# FUZZY LOGIC

BY SOUROURAB MANDAL

# INTRODUCTION

- The concept of Fuzzy Logic was conceived by Lotfi Zadeh, a professor at the University of California at Berkley in 1965.
- It was presented not as a control methodology, but as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership.
- Professor Zadeh reasoned that people do not require precise, numerical information input, and yet they are capable of highly adaptive control.
- If feedback controllers could be programmed to accept noisy, imprecise input, they would be much more effective and perhaps easier to implement.

# WHAT IS FUZZY LOGIC?

- It is a generalization of classical logic which manifests in crisp quantities.
- It represents concepts with unclear boundaries.
- Crisp logic deals with crisp sets having sharp boundaries. The inherent logic is Boolean in nature (i.e. either TRUE or FALSE).
- Fuzzy logic deals with fuzzy sets having indistinct boundaries. The inherent logic is multivalued in nature.
- In essence, fuzzy logic (FL) is focused on modes of reasoning which are approximate rather than exact.

# ..WHAT IS FUZZY LOGIC?

- In fuzzy logic, everything, including truth, is or is allowed to be a matter of degree.
- Fuzzy logic has been and still is, though to a lesser degree, an object of controversy.
- For the most part, the controversies are rooted in misperceptions, especially a misperception of the relation between fuzzy logic and probability theory.
- Fuzzy Logic deals with
  - Partial, i.e., a matter of degree information
  - Imprecise (approximate) information
  - Granular (linguistic) information
  - Perception based information

# WHY WE SHOULD USE FUZZY LOGIC?

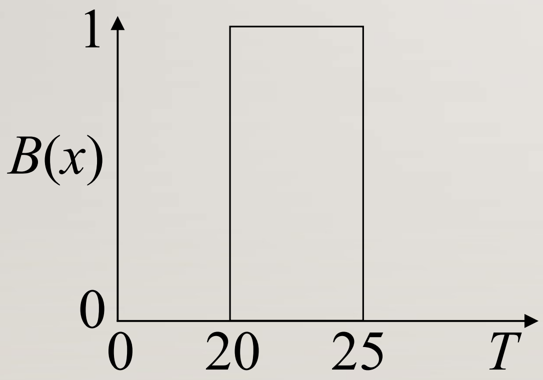
- Systems with uncertainties due to imprecision, vagueness, ambiguity, randomness, partial truth and approximation.
- Black box or gray box systems.

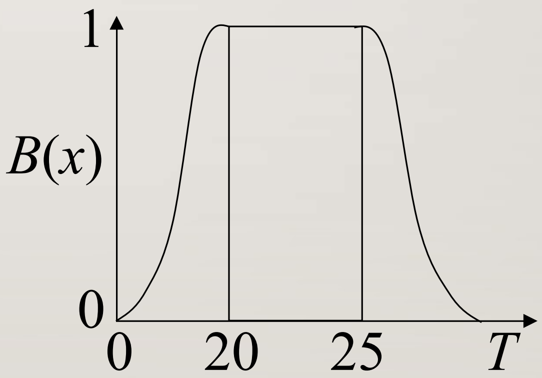
# WHEN WE SHOULD NOT USE FUZZY LOGIC?

- If we are sure that there are no uncertainties due to vagueness, imprecision and ambiguity present.
- White box model.
- Linear systems.
- Systems with moderate non-linearities.
- Systems with moderate complexities.

# FUZZY LOGIC

- Consider the temperature on a sunny day.
- It can either be represented in terms of temperature values e.g. $20^0$-$25^0$C.
- It can also be represented as a "warm" day with temperature, $T \in [20^0, 25^0]$.



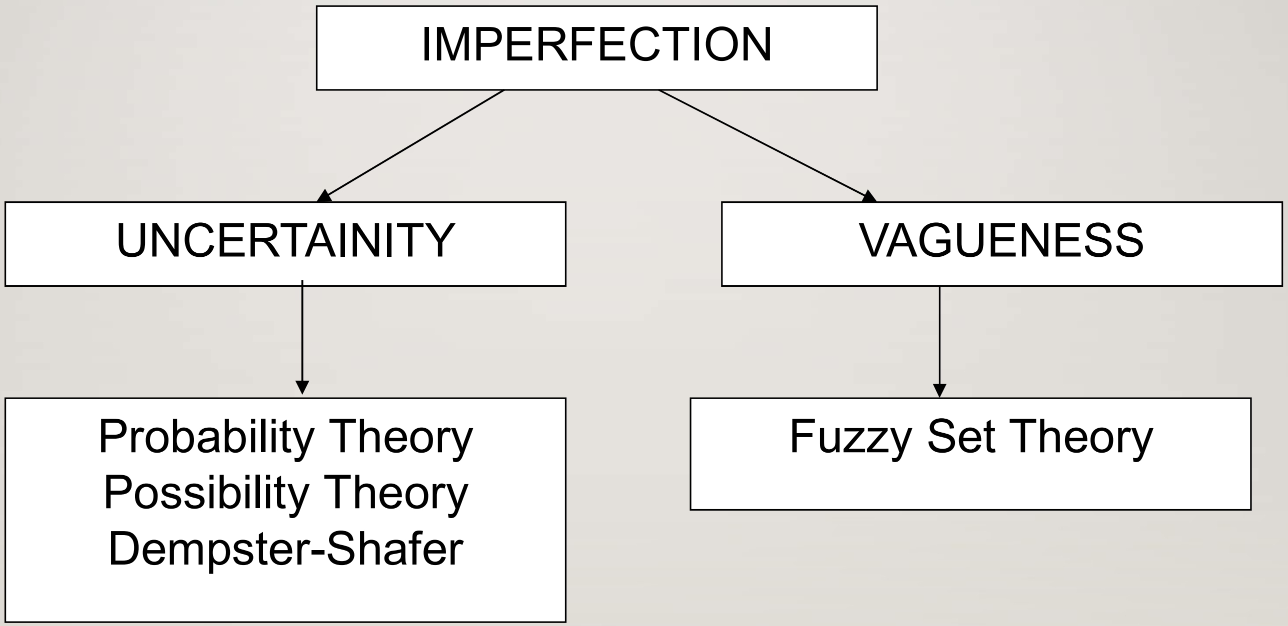
# CONCEPT OF FUZZINESS

- Decreases complexity through abstraction.
- Represents vagueness through qualification.
  - e.g. it is usually warm in the summer vs. usually 26$^0$C.
  - e.g. the weather is sunny meaning 0% cloudy? Or 5% cloudy? Or 10% cloudy? If 15% cloudy, then what about 16% or 15.1%?

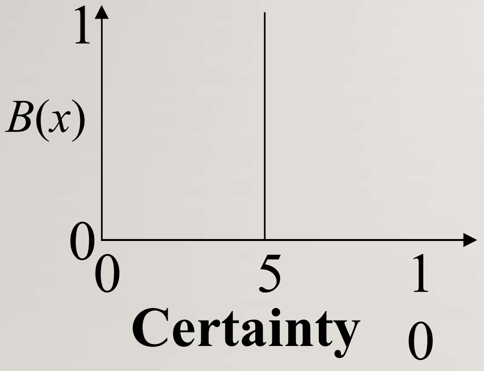
# DEFINITIONS OF FUZZINESS

- Fuzziness measures the degree to which an event occurs, not whether it occurs.
- It exists when the law of non-contradiction [$A \cap \tilde{A} = \Phi$] (or the law of excluded middle [$A \cup \tilde{A} = U$]) is violated.
- Fuzziness primarily describes partial truth or imprecision.
- It means that everything is a matter of degree.
- It is a type of deterministic uncertainty. It deals with the soft meaning of concepts.
- It is related to computing with words, i.e. linguistic variables.

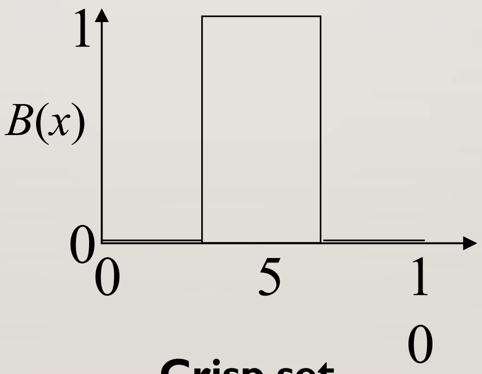
# PROBABILITY AND FUZZY LOGIC



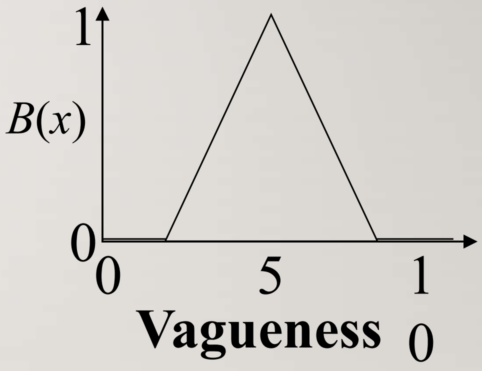
# PROBABILITY AND FUZZY LOGIC




Certainty




Crisp set




Vagueness

# PROBABILITY AND FUZZY LOGIC

| Probability Measure | Membership Function |
| --- | --- |
| Calculates the probability that an ill-known variable x ranging on U hits the well-known set A | Calculates the probability of a well-known variable x ranging on U hits the well-known set A |
| Before an event occurs | After an event occurs |
| Domain is Boolean | Domain is [0.1] |

# PROBABILITY VS FUZZY LOGIC

- Fuzzy ≠ Probability => $\mu_A(x) \neq p_A(x)$
- Both map x to a value in [0, 1].
- $p_A(x)$ measures our knowledge or ignorance of the truth of the event that x belongs to the set A.
  - Probability deals with uncertainty and likelihood.
- $\mu_A(x)$ measures the degree of belongingness of x to set A and there is no interest regarding the uncertainty behind the outcome of the event x.
- Event x has occurred and we are interested in only making observations regarding the degree to which x belongs to A.
  - Fuzzy logic deals with ambiguity and vagueness.

# EXAMPLE

- A bottle of water
- 50% probability of being poisonous means 50% chance.
  - 50% water is clean.
  - 50% water is poisonous.
- 50% fuzzy membership of poisonous means that the water has poison.
  - Water is half poisonous.

# FUZZY LOGIC REPRESENTATION

- FL incorporates a simple, rule-based IF X AND Y THEN Z approach to a solving control problem rather than attempting to model a system mathematically.
- Terms like
  - "IF (process is too cool) AND (process is getting colder) THEN (add heat to the process)"
  - or "IF (process is too hot) AND (process is heating rapidly) THEN (cool the process quickly)"

  are used.

# HOW DOES FUZZY LOGIC WORK?

- FL requires some numerical parameters in order to operate, such as what is considered significant error and significant rate-of-change-of-error, but exact values of these numbers are usually not critical unless very responsive performance is required in which case empirical tuning would determine them.

# FUZZY MEMBERSHIP

- In a crisp set, elements are either strictly contained in the set or are not strictly contained in the set.
- For a crisp set *C*, an element *X* is related as

  $C : X \in \{0,1\}$ implies either accept or reject, the Dichotomy concept.
- A fuzzy set comprises elements with a certain degree of containment in the set, referred to as the membership of that element.
- For a fuzzy set *F*, an element *X* is related as

  $F : X \in [0, 1]$ implies relaxed membership.

# FUZZY SETS

- A fuzzy set A in the universe of discourse X can be defined as a set of ordered pairs and it can be represented mathematically as −

$$\underline{A} = \{(x, \mu_{\underline{A}}(x)) \mid x \in X, \mu_{\underline{A}}(x) \in [0,1]\}$$

$$X = \{x_1, x_2, x_{3,} \ldots, x_n\}$$

Here $\mu_{\underline{A}}(x)$= degree of membership of x in A

# REPRESENTATION OF FUZZY SET

- When universe of discourse is discrete and finite

$$\underline{A} = \{(\frac{\mu_{\underline{A}}(x_i)}{x_i}) \mid x_i \in X\}$$

$$or \quad \underline{A} = \frac{\mu_{\underline{A}}(x_1)}{x_1} + \frac{\mu_{\underline{A}}(x_2)}{x_2} + \frac{\mu_{\underline{A}}(x_3)}{x_3} + \ldots\ldots + \frac{\mu_{\underline{A}}(x_n)}{x_n} = \sum_{i=1}^{n} \frac{\mu_{\underline{A}}(x_i)}{x_i}$$

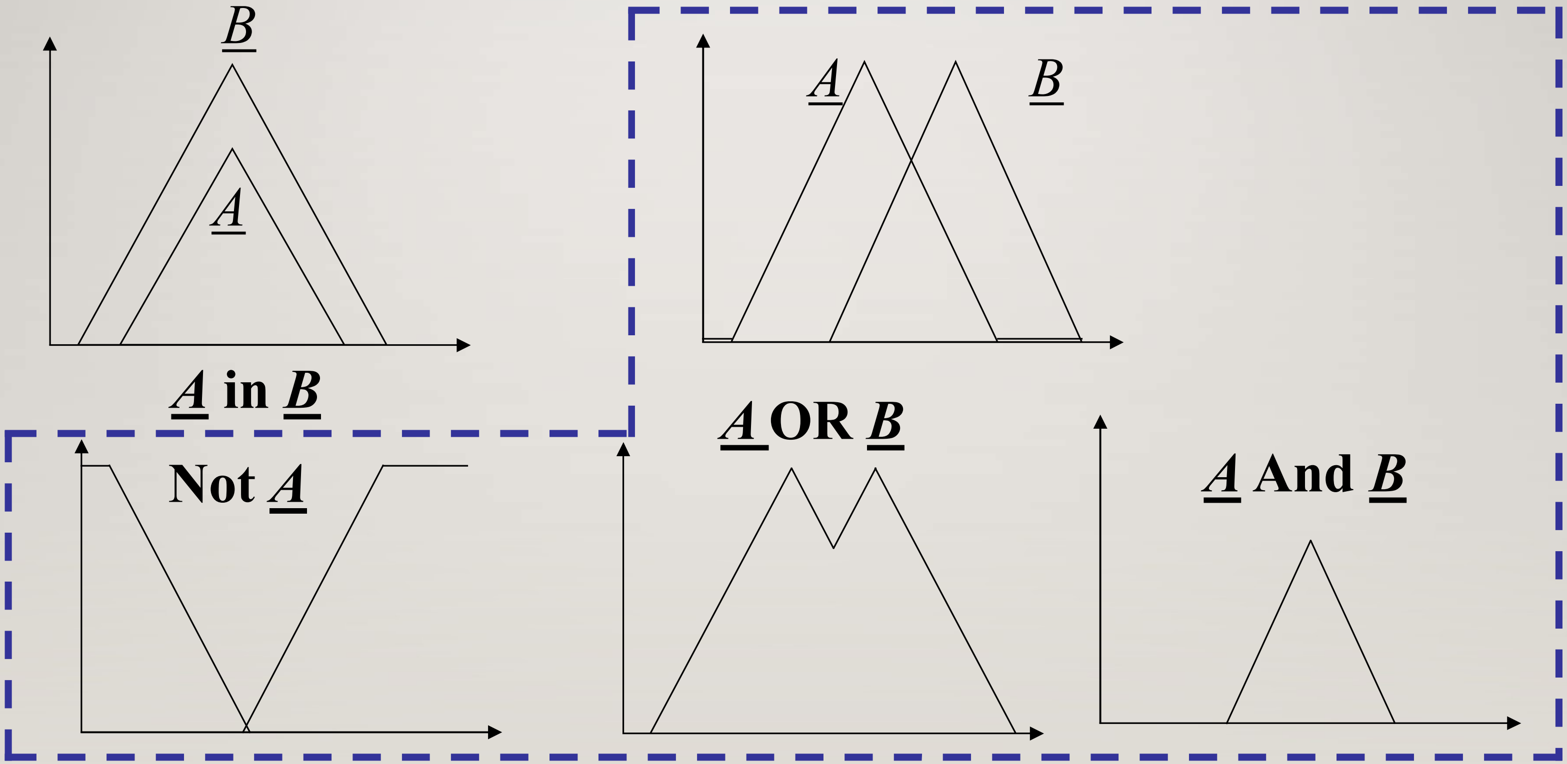
- When universe of discourse is continuous

$$\underline{A}(x) = \int_x \frac{\mu_{\underline{A}}(x)}{x}$$

# SET THEORETIC OPERATIONS ON FUZZY SET

- Cardinality $$|\underline{A}| = \sum_{x \in X} \mu_{\underline{A}}(x)$$
- Subset $$\underline{A} \subseteq \underline{B} \Rightarrow \mu_{\underline{A}} \leq \mu_{\underline{B}}$$
- Complement $$\overline{\underline{A}} = X - \underline{A} \Rightarrow \mu_{\overline{\underline{A}}}(x) = 1 - \mu_{\underline{A}}(x)$$
- Union $$\underline{C} = \underline{A} \cup \underline{B} \Rightarrow \mu_{\underline{C}}(x) = \max(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)) = \mu_{\underline{A}}(x) \vee \mu_{\underline{B}}(x)$$
- Intersection $$\underline{C} = \underline{A} \cap \underline{B} \Rightarrow \mu_{\underline{C}}(x) = \min(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)) = \mu_{\underline{A}}(x) \wedge \mu_{\underline{B}}(x)$$

# SET THEORETIC OPERATIONS ON FUZZY SET

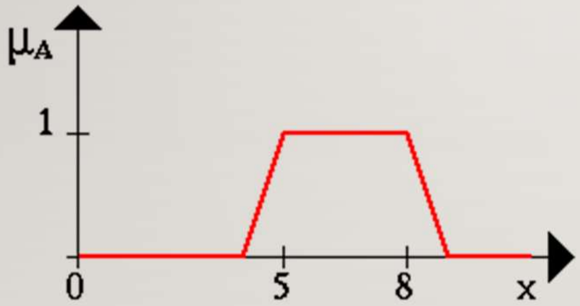


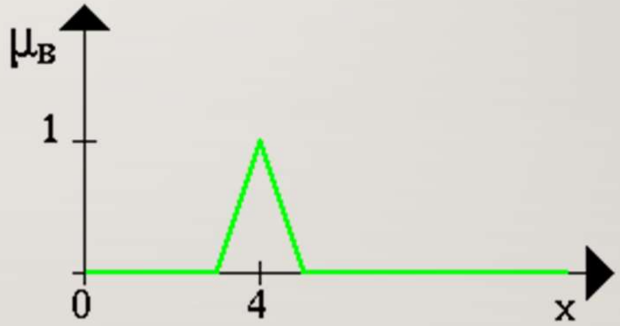
# PROPERTIES OF FUZZY SETS

- Commutativity $\underline{A} \cup \underline{B} = \underline{B} \cup \underline{A}; \quad \underline{A} \cap \underline{B} = \underline{B} \cap \underline{A}$
- Idempotency $\underline{A} \cup \underline{A} = \underline{A}; \quad \underline{A} \cap \underline{A} = \underline{A}$
- Associativity $\underline{A} \cup (\underline{B} \cup \underline{C}) = (\underline{A} \cup \underline{B}) \cup \underline{C}; \quad \underline{A} \cap (\underline{B} \cap \underline{C}) = (\underline{A} \cap \underline{B}) \cap \underline{C}$
- Distributivity $\underline{A} \cup (\underline{B} \cap \underline{C}) = (\underline{A} \cup \underline{B}) \cap (\underline{A} \cup \underline{C}); \quad \underline{A} \cap (\underline{B} \cup \underline{C}) = (\underline{A} \cap \underline{B}) \cup (\underline{A} \cap \underline{C})$
- Involution $\overline{\overline{\underline{A}}} = \underline{A}$
- De Morgan's Law $\overline{\underline{A} \cup \underline{B}} = \overline{\underline{A}} \cap \overline{\underline{B}}; \quad \overline{\underline{A} \cap \underline{B}} = \overline{\underline{A}} \cup \overline{\underline{B}}$
- Law of absorption $\underline{A} \cup (\underline{A} \cap \underline{B}) = \underline{A}; \quad \underline{A} \cap (\underline{A} \cup \underline{B}) = \underline{A}$

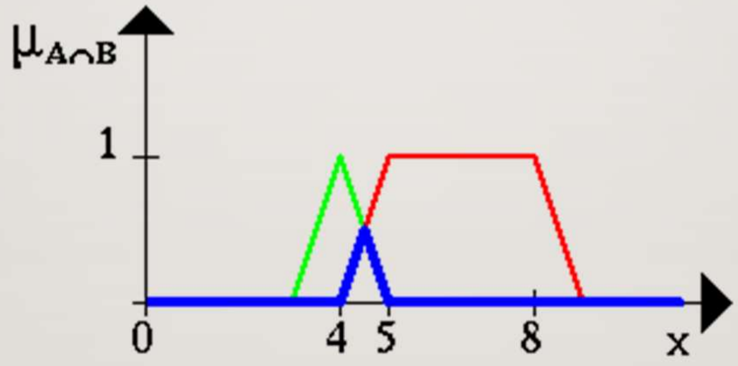
- Let *A* be a fuzzy interval *between* 5 *and* 8 and

  *B* be a fuzzy number *about* 4

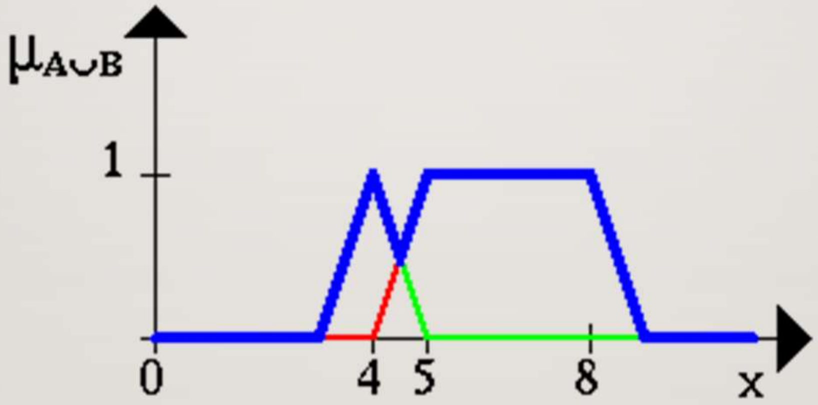


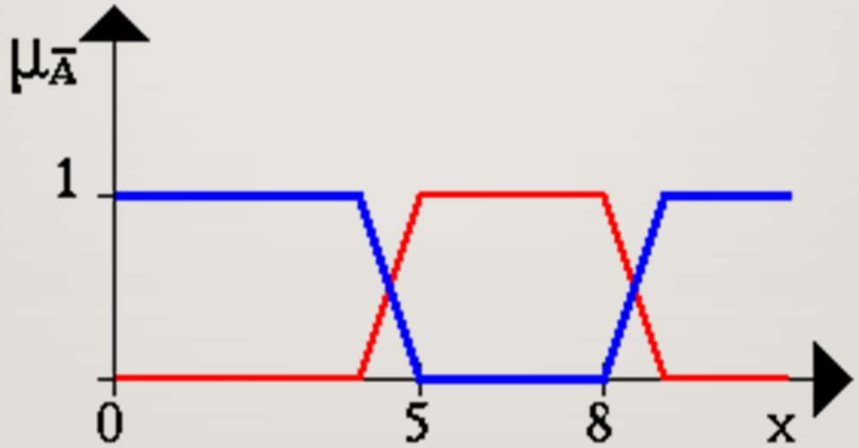
# FUZZY SET *BETWEEN* 5 *AND* 8 **AND** *ABOUT* 4



# THE FUZZY SET *BETWEEN* 5 *AND* 8 **OR** *ABOUT* 4 IS SHOWN IN THE NEXT FIGURE



# NEGATION OF THE FUZZY SET *A*



# THANK YOU