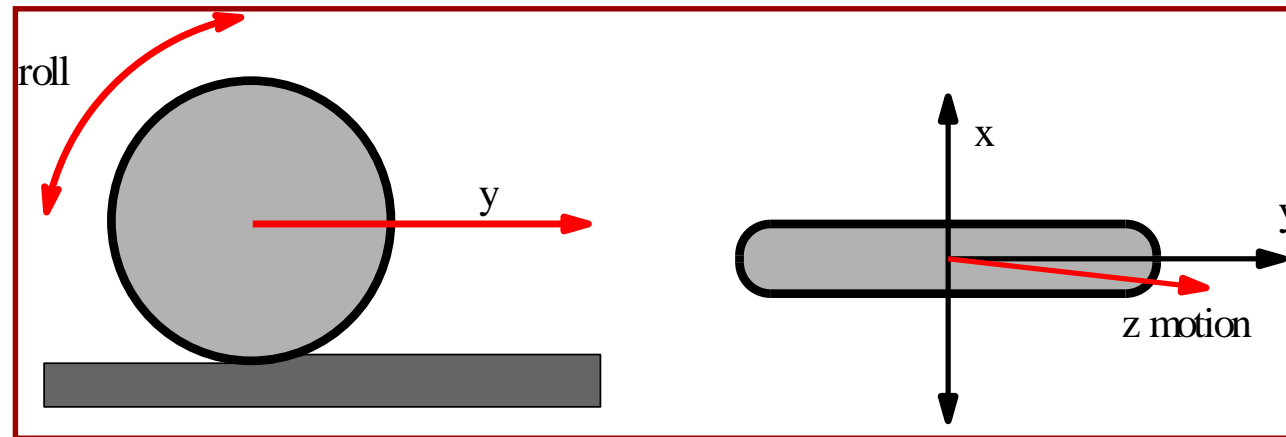


# Locomotion of Wheeled Robots

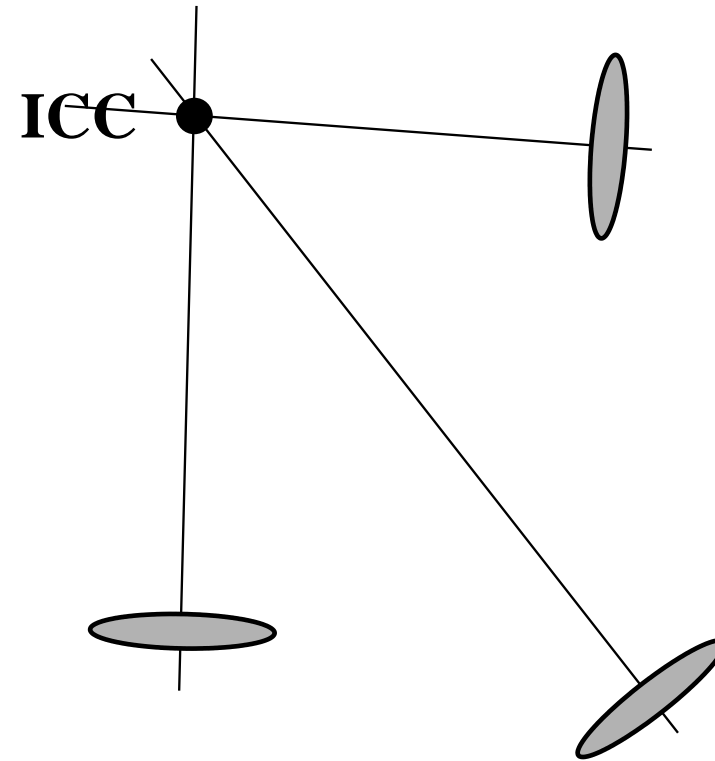
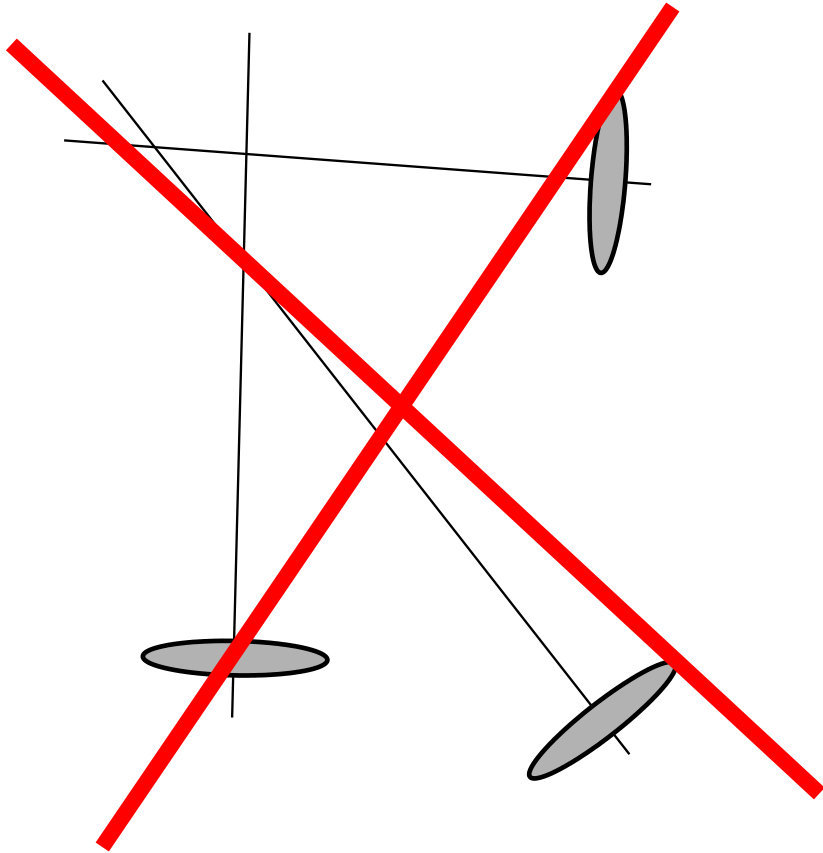
Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- XR4000
- Mecanum wheels



we also allow wheels to rotate around the z axis

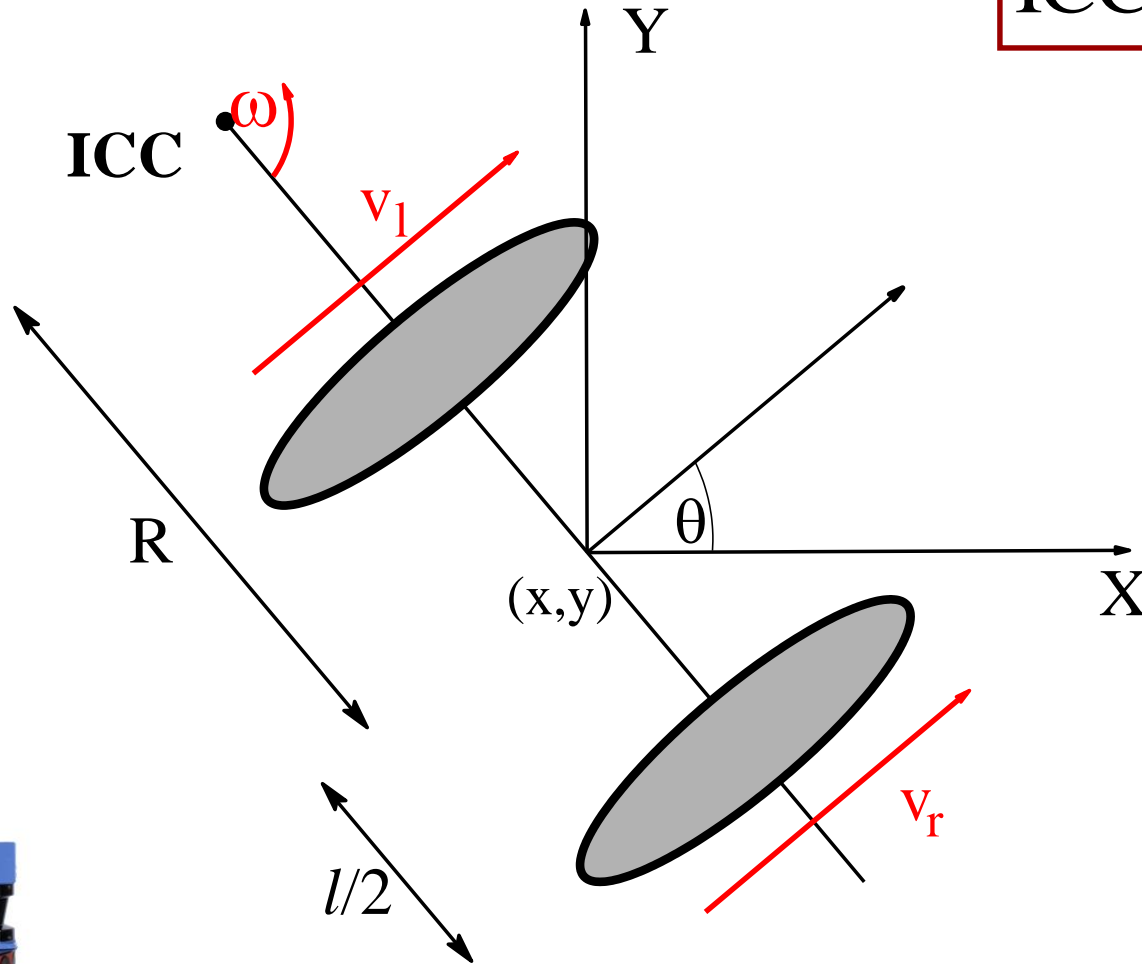
# Instantaneous Center of Curvature



- For rolling motion to occur, each wheel has to move along its y-axis

# Differential Drive

$$\text{ICC} = [x - R \sin \theta, y + R \cos \theta]$$



$$W(R + l / 2) = v_r$$

$$W(R - l / 2) = v_l$$

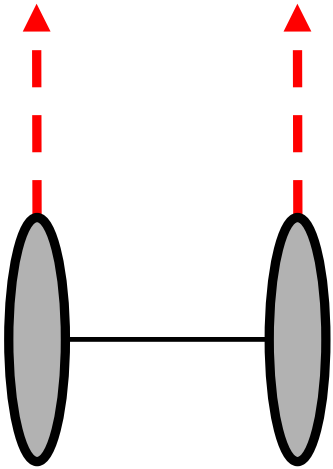
$$R = \frac{l (v_l + v_r)}{2 (v_r - v_l)}$$

$$W = \frac{v_r - v_l}{l}$$

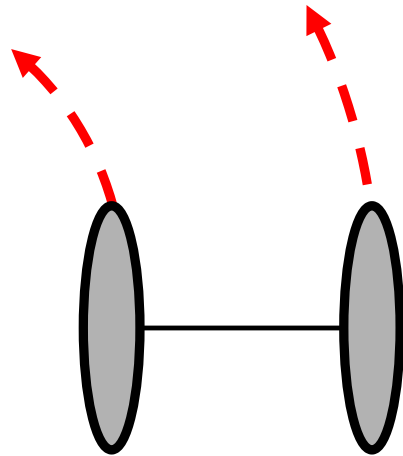
$$v = \frac{v_r + v_l}{2}$$

# Differential Drive Motion Patterns

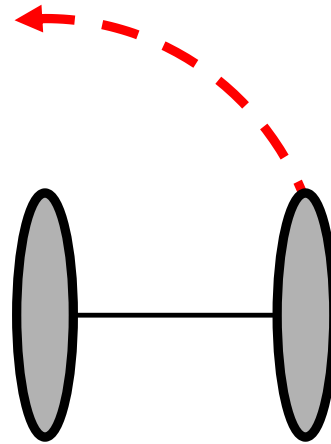
$$R = \frac{l (v_l + v_r)}{2 (v_r - v_l)}, \quad \omega = \frac{v_r - v_l}{l}$$



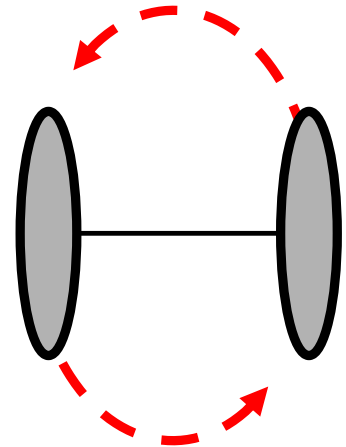
$$v_l = v_r$$



$$v_l < v_r$$
$$v_l > 0$$

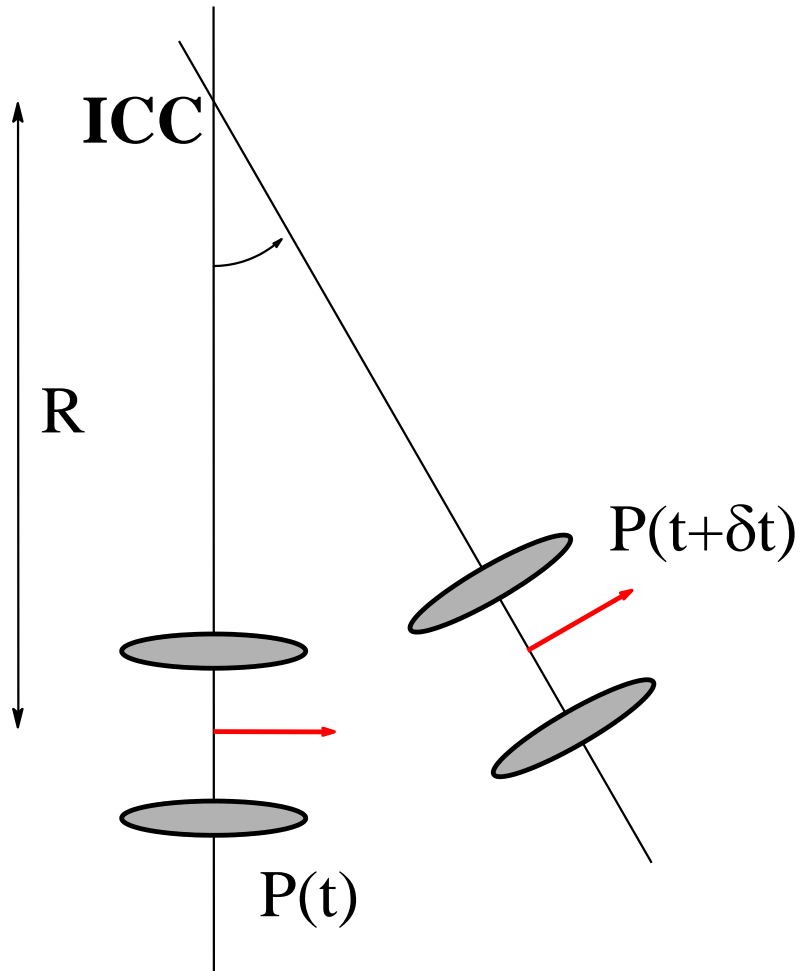


$$v_l = 0$$
$$v_r > 0$$



$$v_l = -v_r$$

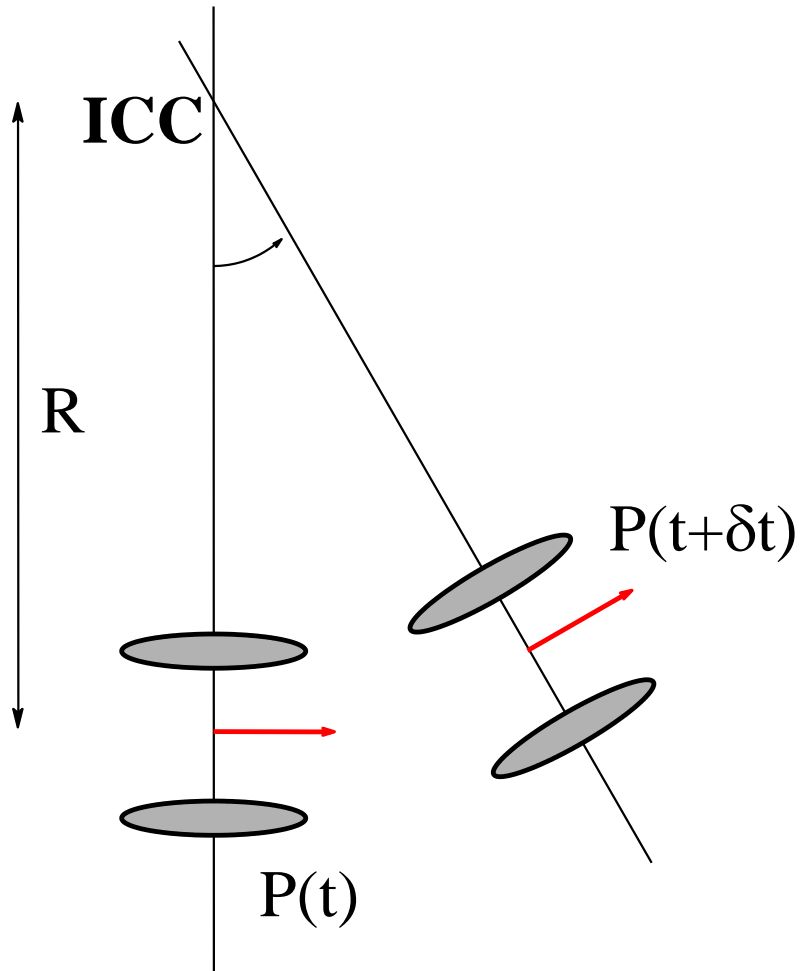
# Differential Drive: Forward Kinematics



$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\ \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega \delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$
$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$
$$\theta(t) = \int_0^t \omega(t') dt'$$

# Differential Drive: Forward Kinematics



$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\ \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega \delta t \end{bmatrix}$$

$$x(t) = \frac{1}{2} \int_0^t [v_r(t') + v_l(t')] \cos[\theta(t')] dt'$$

$$y(t) = \frac{1}{2} \int_0^t [v_r(t') + v_l(t')] \sin[\theta(t')] dt'$$

$$\theta(t) = \frac{1}{l} \int_0^t [v_r(t') - v_l(t')] dt'$$