Bayes Filters: Framework

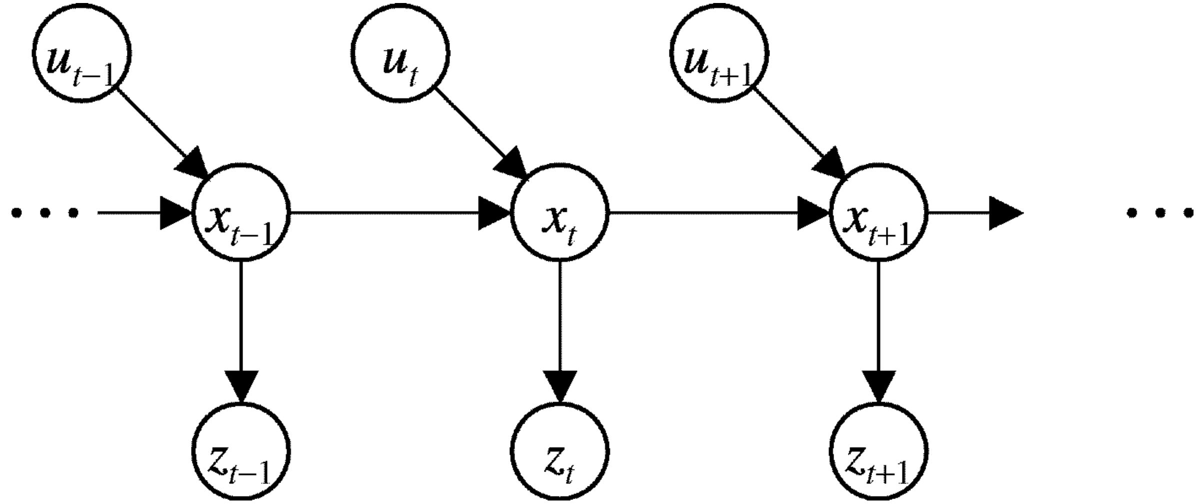
- **Given:**
  - Stream of observations *z* and action data *u:*

$$d_t = \{u_1, z_1, \ldots, u_t, z_t\}$$

  - Sensor model $P(z \mid x)$
  - Action model $P(x \mid u, x')$
  - Prior probability of the system state $P(x)$
- **Wanted:**
  - Estimate of the state X of a dynamical system
  - The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

Markov Assumption



$$P(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = P(z_t \mid x_t)$$
$$P(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = P(x_t \mid x_{t-1}, u_t)$$

Underlying Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors

Bayes Filters

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1, \dots, u_t, z_t)$$

Bayes $$= \eta P(z_t \mid x_t, u_1, z_1, \dots, u_t) P(x_t \mid u_1, z_1, \dots, u_t)$$

Markov $$= \eta P(z_t \mid x_t) P(x_t \mid u_1, z_1, \dots, u_t)$$

Total prob. $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \dots, u_t, x_{t-1}) \, P(x_{t-1} \mid u_1, z_1, \dots, u_t) \, dx_{t-1}$$

Markov $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) P(x_{t-1} \mid u_1, z_1, \dots, u_t) \, dx_{t-1}$$

Markov $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) P(x_{t-1} \mid u_1, z_1, \dots, z_{t-1}) \, dx_{t-1}$$

$$\boxed{= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) Bel(x_{t-1}) \, dx_{t-1}}$$

$$Bel(x_t) = \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

Algorithm **Bayes_filter**($Bel(x), d$):

$\eta = 0$

If d is a perceptual data item z then

For all x do

$Bel'(x) = P(z \mid x) Bel(x)$

$\eta = \eta + Bel'(x)$

For all x do

$Bel'(x) = \eta^{-1} Bel'(x)$

Else if d is an action data item u then

For all x do

$Bel'(x) = \int P(x \mid u, x') \, Bel(x') \, dx'$

Return $Bel'(x)$

Bayes Filters are Familiar!

$$Bel(x_t) = \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)