

Dreierlei Problem

1) a) $\lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6} = [8] = \lim_{x \rightarrow 6} \frac{(x-6)(x+6)}{(x-6)} = \lim_{x \rightarrow 6} (x+6) = -12$

b) $\lim_{x \rightarrow 1} \frac{x^2 + 1}{x + 1} = \lim_{x \rightarrow 1} \frac{(x+1)(x^2 - x + 1)}{(x+1)} = \lim_{x \rightarrow 1} (x^2 - x + 1) = 1$

c) $\lim_{x \rightarrow 0} \frac{3x^2 + 6x - 1}{8x^2 + 12x + 2} = \frac{3+0-1}{0+0+2} = 0,5$

d) $\lim_{x \rightarrow 1} \frac{x^3 + 3x^2 + 2x}{x^2 - x - 6} = \frac{-1+3+2}{1+1-6} = \frac{0}{-6} = 0$

2) a) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{\frac{1}{3}x} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{\frac{x}{2}}\right)^{\frac{1}{3}x \cdot \frac{x}{2} \cdot \frac{2}{x}} = e^{\lim_{x \rightarrow \infty} -\frac{2}{3}} = e^{-\frac{2}{3}} = e^{\frac{1}{3}}$

b) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^x = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{\frac{x}{2}}\right)^{\frac{1}{2} \cdot \frac{x}{2} \cdot 2} = e^{-\frac{2}{2}} = e^{-1}$

c) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{\frac{1}{2}x} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{\frac{x}{2}}\right)^{\frac{1}{2} \cdot \frac{x}{2} \cdot 2} = e^{\frac{2}{2}} = e^1$

d) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{\frac{1}{2}x} = e^{\frac{2}{2}}$

e) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1}\right)^x = \lim_{x \rightarrow \infty} \left(\frac{x-1+2}{x-1}\right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x-1}\right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{\frac{x-1}{2}}\right)^{\frac{x-1}{2} \cdot \frac{2}{x-1} \cdot x} = e^{\lim_{x \rightarrow \infty} \frac{2}{1}} = e^2$

3) a) $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 8x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot \lim_{x \rightarrow 0} \frac{8x}{\tan 8x} = \frac{5x}{8x} = \frac{5}{8}$

b) $\lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{x^2} = \lim_{x \rightarrow 0} \frac{\cos 5x (1 - \cos 3x)}{x^2} = \frac{\cos 5x \cdot \sin^2 3x \cdot 3}{25x^2} = 1 \cdot \lim_{x \rightarrow 0} \frac{2 \sin 3x \cos 3x \cdot 3}{25x^2} = 25$

2) $\lim_{x \rightarrow 0} \frac{1 - \cos 6x}{1 - \cos 2x} = \frac{1-1}{1-1} = \left[\frac{0}{0}\right] = \lim_{x \rightarrow 0} \frac{2 \sin^2 3x}{2 \sin^2 x} = \lim_{x \rightarrow 0} \frac{\sin^2 3x \cdot x^2 \cdot 9x^2}{\sin^2 x \cdot x^2 \cdot 9x^2} = 1 \cdot 1 \cdot$

$\cdot \lim_{x \rightarrow 0} \frac{9x^2}{x^2} = 1 \cdot 1 \cdot 9 = 9$

a) $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 8x} = \lim_{x \rightarrow 0} \frac{\sin 5x \cdot 5x \cdot 8x}{\tan 8x \cdot 8x \cdot 5x} = 1 \cdot 1 \cdot \lim_{x \rightarrow 0} \frac{5x}{8x} = 1 \cdot 1 \cdot \frac{5}{8} = \frac{5}{8}$