

Keras -- MLPs on MNIST

In [0]:

```
# if you keras is not using tensorflow as backend set "KERAS_  
BACKEND=tensorflow" use this command  
from keras.utils import np_utils  
from keras.datasets import mnist  
import seaborn as sns  
from keras.initializers import RandomNormal
```

Using TensorFlow backend.

In [0]:

```
%matplotlib notebook  
%matplotlib inline  
import matplotlib.pyplot as plt  
import numpy as np  
import time  
# https://gist.github.com/greydanus/f6eee59eaf1d90fcb3b534a25  
362cea4  
# https://stackoverflow.com/a/14434334  
# this function is used to update the plots for each epoch an  
d error  
def plt_dynamic(x, vy, ty, ax, colors=['b']):  
    ax.plot(x, vy, 'b', label="Validation Loss")  
    ax.plot(x, ty, 'r', label="Train Loss")  
    plt.legend()  
    plt.grid()  
    fig.canvas.draw()
```

In [0]:

```
# the data, shuffled and split between train and test sets
```

```
(X_train, y_train), (X_test, y_test) = mnist.load_data()
```

Downloading data from [https://s3.amazonaws.com!\[\]\(529949c2c3dadbaa4e538e8c643454bc_img.jpg\)](https://s3.amazonaws.com/img-datasets/mnist.npz)
11493376/11490434 [=====]
===] - 2s 0us/step

In [0]:

```
print("Number of training examples :", X_train.shape[0], "and  
each image is of shape (%d, %d)"%(X_train.shape[1], X_train.  
shape[2]))  
print("Number of training examples :", X_test.shape[0], "and  
each image is of shape (%d, %d)"%(X_test.shape[1], X_test.sha  
pe[2]))
```

Number of training examples : 60000 and each i
mage is of shape (28, 28)

Number of training examples : 10000 and each i
mage is of shape (28, 28)

In [0]:

```
# if you observe the input shape its 2 dimensional vector  
# for each image we have a (28*28) vector  
# we will convert the (28*28) vector into single dimensional  
vector of 1 * 784  
  
X_train = X_train.reshape(X_train.shape[0], X_train.shape[1]*  
X_train.shape[2])  
X_test = X_test.reshape(X_test.shape[0], X_test.shape[1]*X_te  
st.shape[2])
```

In [0]:

```
# after converting the input images from 3d to 2d vectors  
  
print("Number of training examples :", X_train.shape[0], "and  
each image is of shape (%d)"%(X_train.shape[1]))
```

```
print("Number of training examples :", X_test.shape[0], "and  
each image is of shape (%d)"%(X_test.shape[1]))
```

Number of training examples : 60000 and each i
mage is of shape (784)

Number of training examples : 10000 and each i
mage is of shape (784)

In [0]:

```
# An example data point  
print(X_train[0])
```

```
[ 0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  0  0  0  0  
 0  0  0  0  0  0  0  
  0  0  0  0  0  0  0  0  3  18  18  
18 126 136 175  26 166 255  
247 127  0  0  0  0  0  0  0  0  0  0  
 0  0  0  30  36  94 154  
170 253 253 253 253 253 225 172 253 242 195  
64  0  0  0  0  0  0  
  0  0  0  0  0  49 238 253 253 253 253 2  
53 253 253 253 251  93  82  
 82  56  39  0  0  0  0  0  0  0  0  0
```

0	0	0	0	18	219	253						
253	253	253	253	198	182	247	241	0	0	0		
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	80	156	107	2
53	253	205	11	0	43	154						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	14	1	154	253	90	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	139	253	190	2	0						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	0	11	190	253	70	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	35	241						
225	160	108	1	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	0	81	240	2
53	253	119	25	0	0	0						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	0	45	186	253	253	150	27	0	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	16	93	252	253	187						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	249	253	249	64	
0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	46	130	183	253						
253	207	2	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0						
	0	0	0	0	39	148	229	253	253	253	250	1
82	0	0	0	0	0	0						

[illegible]

Normalize the data

In [0]:

```
# if we observe the above matrix each cell is having a value
# between 0-255
# before we move to apply machine learning algorithms lets tr
# y to normalize the data
#  $X \Rightarrow (X - X_{min}) / (X_{max} - X_{min}) = X / 255$ 

X_train = X_train/255
X_test = X_test/255
```

In [0]:

```
# example data point after normalizing
print(X_train[0])
```

[illegible]

.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.01176471	0.07058824	0
.	0.07058824	0.07058824		
.	0.49411765	0.53333333	0.68627451	0.10196078
.	0.65098039	1.		

0.96862745	0.49803922	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.11764706	0.14117647	0
.36862745	0.60392157			
0.66666667	0.99215686	0.99215686	0.99215686	0
.99215686	0.99215686			
0.88235294	0.6745098	0.99215686	0.94901961	0
.76470588	0.25098039			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.19215686			
0.93333333	0.99215686	0.99215686	0.99215686	0
.99215686	0.99215686			
0.99215686	0.99215686	0.99215686	0.98431373	0
.36470588	0.32156863			
0.32156863	0.21960784	0.15294118	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.07058824	0
.85882353	0.99215686			
0.99215686	0.99215686	0.99215686	0.99215686	0
.77647059	0.71372549			
0.96862745	0.94509804	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.31372549	0.61176471	0
.41960784	0.99215686			
0.99215686	0.80392157	0.04313725	0.	0
.16862745	0.60392157			
0.	0.	0.	0.	0

.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.05490196	0.00392157	0.60392157	0
.99215686	0.35294118			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.54509804	0.99215686	0.74509804	0
.00784314	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.04313725			
0.74509804	0.99215686	0.2745098	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.1372549	0.94509804			
0.88235294	0.62745098	0.42352941	0.00392157	0
.	0.			

0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.31764706	0
.94117647	0.99215686			
0.99215686	0.46666667	0.09803922	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.17647059	0.72941176	0
.99215686	0.99215686			
0.58823529	0.10588235	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.0627451	0.36470588	0.98823529	0
.99215686	0.73333333			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.	0.	0.	0
.	0.			
0.	0.97647059	0.99215686	0.97647059	0

.25098039	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.18039216	0.50980392	0	
.71764706	0.99215686				
0.99215686	0.81176471	0.00784314	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.15294118	0.58039216				
0.89803922	0.99215686	0.99215686	0.99215686	0	
.98039216	0.71372549				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.09411765	0.44705882	0.86666667	0.99215686	0	
.99215686	0.99215686				
0.99215686	0.78823529	0.30588235	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.	0.	0	
.	0.				
0.	0.	0.09019608	0.25882353	0	
.83529412	0.99215686				
0.99215686	0.99215686	0.99215686	0.77647059	0	
.31764706	0.00784314				

[illegible]

```

.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
.      0.
0.      0.      0.      0.      0
0.      0.      0.      0.      ]

```

In [0]:

```

# here we are having a class number for each image
print("Class label of first image :", y_train[0])

# lets convert this into a 10 dimensional vector
# ex: consider an image is 5 convert it into 5 => [0, 0, 0, 0
, 0, 1, 0, 0, 0, 0]
# this conversion needed for MLPs

Y_train = np_utils.to_categorical(y_train, 10)
Y_test = np_utils.to_categorical(y_test, 10)

print("After converting the output into a vector : ",Y_train[

```

```
0])
```

```
Class label of first image : 5
```

```
After converting the output into a vector : [  
0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]
```

Softmax classifier

In [0]:

```
# https://keras.io/getting-started/sequential-model-guide/

# The Sequential model is a linear stack of layers.
# you can create a Sequential model by passing a list of layer
# instances to the constructor:

# model = Sequential([
#     Dense(32, input_shape=(784,)),
#     Activation('relu'),
#     Dense(10),
#     Activation('softmax'),
# ])

# You can also simply add layers via the .add() method:

# model = Sequential()
# model.add(Dense(32, input_dim=784))
# model.add(Activation('relu'))

###

# https://keras.io/layers/core/

# keras.layers.Dense(units, activation=None, use_bias=True, k
# ernel_initializer='glorot_uniform',
# bias_initializer='zeros', kernel_regularizer=None, bias_reg
# ularizer=None, activity_regularizer=None,
# kernel_constraint=None, bias_constraint=None)

# Dense implements the operation: output = activation(dot(inp
```

```

ut, kernel) + bias) where
# activation is the element-wise activation function passed as
the activation argument,
# kernel is a weights matrix created by the layer, and
# bias is a bias vector created by the layer (only applicable
if use_bias is True).

# output = activation(dot(input, kernel) + bias) => y = activation(WT.X + b)

####

# https://keras.io/activations/

# Activations can either be used through an Activation layer,
or through the activation argument supported by all forward
layers:

# from keras.layers import Activation, Dense

# model.add(Dense(64))
# model.add(Activation('tanh'))

# This is equivalent to:
# model.add(Dense(64, activation='tanh'))

# there are many activation functions available ex: tanh,
relu, softmax

from keras.models import Sequential
from keras.layers import Dense, Activation

```

In [0]:

```

# some model parameters

```



```
output_dim = 10  
input_dim = X_train.shape[1]
```

```
batch_size = 128  
nb_epoch = 20
```

Model 1 : MLP + ReLU + ADAM

In [0]:

```
model_relu = Sequential()
model_relu.add(Dense(324, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.062, seed=None)))
model_relu.add(Dense(108, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.125, seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_relu.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

WARNING: Logging before flag parsing goes to stderr.

W0728 09:41:24.063638 139911311832960 deprecation_wrapper.py:119] From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow_backend.py:74: The name tf.get_default_graph is deprecated. Please use tf.compat.v1.get_default_graph instead.

W0728 09:41:24.105846 139911311832960 deprecation_wrapper.py:119] From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow_backend.py:517: The name tf.placeholder is deprecated.

ated. Please use `tf.compat.v1.placeholder` instead.

W0728 09:41:24.116193 139911311832960 deprecation_wrapper.py:119] From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow_backend.py:4115: The name `tf.random_normal` is deprecated. Please use `tf.random.normal` instead.

W0728 09:41:24.149391 139911311832960 deprecation_wrapper.py:119] From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow_backend.py:4138: The name `tf.random_uniform` is deprecated. Please use `tf.random.uniform` instead.

W0728 09:41:24.171610 139911311832960 deprecation_wrapper.py:119] From /usr/local/lib/python3.6/dist-packages/keras/optimizers.py:790: The name `tf.train.Optimizer` is deprecated. Please use `tf.compat.v1.train.Optimizer` instead.

W0728 09:41:24.203595 139911311832960 deprecation_wrapper.py:119] From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow_backend.py:3295: The name `tf.log` is deprecated. Please use `tf.math.log` instead.

W0728 09:41:24.345604 139911311832960 deprecation.py:323] From /usr/local/lib/python3.6/dist-packages/tensorflow/python/ops/math_grad.py:1250: `add_dispatch_support.<locals>.wrapper` (from `tensorflow.python.ops.array_ops`) is deprecated and will be removed in a future version. Instructions for updating:
Use `tf.where` in 2.0, which has the same broadcast rule as `np.where`

Layer (type)	Output Shape
Param #	

dense_1 (Dense)	(None, 324)
254340	

dense_2 (Dense)	(None, 108)
35100	

dense_3 (Dense)	(None, 10)
1090	

Total params: 290,530
Trainable params: 290,530
Non-trainable params: 0

None
Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] -
6s 108us/step - loss: 0.2468 - acc: 0.9263 -
val_loss: 0.1157 - val_acc: 0.9659
Epoch 2/20
60000/60000 [=====] -
3s 46us/step - loss: 0.0939 - acc: 0.9720 - v
al_loss: 0.0882 - val_acc: 0.9732
Epoch 3/20
60000/60000 [=====] -
3s 46us/step - loss: 0.0610 - acc: 0.9809 - v

```
al_loss: 0.0834 - val_acc: 0.9738
Epoch 4/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0428 - acc: 0.9869 - v
al_loss: 0.0710 - val_acc: 0.9790
Epoch 5/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0298 - acc: 0.9912 - v
al_loss: 0.0791 - val_acc: 0.9756
Epoch 6/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0242 - acc: 0.9925 - v
al_loss: 0.0730 - val_acc: 0.9788
Epoch 7/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0185 - acc: 0.9942 - v
al_loss: 0.0702 - val_acc: 0.9792
Epoch 8/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0152 - acc: 0.9953 - v
al_loss: 0.0994 - val_acc: 0.9744
Epoch 9/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0129 - acc: 0.9960 - v
al_loss: 0.0846 - val_acc: 0.9778
Epoch 10/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0115 - acc: 0.9962 - v
al_loss: 0.0825 - val_acc: 0.9791
Epoch 11/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0110 - acc: 0.9962 - v
al_loss: 0.0720 - val_acc: 0.9799
Epoch 12/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0112 - acc: 0.9962 - v
al_loss: 0.0788 - val_acc: 0.9791
```

```
Epoch 13/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0116 - acc: 0.9960 - v
al_loss: 0.0926 - val_acc: 0.9787
Epoch 14/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0084 - acc: 0.9973 - v
al_loss: 0.0793 - val_acc: 0.9822
Epoch 15/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0103 - acc: 0.9966 - v
al_loss: 0.1006 - val_acc: 0.9769
Epoch 16/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0072 - acc: 0.9976 - v
al_loss: 0.0944 - val_acc: 0.9797
Epoch 17/20
60000/60000 [=====] -
  3s 45us/step - loss: 0.0028 - acc: 0.9990 - v
al_loss: 0.0888 - val_acc: 0.9804
Epoch 18/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0114 - acc: 0.9962 - v
al_loss: 0.1006 - val_acc: 0.9778
Epoch 19/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0087 - acc: 0.9970 - v
al_loss: 0.0947 - val_acc: 0.9809
Epoch 20/20
60000/60000 [=====] -
  3s 46us/step - loss: 0.0071 - acc: 0.9975 - v
al_loss: 0.1004 - val_acc: 0.9794
```

Train Accuracy = 99.75%

In [0]:

```

score = model_relu.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

model_test_score = score[0]
model_test_acc = score[1]
model_train = history.history['acc']

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

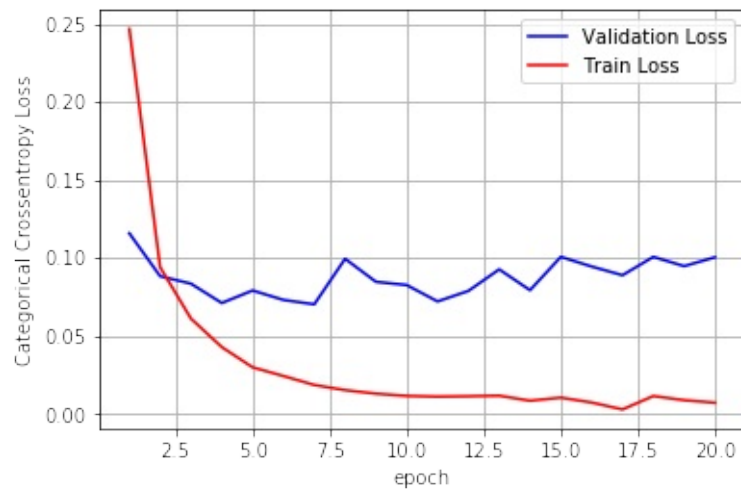
# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)

```

Test score: 0.10035708108908771

Test accuracy: 0.9794



MLP + Batch-Norm on hidden Layers + AdamOptimizer </2>

In [0]:

```
from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_batch.add(Dense(324, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.039, seed=None)))
model_batch.add(BatchNormalization())

model_batch.add(Dense(108, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))

model_batch.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_batch.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] -
5s 86us/step - loss: 0.2165 - acc: 0.9363 - val_loss: 0.1146 - val_acc: 0.9670

Epoch 2/20
60000/60000 [=====] -
4s 74us/step - loss: 0.0809 - acc: 0.9754 - v
al_loss: 0.0907 - val_acc: 0.9728
Epoch 3/20
60000/60000 [=====] -
4s 74us/step - loss: 0.0523 - acc: 0.9845 - v
al_loss: 0.0789 - val_acc: 0.9754
Epoch 4/20
60000/60000 [=====] -
4s 74us/step - loss: 0.0367 - acc: 0.9889 - v
al_loss: 0.0728 - val_acc: 0.9779
Epoch 5/20
60000/60000 [=====] -
4s 75us/step - loss: 0.0301 - acc: 0.9908 - v
al_loss: 0.0721 - val_acc: 0.9773
Epoch 6/20
60000/60000 [=====] -
4s 75us/step - loss: 0.0237 - acc: 0.9928 - v
al_loss: 0.0747 - val_acc: 0.9777
Epoch 7/20
60000/60000 [=====] -
5s 76us/step - loss: 0.0216 - acc: 0.9929 - v
al_loss: 0.0773 - val_acc: 0.9770
Epoch 8/20
60000/60000 [=====] -
5s 76us/step - loss: 0.0137 - acc: 0.9959 - v
al_loss: 0.0931 - val_acc: 0.9746
Epoch 9/20
60000/60000 [=====] -
4s 75us/step - loss: 0.0149 - acc: 0.9955 - v
al_loss: 0.0835 - val_acc: 0.9765
Epoch 10/20
60000/60000 [=====] -
4s 74us/step - loss: 0.0125 - acc: 0.9961 - v
al_loss: 0.0849 - val_acc: 0.9767
Epoch 11/20

```
60000/60000 [=====] -  
  4s 75us/step - loss: 0.0129 - acc: 0.9957 - v  
al_loss: 0.0830 - val_acc: 0.9770  
Epoch 12/20  
60000/60000 [=====] -  
  4s 75us/step - loss: 0.0109 - acc: 0.9966 - v  
al_loss: 0.0794 - val_acc: 0.9797  
Epoch 13/20  
60000/60000 [=====] -  
  4s 74us/step - loss: 0.0086 - acc: 0.9973 - v  
al_loss: 0.0820 - val_acc: 0.9789  
Epoch 14/20  
60000/60000 [=====] -  
  4s 74us/step - loss: 0.0089 - acc: 0.9971 - v  
al_loss: 0.0794 - val_acc: 0.9783  
Epoch 15/20  
60000/60000 [=====] -  
  4s 74us/step - loss: 0.0091 - acc: 0.9968 - v  
al_loss: 0.0839 - val_acc: 0.9790  
Epoch 16/20  
60000/60000 [=====] -  
  4s 74us/step - loss: 0.0086 - acc: 0.9973 - v  
al_loss: 0.0779 - val_acc: 0.9810  
Epoch 17/20  
60000/60000 [=====] -  
  4s 73us/step - loss: 0.0081 - acc: 0.9976 - v  
al_loss: 0.0760 - val_acc: 0.9814  
Epoch 18/20  
60000/60000 [=====] -  
  4s 73us/step - loss: 0.0064 - acc: 0.9979 - v  
al_loss: 0.0837 - val_acc: 0.9792  
Epoch 19/20  
60000/60000 [=====] -  
  4s 74us/step - loss: 0.0081 - acc: 0.9974 - v  
al_loss: 0.0788 - val_acc: 0.9804  
Epoch 20/20  
60000/60000 [=====] -
```

```
4s 75us/step - loss: 0.0071 - acc: 0.9977 - v
al_loss: 0.0793 - val_acc: 0.9802
```

Train Accuracy = 99.77%

In [0]:

```
score = model_batch.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

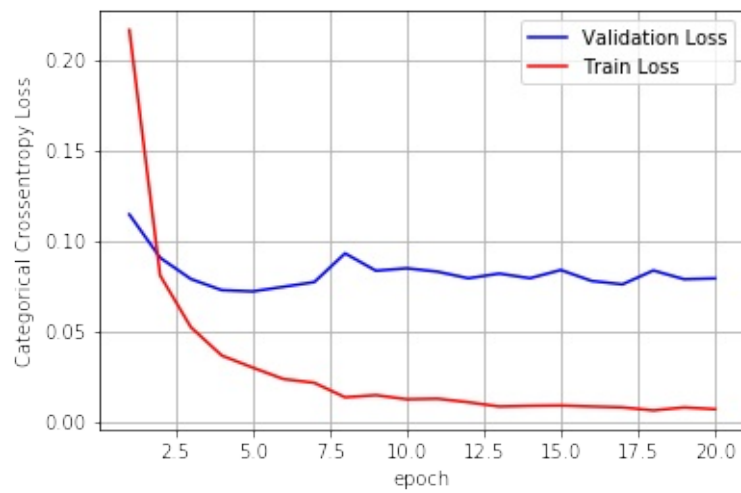
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.07929278038347275

Test accuracy: 0.9802



5. MLP + Dropout + AdamOptimizer

In [0]:

```
# https://stackoverflow.com/questions/34716454/where-do-i-call-the-batchnormalization-function-in-keras
```

```
from keras.layers import Dropout
```

```
model_drop = Sequential()
```

```
model_drop.add(Dense(324, activation='relu', input_shape=(input_dim,), kernel_initializer=RandomNormal(mean=0.0, stddev=0.039, seed=None)))
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(108, activation='relu', kernel_initializer=RandomNormal(mean=0.0, stddev=0.55, seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(output_dim, activation='softmax'))
```

```
model_drop.summary()
```

```
W0728 10:01:15.132498 139911311832960 deprecation.py:506] From /usr/local/lib/python3.6/dist-packages/keras/backend/tensorflow_backend.py:3445: calling dropout (from tensorflow.python.ops.nn_ops) with keep_prob is deprecated and will be removed in a future version.
Instructions for updating:
```

Please use `rate` instead of `keep_prob`. Rate should be set to `rate = 1 - keep_prob`.

Layer (type)	Output Shape
Param #	
=====	
=====	
dense_7 (Dense)	(None, 324)
254340	
=====	
batch_normalization_3 (Batch Normalization)	(None, 324)
1296	
=====	
dropout_1 (Dropout)	(None, 324)
0	
=====	
dense_8 (Dense)	(None, 108)
35100	
=====	
batch_normalization_4 (Batch Normalization)	(None, 108)
432	
=====	
dropout_2 (Dropout)	(None, 108)
0	
=====	
dense_9 (Dense)	(None, 10)
1090	
=====	
=====	

Total params: 292,258
Trainable params: 291,394
Non-trainable params: 864

In [0]:

```
model_drop.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] -
5s 90us/step - loss: 0.5430 - acc: 0.8353 - val_loss: 0.1843 - val_acc: 0.9431

Epoch 2/20

60000/60000 [=====] -
5s 78us/step - loss: 0.2851 - acc: 0.9163 - val_loss: 0.1363 - val_acc: 0.9570

Epoch 3/20

60000/60000 [=====] -
5s 78us/step - loss: 0.2328 - acc: 0.9307 - val_loss: 0.1152 - val_acc: 0.9632

Epoch 4/20

60000/60000 [=====] -
5s 78us/step - loss: 0.2003 - acc: 0.9401 - val_loss: 0.1024 - val_acc: 0.9694

Epoch 5/20

60000/60000 [=====] -
5s 78us/step - loss: 0.1782 - acc: 0.9463 - val_loss: 0.0930 - val_acc: 0.9711

Epoch 6/20


```
60000/60000 [=====] -  
  5s 79us/step - loss: 0.1635 - acc: 0.9519 - v  
al_loss: 0.0893 - val_acc: 0.9735  
Epoch 7/20  
60000/60000 [=====] -  
  5s 78us/step - loss: 0.1514 - acc: 0.9537 - v  
al_loss: 0.0866 - val_acc: 0.9745  
Epoch 8/20  
60000/60000 [=====] -  
  5s 79us/step - loss: 0.1431 - acc: 0.9576 - v  
al_loss: 0.0796 - val_acc: 0.9765  
Epoch 9/20  
60000/60000 [=====] -  
  5s 79us/step - loss: 0.1302 - acc: 0.9608 - v  
al_loss: 0.0728 - val_acc: 0.9777  
Epoch 10/20  
60000/60000 [=====] -  
  5s 79us/step - loss: 0.1252 - acc: 0.9621 - v  
al_loss: 0.0777 - val_acc: 0.9761  
Epoch 11/20  
60000/60000 [=====] -  
  5s 78us/step - loss: 0.1177 - acc: 0.9654 - v  
al_loss: 0.0776 - val_acc: 0.9775  
Epoch 12/20  
60000/60000 [=====] -  
  5s 78us/step - loss: 0.1114 - acc: 0.9659 - v  
al_loss: 0.0704 - val_acc: 0.9788  
Epoch 13/20  
60000/60000 [=====] -  
  5s 78us/step - loss: 0.1067 - acc: 0.9667 - v  
al_loss: 0.0730 - val_acc: 0.9774  
Epoch 14/20  
60000/60000 [=====] -  
  5s 77us/step - loss: 0.1051 - acc: 0.9673 - v  
al_loss: 0.0735 - val_acc: 0.9775  
Epoch 15/20  
60000/60000 [=====] -
```

```
5s 78us/step - loss: 0.0994 - acc: 0.9693 - v
al_loss: 0.0686 - val_acc: 0.9800
Epoch 16/20
60000/60000 [=====] -
5s 78us/step - loss: 0.0928 - acc: 0.9717 - v
al_loss: 0.0639 - val_acc: 0.9805
Epoch 17/20
60000/60000 [=====] -
5s 78us/step - loss: 0.0937 - acc: 0.9712 - v
al_loss: 0.0722 - val_acc: 0.9779
Epoch 18/20
60000/60000 [=====] -
5s 78us/step - loss: 0.0915 - acc: 0.9725 - v
al_loss: 0.0681 - val_acc: 0.9808
Epoch 19/20
60000/60000 [=====] -
5s 78us/step - loss: 0.0827 - acc: 0.9741 - v
al_loss: 0.0636 - val_acc: 0.9808
Epoch 20/20
60000/60000 [=====] -
5s 78us/step - loss: 0.0864 - acc: 0.9739 - v
al_loss: 0.0689 - val_acc: 0.9796
```

Train Accuracy = 97.39%

In [0]:

```
score = model_drop.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

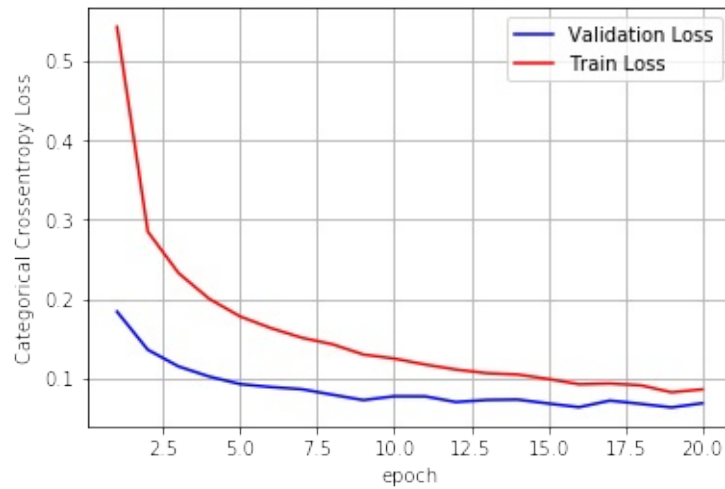
# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
```

```
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.06887181369569152

Test accuracy: 0.9796



Model 2 : (3 Layered) MLP + ReLU + ADAM

In [0]:

```
from keras.initializers import he_normal

model_relu = Sequential()
model_relu.add(Dense(356, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(105, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(51, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_relu.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Layer (type)	Output Shape
Param #	
=====	
=====	
dense_10 (Dense)	(None, 356)
279460	

dense_11 (Dense)	(None, 105)
37485	

dense_12 (Dense)	(None, 51)
5406	

dense_13 (Dense)	(None, 10)
520	

=====
=====

Total params: 322,871
Trainable params: 322,871
Non-trainable params: 0

None

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] -
4s 58us/step - loss: 0.2576 - acc: 0.9233 - val_loss: 0.1140 - val_acc: 0.9645

Epoch 2/20

60000/60000 [=====] -
3s 50us/step - loss: 0.0955 - acc: 0.9711 - val_loss: 0.0838 - val_acc: 0.9735

Epoch 3/20

60000/60000 [=====] -
3s 50us/step - loss: 0.0624 - acc: 0.9809 - val_loss: 0.0809 - val_acc: 0.9764

Epoch 4/20

60000/60000 [=====] -
3s 50us/step - loss: 0.0447 - acc: 0.9861 - v

```
al_loss: 0.0752 - val_acc: 0.9768
Epoch 5/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0322 - acc: 0.9899 - v
al_loss: 0.0765 - val_acc: 0.9768
Epoch 6/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0268 - acc: 0.9914 - v
al_loss: 0.0789 - val_acc: 0.9777
Epoch 7/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0198 - acc: 0.9934 - v
al_loss: 0.0770 - val_acc: 0.9798
Epoch 8/20
60000/60000 [=====] -
  3s 51us/step - loss: 0.0193 - acc: 0.9935 - v
al_loss: 0.0658 - val_acc: 0.9817
Epoch 9/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0163 - acc: 0.9946 - v
al_loss: 0.0895 - val_acc: 0.9768
Epoch 10/20
60000/60000 [=====] -
  3s 51us/step - loss: 0.0133 - acc: 0.9957 - v
al_loss: 0.0781 - val_acc: 0.9804
Epoch 11/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0156 - acc: 0.9949 - v
al_loss: 0.1048 - val_acc: 0.9751
Epoch 12/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0130 - acc: 0.9955 - v
al_loss: 0.0855 - val_acc: 0.9815
Epoch 13/20
60000/60000 [=====] -
  3s 50us/step - loss: 0.0122 - acc: 0.9958 - v
al_loss: 0.0836 - val_acc: 0.9814
```

Epoch 14/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0109 - acc: 0.9965 - v
al_loss: 0.0883 - val_acc: 0.9810
Epoch 15/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0119 - acc: 0.9964 - v
al_loss: 0.0764 - val_acc: 0.9834
Epoch 16/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0084 - acc: 0.9975 - v
al_loss: 0.0988 - val_acc: 0.9787
Epoch 17/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0139 - acc: 0.9954 - v
al_loss: 0.0918 - val_acc: 0.9795
Epoch 18/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0064 - acc: 0.9980 - v
al_loss: 0.0766 - val_acc: 0.9841
Epoch 19/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0035 - acc: 0.9990 - v
al_loss: 0.0956 - val_acc: 0.9806
Epoch 20/20
60000/60000 [=====] -
3s 50us/step - loss: 0.0105 - acc: 0.9964 - v
al_loss: 0.0864 - val_acc: 0.9830

Train Accuracy=99.64%

In [0]:

```
score = model_relu.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

model_test_score = score[0]
model_test_acc = score[1]
model_train = history.history['acc']

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

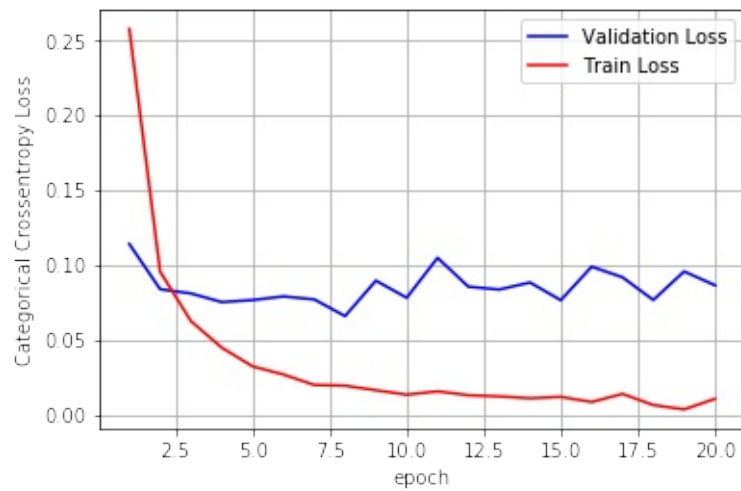
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs
```

```
vy = history.history['val_loss']  
ty = history.history['loss']  
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.08639832216318823

Test accuracy: 0.983



MLP + Batch-Norm on hidden Layers + AdamOptimizer

In [0]:

```
from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_batch.add(Dense(356, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())

model_batch.add(Dense(105, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(51, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))

model_batch.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_batch.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Train on 60000 samples, validate on 10000 samples
Epoch 1/20

```
60000/60000 [=====] -  
  7s 120us/step - loss: 0.2287 - acc: 0.9336 -  
val_loss: 0.1145 - val_acc: 0.9643  
Epoch 2/20  
60000/60000 [=====] -  
  6s 96us/step - loss: 0.0844 - acc: 0.9748 - v  
al_loss: 0.0870 - val_acc: 0.9730  
Epoch 3/20  
60000/60000 [=====] -  
  6s 96us/step - loss: 0.0545 - acc: 0.9829 - v  
al_loss: 0.0788 - val_acc: 0.9755  
Epoch 4/20  
60000/60000 [=====] -  
  6s 95us/step - loss: 0.0419 - acc: 0.9863 - v  
al_loss: 0.0909 - val_acc: 0.9719  
Epoch 5/20  
60000/60000 [=====] -  
  6s 97us/step - loss: 0.0343 - acc: 0.9892 - v  
al_loss: 0.0789 - val_acc: 0.9767  
Epoch 6/20  
60000/60000 [=====] -  
  6s 95us/step - loss: 0.0272 - acc: 0.9913 - v  
al_loss: 0.0786 - val_acc: 0.9774  
Epoch 7/20  
60000/60000 [=====] -  
  6s 97us/step - loss: 0.0231 - acc: 0.9927 - v  
al_loss: 0.0800 - val_acc: 0.9768  
Epoch 8/20  
60000/60000 [=====] -  
  6s 97us/step - loss: 0.0231 - acc: 0.9921 - v  
al_loss: 0.0824 - val_acc: 0.9745  
Epoch 9/20  
60000/60000 [=====] -  
  6s 96us/step - loss: 0.0176 - acc: 0.9942 - v  
al_loss: 0.0811 - val_acc: 0.9785  
Epoch 10/20  
60000/60000 [=====] -
```

```
6s 97us/step - loss: 0.0186 - acc: 0.9938 - v
al_loss: 0.0731 - val_acc: 0.9793
Epoch 11/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0145 - acc: 0.9955 - v
al_loss: 0.0718 - val_acc: 0.9806
Epoch 12/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0154 - acc: 0.9949 - v
al_loss: 0.0859 - val_acc: 0.9774
Epoch 13/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0144 - acc: 0.9953 - v
al_loss: 0.0721 - val_acc: 0.9817
Epoch 14/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0117 - acc: 0.9963 - v
al_loss: 0.0764 - val_acc: 0.9804
Epoch 15/20
60000/60000 [=====] -
6s 98us/step - loss: 0.0128 - acc: 0.9954 - v
al_loss: 0.0859 - val_acc: 0.9783
Epoch 16/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0118 - acc: 0.9957 - v
al_loss: 0.0766 - val_acc: 0.9819
Epoch 17/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0097 - acc: 0.9968 - v
al_loss: 0.0654 - val_acc: 0.9833
Epoch 18/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0088 - acc: 0.9970 - v
al_loss: 0.0834 - val_acc: 0.9814
Epoch 19/20
60000/60000 [=====] -
6s 96us/step - loss: 0.0093 - acc: 0.9967 - v
```

al_loss: 0.0786 - val_acc: 0.9807

Epoch 20/20

60000/60000 [=====] -

6s 96us/step - loss: 0.0119 - acc: 0.9960 - v

al_loss: 0.0803 - val_acc: 0.9810

Train Accuracy=99.60%

In [0]:

```
score = model_batch.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

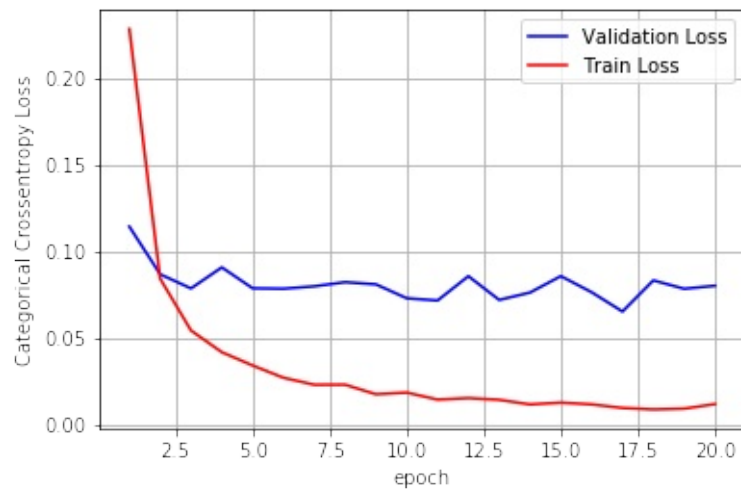
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.08030926787399022

Test accuracy: 0.981



5. MLP + Dropout + AdamOptimizer

In [0]:

```
# https://stackoverflow.com/questions/34716454/where-do-i-call-the-batchnormalization-function-in-keras
```

```
from keras.layers import Dropout
```

```
model_drop = Sequential()
```

```
model_drop.add(Dense(356, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal(seed=None)))
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(105, activation='relu', kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(51, activation='relu', kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(output_dim, activation='softmax'))
```

```
model_drop.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
```

```
history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] -
8s 129us/step - loss: 0.7857 - acc: 0.7579 -
val_loss: 0.2015 - val_acc: 0.9392

Epoch 2/20

60000/60000 [=====] -
6s 101us/step - loss: 0.3409 - acc: 0.9055 -
val_loss: 0.1470 - val_acc: 0.9549

Epoch 3/20

60000/60000 [=====] -
6s 100us/step - loss: 0.2589 - acc: 0.9297 -
val_loss: 0.1247 - val_acc: 0.9633

Epoch 4/20

60000/60000 [=====] -
6s 101us/step - loss: 0.2181 - acc: 0.9413 -
val_loss: 0.1096 - val_acc: 0.9680

Epoch 5/20

60000/60000 [=====] -
6s 101us/step - loss: 0.1917 - acc: 0.9476 -
val_loss: 0.1011 - val_acc: 0.9707

Epoch 6/20

60000/60000 [=====] -
6s 100us/step - loss: 0.1708 - acc: 0.9537 -
val_loss: 0.0910 - val_acc: 0.9740

Epoch 7/20

60000/60000 [=====] -
6s 101us/step - loss: 0.1654 - acc: 0.9554 -
val_loss: 0.0871 - val_acc: 0.9738

Epoch 8/20

60000/60000 [=====] -
6s 100us/step - loss: 0.1531 - acc: 0.9587 -
val_loss: 0.0799 - val_acc: 0.9772

Epoch 9/20

60000/60000 [=====] -
6s 100us/step - loss: 0.1383 - acc: 0.9612 -

```
val_loss: 0.0847 - val_acc: 0.9743
Epoch 10/20
60000/60000 [=====] -
  6s 100us/step - loss: 0.1316 - acc: 0.9635 -
val_loss: 0.0778 - val_acc: 0.9778
Epoch 11/20
60000/60000 [=====] -
  6s 102us/step - loss: 0.1238 - acc: 0.9659 -
val_loss: 0.0838 - val_acc: 0.9760
Epoch 12/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.1150 - acc: 0.9683 -
val_loss: 0.0754 - val_acc: 0.9783
Epoch 13/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.1154 - acc: 0.9688 -
val_loss: 0.0772 - val_acc: 0.9785
Epoch 14/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.1121 - acc: 0.9684 -
val_loss: 0.0745 - val_acc: 0.9796
Epoch 15/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.1095 - acc: 0.9712 -
val_loss: 0.0755 - val_acc: 0.9793
Epoch 16/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.1032 - acc: 0.9716 -
val_loss: 0.0758 - val_acc: 0.9803
Epoch 17/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.1026 - acc: 0.9721 -
val_loss: 0.0680 - val_acc: 0.9811
Epoch 18/20
60000/60000 [=====] -
  6s 101us/step - loss: 0.0977 - acc: 0.9728 -
val_loss: 0.0704 - val_acc: 0.9800
```

Epoch 19/20

60000/60000 [=====] -

6s 101us/step - loss: 0.0926 - acc: 0.9738 -

val_loss: 0.0712 - val_acc: 0.9814

Epoch 20/20

60000/60000 [=====] -

6s 101us/step - loss: 0.0899 - acc: 0.9751 -

val_loss: 0.0664 - val_acc: 0.9822

Train Accuracy=97.51%

In [0]:

```
score = model_batch.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

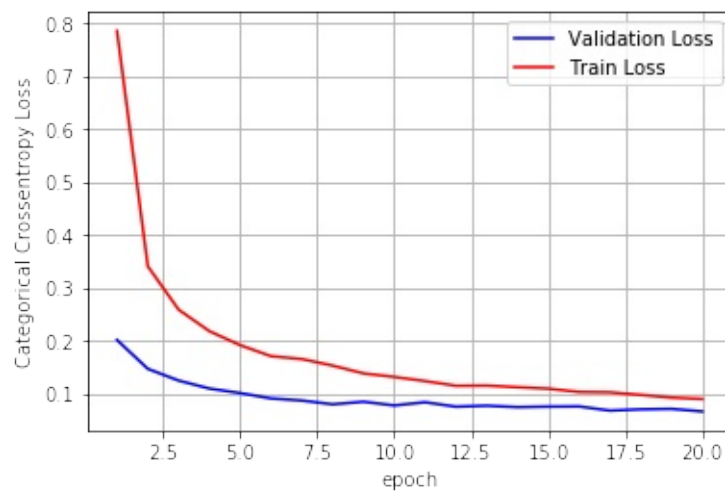
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.08030926787399022

Test accuracy: 0.981



Model 3 : (5 Layered) MLP + ReLU + ADAM

In [0]:

```
from keras.initializers import he_normal

model_relu = Sequential()
model_relu.add(Dense(507, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(312, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(212, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(126, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(84, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_relu.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Layer (type)	Output Shape
Param #	

```

=====
=====
dense_22 (Dense)          (None, 507)
      397995

-----

dense_23 (Dense)          (None, 312)
      158496

-----

dense_24 (Dense)          (None, 212)
      66356

-----

dense_25 (Dense)          (None, 126)
      26838

-----

dense_26 (Dense)          (None, 84)
      10668

-----

dense_27 (Dense)          (None, 10)
      850
=====
=====
Total params: 661,203
Trainable params: 661,203
Non-trainable params: 0

-----

None
Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] -
  5s 87us/step - loss: 0.2285 - acc: 0.9311 - v

```



```
al_loss: 0.1176 - val_acc: 0.9640
Epoch 2/20
60000/60000 [=====] -
  4s 70us/step - loss: 0.0901 - acc: 0.9722 - v
al_loss: 0.0963 - val_acc: 0.9717
Epoch 3/20
60000/60000 [=====] -
  4s 70us/step - loss: 0.0604 - acc: 0.9811 - v
al_loss: 0.0991 - val_acc: 0.9703
Epoch 4/20
60000/60000 [=====] -
  4s 69us/step - loss: 0.0468 - acc: 0.9852 - v
al_loss: 0.0948 - val_acc: 0.9725
Epoch 5/20
60000/60000 [=====] -
  4s 69us/step - loss: 0.0349 - acc: 0.9888 - v
al_loss: 0.0800 - val_acc: 0.9759
Epoch 6/20
60000/60000 [=====] -
  4s 71us/step - loss: 0.0338 - acc: 0.9895 - v
al_loss: 0.0943 - val_acc: 0.9751
Epoch 7/20
60000/60000 [=====] -
  4s 70us/step - loss: 0.0270 - acc: 0.9914 - v
al_loss: 0.0792 - val_acc: 0.9791
Epoch 8/20
60000/60000 [=====] -
  4s 68us/step - loss: 0.0291 - acc: 0.9907 - v
al_loss: 0.0827 - val_acc: 0.9791
Epoch 9/20
60000/60000 [=====] -
  4s 70us/step - loss: 0.0198 - acc: 0.9933 - v
al_loss: 0.0813 - val_acc: 0.9803
Epoch 10/20
60000/60000 [=====] -
  4s 69us/step - loss: 0.0192 - acc: 0.9940 - v
al_loss: 0.0825 - val_acc: 0.9805
```

Epoch 11/20
60000/60000 [=====] -
4s 70us/step - loss: 0.0187 - acc: 0.9942 - v
al_loss: 0.0992 - val_acc: 0.9739
Epoch 12/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0185 - acc: 0.9941 - v
al_loss: 0.0894 - val_acc: 0.9785
Epoch 13/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0154 - acc: 0.9954 - v
al_loss: 0.0873 - val_acc: 0.9803
Epoch 14/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0163 - acc: 0.9949 - v
al_loss: 0.0815 - val_acc: 0.9817
Epoch 15/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0109 - acc: 0.9969 - v
al_loss: 0.1100 - val_acc: 0.9771
Epoch 16/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0127 - acc: 0.9962 - v
al_loss: 0.0869 - val_acc: 0.9807
Epoch 17/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0153 - acc: 0.9952 - v
al_loss: 0.0987 - val_acc: 0.9795
Epoch 18/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0151 - acc: 0.9956 - v
al_loss: 0.0793 - val_acc: 0.9826
Epoch 19/20
60000/60000 [=====] -
4s 69us/step - loss: 0.0097 - acc: 0.9971 - v
al_loss: 0.0884 - val_acc: 0.9817
Epoch 20/20

```
60000/60000 [=====] -  
 4s 68us/step - loss: 0.0091 - acc: 0.9973 - v  
al_loss: 0.0729 - val_acc: 0.9835
```

Train Accuracy=99.73%

In [0]:

```
score = model_batch.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

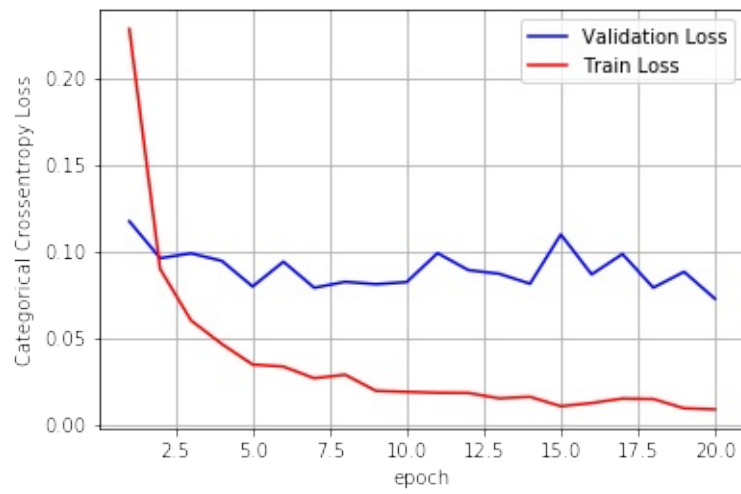
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.08030926787399022

Test accuracy: 0.981



MLP + Batch-Norm on hidden Layers + AdamOptimizer

In [0]:

```
from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_batch.add(Dense(507, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())

model_batch.add(Dense(312, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(212, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(126, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(84, activation='relu', kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))

model_batch.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
```

```
history = model_batch.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))
```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

```
60000/60000 [=====] -  
 11s 183us/step - loss: 0.2148 - acc: 0.9356 -  
 val_loss: 0.1066 - val_acc: 0.9677
```

Epoch 2/20

```
60000/60000 [=====] -  
 9s 147us/step - loss: 0.0867 - acc: 0.9731 -  
 val_loss: 0.0811 - val_acc: 0.9742
```

Epoch 3/20

```
60000/60000 [=====] -  
 9s 148us/step - loss: 0.0587 - acc: 0.9815 -  
 val_loss: 0.0947 - val_acc: 0.9702
```

Epoch 4/20

```
60000/60000 [=====] -  
 9s 149us/step - loss: 0.0505 - acc: 0.9831 -  
 val_loss: 0.0926 - val_acc: 0.9711
```

Epoch 5/20

```
60000/60000 [=====] -  
 9s 149us/step - loss: 0.0407 - acc: 0.9867 -  
 val_loss: 0.0816 - val_acc: 0.9761
```

Epoch 6/20

```
60000/60000 [=====] -  
 9s 148us/step - loss: 0.0365 - acc: 0.9879 -  
 val_loss: 0.0848 - val_acc: 0.9742
```

Epoch 7/20

```
60000/60000 [=====] -  
 9s 149us/step - loss: 0.0302 - acc: 0.9899 -  
 val_loss: 0.0784 - val_acc: 0.9779
```

Epoch 8/20

```
60000/60000 [=====] -
```

```
9s 148us/step - loss: 0.0270 - acc: 0.9908 -  
val_loss: 0.0772 - val_acc: 0.9766  
Epoch 9/20  
60000/60000 [=====] -  
9s 148us/step - loss: 0.0248 - acc: 0.9919 -  
val_loss: 0.0743 - val_acc: 0.9766  
Epoch 10/20  
60000/60000 [=====] -  
9s 150us/step - loss: 0.0207 - acc: 0.9933 -  
val_loss: 0.0866 - val_acc: 0.9755  
Epoch 11/20  
60000/60000 [=====] -  
9s 149us/step - loss: 0.0235 - acc: 0.9920 -  
val_loss: 0.0823 - val_acc: 0.9757  
Epoch 12/20  
60000/60000 [=====] -  
9s 148us/step - loss: 0.0221 - acc: 0.9923 -  
val_loss: 0.0769 - val_acc: 0.9786  
Epoch 13/20  
60000/60000 [=====] -  
9s 148us/step - loss: 0.0178 - acc: 0.9942 -  
val_loss: 0.0716 - val_acc: 0.9803  
Epoch 14/20  
60000/60000 [=====] -  
9s 149us/step - loss: 0.0171 - acc: 0.9946 -  
val_loss: 0.0803 - val_acc: 0.9796  
Epoch 15/20  
60000/60000 [=====] -  
9s 148us/step - loss: 0.0164 - acc: 0.9944 -  
val_loss: 0.0763 - val_acc: 0.9790  
Epoch 16/20  
60000/60000 [=====] -  
9s 149us/step - loss: 0.0158 - acc: 0.9948 -  
val_loss: 0.0727 - val_acc: 0.9808  
Epoch 17/20  
60000/60000 [=====] -  
9s 148us/step - loss: 0.0148 - acc: 0.9951 -
```


val_loss: 0.0857 - val_acc: 0.9761

Epoch 18/20

60000/60000 [=====] -

9s 149us/step - loss: 0.0132 - acc: 0.9955 -

val_loss: 0.0741 - val_acc: 0.9820

Epoch 19/20

60000/60000 [=====] -

9s 149us/step - loss: 0.0128 - acc: 0.9958 -

val_loss: 0.0828 - val_acc: 0.9802

Epoch 20/20

60000/60000 [=====] -

9s 149us/step - loss: 0.0132 - acc: 0.9957 -

val_loss: 0.0658 - val_acc: 0.9831

Train Accuracy=99.57%

In [0]:

```
score = model_batch.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

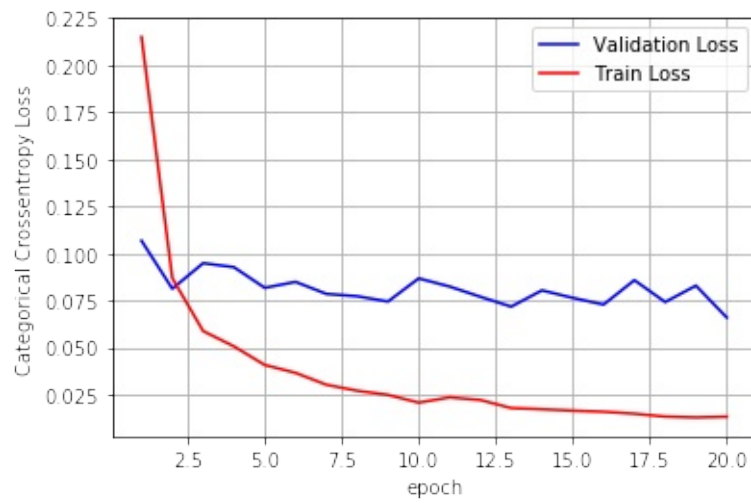
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)
```

Test score: 0.0658267695394985

Test accuracy: 0.9831



5. MLP + Dropout + AdamOptimizer

In [0]:

```
# https://stackoverflow.com/questions/34716454/where-do-i-call-the-batchnormalization-function-in-keras
```

```
from keras.layers import Dropout
```

```
model_drop = Sequential()
```

```
model_drop.add(Dense(507, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal(seed=None)))
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(312, activation='relu', kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(212, activation='relu', kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(126, activation='relu', kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```
model_drop.add(Dropout(0.5))
```

```
model_drop.add(Dense(84, activation='relu', kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
```

```

model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))

model_drop.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

```

60000/60000 [=====] -
 12s 201us/step - loss: 1.1379 - acc: 0.6438 -
 val_loss: 0.2566 - val_acc: 0.9249

```

Epoch 2/20

```

60000/60000 [=====] -
 9s 156us/step - loss: 0.3986 - acc: 0.8896 -
 val_loss: 0.1720 - val_acc: 0.9503

```

Epoch 3/20

```

60000/60000 [=====] -
 9s 156us/step - loss: 0.2922 - acc: 0.9221 -
 val_loss: 0.1444 - val_acc: 0.9598

```

Epoch 4/20

```

60000/60000 [=====] -
 9s 156us/step - loss: 0.2413 - acc: 0.9353 -
 val_loss: 0.1243 - val_acc: 0.9661

```

Epoch 5/20

```

60000/60000 [=====] -
 9s 157us/step - loss: 0.2142 - acc: 0.9436 -
 val_loss: 0.1076 - val_acc: 0.9711

```

Epoch 6/20

```

60000/60000 [=====] -
 9s 157us/step - loss: 0.1873 - acc: 0.9498 -
 val_loss: 0.1088 - val_acc: 0.9701

```

Epoch 7/20
60000/60000 [=====] -
9s 157us/step - loss: 0.1751 - acc: 0.9537 -
val_loss: 0.0930 - val_acc: 0.9749

Epoch 8/20
60000/60000 [=====] -
9s 156us/step - loss: 0.1660 - acc: 0.9568 -
val_loss: 0.0918 - val_acc: 0.9758

Epoch 9/20
60000/60000 [=====] -
9s 157us/step - loss: 0.1491 - acc: 0.9610 -
val_loss: 0.0863 - val_acc: 0.9770

Epoch 10/20
60000/60000 [=====] -
9s 156us/step - loss: 0.1431 - acc: 0.9621 -
val_loss: 0.0819 - val_acc: 0.9774

Epoch 11/20
60000/60000 [=====] -
9s 158us/step - loss: 0.1352 - acc: 0.9640 -
val_loss: 0.0807 - val_acc: 0.9787

Epoch 12/20
60000/60000 [=====] -
9s 157us/step - loss: 0.1289 - acc: 0.9667 -
val_loss: 0.0772 - val_acc: 0.9796

Epoch 13/20
60000/60000 [=====] -
9s 155us/step - loss: 0.1282 - acc: 0.9663 -
val_loss: 0.0778 - val_acc: 0.9804

Epoch 14/20
60000/60000 [=====] -
9s 157us/step - loss: 0.1235 - acc: 0.9679 -
val_loss: 0.0799 - val_acc: 0.9793

Epoch 15/20
60000/60000 [=====] -
9s 157us/step - loss: 0.1164 - acc: 0.9699 -
val_loss: 0.0785 - val_acc: 0.9803

Epoch 16/20

```
60000/60000 [=====] -  
  9s 156us/step - loss: 0.1118 - acc: 0.9702 -  
val_loss: 0.0692 - val_acc: 0.9820  
Epoch 17/20  
60000/60000 [=====] -  
  9s 156us/step - loss: 0.1055 - acc: 0.9726 -  
val_loss: 0.0700 - val_acc: 0.9823  
Epoch 18/20  
60000/60000 [=====] -  
  9s 156us/step - loss: 0.1022 - acc: 0.9731 -  
val_loss: 0.0752 - val_acc: 0.9812  
Epoch 19/20  
60000/60000 [=====] -  
  9s 156us/step - loss: 0.0957 - acc: 0.9750 -  
val_loss: 0.0721 - val_acc: 0.9820  
Epoch 20/20  
60000/60000 [=====] -  
 10s 158us/step - loss: 0.0923 - acc: 0.9760 -  
val_loss: 0.0721 - val_acc: 0.9820
```

Train Accuracy=97.60%

In [0]:

```
score = model_batch.evaluate(X_test, Y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax = plt.subplots(1, 1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1, nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(X_train, Y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(X_test, Y_test))

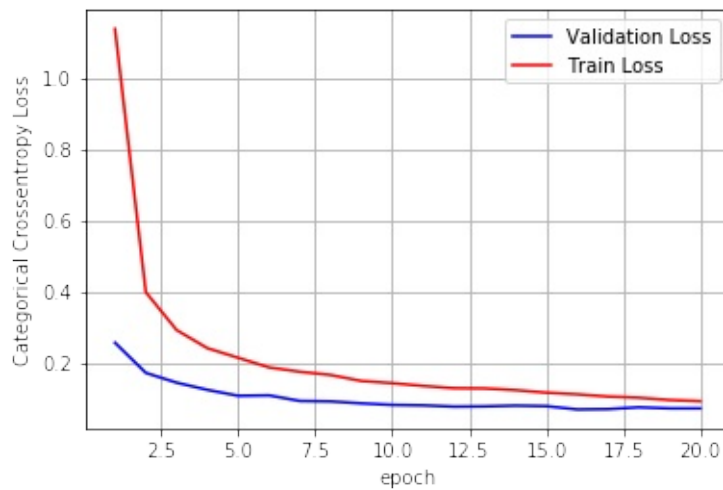
# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy = history.history['val_loss']
ty = history.history['loss']
plt_dynamic(x, vy, ty, ax)
```


Test score: 0.0658267695394985

Test accuracy: 0.9831



In [1]:

```
# Please compare all your models using Prettytable library
# http://zetcode.com/python/prettytable/

from prettytable import PrettyTable

#If you get a ModuleNotFoundError error , install prettytable
using: pip3 install prettytable

x = PrettyTable()
x.field_names = ["Hidden Layers", "Activation Unit", "Optimise
r", "Batch Normalisation", "DropOut(0.5)", " Train Accuracy", "
Test_Accuracy"]
x.add_row(["2", "ReLU", "Adam", "-", "-", "99.75%", "97.94%"])
x.add_row(["2", "ReLU", "Adam", "Yes", "-", "99.77%", "98.02%"])
x.add_row(["2", "ReLU", "Adam", "Yes", "0.5", "97.39%", "97.96%
"])

x.add_row(["2", "ReLU", "Adam", "-", "-", "99.64%", "98.3%"])
x.add_row(["2", "ReLU", "Adam", "Yes", "-", "99.60%", "98.1%"])
x.add_row(["2", "ReLU", "Adam", "Yes", "0.5", "97.51%", "98.1%"])
```

```
)

x.add_row(["2", "ReLU", "Adam", "-", "-", "99.73%", "98.1%"])
x.add_row(["2", "ReLU", "Adam", "Yes", "-", "99.57%", "97.31%"])
x.add_row(["2", "ReLU", "Adam", "Yes", "0.5", "97.60%", "98.31%"])

print(x)
```

```
+-----+-----+-----+
+-----+-----+-----+
-----+-----+
| Hidden Layers | Activation Unit | Optimiser
| Batch Normalisation | DropOut(0.5) | Train
Accuracy | Test_Accuracy |
+-----+-----+-----+
+-----+-----+-----+
-----+-----+
|      2      |      ReLU      |      Adam
|      -      |      -      |      99
.75% |      97.94% |
|      2      |      ReLU      |      Adam
|      Yes     |      -      |      99
.77% |      98.02% |
|      2      |      ReLU      |      Adam
|      Yes     |      0.5     |      97
.39% |      97.96% |
|      2      |      ReLU      |      Adam
|      -      |      -      |      99
.64% |      98.3%   |
|      2      |      ReLU      |      Adam
|      Yes     |      -      |      99
.60% |      98.1%   |
|      2      |      ReLU      |      Adam
|      Yes     |      0.5     |      97
.51% |      98.1%   |
|      2      |      ReLU      |      Adam
```

	-		-		99
.73%		98.1%			
	2		ReLU		Adam
	Yes		-		99
.57%		97.31%			
	2		ReLU		Adam
	Yes		0.5		97
.60%		98.31%			
+-----+-----+-----					
+-----+-----+-----					
-----+-----+					

Conclusion:

1. While using Batch Normalisation and Dropouts together, we get better results with good accuracy.
2. Model 3 with many layers gave better results compared to Model 1 and Model 2.