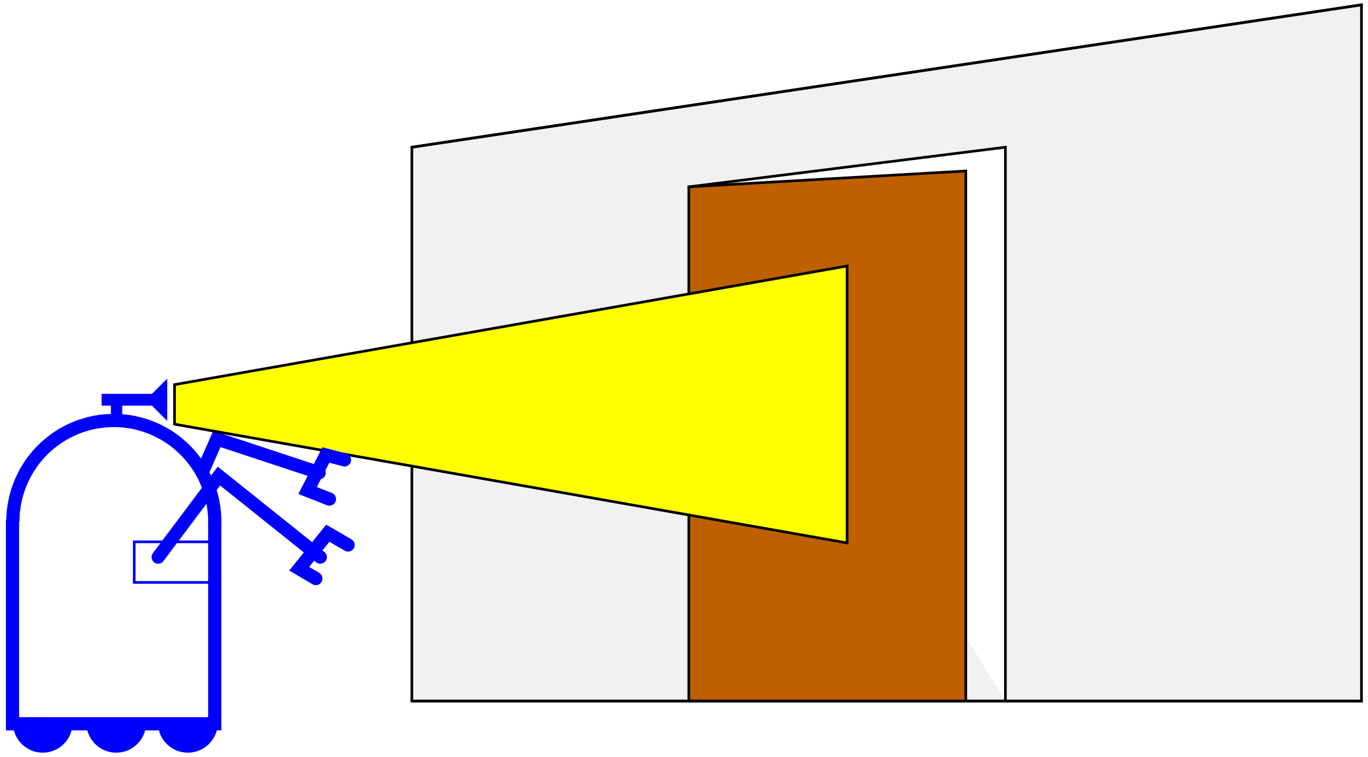
Simple Example of State Estimation

- Suppose a robot obtains measurement z
- What is $P(open \mid z)$?



Causal vs. Diagnostic Reasoning

- $P(open \mid z)$ is diagnostic
- $P(z \mid open)$ is causal
- In some situations, causal knowledge is easier to obtain
- Bayes rule allows us to use causal knowledge:

count frequencies!

$$P(open \mid z) = \frac{P(z \mid open) P(open)}{P(z)}$$

Example

- $P(z \mid open) = 0.6$ $\qquad P(z \mid \neg open) = 0.3$
- $P(open) = P(\neg open) = 0.5$
- $P(open \mid z) = \frac{P(z|open)P(open)}{P(z)} = \frac{0.6*0.5}{0.6*0.5+0.3*0.5} = \frac{2}{3}$
- z raises the probability that the door is open

Combining Evidence

- Suppose our robot obtains another observation z_2
- How can we integrate this new information?
- More generally, how can we estimate $P(x \mid z_1, \ldots, z_n)$?

Recursive Bayesian Updating

$$P(x \mid z_1, \ldots, z_n) = \frac{P(z_n \mid x, z_1, \ldots, z_{n-1})P(x \mid z_1, \ldots, z_{n-1})}{P(z_n \mid z_1, \ldots, z_{n-1})}$$

Markov assumption:

z_n is independent of $z_1, \ldots, z_{n-1}$ given we know x

$$P(x \mid z_1, \ldots, z_n) = \frac{P(z_n \mid x)P(x \mid z_1, \ldots, z_{n-1})}{P(z_n \mid z_1, \ldots, z_{n-1})}$$

$$= \alpha P(z_n \mid x)P(x \mid z_1, \ldots, z_{n-1})$$

$$= \alpha P(x) \prod_{i=1\ldots n} P(z_i \mid x)$$

Example: Second Measurement

- $P(z_2 \mid open) = 0.25$ $\quad P(z_2 \mid \neg open) = 0.3$
- $P(open \mid z_1) = \frac{2}{3}$

$$P(open \mid z_2, z_1) = \frac{P(z_2 \mid open)P(open \mid z_1)}{P(z_2 \mid open)P(open \mid z_1) + P(z_2 \mid \neg open)P(\neg open \mid z_1)}$$

$$= \frac{\frac{1}{4}\cdot\frac{2}{3}}{\frac{1}{4}\cdot\frac{2}{3} + \frac{3}{10}\cdot\frac{1}{3}} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{10}} = \frac{\frac{1}{6}}{\frac{4}{15}} = \frac{5}{8} = 0.625$$

- z_2 lowers the probability that the door is open

Actions

- Often the world is **dynamic** since
  - **actions carried out by the robot**,
  - **actions carried out by other agents**,
  - or just the **time** passing by

  change the world

- How can we **incorporate** such **actions**?

Typical Actions

- The robot **turns its wheels** to move
- The robot **uses its manipulator** to grasp an object
- Plants grow over **time** …

- Actions are **never carried out with absolute certainty**
- In contrast to measurements, **actions generally increase the uncertainty**

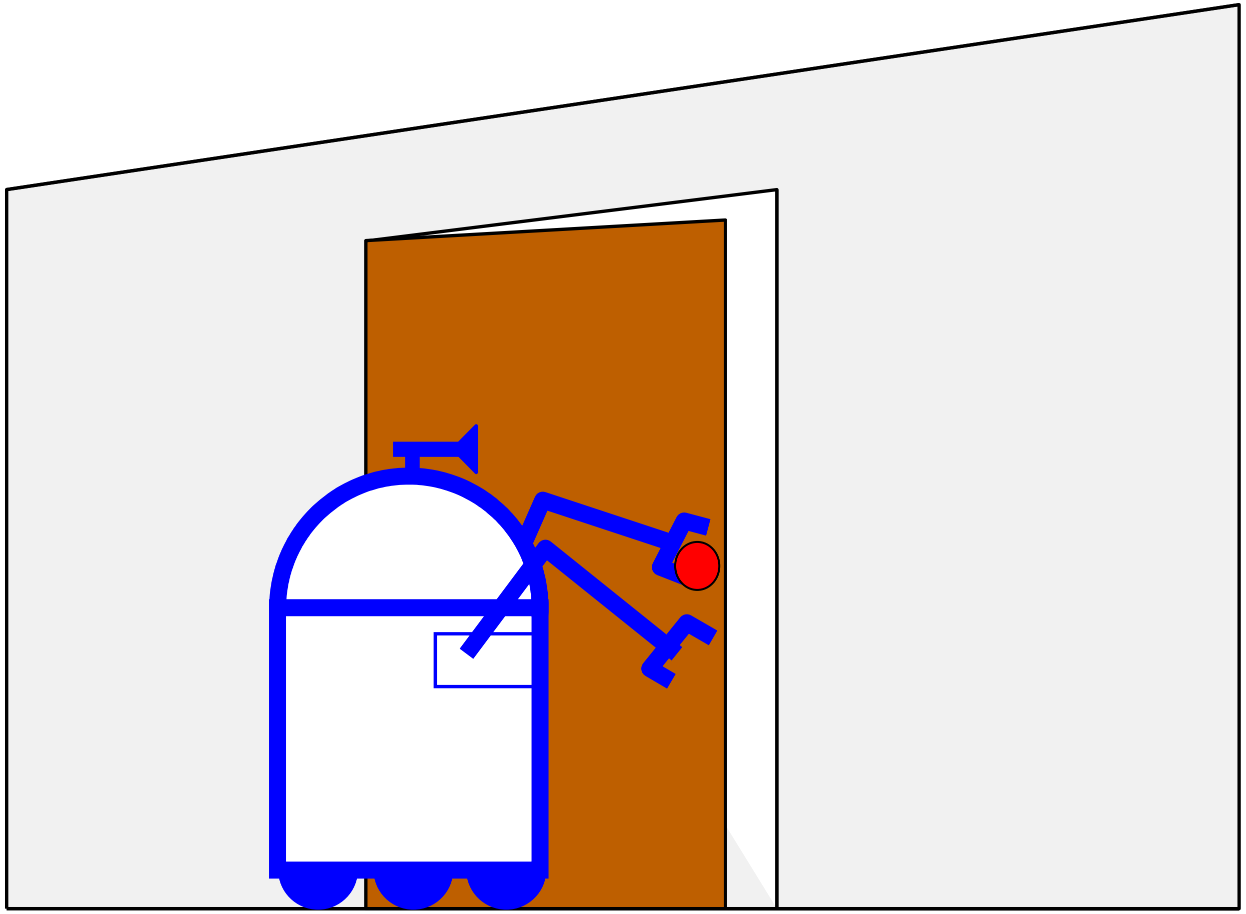
Modeling Actions

- To incorporate the outcome of an action *u* into the current "belief", we use the conditional pdf

$$P(x \mid u, x')$$

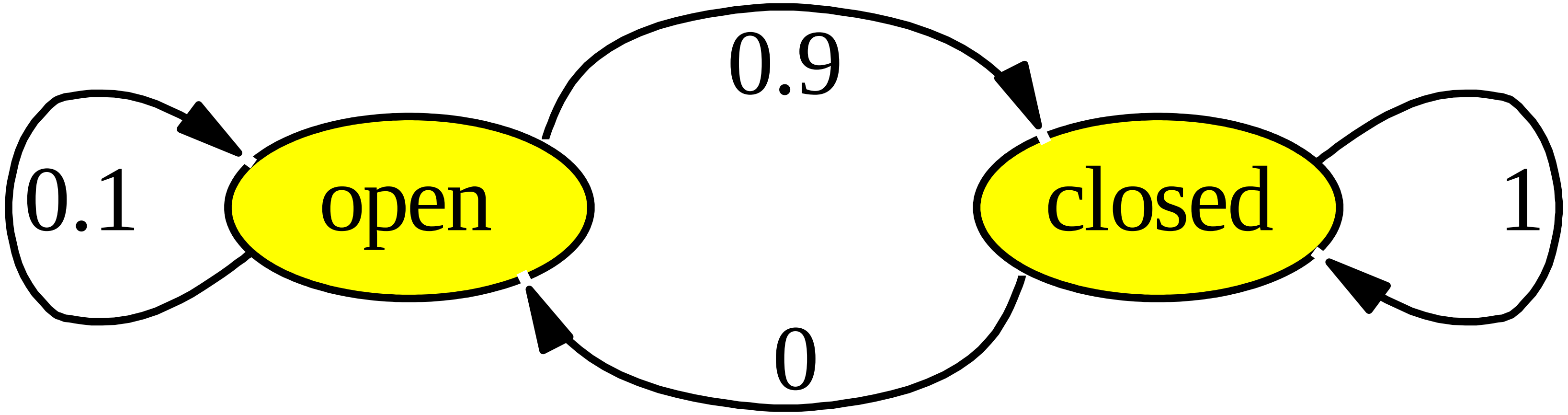
- This term specifies the pdf that **executing *u* changes the state from x' *to* x.**

Example: Closing the door



State Transitions

$P(x \mid u, x')$ for u = “close door”:



If the door is open, the action “close door” succeeds in 90% of all cases

Integrating the Outcome of Actions

Continuous case:

$$P(x \mid u) = \int P(x \mid u, x')\, P(x' \mid \cancel{u})\, dx'$$

Discrete case:

$$P(x \mid u) = \sum P(x \mid u, x')\, P(x' \mid \cancel{u})$$

We will make an independence assumption to get rid of the u in the second factor in the sum.

Example: The Resulting Belief

$$P(closed \mid u) = \sum P(closed \mid u, x')P(x')$$

$$= P(closed \mid u, open)P(open) + P(closed \mid u, closed)P(closed)$$

$$= \frac{9}{10} \cdot \frac{5}{8} + \frac{1}{1} \cdot \frac{3}{8} = \frac{15}{16}$$

$$P(open \mid u) = \sum P(open \mid u, x')P(x')$$

$$= P(open \mid u, open)P(open) + P(open \mid u, closed)P(closed)$$

$$= \frac{1}{10} \cdot \frac{5}{8} + \frac{0}{1} \cdot \frac{3}{8} = \frac{1}{16}$$

$$= 1 - P(closed \mid u)$$