Student Name: Abhimanyu Bhowmik
Course Code: MR2022P (Fall 2022)

Mobile Robotics
Individual Assignment: Discrete and Particle Filter

Question

In the course material, it was mentioned that the particle filter is more efficient than the discrete filter. In your own words, briefly justify this statement. Please also find one scenario in which the discrete filter would have an advantage over the basic particle filter.

Section 1

For the discrete bayesian filter, if we consider localising a robot in a 1-dimensional space, the initial belief can be taken as

$$Bel_0(s_k) = 1/n$$

where s is the total state space divided into n equal intervals and s_k is the centre of the kth interval. Moreover, if we take into account door observations as o, then the continuous measurement likelihood for the centre of the kth interval will be $p(0|s_k)$. Thus, the belief can be updated as

$$Bel_1(s_k) = p(0|s_k)Bel_0(s_k)$$

Further, when motion action u is introduced, let's say for an interval of i to k when the robot is moving, the updated belief becomes

$$Bel_2(s_k) = \sum_i p(0|s_k, u, s_i)Bel_1(s_i)$$

Thus, for calculating the motion update, the computational complexity turns out to be $O(n^2)$. When applied to higher-dimensional state spaces, the computational complexity and the need for memory and hardware resources increase drastically. So, to handle high dimensional spaces, the particle filter has a massive advantage over the discrete bayesian filter since its computational requirement is almost the same with a much better result than the latter one. This is due to the fact that we primarily use Stochastic universal sampling for the re-sampling step in particle filter algorithm which has Linear time complexity ($O(n)$). Particle filters are also extremely easy to implement compared to the Dynamic decomposition in histogram filters for continuous state estimation.

Section 2

The particle filter has advantage over the discrete bayesian filter for high-dimensional spaces in most scenarios. However, for particle filter, one of the prime conditions is that the proposed distribution(π) has to be positive, i.e., $f(x) > 0 \rightarrow \pi(x) > 0$, where f is the target distribution. To account for the differences between π and f, we introduce importance weight (w) where $w = f/\pi$. From this, we can conclude that,

In cases where π is a negative value, particle filter will not be a good option. Moreover, if some proposal distribution exists which is 0, where the actual distribution is non-0, we couldn't represent them with importance sampling. It can be a problem for state estimation with particle filter.

The other problem with the particle filter algorithm is the choice of proposal distribution. Although in practice, we tend to use the motion model as our proposal distribution in particle filter algorithm, if somehow the actual state changes a lot than our suggested proposal distribution and we didn't use infinitely many samples (which is actually the case for state estimation) we may end up with very different state estimation than actual reality.

Additionally, the discrete bayesian filter chooses its points based on an algorithm, whereas the particle filter chooses its points at random. As a result, a particle filter often requires a substantially higher number of points. In case we could not generate enough sample points due to a lack of computational resources, especially for 1- or 2- dimensional spaces, the discrete bayesian filter might be a better option.