Control Theory Report

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Control Theory

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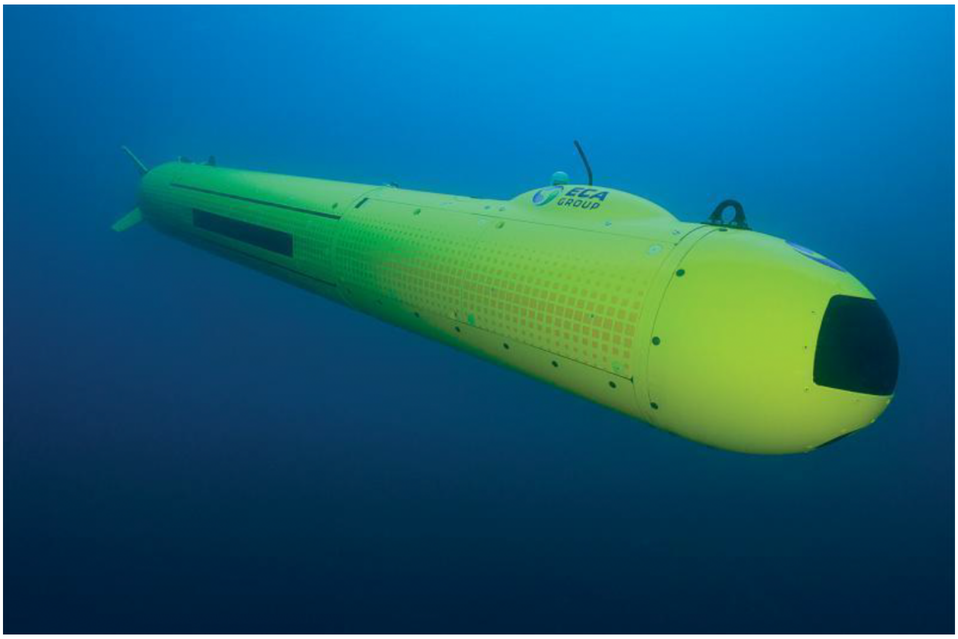

Figure 1: Autonomous Underwater Vehicle (A-18)

1.1 Aim

This report aims to show the procedure of creating an optimal control command law on the vertical plane for an Autonomous Underwater Vehicle (AUV).

1.2 Objectives

To improve the vertical stability of the AUV, we propose using a Linear Quadratic Regulator(LQR) controller instead of the existing Proportional-Integral-Derivative(PID) controller. Our specific objectives to implement the LQR controller are:

a) Identify the missing hydrodynamic parameters using the provided data.

b) Linearize the simplified nonlinear AUV model.

c) Select and implement a motion control system.

d) Refine the LQR's tuning for vertical commands.

1.3 Background

Autonomous underwater vehicles (AUVs) have emerged as versatile tools for ocean exploration, scientific research, and marine surveillance. These unmanned vehicles operate independently in the underwater environment, navigating through complex currents and obstacles to gather valuable data. However, achieving effective control and stability of AUVs is crucial for their safe operation and mission success.

ECA Robotics developed autonomous underwater vehicles (AUVs) designed to independently survey underwater areas. Equipped with sonar, AUVs effectively detect underwater mines, which pose a significant threat to military ships and submarines. However, maintaining high stability in all dimensions – vertical, horizontal, and roll – is crucial for the sonar to function properly.

Proportional-integral-derivative (PID) controllers have traditionally been employed for AUV control due to their simplicity and adaptability. PID controllers adjust control inputs based on the error between the desired and actual states of the AUV. However, PID controllers often struggle with nonlinearities and disturbances in the underwater environment, leading to suboptimal performance.

Linear quadratic regulator (LQR) controllers offer a more advanced approach to AUV control. LQR controllers optimize the control inputs by minimizing a quadratic cost function that considers both the system state and control effort. This approach leads to more robust and efficient control, particularly in the presence of nonlinearities and disturbances.

This report provides a simplified AUV simulator, currently utilizing a PID controller with limitations in robustness and stability. This practical work will focus on the development of optimal control law for movement planes of A18M, one of the AUV models.

2 Identification:

Since all the parameters are provided except the damping parameters in the horizontal and vertical plane, as $K_{sh}, CY_{uv}, CY_{ur}, CN_{uv}, CN_{ur}, CZuw, CZ_{uq}, CM_{uw}, CM_{uq}$.

Here,

- CY_{uv} : Force damping coefficient F_Y by the product of $u \cdot v$.
- CY_{ur} : Force damping coefficient F_Y by the product of $u \cdot r$.
- CN_{uv} : Damping coefficient of the moment M_Z by the product of $u \cdot v$.
- CN_{ur} : Damping coefficient of the moment M_Z by the product of $u \cdot r$.
- CZ_{uw} : Force damping coefficient F_Z by the product of $u \cdot w$.
- CZ_{uq} : Force damping coefficient F_Z by the product of $u \cdot q$.
- CM_{uw} : Damping coefficient of the moment M_Y by the product of $u \cdot w$.
- CM_{uq} : Damping coefficient of the moment M_Y by the product of $u \cdot q$.

2.1 Ksh

To identify K_{sh},

Since we already have some data from previous missions, We use the speed step data to calculate K_{sh}.

Speed step: In the mission, the AUV makes pure advance speed steps ("u") with all the other speeds so small that we can consider them null.

To calculate K_{sh}, we need to first model drag force F_x which is a function of (u),

$$F_x(u) = \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot C_{X0} \cdot u \cdot |u| \tag{1}$$

Here, ρ is the water density, S_{ref} is the AUV's reference surface area and the term C_{X0} is defined as,

$$C_{X0} = K_{sh} \cdot C_{XF} \tag{2}$$

Here, C_{XF} is derived using the Reynolds number and it fits F_x to account for the flow dynamics. It is defined as,

$$C_{XF} = \frac{0.075}{(\log_{10} R_e - 2)^2} \tag{3}$$

And Reynolds's number is defined as,

$$R_e = \frac{|u| \cdot L_{ref}}{\nu} \tag{4}$$

Now substituting 2, 3 and 4 in 1.

$$F_x = K_{sh} \cdot (\frac{1}{2} \cdot \rho \cdot S_{ref} \cdot u \cdot |u| \cdot \frac{0.075}{(\log_{10}(|u| \cdot \frac{L_{ref}}{\nu}) - 2)^2}) \tag{5}$$

Since we are using the Least Square method to Calculate K_{sh}. We would have to define A and B which are fed into $K_{sh} = lsqr(A, B)$.

$$A = \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot u \cdot |u| \cdot \frac{0.075}{(\log_{10}(|u| \cdot \frac{L_{ref}}{\nu}) - 2)^2} \tag{6}$$

and,

$$B = F_x \tag{7}$$

At last, we will get the perfect K_{sh} using which F_x is calculated and it will perfectly fit into the known F_x value. ($K_{sh} = -43.9863$)

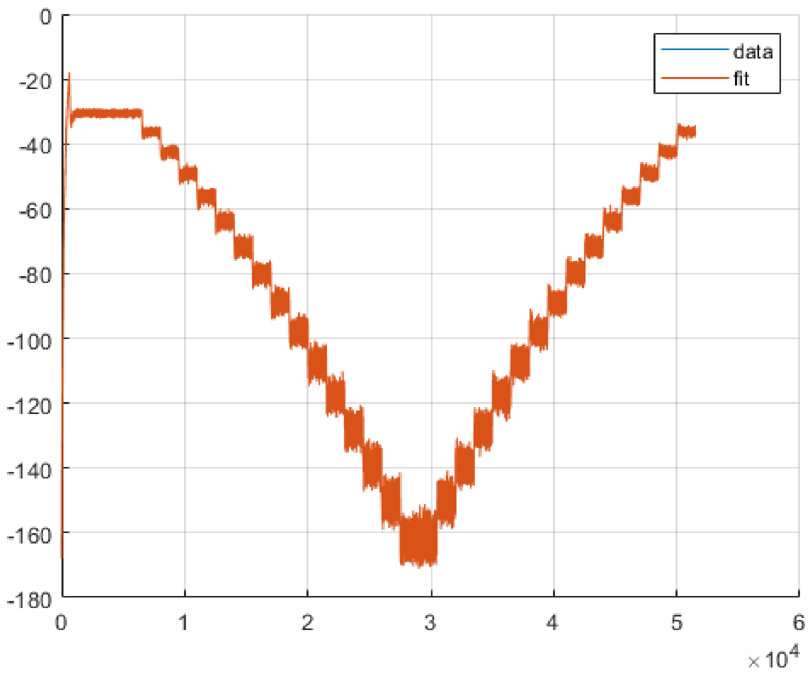

Figure 2: Comparison between the measured force data and the estimated force data using K_{sh}

2.2 Computing CZ, CM

To identify $CZ_{uw}, CZ_{uq}, CM_{uw}, CM_{uq}$,

Since we already have some data from previous missions, We use the pitch step data to calculate $CZ_{uw}, CZ_{uq}, CM_{uw}, CM_{uq}$.

Pitch step: In the mission, the AUV makes pitch steps ("θ") that implies speeds steps as "w" and "q", all other speeds are so small that we consider them as null.

For calculating CZ values, the F_Z i.e. Hydrodynamic force in the $z - dirction$, is fundamental to the model. Following similar steps from the previous section, we need to find A and B such that,

$$\begin{bmatrix} CZ_{uw} \\ CZ_{uq} \end{bmatrix} = lsqr(A, B) \tag{8}$$

After solving the equations,

$$A = \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot u \cdot \begin{bmatrix} w \\ L_{ref} \cdot q \end{bmatrix} \tag{9}$$

and,

$$B = F_Z - \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot C_{Z0} \cdot u \cdot |u| \tag{10}$$

At last, we will get the perfect CZ_{uw}, CZ_{uq} using which F_Z is calculated and it will perfectly fit into the known F_Z value. ($CZ_{uw} = -3.1582, CZ_{uq} = -1.9394$)

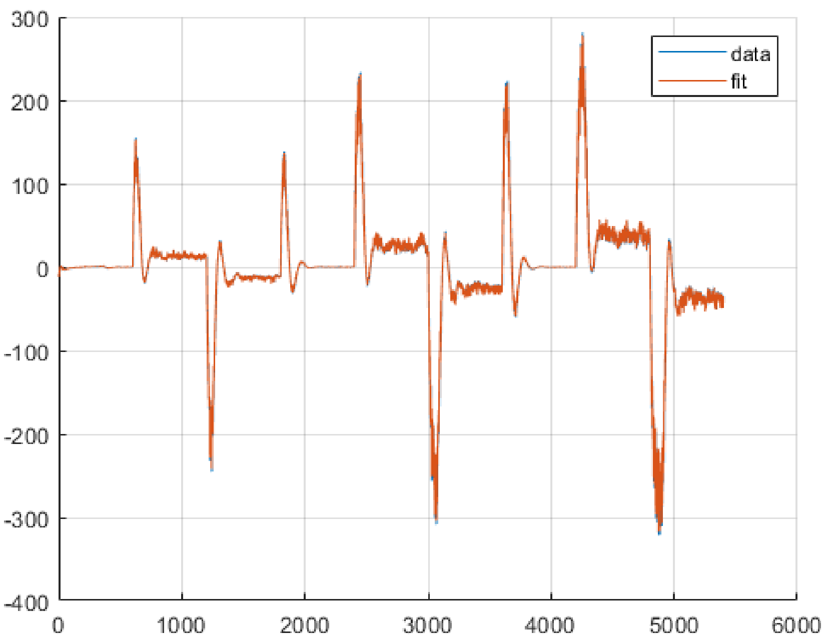

Figure 3: Comparison between the measured F_Z and the estimated F_Z using CZ_{uw}, CZ_{uq}

For calculating CM values, the M_Y i.e. Hydrodynamic moment in the $y - dirction$, is fundamental to the model. Following similar steps from the previous section, we need to find A and B such that,

$$\begin{bmatrix} CM_{uw} \\ CM_{uq} \end{bmatrix} = lsqr(A, B) \tag{11}$$

After solving the equations,

$$A = \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot L_{ref} \cdot u \cdot \begin{bmatrix} w \\ L_{ref} \cdot q \end{bmatrix} \tag{12}$$

and,

$$B = M_Y - \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot L_{ref} \cdot C_{M0} \cdot u \cdot |u| \tag{13}$$

At last, we will get the perfect CM_{uw}, CM_{uq} using which M_Y is calculated and it will perfectly fit into the known M_Y value. ($CM_{uw} = 0.9782, CM_{uq} = -0.8908$)

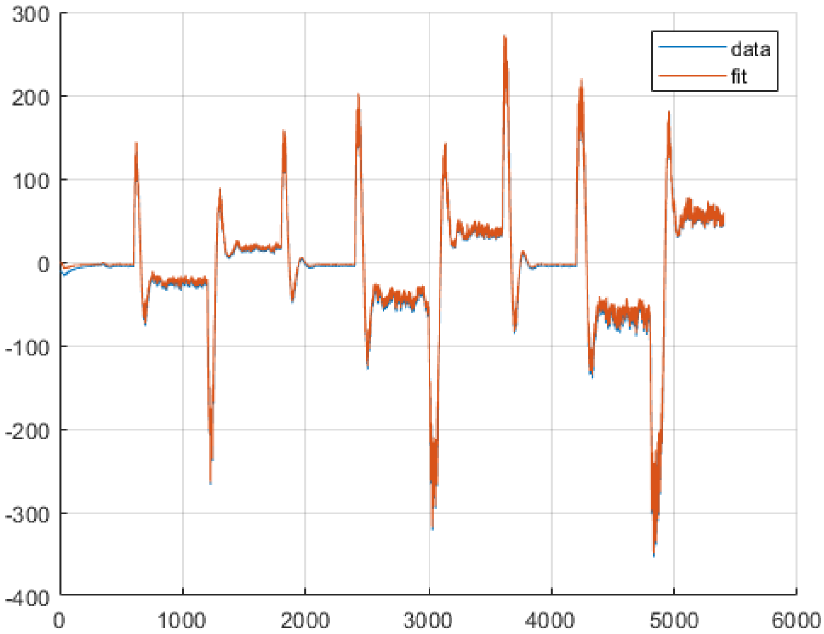

Figure 4: Comparison between the measured M_Y and the estimated M_Y using CM_{uw}, CM_{uq}

2.3 Computing CY, CN

To identify $CY_{uv}, CY_{ur}, CN_{uv}, CN_{ur}$,

It is different and tricky to calculate these values because the least square method does not work here due to several local minima and if we want to use the least square method, we would need correct starting values. In this case, we use ”fminsearchbnd”.

To use this method we would need a function, starting values, and upper and lower bounds.

For the starting values, because of the symmetry $CY_{uv}, CY_{ur}, CN_{uv}, CN_{ur}$ is comparable to $CZ_{uw}, -CZ_{uq}, -CM_{uw}, CM_{uq}$. So we can use these values.

Since we already have some data from previous missions, We use the Dieudinnée data to calculate $CY_{uv}, CY_{ur}, CN_{uv}, CN_{ur}$.

Dieudinnée: In the mission, the AUV makes virtual helms steps ("A") that implies speeds steps as "v" and "r", all other speeds are so small that we consider them as null.

For calculating CY values, the F_Y i.e. Hydrodynamic force in the $y - dirction$, is fundamental to the model. We need to find function, starting points, lower and upper bounds such that,

$$\begin{bmatrix} CY_{uv} \\ CY_{ur} \end{bmatrix} = fminsearchbnd(fun, x_0, lowerbounds, upperbounds) \tag{14}$$

After solving the equations,

$$fun(CY) = \sum (F_Y - \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot (CY_{uv} \cdot v \cdot u + L_{ref} \cdot CY_{ur} \cdot r \cdot u))^2 \tag{15}$$

Next, the starting points which are already explained above, and after that lower and upper bound. For this, we add and subtract the mentioned error percentages from the starting points.

For example, the starting points, the lower and upper bounds are as follows:

$$x_0 = \begin{bmatrix} CZ_{uw} & -CZ_{uq} \end{bmatrix} \tag{16}$$

$$lowerbound = \begin{bmatrix} 1.2 \cdot CZ_{uw} & -0.975 \cdot CZ_{uq} \end{bmatrix} \tag{17}$$

$$upperbound = \begin{bmatrix} 0.8 \cdot CZ_{uw} & -1.025 \cdot CZ_{uq} \end{bmatrix} \tag{18}$$

At last, we will get the perfect CY_{uv}, CY_{ur} using which F_Y is calculated and it will perfectly fit into the known F_Y value. ($CY_{uv} = -3.7898, CY_{ur} = 1.9450$)

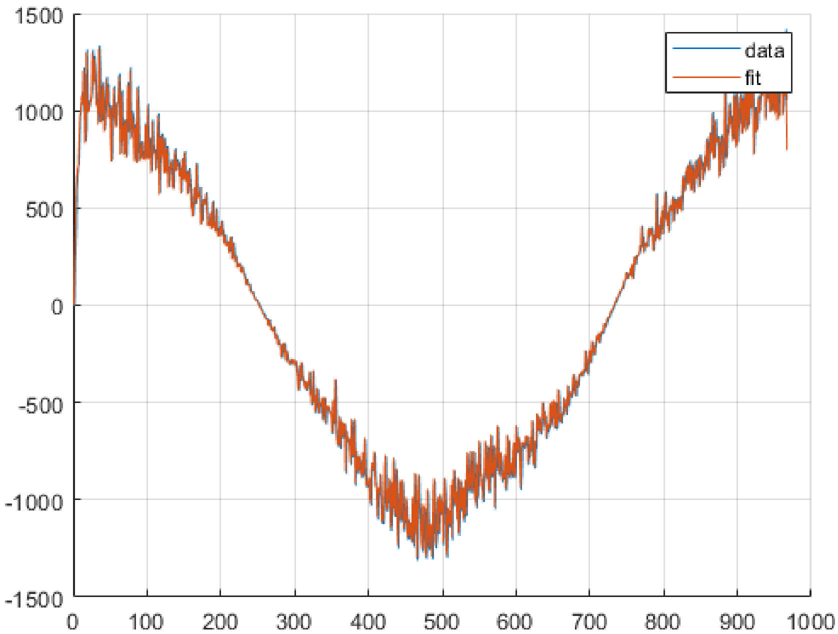

Figure 5: Comparison between the measured F_Y and the estimated F_Y using CY_{uv}, CY_{ur}

For calculating CN values, the M_Z i.e. Hydrodynamic moment in the $z-dirction$, is fundamental to the model. Following similar steps from the previous section, we need to find function, starting points, lower and upper bounds such that,

$$\begin{bmatrix} CN_{uv} \\ CN_{ur} \end{bmatrix} = fminsearchbnd(fun, x_0, lowerbounds, upperbounds) \tag{19}$$

After solving the equations,

$$fun(CN) = \sum (M_Z - \frac{1}{2} \cdot \rho \cdot S_{ref} \cdot (CN_{uv} \cdot v \cdot u + L_{ref}^2 \cdot CN_{ur} \cdot r \cdot u))^2 \tag{20}$$

Next, the starting points which are already explained above, and after that lower and upper bound. For this, we add and subtract the mentioned error percentages from the starting points.

For example, the starting points, the lower and upper bounds are as follows:

$$x_0 = \begin{bmatrix} -CM_{uw} & CM_{uq} \end{bmatrix} \tag{21}$$

$$lowerbound = \begin{bmatrix} -1.1 \cdot CM_{uw} & 1.1 \cdot CM_{uq} \end{bmatrix} \tag{22}$$

$$upperbound = \begin{bmatrix} -0.9 \cdot CM_{uw} & 0.9 \cdot CM_{uq} \end{bmatrix} \tag{23}$$

At last, we will get the perfect CN_{uv}, CN_{ur} using which M_Z is calculated and it will perfectly fit into the known M_Z value. ($CN_{uv} = -1.0761, CN_{ur} = -0.8018$)

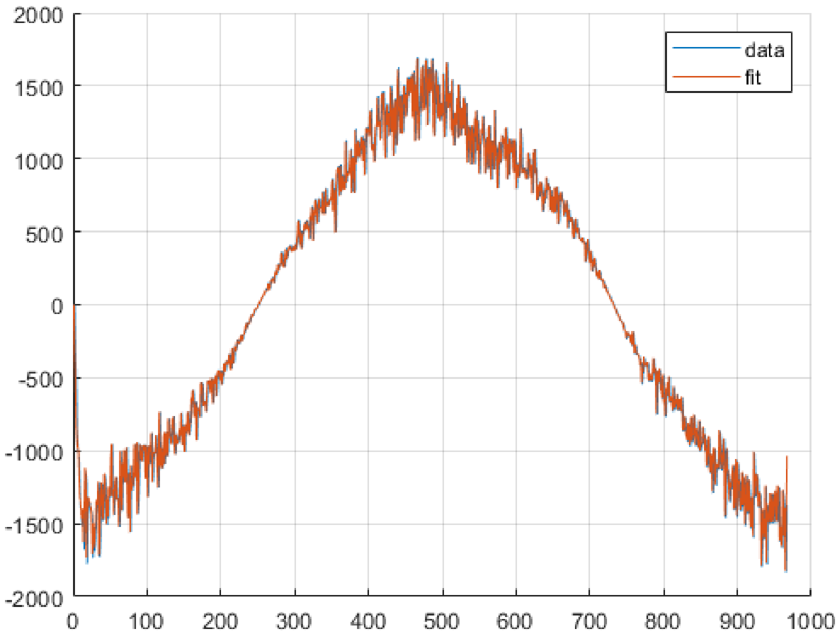

Figure 6: Comparison between the measured M_Z and the estimated M_Z using CN_{uv}, CN_{ur}

3 Optimal control

The state space equation will be simplified by linearizing it at the equilibrium point. This is because most control problems in the real world are non-linear, and it isn't easy to solve them without linearization.

An existing PID control system has been implemented for all four planes of motion: advance, roll, horizontal, and vertical. However, this PID control lacks stability, hindering the AUV's ability to fulfil its mission objectives. To address this issue, a more robust and effective LQR control method is required. In this assignment, we tried to enhance the vertical plane control, as it is deemed more critical for the AUV's operation.

3.1 Requirements for control law (we chose motion in the vertical plane)

To make the vertical plane control law, there are some important aspects to consider:

- State to be controlled (z).
- Pitch oscillations need to be minimized (low "q").
- The small final error is highly preferable, and above all not to oscillate around the command (low "θ" and "$\int z$").
- Lift speed oscillations need to be minimized (low "w")
- Maximum overshoot is 2 metres.

Considering all the aspects above, we can specify all states that we need to control, which are:

$$X = \begin{bmatrix} q \\ w \\ z \\ \theta \\ \int z \end{bmatrix}$$

3.2 Linearization

First, we have to linearise the system at the equilibrium point. To evaluate the $\dot{X}$, we need to compute the corresponding continuous tangent of the linearized control system. Jacobian Matrix is used to solve this problem. Then the system becomes:

$$\dot{X} = AX + BU$$

where,

$$A = \begin{bmatrix} \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial w} & \frac{\partial \dot{q}}{\partial z} & \frac{\partial \dot{q}}{\partial \theta} & \frac{\partial \dot{q}}{\partial \int z} \\ \frac{\partial \dot{w}}{\partial q} & \frac{\partial \dot{w}}{\partial w} & \frac{\partial \dot{w}}{\partial z} & \frac{\partial \dot{w}}{\partial \theta} & \frac{\partial \dot{w}}{\partial \int z} \\ \frac{\partial \dot{z}}{\partial q} & \frac{\partial \dot{z}}{\partial w} & \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial \theta} & \frac{\partial \dot{z}}{\partial \int z} \\ \frac{\partial \dot{\theta}}{\partial q} & \frac{\partial \dot{\theta}}{\partial w} & \frac{\partial \dot{\theta}}{\partial z} & \frac{\partial \dot{\theta}}{\partial \theta} & \frac{\partial \dot{\theta}}{\partial \int z} \\ \frac{\partial \int \dot{z}}{\partial q} & \frac{\partial \int \dot{z}}{\partial w} & \frac{\partial \int \dot{z}}{\partial z} & \frac{\partial \int \dot{z}}{\partial \theta} & \frac{\partial \int \dot{z}}{\partial \int z} \end{bmatrix}$$

and,

$$B = \begin{bmatrix} \frac{\partial q}{\partial BAR} \\ \frac{\partial w}{\partial BAR} \\ \frac{\partial z}{\partial BAR} \\ \frac{\partial \theta}{\partial BAR} \\ \frac{\partial \int z}{\partial BAR} \end{bmatrix}$$

3.3 Calculation of the equilibrium point

A system's equilibrium point represents a state where the system's state variables converge to zero when evaluated at that specific point.

$$f(x^e, u^e) = 0$$

We have calculated the equilibrium point at θ_0 and BAR_0,

$$\begin{aligned} \alpha_1 &= Det_{33}\frac{1}{2}\rho S_{ref} C_Z u_0 |u_0| \\ \alpha_3 &= Det_{33}\frac{1}{2}\rho S_{ref} C_{Z0} |u_0| u_0 \\ \alpha_4 &= Det_{33}\frac{1}{2}\rho S_{ref} C_{Zuw} u_0 \\ \alpha_5 &= Det_{33}(W - B) \end{aligned}$$

$$\begin{aligned} \beta_1 &= Det_{55}\frac{1}{2}\rho S_{ref} L_{ref} C_M u_0 |u_0| \\ \beta_2 &= Det_{55}\frac{1}{2}\rho S_{ref} C_{Muw} L_{ref} u_0 \\ \beta_4 &= Det_{55}\frac{1}{2}\rho S_{ref} L_{ref} CM0 |u_0| u_0 \\ \beta_6 &= -Det_{55}(z_g W - z_v B) \end{aligned}$$

$$\theta_0 = \frac{-\alpha_1\beta_4 + \alpha_3\beta_1 + \alpha_5\beta_1}{\alpha_1\beta_2u_0 + \alpha_1\beta_6 - \alpha_4\beta_1u_0}$$

$$BAR_0 = \frac{-\beta_2u_0\theta_0 - \beta_4 - \beta_6\theta_0}{\beta_1}$$

Here M^{-1} can be simplified as :

$$M^{-1} = \begin{bmatrix} Det_{11} & 0 & 0 \\ 0 & Det_{33} & 0 \\ 0 & 0 & Det_{55} \end{bmatrix}$$

$$\begin{aligned}
Det_{11} &= \frac{m33_t \cdot m55_t - m35^2}{Det_M} \\
Det_{33} &= \frac{m11_t \cdot m55_t - m15^2}{Det_M} \\
Det_{55} &= \frac{m11_t \cdot m33_t - m13^2}{Det_M} \\
Det_M &= m11_t \cdot m33_t \cdot m55_t - m11_t \cdot m35^2 - m13^2 \cdot m55_t + 2m13 \cdot m15 \cdot m35 - m15^2 \cdot m33_t
\end{aligned}$$

where, $mij \in M$

For matrix A,

$$\begin{aligned}
\frac{\partial \dot{q}}{\partial q} &= Det_{55}[-mz_gw + \frac{1}{2}\rho S_{ref}L_{ref}^2C_{Muq}u_0] \\
\frac{\partial \dot{q}}{\partial w} &= Det_{55}[-mz_gq + \frac{1}{2}\rho S_{ref}C_{Muw}L_{ref}u_0] \\
\frac{\partial \dot{q}}{\partial z} &= 0 \\
\frac{\partial \dot{q}}{\partial \theta} &= -Det_{55}[z_gW - z_vB]\cos(\theta_0) \\
\frac{\partial \dot{q}}{\partial \int z} &= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \dot{w}}{\partial q} &= Det_{33}[2qmz_g + mu_0 + \frac{1}{2}\rho S_{ref}C_{Zuq}L_{ref}u_0] \\
\frac{\partial \dot{w}}{\partial w} &= Det_{33}\frac{1}{2}\rho S_{ref}C_{Zuw}u_0 \\
\frac{\partial \dot{w}}{\partial z} &= 0 \\
\frac{\partial \dot{w}}{\partial \theta} &= -Det_{33}(W - B)\sin(\theta_0) \\
\frac{\partial \dot{w}}{\partial \int z} &= 0
\end{aligned}$$

$$\frac{\partial \dot{z}}{\partial q} = 0$$
$$\frac{\partial \dot{z}}{\partial w} = \cos(\theta_0)$$
$$\frac{\partial \dot{z}}{\partial z} = 0$$
$$\frac{\partial \dot{z}}{\partial \theta} = -u_0 * \cos(\theta_0) - w * \sin(\theta_0)$$
$$\frac{\partial \dot{z}}{\partial \int z} = 0$$

$$\frac{\partial \dot{\theta}}{\partial q} = 1 \;\; ; \;\; \frac{\partial \int \dot{z}}{\partial q} = 0$$
$$\frac{\partial \dot{\theta}}{\partial w} = 0 \;\; ; \;\; \frac{\partial \int \dot{z}}{\partial w} = 0$$
$$\frac{\partial \dot{\theta}}{\partial z} = 0 \;\; ; \;\; \frac{\partial \int \dot{z}}{\partial z} = 1$$
$$\frac{\partial \dot{\theta}}{\partial \theta} = 0 \;\; ; \;\; \frac{\partial \int \dot{z}}{\partial \theta} = 0$$
$$\frac{\partial \dot{\theta}}{\partial \int z} = 0 \;\; ; \;\; \frac{\partial \int \dot{z}}{\partial \int z} = 0$$

For matrix B,

$$B = \begin{bmatrix} \frac{1}{2}M^{-1}\rho S_{ref} L_{ref} C_M |u_0| u_0 \\ \frac{1}{2}M^{-1}\rho S_{ref} C_z |u_0| u_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3.4 Calculation of Kd matrix

The Linear-Quadratic Regulator (LQR) method is a technique for designing feedback control systems. It works by calculating the optimal feedback gain matrix K, which determines how the control input u should be adjusted based on the current state of the system y. The goal of the LQR method is to minimize a cost function J, which represents the sum of the squared errors between the desired state and the actual state, and the squared control inputs.

$$J = \int_0^{\infty} [y^T Q y + u^T R u] dt$$

The MATLAB ‘lqrd‘ command is used for discrete full-state feedback regulation from a continuous system.

Here we have chosen the Q and R matrix as follows.

$$Q = \begin{bmatrix} 10000 & 0 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 100000 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = 100$$

We find K_d at each time step using Matlab's 'lqrd' command.

4 Results of the simulation, Q and R tuning

After running the simulation with all the parameters previously discussed, the following results were plotted, describing the behaviour of the system modelled.

4.0.1 Depth profile

The graph illustrates the AUV's vertical trajectory during the ascent from a depth of 40 meters to 10 meters. As evident from the graph, it took approximately 50 seconds for the AUV to reach the 40-meter depth, exhibiting a slight overshoot. The AUV then stabilized within the desired depth range within approximately 100 seconds.

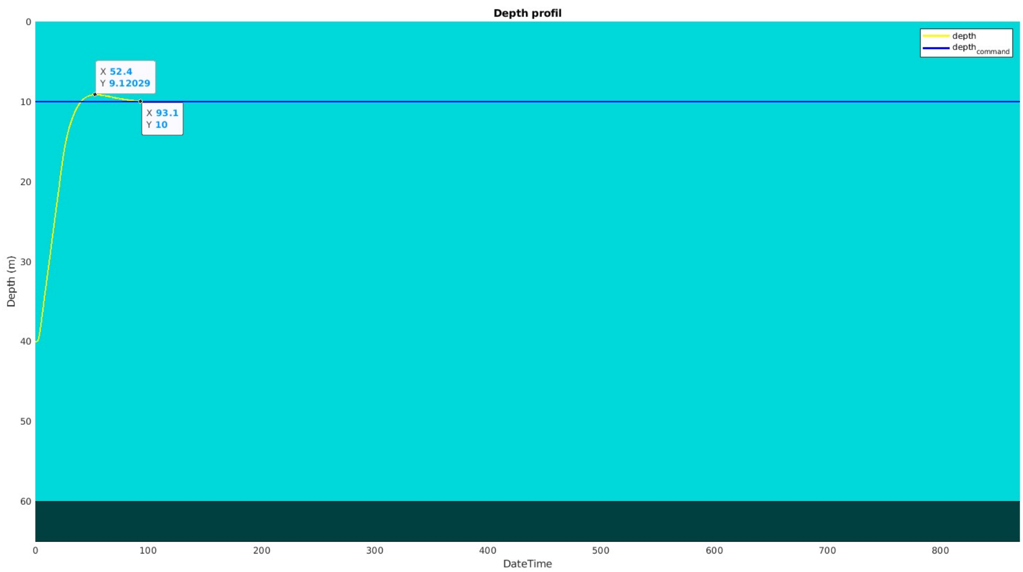

Figure 7: Depth Profile

4.0.2 XY plot

This shows the trajectory of the AUV as viewed from the ocean surface.

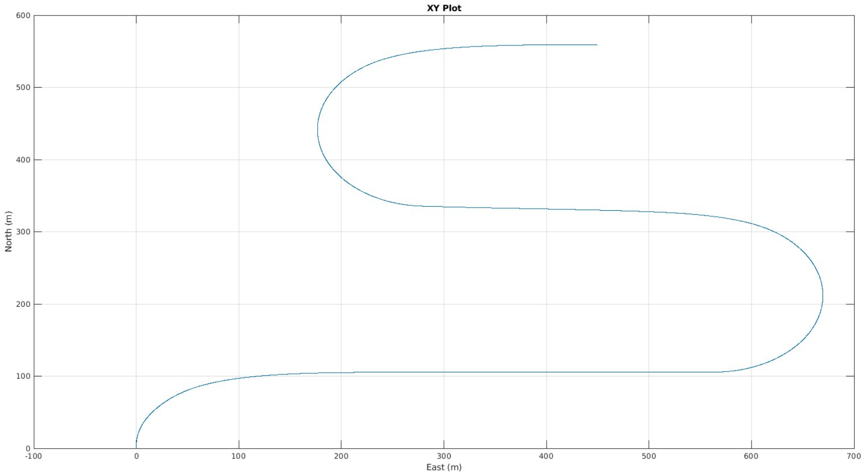

Figure 8: XY plot

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4.0.3 Euler angles

In the vertical section, the rollϕ and yawψ angles are negligible, while the pitch angle θ plays a significant role. The roll angle increases quite a lot in the beginning but quickly reduces to ≤ 0.2 at around 100 seconds. The thrusters are adjusted to maintain a positive pitch angle, causing the AUV to ascend initially. As the AUV approaches the desired depth of 10 meters after about 100 seconds, the pitch angle θ is reduced, eventually reaching zero at around 100 seconds when the desired depth is reached.

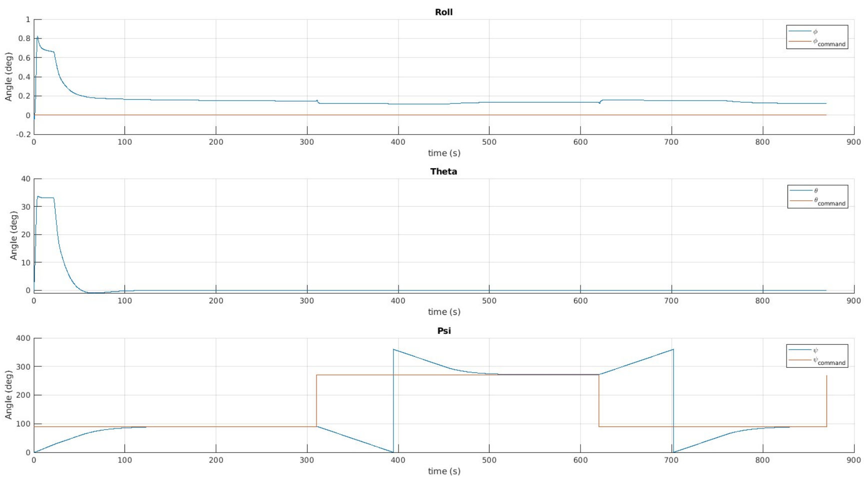

Figure 9: Euler angles

4.0.4 Linear speed

The AUV's longitudinal speed remained relatively steady throughout the ascent, maintaining a constant propulsion speed of 2ms−1. This forward motion, combined with the controlled inclination angle , facilitated the AUV's ascent. The lift speed, while small in magnitude, provided a subtle but significant contribution to the AUV's vertical control. This control was effectively adjusted until the desired depth of 10m was achieved, which occurred around 380 seconds into the ascent. Notably, the lift speed remained stable without any oscillations, a result attributed to the low value of the parameter ”w”.

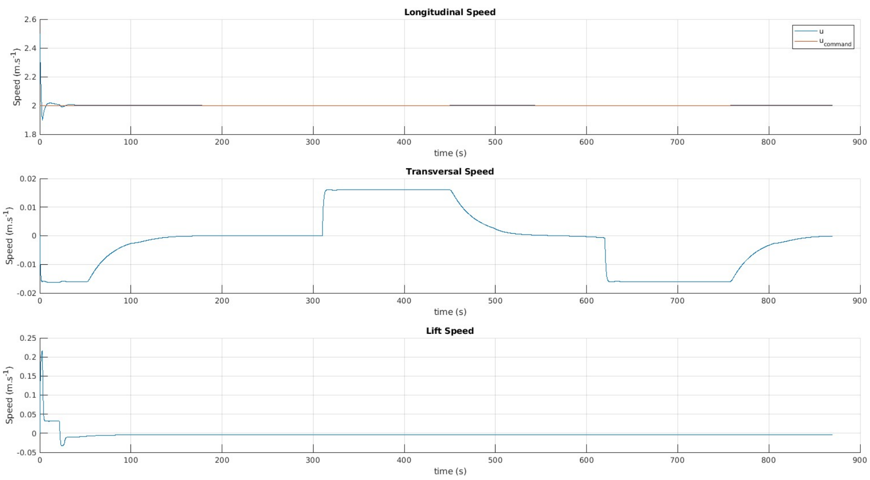

Figure 10: Linear speed

4.0.5 Angular speed

The pitch rate graph depicts the AUV's attitude adjustments to attain the desired depth. The rate of change in the pitch angle closely mirrors the depth command, demonstrating effective control without excessive oscillations. This outcome can be attributed to the low value of the parameter ”q”.

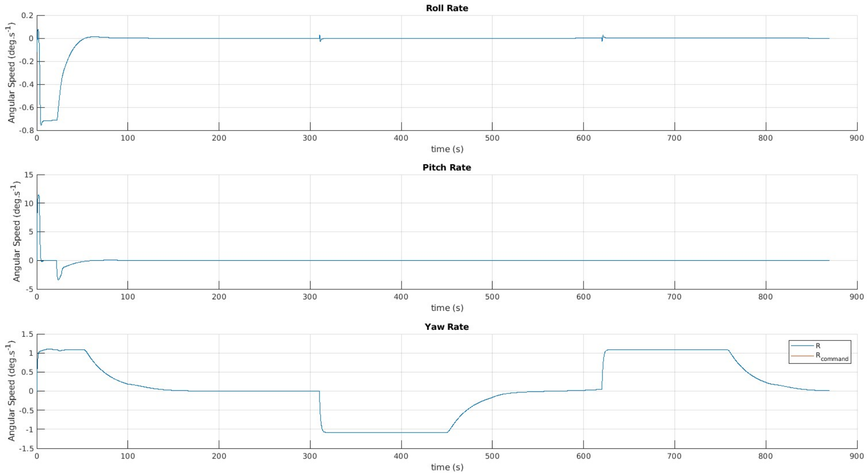

Figure 11: Angular speed

4.0.6 Actuators

The BAR actuator, responsible for regulating the AUV's vertical movement, exhibited a gentle trajectory on the graph, suggesting that the actuators are operating in a non-aggressive manner. This gentle response could potentially lead to excessive wear and tear on the actuators, jeopardizing the system's integrity.

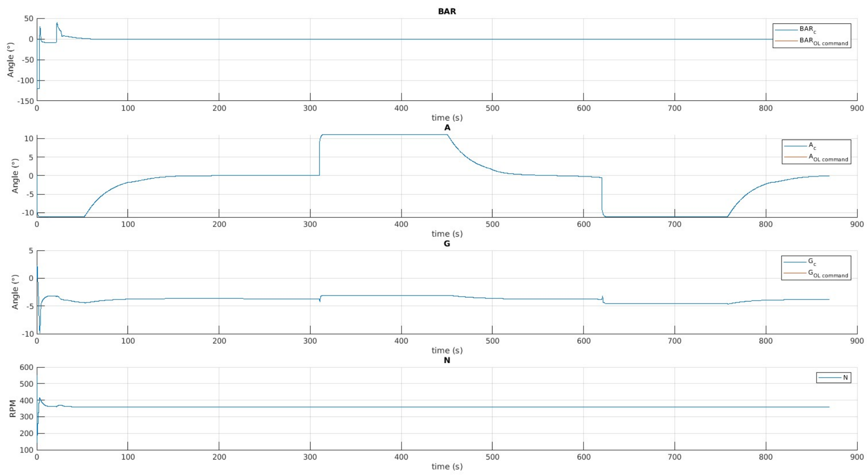

Figure 12: Actuators