1. Find:

$$\int (x+3)\sqrt{3-4x-x^2}\,dx$$

2. Find the particular solution of differential equation:

$$\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$$

   given that $y = 1$ when $x = 0$.

3. Find the particular solution of the differential equation

$$2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$$

   given that $x = 0$ when $y = 1$.

4. Show that the four points $A\,(4,5,1), B\,(0,-1,-1), C\,(3,9,4)$ and $D\,(-4,4,4)$ are coplanar.

5. Find the coordinates of the foot of perpendicular drawn from the point $A\,(-1,8,4)$ to the line joining the points $B\,(0,-1,3)$ and $C\,(2,-3,-1)$. Hence find the image of the point A in the line BC.

6. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn(without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

7. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

8. Three numbers are selected at randomm (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained.Find the probability distribution of X. Also, find the mean and variance of the distribution.

9. Let $A = R \times R$ and $*$ be a binary operation on A defined by
   $(a,b) * (c,d) = (a+c, b+d)$

Show that $*$ is commutative and associative. Find the identity element for $*$ on A. Also find the inverse of every element $(a, b) \in A$.

10. Prove that

$$y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$$

is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$.

11. show that semi-vertical angle of cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

12. Using the method of integration, find the area of the triangular region whose vertices are $(2, -2), (4, 3)$ and $(1, 2)$.

13. Find the equation of the plane which contains the line of intersection of the planes
$\vec{r}.\left(\hat{i} - 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and
$\vec{r}.\left(-2\hat{i} = \hat{j} + \hat{k}\right) + 5 = 0$
and whose intercept on x-axis is equal to that of on y-axis.

14. A retired person wants to invest an amount of ₹50,000. His broker recommands investing in two type of bonds $'A'$ and $'B'$ yielding 10% and 9% return respectively on the invested amount. He decides to invest at least₹20,000. in bond $'A'$ and at least ₹10,000. in bond $'B'$.He also wants to inverst at least as much in bond $'A'$ as in bond $'B'$.Slove this linear programming problem graphically to maximise his retuns.

15. Using properties of determinants, prove that

$$\begin{vmatrix} (x + y)^2 & zx & zy \\ zx & (z + y)^2 & xy \\ zy & xy & (z + x)^2 \end{vmatrix} = 2xyz(x + y + z)^3$$

16. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$ find k.