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Course Code: MR2022P (Fall 2022)

Mobile Robotics
Individual Assignment: Bayes Filter

Question

Consider a household robot equipped with a camera. It operates in an apartment with two rooms: a living room and a bedroom. The robot runs an artificial neural network that can recognize a living room in the camera image. Further, the robot can perform a switch-room action, i.e., it moves to the living room if it is in the bedroom, and vice versa. Neither the recognition nor the motion controller is perfect.

From previous experience, you know that the robot succeeds in moving from the living room to the bedroom with a probability of 0.7, and with a probability of 0.8 in the other direction:

$$p(x_{t+1} = bedroom|x_t = living\_room, u_{t+1} = switch - room) = 0.7$$

$$p(x_{t+1} = living\_room|x_t = bedroom, u_{t+1} = switch - room) = 0.8$$

The probability that the neural network indicates that the robot is in the living room although it is in the bedroom is given by $p(z = living\_room|x = bedroom) = 0.3$ and the probability that the network correctly detects the living room is given by $p(z = living\_room|x = living\_room) = 0.9$.

Unfortunately, you have no knowledge about the current location of the robot. However, after performing the switch-room action, the neural network indicates that the robot is not in the living room. After performing the switch-room action for the second time, the network again indicates not seeing a living room.

1. Use the Bayes filter algorithm to compute the probability that the robot is in the bedroom after performing the two actions. Use an appropriate prior distribution and justify your choice.
2. Which prior minimizes that probability? Briefly explain your answer.

Section 1

Let's first consider Living Room = L, Bed Room = B and Switch Room = SR then our probabilities for the action model are as follows:

$$p(x_{t+1} = B|x_t = L, u_{t+1} = SR) = 0.7$$

$$p(x_{t+1} = B|x_t = B, u_{t+1} = SR) = 0.3$$

$$p(x_{t+1} = L|x_t = B, u_{t+1} = SR) = 0.8$$

$$p(x_{t+1} = L|x_t = L, u_{t+1} = SR) = 0.2$$

We can visualise these probabilities in Markov-Chain representation which is shown in the figure 1.

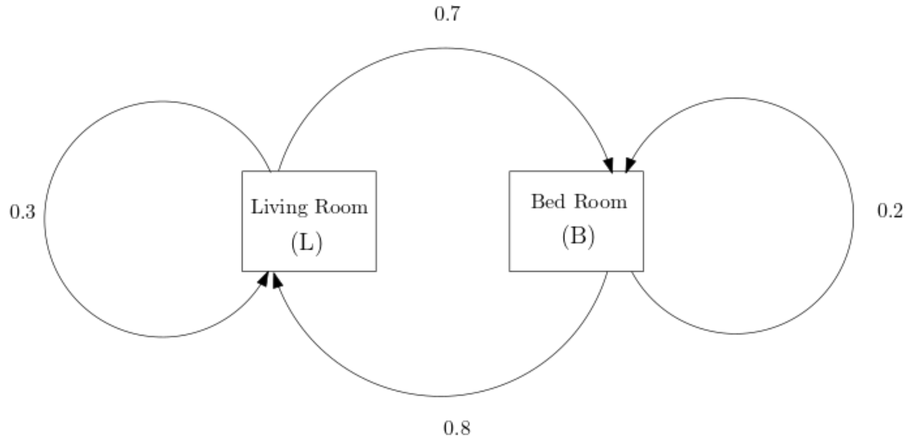


Figure 1: Markov Chain representation of probabilities

Moving on to the Sensor model, according to our question we have,

$$p(z = L|x = B) = 0.3$$

$$p(z = L|x = L) = 0.9$$

These probabilities suggest that the robot's sensors are relatively reliable in detecting a Living Room, in that the error probability is 0.2. However, in the case of the Bedroom, it has a probability of 0.3 to be wrong, which is a relatively error prone measurement.

According to our problem, we have no knowledge about the current situation. But the measurement of the robot before and after the action SR are both L. So, we can model our system as displayed in the figure 2.

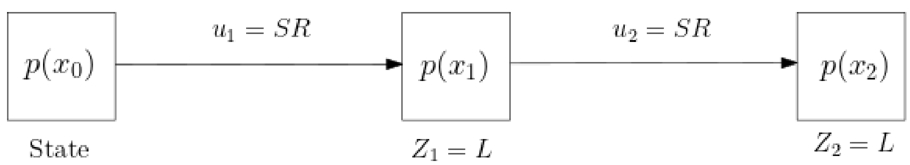


Figure 2: State and Action representation of the problem

Let us first assume that the robot does not know the state of the room initially. So, it assigns an equal prior probability to the two possible initial states:

$$bel(x_0 = L) = 0.5$$

$$bel(x_0 = B) = 0.5$$

In accordance with probabilistic Bayesian filtering the *prediction* of x_1 state is as follows:

$$\overline{bel}(x_1) = \int p(x_1|x_0, u_1) bel(x_0) dx_0$$

$$\Rightarrow \sum_{x_0} p(x_1|x_0, u_1) bel(x_0)$$

$$= p(x_1|x_0 = L, u_1 = SR)bel(x_0 = L) + p(x_1|x_0 = B, u_1 = SR)bel(x_0 = B)$$

We can now substitute the two possible values for the state variable x_1 . For the hypothesis $x_1 =$ L, we obtain

$$\begin{aligned}
&\overline{bel}(x_1 = L) \\
&\quad = p(x_1 = L|x_0 = L, u_1 = SR)bel(x_0 = L) + p(x_1 = L|x_0 = B, u_1 = SR)bel(x_0 = B) \\
&\quad = 0.2 \times 0.5 + 0.8 \times 0.5 = 0.5
\end{aligned}$$

Likewise, for $x_1 = B$ we get,

$$\begin{aligned}
&\overline{bel}(x_1 = B) \\
&\quad = p(x_1 = B|x_0 = L, u_1 = SR)bel(x_0 = L) + p(x_1 = B|x_0 = B, u_1 = SR)bel(x_0 = B) \\
&\quad = 0.7 \times 0.5 + 0.3 \times 0.5 = 0.5
\end{aligned}$$

The fact that $\overline{bel}(x_1)$ is same as our prior belief $bel(x_0)$ is because it's evident that both for the case, the robot in the bedroom or living room, if it performs the action 'switch room' the probability of being in bedroom and living room will be same as it's prior distribution. Incorporating the measurement, however, can change the belief. So, according to Bayes filter algorithm,

$$bel(x_1) = \eta p(z_1 = L|x_1)\overline{bel}(x_1)$$

For the two possible cases, $x_1 = L$ and $x_1 = B$, we get

$$\begin{aligned}
&\mathrm{bel}(\mathrm{x}_1 = L) \\
&\quad = \eta p(z_1 = L|x_1 = L)\overline{bel}(x_1 = L) \\
&\quad = \eta \times 0.9 \times 0.5 = \eta \times 0.45
\end{aligned}$$

and,

$$\begin{aligned}
&\mathrm{bel}(\mathrm{x}_1 = B) \\
&\quad = \eta p(z_1 = L|x_1 = B)\overline{bel}(x_1 = B) \\
&\quad = \eta \times 0.3 \times 0.5 = \eta \times 0.15
\end{aligned}$$

The normalizer η can be calculated as

$$\eta = (0.15 + 0.45)^{-1} = 1.6666 \approx 1.67$$

Hence, we have

$$bel(x_1 = L) = 1.67 \times 0.45 = 0.7515 \approx 0.75$$

$$bel(x_1 = B) = 1.67 \times 0.15 = 0.2505 \approx 0.25$$

For the second step we iterate the same procedure, and can we can write,

$$\overline{bel}(x_2 = L) = 0.2 \times 0.75 + 0.8 \times 0.25 = 0.35$$

$$\overline{bel}(x_2 = B) = 0.7 \times 0.75 + 0.3 \times 0.25 = 0.6$$

and

$$\eta = (0.315 + 0.18)^{-1} \approx 2.02$$

$$bel(x_2 = L) = \eta \times 0.9 \times 0.35 \approx 0.64$$

$$bel(x_2 = B) = \eta \times 0.3 \times 0.6 \approx 0.36$$

At this point, the robot believes that with 0.64 probability it is in the Living Room.

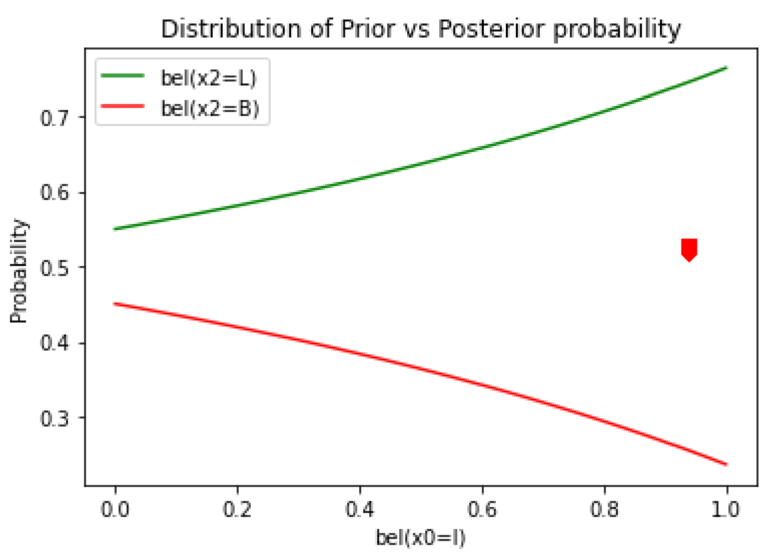
Section 2

For the second part of the question first, we can run an experiment and visualise how the posterior probability distribution for state x_2 changes if the prior probability distribution for state x_0 changes. Here the independent prior probability is set as $p(x_0 = L)$ and $p(x_0 = B)$ is derived from the fact the total probability sums up to 1 for any given state. The code is written in Python and Matplotlib Library is used to visualise the distribution. The code snippet and its output are given as follows:

```python
def plot_distribution(step):
    prob_list = []
    X =  np.linspace(0,1,step)
    for i in X:
        bel_x0_is_l = i
        bel_x0_is_b = 1 - i
        bel_x2_is_l, bel_x2_is_b = bayesian_filtering(bel_x0_is_l,bel_x0_is_b)
        prob_list.append([bel_x0_is_l,bel_x0_is_b,bel_x2_is_l,bel_x2_is_b])
    prob_array = np.array(prob_list).reshape(step,4)

    plt.plot(X, prob_array[:,2], color='g', label='bel(x2=L)')
    plt.plot(X, prob_array[:,3], color='r', label='bel(x2=B)')
    plt.xlabel("bel(x0=l)")
    plt.ylabel("Probability")
    plt.title("Distribution of Prior vs Posterior probability")
    plt.legend()
    plt.show()

# Plotting Results
plot_distribution(100)
```



Now, as it is clear from the figure that for the distribution $p(x_0 = L) = 0$ and $p(x_0 = B) = 1$ minimises the posterior probability distribution. If we do the calculations, it turns out to be :

$$bel(x_2 = L) \approx 0.55, bel(x_2 = B) \approx 0.45$$

Now, moving on to the explanation of the result, we can infer from the sensor model that the robot can precisely measure the Living Room. However, It can wrongly estimate Bedroom sometimes. So, If we want to maximise the error which as result minimises the prior probabilities we should consider the prior state as the bedroom.

Also, another analogy we can state is that the measurement from the current state is far more important than the previous ones. So, if the robot is in the Bedroom in state x_2 and sens as it is the Living Room it will impact more on the posterior probability than if the error occurs in the x_1 state. This is due to the fact that previous measurement construct our belief of the previous state which act as the prior to our distribution. By incorporating the current state measurement we can influence its belief. Therefore, if we take the initial state as a bedroom after two Switch Room actions (with 70% and 80% probability of successful outcome) the probability of being in bedroom maximises which translates to maximum error and minimum probability distribution.