= 21.612