CS 480

Introduction to Artificial Intelligence

September 8, 2022

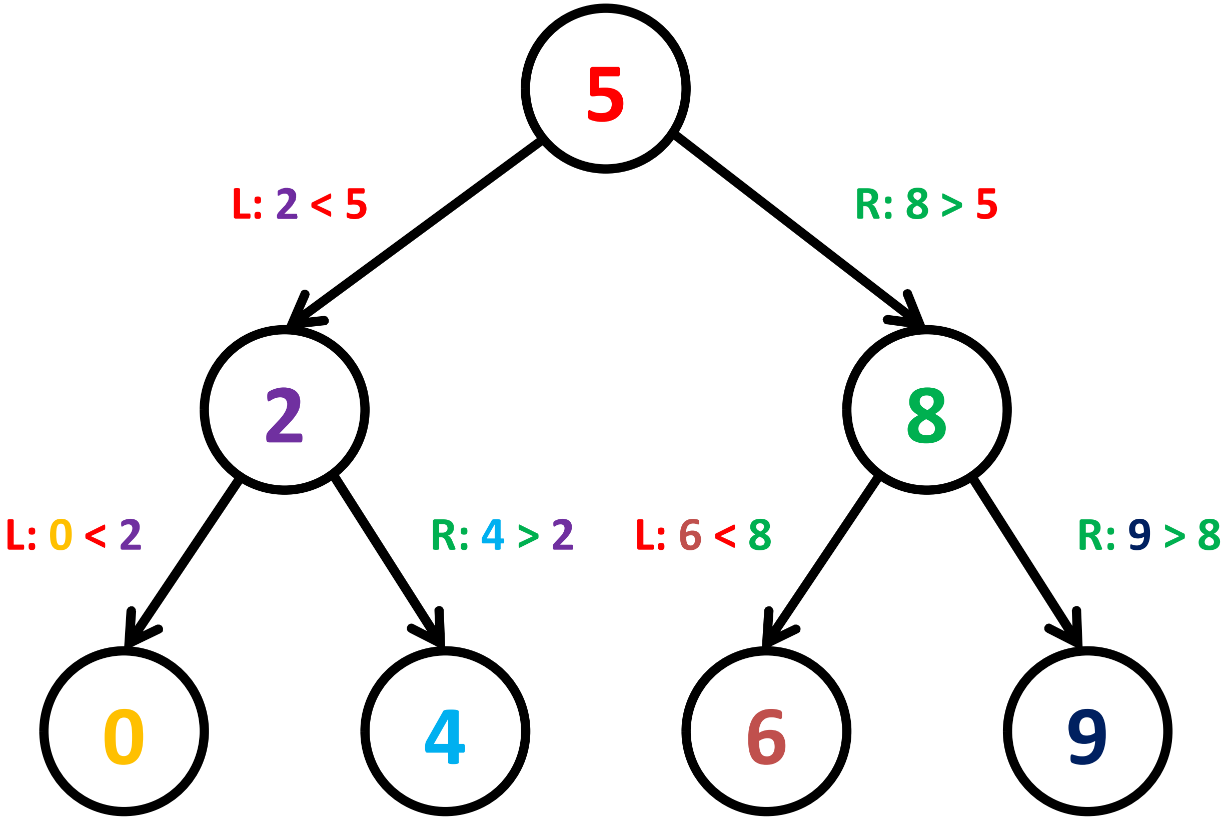
Announcements / Reminders

- Please follow the Week 03 To Do List instructions
- Quiz #01 will be posted on Friday
  - a separate announcement will be sent to you to let you know that it is available
  - you will have a week or more to complete it
- Written Assignment #01 will be posted this week
- Midterm Exam (consider fixed):
  - October 13th, 2022 during (Thursday) lecture time

Plan for Today

- **Data Structures: Review [OPTIONAL MATERIAL]**

Binary Search Tree



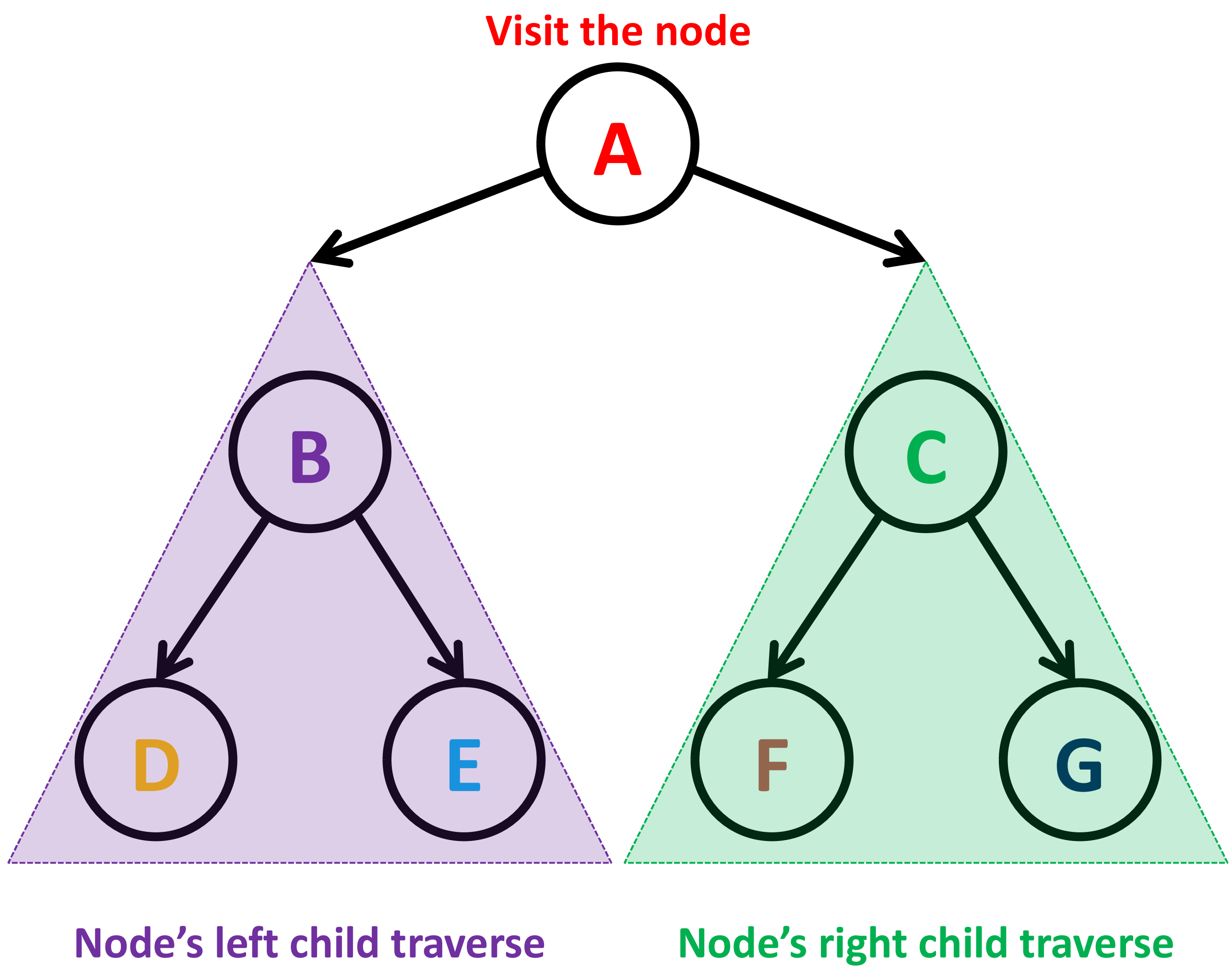
(Binary) Tree Traversals

- Tree Traversal: a process to visit all nodes in a tree is called a tree traversal. Typically used to find a key or process all or some keys.
- Traversal always starts at the root of the tree

Binary Tree Traversal involves three “visits”:

- The node is visited (the root)
- The node’s left child is traversed
- The node’s right child is traversed

Binary Tree Traversal



Binary Tree Traversals

The sequence in which those three "visits" are executed determins what kind of traversal it is.

Typical traversals:

- **Pre-order Traversal (Node, Left Child, Right Child)**
- **In-order Traversal (Left Child, Node, Right Child)**
- **Post-order Traversal (Left Child, Right Child, Node)**

- **Level-order (Level-by-level | left-to-right)**

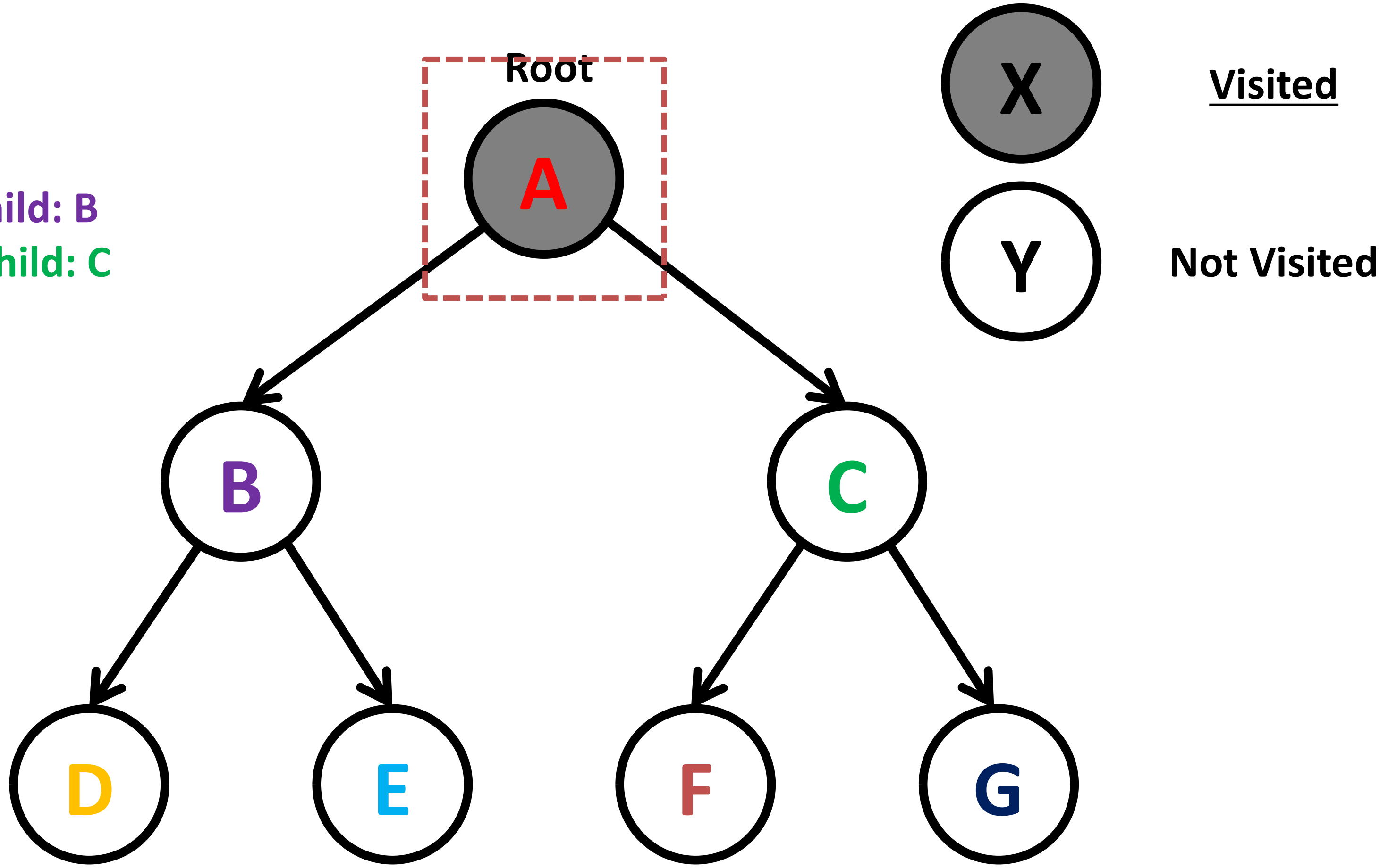
Binary Tree Traversals

- **Pre-order Traversal (Node, Left Child, Right Child)**
  - **See parent first, then look at left and right branches**
- **In-order Traversal (Left Child, Node, Right Child)**
  - **See left branch first, then look at the parent, and finally look at the right branch**
- **Post-order Traversal (Left Child, Right Child, Node)**
  - **Look at left and right branches first, and then see parent**

Pre-order (Node, Left, Right)

Step 1:

Node: A
Node's Left Child: B
Node's Right Child: C

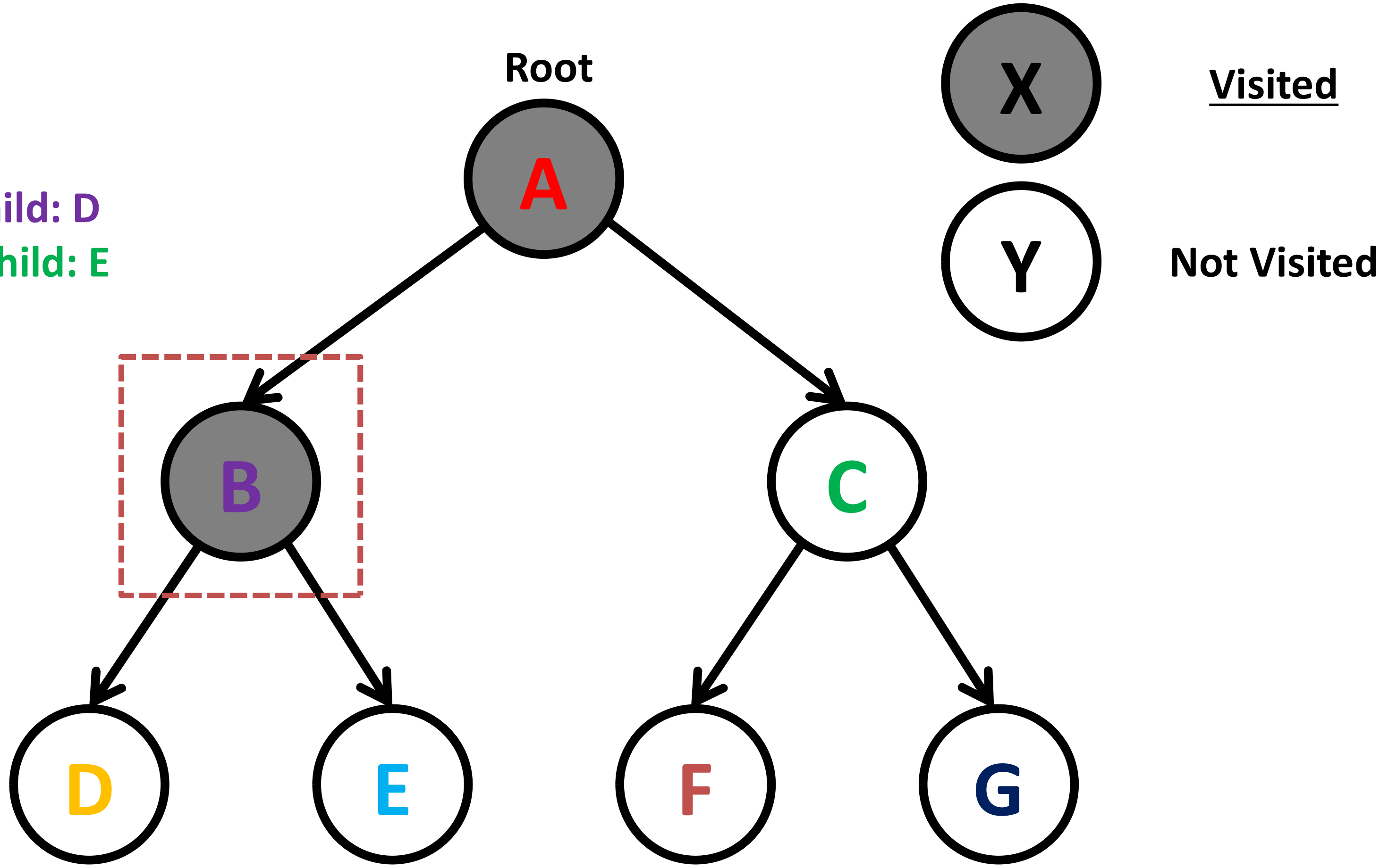


Nodes traversed so far: A

Pre-order (Node, Left, Right)

Step 2:

Node: B
Node's Left Child: D
Node's Right Child: E

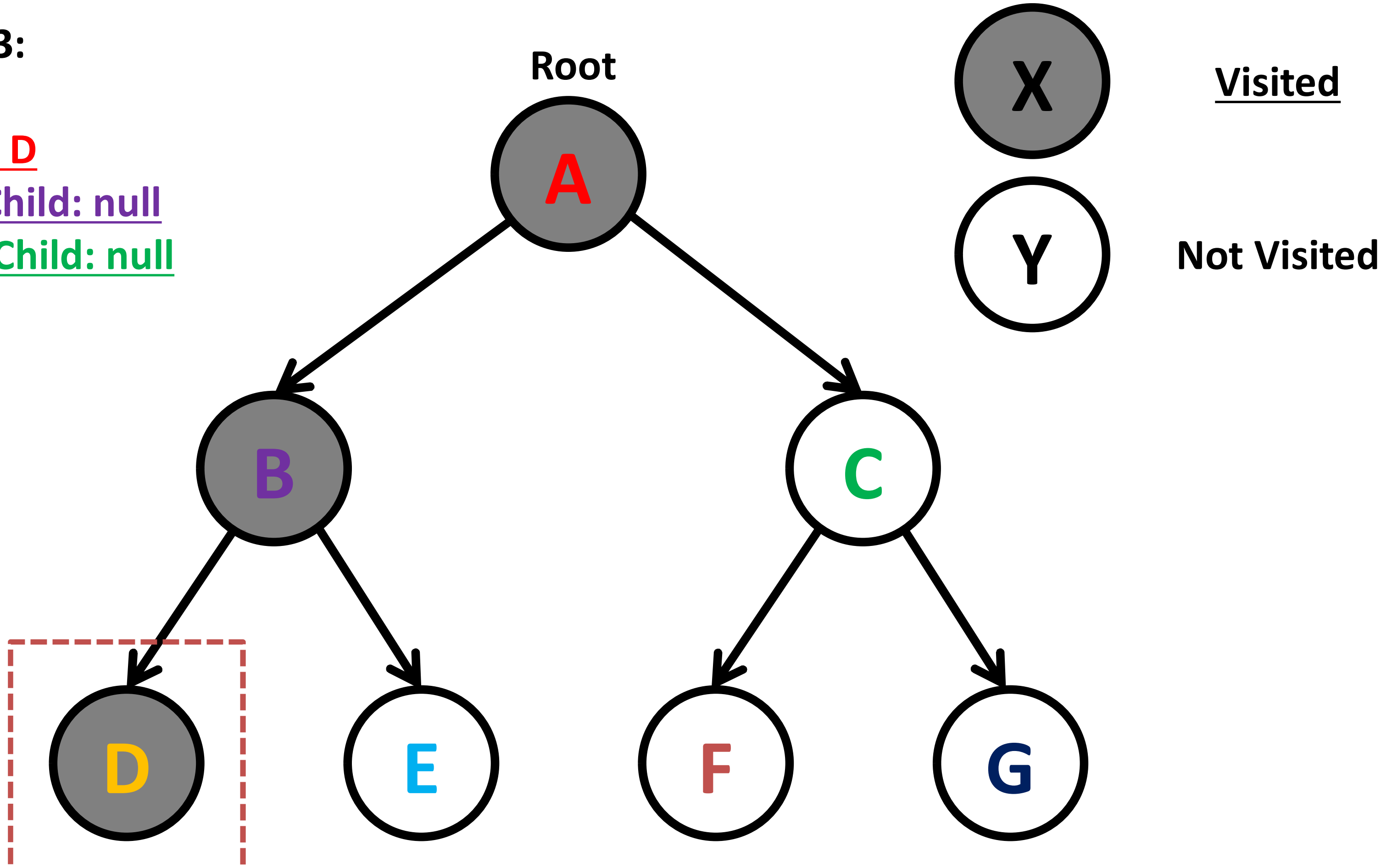


Nodes traversed so far: A B

Pre-order (Node, Left, Right)

Step 3:

Node: D
Node's Left Child: null
Node's Right Child: null

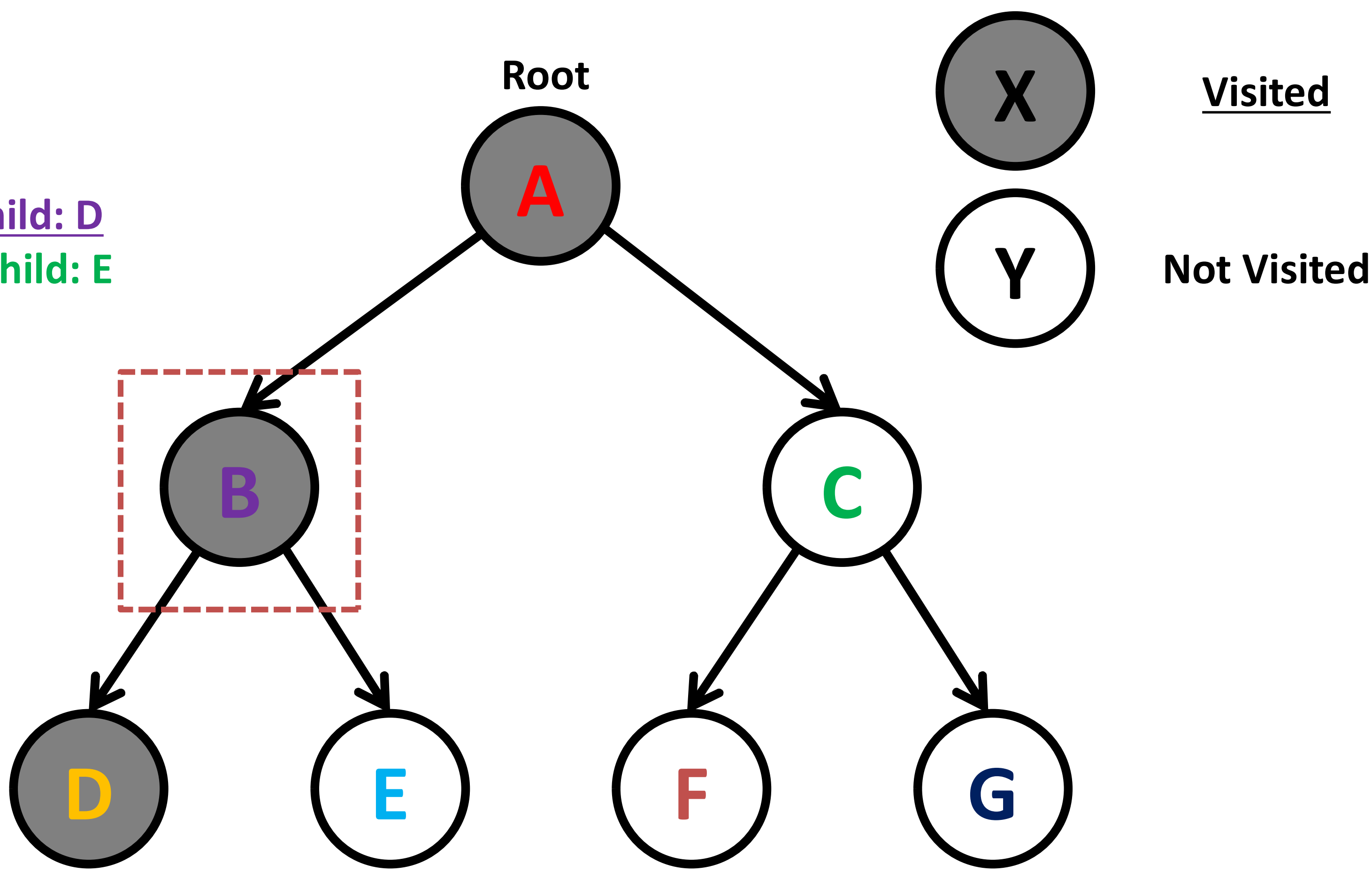


Nodes traversed so far: A B D

Pre-order (Node, Left, Right)

Step 4:

Node: B
Node's Left Child: D
Node's Right Child: E

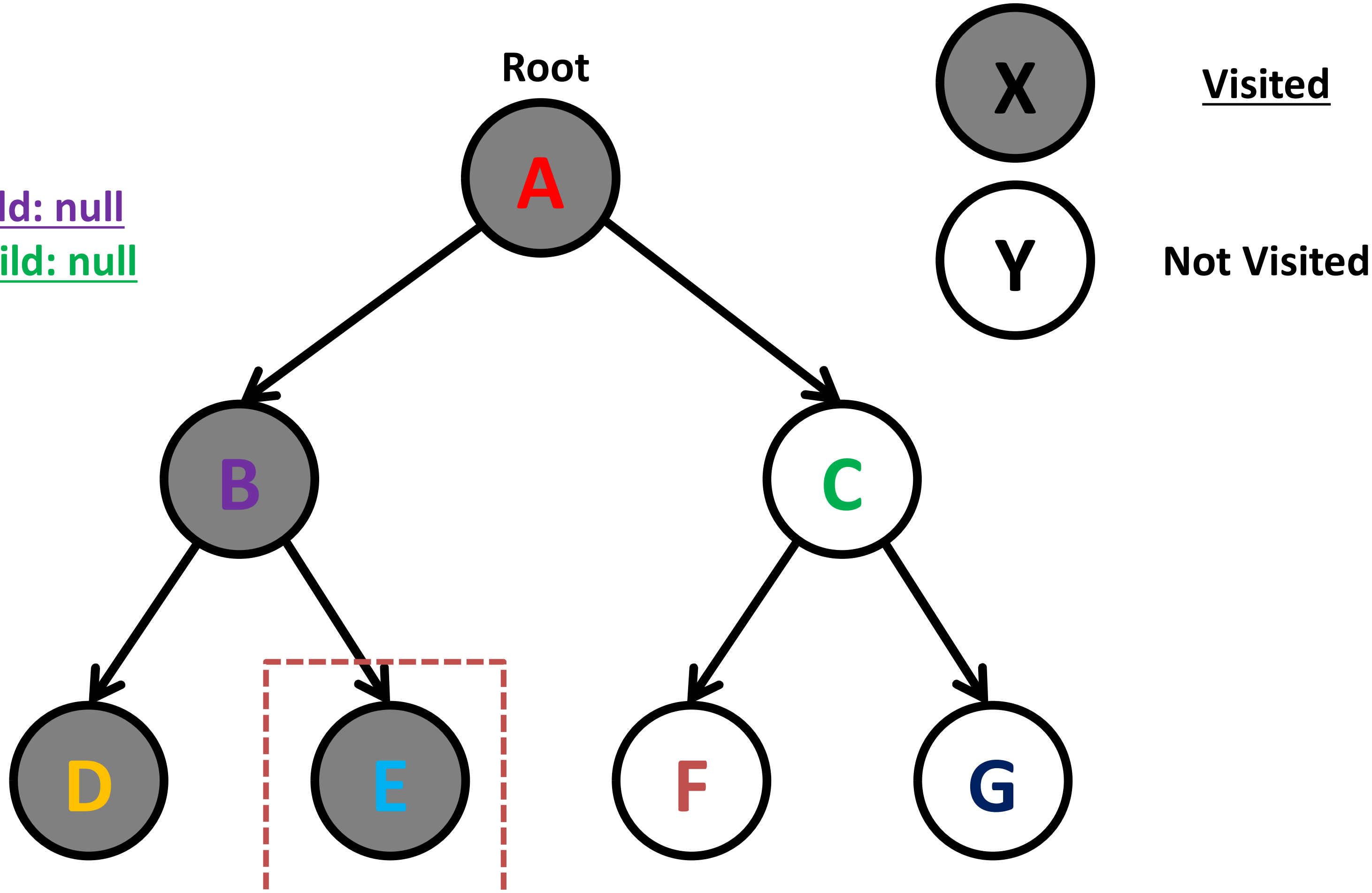


Nodes traversed so far: A B D

Pre-order (Node, Left, Right)

Step 5:

Node: E
Node's Left Child: null
Node's Right Child: null

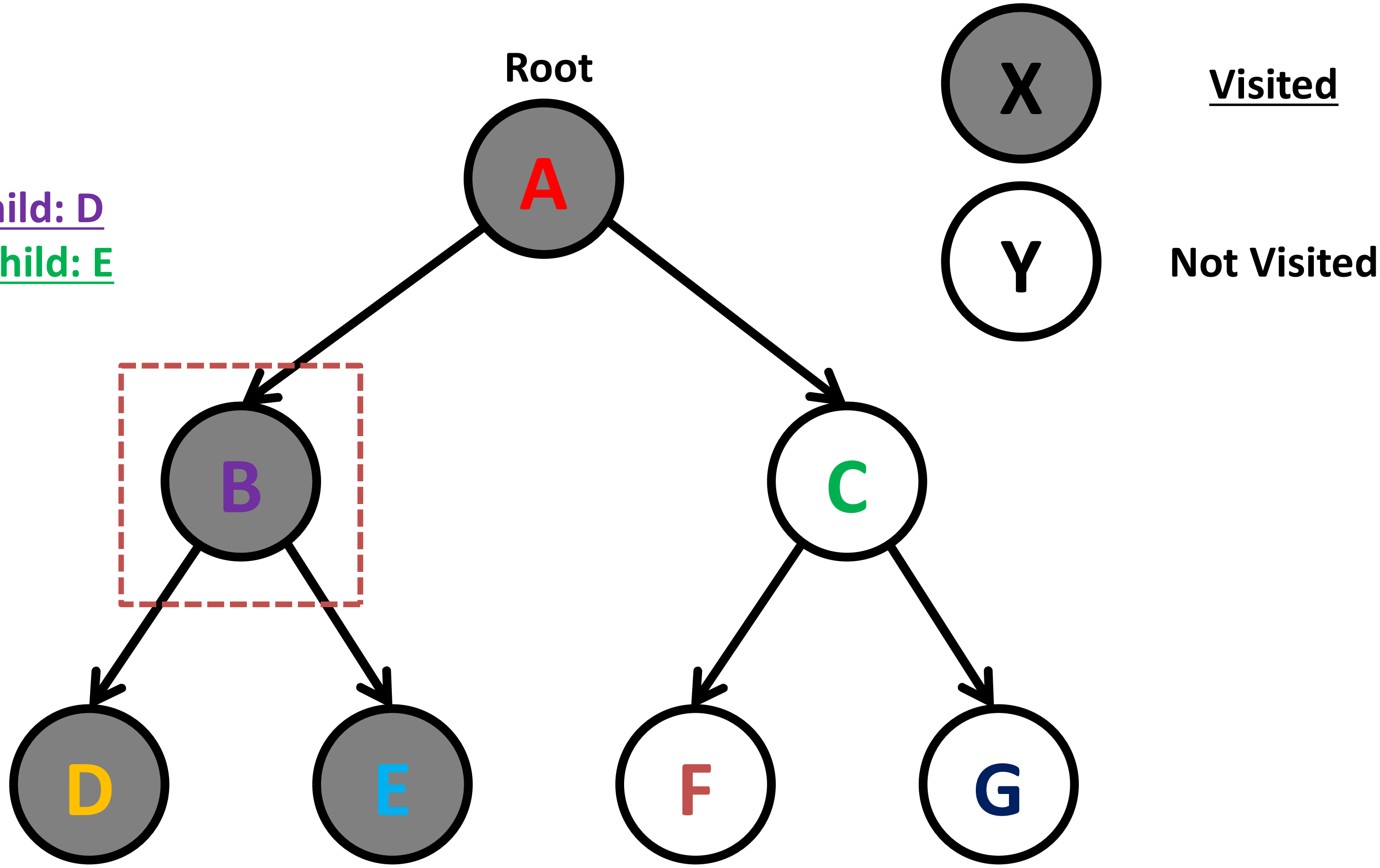


Nodes traversed so far: A B D E

Pre-order (Node, Left, Right)

Step 6:

Node: B
Node's Left Child: D
Node's Right Child: E

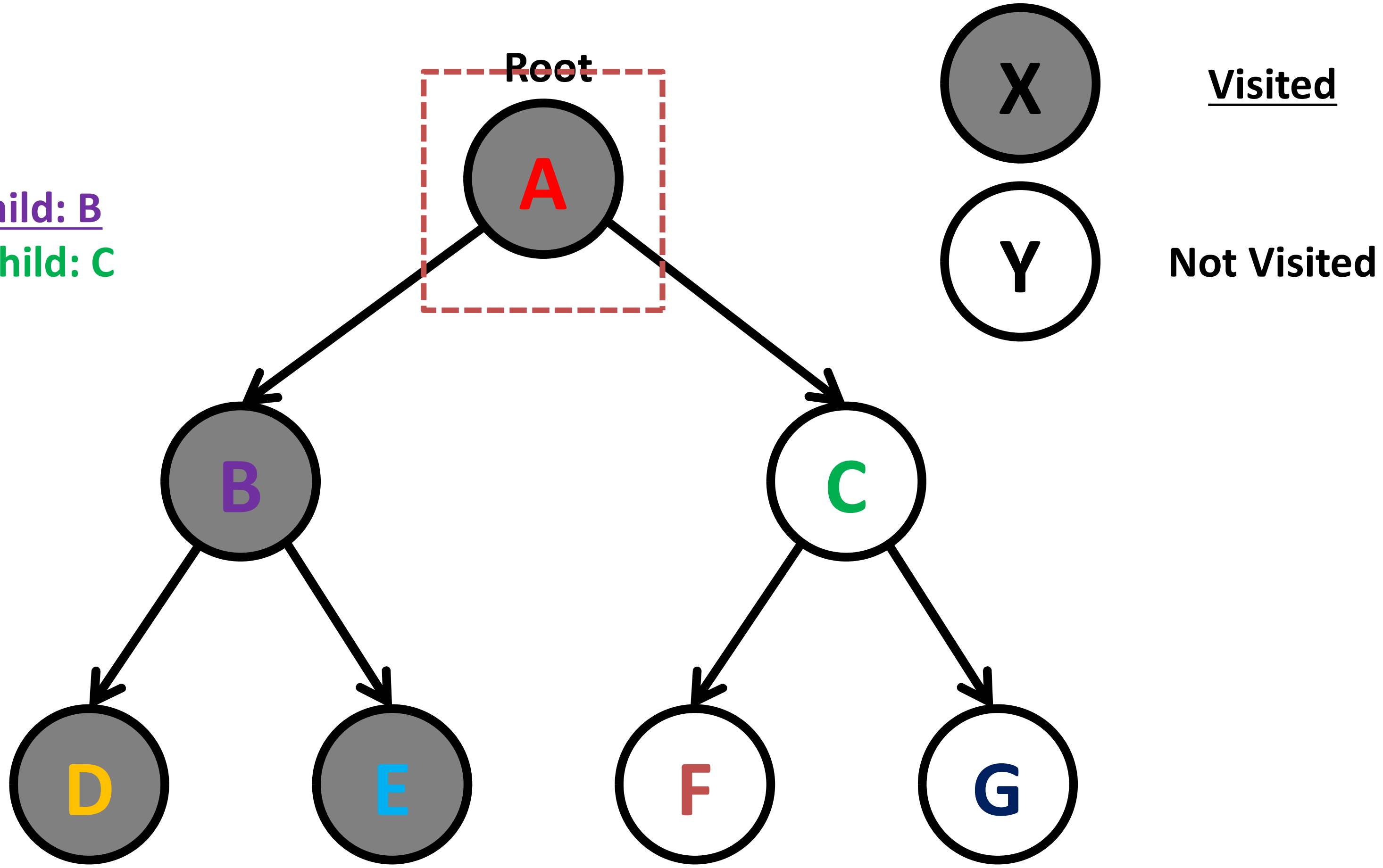


Nodes traversed so far: A B D E

Pre-order (Node, Left, Right)

Step 7:

Node: A
Node's Left Child: B
Node's Right Child: C

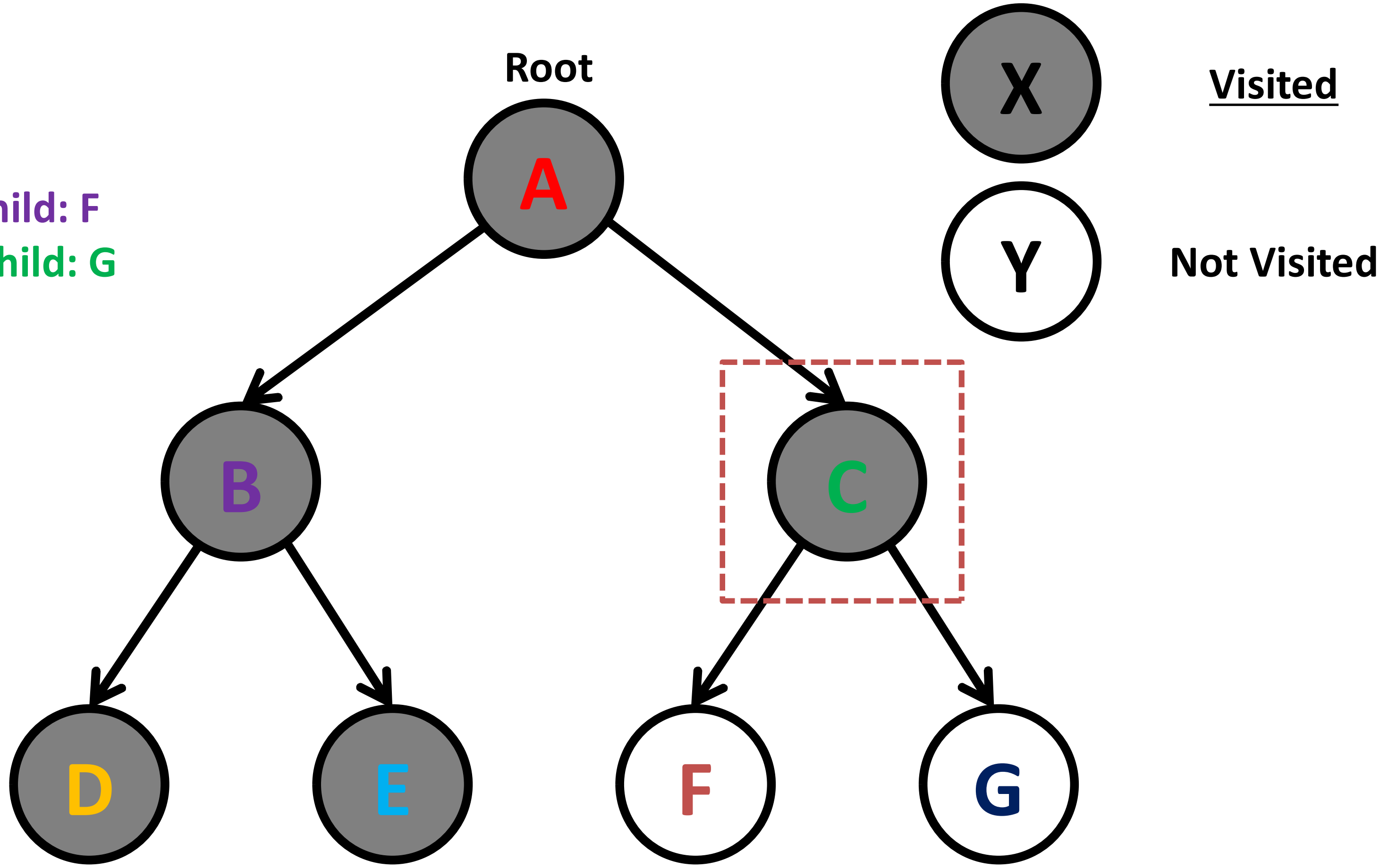


Nodes traversed so far: A B D E

Pre-order (Node, Left, Right)

Step 8:

Node: C
Node's Left Child: F
Node's Right Child: G

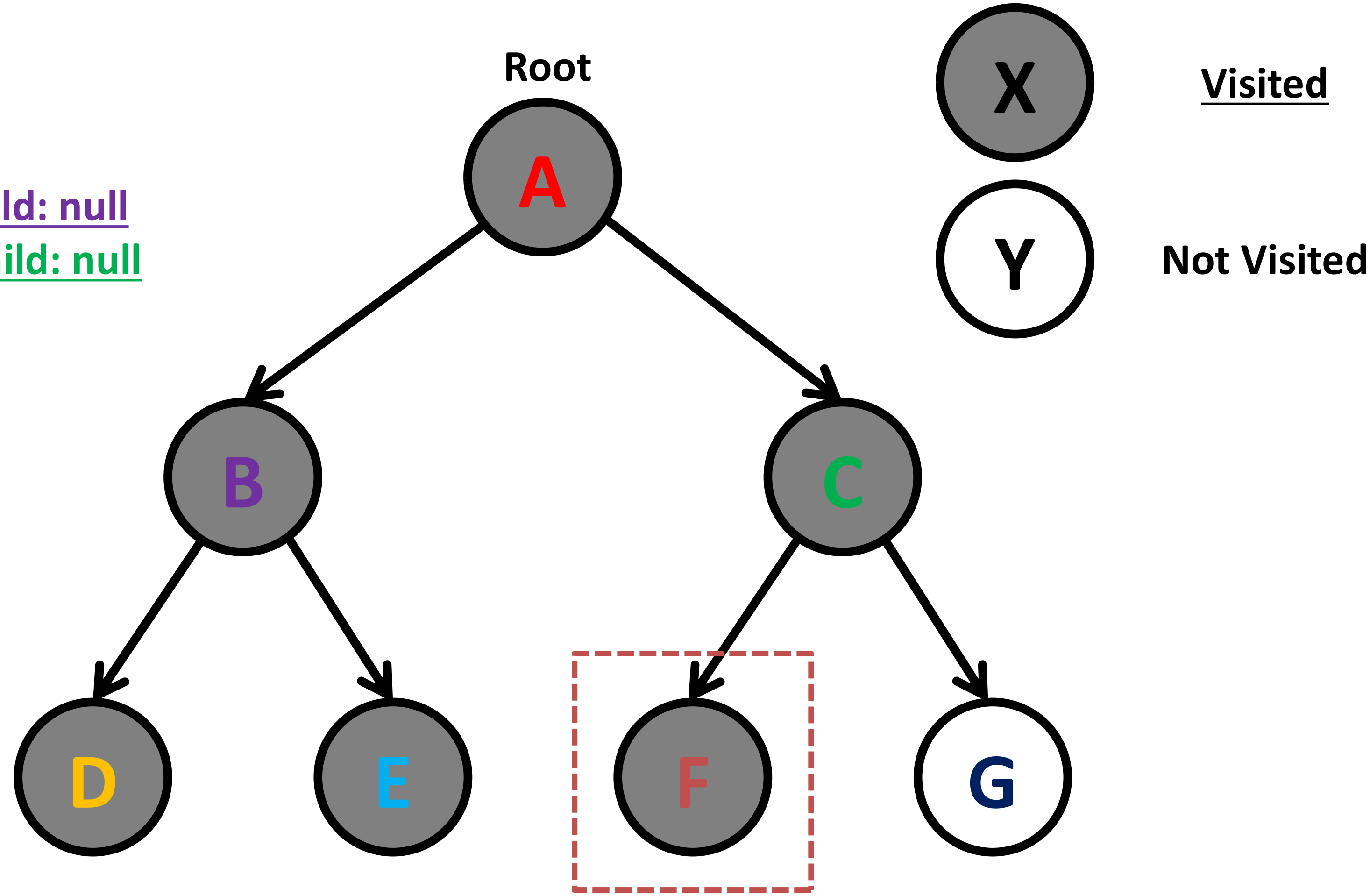


Nodes traversed so far: A B D E C

Pre-order (Node, Left, Right)

Step 9:

Node: F
Node's Left Child: null
Node's Right Child: null

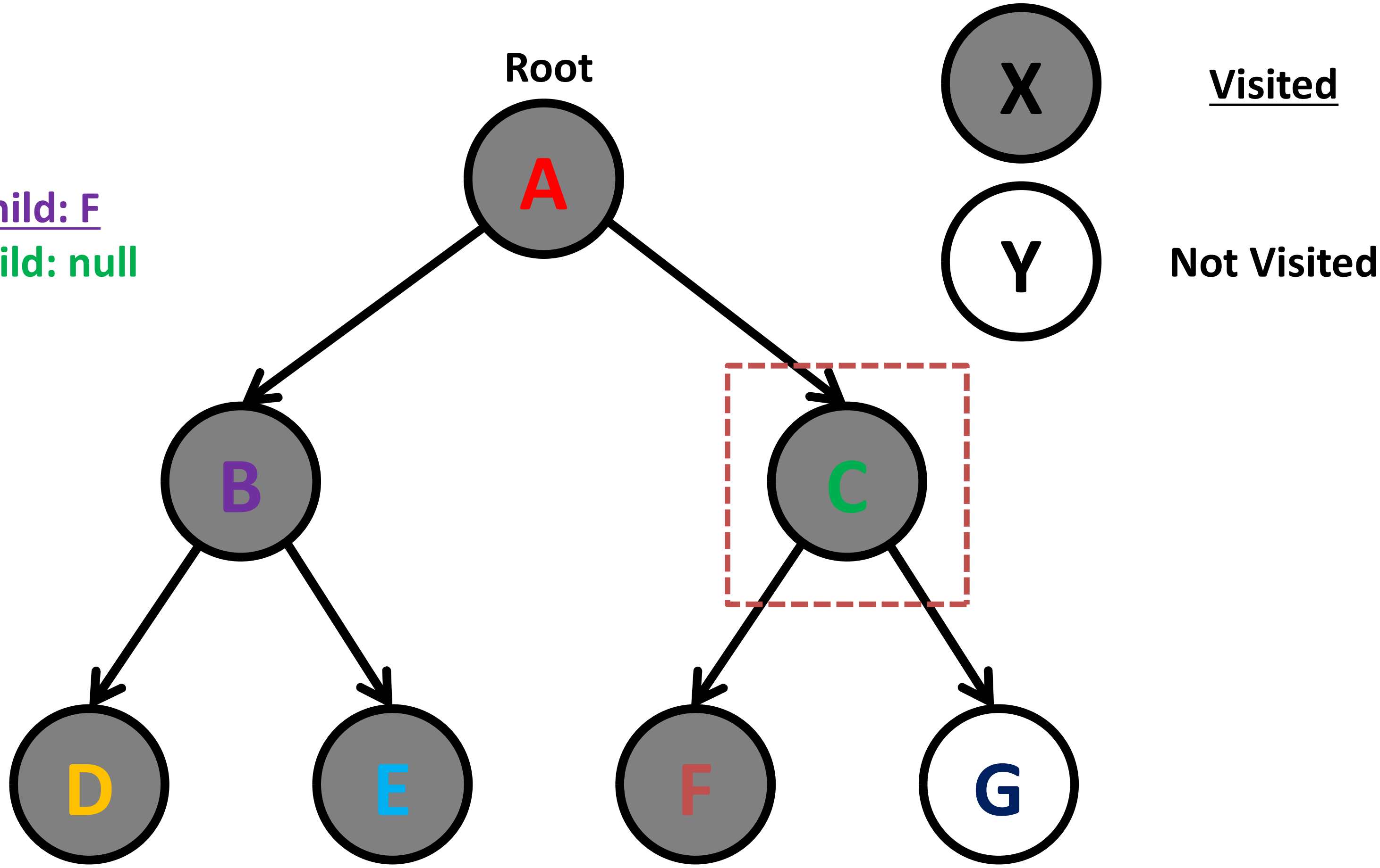


Nodes traversed so far: A B D E C F

Pre-order (Node, Left, Right)

Step 10:

Node: C
Node's Left Child: F
Node's Right Child: null

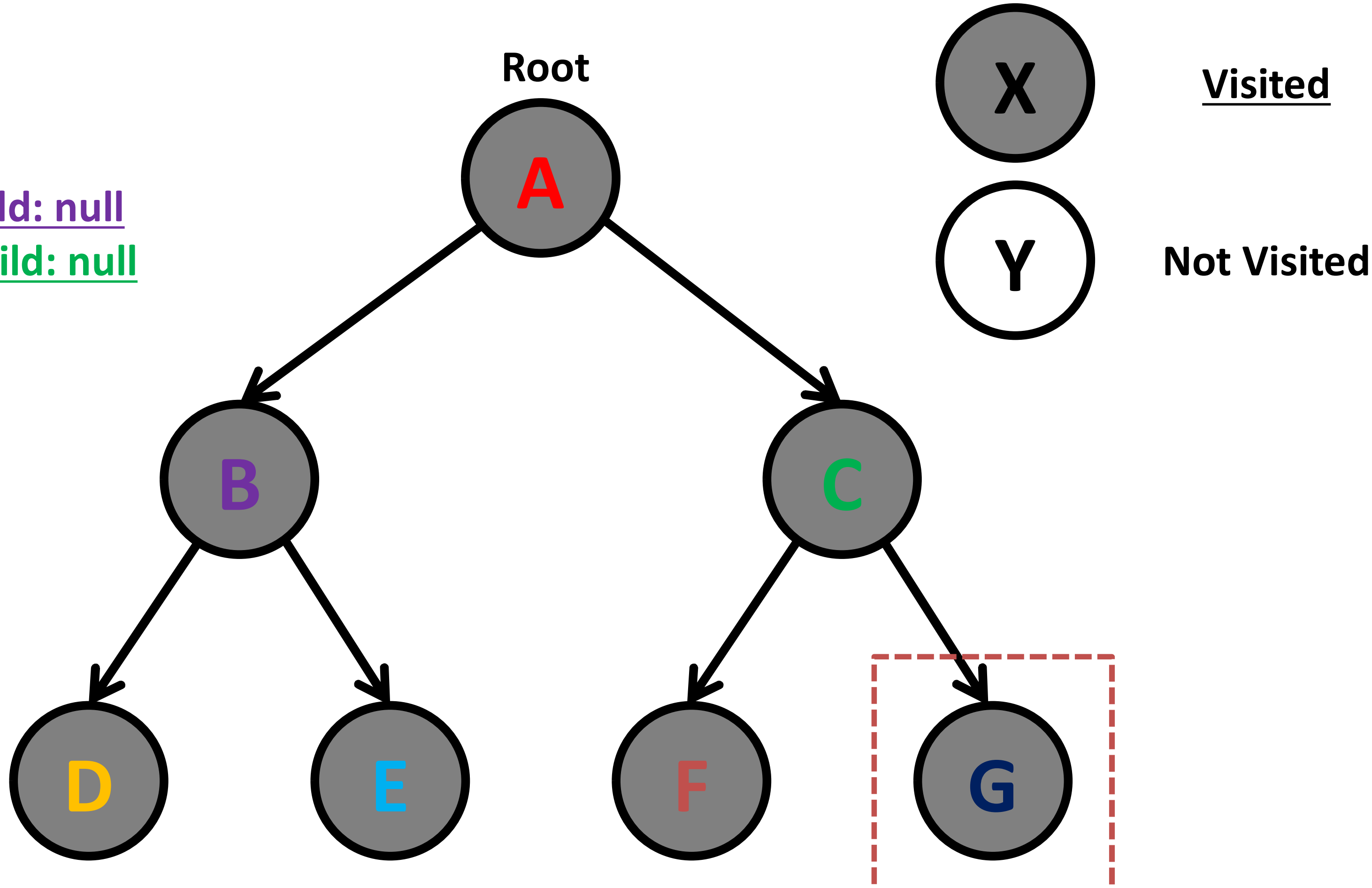


Nodes traversed so far: A B D E C F

Pre-order (Node, Left, Right)

Step 11:

Node: G
Node's Left Child: null
Node's Right Child: null

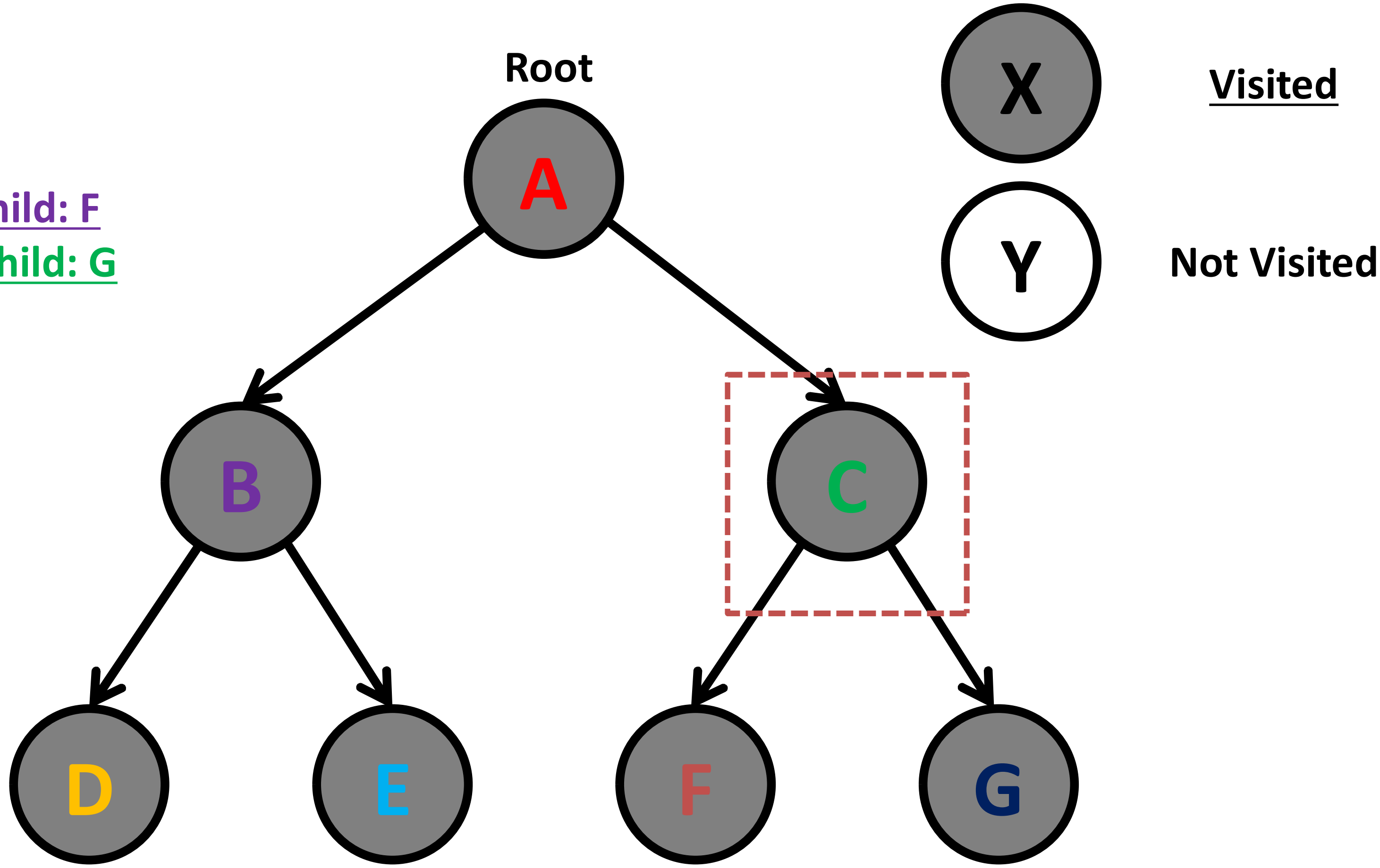


Nodes traversed so far: A B D E C F G

Pre-order (Node, Left, Right)

Step 12:

Node: C
Node's Left Child: F
Node's Right Child: G

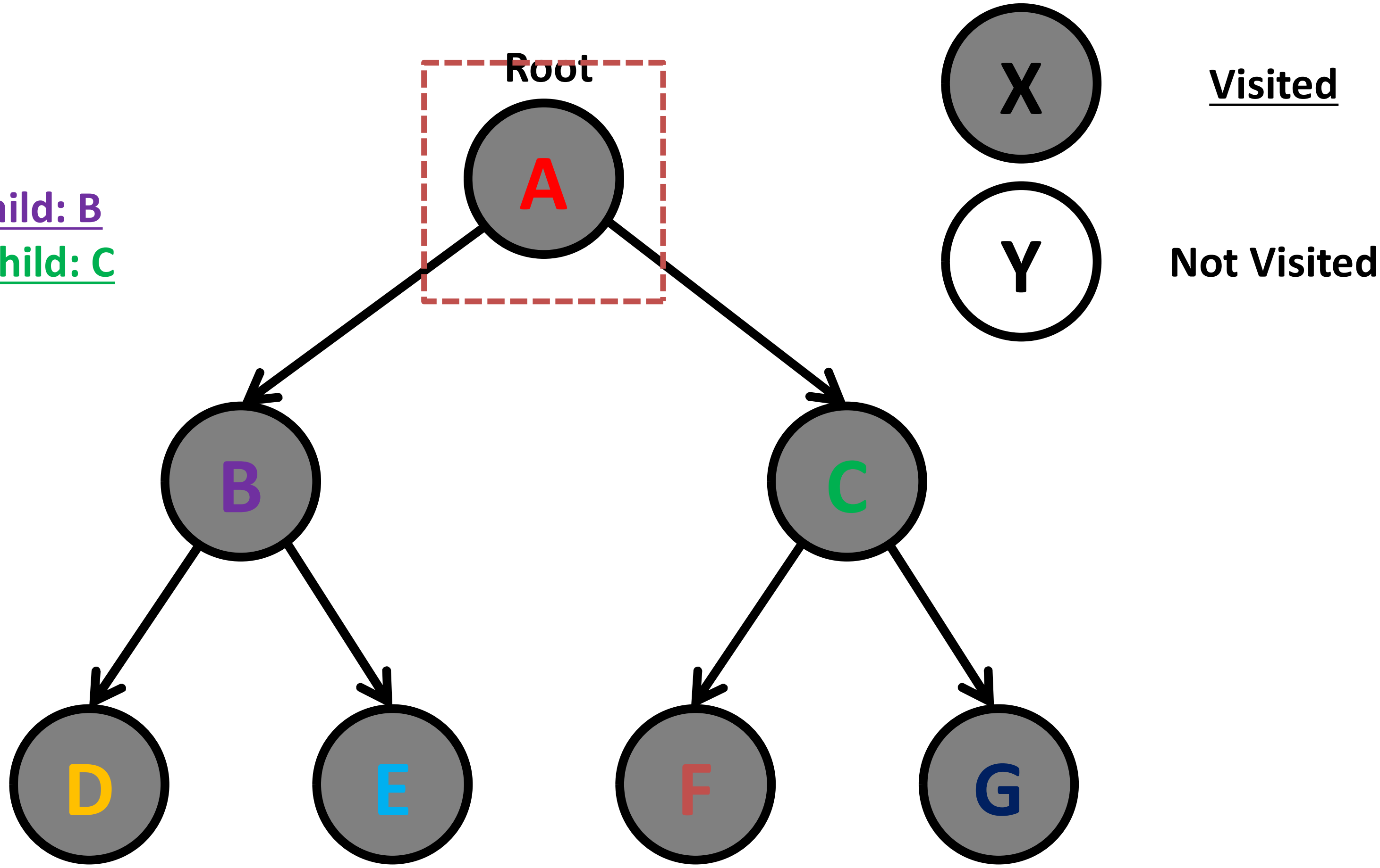


Nodes traversed so far: A B D E C F G

Pre-order (Node, Left, Right)

Step 13:

Node: A
Node's Left Child: B
Node's Right Child: C

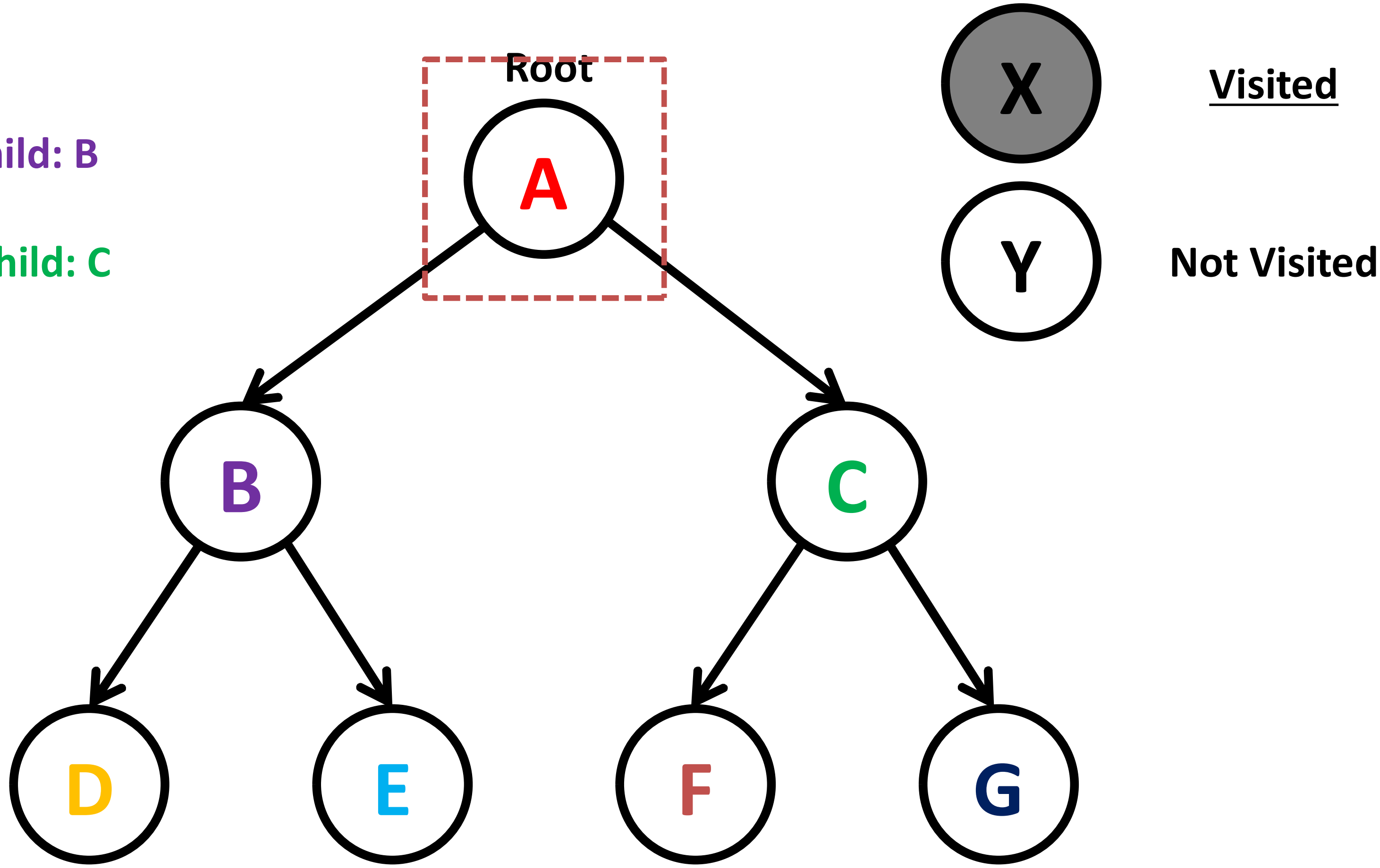


Nodes traversed so far: A B D E C F G

In-order (Left, Node, Right)

Step 1:

Node’s Left Child: B
Node: A
Node’s Right Child: C

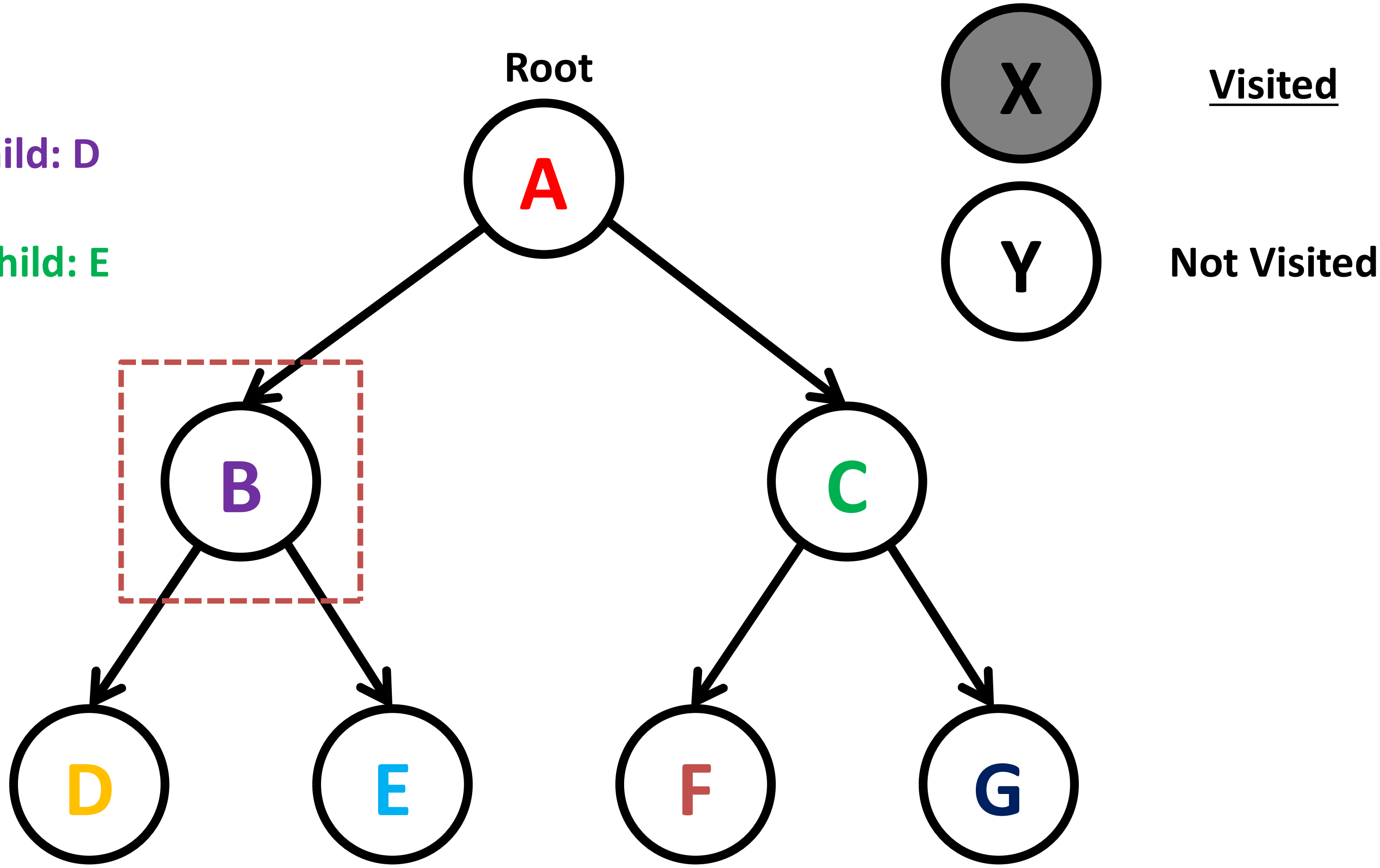


Nodes traversed so far:

In-order (Left, Node, Right)

Step 2:

Node's Left Child: D
Node: B
Node's Right Child: E

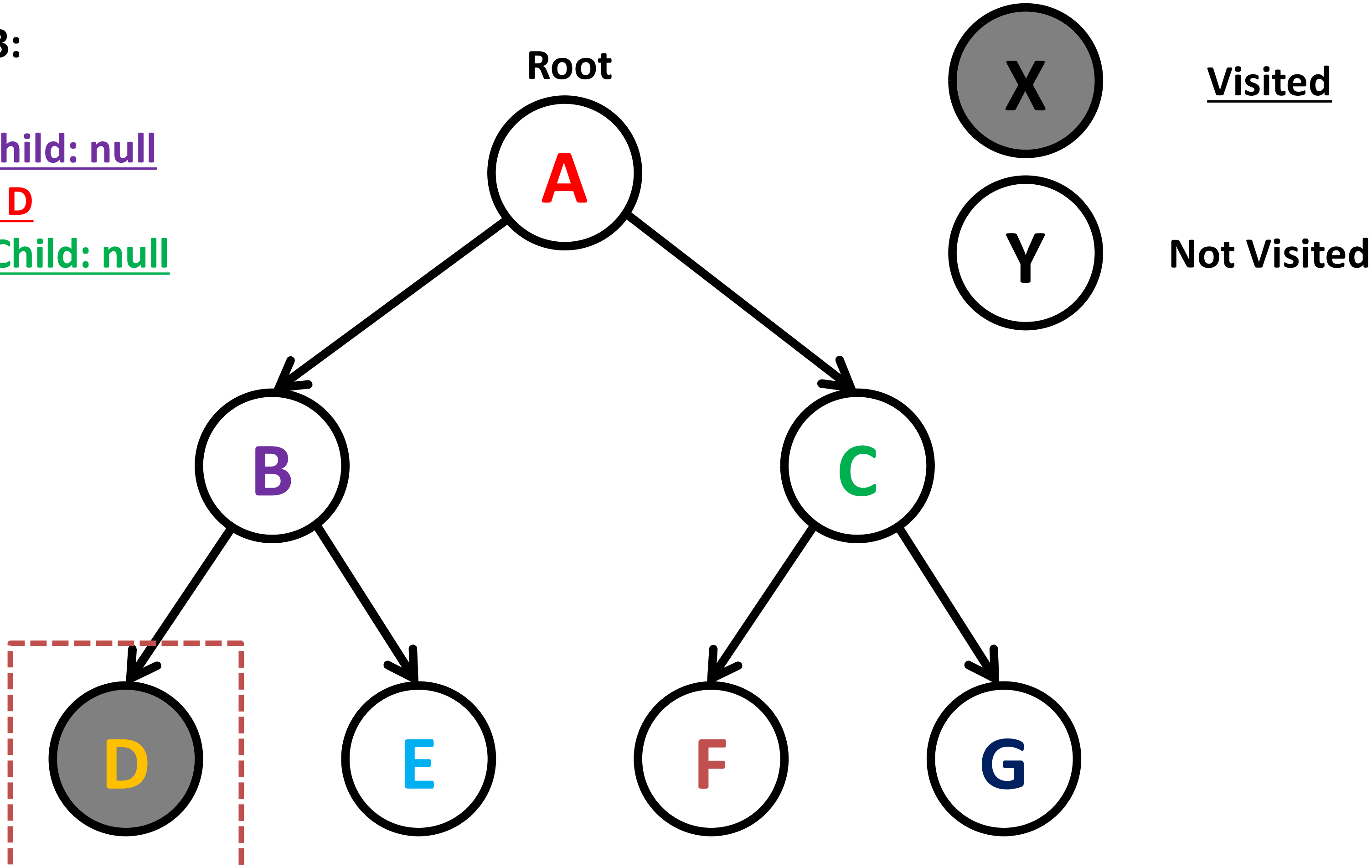


Nodes traversed so far:

In-order (Left, Node, Right)

Step 3:

Node's Left Child: null
Node: D
Node's Right Child: null

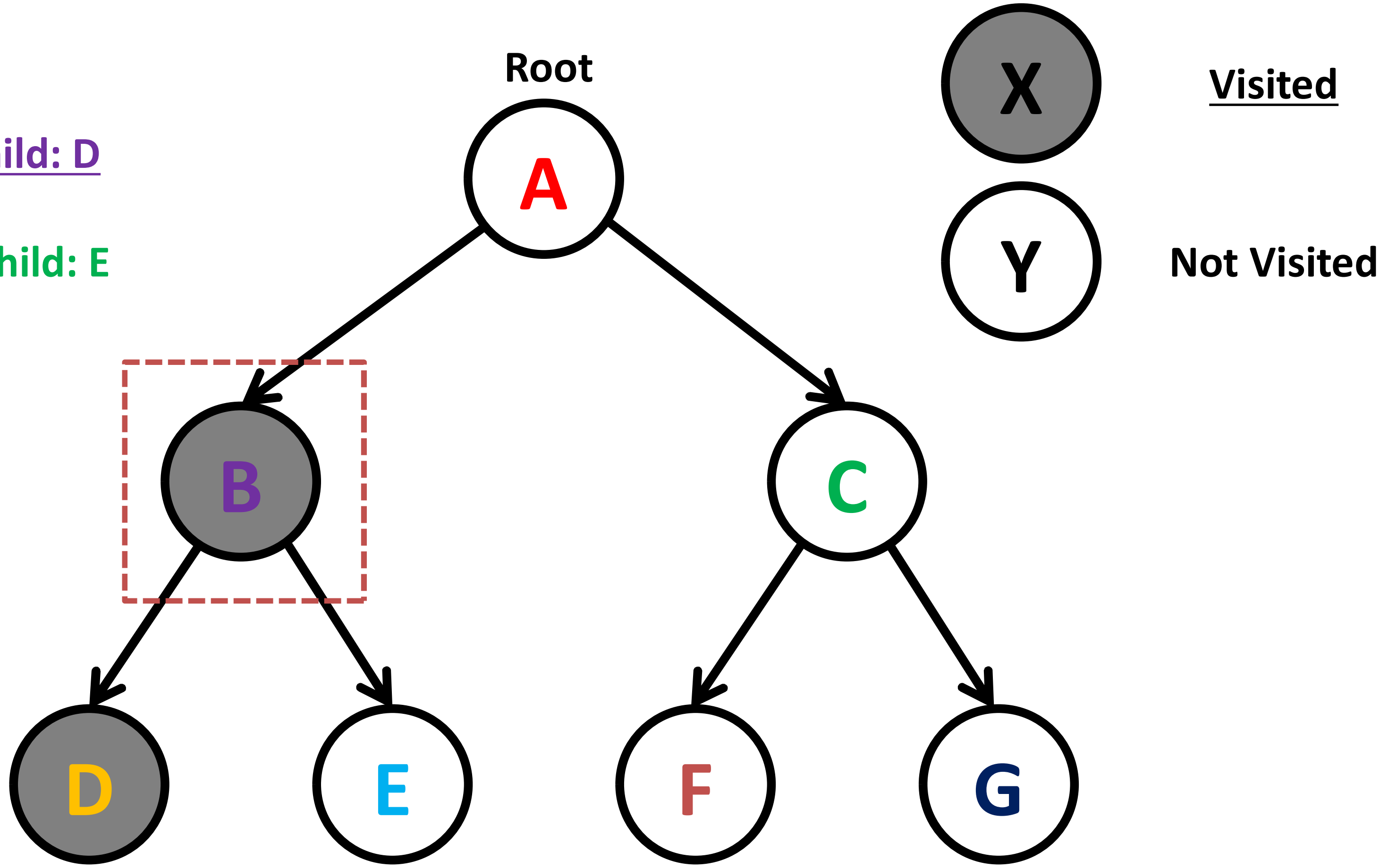


Nodes traversed so far: D

In-order (Left, Node, Right)

Step 4:

Node's Left Child: D
Node: B
Node's Right Child: E

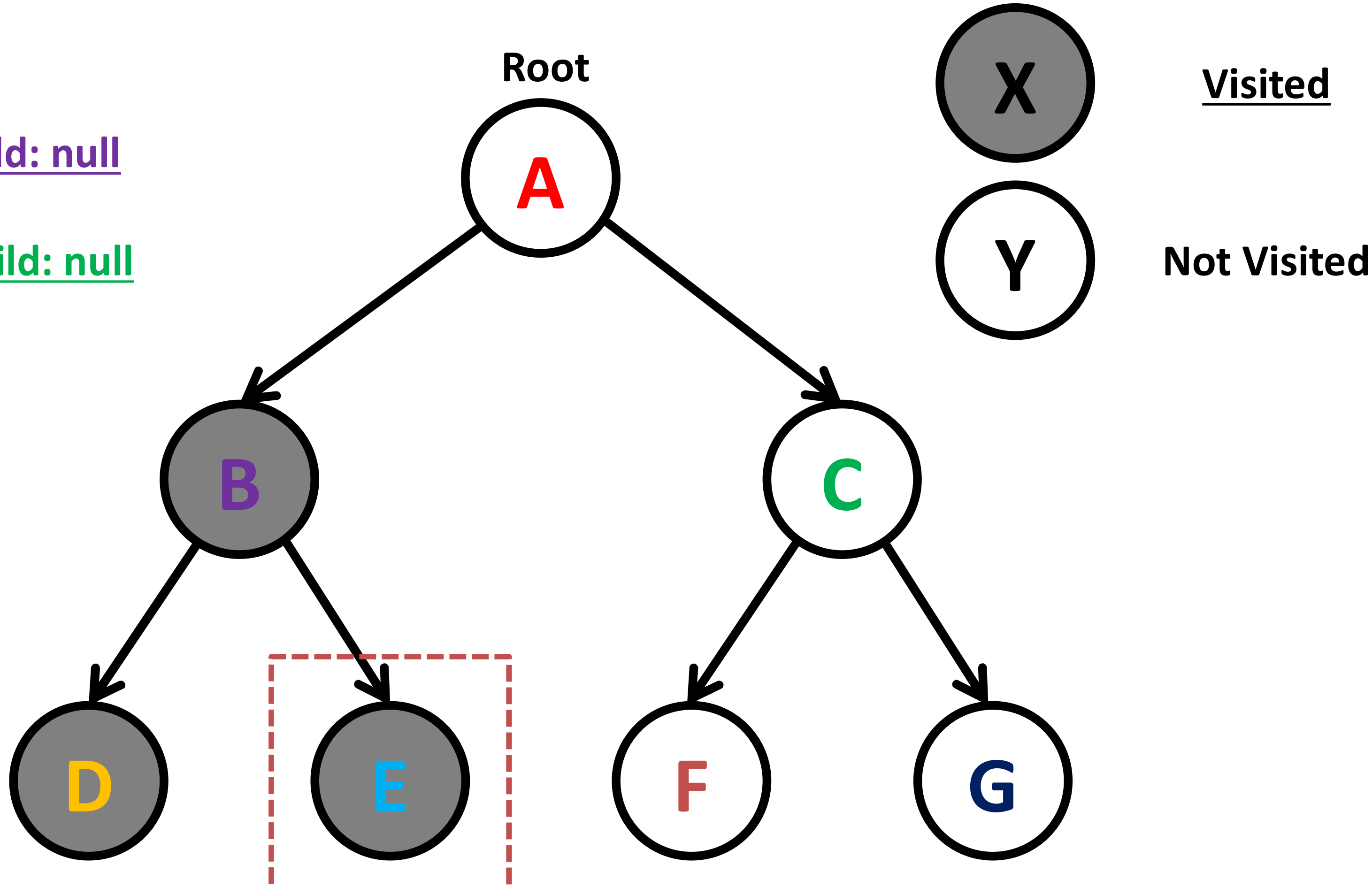


Nodes traversed so far: D B

In-order (Left, Node, Right)

Step 5:

Node's Left Child: null
Node: E
Node's Right Child: null

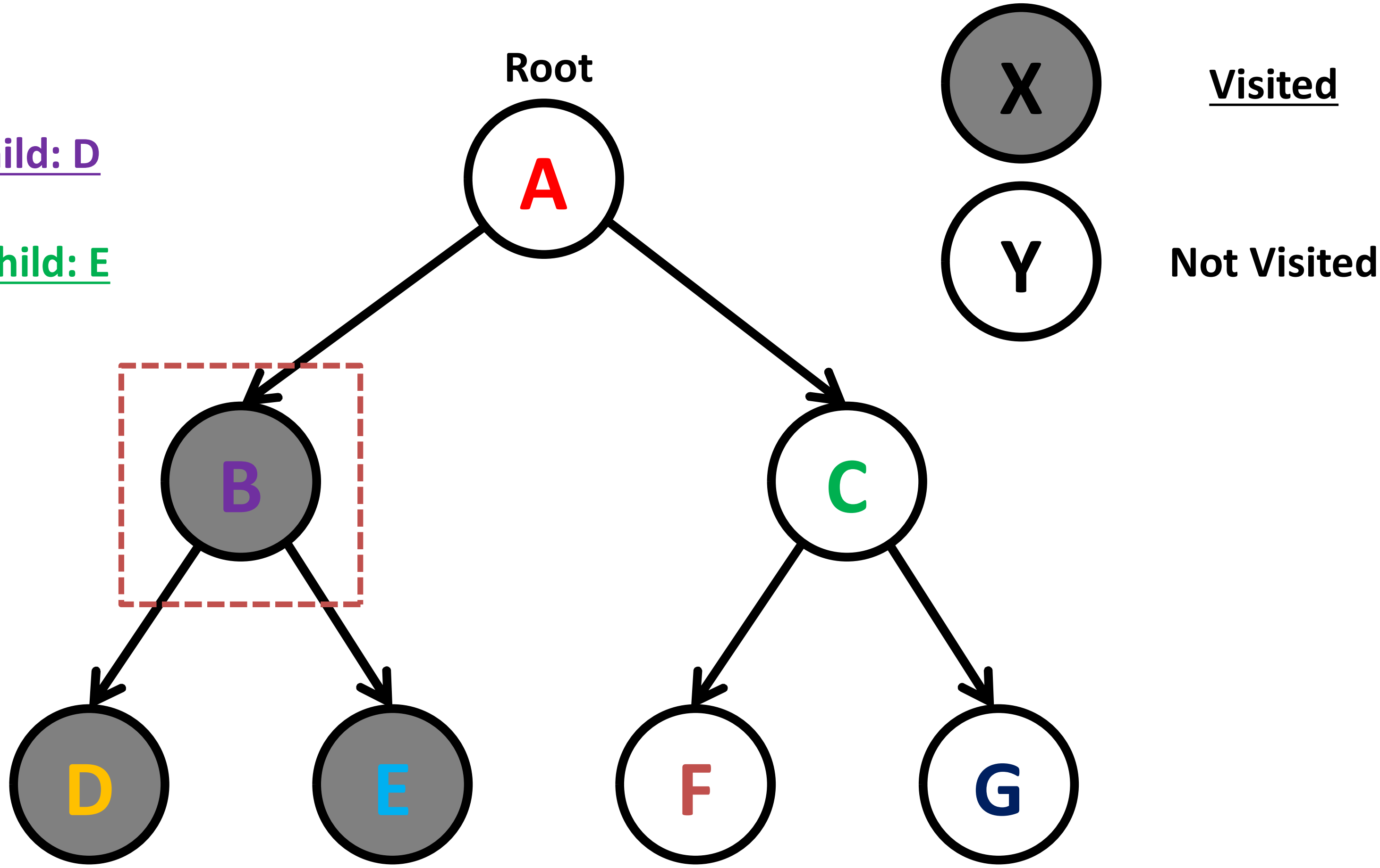


Nodes traversed so far: D B E

In-order (Left, Node, Right)

Step 6:

Node's Left Child: D
Node: B
Node's Right Child: E

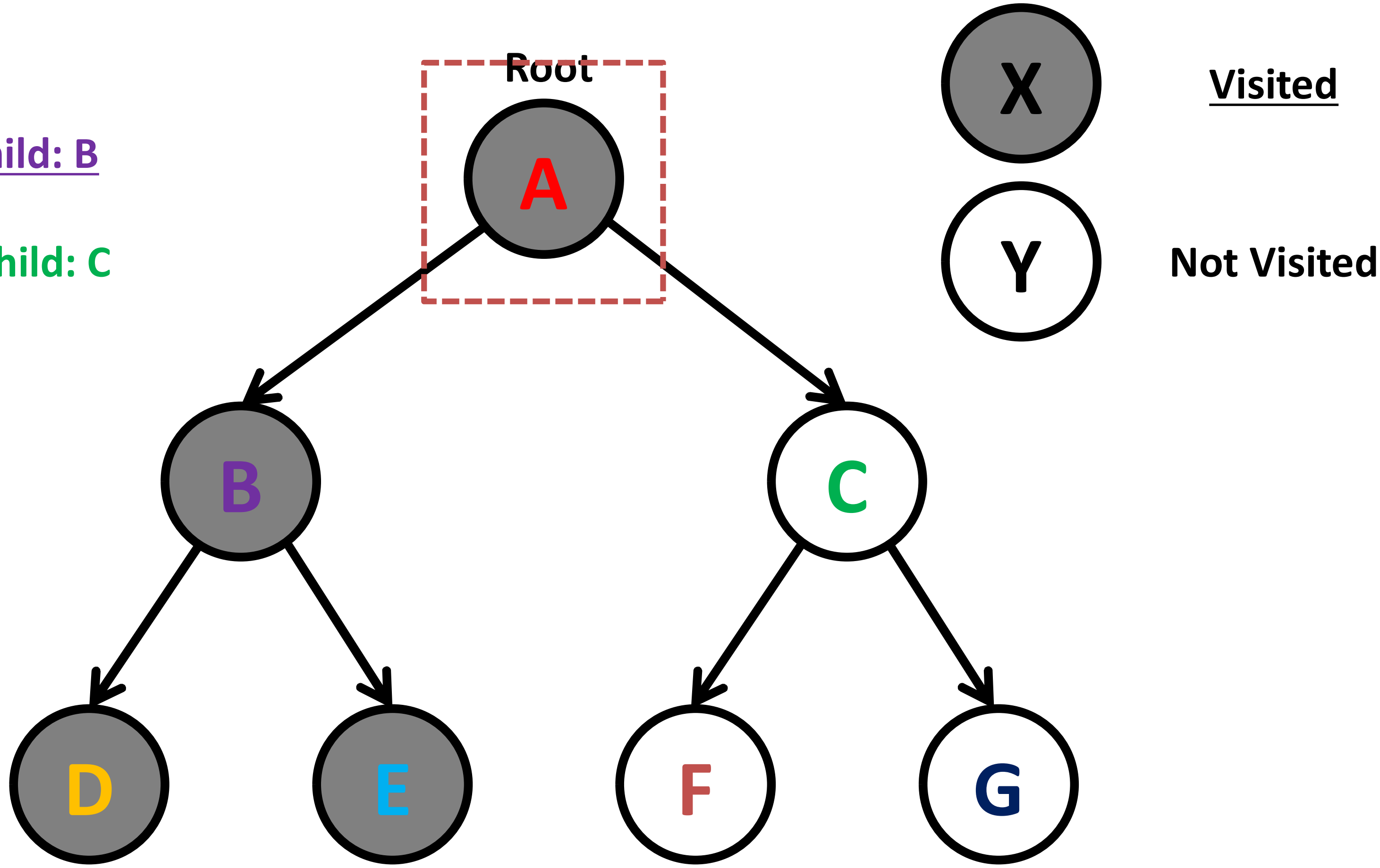


Nodes traversed so far: D B E

In-order (Left, Node, Right)

Step 7:

Node's Left Child: B
Node: A
Node's Right Child: C

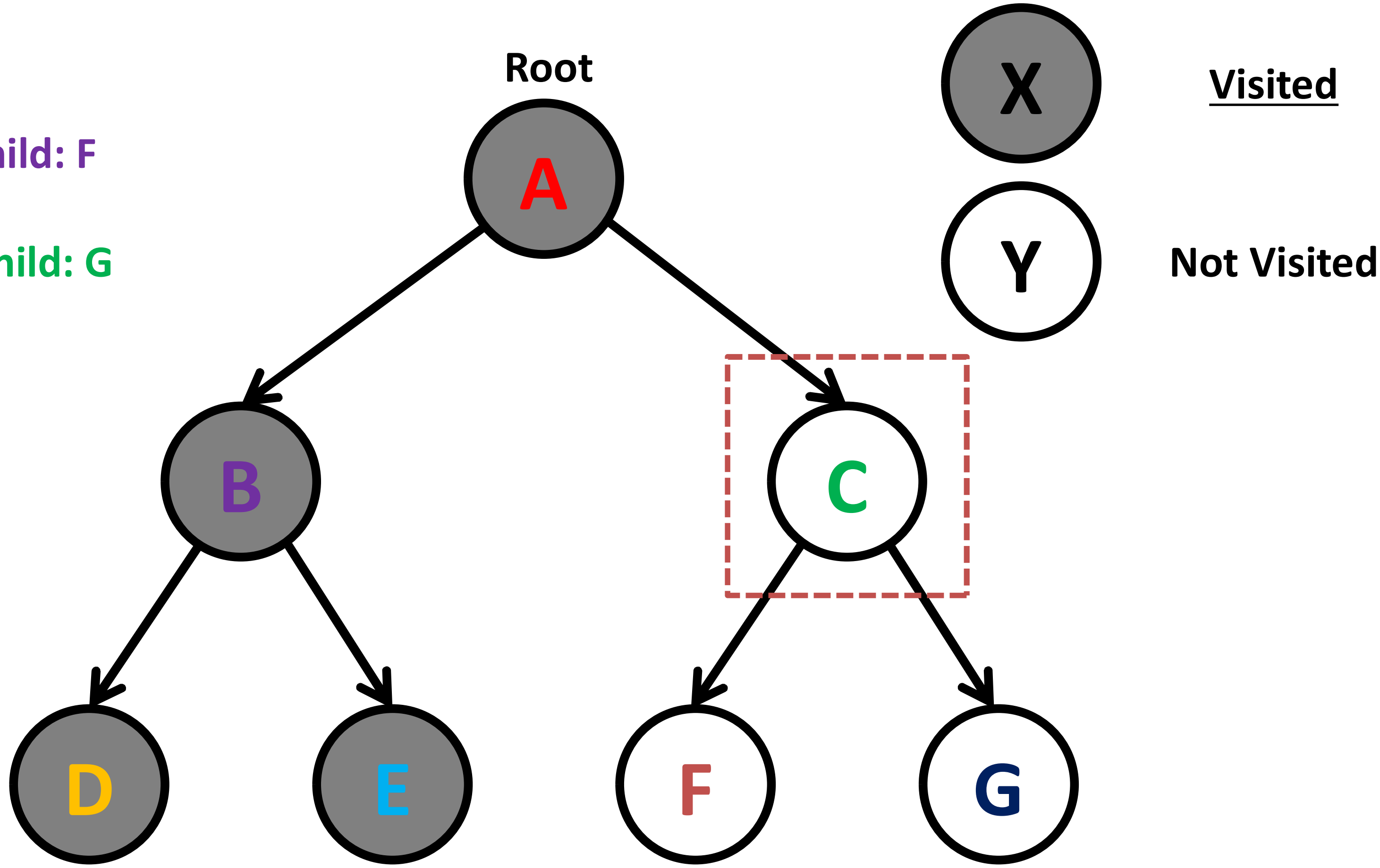


Nodes traversed so far: D B E A

In-order (Left, Node, Right)

Step 8:

Node's Left Child: F
Node: C
Node's Right Child: G

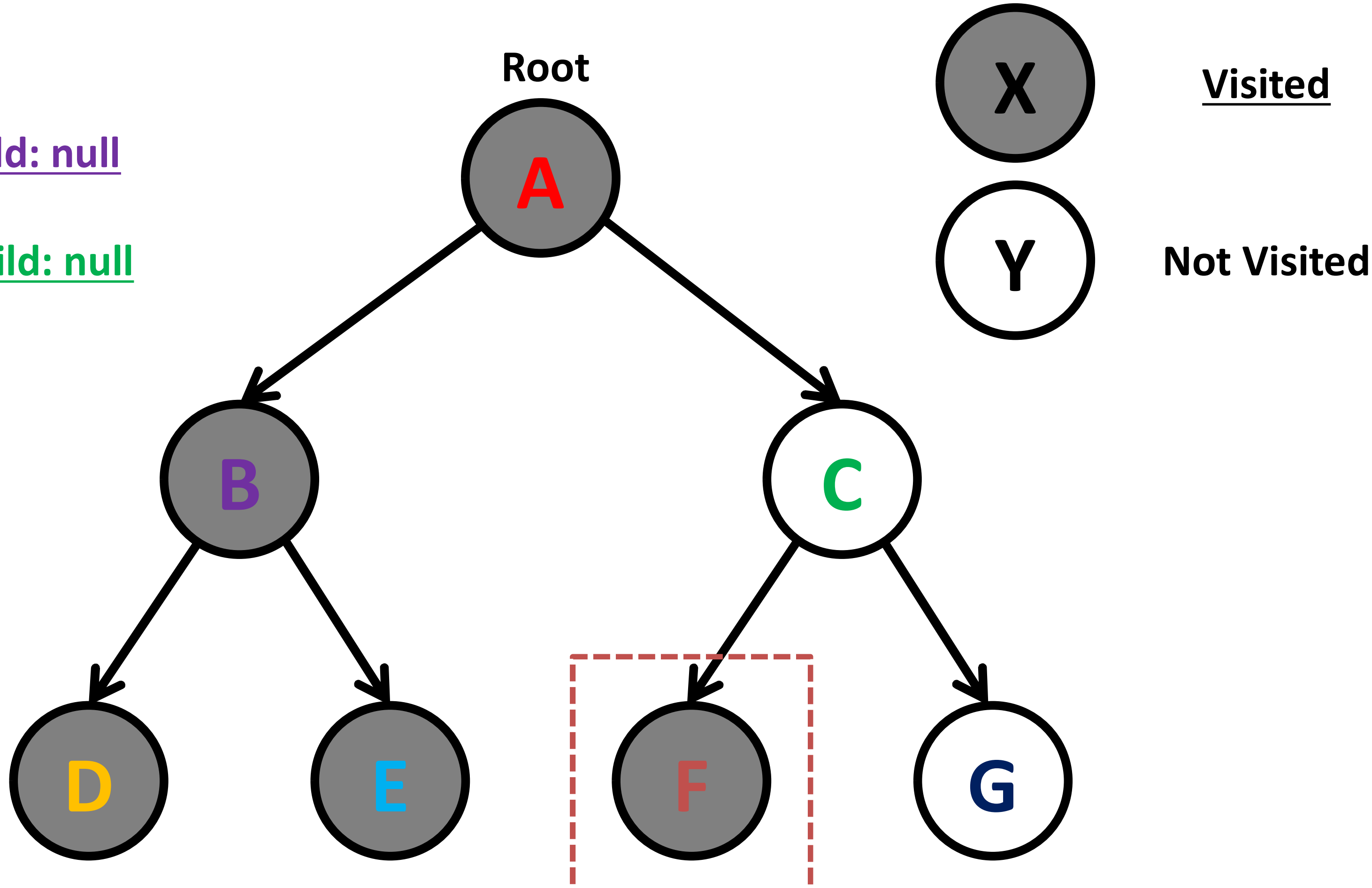


Nodes traversed so far: D B E A

In-order (Left, Node, Right)

Step 9:

Node's Left Child: null
Node: F
Node's Right Child: null



Nodes traversed so far: D B E A F

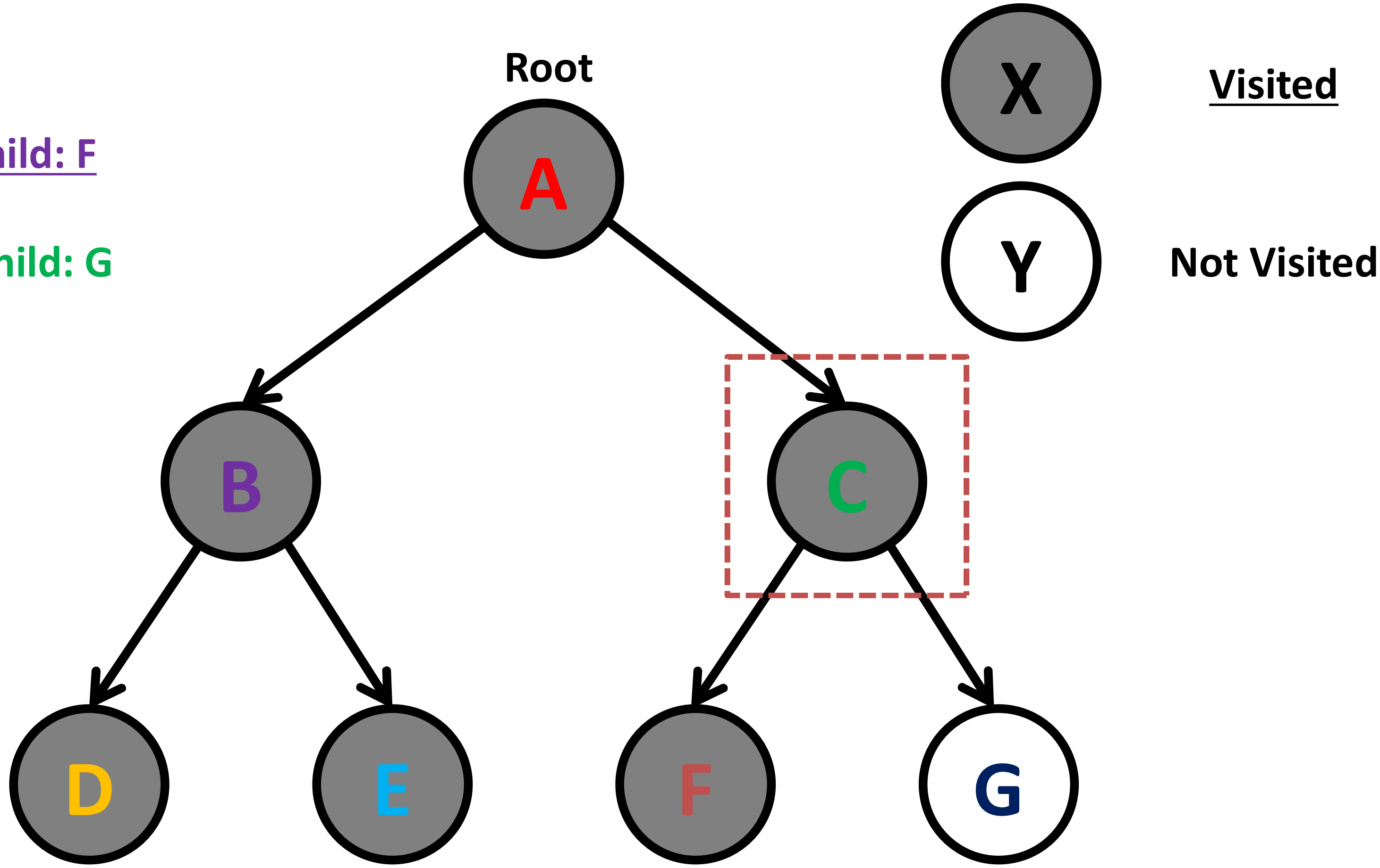
In-order (Left, Node, Right)

Step 10:

Node's Left Child: F

Node: C

Node's Right Child: G

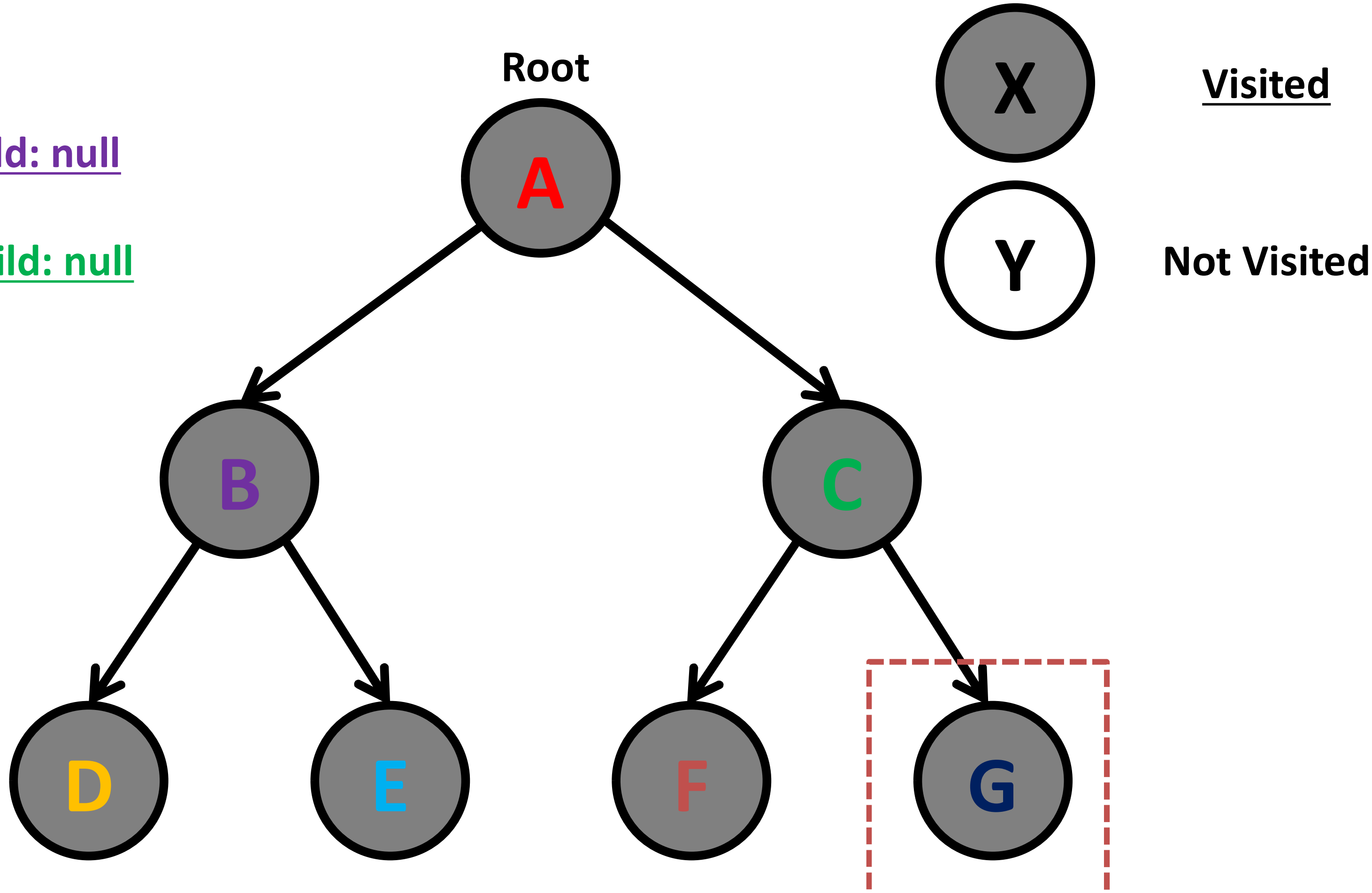


Nodes traversed so far: D B E A F C

In-order (Left, Node, Right)

Step 11:

Node’s Left Child: null
Node: G
Node’s Right Child: null

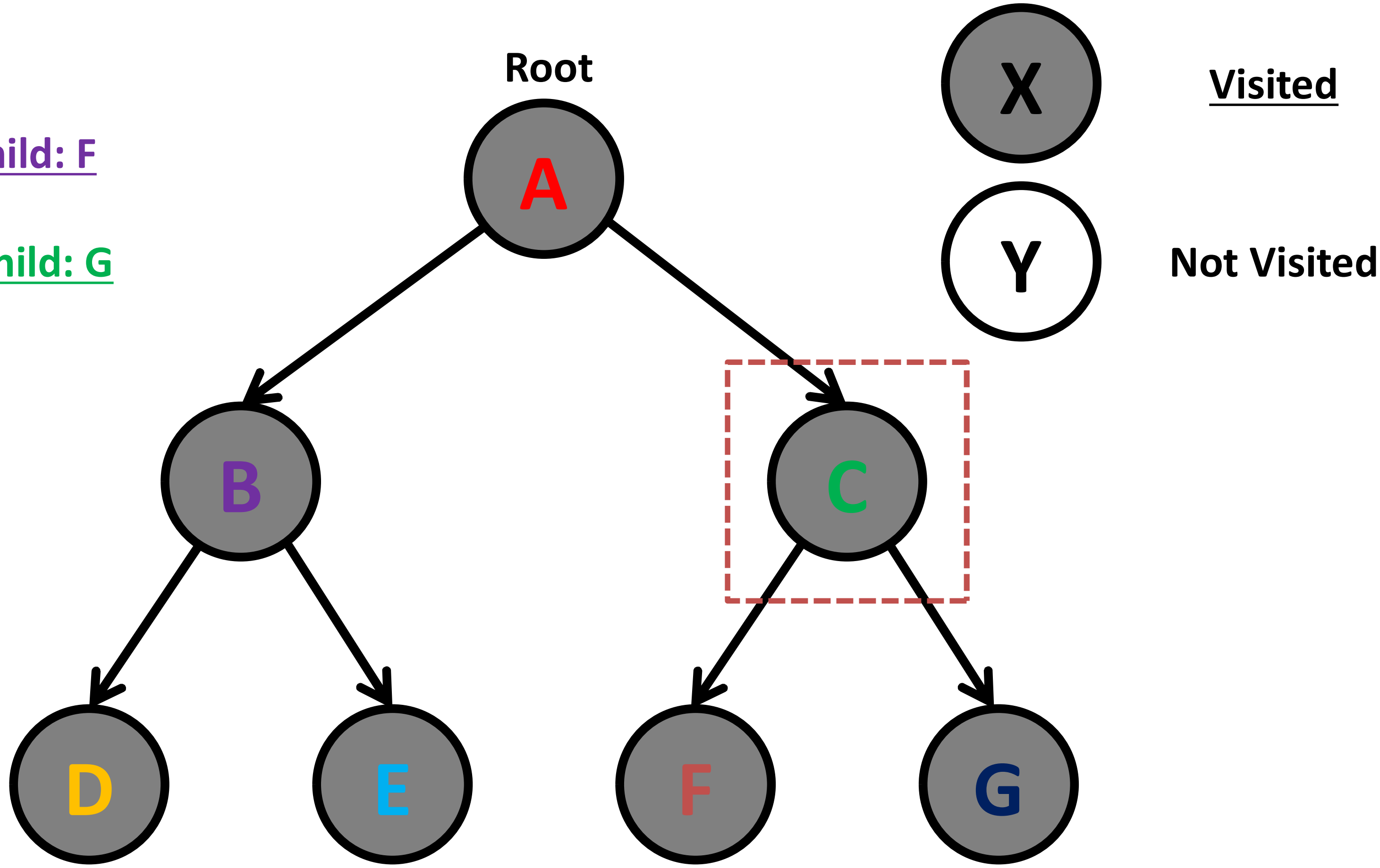


Nodes traversed so far: D B E A F C G

In-order (Left, Node, Right)

Step 12:

Node's Left Child: F
Node: C
Node's Right Child: G

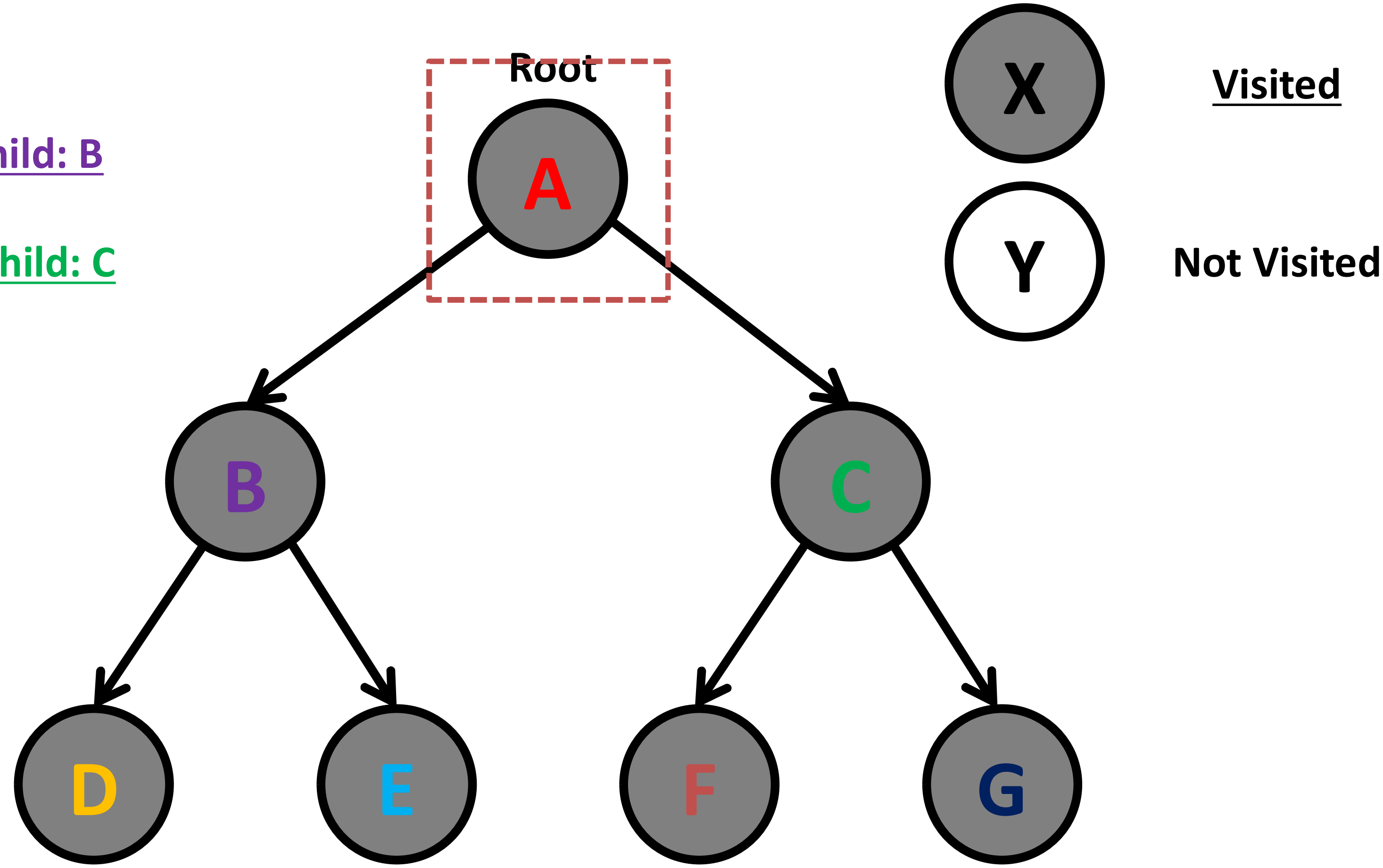


Nodes traversed so far: D B E A F C G

In-order (Left, Node, Right)

Step 13:

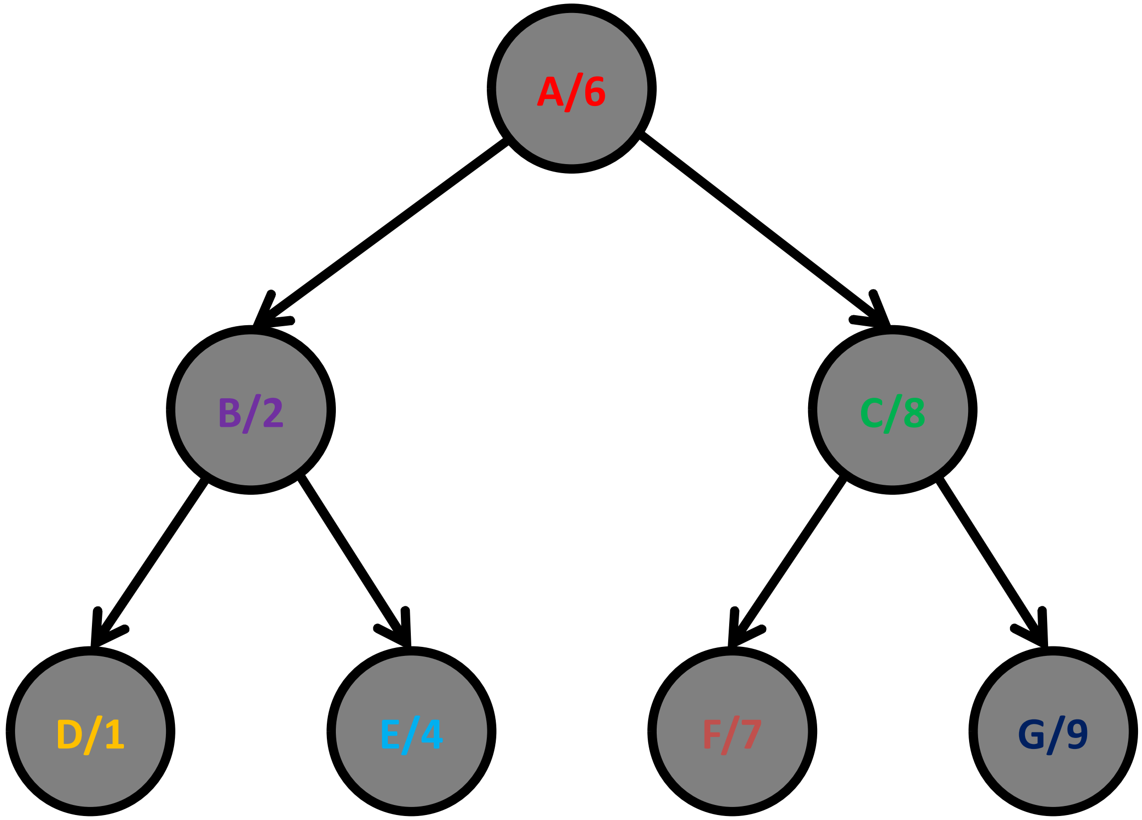
Node's Left Child: B
Node: A
Node's Right Child: C



Nodes traversed so far: D B E A F C G

In-order (Left, Node, Right)

If we had BST with numerical keys -> sorted order!

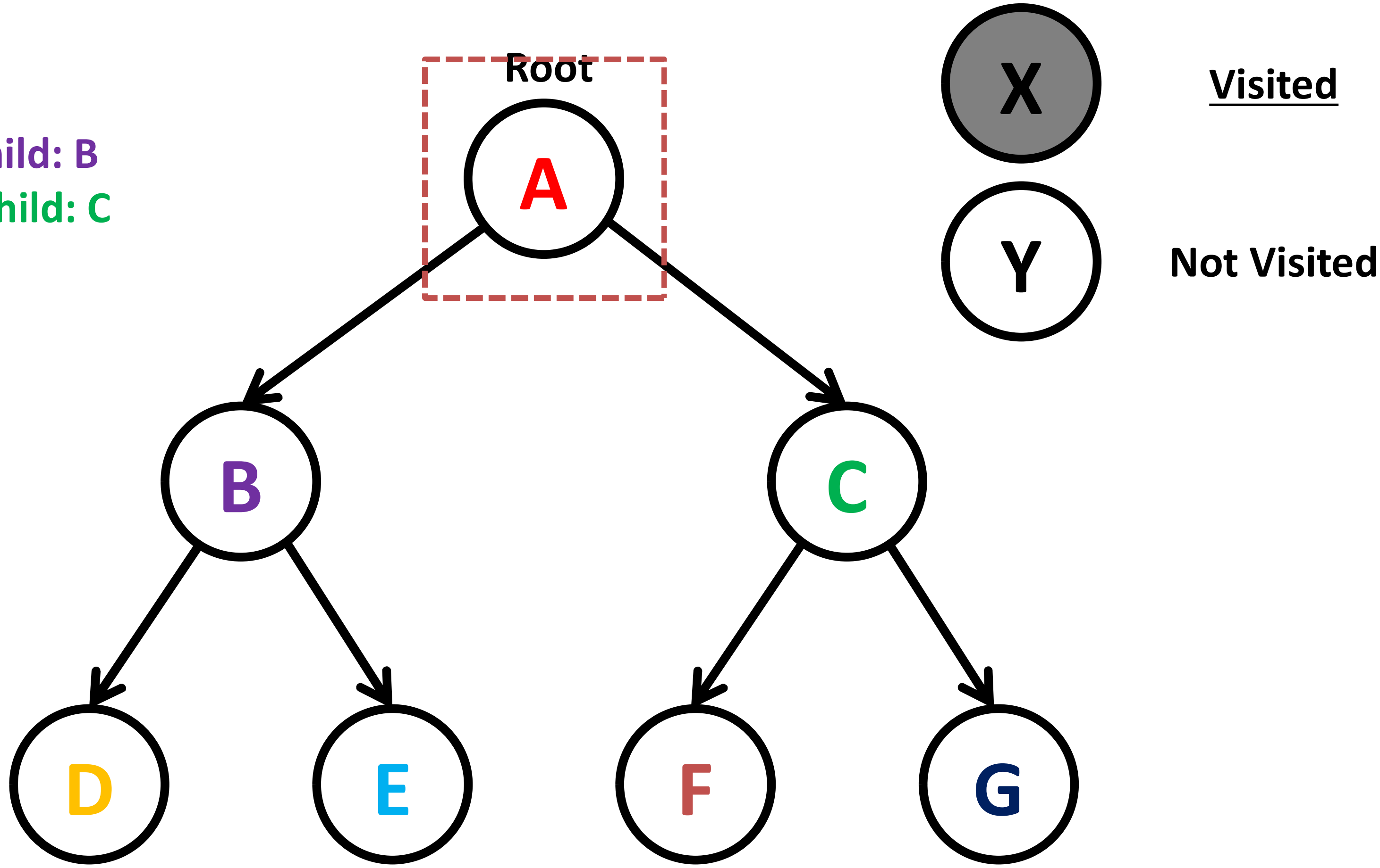


In-order traversal: D/1 B/2 E/4 A/6 F/7 C/8 G/9

Post-order (Left, Right, Node)

Step 1:

Node's Left Child: B
Node's Right Child: C
Node: A

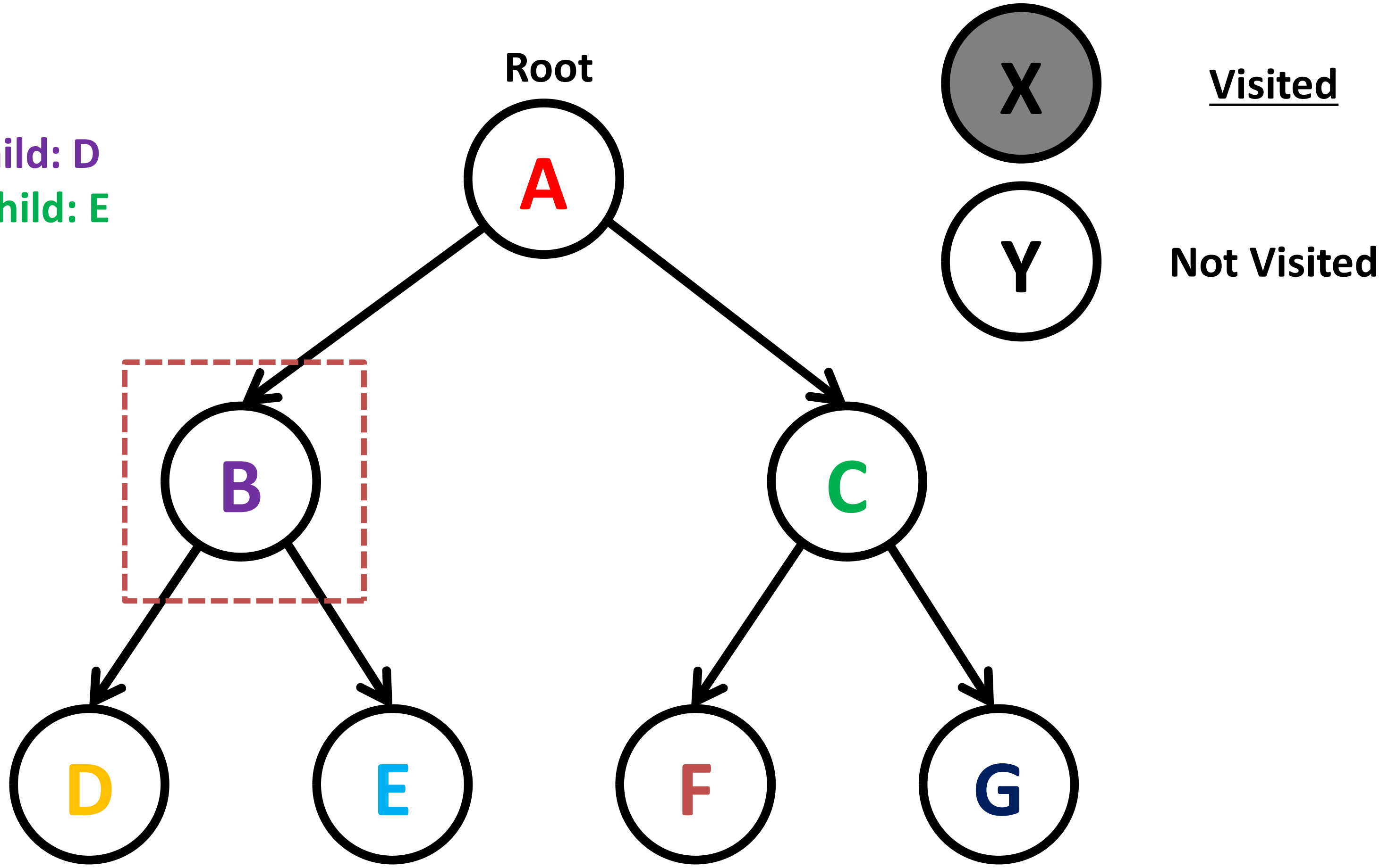


Nodes traversed so far:

Post-order (Left, Right, Node)

Step 2:

Node's Left Child: D
Node's Right Child: E
Node: B



Nodes traversed so far:

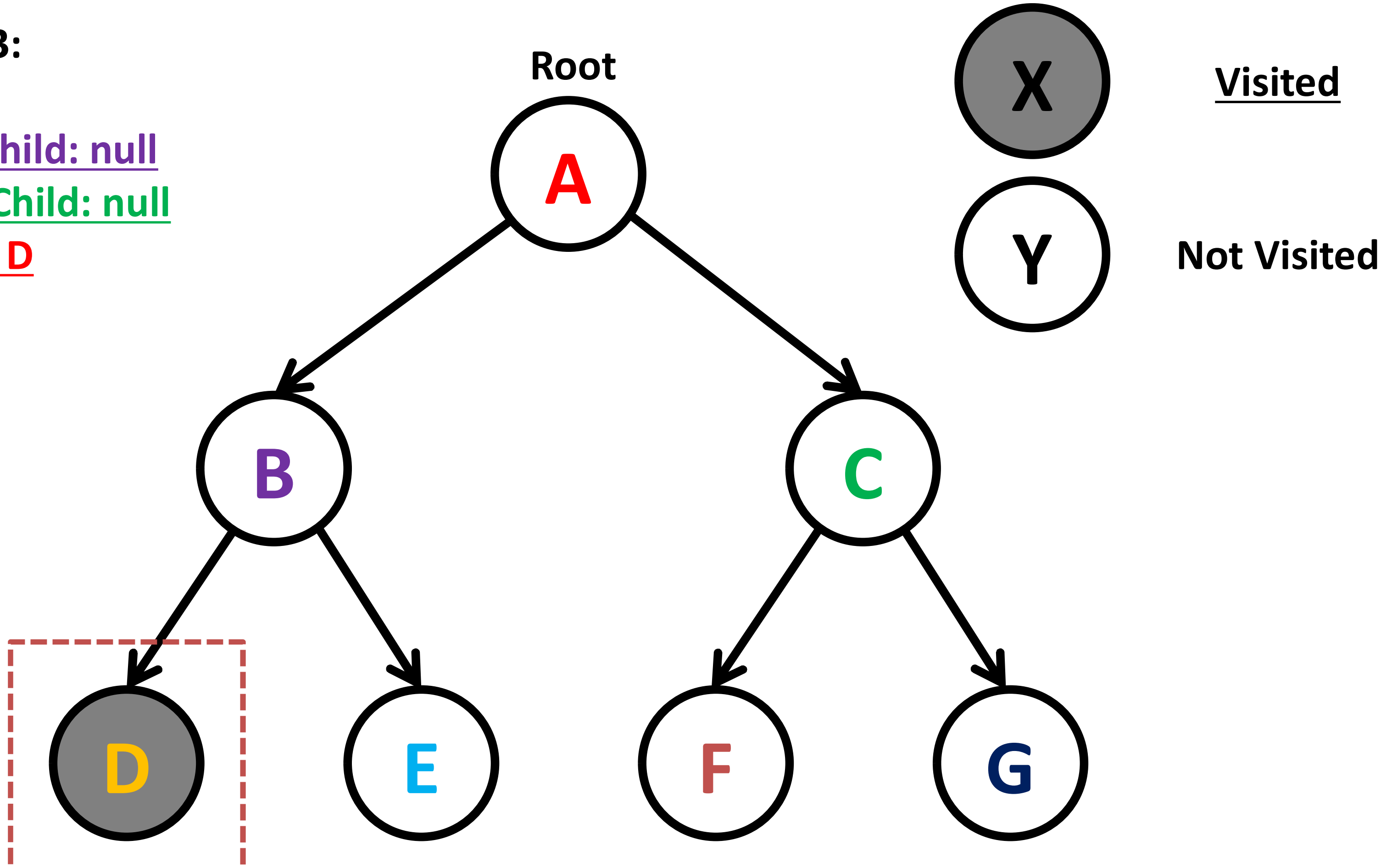
Post-order (Left, Right, Node)

Step 3:

<u>Node’s Left Child: null</u>

<u>Node’s Right Child: null</u>

<u>Node: D</u>

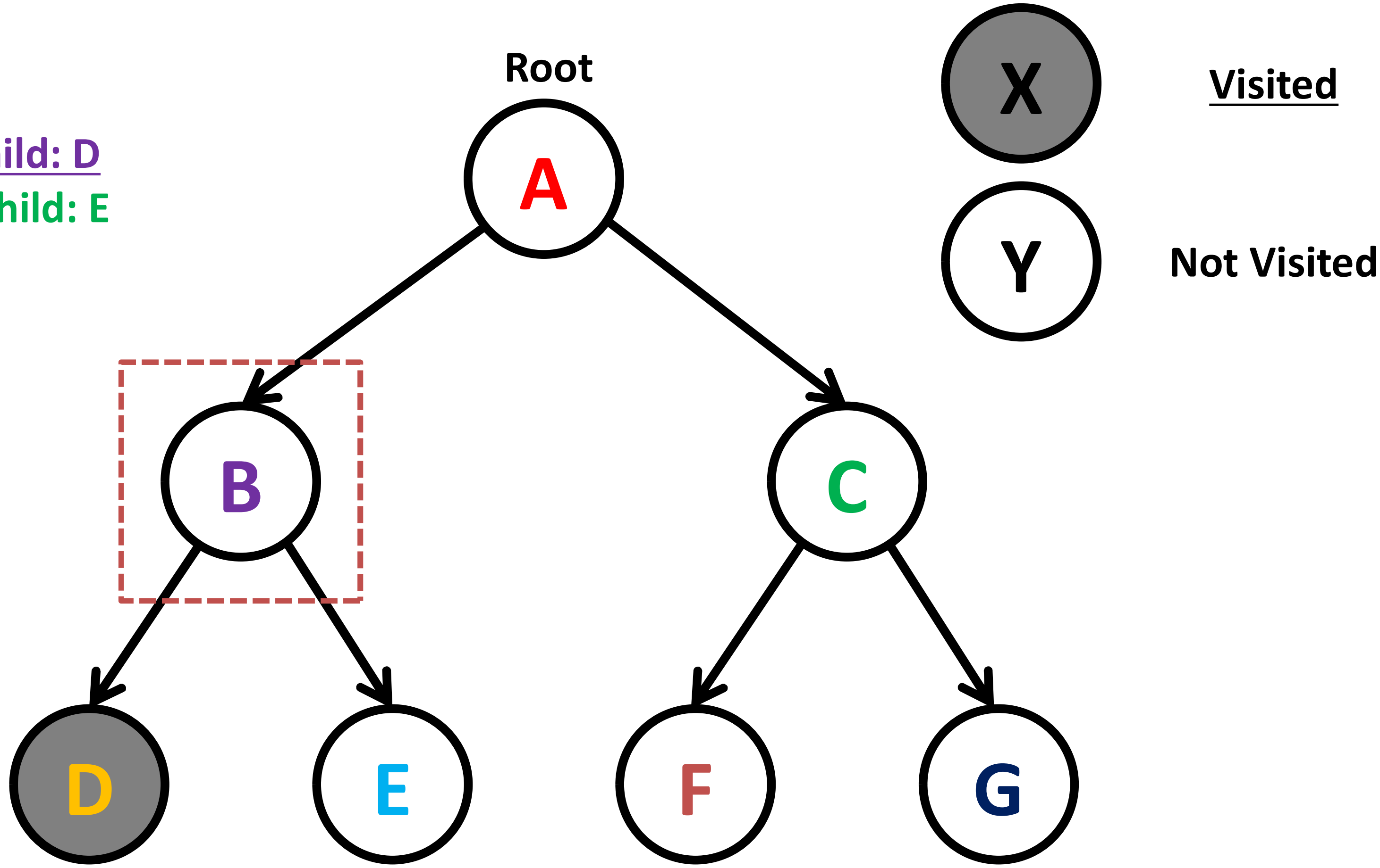


Nodes traversed so far: D

Post-order (Left, Right, Node)

Step 4:

Node's Left Child: D
Node's Right Child: E
Node: B

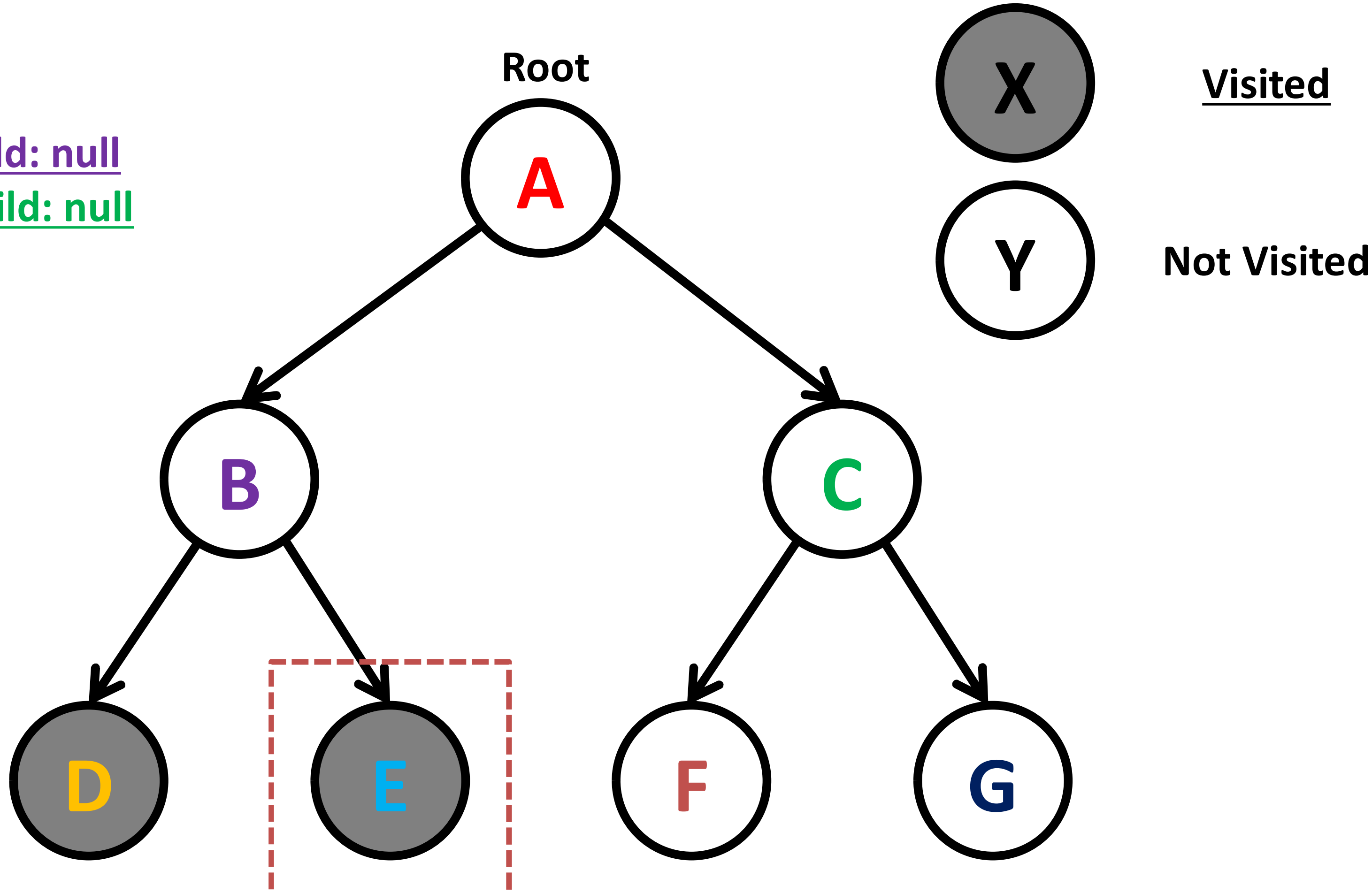


Nodes traversed so far: D

Post-order (Left, Right, Node)

Step 5:

Node’s Left Child: null
Node’s Right Child: null
Node: E

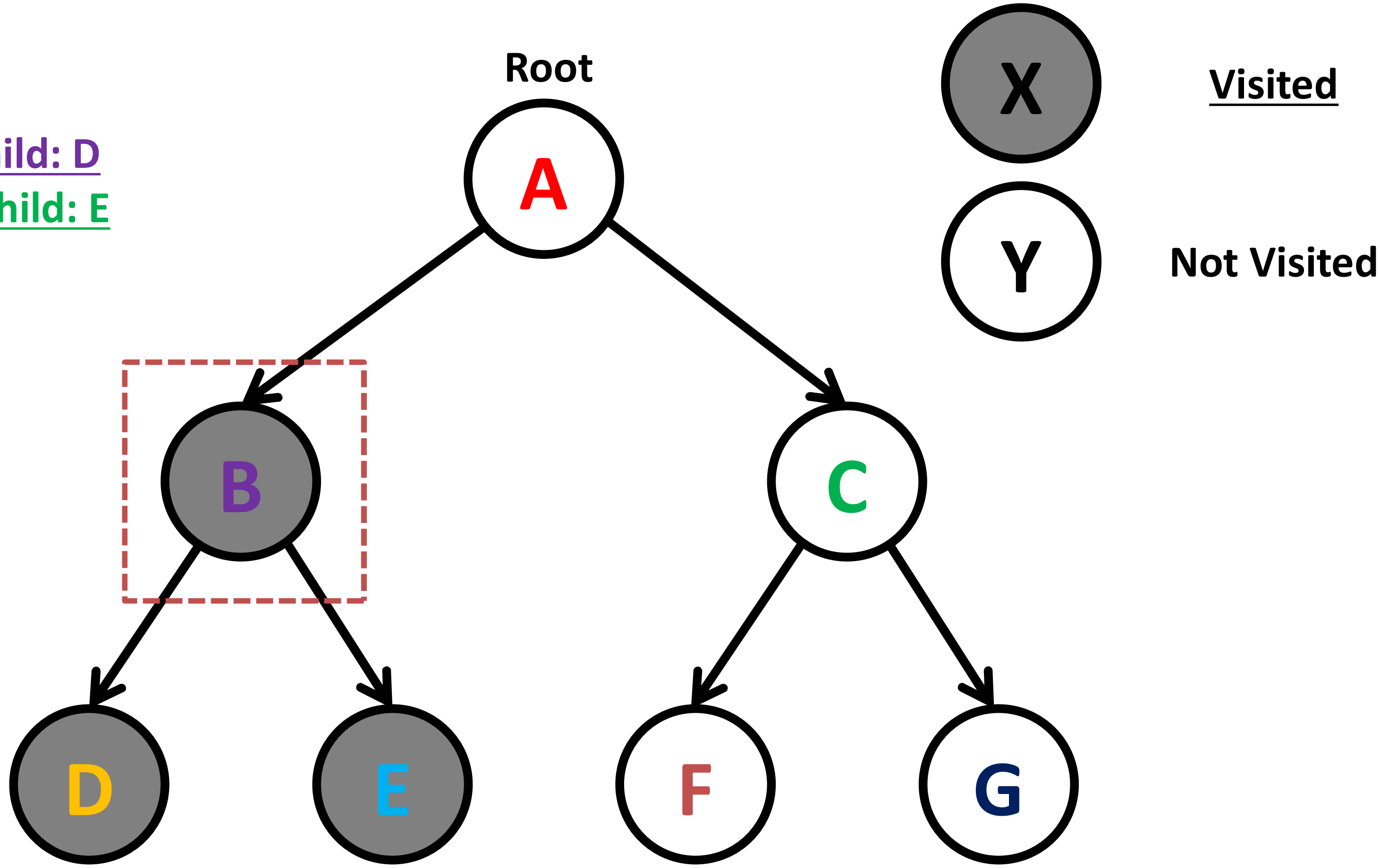


Nodes traversed so far: D E

Post-order (Left, Right, Node)

Step 6:

Node's Left Child: D
Node's Right Child: E
Node: B

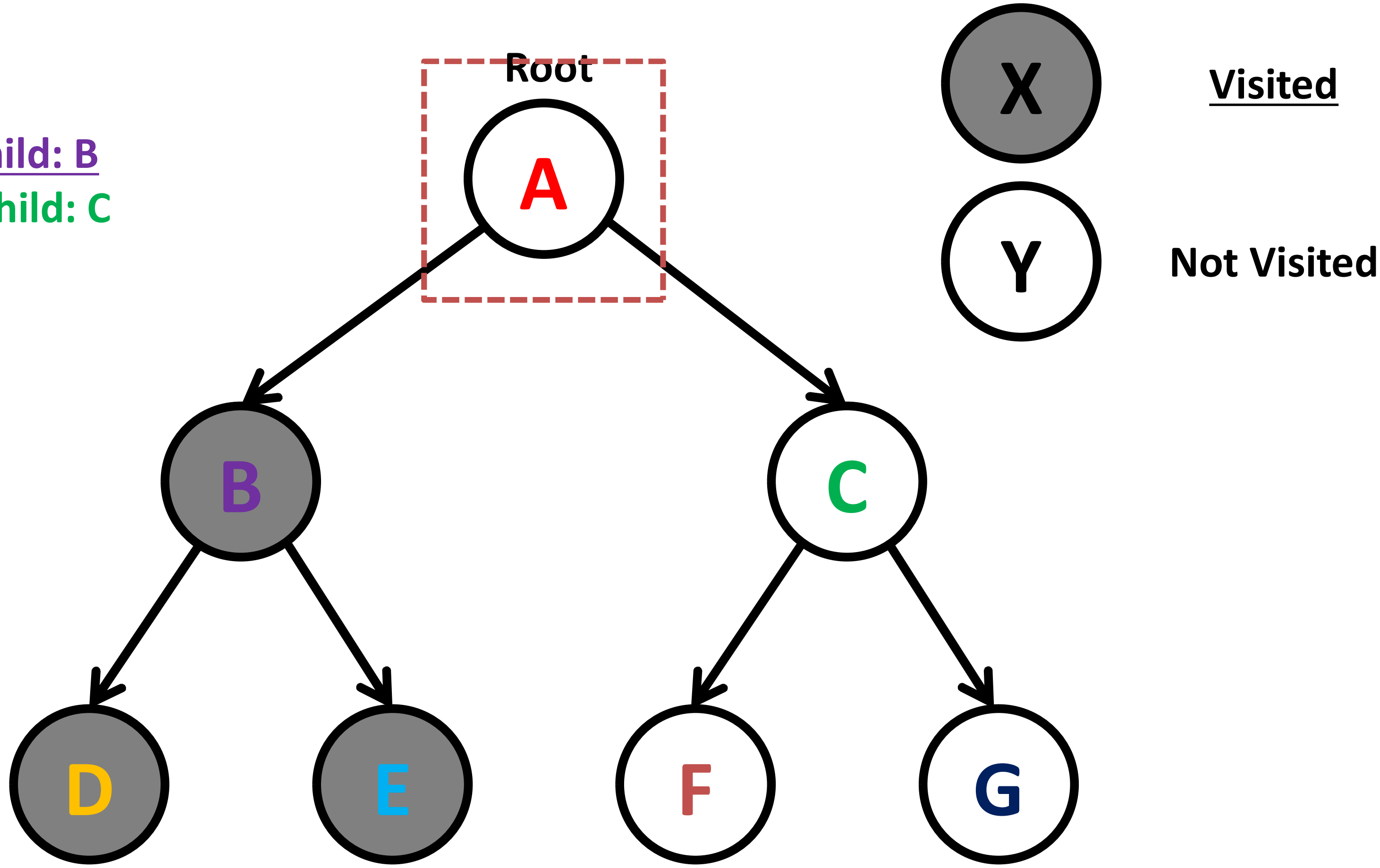


Nodes traversed so far: D E B

Post-order (Left, Right, Node)

Step 7:

Node's Left Child: B
Node's Right Child: C
Node: A

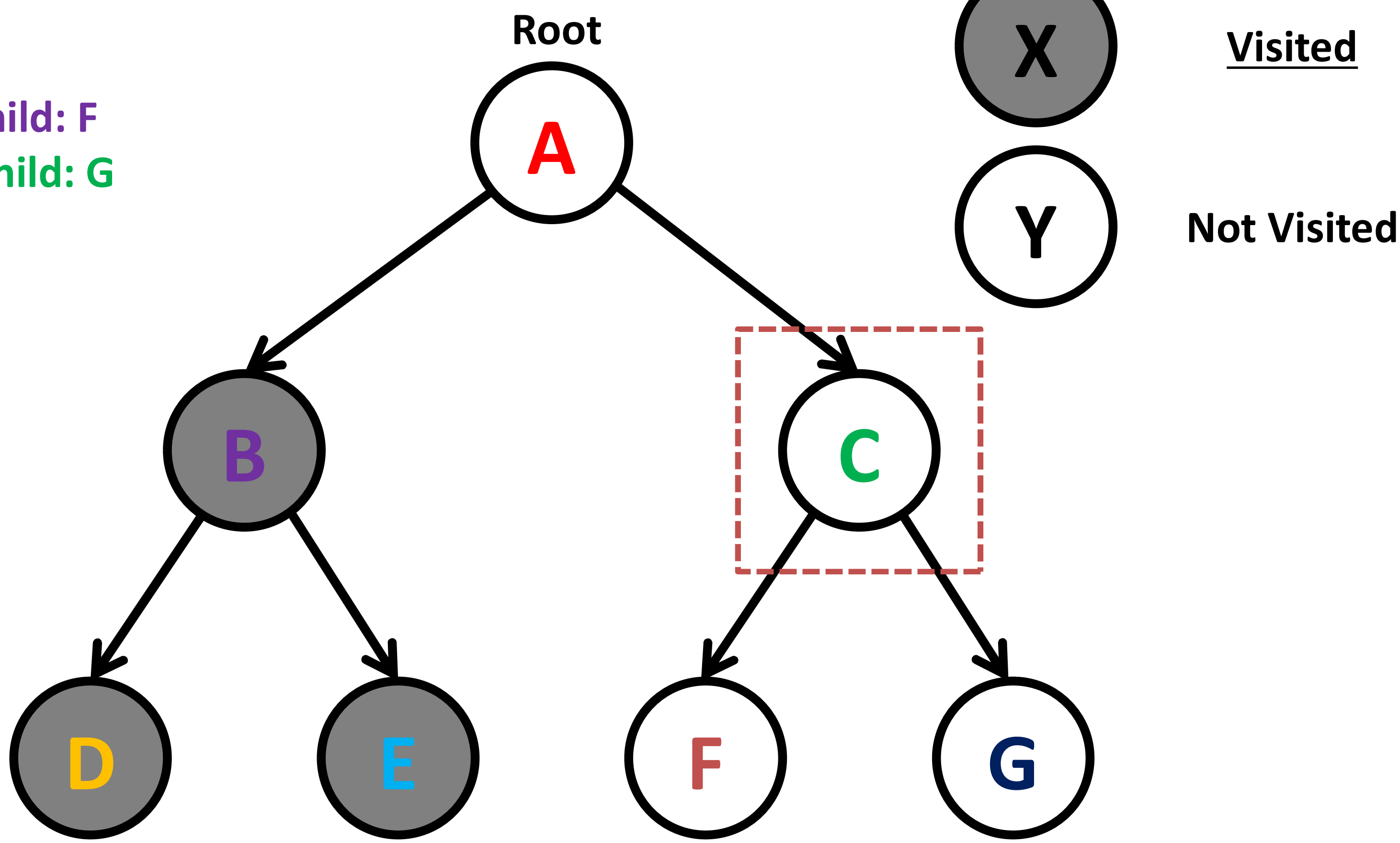


Nodes traversed so far: D E B

Post-order (Left, Right, Node)

Step 8:

Node's Left Child: F
Node's Right Child: G
Node: C

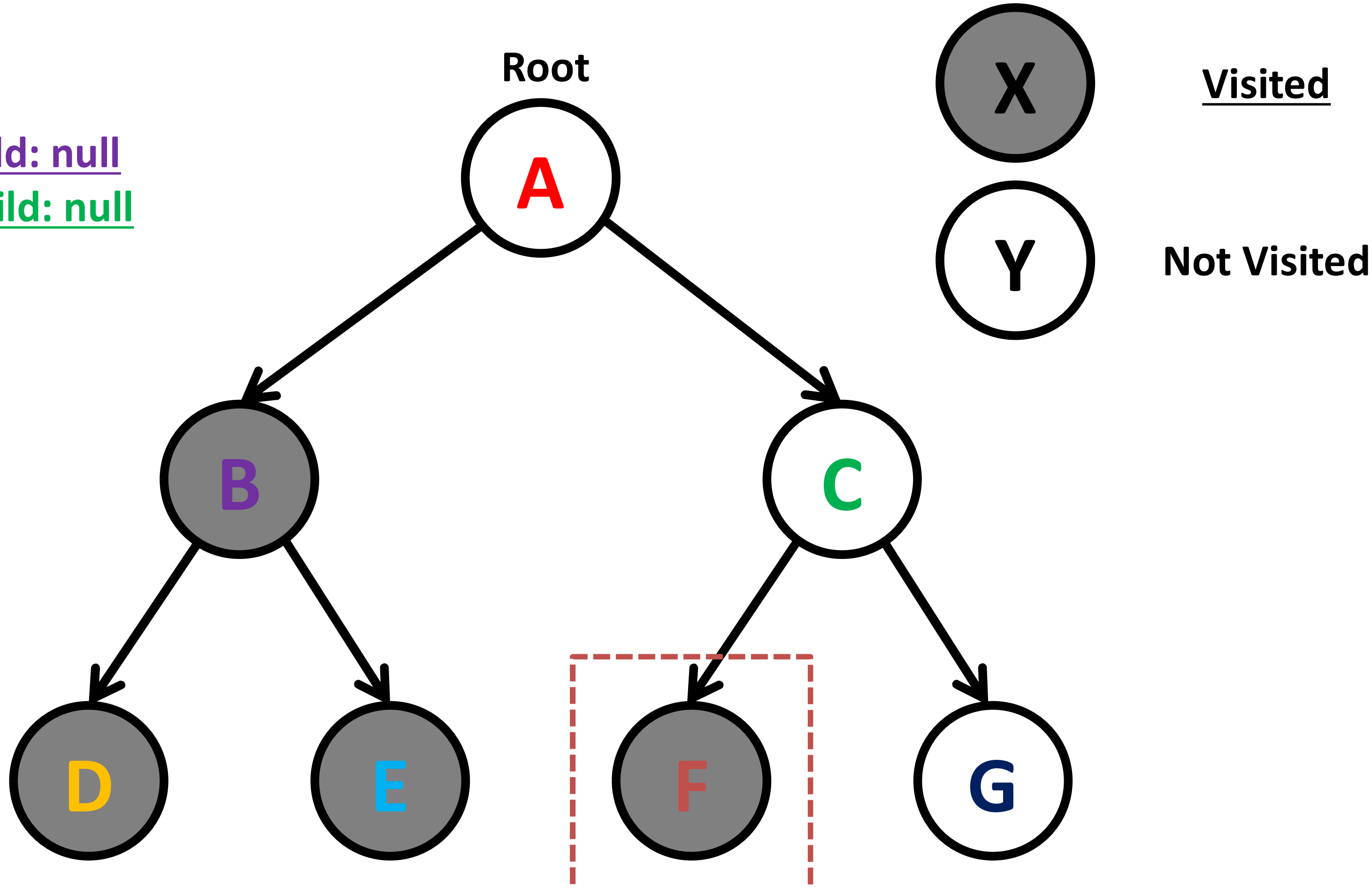


Nodes traversed so far: D E B

Post-order (Left, Right, Node)

Step 9:

Node's Left Child: null
Node's Right Child: null
Node: F

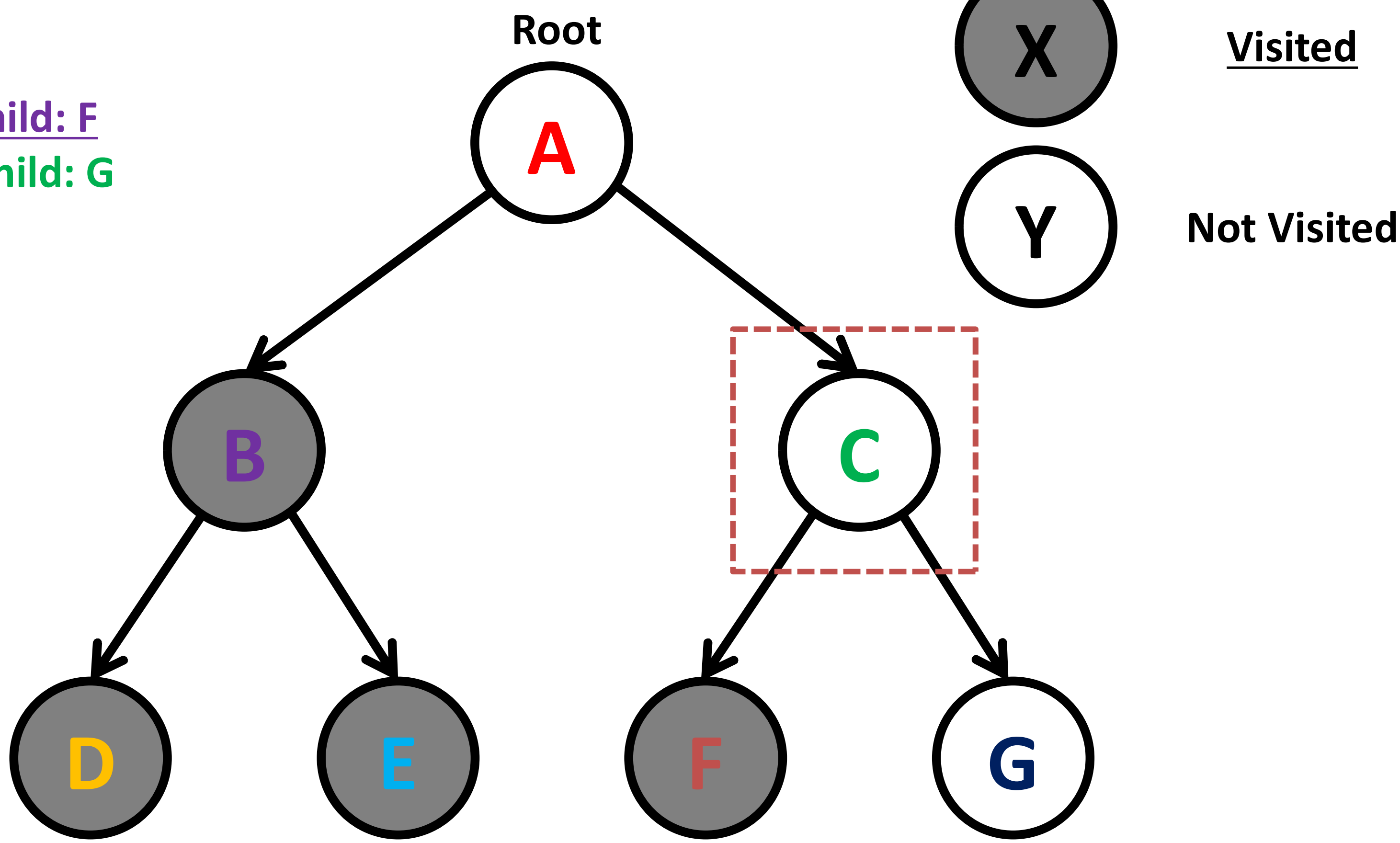


Nodes traversed so far: D E B F

Post-order (Left, Right, Node)

Step 10:

Node's Left Child: F
Node's Right Child: G
Node: C

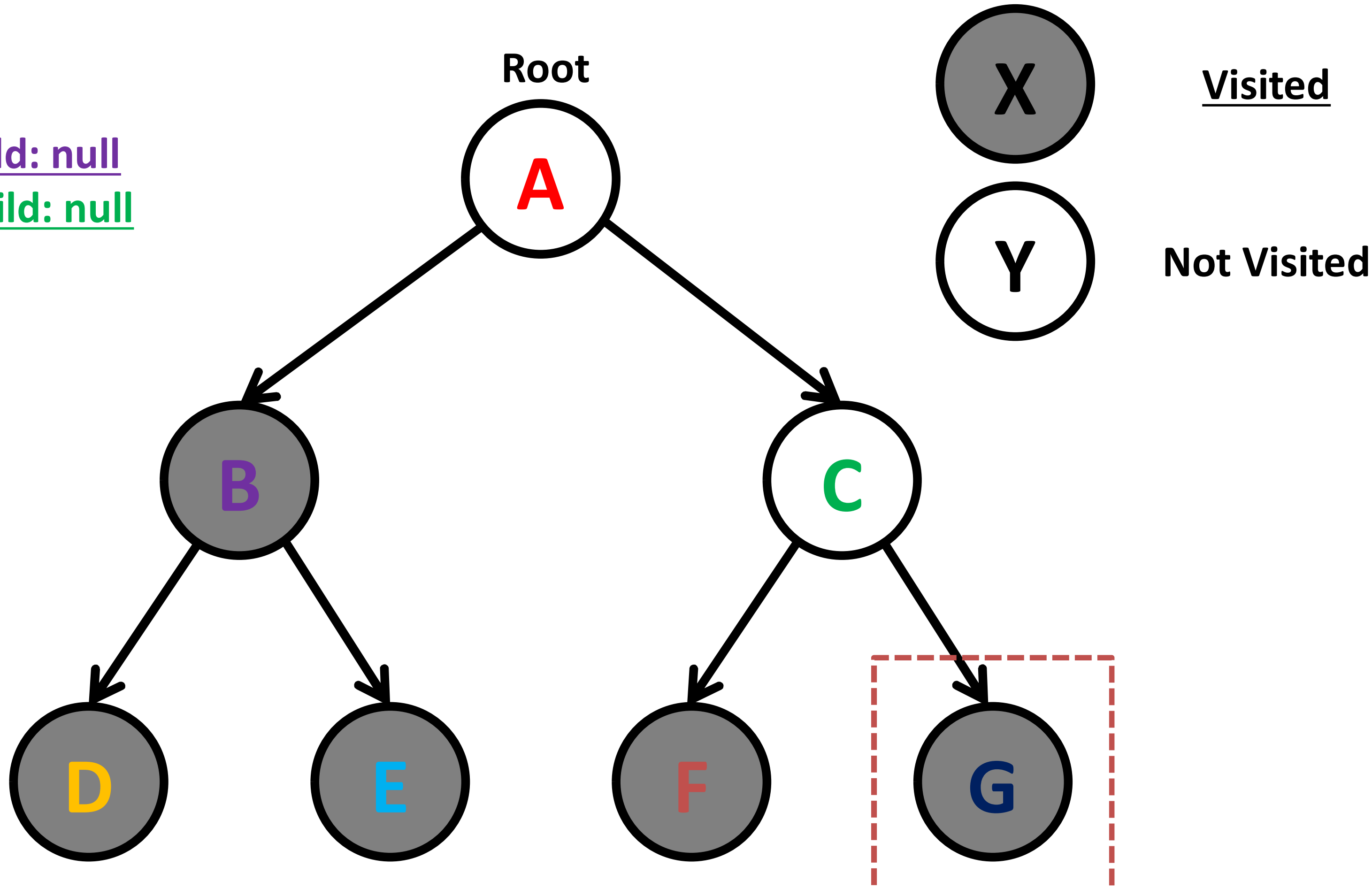


Nodes traversed so far: D E B F

Post-order (Left, Right, Node)

Step 11:

Node's Left Child: null
Node's Right Child: null
Node: G

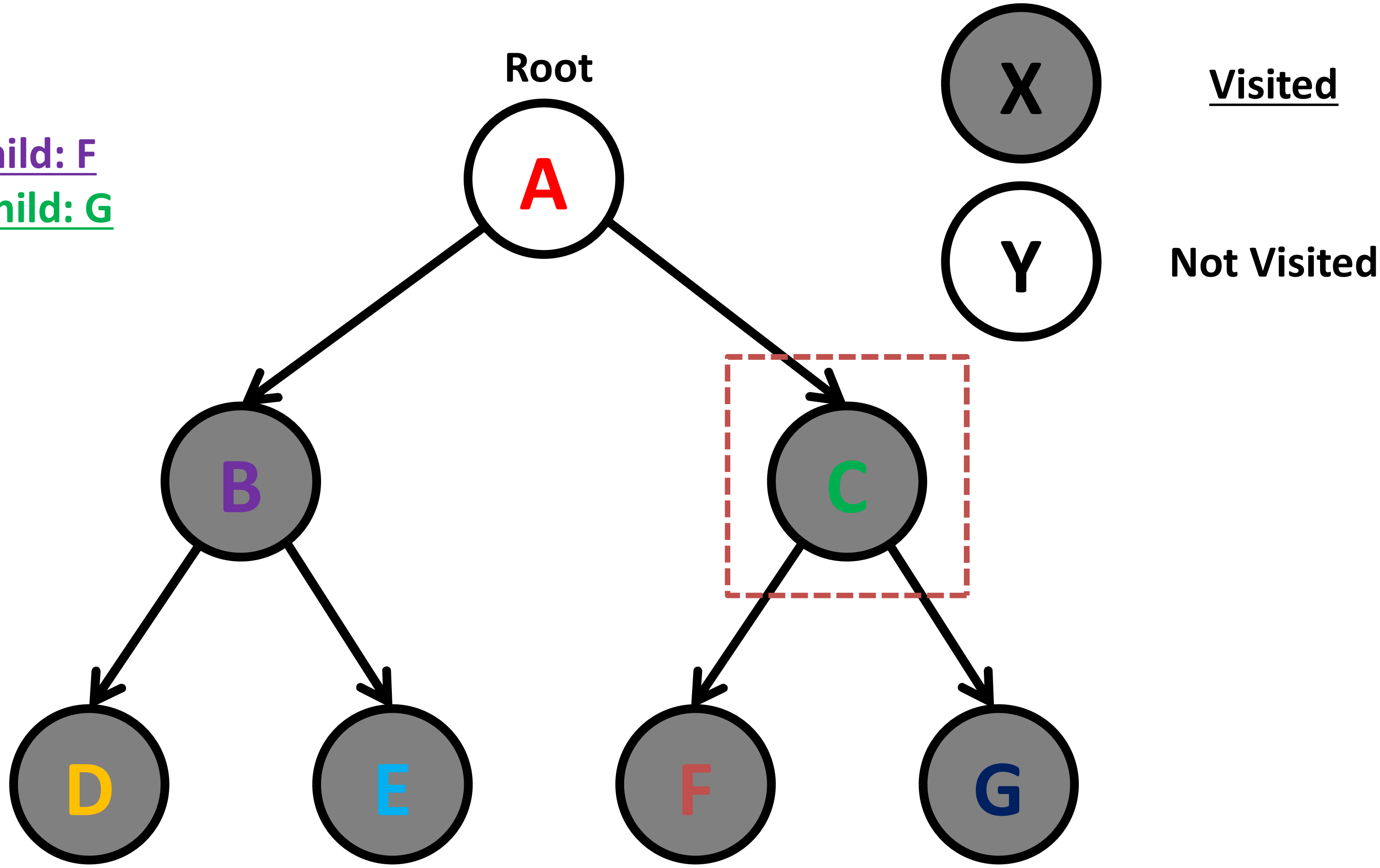


Nodes traversed so far: D E B F G

Post-order (Left, Right, Node)

Step 12:

Node’s Left Child: F
Node’s Right Child: G
Node: C

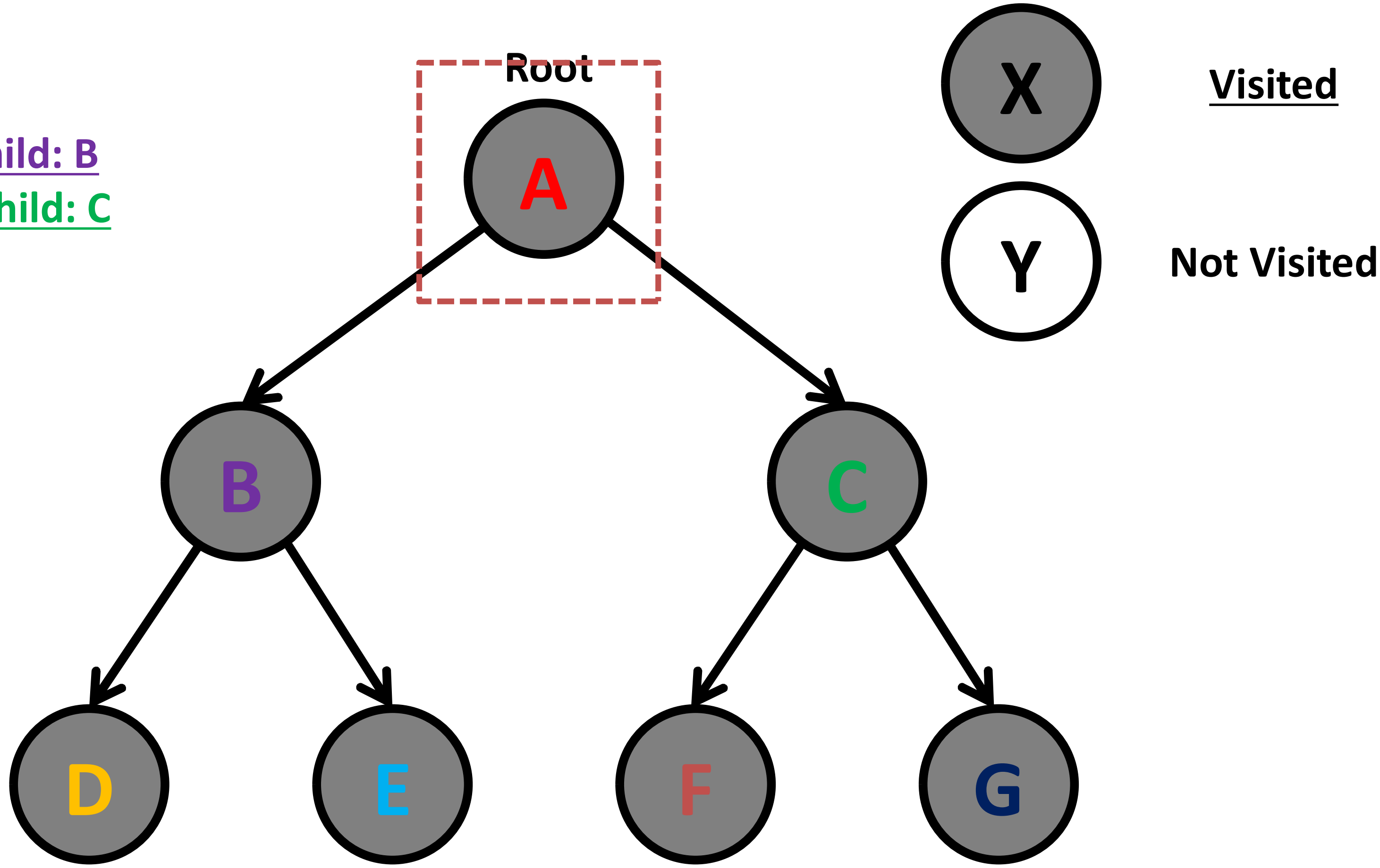


Nodes traversed so far: D E B F G C

Post-order (Left, Right, Node)

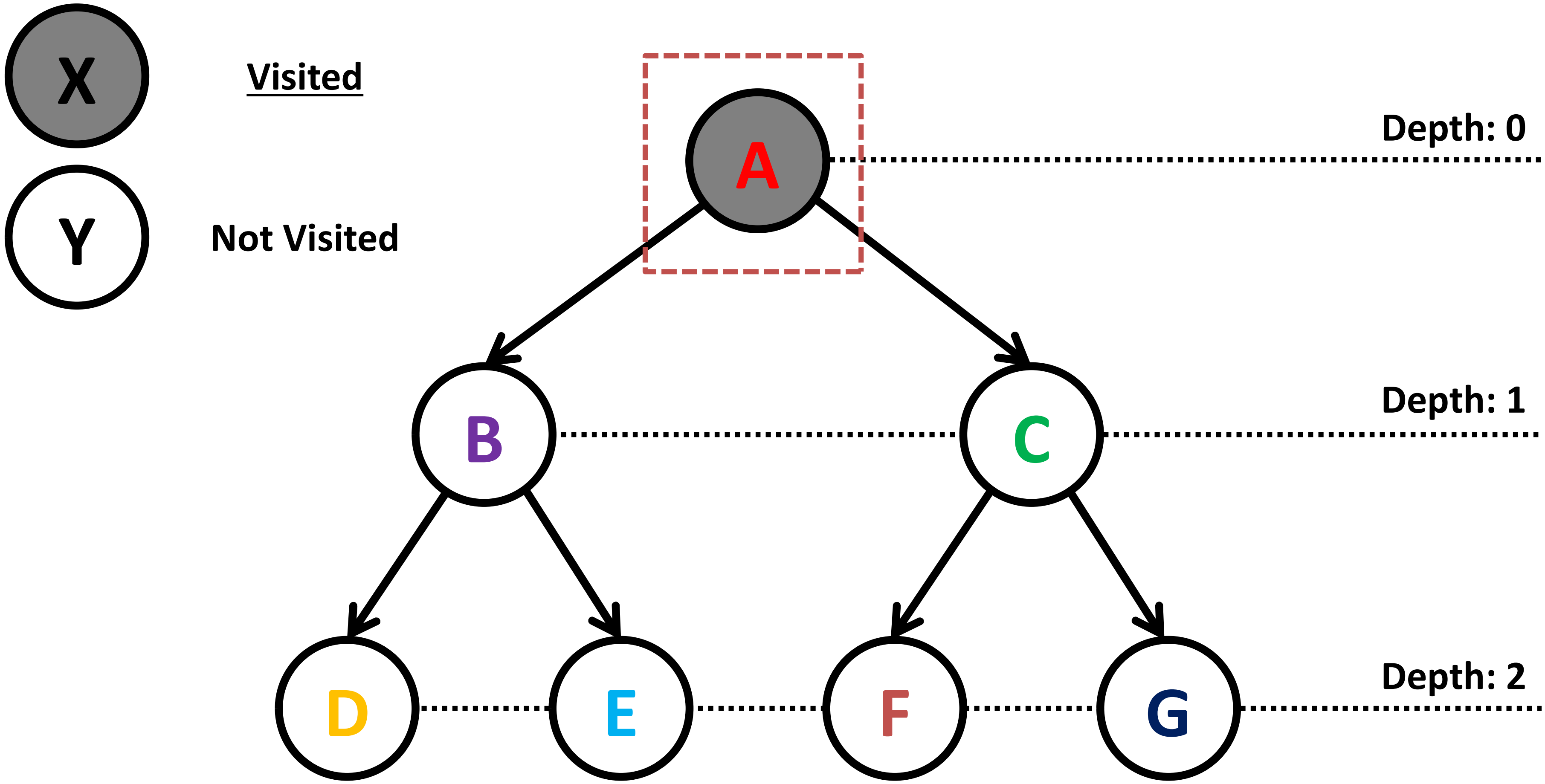
Step 13:

Node's Left Child: B
Node's Right Child: C
Node: A



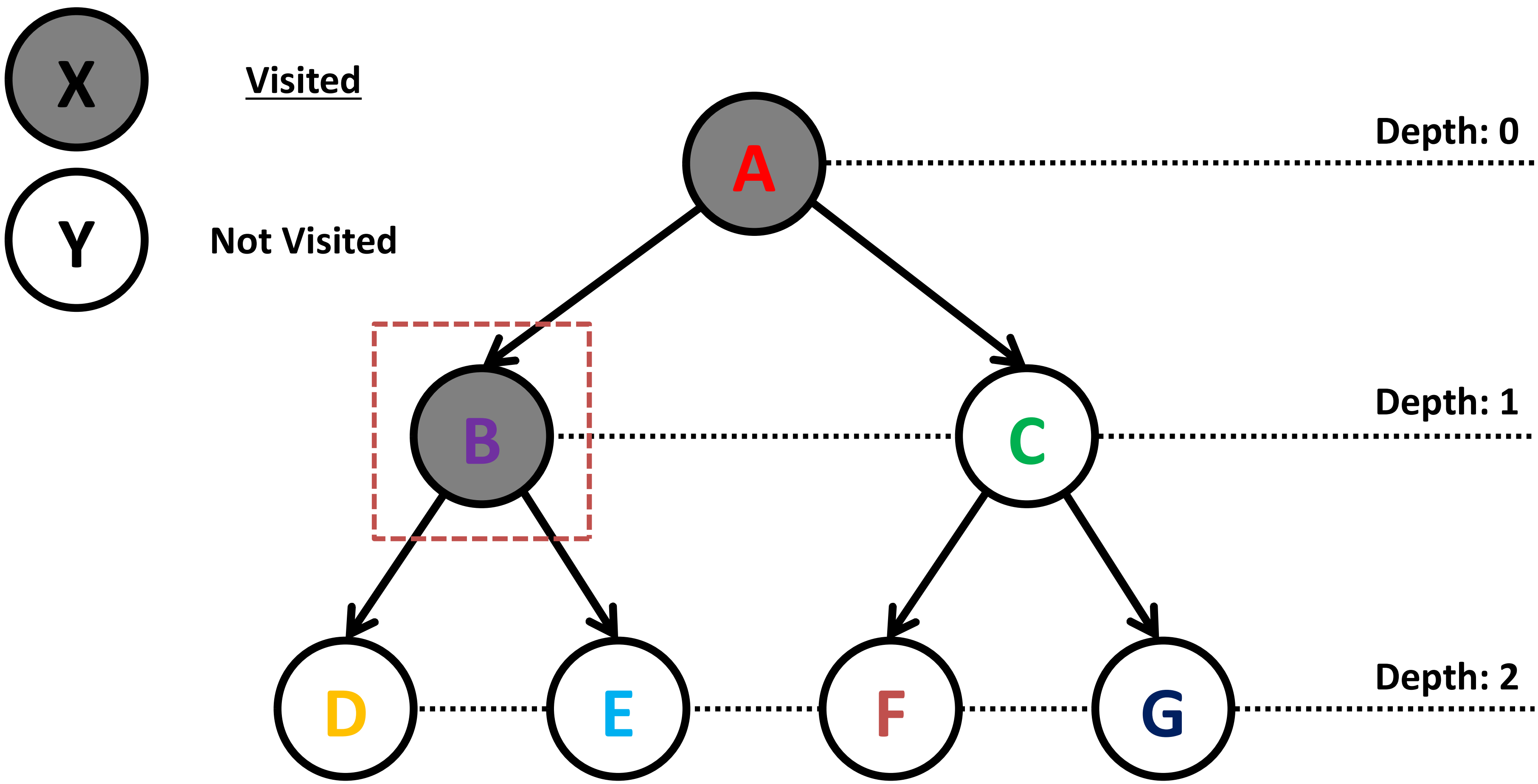
Nodes traversed so far: D E B F G C A

Level-order (Level-first | Left-Right)



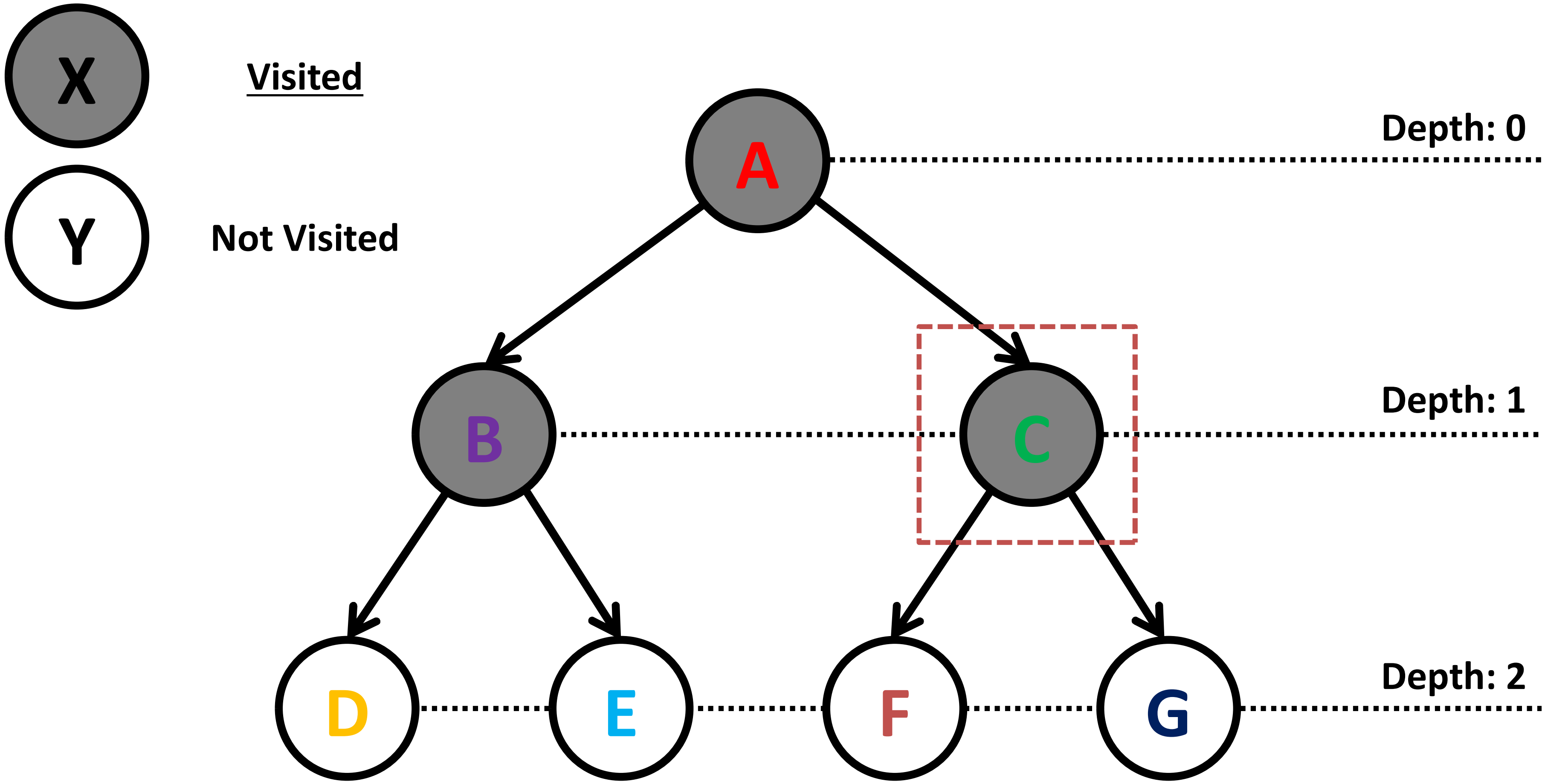
Nodes traversed so far: A

Level-order (Level-first | Left-Right)



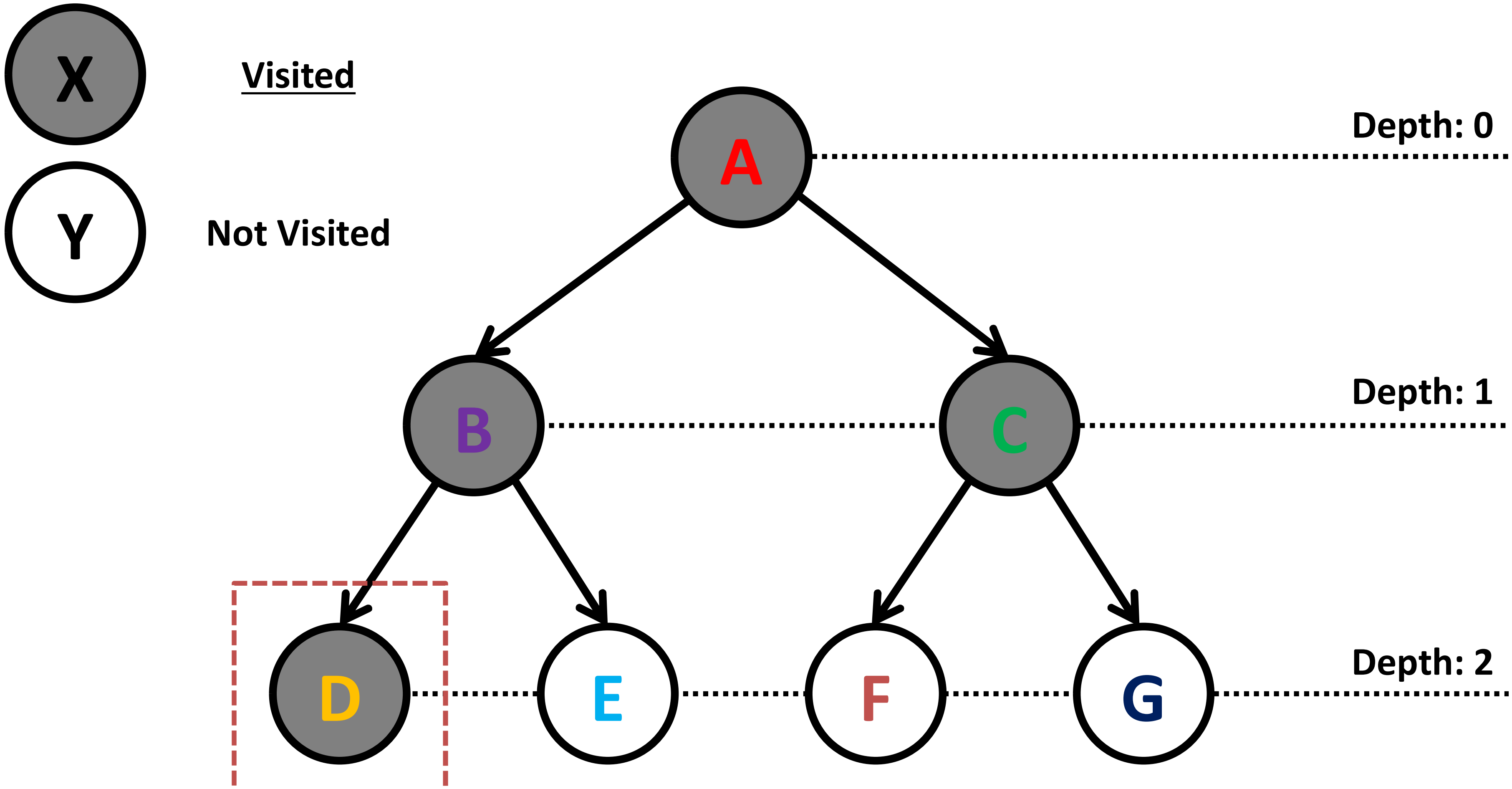
Nodes traversed so far: A B

Level-order (Level-first | Left-Right)



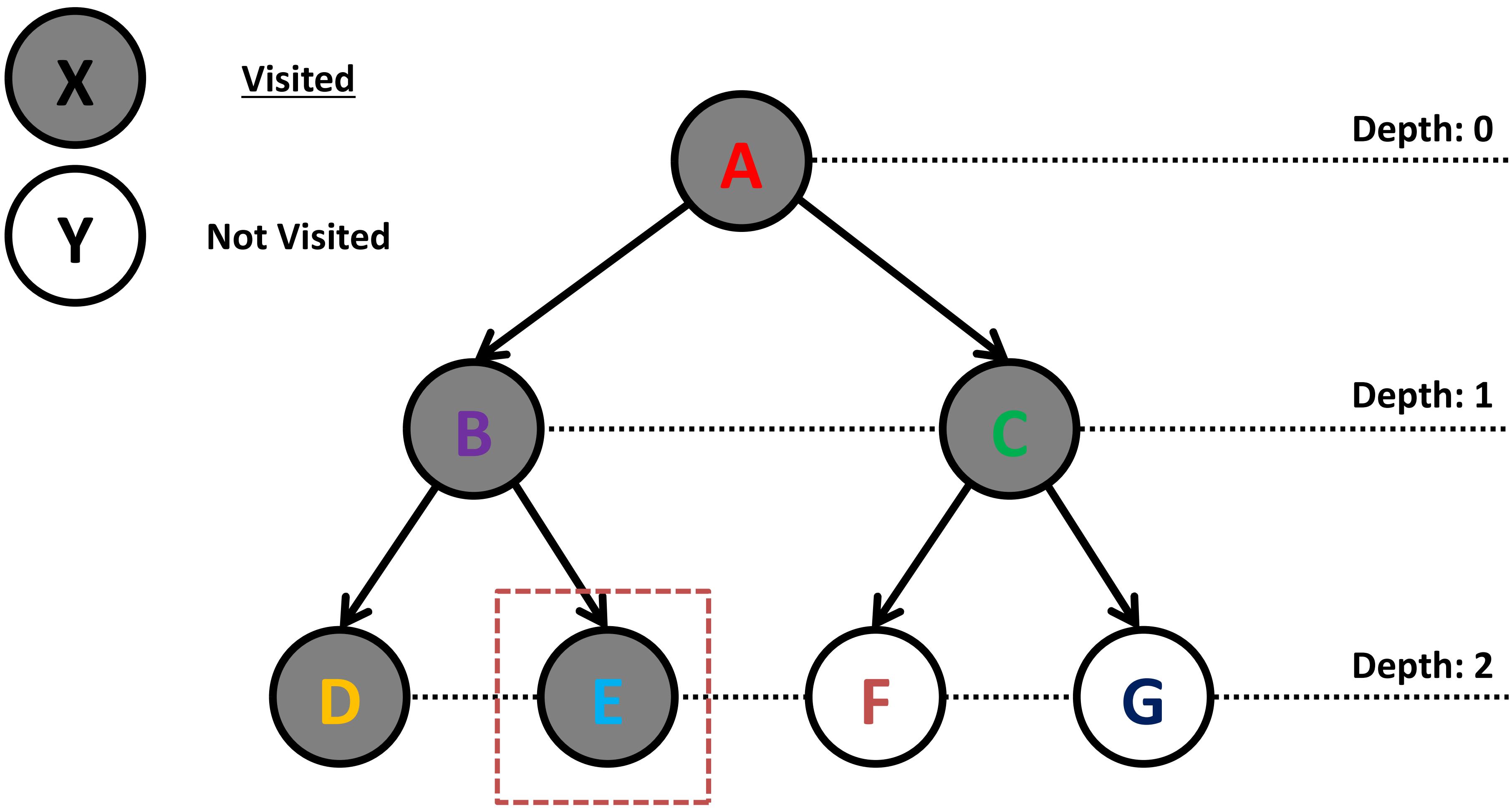
Nodes traversed so far: A B C

Level-order (Level-first | Left-Right)



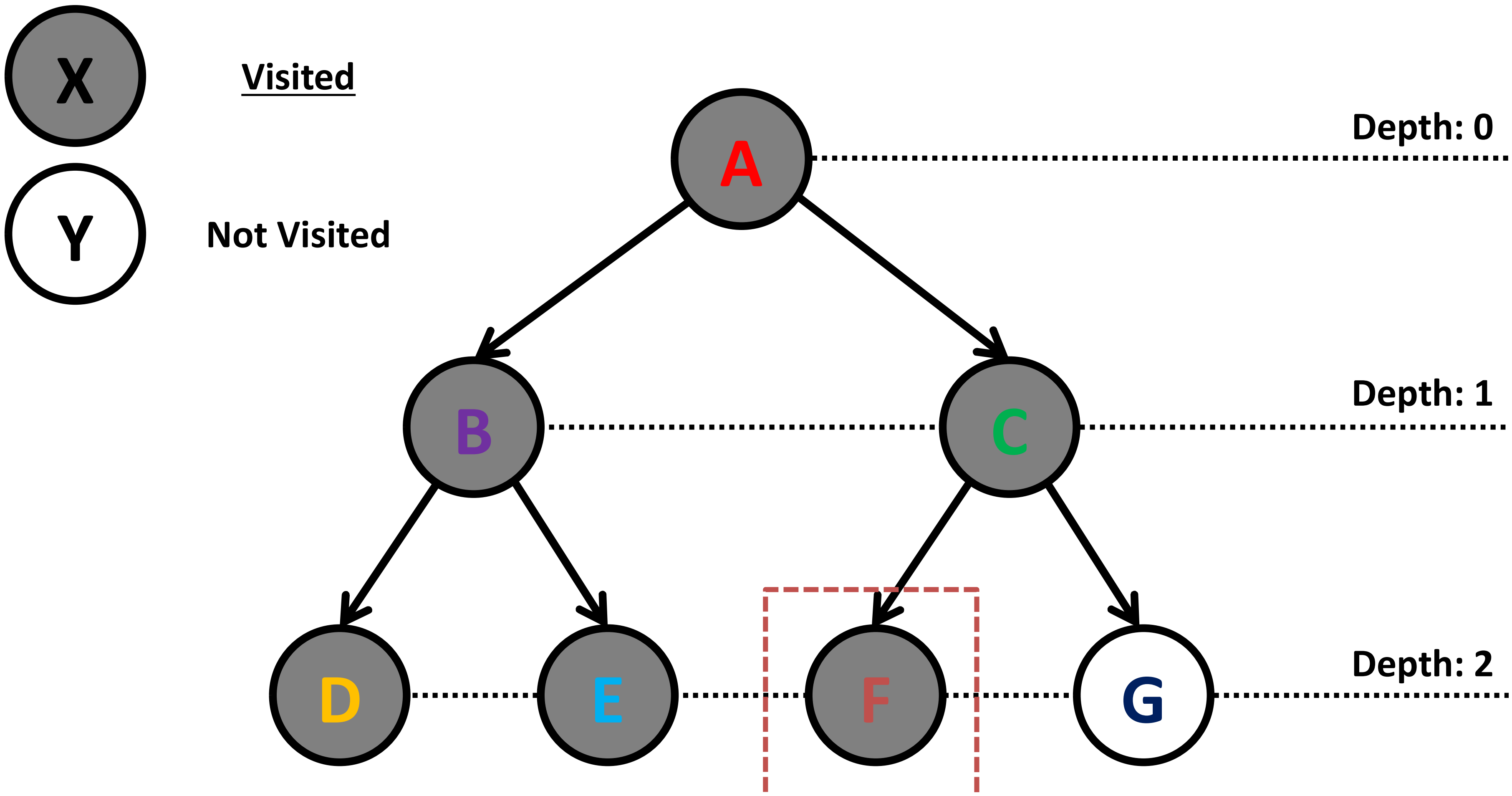
Nodes traversed so far: A B C D

Level-order (Level-first | Left-Right)



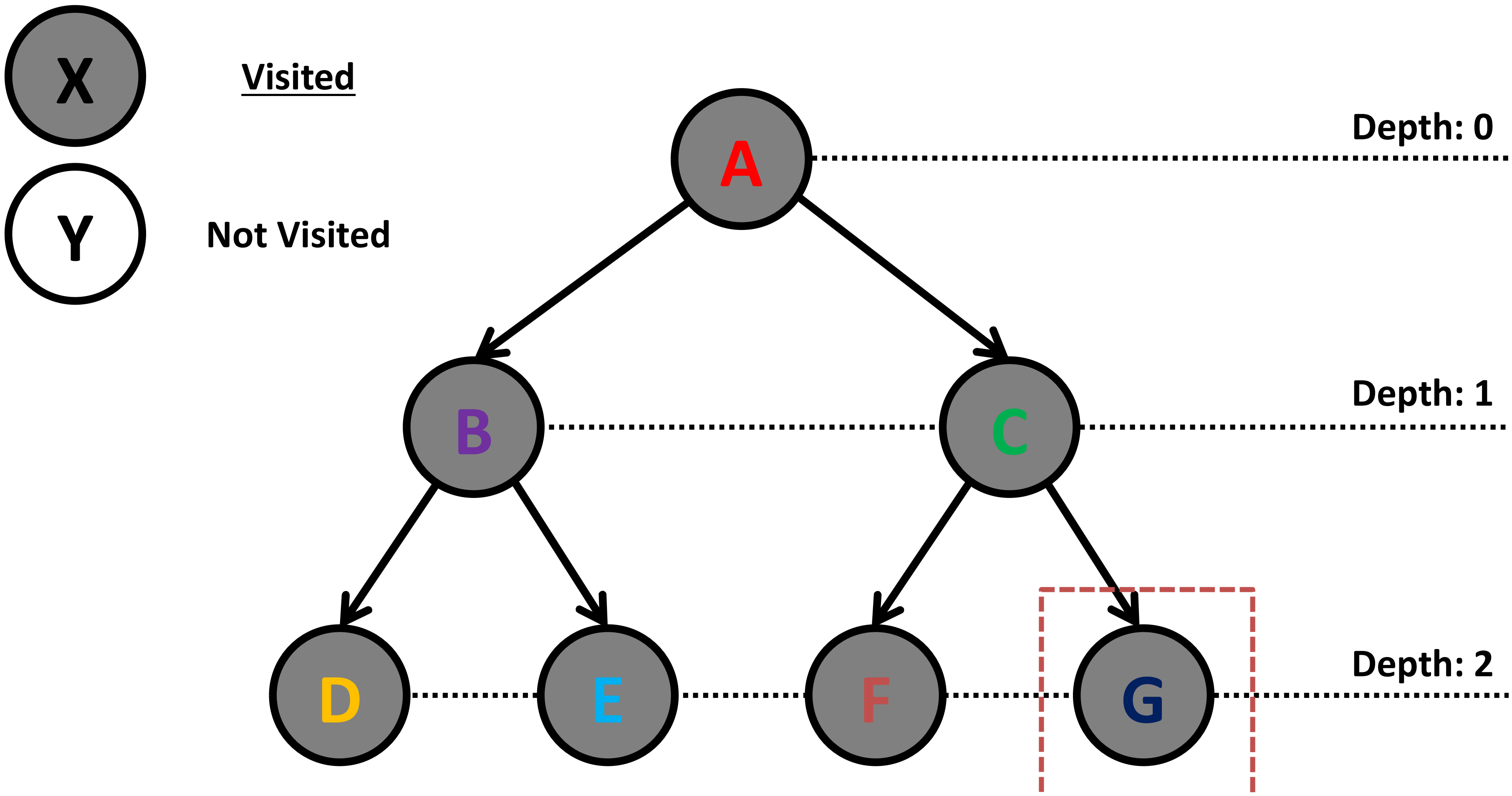
Nodes traversed so far: A B C D E

Level-order (Level-first | Left-Right)



Nodes traversed so far: A B C D E F

Level-order (Level-first | Left-Right)



Nodes traversed so far: A B C D E F G

Depth First vs Breadth (Width) First

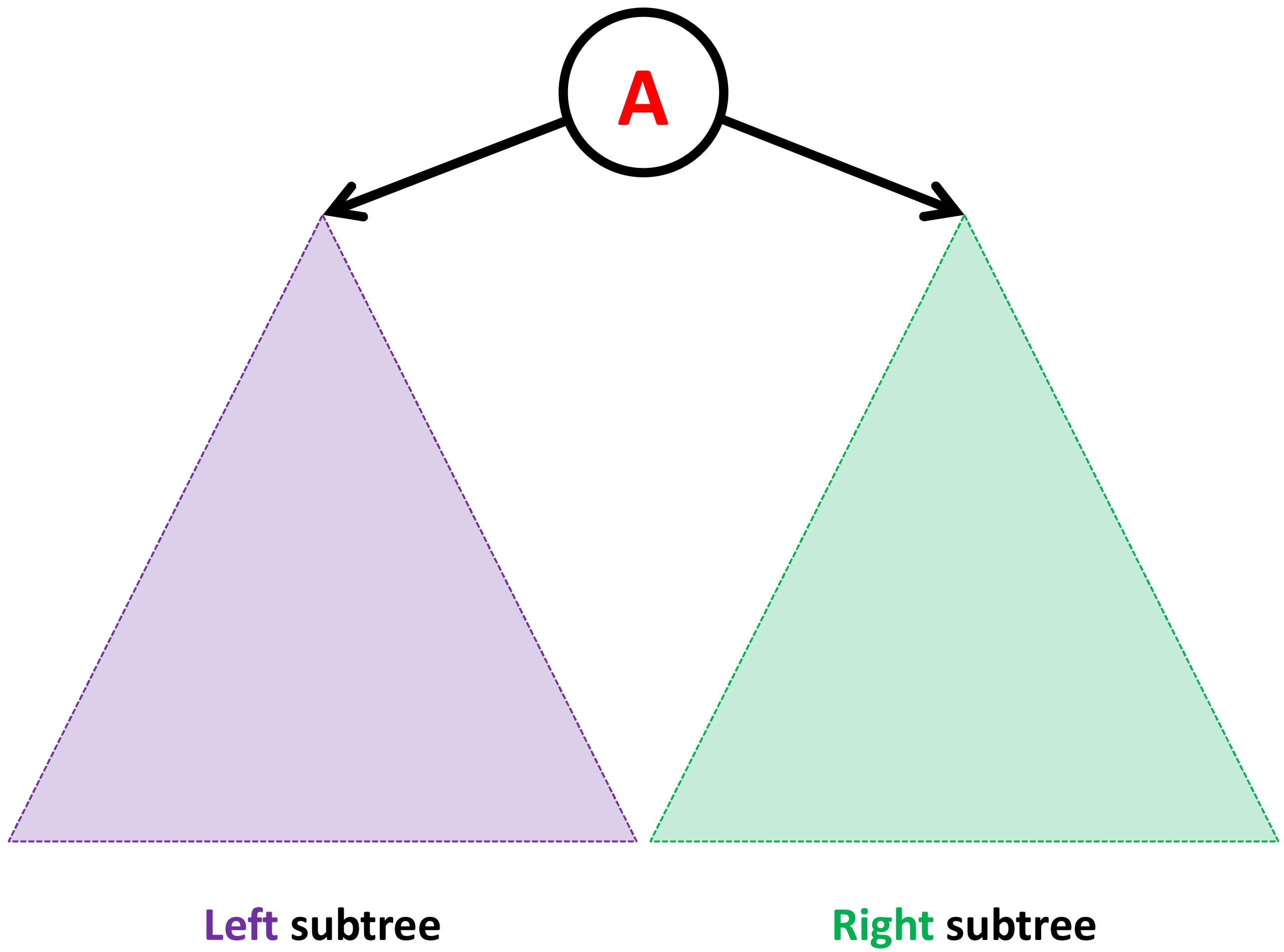
Depth First Traversals:

- Pre-order Traversal (Node, Left Child, Right Child)
- In-order Traversal (Left Child, Node, Right Child)
- Post-order Traversal (Left Child, Right Child, Node)

Breadth (Width) First

- Level-order (Level-by-level | left-to-right)

Binary Tree: Recursive Structure



Traversals: Algorithms

Pre-order Traversal (recursive)

```
void preOrder(Node node){
  if (node != null){
      System.out.print(node.getKey());
      preOrder(node.getLeftChild());
      preOrder(node.getRightChild());
  } else {
    return;
  }
}
```

In-Order Traversal (recursive)

```
void inOrder(Node node){
  if (node != null){
      inOrder(node.getLeftChild());
      System.out.print(node.getKey());
      inOrder(node.getRightChild());
  } else {
    return;
  }
}
```

Post-order Traversal (recursive)

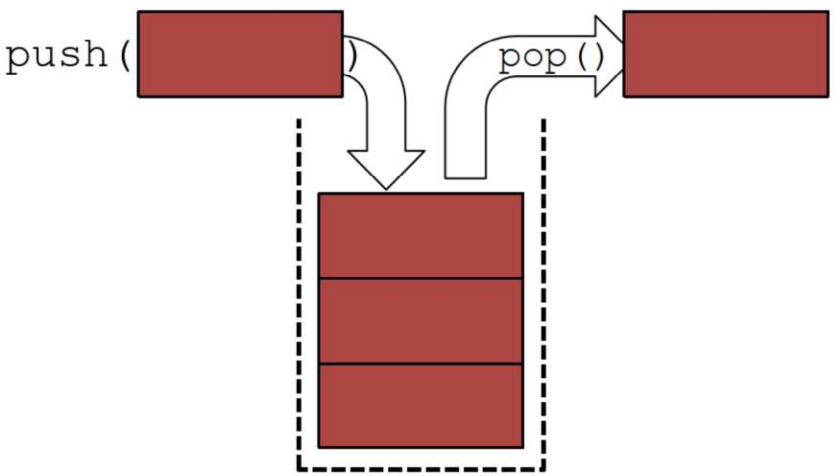
```
void postOrder(Node node){
  if (node != null){
      postOrder(node.getLeftChild());
      postOrder(node.getRightChild());
      System.out.print(node.getKey());
  } else {
    return;
  }
}
```

Level-order Traversal

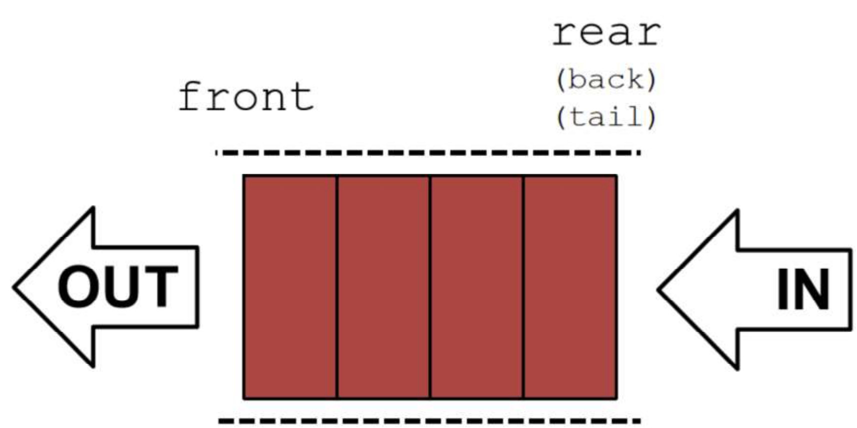
```
void levelOrder(Node node){
  if (node != null){
     queue.add(node);
     while(queue.isEmpty() == null)
       current = queue.dequeue();
       System.out.println(current);
       queue.enqueue(current children)
  }
}
```

Traversals: Data Structure Use

- **Pre-order Traversal**
- **In-order Traversal**
- **Post-order Traversal**
- **Level-order (Level-by-level | left-to-right)**

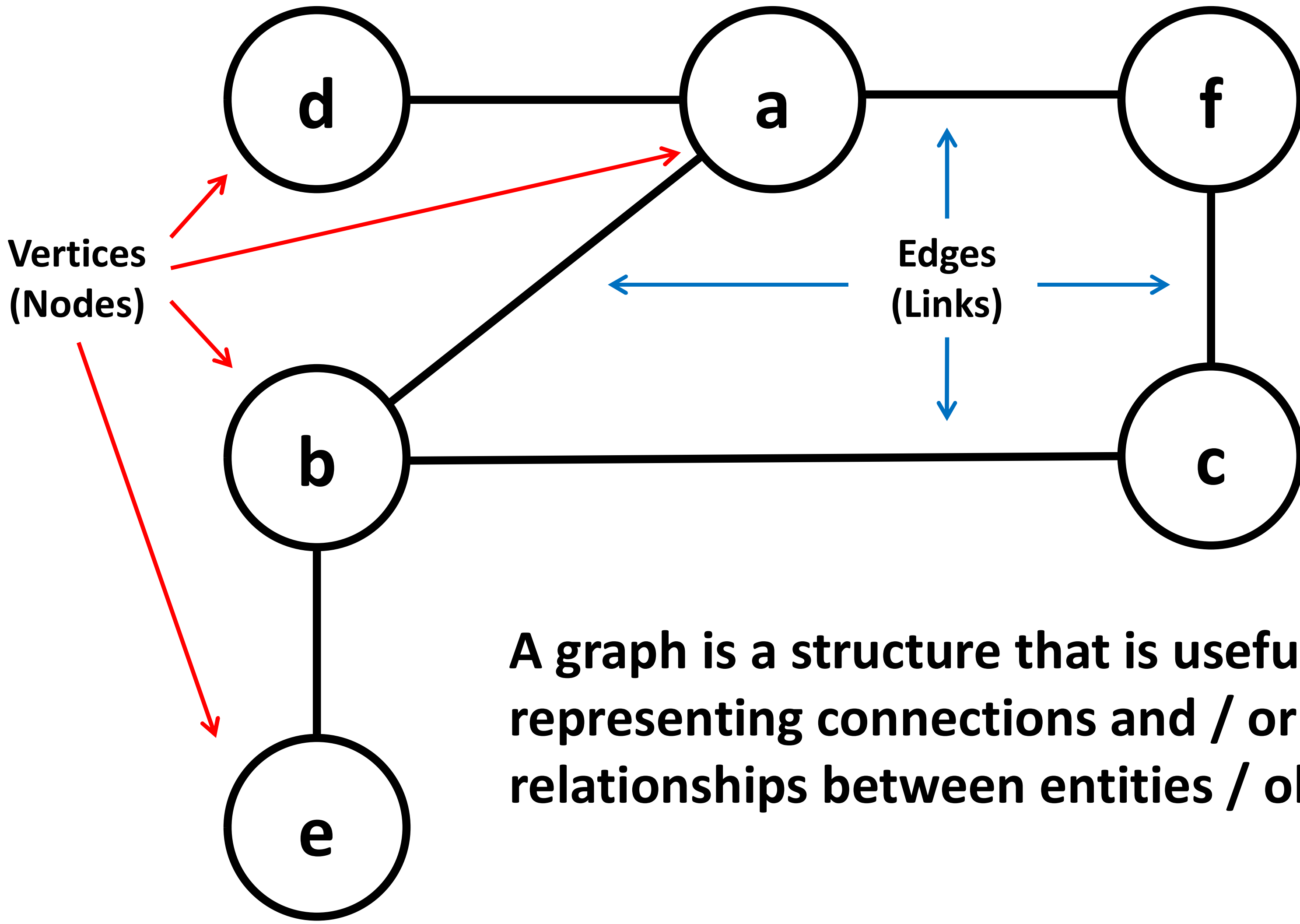


Use a stack



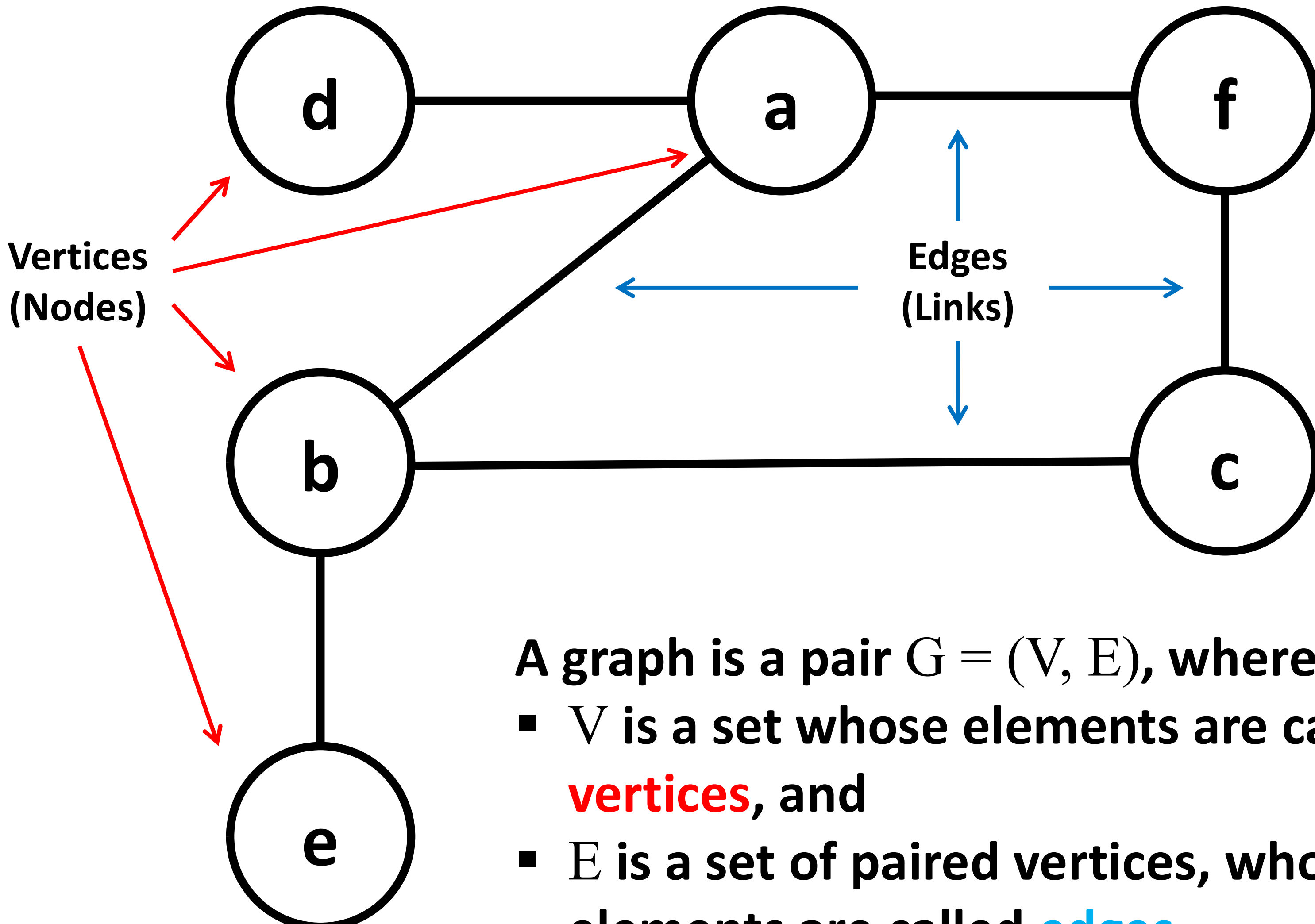
Use a queue

Graph = Vertices + Edges



A graph is a structure that is useful in representing connections and / or relationships between entities / objects

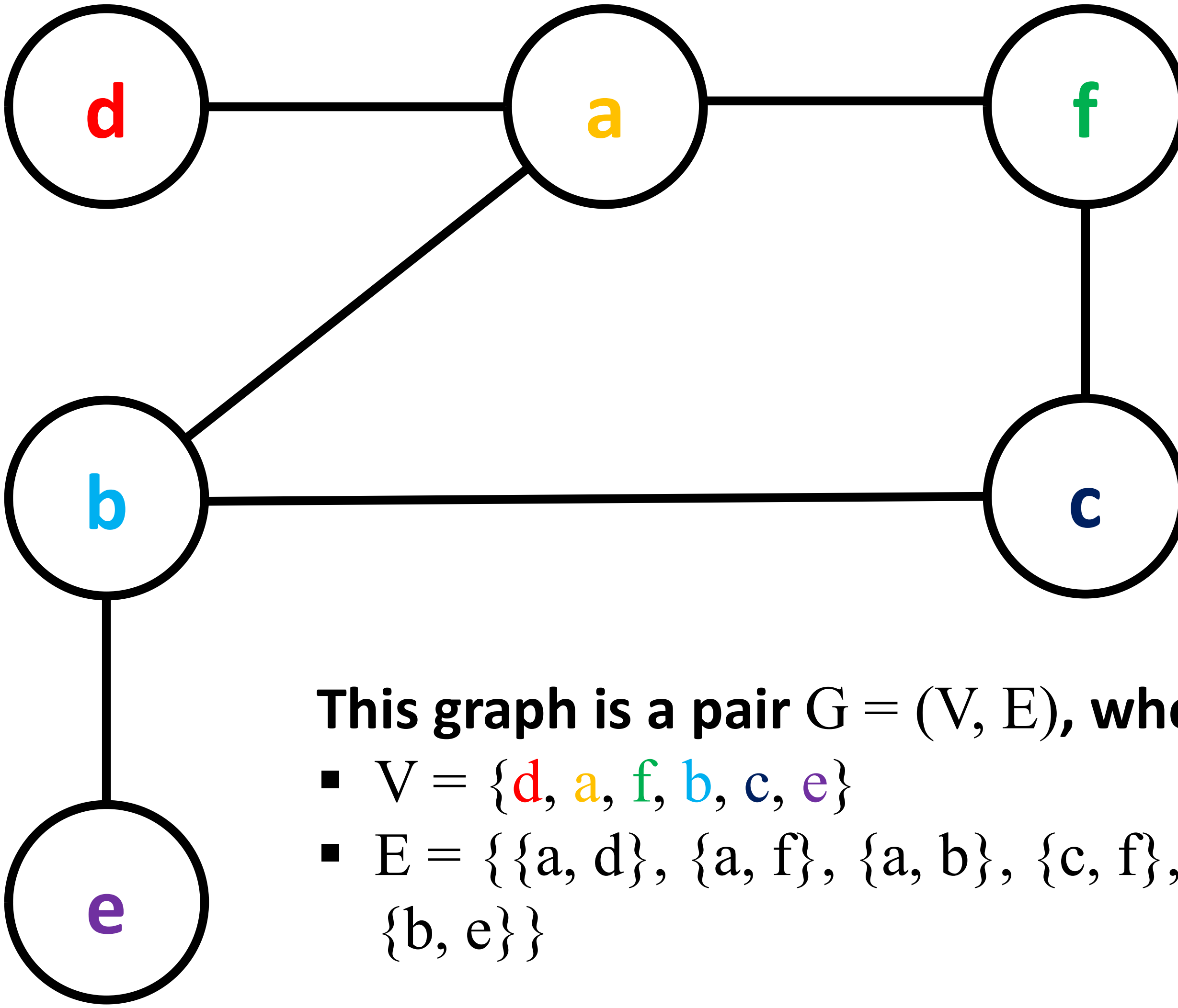
Graph: $G = (V, E)$



A graph is a pair $G = (V, E)$, where:

- V **is a set whose elements are called vertices, and**
- E **is a set of paired vertices, whose elements are called edges**

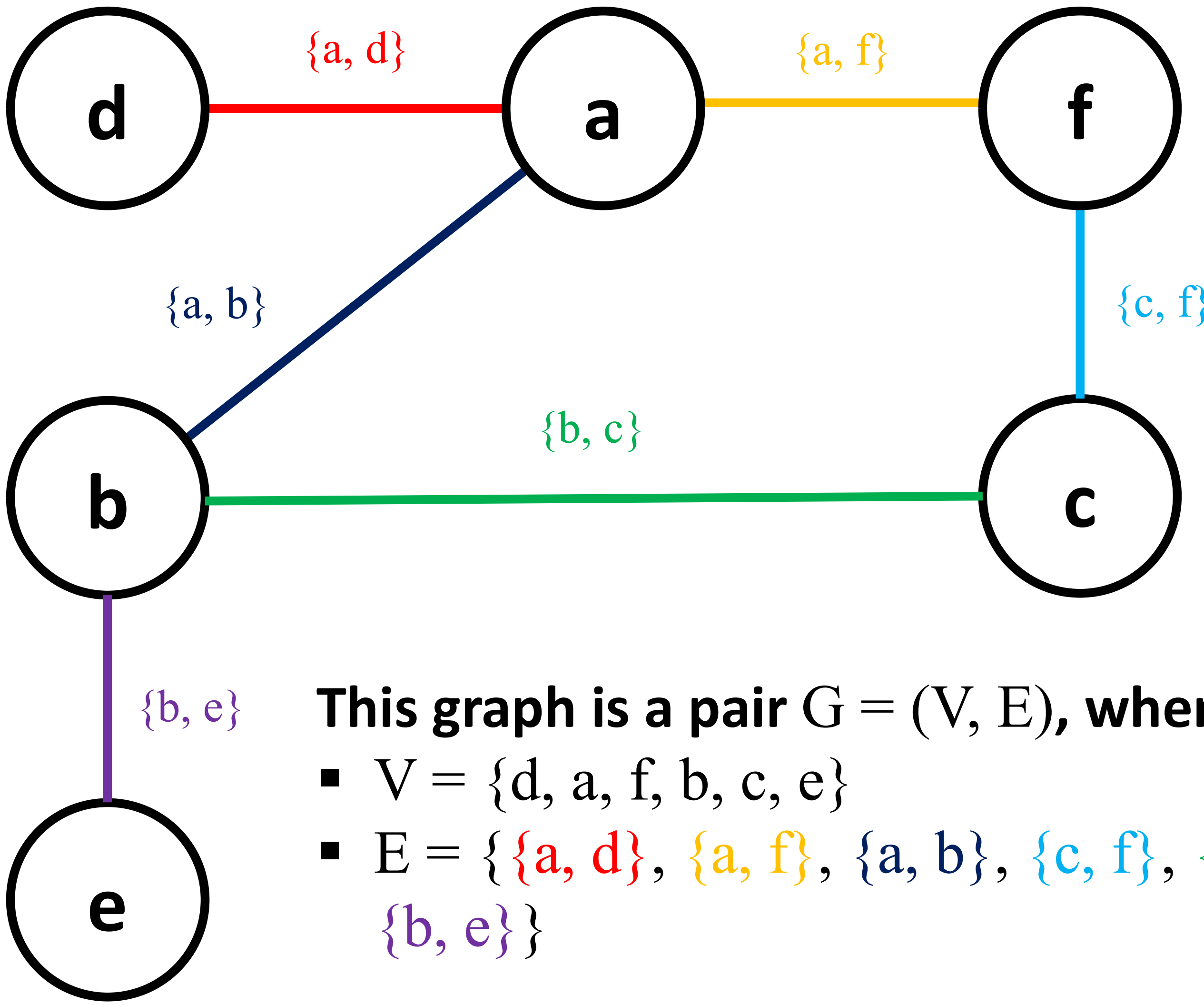
Graph: $G = (V, E)$



This graph is a pair $G = (V, E)$**, where:**

- $V = \{d, a, f, b, c, e\}$
- $E = \{\{a, d\}, \{a, f\}, \{a, b\}, \{c, f\}, \{b, c\}, \{b, e\}\}$

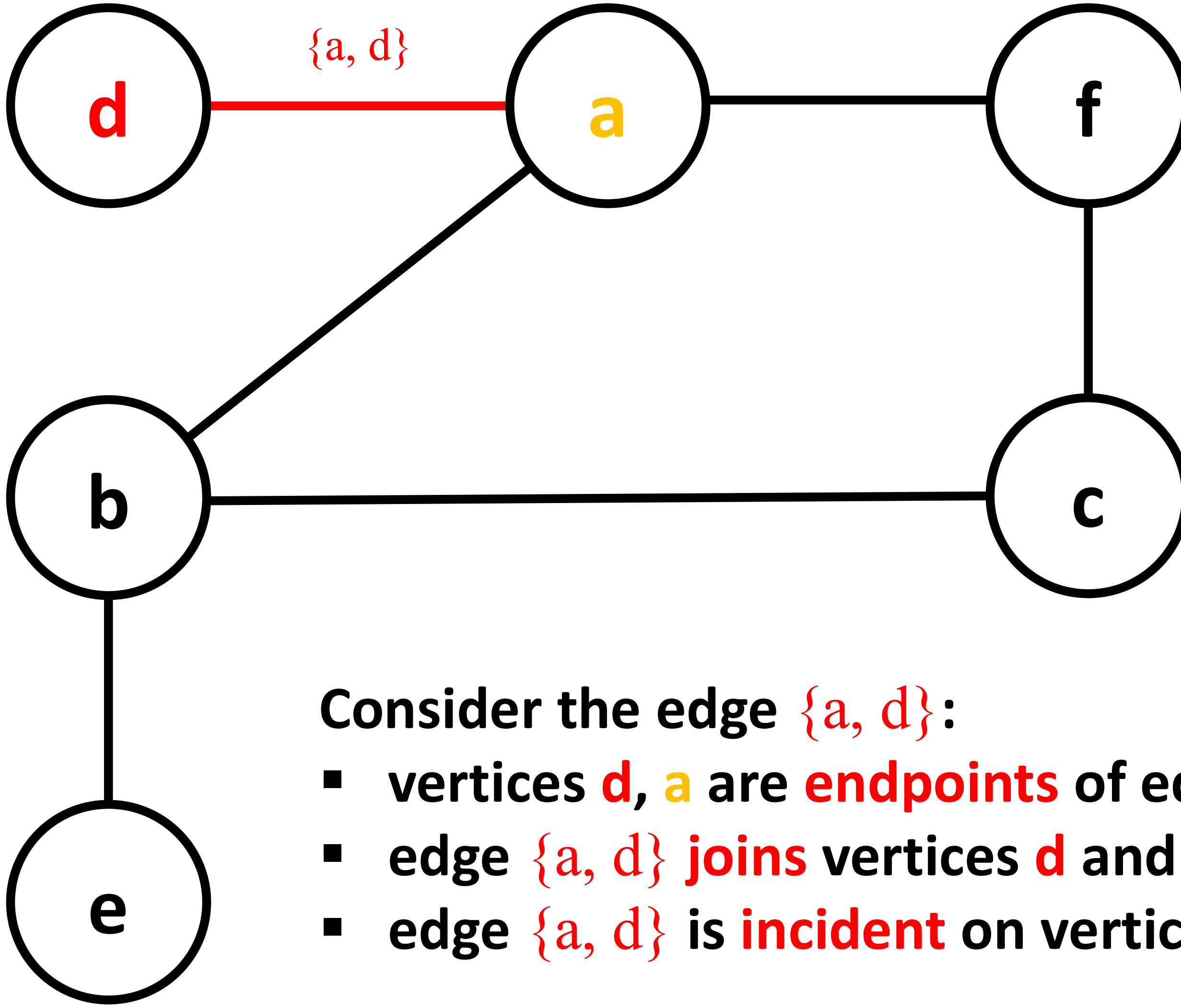
Graph: $G = (V, E)$



This graph is a pair $G = (V, E)$**, where:**

- $V = \{d, a, f, b, c, e\}$
- $E = \{\{a, d\}, \{a, f\}, \{a, b\}, \{c, f\}, \{b, c\}, \{b, e\}\}$

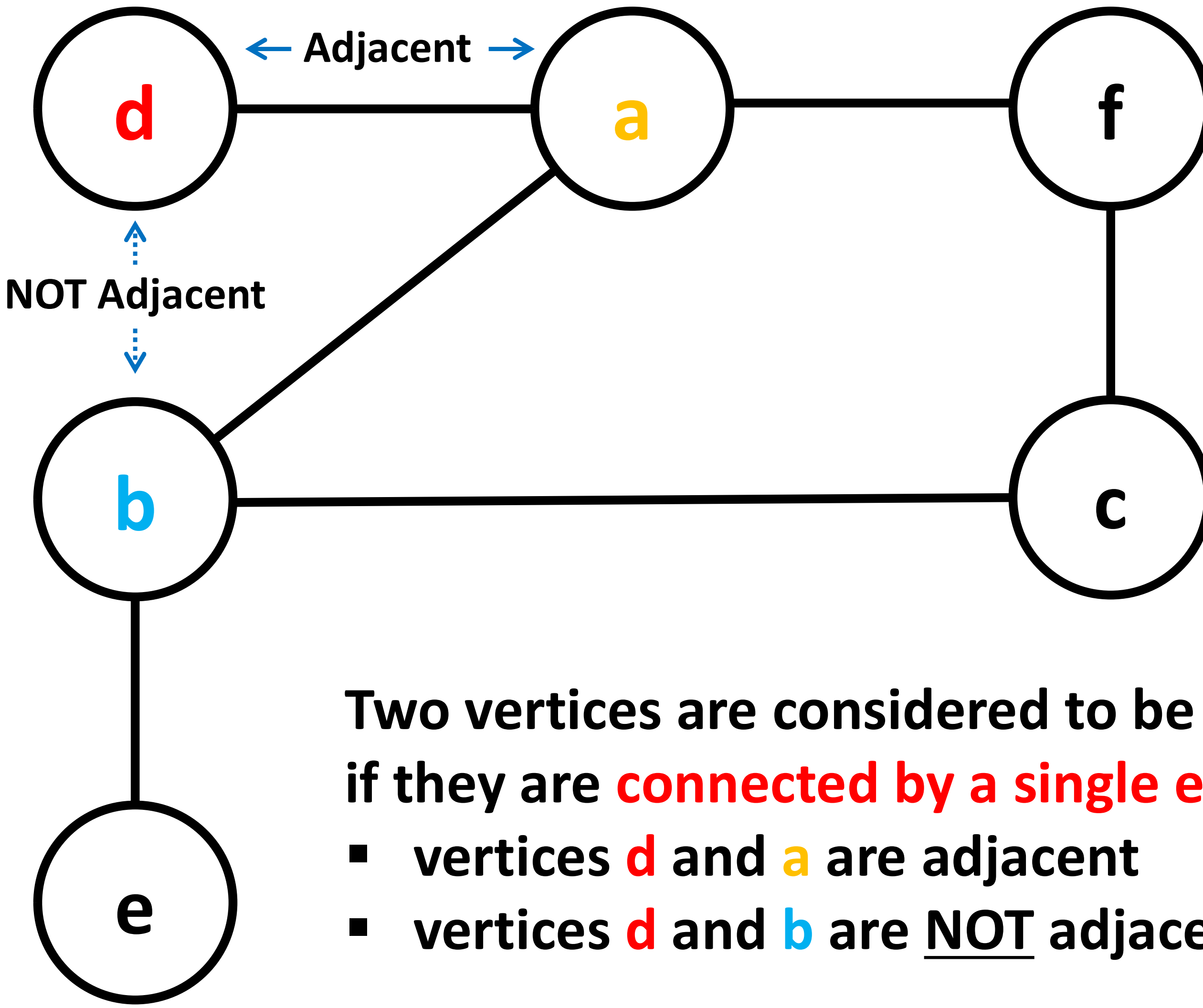
Graph: Edges



Consider the edge $\{a, d\}$:

- vertices **d**, **a** are **endpoints** of edge $\{a, d\}$
- edge $\{a, d\}$ **joins** vertices **d** and **a**
- edge $\{a, d\}$ is **incident** on vertices **d** and **a**

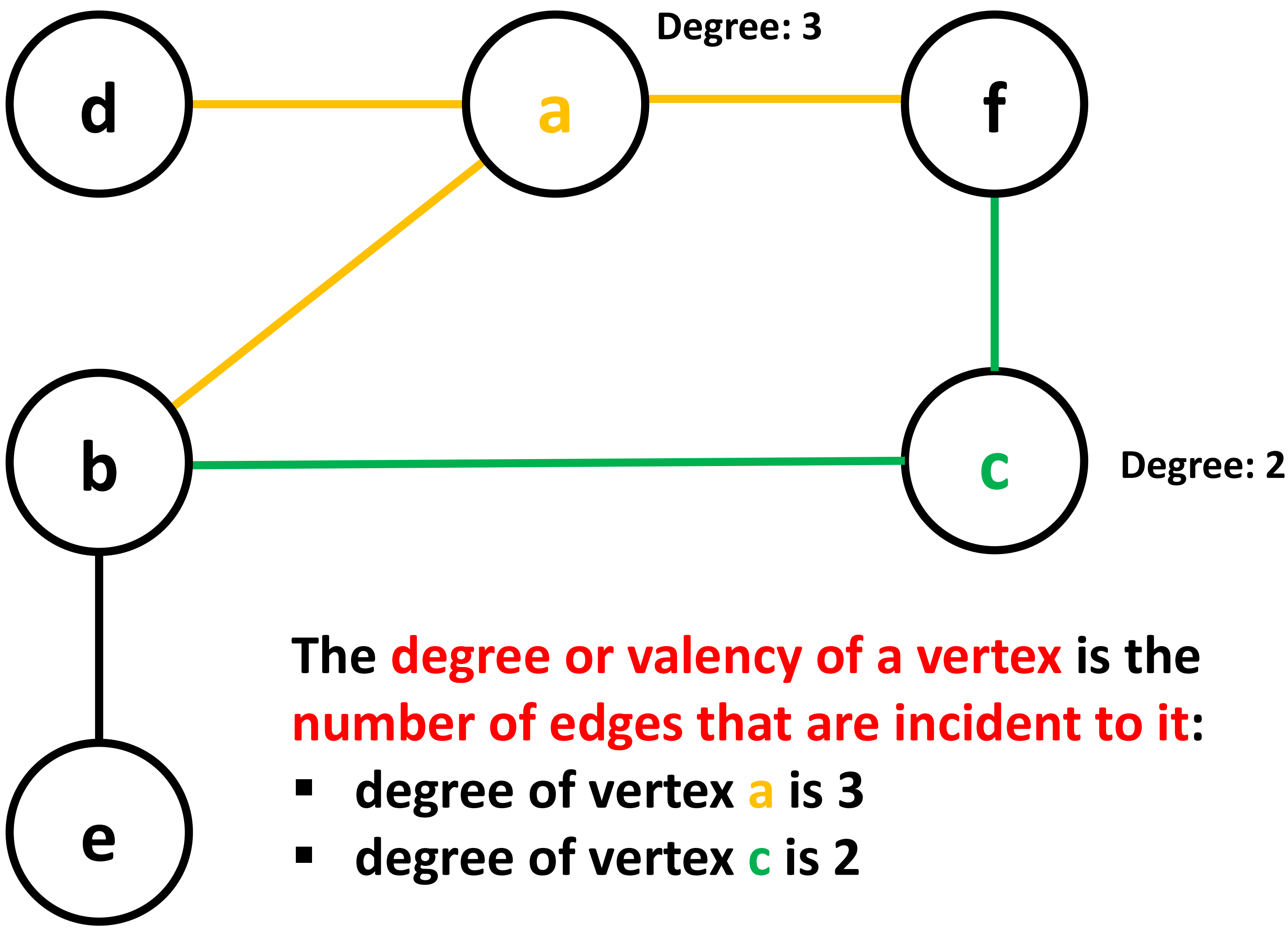
Graph: Vertex Adjacency



Two vertices are considered to be adjacent if they are connected by a single edge:

- vertices d and a are adjacent
- vertices d and b are NOT adjacent

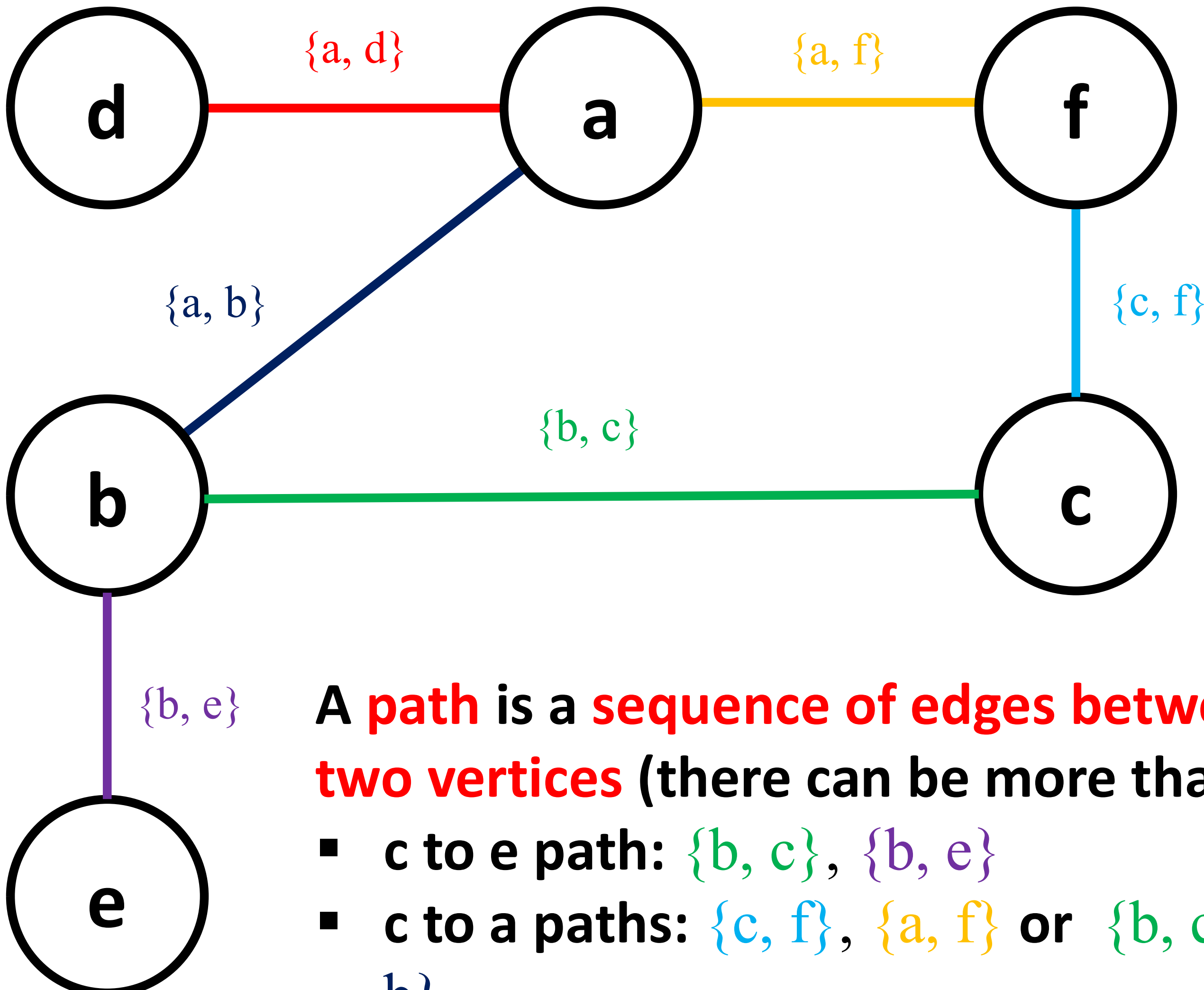
Graph: Vertex Degree / Valency



The **degree or valency of a vertex** is the **number of edges that are incident to it**:

- degree of vertex a is 3
- degree of vertex c is 2

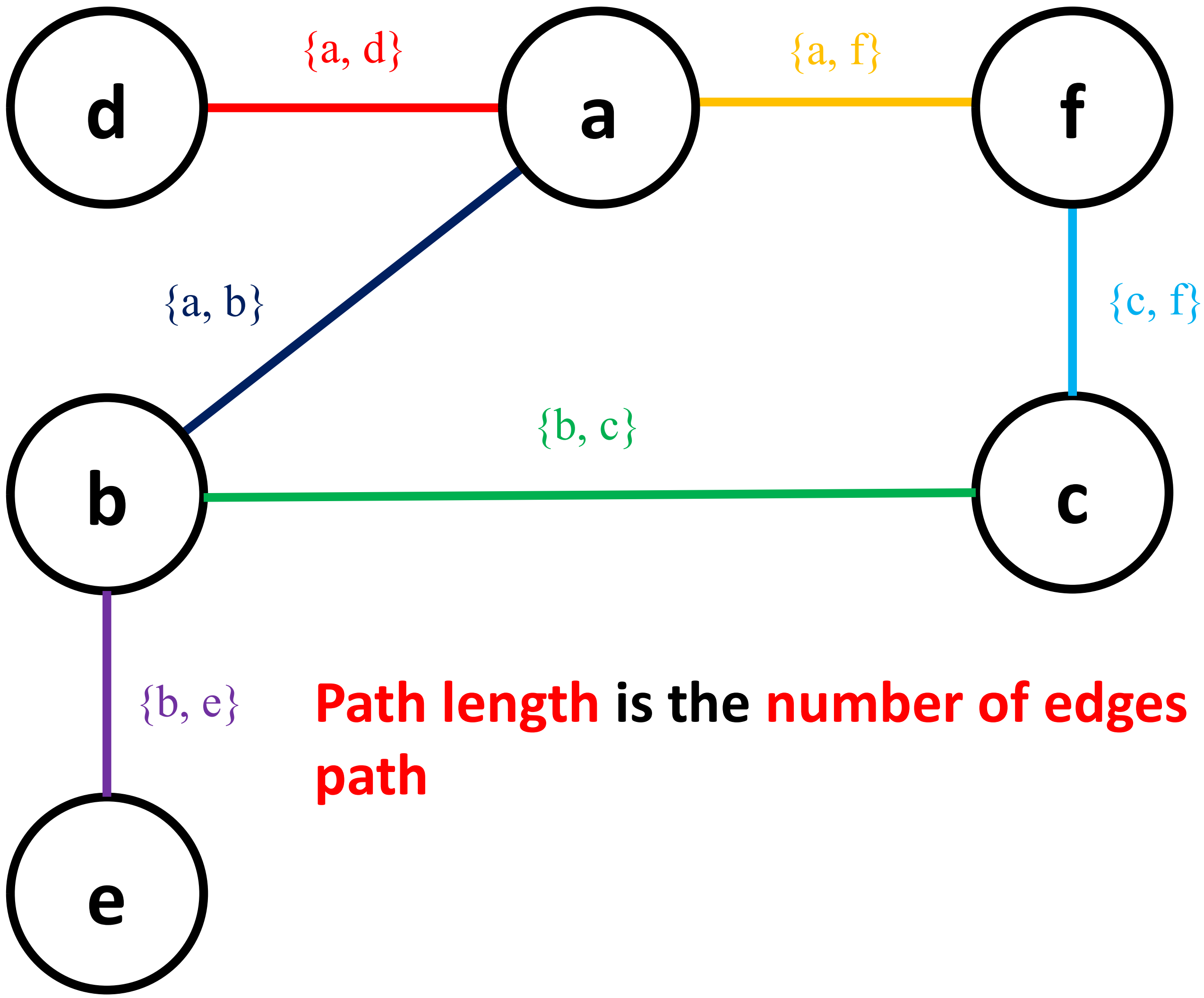
Graph: A Path



A **path** is a **sequence of edges between two vertices** (there can be more than one):

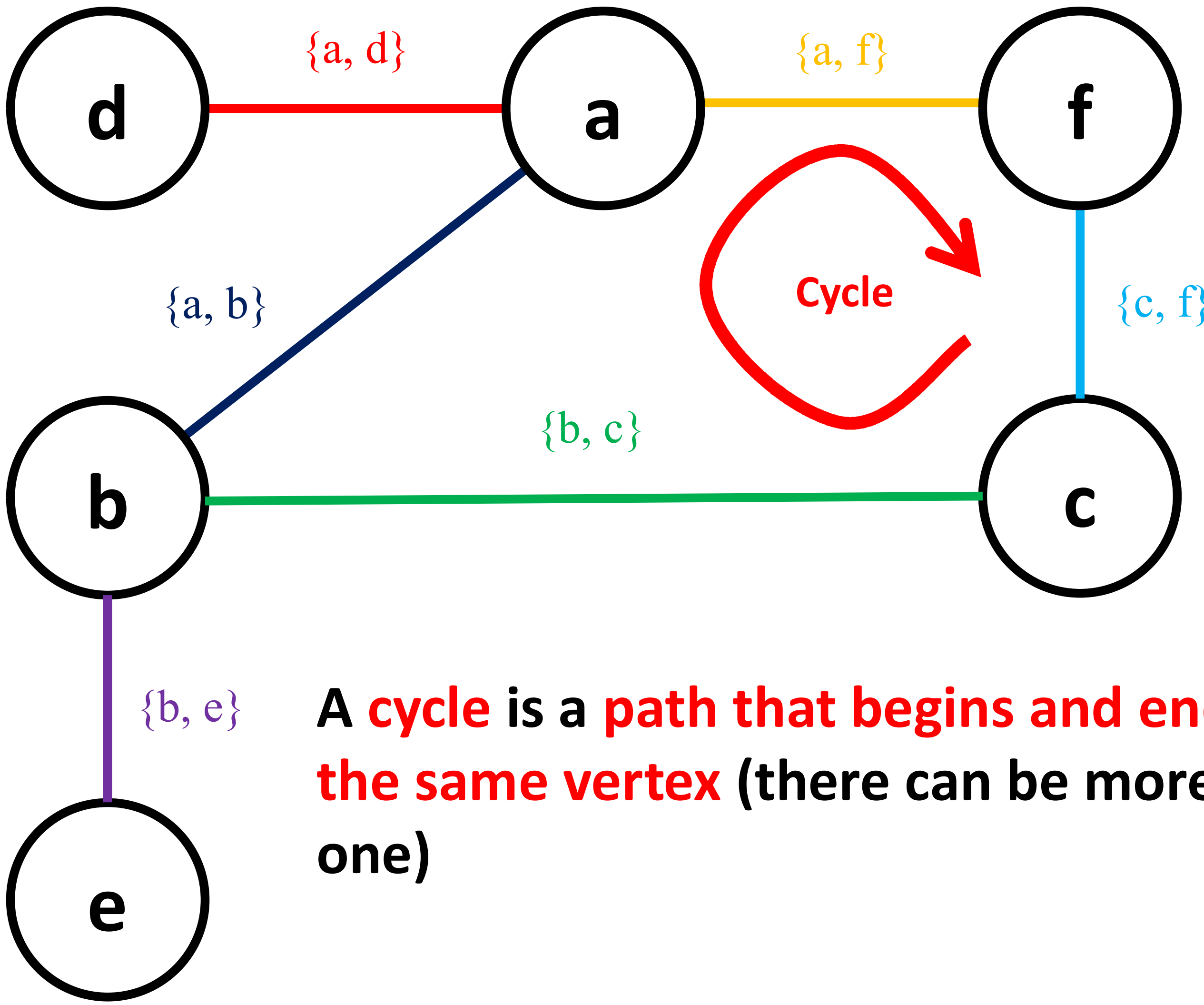
- c to e path: {b, c}, {b, e}
- c to a paths: {c, f}, {a, f} or {b, c}, {a, b}

Graph: Path Length



Path length is the **number of edges in a path**

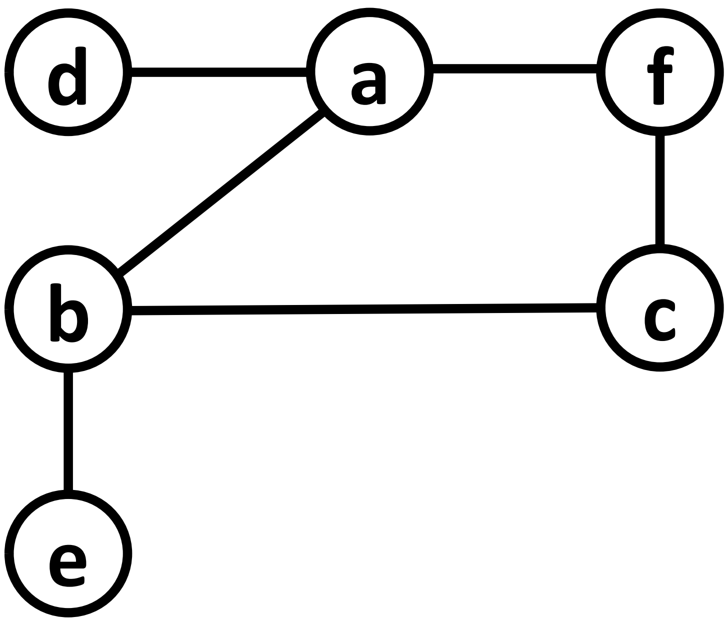
Graph: A Cycle



A **cycle** is a **path that begins and ends at the same vertex** (there can be more than one)

Connected vs. Non-Connected

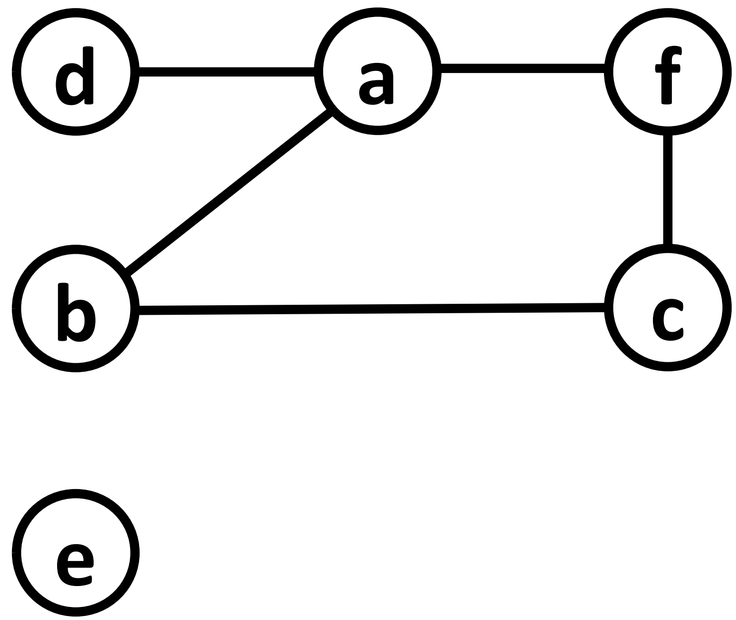
Connected Graph



Connected graph:

- a graph is said to be connected if there exist at least one path from every vertex to every other vertex

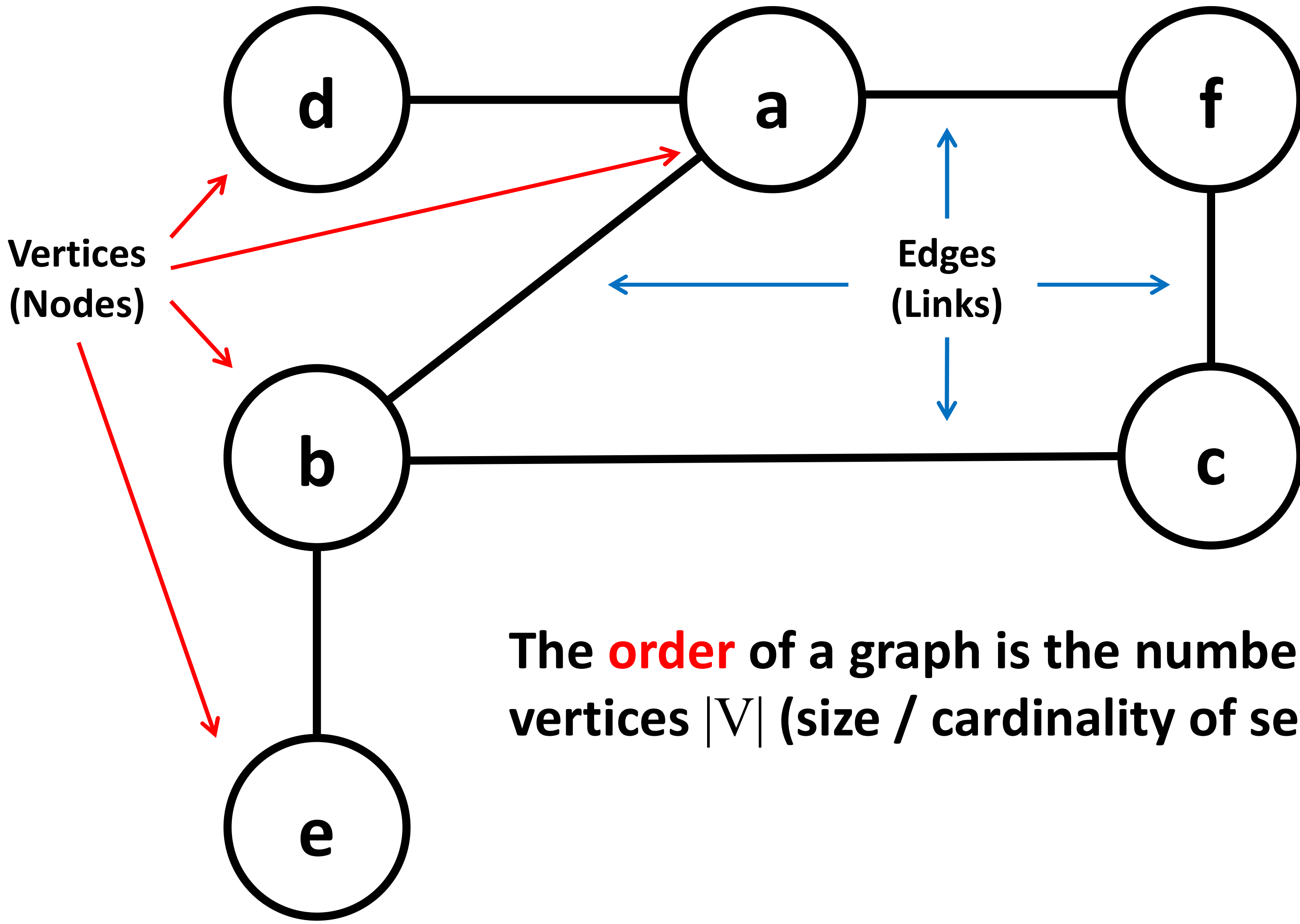
Non-Connected Graph



Non-connected graph:

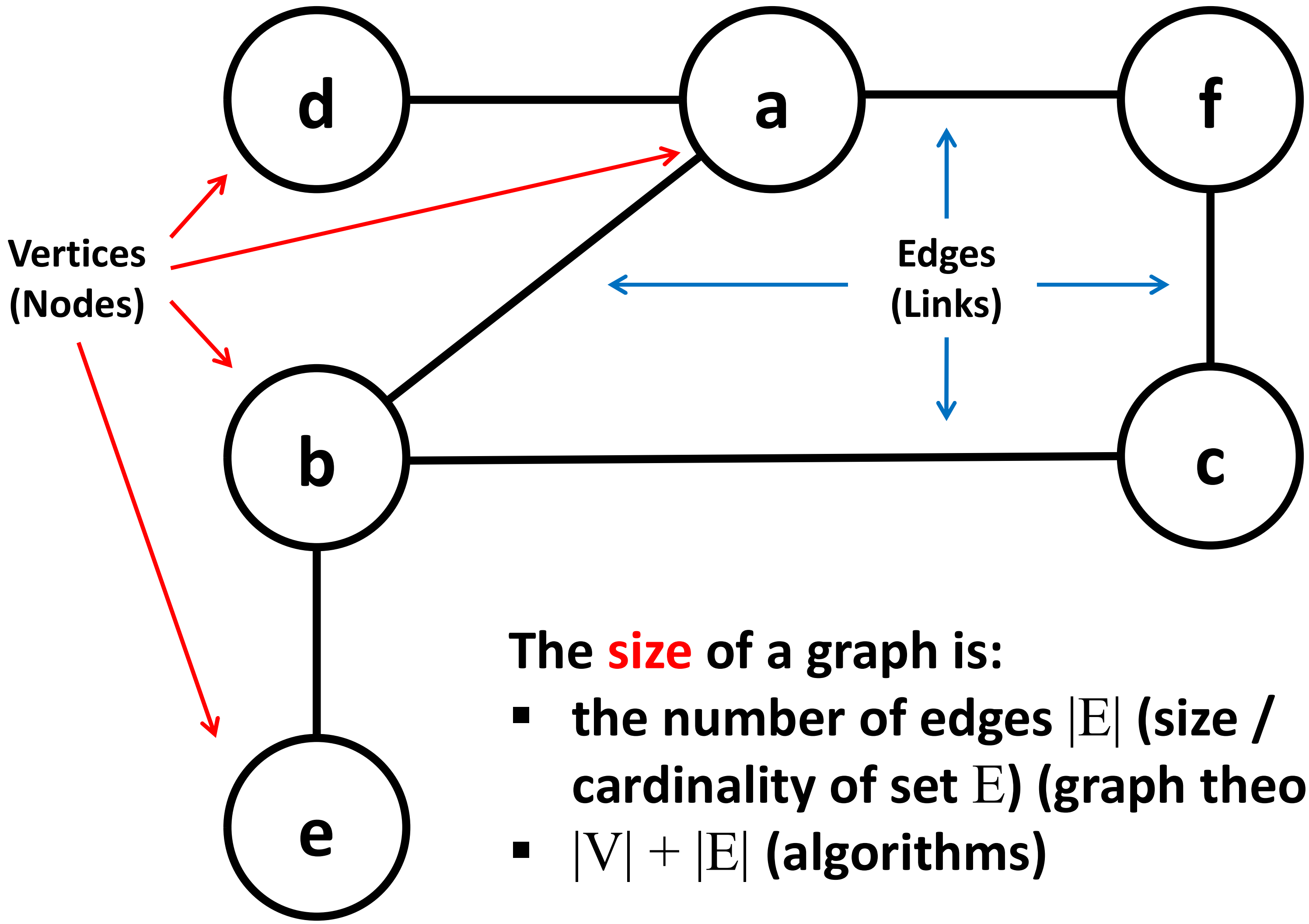
- a graph is non-connected if there exist at least one vertex such there there is no path to it from all the other vertices (vertex e in this example)
- vertex e is not "reachable"

Graph: Order



The order of a graph is the number of vertices $|V|$ (size / cardinality of set V)

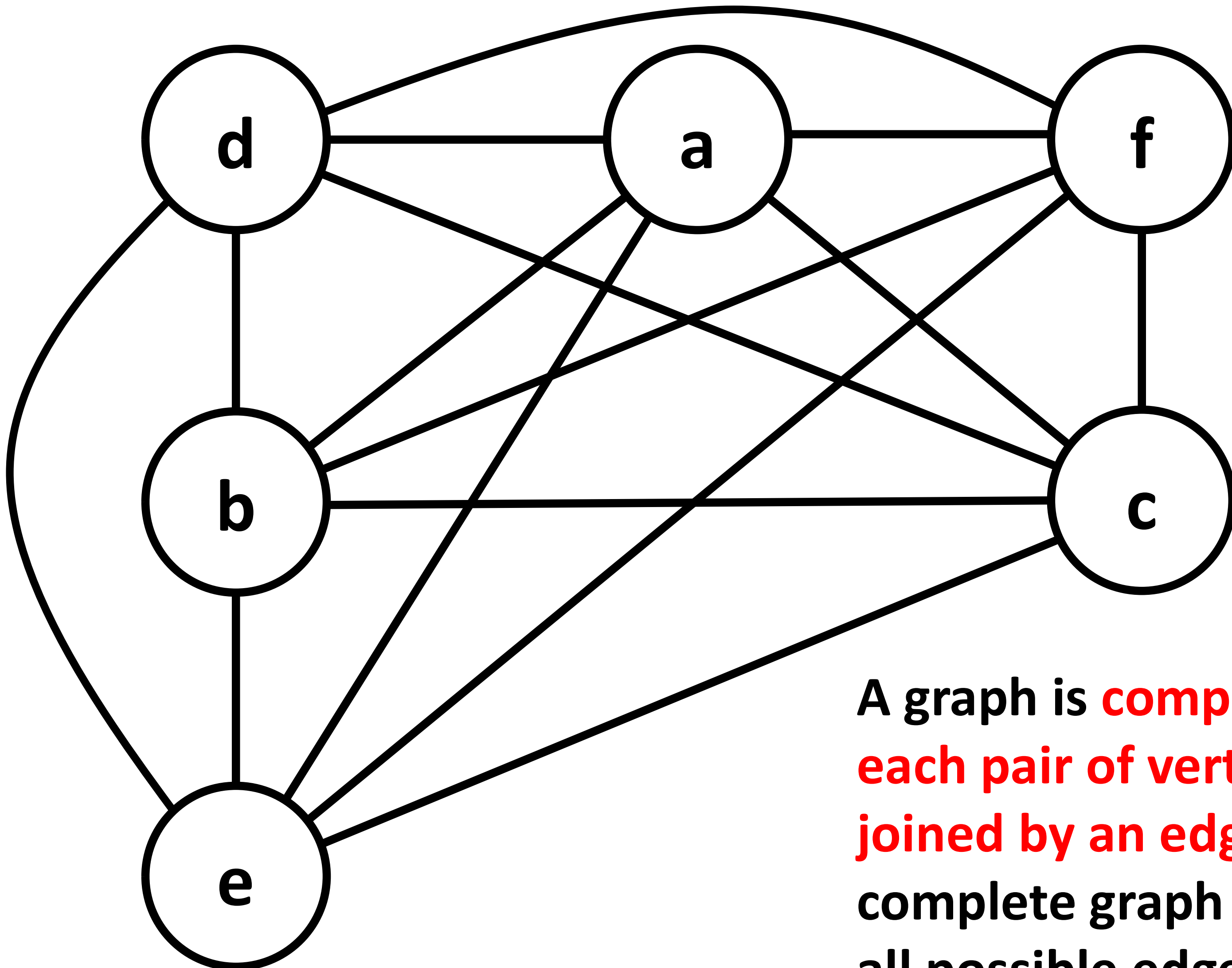
Graph: Size



The **size** of a graph is:

- the number of edges $|E|$ (size / cardinality of set E) (graph theory)
- $|V| + |E|$ (algorithms)

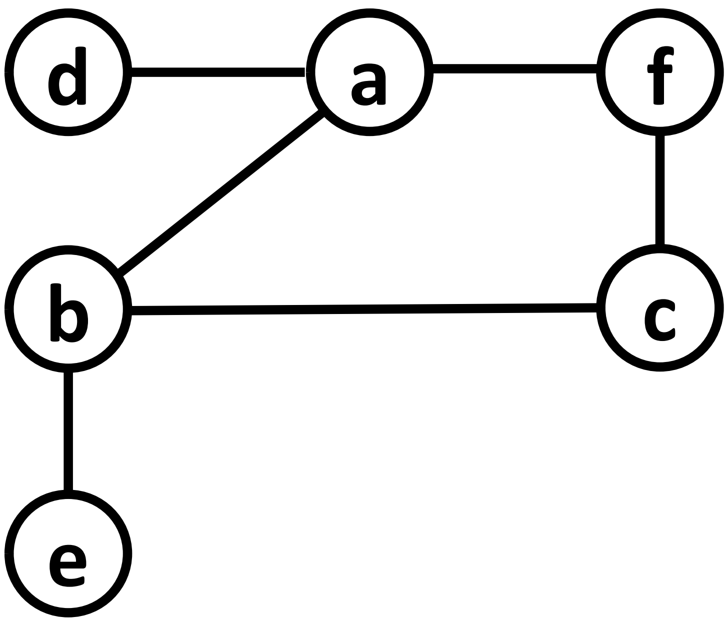
A Complete Graph



A graph is **complete** if **each pair of vertices is joined by an edge**. A complete graph contains all possible edges

Representation: Adjacency Matrix

Graph G



A graph $G = \{V, E\}$:

- $N = |V|$ vertices / nodes, and
- $|E|$ edges

Adjacency Matrix for Graph G

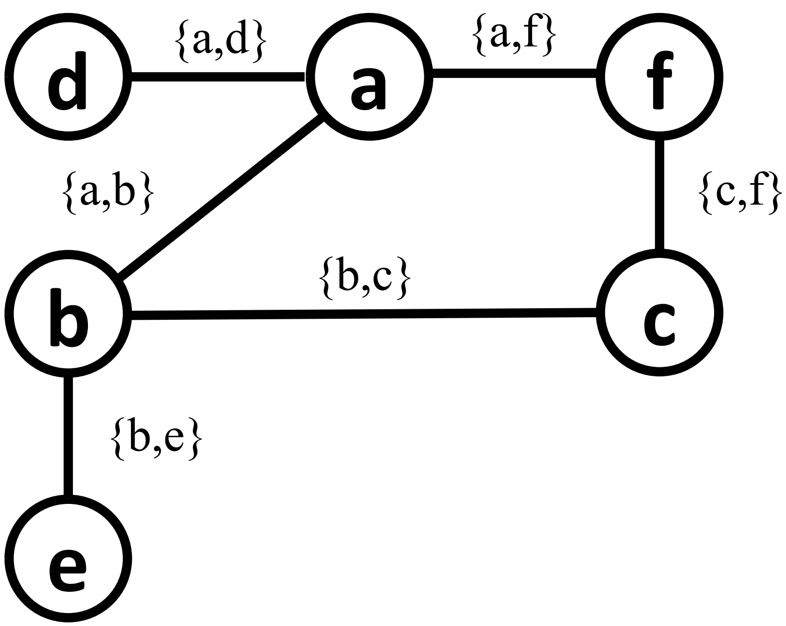
	a	b	c	d	e	f
a	0	1	0	1	0	1
b	1	0	1	0	1	0
c	0	1	0	0	0	1
d	1	0	0	0	0	0
e	0	1	0	0	0	0
f	1	0	1	0	0	0

Adjacency matrix for graph G:

- a 2D N x N array with:
  - 1s indicating an edge / connection between two vertices
  - 0s where there is no edge / connection between vertices

Representation: Incidence Matrix

Graph G



A graph $G = \{V, E\}$**:**

- $N = |V|$ **vertices / nodes, and**
- $|E|$ **edges**

Incidence Matrix for Graph G

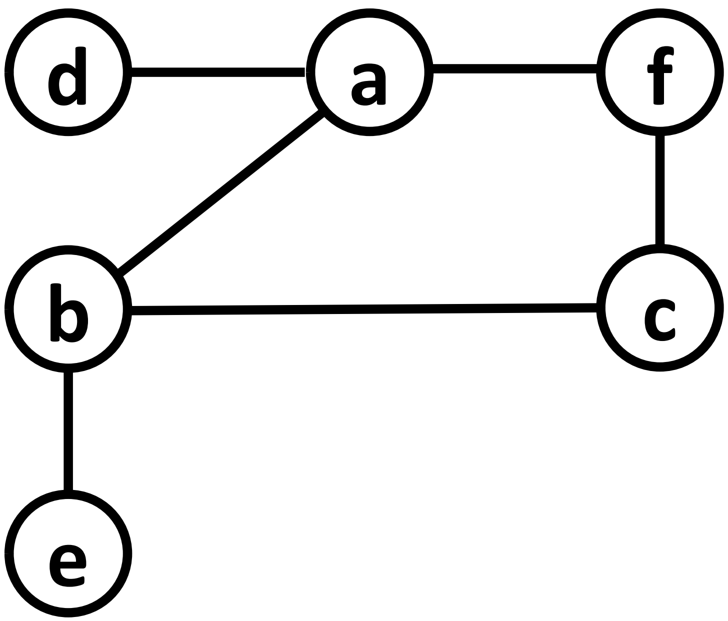
	{a,b}	{a,d}	{a,f}	{b,c}	{b,e}	{c,f}
a	1	1	1	0	0	0
b	1	0	0	1	1	0
c	0	0	0	1	0	1
d	0	1	0	0	0	0
e	0	0	0	0	1	0
f	0	0	1	0	0	1

Incidence matrix for graph G:

- **a 2D N x N array with:**
  - **1s indicating that an edge** i **is incident on vertex** j
  - **0s otherwise**

Representation: Adjacency List

Graph G



A graph $G = \{V, E\}$:
- $N = |V|$ vertices / nodes, and
- $|E|$ edges

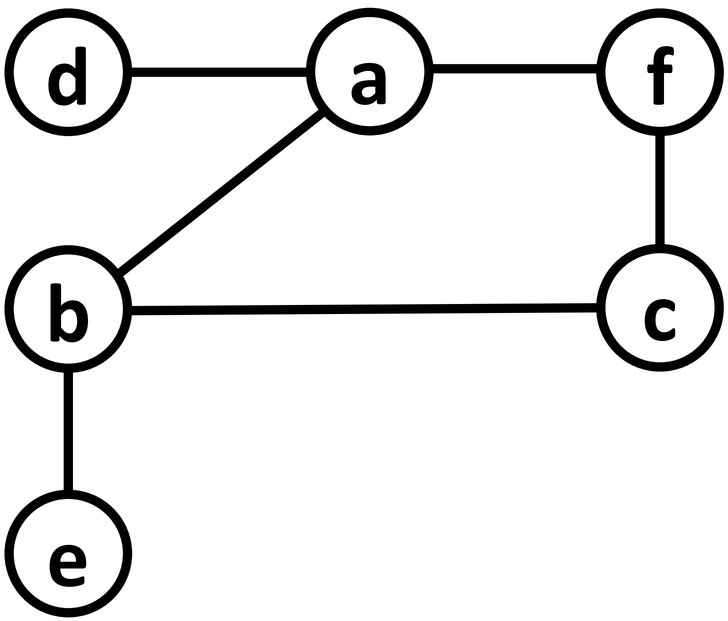
Adjacency List for Graph G

Vertex	List containg adjacent vertices
a	b $\rightarrow$ d $\rightarrow$ f
b	a $\rightarrow$ c $\rightarrow$ e
c	b $\rightarrow$ f
d	a
e	b
f	a $\rightarrow$ c

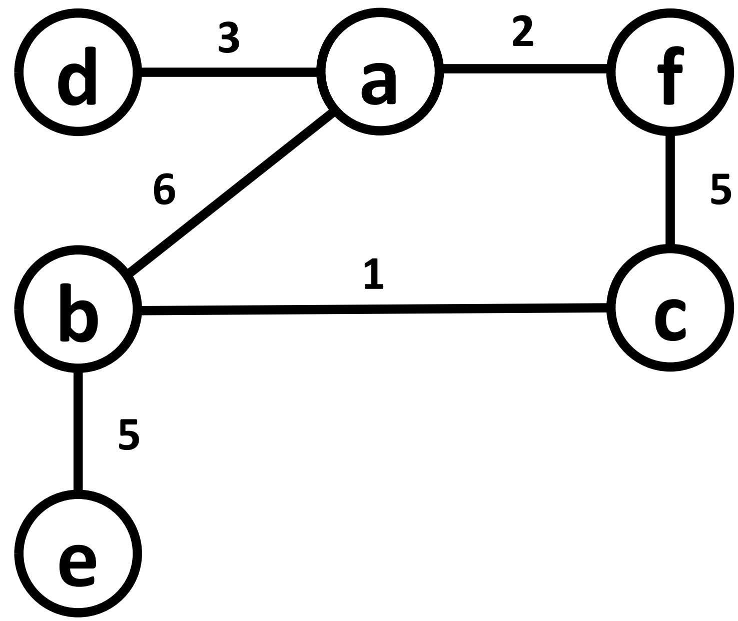
Adjacency list for graph G:
- an array (or a list) of lists, where:
  - each individual list corresponds to a single vertex i and includes all vertices adjacent to i
  - $\rightarrow$ indicates a link in a list

Non-Weighted vs. Weighted

Non-weighted Graph



Weighted Graph

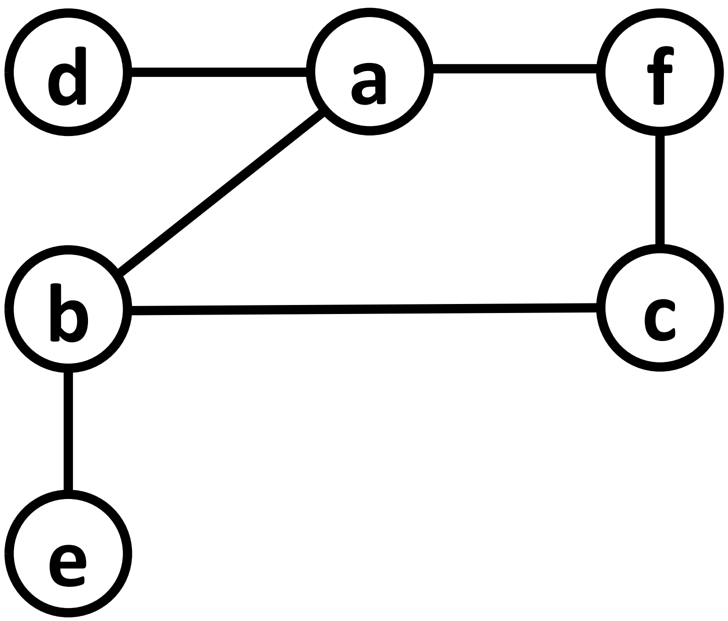


Weighted graph:

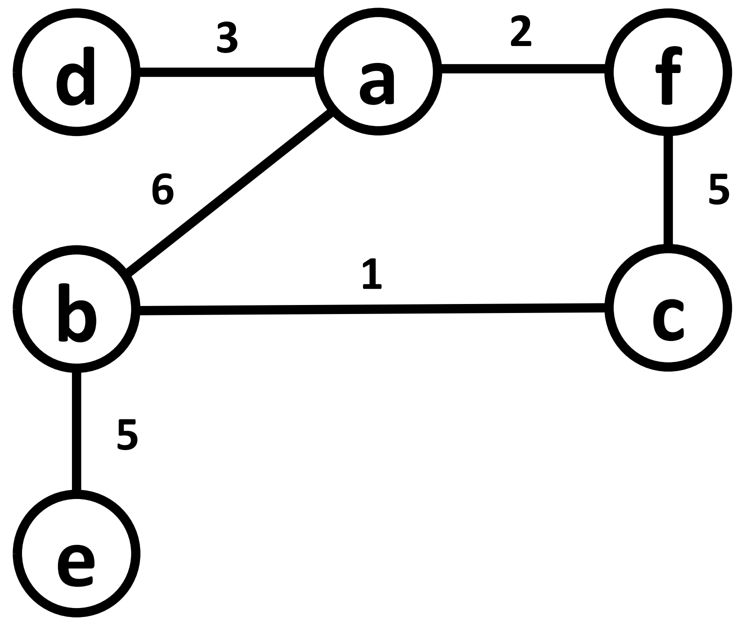
- a graph (sometimes called a network) is in which a number (weight) is assigned to each edge is considered to be weighted
- $G = \{V, E, w\}$

Non-Weighted vs. Weighted

Non-weighted Graph



Weighted Graph

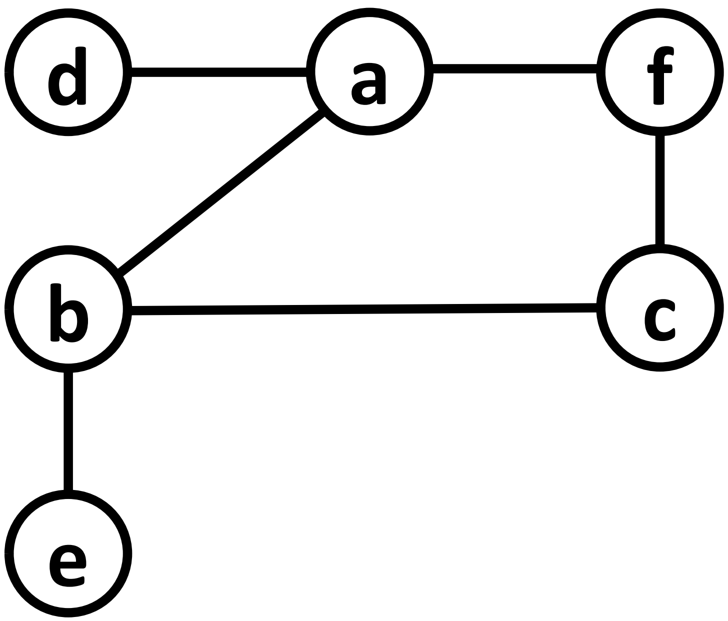


Weighted graph:

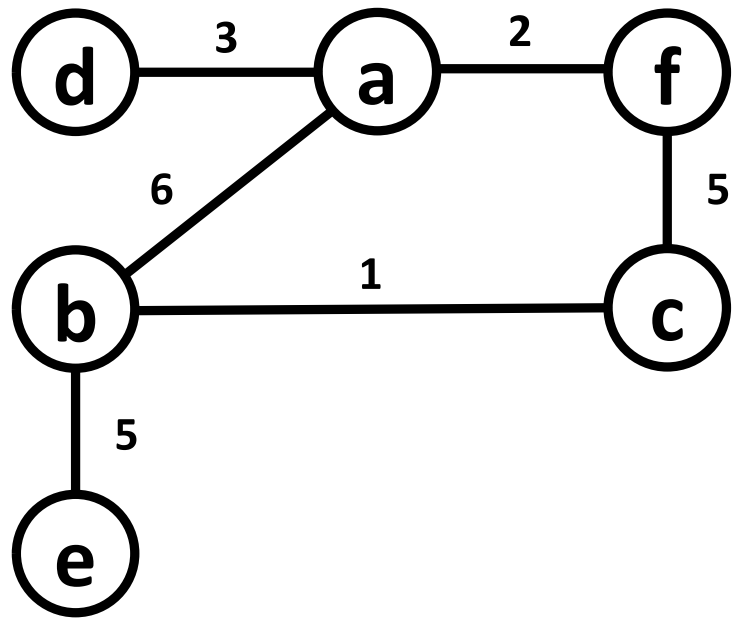
- weights can represent for costs, lengths or capacities, depending on the context / problem

Non-Weighted vs. Weighted

Non-weighted Graph



Weighted Graph

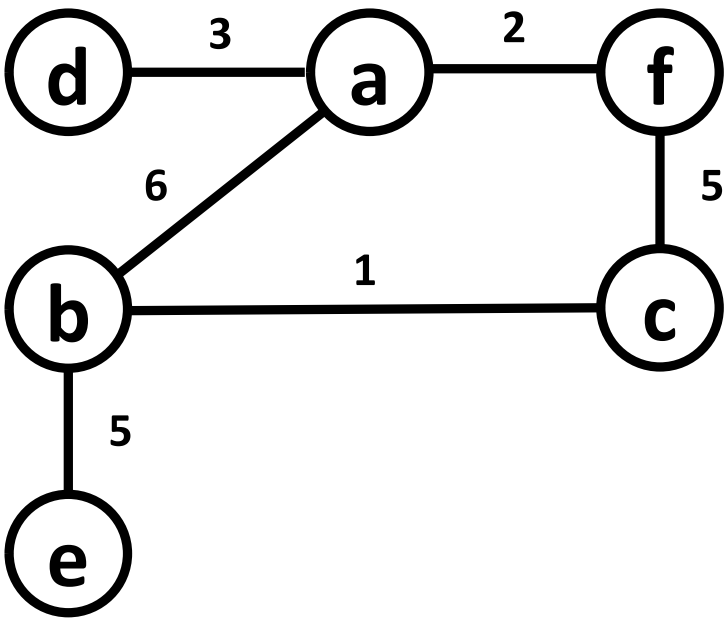


Weighted graph examples:

- maps (distances)
- sea shipping routes (cost)
- computer networks (throughput)
- airline route map (distances)
- Your CS MSc course plan (difficulty level)

Weighted Graph: Adjacency Matrix

Weighted Graph G



A graph $G = \{V, E, w\}$:

- $N = |V|$ vertices / nodes, and
- $|E|$ edges
- $|E|$ weights

Adjacency Matrix for Graph G

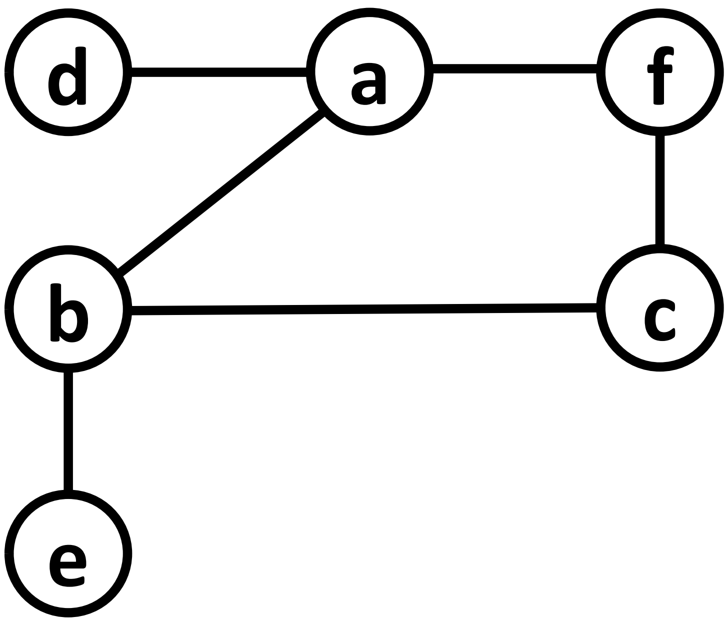
	a	b	c	d	e	f
a	0	6	0	3	0	2
b	6	0	1	0	5	0
c	0	1	0	0	0	5
d	3	0	0	0	0	0
e	0	5	0	0	0	0
f	2	0	5	0	0	0

Adjacency matrix for graph G:

- a 2D N x N array with:
  - weights indicating an edge / connection between two vertices
  - 0s where there is no edge / connection between vertices

Undirected vs. Directed Graph

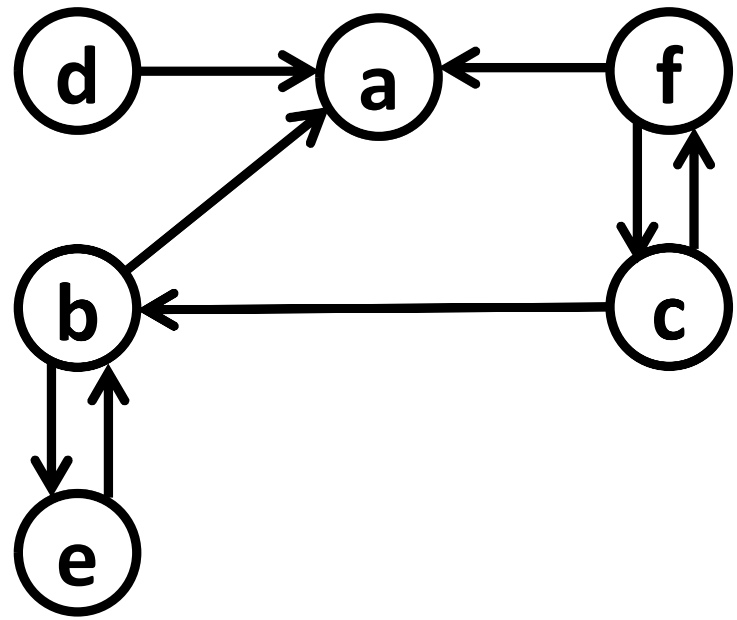
Undirected Graph



Undirected graph:

- a graph in which edges DO NOT have orientations / directions is called an undirected graph
- edge $\{b, e\}$ is the same as $\{e, b\}$

Directed Graph

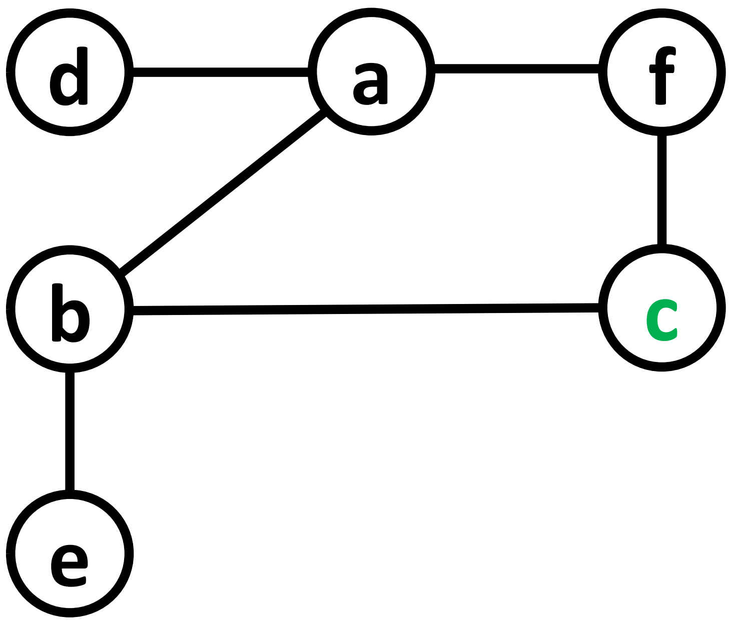


Directed graph:

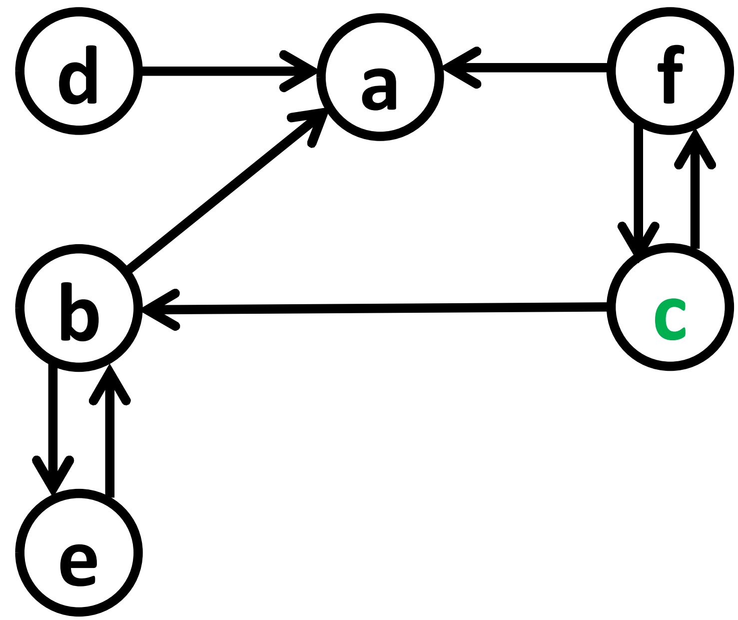
- a graph in which edges have orientations / directions is called a directed graph (or digraph)
- edge $\{b, e\}$ is NOT the same as $\{e, b\}$

Undirected vs. Directed Graph

Undirected Graph



Directed Graph



Undirected graph:

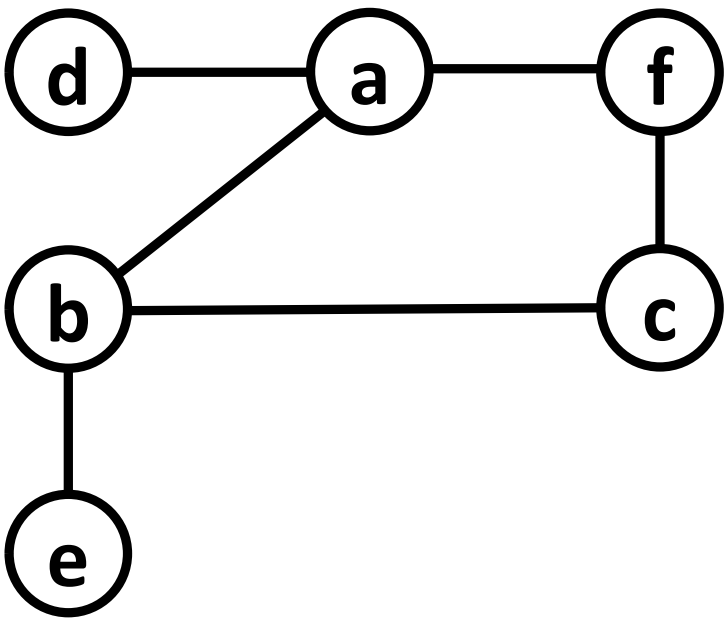
- the degree of a vertex is the number of edges that are incident to it (degree of vertex c: 2)

Directed graph:

- the in-degree of a vertex is the number of edges that are ”entering” it (in-degree of vertex c: 1)
- the out-degree of a vertex is the number of edges that are ”leaving” it (out-degree of vertex c: 2)

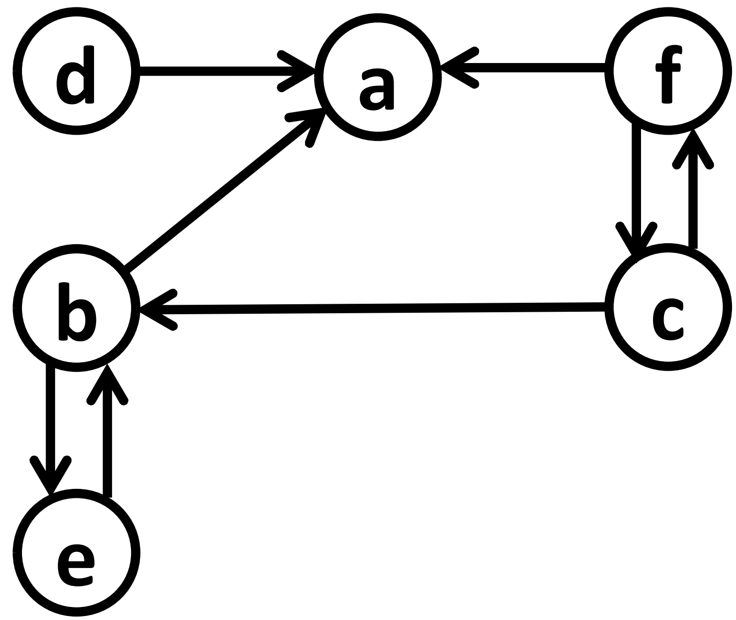
Undirected vs. Directed Graph

Undirected Graph



Undirected graph examples:
- two-way streets
- US interstate system
- computer networks
- Facebook friends
- academic paper collaborators

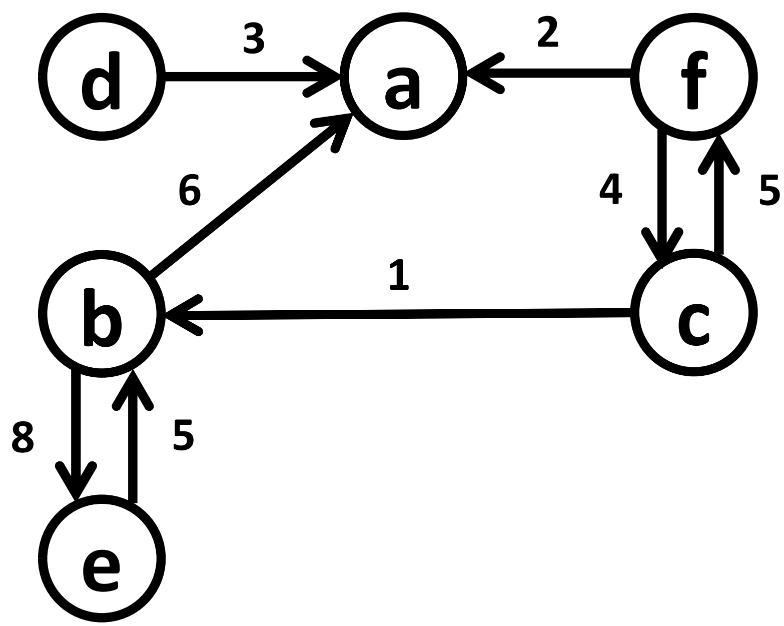
Directed Graph



Directed graph examples:
- one-way streets
- social media followers
- computer networks / World Wide Web
- airline route map
- emergency exit routes

Directed Weighted Graph

Directed Weighted Graph G



A graph $G = \{V, E, w\}$:

- $N = |V|$ vertices / nodes, and
- $|E|$ edges
- $|E|$ weights

Adjacency Matrix for Graph G

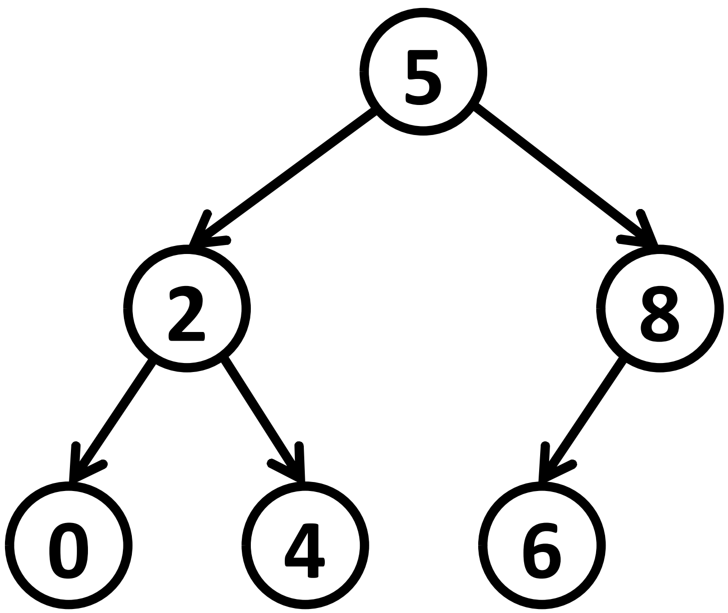
	a	b	c	d	e	f
a	0	0	0	0	0	0
b	6	0	0	0	8	0
c	0	1	0	0	0	5
d	3	0	0	0	0	0
e	0	5	0	0	0	0
f	2	0	4	0	0	0

Adjacency matrix for graph G:

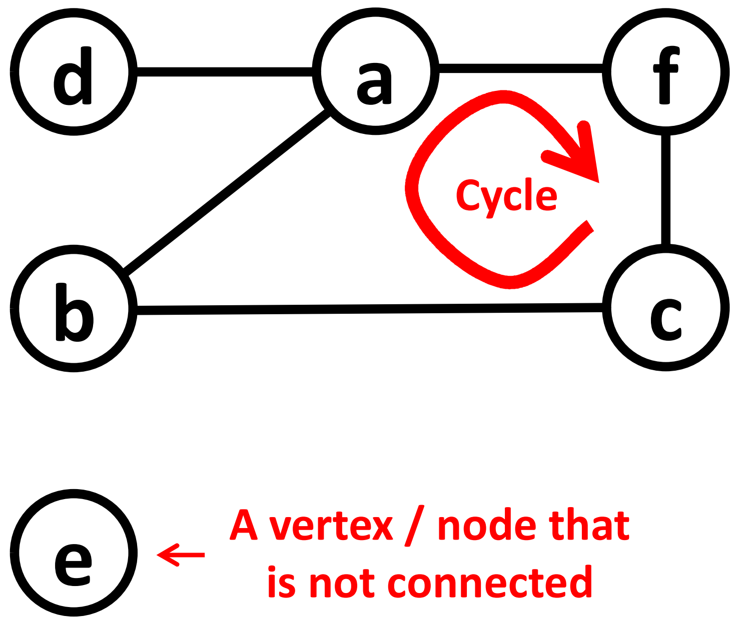
- a 2D N x N array with:
  - weights indicating an edge / connection between two vertices
  - 0s where there is no edge / connection between vertices

Graphs vs. Trees: Key Differences

Tree



Graph



A tree:

- is a graph where all vertices / nodes are connected to the root
- is a graph without any cycles (a tree is an acyclic graph)

A graph:

- a graph can be non-connected (some vertices are not "reachable")
- a graph can have one or more cycle
- a graph does not have to be directed

Graphs: Traversals / Searches

Key difference between Tree and Graph Traversal:

- **Graphs: need to keep track of all visited vertices / nodes**

Purpose:

- **Finding a vertex / node**
- **Traversing the graph**

Types:

- **Depth First Traversal / Search**
- **Breadth First Traversal / Search**

Graph Traversals: Algorithms

General Depth-First Traversal (iterative)

```
dfs(Graph G, vertex v) //v start point
   initialize stack S
   S.push(v)
   mark v as visited
   while (S is not empty)
      m = S.pop()
      for all m's adjacent vertices k
          if k is not visited
              S.push(k)
              mark k as visited
          end
      end
   end
end
```

Applications:

- Problems that require backtracking
- Path finding
- Solving puzzles with a single solution
- Detecting cycles in a graph

General Breadth-First Traversal (iterative)

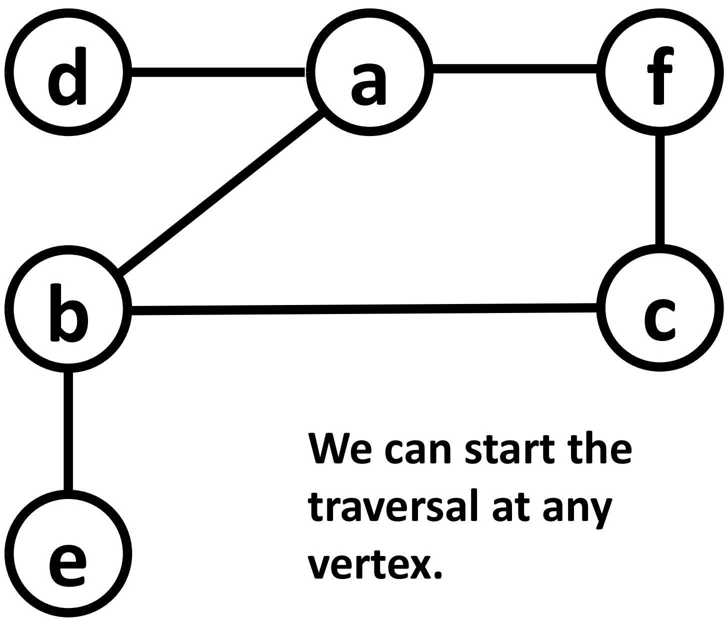
```
bfs(Graph G, vertex v) //v start point
   initialize queue Q
   S.enqueue(v)
   mark v as visited
   while (Q is not empty)
      m = Q.dequeue()
      for all m's adjacent vertices k
          if k is not visited
              Q.enqueue(k)
              mark k as visited
          end
      end
   end
end
```

Applications:

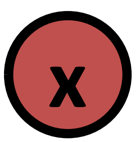
- Nearest neighbor finding (for example in peer-to-peer networks or social networks)
- Web crawling / indexing
- Broadcasting information over a network

Graph: Depth First Traversal

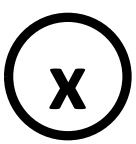
Graph G



We can start the traversal at any vertex.



Already visited vertex



Not visited yet vertex

Depth First Traversal For G

Current Vertex	

Already Visited Vertices
None

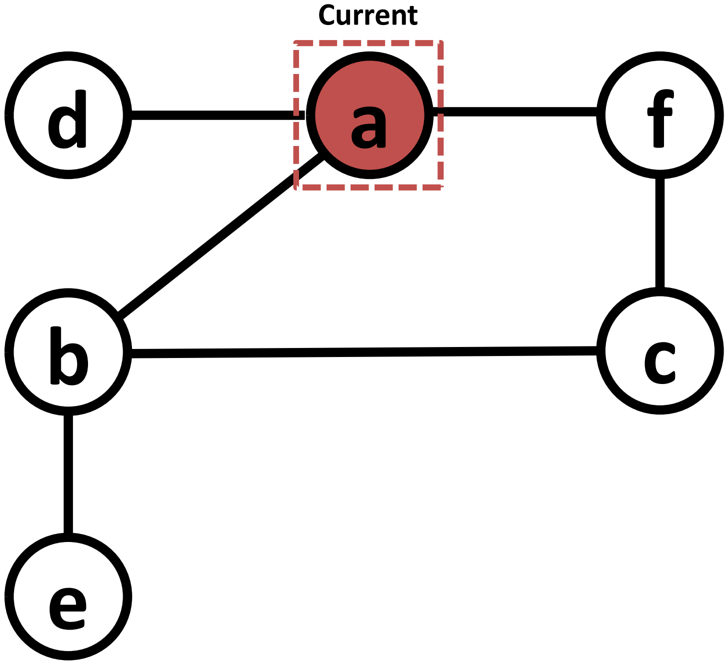
Adjacency List for Current Vertex	

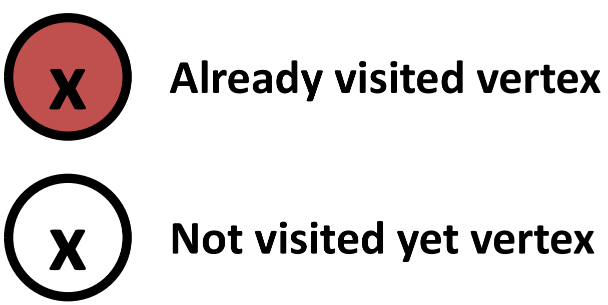
Vertex Stack (Top to Bottom)
Empty

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G





Depth First Traversal For G

Current Vertex	a

Already Visited Vertices
a

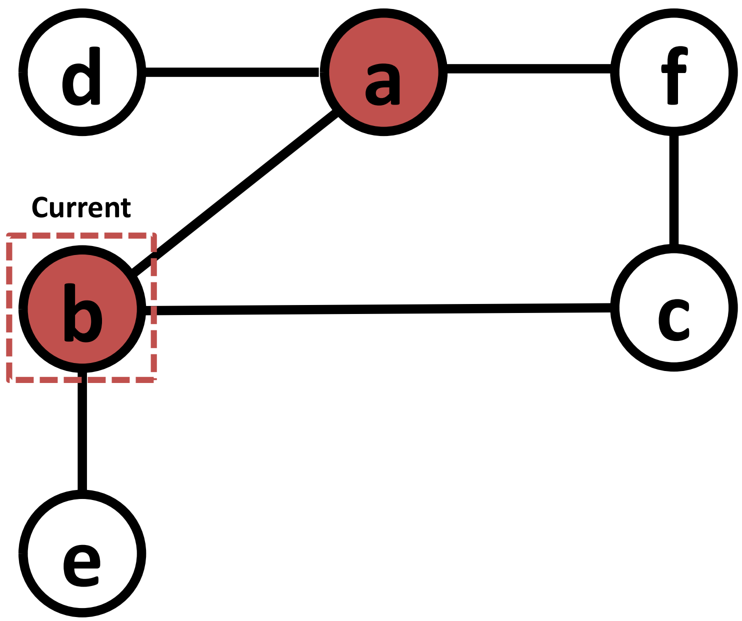
Adjacency List for Current Vertex	
a	b → d → f

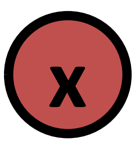
Vertex Stack (Top to Bottom)
a

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

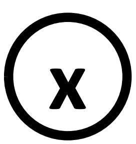
Graph: Depth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Depth First Traversal For G

Current Vertex	b

Already Visited Vertices
a, b

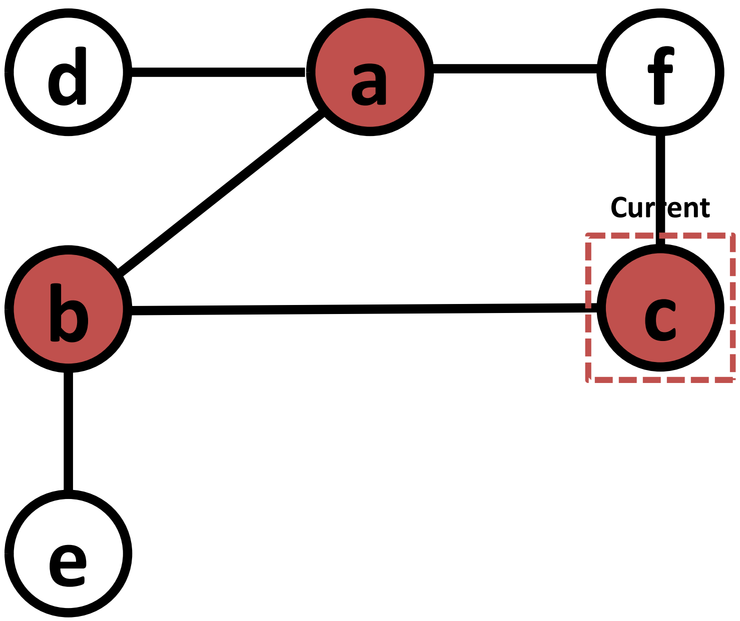
Adjacency List for Current Vertex	
b	a → c → e

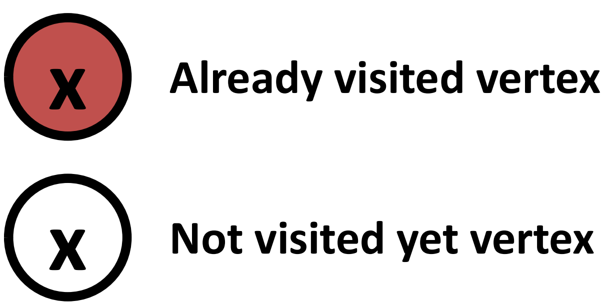
Vertex Stack (Top to Bottom)
b, a

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G





Depth First Traversal For G

Current Vertex	c

Already Visited Vertices
a, b, c

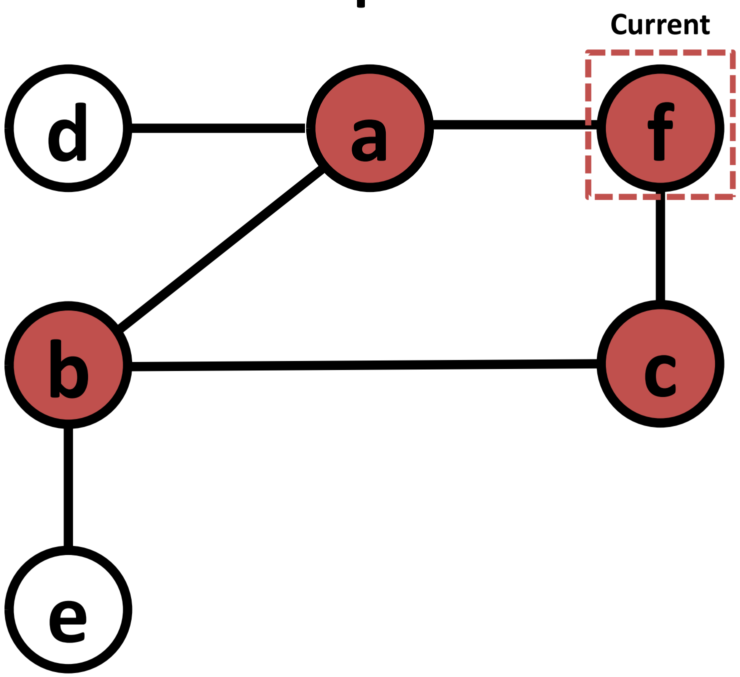
Adjacency List for Current Vertex	
c	b → f

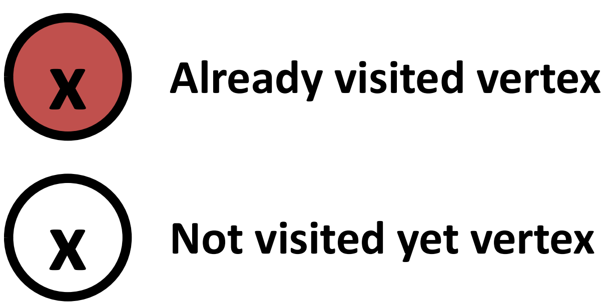
Vertex Stack (Top to Bottom)
c, b, a

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G





Depth First Traversal For G

Current Vertex	f

Already Visited Vertices
a, b, c, f

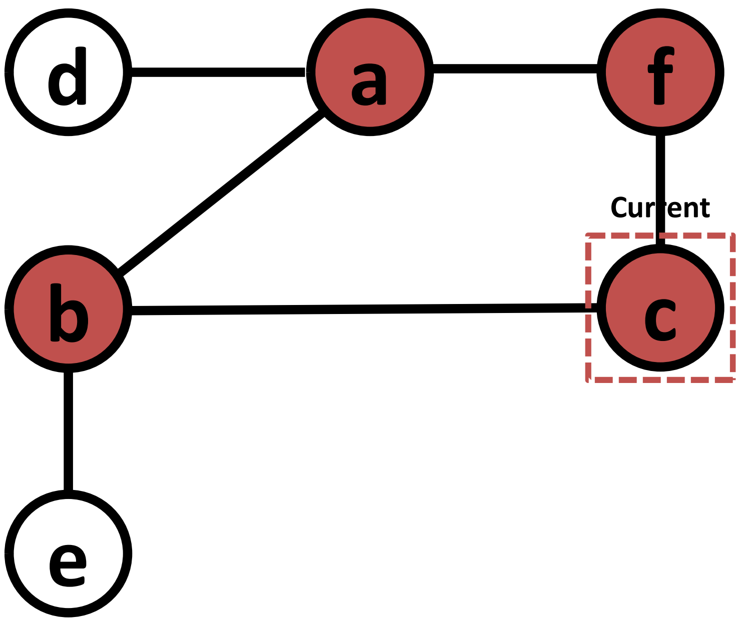
Adjacency List for Current Vertex	
f	a → c [All adjacent visited]

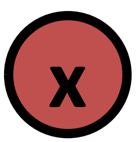
Vertex Stack (Top to Bottom)
f, c, b, a [pop() and backtrack]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

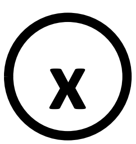
Graph: Depth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Depth First Traversal For G

Current Vertex	**c**

Already Visited Vertices
a, b, c, f

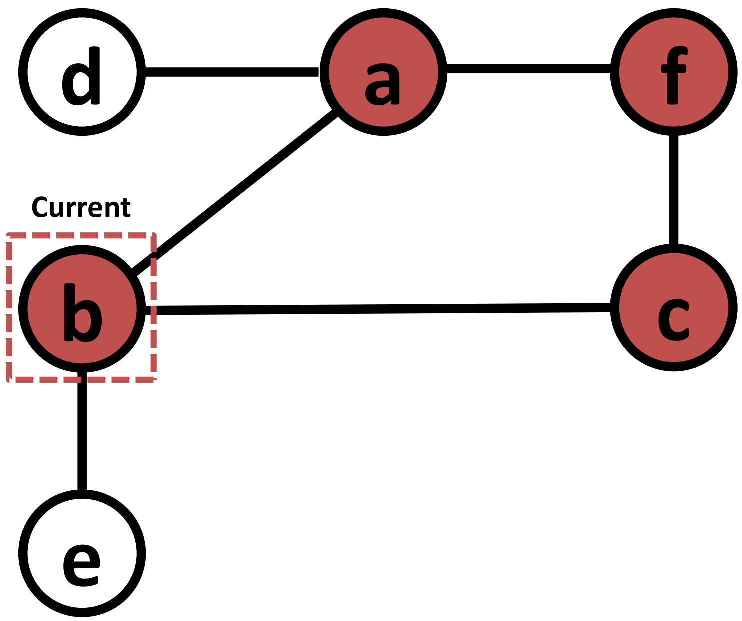
Adjacency List for Current Vertex	
c	**b → f** **[All adjacent visited]**

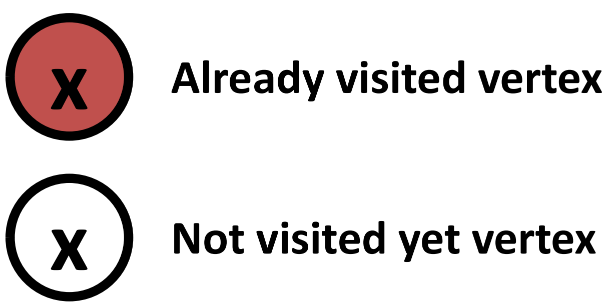
Vertex Stack (Top to Bottom)
c, b, a **[pop() and backtrack]**

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G





Depth First Traversal For G

Current Vertex	b

Already Visited Vertices
a, b, c, f

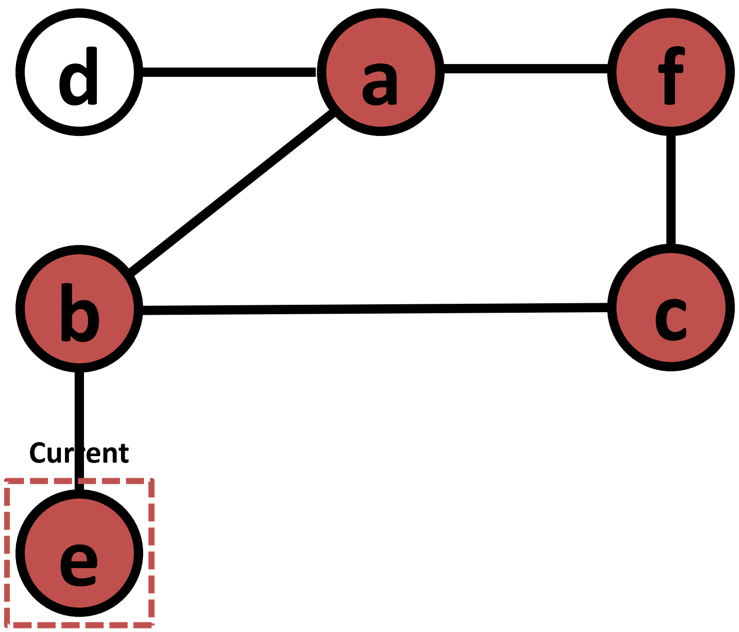
Adjacency List for Current Vertex	
b	a → c → e

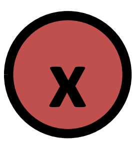
Vertex Stack (Top to Bottom)
b, a

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

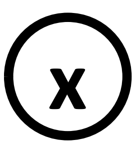
Graph: Depth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Depth First Traversal For G

Current Vertex	e

Already Visited Vertices
a, b, c, f, e

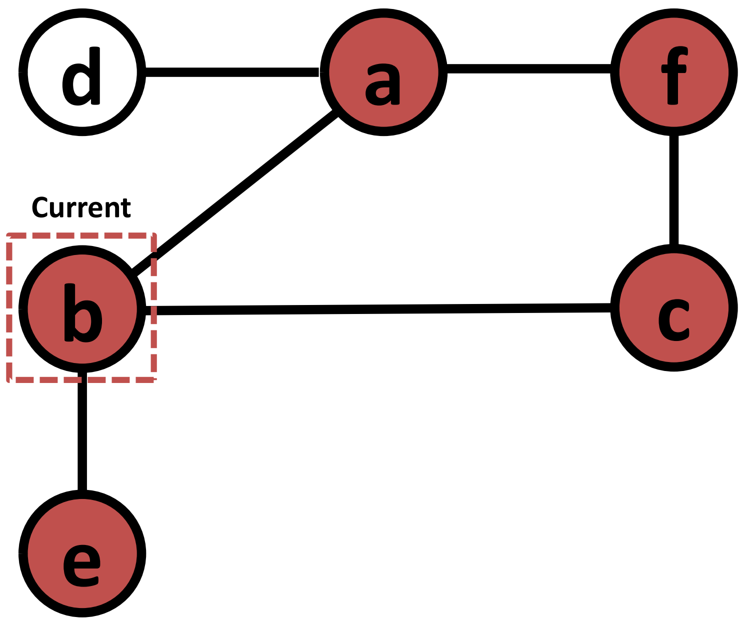
Adjacency List for Current Vertex	
e	b [All adjacent visited]

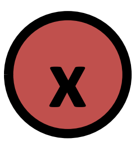
Vertex Stack (Top to Bottom)
e, b, a [pop() and backtrack]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

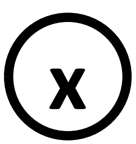
Graph: Depth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Depth First Traversal For G

Current Vertex	b

Already Visited Vertices
a, b, c, f, e

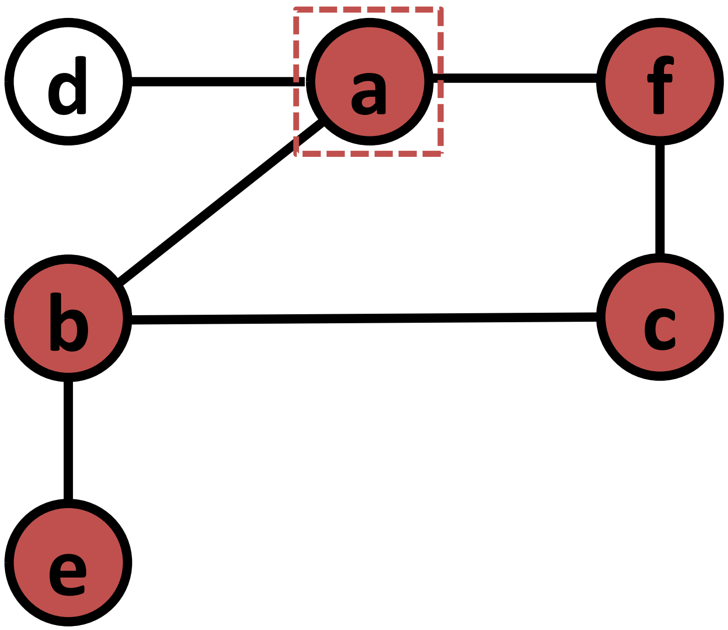
Adjacency List for Current Vertex	
b	a → c → e [All adjacent visited]

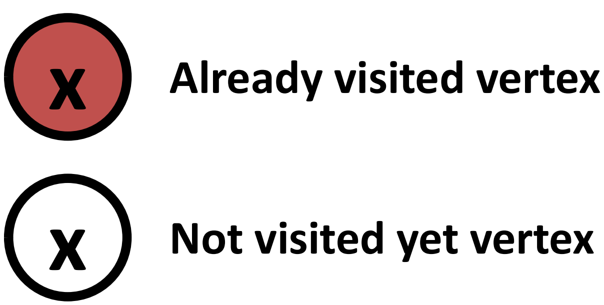
Vertex Stack (Top to Bottom)
b, a [pop() and backtrack]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G





Depth First Traversal For G

Current Vertex	a

Already Visited Vertices
a, b, c, f, e

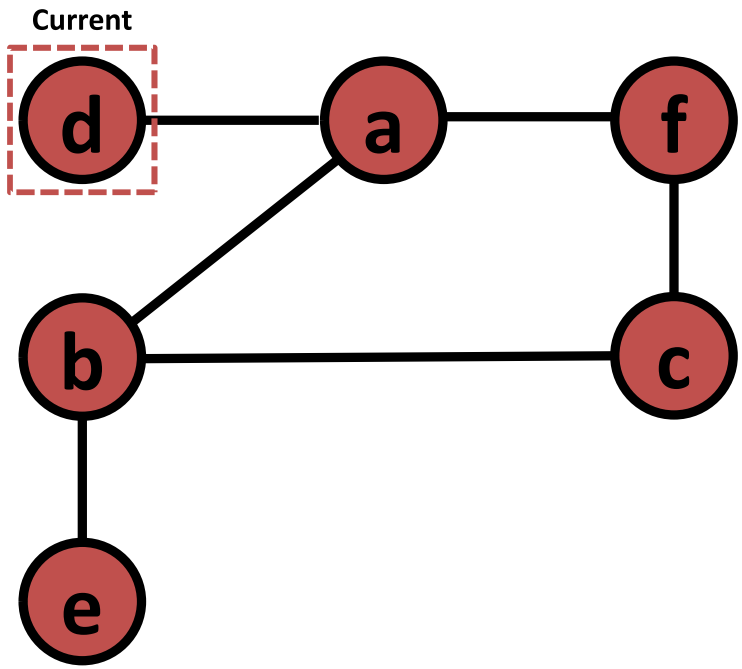
Adjacency List for Current Vertex	
a	b → d → f

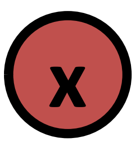
Vertex Stack (Top to Bottom)
a

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

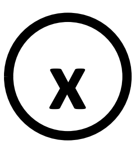
Graph: Depth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Depth First Traversal For G

Current Vertex	d

Already Visited Vertices
a, b, c, f, e, d

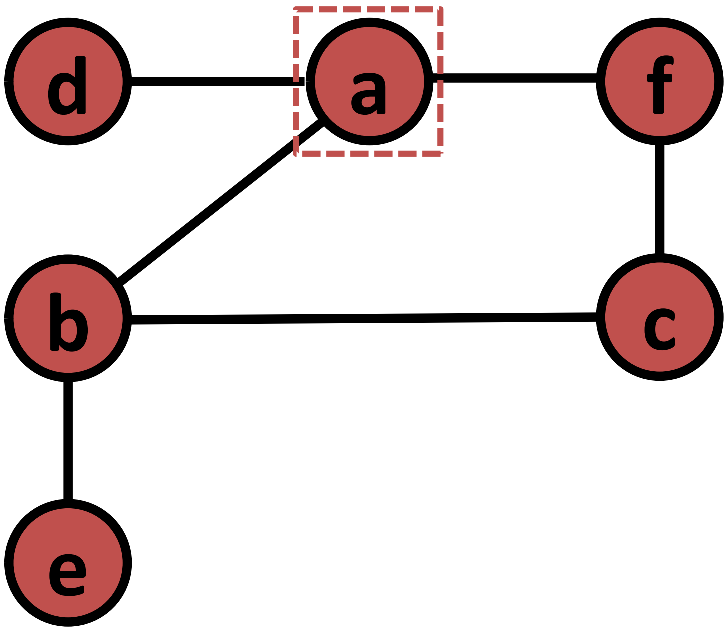
Adjacency List for Current Vertex		
d	a	[All adjacent visited]

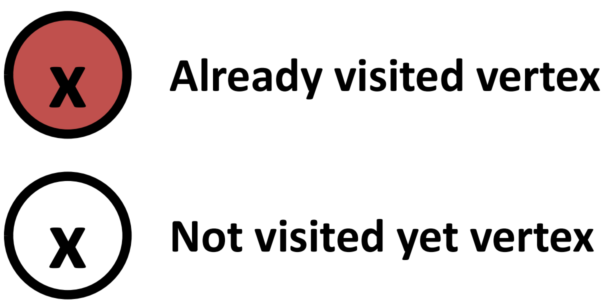
Vertex Stack (Top to Bottom)	
d, a	[pop() and backtrack]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G





Depth First Traversal For G

Current Vertex	a

Already Visited Vertices
a, b, c, f, b, e

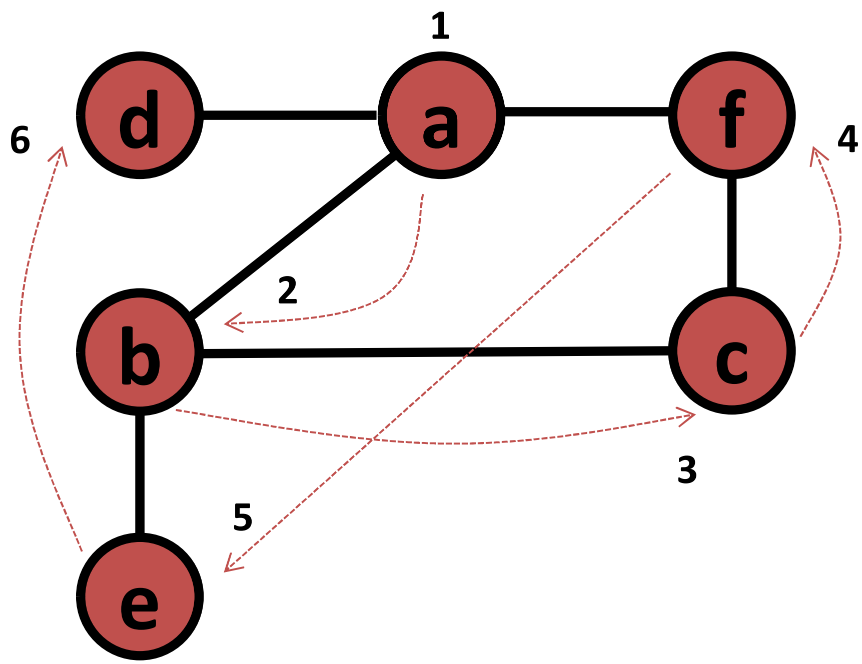
Adjacency List for Current Vertex	
a	b → d → f [All adjacent visited]

Vertex Stack (Top to Bottom)
a [pop() and Done!]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Depth First Traversal

Graph G



x Already visited vertex

x Not visited yet vertex

Depth First Traversal For G

Current Vertex	

Already Visited Vertices
a, b, c, f, b, e

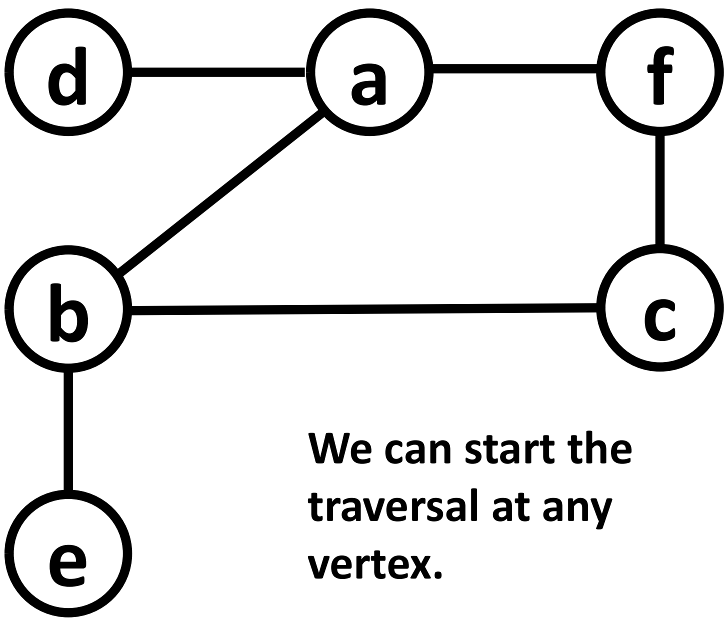
Adjacency List for Current Vertex	

Vertex Stack (Top to Bottom)

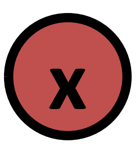
Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, push it on the stack
Rule 2: If no unvisited vertices (adjacent to current), pop(), current vertex = top()/peek()
Rule 3: If Rules 1/2 impossible, you are done

Graph: Breadth First Traversal

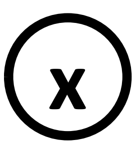
Graph G



We can start the traversal at any vertex.



Already visited vertex



Not visited yet vertex

Breadth First Traversal For G

Current Vertex	

Already Visited Vertices
None

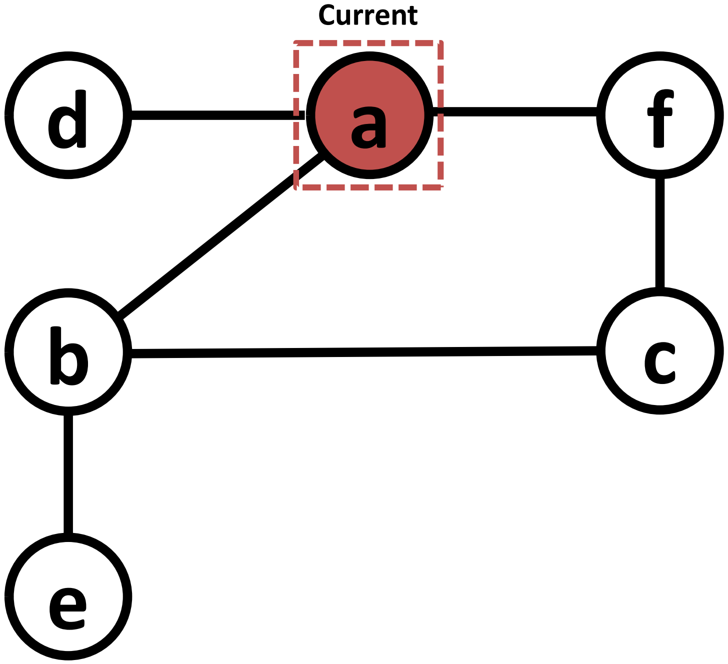
Adjacency List for Current Vertex	

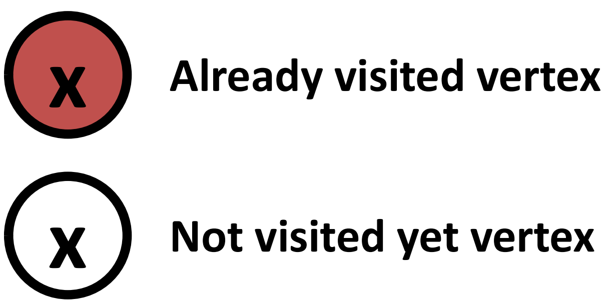
Vertex Queue (Front to Rear)
Empty

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited vertices (adjacent to current), current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G





Breadth First Traversal For G

Current Vertex	a

Already Visited Vertices
a

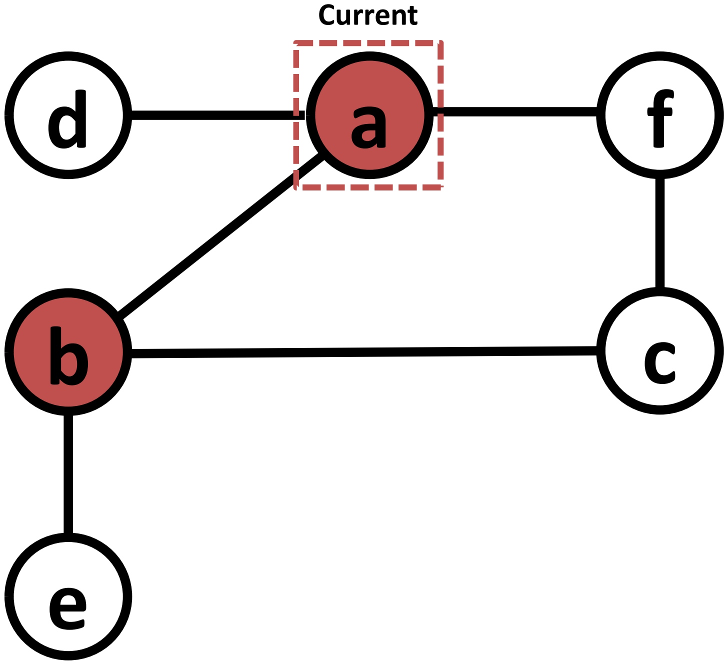
Adjacency List for Current Vertex	
a	b → d → f

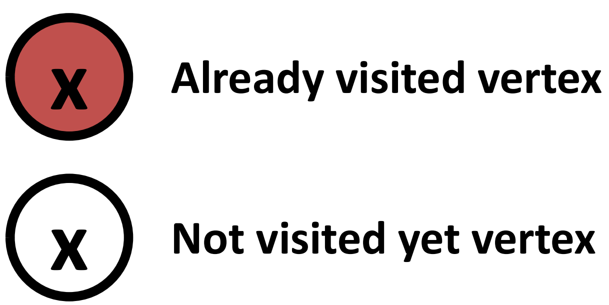
Vertex Queue (Front to Rear)
Empty

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G





Breadth First Traversal For G

Current Vertex	a

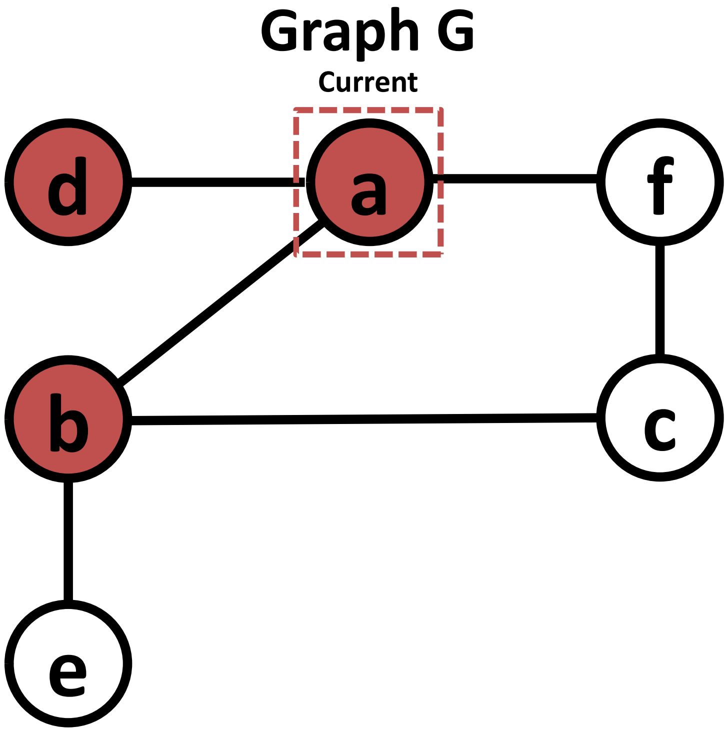
Already Visited Vertices
a, b

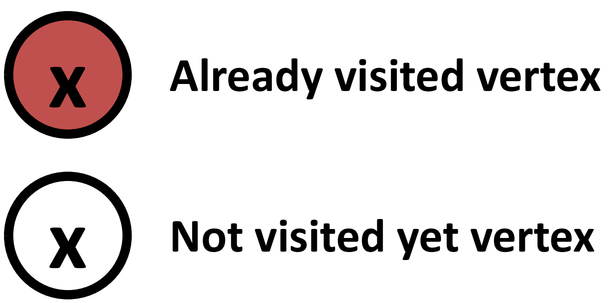
Adjacency List for Current Vertex	
a	b → d → f

Vertex Queue (Front to Rear)
b

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal





Breadth First Traversal For G

Current Vertex	a

Already Visited Vertices
a, b, d

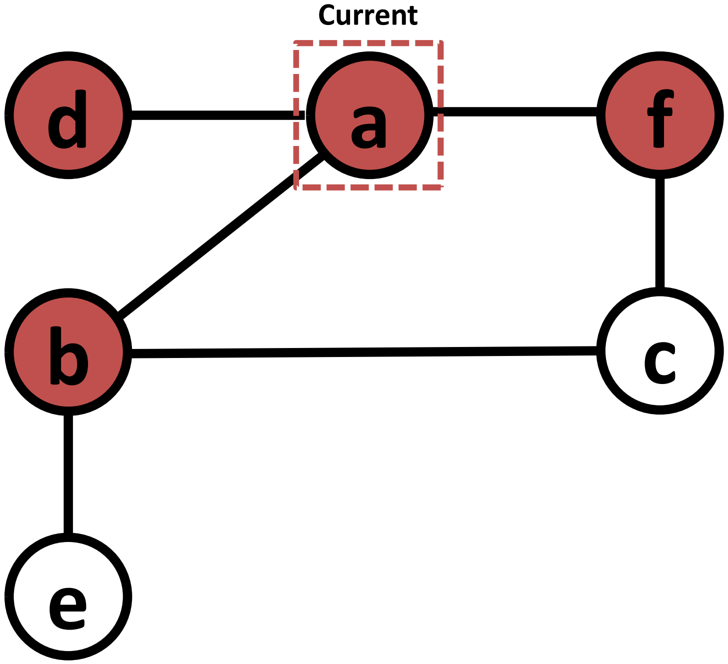
Adjacency List for Current Vertex	
a	b → d → f

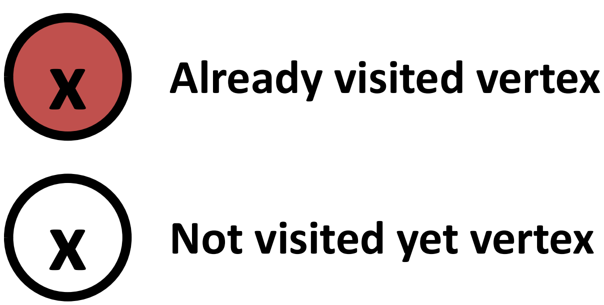
Vertex Queue (Front to Rear)
b, d

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G





Breadth First Traversal For G

Current Vertex	a

Already Visited Vertices
a, b, d, f

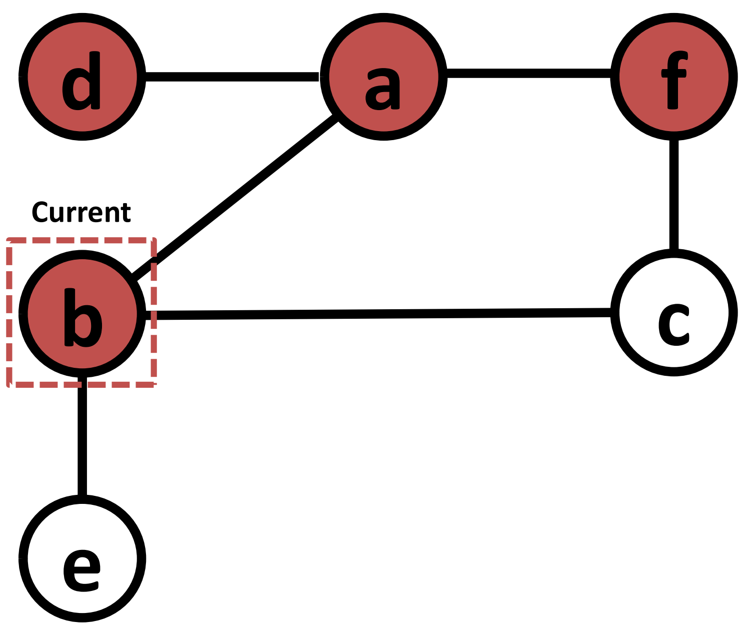
Adjacency List for Current Vertex	
a	b → d → f [All visited now!]

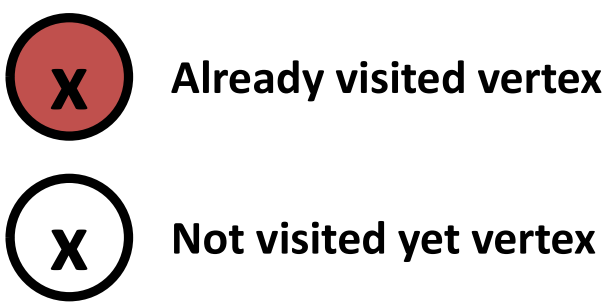
Vertex Queue (Front to Rear)
b, d, f [dequeue()]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G





Breadth First Traversal For G

Current Vertex	b

Already Visited Vertices
a, b, d, f

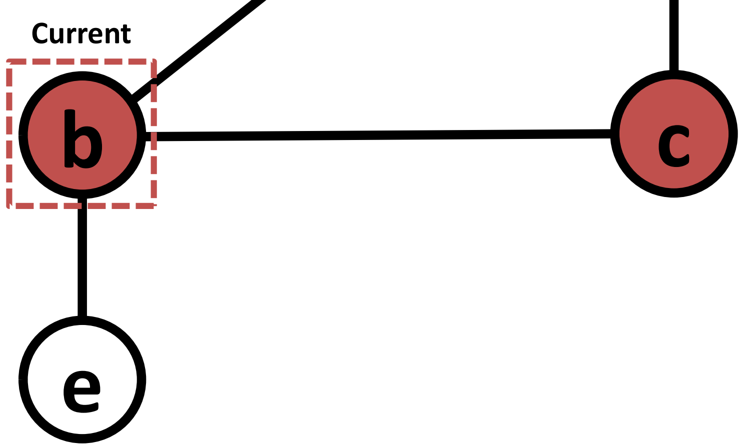
Adjacency List for Current Vertex	
b	a → c → e

Vertex Queue (Front to Rear)
d, f

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G



x Already visited vertex

x Not visited yet vertex

Breadth First Traversal For G

Current Vertex	b

Already Visited Vertices
a, b, d, f, c

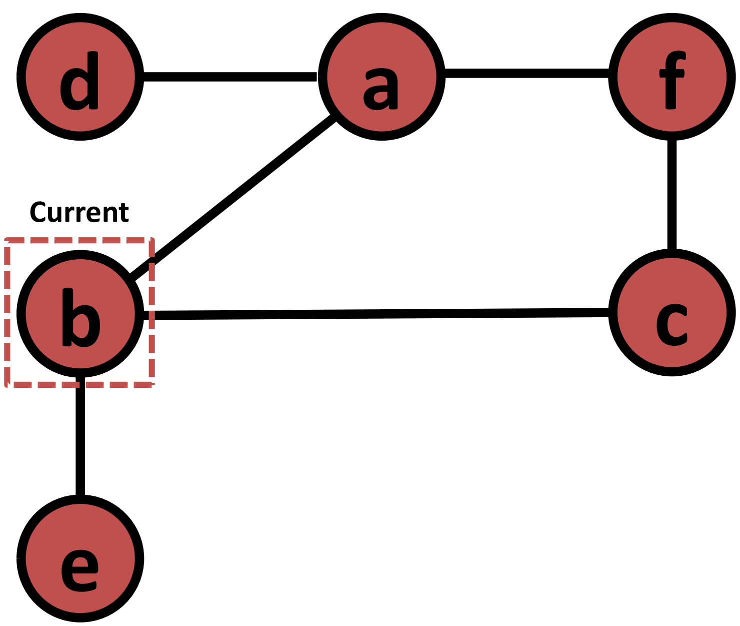
Adjacency List for Current Vertex	
b	a → c → e

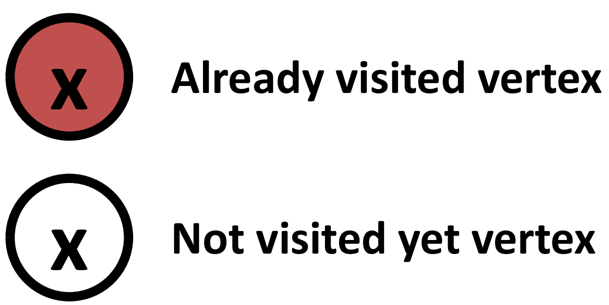
Vertex Queue (Front to Rear)
d, f, c

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G





Breadth First Traversal For G

Current Vertex	b

Already Visited Vertices
a, b, d, f, c, e

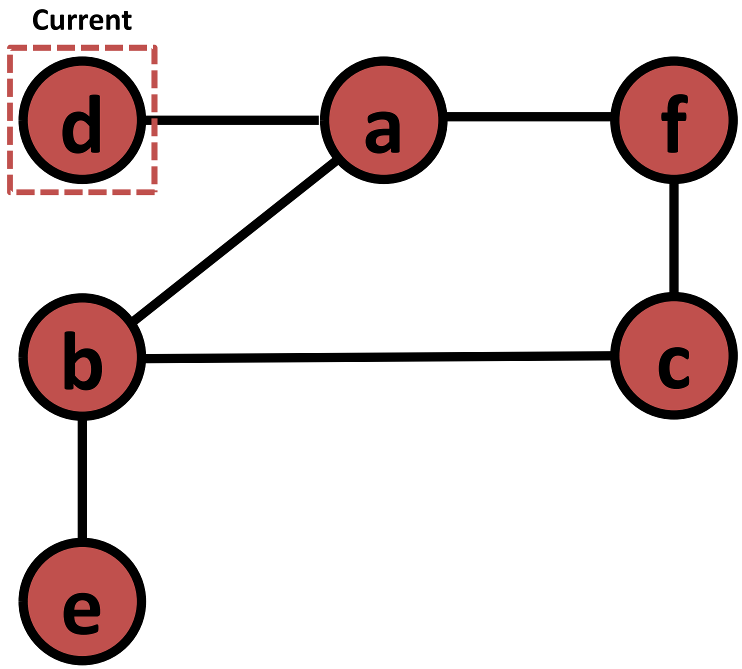
Adjacency List for Current Vertex	
b	a → c → e [All visited now!]

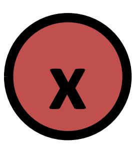
Vertex Queue (Front to Rear)
d, f, c, e [dequeue()]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

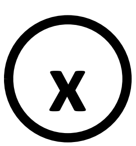
Graph: Breadth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Breadth First Traversal For G

Current Vertex	d

Already Visited Vertices
a, b, d, f, c, e

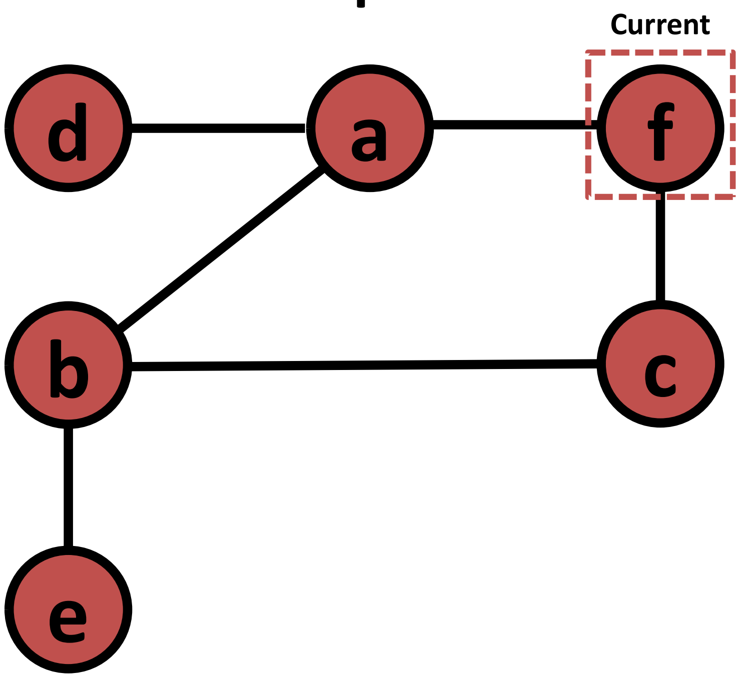
Adjacency List for Current Vertex	
d	a [All visited now!]

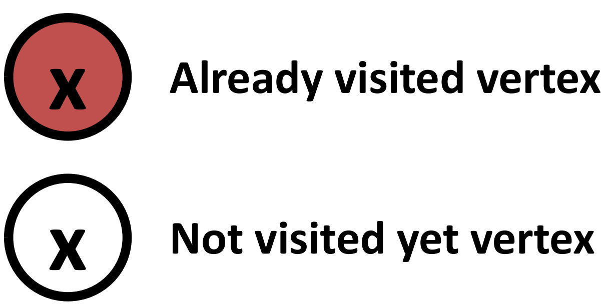
Vertex Queue (Front to Rear)
f, c, e [dequeue()]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph: Breadth First Traversal

Graph G





Breadth First Traversal For G

Current Vertex	f

Already Visited Vertices
a, b, d, f, c, e

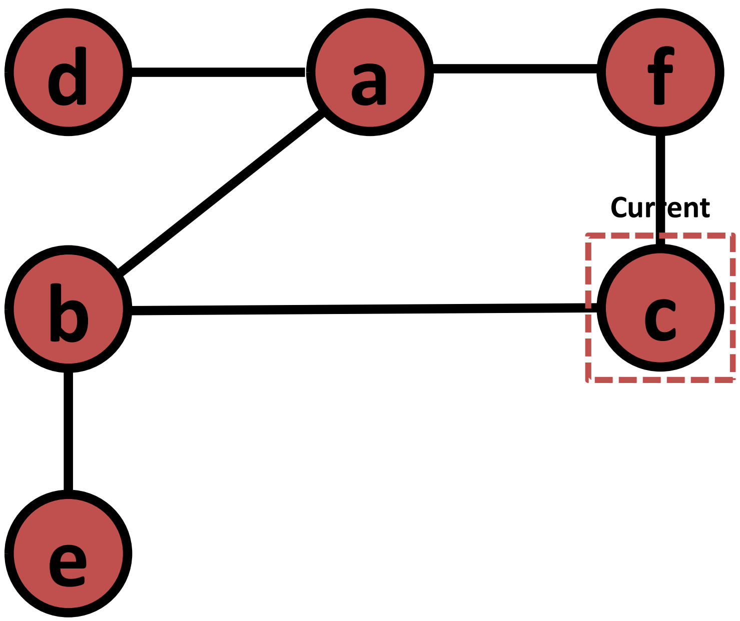
Adjacency List for Current Vertex	
f	a → c [All visited now!]

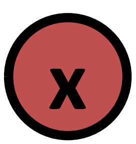
Vertex Queue (Front to Rear)
c, e [dequeue()]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

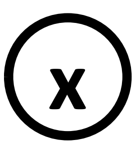
Graph: Breadth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Breadth-First Traversal For G

Current Vertex	**c**

Already Visited Vertices
a, b, d, f, c, e

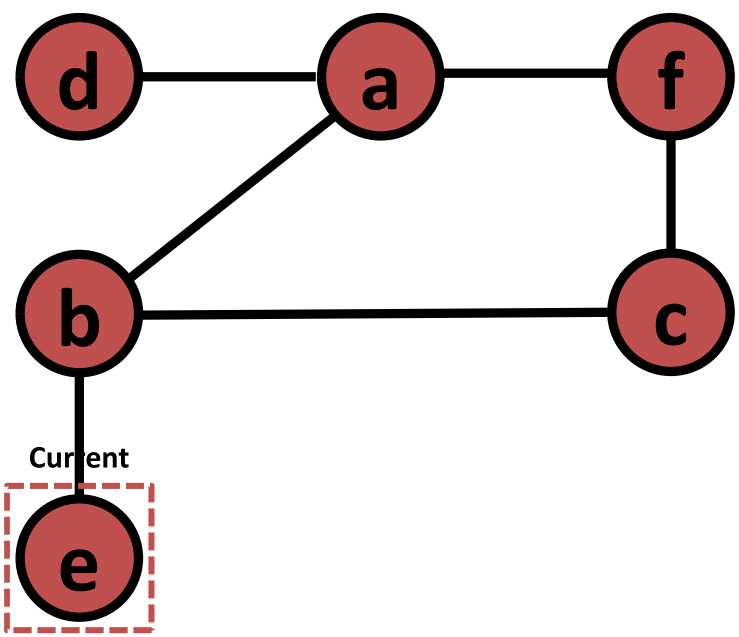
Adjacency List for Current Vertex	
c	**b → f** **[All visited now!]**

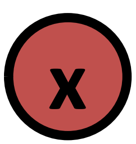
Vertex Queue (Front to Rear)
e **[dequeue()]**

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

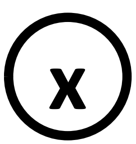
Graph: Breadth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Breadth First Traversal For G

Current Vertex	e

Already Visited Vertices
a, b, d, f, c, e

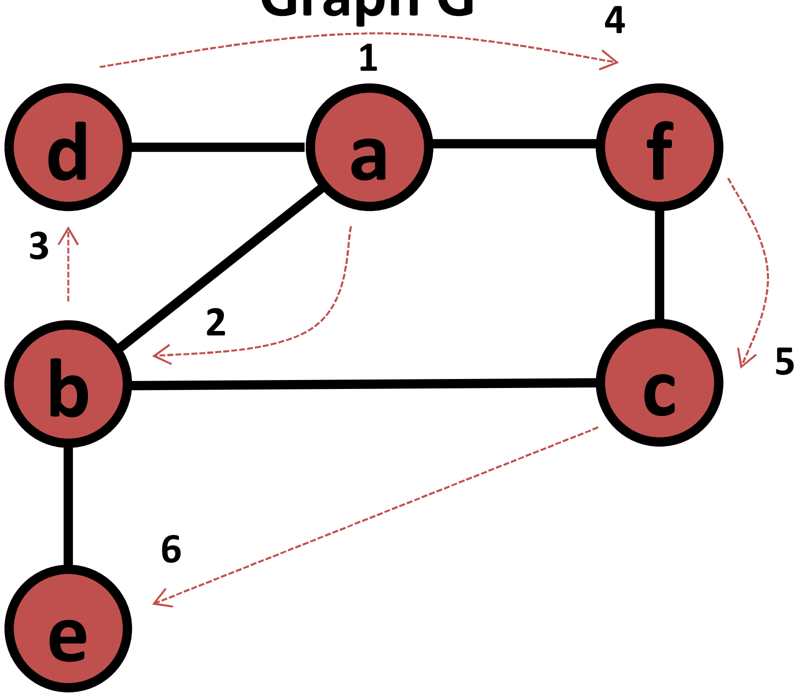
Adjacency List for Current Vertex		
e	b	[All visited now!]

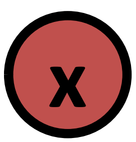
Vertex Queue (Front to Rear)
[empty - Done!]

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

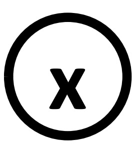
Graph: Breadth First Traversal

Graph G





Already visited vertex



Not visited yet vertex

Breadth First Traversal For G

Current Vertex	

Already Visited Vertices
a, b, d, f, c, e

Adjacency List for Current Vertex	

Vertex Queue (Front to Rear)

Rule 1: Visit next unvisited vertex adjacent to current vertex, mark it, add to the queue
Rule 2: If no unvisited (adjacent to current) vertices, current vertex = dequeue()
Rule 3: If queue is empty, you are done

Graph Traversals: Time Complexity

General Depth-First Traversal (iterative)

```
dfs(Graph G, vertex v) //v start point
   initialize stack S
   S.push(v)
   mark v as visited
   while (S is not empty)
      m = S.pop()
      for all (m, k) edges
         if k is not visited
            S.push(k)
            mark k as visited
         end
      end
   end
end
```

General Breadth-First Traversal (iterative)

```
bfs(Graph G, vertex v) //v start point
   initialize queue Q
   S.enqueue(v)
   mark v as visited
   while (Q is not empty)
      m = Q.dequeue()
      for all (m, k) edges
         if k is not visited
            Q.enqueue(k)
            mark k as visited
         end
      end
   end
end
```

Sequence of exploration (BFS):

v1 + (edges incident on v1) + v2 + (edges incident on v2) + ... + vn + (edges incident on vn)

[v1 + v2 + ... + vn] + [(edges incident on v1) + (edges incident on v2) + ... + (edges incident on vn)]

which is:

|V| + at most 2*|E| (each vertex is visited once, each edge is visited at most twice)

Big-O notation (assuming adjacency list representation):

O(|V| + |E|)