Investcoin: A System for Privacy-Preserving Investments *

Filipp Valovich Email: filipp.valovich@rub.de

Horst Görtz Institute for IT Security Faculty of Mathematics

Ruhr-Universität Bochum, Universitätsstraße 150, 44801 Bochum, Germany Abstract. This work presents a new framework for Privacy-Preserving Investment systems in a distributed model. In this model, independent investors can transfer funds to independent projects, in the same way as it works on crowdfunding platforms. The framework protects the investors' single payments from being detected (by any other party), only the sums of each investor's payments are revealed (e.g to the system). Likewise, the projects single incoming payments are concealed and only the final sums of the incoming payments for every project are revealed. In this way, no other party than the investor (not even the system administration) can detect how much she paid to any single project. Though it is still possible to confidentially exchange any part of an investment between any pair of investors, such that market liquidity is unaffected by the system. On top, our framework allows a privacy-preserving return of a multiple of all the held investments (e.g. interest payments or dividends) to the individual investors while still revealing nothing else than the sum of all returns for every investor. We provide reasonable security guarantees for this framework that are based on common notions from the Secure Multi-Party Computation (SMPC) literature. As an instantiation for this framework we present Investcoin. This is a proper combination of three cryptographic protocols, namely a Private Stream Aggregation scheme, a Commitment scheme and a Range test and it is usable in connection with any existing currency. The security of the three protocols is based on the DDH assumption. Thus, by a composition theorem from the SMPC literature, the security of the resulting Investoin protocol is also based on the DDH assumption. Furthermore, we provide a simple decentralised key generation protocol for Investoin that supports dynamic join, dynamic leave and fault-tolarance of investors and moreover achieves some security guarantees against malicious investors.

1 Introduction

The promise of performance benefit by using technologies like online-outsourcing and cloud-computing goes along with the loss of control over individual data. Therefore the public awareness of data protection increases. We use encryption and privacy technologies to protect our electronic messages, our consumer behaviour or patient records. In this work, we put the following question up for discussion: why is there only minor attention paid to the protection of sensitive *financial data* in the public? Indeed the requirement to trust in financial institutes may be an obstacle for the trade secrecy of companies. On the one hand, transactions on organised markets are registered by electronic systems, audited and eventually get under the control of the system administration (e.g. it can refuse a transaction). In some cases this is desired: e.g. it should be possible to detect a company financing criminal activities. On the

 $^{^{\}star}$ The research was supported by the DFG Research Training Group GRK 1817/1

other hand, we would like to protect the trade secrecy of the companies. In this sense, there is a transparency/confidentiality trade-off in organised markets, such as exchange or to some extent also crowdfunding platforms.

In this work we address the problem of providing adequate privacy guarantees to investors. As observed by Nofer [18], although there is no observable significant effect concerning "the impact of privacy violations on the investment amount, (...) one has to remember that trust influences behavior (...) and privacy issues influences trust (...) and therefore an indirect influence still exists". Conversely, this means that individuals would participate more in investments if their privacy is proteted. As an effect, the reporting of investors concerning wins and losses (and therefore risks) becomes more reliable [14,9,3]. As further motivation of our work, the possibility to circumvent certain regulatories, such as financial sanctions by order of "repressive" countries, may be desired. Investors may look for a way to invest in sanctioned companies without being traced by their home country.

Consequently, the objective of this work is to solve privacy issues by concealing particular investment decisions but offering transparency of "aggregated" investment decisions. In this regard we introduce a Distributed Investment Encryption (DIE) scheme for the aggregation of the investments of a number of different investors funding different projects on an electronic platform. A DIE scheme maintains market liquidity, i.e. the scheme does not affect the possibility to trade assets among investors. Its cryptographic security is set through the definition of the Privacy-Preserving Investment (PPI) system. Informally, a PPI system conceals the single payments of investors from being traced but reveals to the system only the aggregates of the investors' payments. Similarly, the projects' single incoming payments are concealed and only the final sums of the incoming payments for every project are revealed. Moreover, a PPI system conceals the single returns (representing interest payments, coupons or dividends) from every single project to every single investor but reveals (to the system administration) the aggregated return of every single investor. Therefore, up to a certain extent, a PPI system simultaneously maintains transparency (e.g. taxes on the final return of every investor can be raised) and trade secrecy. As a particular PPI system we present Investcoin, a combination of three cryptographic protocols: a Private Stream Aggregation (PSA) scheme, first introduced by Shi et al. [23], a (homomorphic) Commitment scheme and a Range test for committed values, all secure under the Decisional Diffie-Hellman assumption. Informally, the PSA scheme is used for the secure aggregation of funds for every particular project and the homomorphic Commitment scheme is used for the secure aggregation of all investments and returns of every particular investor. The Range test ensures that investments are not negative. We provide a simple secret sharing key generation protocol for Investoin, that allows investors to dynamically join, leave or fail during the protocol execution and prevents investors from some malicious cheating.

Related work. The notion of Commitment schemes (first in [4,6]) is well-established in the literature. The notion of PSA was introduced in [23]. A PSA scheme is a cryptographic protocol which enables a number of users to individually and securely send encrypted time-series data to an untrusted aggregator requiring each user to send exactly one message per time-step. The aggregator is able to decrypt the aggregate of all data per time-step, but cannot retrieve any further information about the individual data. In [23] a security definition for PSA and a secure instantiation were provided. Joye and Libert [15] provided a scheme with a tighter security reduction and Benhamouda et al. [2] generalised the scheme in [15]. By lowering the security requirements established in [23], Valovich and Aldà [25] provided general conditions

for the existence of secure PSA schemes, based on key-homomorphic weak PRFs. Investorin is not a classical cryptocurrency. It can be thought of as a cryptographic layer on top of any currency used for investments, similar to what Zerocoin is intended to be for Bitcoin. Bitcoin is the first cryptocurrency, introduced by Satoshi Nakamoto [17]. Currently it is the cryptocurrency with the largest market capitalisation. Bitcoin is as a peer-to-peer payment system where transactions are executed directly between users without interaction of any intermediate party. The transactions are verified by the users of the network and publicly recorded in a blockchain, a distributed database. Zerocoin was proposed by Miers et al. [16] as an extension for Bitcoin (or any other cryptocurrency) providing cryptographic anonymity to recorded transactions in the blockchain (Bitcoin itself provides only pseudonymity). This is achieved by the use of a seperate mixing procedure based on Commitment schemes. Therefore particular transactions cannot be publicly traced back to particular Bitcoin adresses anymore. This is also the main principle of a PPI system: no investment in a particular project can be traced back to a particular investor. In this regard Investoin has similarities with Zerocoin.

Methods for market regulation through aggregated privacy-preserving risk reporting were studied by Abbe et al. [1]. They constructed protocols allowing a number of users to securely compute aggregated risk measures based on summations and inner products. Flood et al. [12] considered balancing transparency and confidentiality for financial regulation by investigating cryptographic tools for statistical data privacy.

2 Preliminaries

In this section we provide the description of our investment model, the basic protocols underlying Investoin and their corresponding security definitions.

2.1 Model

As initial situation we consider a network consisting of n investors and λ projects to be funded by the investors. As an analogy from the real world one can think of a crowdfunding platform or an exchange system where projects or companies try to collect funds from various individual investors. Each investor N_i , i = 1, ..., n, is willing to invest the amount $x_{i,j} \geq 0$ to the project P_j , $j = 1, ..., \lambda$, thus the total amount invested by N_i is $\sum_{j=1}^{\lambda} x_{i,j}$ and the total amount received by project P_j is $\sum_{i=1}^{n} x_{i,j}$. Moreover, there exists an administration (which may be the administration of the crowdfunding platform). The investors and the project managements are not required to trust the administration.

We consider a series of investment rounds. An investment round denotes the moment when the payments of all participating investors are registered by the administration of the system. From round to round the values n and λ may change, i.e. investors and projects may join or leave the network before any round.

After an investment round is over and the time comes to give a return to the investors (i.e. at maturity), the management of each project P_j publishes some value α_j defining the return for each investor (i.e. an indicator of economic growth, interest yield, dividend yield or similar). The untrusted system administration (or simply system) serves as a pool for the distribution of investments to the projects and of returns to the investors: first, for all $i=1,\ldots,n$ it collects the total amount $\sum_{j=1}^{\lambda} x_{i,j}$ invested by investor N_i and rearranges the union of the total amounts into the aggregated investment $\sum_{i=1}^{n} x_{i,j}$ for project P_j for all $j=1,\ldots,\lambda$; at maturity date, for all

 $j=1,\ldots,\lambda$ it collects the total returns $\alpha_j \sum_{i=1}^n x_{i,j}$ of the projects and rearranges the union of the total returns into the returns $\sum_{j=1}^{\lambda} \alpha_j x_{i,j}$ of the investors.

While the investors do not have to trust each other nor the system (i.e. an investor doesn't want the others to know her financial data), we consider a honest-but-curious model where the untrusted system administration tries to compromise investors to build a coalition. This coalition tries to infer additional information about uncompromised investors, but under the constraint of honestly following the investment protocol. On the other hand, we allow investors that are not part of the coalition, to execute some malicious behaviour, that we will concretise later. Thereby we have the following objectives in each investment round:

Security

- Hiding: For all $i=1,\ldots,n$ the only financial data of investor N_i (if uncompromised) known to the system is $C_i = \sum_{j=1}^{\lambda} x_{i,j}$ and $E_i = \sum_{j=1}^{\lambda} \alpha_j x_{i,j}$. For all $j=1,\ldots,\lambda$ the only financial data of project P_j known to the system is $X_j = \sum_{i=1}^n x_{i,j}$ and α_j . Particularly, the single investments of N_i to P_1,\ldots,P_{λ} should remain concealed.
- Binding: Investors may not announce an incorrect investment, i.e. if N_i has send $x_{i,j}$ to P_j , then P_j should also receive $x_{i,j}$ from N_i .
- For all i, j it holds that $x_{i,j} \ge 0$, i.e. no investor can 'steal' money from a project.

Correctness

- For all i, if C_i is the real aggregate of N_i 's investments, then $C_i = \sum_{j=1}^{\lambda} x_{i,j}$, the system knows C_i and can charge the bank account of N_i with amount C_i .
- For all j, if X_j is the real aggregate of P_j 's funds, then $X_j = \sum_{i=1}^n x_{i,j}$, the system knows X_j and transfers the amount X_j to the bank account of P_j .
- For all i, if E_i is the real aggregate of N_i 's returns, then $E_i = \sum_{j=1}^{\lambda} \alpha_j x_{i,j}$, the system knows E_i and transfers the amount E_i to the bank account of N_i .
- If one of these conditions is violated (e.g. on the purpose of stealing money), then the injured party should be able to detect this fact and to prove it to the network latest after the end of the corresponding investment round.

Now we provide the building blocks for a scheme satisfying these objectives.

2.2 Private Stream Aggregation

In this section, we define Private Stream Aggregation (PSA) and provide a security definition. The notion of PSA was introduced by Shi et al. [23].

The definition of Private Stream Aggregation. A PSA scheme is a protocol for safe distributed time-series data transfer which enables the receiver (here: the system administrator) to learn nothing else than the sums $\sum_{i=1}^{n} x_{i,j}$ for $j=1,2,\ldots$, where $x_{i,j}$ is the value of the *i*th participant in (time-)step j and n is the number of participants (here: investors). Such a scheme needs a key exchange protocol for all n investors together with the administrator as a precomputation, and requires each investor to send exactly one message (namely the amount to spend for a particular project) in each step $j=1,2,\ldots$

Definition 1 (Private Stream Aggregation [23]) Let κ be a security parameter, \mathcal{D} a set and $n, \lambda \in \mathbb{N}$ with $n = poly(\kappa)$ and $\lambda = poly(\kappa)$. A Private Stream Aggregation (PSA) scheme $\Sigma = (\mathsf{Setup}, \mathsf{PSAEnc}, \mathsf{PSADec})$ is defined by three ppt algorithms:

Setup: $(pp, T, s_0, s_1, \dots, s_n) \leftarrow Setup(1^{\kappa})$ with public parameters $pp, T = \{t_1, \dots, t_{\lambda}\}$ and secret keys s_i for all i = 1, ..., n.

PSAEnc: For $t_j \in T$ and all i = 1, ..., n: $c_{i,j} \leftarrow \mathsf{PSAEnc}_{s_i}(t_j, x_{i,j})$ for $x_{i,j} \in \mathcal{D}$.

PSADec: Compute $\sum_{i=1}^{n} x'_{i,j} = \mathsf{PSADec}_{s_0}(t_j, c_{1,j}, \dots, c_{n,j})$ for $t_j \in T$ and ciphers $c_{1,j}, \dots, c_{n,j}$. For all $t_j \in T$ and $x_{1,j}, \dots, x_{n,j} \in \mathcal{D}$ the following holds:

$$\mathsf{PSADec}_{s_0}(t_j, \mathsf{PSAEnc}_{s_1}(t_j, x_{1,j}), \dots, \mathsf{PSAEnc}_{s_n}(t_j, x_{n,j})) = \sum_{i=1}^n x_{i,j}.$$

The system parameters pp are public and constant for all t_i with the implicit understanding that they are used in Σ . Every investor encrypts her amounts $x_{i,j}$ with her own secret key s_i and sends the ciphertext to the administrator. If the administrator receives the ciphertexts of all investors for some t_j , it can compute the aggregate of the investors' data using the decryption key s_0 .

While in [23], the $t_i \in T$ were considered to be time-steps within a time-series (e.g. for analysing time-series data of a smart meter), in our work the $t_i \in T$ are associated with projects P_j , $j = 1, ..., \lambda$, to be funded in a particular investment round.

Security of Private Stream Aggregation. Our model allows an attacker to compromise investors. It can obtain auxiliary information about the values of investors or their secret keys. Even then a secure PSA scheme should release no more information than the aggregates of the uncompromised investors' values.

Definition 2 (Aggregator Obliviousness [23]) Let κ be a security parameter. Let \mathcal{T} be a ppt adversary for a PSA scheme $\Sigma = (Setup, PSAEnc, PSADec)$ and let \mathcal{D} be a set. We define a security game between a challenger and the adversary \mathcal{T} .

Setup. The challenger runs the Setup algorithm on input security parameter κ and returns public parameters pp. public encryption parameters T with $|T| = \lambda =$ $poly(\kappa)$ and secret keys s_0, s_1, \ldots, s_n . It sends κ, pp, T, s_0 to \mathcal{T} .

Queries. T is allowed to query $(i, t_j, x_{i,j})$ with $i \in \{1, ..., n\}, t_j \in T, x_{i,j} \in D$ and the challenger returns $c_{i,j} \leftarrow \mathsf{PSAEnc}_{s_i}(t_j, x_{i,j})$. Moreover, \mathcal{T} is allowed to make compromise queries $i \in \{1, ..., n\}$ and the challenger returns s_i .

Challenge. \mathcal{T} chooses $U \subseteq \{1, \ldots, n\}$ such that no compromise query for $i \in U$ was made and sends U to the challenger. \mathcal{T} chooses $t_{j*} \in T$ such that no encryption query with t_{j*} was made. (If there is no such t_{j*} then the challenger simply aborts.) \mathcal{T} queries two different tuples $(x_{i,j*}^{[0]})_{i\in U}, (x_{i,j*}^{[1]})_{i\in U}$ with

$$\sum_{i \in U} x_{i,j*}^{[0]} = \sum_{i \in U} x_{i,j*}^{[1]}.$$

The challenger flips a random bit $b \leftarrow_R \{0,1\}$. For all $i \in U$ the challenger returns $c_{i,j*} \leftarrow \mathsf{PSAEnc}_{s_i}(t_{j*}, x_{i,j*}^{[b]})$. Queries. \mathcal{T} is allowed to make the same type of queries as before restricted to

encryption queries with $t_j \neq t_{j*}$ and compromise queries for $i \notin U$.

Guess. \mathcal{T} outputs a guess about b.

The adversary wins the game if it correctly guesses b. A PSA scheme achieves Aggregator Obliviousness (AO) or is secure if no ppt adversary \mathcal{T} has more than negligible advantage (with respect to the parameter κ) in winning the above game.

Encryption queries are made only for $i \in U$, since knowing the secret key for all $i \notin U$ the adversary can encrypt a value autonomously. If encryption queries in time-step t_{j*} were allowed, then no deterministic scheme would be secure. The adversary \mathcal{T} can determine the original data of all $i \notin U$, since it knows $(s_i)_{i\notin U}$. Then \mathcal{T} can compute the sum $\sum_{i\in U} x_{i,j} = \mathsf{PSADec}_{s_0}(t_j, c_{1,j}, \ldots, c_{n,j}) - \sum_{i\notin U} x_{i,j}$ of the uncompromised investors' values. If there is an investor's cipher which \mathcal{T} does not receive, then it cannot compute the sum for the corresponding t_i .

It is also possible to define AO in the non-adaptive model as in [25,24]: here an adversary may not compromise investors adaptively, but has to specify the coalition U of compromised investors before making any query.

Feasibility of AO. In the random oracle model we can achieve AO for some constructions [23,2,25]. Because of its simplicity and efficient decryption, we use the PSA scheme proposed in [25] and present it in Figure 1. It achieves AO based on the DDH assumption.

PSA scheme

Setup: The public parameters are a primes $q > m \cdot n, p = 2q + 1$ and a hash function $H: T \to \mathcal{QR}_{p2}$ modelled as a random oracle. The secret keys are $s_0, \ldots, s_n \leftarrow_R \mathbb{Z}_{pq}$ with $\sum_{i=0}^n s_i = 0 \mod pq$. PSAEnc: For $t_j \in T$ and $i = 1, \ldots, n$, encrypt $x_{i,j} \in [-m,m]$ by $c_{i,j} \leftarrow (1+p \cdot x_{i,j}) \cdot H(t_j)^{s_i} \mod p^2$. PSADec: For $t_j \in T$ and ciphers $c_{1,j}, \ldots, c_{n,j}$ compute $V_j \in \{1-p \cdot mn, \ldots, 1+p \cdot mn\}$ with

$$V_j \equiv H(t_j)^{s_0} \cdot \prod_{i=1}^n c_{i,j} \equiv \prod_{i=1}^n (1 + p \cdot x_{i,j}) \equiv 1 + p \cdot \sum_{i=1}^n x_{i,j} \text{ mod } p^2$$

and compute $\sum_{i=1}^{n} x_{i,j} = (V_j - 1)/p$ over the integers (this holds if the $c_{i,j}$ are encryptions of the $x_{i,j}$)

Fig. 1. PSA scheme secure in the random oracle model.

The scheme proposed in [23] is similar to the one in Figure 1, but is inefficient, if the range of possible decryption outcomes in is super-polynomially large in the security parameter. The scheme in Figure 1 also achieves the non-adaptive version of AO in the standard model.

2.3 Commitment schemes

A Commitment scheme allows a party to publicly commit to a value such that the value cannot be changed after it has been committed to (binding) and the value itself stays hidden to other parties until the owner reveals it (hiding). For the basic definitions we refer to [4] and [6]. Here we just recall the Pedersen Commitment introduced in [19] (Figure 2), which is computationally binding under the dlog assumption and perfectly hiding. In Section 3 we will combine the Pedersen Commitment with the PSA scheme from Figure 1 for the construction of Investcoin and thereby consider the input data $x = x_{i,j}$ to the Commitment scheme as investment amounts from investor N_i to project P_j . An essential property for the construction of Investcoin is that the Pedersen Commitment contains a homomorphic commitment algorithm, i.e. $Com_{pk}(x,r) * Com_{pk}(x',r') = Com_{pk}(x+x',r+r')$.

2.4 Range test

To allow the honest verifier to verify in advance, that the (possibly malicious) prover commits to a an integer x in a certain range, a Range test must be applied. Range

Commitment scheme

GenCom: (pp, pk) \leftarrow GenCom(1^{κ}), with public parameters pp describing a cyclic group G of order q and public key $pk = (h_1, h_2)$ for two generators h_1, h_2 of G. (where $\text{dlog}_{h_1}(h_2)$ is not known to the committing party).

Com: For $x \in \mathbb{Z}_q$ choose $r \leftarrow_R \mathbb{Z}_q$ and compute

 $com = \mathsf{Com}_{pk}(x, r) = h_1^x \cdot h_2^r \in G.$

Unv: For $pk = (h_1, h_2)$, commitment com and opening (x, r) it holds that

 $\mathsf{Unv}_{pk}(com, x, r) = 1 \Leftrightarrow h_1^x \cdot h_2^r = com.$

Fig. 2. The Pedersen Commitment.

tests were studied in [5,7,20,21]. For Investcoin, an interactive proof procedure can be applied. It is a combination of the Pedersen Commitment to the binary representation of x and the extended Schnorr proof of knowledge [22] (Figure 3) applied to proving knowledge of one out of two secrets as described by Cramer [10]. Its basic idea was described by Boudot [5].

Let W(x) be a set of witnesses for x. We define the following relation.

$$R = \{(x, w) \mid x \in L, w \in W(x)\}.$$

An interactive proof procedure has the following properties. First, it is complete, meaning that if $(x,w) \in R$ and the (possibly malicious) prover P knows the witness w for x, then it is able to convince the honest verifier V (i.e. V honestly follows the procedure). Second, it is sound, meaning that if $(x,w) \notin R$, then no ppt algorithm P is able to convince the honest V except with negligible probability. For a proof of knowledge, instead of soundness we need $special\ soundness$, i.e. there exists a ppt algorithm that extracts the witness w given a pair of different accepting conversations on the same input x such that $(x,w) \in R$. Thus, no prover without knowledge of the witness can convince the verifier. Third, for a proof of knowledge, we need the $special\ honest\ verifier\ zero\ knowledge\ property$, meaning that there exists a ppt algorithm that on input x outputs an accepting conversation with the same probability distribution as the conversations between P and a honest V on input x. Thus, the verifier is not able to distinguish the witness, if it acts honestly. For the extended Schnorr proof in Figure 3, completeness, special soundness and special honest-verifier zero-knowledge hold as can be shown by similar arguments as in [10].

By the interactive procedure from Figure 4, the prover shows to the verifier, that the committed value x lies in the interval $[0, 2^l - 1]$ without revealing anything else about x. For the security of the construction in Figure 3 we refer to [10], where a more general protocol was considered (particularly the special honest verifier zero-knowledge property is needed). We use the Fiat-Shamir heuristic [11] to make the Range test non-interactive.

2.5 Secure Computation

Our security definition for Investcoin will be twofold. In the first part, we consider a honest-but-curious coalition consisting of the untrusted system administration together with its compromised investors and the group of honest investors. Here we refer to notions from Secure Multi-Party Computation (SMPC). In the second part, we identify reasonable malicious behaviour that an investor could possibly execute and show, how the system can be secured against such malicious investors.

In this Section we focus on defining security against the honest-but-curious coalition.

Extended Schnorr Proof of Knowledge

Let G be a cyclic group of order q and h a generator of G. The prover wants to show to the verifier that she knows the discrete logarithms of either $R = h^r$ or $S = h^s$ without revealing its value and position. W.l.o.g. the prover knows r.

Com: The prover chooses random $v_2, w_2, z_1 \leftarrow_R \mathbb{Z}_q^*$ and sends $a_1 = h^{z_1}, a_2 = h^{w_2} \cdot S^{-v_2}$ to the verifier.

Chg: The verifier sends a random $v \leftarrow_R \mathbb{Z}_q^*$ to the prover. Opn: The prover sends $v_1 = v - v_2, w_1 = z_1 + v_1 r, v_2$ and w_2 to the verifier. Chk: The verifier verifies that $v = v_1 + v_2$ and that $h^{w_1} = a_1 R^{v_1}, h^{w_2} = a_2 S^{v_2}$.

Fig. 3. Schnorr Proof of Knowledge of one out of two secrets.

Range test

 $\textbf{Gen:} \ (\mathsf{pp}, pk) \leftarrow \mathsf{Gen}(1^\kappa), \ \text{with public parameters } \mathsf{pp} \ \text{describing the cyclic group} \ G \ \text{of order} \ q, \ \text{a test range}$

[0, 2^l-1] \subset G and public key pk=(g,h) for generators g,h of G (the prover does not know $dlog_g(h)$). **Com:** For $x=\sum_{k=0}^{l-1}x^{(k)}\cdot 2^k$ with $x^{(k)}\in\{0,1\}$ for all $k=0,\ldots,l-1$, the prover chooses random $r^{(k)},v_2^{(k)},w_2^{(k)},z_1^{(k)}\leftarrow_R \mathbb{Z}_q^*, k=0\ldots,l-1$, computes $r\equiv\sum_{k=0}^{l-1}r^{(k)}\cdot 2^k$ mod q and sends

$$\begin{split} com &= g^x \cdot h^r \bmod q, \ com^{(k)} = g^{x^{(k)}} \cdot h^{r^{(k)}} \bmod q \\ & a_1^{(k)} = h^{z_1^{(k)}} \\ & a_2^{(k)} = h^{w_2^{(k)}} \cdot \left(h^{r^{(k)}} g^{2x^{(k)} - 1}\right)^{-v_2^{(k)}} \end{split}$$

for $k = 0, \ldots, l-1$ to the verifier.

Chg: The verifier verifies that $com \equiv \prod_{k=0}^{l-1} (com^{(k)})^{2^k} \mod q$. The verifier sends random $v^{(k)} \leftarrow_R \mathbb{Z}_q^*$ for all $k=0\ldots,l-1$ to the prover.

Opn: The prover sends $v_1^{(k)} = v^{(k)} - v_2^{(k)}, w_1^{(k)} = z_1^{(k)} + v_1^{(k)} r^{(k)}, v_2^{(k)}$ and $w_2^{(k)}$ for all $k=0\ldots,l-1$ to

 $\begin{array}{l} \text{Opn: The prover senus } v_1 = v = v_2 = v_1 = 1 \\ \text{the verifier.} \\ \text{Chk: For all } k = 0 \dots, l-1, \text{ the verifier verifies that } v^{(k)} = v_1^{(k)} + v_2^{(k)} \text{ and that either } \\ \left(h^{w_1^{(k)}}, h^{w_2^{(k)}}\right) = \left(a_1^{(k)} (com^{(k)})^{v_1^{(k)}}, a_2^{(k)} (com^{(k)} \cdot g^{-1})^{v_2^{(k)}}\right) \text{ or } \\ \left(h^{w_1^{(k)}}, h^{w_2^{(k)}}\right) = \left(a_1^{(k)} (com^{(k)} \cdot g^{-1})^{v_1^{(k)}}, a_2^{(k)} (com^{(k)})^{v_2^{(k)}}\right). \end{array}$

Fig. 4. Range test for a committed value.

Definition 3 Let κ be a security parameter and $n, \lambda \in \mathbb{N}$ with $n = poly(\kappa)$. Let ρ be a protocol executed by a group of size $u \leq n$ and a coalition of honest-but-curious adversaries of size n - u + 1 for computing the deterministic functionality f_{ρ} . The protocol ρ performs a secure computation (or securely computes f_{ρ}), if there exists a ppt algorithm \mathcal{S} , such that $\{\mathcal{S}(1^{\kappa}, y, f_{\rho}(x, y))\}_{x,y,\kappa} \approx_{c} \{\text{view}^{\rho}(x, y, \kappa)\}_{x,y,\kappa}$, where $\text{view}^{\rho}(x, y, \kappa) = (y, r, m)$ is the view of the coalition during the execution of the protocol ρ on input (x, y), x is the input of the group, y is the input of the coalition, r is its random tape and m is its vector of received messages.

This definition follows standard notions from SMPC (as in [13]) and is adapted to our environment: first, we consider two-party protocols where each party consists of multiple individuals (each individual in a party has the same goals) and second, we do not consider security of the coalition against the group, since the system administration has no input and thus its security against honest-but-curious investors is trivial. Rather we will later consider its security against malicious investors. Since Investcoin is the combination of various protocols, we will prove the security of these protocols separately and then use the composition theorem by Canetti [8].

Theorem 1 (Composition Theorem in the Honest-but-Curious Model [8]) Let κ be a security parameter and let $m = poly(\kappa)$. Let π be a protocol that computes a functionality f_{π} by making calls to a trusted party computing the functionalities f_1, \ldots, f_m . Let ρ_1, \ldots, ρ_m be protocols computing the functionalities f_1, \ldots, f_m respectively. Denote by $\pi^{\rho_1, \ldots, \rho_m}$ the protocol π , where the calls to a trusted party are replaced by executions of ρ_1, \ldots, ρ_m . If $\pi, \rho_1, \ldots, \rho_m$ non-adaptively perform secure computations, then also $\pi^{\rho_1, \ldots, \rho_m}$ non-adaptively performs a secure computation.

3 Investcoin

In this section, we define Distributed Investment Encryption (DIE) and introduce Investcoin. This protocol is build from a combination of the PSA scheme from Figure 1, the homomorphic Commitment scheme from Figure 2 and the Range test from Figure 4. Moreover, we provide a simple key-generation protocol for the Investcoin protocol that allows the dynamic join and leave of investors and is fault-tolerant towards investors.

3.1 Definition of DIE

A DIE scheme is a SMPC protocol that allows a group of n investors to aggregate their investments for a set of λ projects in a way that only the sums of the investments for every single project and the sums of the investments of every single investor are revealed. In particular, no sum of investments from a set of investors of size smaller than n to any set of projects is revealed. Additionally, each investor may receive back a return for her particular investments in a way that only the sum of her returns are revealed.

Definition 4 (Distributed Investment Encryption) Let κ be a security parameter and $n, \lambda \in \mathbb{N}$ with $n = \mathsf{poly}(\kappa)$ and $\lambda = \mathsf{poly}(\kappa)$. A Distributed Investment Encryption (DIE) scheme $\Omega = (\mathsf{Setup}, \mathsf{DIEEnc}, \mathsf{DIECom}, \mathsf{DIETes}, \mathsf{DIEUnvPay}, \mathsf{DIEDec}, \mathsf{DIEUnvRet})$ is defined by the following ppt algorithms:

Setup: $(pp, T, sk_0, sk_1, ..., sk_n, pk_1, ..., pk_n) \leftarrow Setup(1^{\kappa})$ with public parameters $pp, T = \{t_1, ..., t_{\lambda}\}$, secret keys $sk_0, sk_1, ..., sk_n$ and public keys $pk_1, ..., pk_n$.

DIEEnc: For all $j = 1, ..., \lambda$ and i = 1, ..., n, choose $x_{i,j} \in \mathcal{D}$ (from some finite set \mathcal{D} parameterised by pp) and compute:

$$c_{i,j} \leftarrow \mathsf{DIEEnc}_{sk_i}(t_j, x_{i,j}).$$

DIECom: For all $j = 1, ..., \lambda$ and i = 1, ..., n, choose $r_{i,j} \leftarrow \mathcal{U}(R)$ (i.e. a value chosen uniformly at random from some finite set R parameterised by pp) and compute

$$(com_{i,j}, \tilde{c}_{i,j}) \leftarrow \mathsf{DIECom}_{pk_i, sk_i}(x_{i,j}, r_{i,j}).$$

DIETes: For all $j = 1, ..., \lambda$ and i = 1, ..., n: for a public key pk_i and data value $x_{i,j}$, compute

$$b_{T,i,j} \leftarrow \mathsf{DIETes}_{pk_i}(x_{i,j}), \ b_{T,i,j} \in \{0,1\}.$$

DIEUnvPay: For all i = 1, ..., n: for a public commitment key pk_i , commitment values $com_{i,1}, ..., com_{i,\lambda}$, a data value C_i and an opening value D_i , compute

$$b_{P,i} \leftarrow \mathsf{DIEUnvPay}_{pk_i} \left(\prod_{i=1}^{\lambda} com_{i,j}, C_i, D_i \right), \, b_{P,i} \in \{0,1\}.$$

DIEDec: For all $j = 1, ..., \lambda$, ciphertexts $c_{1,j}, ..., c_{n,j}$, compute

$$X'_j = \mathsf{DIEDec}_{sk_0}(t_j, c_{1,j}, \dots, c_{n,j}).$$

DIEUnvRet: Generate a public return factor α_j for every $j \in \{1, ..., \lambda\}$. For all i = 1, ..., n: for a public commitment key pk_i , commitment values $com_{i,1}, ..., com_{i,\lambda}$, a data value E_i and an opening value F_i , compute

$$b_{R,i} \leftarrow \mathsf{DIEUnvRet}_{pk_i} \left(\prod_{j=1}^{\lambda} com_{i,j}^{\alpha_j}, E_i, F_i \right), \ b_{R,i} \in \{0,1\}.$$

The system parameters pp are public and constant for all $t_i \in T$. Every investor N_i encrypts her investment amount $x_{i,j}$ for project P_j with the secret key sk_i in the DIEEnc algorithm and sends the ciphertext to the system administrator. If the system administrator receives the ciphertexts from all investors for some t_j , it can compute the sum of the investment amounts for P_i using the decryption key sk_0 in the decryption algorithm DIEDec. After having computed the overall investment amounts for every project, the system administrator needs to know how much each investor has invested in total in order to collect this total amount from the investor's bank account. Each investor N_i claims to the system administrator her total amount C_i invested in all the projects P_1, \ldots, P_{λ} together. In order to prove that $C_i = \sum_{j=1}^{\lambda} x_{i,j}, N_i$ sends a DIE commitment to the system administrator with the DIECom algorithm, where the commitment value $com_{i,j}$ is generated using the public key and the commitment cipher $\tilde{c}_{i,j}$ is generated using the secret key. The system administrator verifies that a combination of all commitments of each investor is valid for C_i with the payment verification algorithm DIEUnvPay. Furthermore, DIETes is executed in order to prove that the amounts are larger than or equal 0. At maturity, for each $j = 1, \ldots, \lambda$, each investor N_i should receive back a multiple $\alpha_j x_{i,j}$ of her amount invested in project P_j (e.g. a ROI). The factor α_j is publicly released by the management of project P_j and may denote a rate of return, interest or similar. Each investor N_i claims to the system administrator her total amount E_i to receive back from all projects P_1, \ldots, P_{λ} together. In order to prove that $E_i = \sum_{j=1}^{\lambda} \alpha_j x_{i,j}$, for all $i = 1, \ldots, n$, the system administrator verifies that a combination of all commitments provided previously during the commitment phase (see DIECom algorithm) by each investor together with the corresponding return factors $\alpha_1, \ldots, \alpha_{\lambda}$ is valid for E_i with the return verification algorithm DIEUnvRet.

3.2 Construction of Investoin

The DIESet algorithm in Figure 5 executes the Setup algorithms of the underlying schemes. Additionally, DIESet generates a verification parameter β_i for each project P_i (and an additional β_0 - this will be used for the security against malicious investors) which is only known to the system administration. In Section 3.3 we provide a simple protocol for generating the secrets. The encryption algorithm DIEEnc executes the encryption algorithm of Σ and encrypts the amounts invested by N_i into P_j . In order to prove that $C_i = \sum_{j=1}^{\lambda} x_{i,j}$, the N_i execute the commitment algorithm DIECom committing to the amounts $x_{i,j}$ invested using the randomness $r_{i,j}$ by executing the commitment algorithm of Γ and encrypting the $r_{i,j}$ with Σ . The Range test algorithm DIETes ensures that the investments are larger or equal 0. The payment verification algorithm DIEUnvPay first verifies that the combination of the committed amounts in the correct order is valid for the same combination of amounts encrypted in the correct order by executing the verification algorithm of Γ . If the investor has not cheated, this verification will output 1 by the homomorphy of Γ and the fact that $\prod_{j=0}^{\lambda} H(t_j)^{\beta_j} = \prod_{j=1}^{\lambda} H(\tilde{t}_j)^{\beta_j} = 1$. The DIEUnvPay algorithm verifies that the combination of commitments is valid for the aggregate C_i of the investments of N_i . The decryption algorithm DIEDec then decrypts the aggregated amounts for every project by executing the decryption algorithm of Σ . After the projects are realised, each investor N_i should receive back a multiple $\alpha_i x_{i,i}$ of her amount invested in each project P_j (e.g. a ROI). The factor α_j is publicly released by the management of project P_j and denotes a rate of return, interest or similar. This value is equal for every investor, since only the investor's stake should determine how much her profit from that project is. If the first check in the DIEUnvPay algorithm has output 1, the return verification algorithm DIEUnvRet verifies that the combination of commitments and return factors is valid for the claimed aggregate E_i of the returns to receive by N_i .

We emphasize the low communication effort needed after the DIESet algorithm: every investor sends the messages for DIEEnc, DIECom, DIETes and DIEUnvPay in one shot to the system, later only the messages for DIEUnvRet have to be sent. Thus, there are only two rounds of communication between the investors and the system.

Theorem 2 (Correctness of Investcoin) Let Ω be the protocol in Figure 5. Then the following properties hold.

1. For all $j = 1, ..., \lambda$ and $x_{1,j}, ..., x_{n,j} \in [0, m]$:

$$\mathsf{DIEDec}_{sk_0}(t_j,\mathsf{DIEEnc}_{sk_1}(t_j,x_{1,j}),\ldots,\mathsf{DIEEnc}_{sk_n}(t_j,x_{n,j})) = \sum_{i=1}^n x_{i,j}.$$

2. For all i = 1, ..., n and $x_{i,1}, ..., x_{i,\lambda} \in [0, m]$:

$$\begin{split} \mathsf{DIEUnvPay}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}, \sum_{j=1}^{\lambda} x_{i,j}, \sum_{j=1}^{\lambda} r_{i,j} \right) &= 1 \\ \Leftrightarrow &\exists \ (\tilde{c}_{i,1}, \dots, \tilde{c}_{i,\lambda}) \ : \ (com_{i,j}, \tilde{c}_{i,j}) \leftarrow \mathsf{DIECom}_{pk, sk_i} (x_{i,j}, r_{i,j}) \ \forall \ j = 1, \dots, \lambda. \end{split}$$

3. For all i = 1, ..., n, public return factors $\alpha_1, ..., \alpha_{\lambda}$ and $x_{i,1}, ..., x_{i,\lambda} \in [0, m]$:

$$\begin{split} \mathsf{DIEUnvRet}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}^{\alpha_{j}}, \sum_{j=1}^{\lambda} \alpha_{j} x_{i,j}, \sum_{j=1}^{\lambda} \alpha_{j} r_{i,j} \right) &= 1 \\ \Leftrightarrow &\exists \left(\tilde{c}_{i,1}, \dots, \tilde{c}_{i,\lambda} \right) : \left(com_{i,j}, \tilde{c}_{i,j} \right) \leftarrow \mathsf{DIECom}_{pk,sk_{i}} \left(x_{i,j}, r_{i,j} \right) \forall \, j = 1, \dots, \lambda. \end{split}$$

Investcoin

Let κ be a security parameter and $n, \lambda \in \mathbb{N}$ with $n = \mathsf{poly}(\kappa)$ and $\lambda = \mathsf{poly}(\kappa)$. Let $\Sigma = (\mathsf{Setup}, \mathsf{PSAEnc}, \mathsf{PSADec})$ be the PSA scheme from Figure 1 and let $\Gamma = (\mathsf{GenCom}, \mathsf{Com}, \mathsf{Unv})$ be the Commitment scheme from Figure 2. We define Investoin $\Omega = (\mathsf{DIESet}, \mathsf{DIEEnc}, \mathsf{DIECom}, \mathsf{DIECom}, \mathsf{DIEDec}, \mathsf{DIEUnvPay}, \mathsf{DIEDec}, \mathsf{DIEUnvPay})$ as follows.

- The system administrator generates public parameters $pp = \{q, p, H\}$ with primes $q > m \cdot n, p = 2q + 1$ (where m = 1 2^l-1 is the maximum possible amount to invest into a single project by an investor, $l=\mathsf{poly}(\kappa)$) and parameters $T=\{t_0,t_1,\ldots,t_\lambda\},\ \tilde{T}=\{\tilde{t}_1,\ldots,\tilde{t}_\lambda\}.$
- The system administrator generates a pair of generators $(h_1, h_2) \in \mathbb{Z}_p^* \times \mathbb{Z}_p^*$ with $\operatorname{ord}(h_1) = \operatorname{ord}(h_2) = pq$, sets the public key $pk = (\tilde{T}, h_1, h_2)$ (which is the same for all investors) and defines a hash function $H: T \cup \tilde{T} \to \mathcal{QR}_{p^2}$.
- The system administrator and the investors together generate secret keys s_0 and $sk_i = (s_i, \hat{s}_i) \leftarrow_R \mathbb{Z}_{pq} \times \mathbb{Z}_{pq}$ for all $i = 1, \ldots, n$ with $s_0 \equiv -\sum_{i=1}^n s_i \bmod pq$.

 The system administrator generates secret parameters $\beta_0, \ldots, \beta_\lambda \leftarrow_R [-q', q'], q' < q/(m\lambda)$,
- such that $\prod_{j=0}^{\lambda} H(t_j)^{\beta j} \equiv \prod_{j=1}^{\lambda} H(\tilde{t}_j)^{\beta j} \equiv 1 \mod p^2$ (see Section 3.3 for the details). It sets
- $sk_0 = (s_0, \beta_0, \dots, \beta_{\lambda}).$ DIEEnc: For all $j = 1, \dots, \lambda$, each investor N_i chooses $x_{i,j} \in [0, m]$ and $x_{i,0} = 0$ and for all $j = 0, \dots, \lambda$, N_i sends the following ciphertexts to the system administrator:

$$c_{i,j} \leftarrow \mathsf{DIEEnc}_{sk_i}(t_j, x_{i,j}) = \mathsf{PSAEnc}_{s_i}(t_j, x_{i,j}).$$

 $\textbf{DIECom:} \ \text{For all} \ j=1,\ldots,\lambda, \ \text{each} \ N_i \ \text{chooses} \ r_{i,j} \leftarrow_R [0,m] \ \text{and sends the following to the system administrator:}$

$$(com_{i,j},\tilde{c}_{i,j}) \leftarrow \mathsf{DIECom}_{pk,sk_i}(x_{i,j},r_{i,j}) = (\mathsf{Com}_{pk}(x_{i,j},r_{i,j}),\mathsf{PSAEnc}_{\tilde{s}_i}(\tilde{t}_j,r_{i,j})).$$

DIETes: For all $j=1,\ldots,\lambda$, the algorithm from Figure 4 on $(x_{i,j},r_{i,j})$ in \mathcal{QR}_{p^2} is executed between the system administrator (as verifier) and each investor N_i (as prover) using pk to compute $b_{T,i,j} \in \{0,1\}$, where $b_{T,i,j} = 1$, if the proof is accepted and $b_{T,i,j} = 0$, if not. Note that there always exists a random representation $r_{i,j}^{(0)}, \ldots, r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)}$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j} \equiv 1$, $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, and $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, and $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, such that $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, and $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$, and $r_{i,j}^{(l-1)} \leftarrow_R [0,m]$. $\sum_{k=0}^{l-1} r_{i,j}^{(k)} \cdot 2^k \mod pq.$ DIEUnvPay: The system administrator verifies for each investor N_i with commitment values $(com_{i,1}, \tilde{c}_{i,1}), \ldots, (com_{i,\lambda}, \tilde{c}_{i,\lambda})$ and

ciphers $c_{i,1}, \ldots, c_{i,\lambda}$ that

$$\operatorname{Unv}_{pk}\left(\prod_{j=1}^{\lambda}\operatorname{com}_{i,j}^{\beta_{j}},A_{i},B_{i}\right)=1,$$

where $A_i = \left(\left(\prod_{j=0}^{\lambda} c_{i,j}^{\beta_j} \mod p^2 \right) - 1 \right) / p$ and $B_i = \left(\left(\prod_{j=1}^{\lambda} \tilde{c}_{i,j}^{\beta_j} \mod p^2 \right) - 1 \right) / p$ (else it aborts). Then each investor N_i sends $C_i = \sum_{j=0}^{\lambda} x_{i,j}$ and $D_i = \sum_{j=1}^{\lambda} r_{i,j}$ to the system administrator which computes

$$b_{P,i} = \mathsf{DIEUnvPay}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}, C_i, D_i \right) = \mathsf{Unv}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}, C_i, D_i \right), b_{P,i} \in \{0,1\}$$

and charges the bank account of N_i with the amount C_i , if $b_{P,i}=1$. DIEDec: For all $j=0,\ldots,\lambda$ and ciphertexts $c_{1,j},\ldots,c_{n,j}$, the system administrator computes

$$X_j = \mathsf{DIEDec}_{sk_0}(t_j, c_{1,j}, \dots, c_{n,j}) = \mathsf{PSADec}_{s_0}(t_j, c_{1,j}, \dots, c_{n,j})$$

and verifies that $X_0 = 0$ (else it aborts).

DIEUnvRet: The management of each project P_j publishes a return factor $\alpha_j \in [-q', q']$. The system administrator charges the bank account of the management of each project P_j with the amount $\alpha_j X_j$. Each investor N_i sends $E_i = \sum_{j=1}^{\lambda} \alpha_j x_{i,j}$ and $F_i = \sum_{j=1}^{\lambda} \alpha_j r_{i,j}$ to the system administrator. If the verification in the DIEUnvPay algorithm has output 1, the system administrator computes for all $i=1,\ldots,n$:

$$b_{R,i} = \mathsf{DIEUnvRet}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}^{\alpha_j}, E_i, F_i \right) = \mathsf{Unv}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}^{\alpha_j}, E_i, F_i \right), b_{R,i} \in \{0,1\}$$

and transfers the amount E_i to the bank account of investor N_i , if $b_{R,i} = 1$.

Fig. 5. The Investoin protocol.

Proof. The first correctness property is given by the correctness of the PSA scheme from Figure 1. The second and third correctness properties are given by the correctness and the homomorphy of the Commitment scheme from Figure 2.

By the first property, the decryption of all ciphers results in the sum of the amounts they encrypt. So the projects receive the correct investments. By the second property, the total investment amount of each investor is accepted by the system if the investor has committed to it. Thus, the investor's account will be charged with the correct amount. By the third property, the total return to each investor is accepted by the system if the investor has committed to the corresponding investment amount before. Thus, the investor will receive the correct return on investment (ROI).

3.3 Generation of public parameters and secret key generation protocol

In this section, we show how the system sets the random oracle $H: T \to \mathcal{QR}_{p^2}$ and we provide a decentralised key generation protocol for Investcoin. It supports dynamic join, dynamic leave and fault-tolarance of investors using one round of communication between the investors. In the Section 4.2, we will show how to use the public parameters and the secret key generation protocol for the security of Investcoin.

Setting the random oracle. Recall that we need to generate public parameters, a random oracle $H: T \to \mathcal{QR}_{p^2}$ and secret parameters $\beta_0, \ldots, \beta_{\lambda} \leftarrow_R [-q', q'], q' < q/(m\lambda)$, such that for $t_0, \ldots, t_{\lambda}, \tilde{t}_1, \ldots, \tilde{t}_{\lambda} \in T$ the following equation holds.

$$\prod_{j=0}^{\lambda} H(t_j)^{\beta_j} = \prod_{j=1}^{\lambda} H(\tilde{t}_j)^{\beta_j} = 1.$$
 (1)

First, for $j=0,\ldots,\lambda-2$, on input t_j let $H(t_j)$ be random in \mathcal{QR}_{p^2} and for $j=1,\ldots,\lambda-2$, on input \tilde{t}_j let $H(\tilde{t}_j)$ be random in \mathcal{QR}_{p^2} . The system chooses $\beta_0,\ldots,\beta_{\lambda-2}\leftarrow_R[-q',q'],\beta_{\lambda-1}\leftarrow_R[-q',q'-1]$ as part of its secret key (note that choosing these values according to a different distribution gives no advantage to the system). Then it computes

$$(H(t_{\lambda-1}), H(\tilde{t}_{\lambda-1})) = \left(\prod_{j=0}^{\lambda-2} H(t_j)^{\beta_j}, \prod_{j=1}^{\lambda-2} H(\tilde{t}_j)^{\beta_j}\right),$$

$$(H(t_{\lambda}), H(\tilde{t}_{\lambda}), \beta_{\lambda}) = (H(t_{\lambda-1}), H(\tilde{t}_{\lambda-1}), -1 - \beta_{\lambda-1}),$$

instructs each investor N_i to set $x_{i,\lambda-1} = x_{i,\lambda} = 0$ and sets $\alpha_{\lambda-1} = \alpha_{\lambda} = 1$. In this way Equation (1) is satisfied. The projects $P_{\lambda-1}, P_{\lambda}$ deteriorate to 'dummy-projects' (e.g. if any investor decides to set $x_{i,\lambda} > 0$, then the system simply collects $X_{\lambda} > 0$).

Key generation for PSA within Investcoin. The building block for a key generation protocol is a n-1 out of n secret sharing scheme between the investors and the system. It is executed before the first investment round in the following way. For all $i=1,\ldots,n$, investor N_i generates uniformly random values $s_{i,1},\ldots,s_{i,n}$ from the key space and sends $s_{i,i'}$ to $N_{i'}$ for all $i'=1,\ldots,n$ via secure channel. Accordingly, each investor $N_{i'}$ obtains the shares $s_{1,i'},\ldots,s_{n,i'}$. Then each investor

 N_i sets the own secret key $s_i = \sum_{i'=1}^n s_{i,i'}$ and each investor $N_{i'}$ sends $\sum_{i=1}^n s_{i,i'}$ to the system. The system then computes

$$s_0 = -\sum_{i'=1}^n \left(\sum_{i=1}^n s_{i,i'}\right) = -\sum_{i=1}^n \left(\sum_{i'=1}^n s_{i,i'}\right) = -\sum_{i=1}^n s_i.$$

By the secret sharing property this is a secure key generation protocol in the sense that only N_i knows s_i for all i = 1, ..., n and only the system knows s_0 .

For key generation, each investor has to send one message to every other investor and one message to the system which makes n^2 messages for the total network.

As a drawback, note that the key of each single investor is controlled by the other investors together with the system: for example, if N_1, \ldots, N_{n-1} (maliciously) send the shares $s_{1,n}, \ldots, s_{n-1,n}$ and $s_{n,1}, \ldots, s_{n,n-1}$ to the system, it can compute the entire key s_n of N_n .

Assume that before the start of an arbitrary investment round, new investors want to join the network or some investors want to leave the network or some investors fail to send the required ciphers. In order to be able to carry out the protocol execution, the network can make a key update that requires O(n) messages (rather than $O(n^2)$ messages for a new key setup) using the established secret sharing scheme. Due to space limitations, we omit the (simple) details.

4 Security of Investcoin

4.1 Definition of security

The administration is honest-but-curious and may compromise investors to build a coalition for learning the values of uncompromised investors. Investors who are not in the coalition may try to execute the following (reasonable) malicious behaviour:

- 1. Use different values for $x_{i,j}$ in DIEEnc and DIECom in order to have a larger profit than allowed.
- 2. Invest negative amounts $x_{i,j} < 0$ in order to 'steal' funds from the projects.
- 3. Use different parameters than generated in the Setup-phase (i.e. send inconsistent or simply random messages) in order to distort the whole computation.

Definition 5 (Privacy-Preserving Investment system) Let κ be a security parameter and $n, \lambda \in \mathbb{N}$ with $n = \mathsf{poly}(\kappa)$ and $\lambda = \mathsf{poly}(\kappa)$. A DIE scheme $\Omega = (\mathsf{DIESet}, \mathsf{DIEEnc}, \mathsf{DIECom}, \mathsf{DIETes}, \mathsf{DIEUnvPay}, \mathsf{DIEDec}, \mathsf{DIEUnvRet})$ consisting of ppt algorithms and executed between a group of uncorrupted parties of size $u \leq n$ and a coalition of honest-but-curious adversaries of size n-u+1 is a Privacy-Preserving Investment (PPI) system, if the following properties hold.

- 1. Let f_{Ω} be the deterministic functionality computed by Ω . Then Ω performs a secure computation (according to Definition 3).
- 2. Ω provides linkage, i.e. for all $i=1,\ldots,n,\ sk_i,pk_i,C_i,D_i,E_i,F_i,$ $T=\{t_1,\ldots,t_{\lambda}\},\ (x_{i,j})_{j=1,\ldots,\lambda},\ (com_{i,j})_{j=1,\ldots,\lambda},\ (\alpha_j)_{j=1,\ldots,\lambda}\ the\ following\ holds:$ if

$$\begin{split} c_{i,j} \leftarrow \mathsf{DIEEnc}_{sk_i}(t_j, x_{i,j}) \, \forall \, j = 1, \dots, \lambda \\ \wedge \mathsf{DIEUnvPay}_{pk_i} \left(\prod_{j=1}^{\lambda} com_{i,j}, C_i, D_i \right) = 1 \end{split}$$

$$\land \mathsf{DIEUnvRet}_{pk_i} \left(\prod_{j=1}^{\lambda} com_{i,j}^{\alpha_j}, E_i, F_i \right) = 1,$$

then

$$C_i = \sum_{j=1}^{\lambda} x_{i,j} \wedge E_i = \sum_{j=1}^{\lambda} \alpha_j x_{i,j}.$$

- 3. For all i = 1, ..., n and $j = 1, ..., \lambda$: DIETes_{pk_i} $(x_{i,j}) = 1$ iff $x_{i,j} \ge 0$.
- 4. For all $i=1,\ldots,n$, there is a ppt distinguisher \mathcal{D}_i , such that the following holds. For any $x_{i,j}, r_{i,j} \in [0,m]$, let $c_{i,j} \leftarrow \mathsf{DIEEnc}_{sk_i}(t_j, x_{i,j})$ and $(com_{i,j}, \tilde{c}_{i,j}) \leftarrow \mathsf{DIECom}_{pk_i, sk_i}(x_{i,j}, r_{i,j})$ for all $j=1,\ldots,\lambda$. For any $x'_{i,j}, r'_{i,j} \in [0,m]$, let $c'_{i,j} \leftarrow \mathsf{DIEEnc}_{sk_i^{(j)}}(t'_j, x'_{i,j})$ and $(com'_{i,j}, \tilde{c}'_{i,j}) \leftarrow \mathsf{DIECom}_{pk_i^{(j)}, sk_i^{(j)}}(x'_{i,j}, r'_{i,j})$, for all $j=1,\ldots,\lambda$, such that $(pk_i^{(j)}, sk_i^{(j)}, t'_j) \neq (pk_i, sk_i, t_j)$ for at least one $j \in [\lambda]$, i.e. at least one entry of at least one tuple is different. Then

$$\begin{split} & \left| \Pr \left[\mathcal{D}_i \left(1^{\kappa}, sk_0, c_{i,1}, com_{i,1}, \tilde{c}_{i,1}, \dots, c_{i,\lambda}, com_{i,\lambda}, \tilde{c}_{i,\lambda} \right) = 1 \right] \\ & - \Pr \left[\mathcal{D}_i \left(1^{\kappa}, sk_0, c'_{i,1}, com'_{i,1}, \tilde{c}'_{i,1}, \dots, c'_{i,\lambda}, com'_{i,\lambda}, \tilde{c}'_{i,\lambda} \right) = 1 \right] \right| \\ \geqslant & 1 - \mathsf{neg}(\kappa). \end{split}$$

The probability space is defined over the internal randomness of \mathcal{D}_i .

The definition is twofold: on the one hand it covers the security of honest investors against a honest-but-curious coalition consisting of the untrusted system administration and compromised investors (Property 1) and on the other hand it covers the security of the system against maliciously behaving investors (Properties 2, 3, 4). Note that we have to distinguish between these two requirements, since we assume different behaviours for the two groups of participants, i.e. we cannot simply give a real-world-ideal-world security proof as in the SMPC literature in the malicious model. Instead, we follow the notions of the SMPC literature [13] for the security of honest investors (only) in the honest-but-curious model and additionally provide security notions against some reasonably assumable behaviour of malicious investors. For the security against a honest-but-curious coalition, the first property ensures that from the outcome of the decryption no other information than X_i can be detected for all $j=1,\ldots,\lambda$ and that the single amounts comitted to by the investors for payment and return are hidden. For the security of the system against maliciously behaving investors, imagine the situation where an investor N_i claims to having payed amount $x_{i,j}$ to project P_j (in the DIECom algorithm) but in fact has only payed $\tilde{x}_{i,j} < x_{i,j}$ (in the DIEEnc algorithm). If the return factor α_i is larger than 1, then N_i would unjustly profit more from her investment than she actually should and the system would have a financial damage. Therefore the second property says that for all $i = 1, \ldots, n$ with overwhelming probability, whenever $x_{i,1}, \ldots, x_{i,\lambda}$ were send by N_i using the DIEEnc algorithm and DIEUnvPay, DIEUnvRet accept C_i , E_i respectively, then C_i and E_i must be the legitimate amounts that N_i respectively has invested in total and has to get back as return in total. The third property ensures that no investor is able to perform a negative investment. The fourth property ensures that all investors use the correct parameters as generated by the DIESet algorithm.²

Usually the investor cannot know if $\alpha_j > 1$ at the time of cheating, since it becomes public in the future. However, in the scenario where a priori information about α_j is known to some investors or where investors simply act maliciously, we need to protect the system from beeing cheated.

² More precisely, it ensures that a cheating investor will be identified by the system.

4.2 Proof of security

First, we concentrate on the security against the honest-but-curious coalition (Property 1) and then show security against malicious investors (Propertis 2, 3, 4).

Theorem 3 By the DDH assumption in the group QR_{p^2} of quadratic residues modulo p^2 for a safe prime p, Investoin is a PPI system in the random oracle model.

Proof. The proof follows from Lemma 7, Lemma 8, Lemma 9 and Lemma 10 below. \Box

Security against honest-but-curious coalition. Investcoin is the combination of the protocols described in Figures 1, 2, 3. We first show the security of the underlying protocols seperately and then use Theorem 1 in order to show composition. In this section, assume without loss of generality that the indices i = 1, ..., u belong to the group of uncorrupted investors and the indices i = u+1, ..., n belong to the investors corrupted by the system administrator. Before showing security, we briefly explain the functionalities used in the proofs. In the following, computations are performed modulo p^2 .

– The functionality f_{Σ} is computed by the protocol in Figure 1. It takes as input n values x_1, \ldots, x_n and outputs their sum

$$f_{\Sigma}(x_1,\ldots,x_n)=\sum_{i=1}^n x_i.$$

- The functionality f_{Γ} is computed by the protocol in Figure 2. It takes as input three values com, x, r and outputs b = 1 iff (x, r) is a valid opening for the commitment com, else b = 0.
- The functionality f_{Υ} is computed by the protocol in Figure 3. It takes as input the values r, R and outputs b = 1 iff r is a discrete logarithm of either R or $S = R \cdot g^{-1}$ with base h, else b = 0.
- For $\beta = (\beta_0, \ldots, \beta_{\lambda})$, the functionality f_{β} is computed by the DIEUnvPay algorithm in Figure 5 (using $\beta_0, \ldots, \beta_{\lambda}$ and $(c_{i,j}, \tilde{c}_{i,j})$ for all $i = 1, \ldots, n, j = 1, \ldots, \lambda$ generated by DIEEnc, DIECom respectively). It takes as input the values $x_0, x_1, r_1, \ldots, x_{\lambda}, r_{\lambda}$ and outputs $A = \sum_{j=0}^{\lambda} \beta_j x_j$ and $B = \sum_{j=1}^{\lambda} \beta_j r_j$.

Lemma 4 Under the DDH assumption, the protocol Σ in Figure 1 securely computes the functionality f_{Σ} in the random oracle model.

Proof. Assume the coalition is corrupted. Let $\{x_{i,j}: i=1,\ldots,n, j=1,\ldots,\lambda\}$ be the input messages of all investors. We construct a simulator \mathcal{S} for the

$$\mathsf{view}^{\varSigma}((x_{i,j})_{i=1,\ldots,n,j=1,\ldots,\lambda},\kappa) = ((x_{i,j})_{i=u+1,\ldots,n,j=1,\ldots,\lambda},\bot,(c_{i,j})_{i=1,\ldots,u,j=1,\ldots,\lambda})$$

of the coaltion, where $c_{i,j} = \mathsf{PSAEnc}_{s_i}(t_j, x_{i,j})$ for all i, j and $\mathsf{PSAEnc}_{s_0}(t_j, 0) \cdot \prod_i c_{i,j} = 1 + p \cdot f_{\Sigma}((x_{i,j})_{i=1,\dots,n}) \bmod p^2$ for all j, as follows.⁴

 $^{^3}$ The Range test in Figure 4 is the combination of the protocols in Figure 2 and 3. Therefore it suffices to show the security of these protocols.

 $^{^4}$ There is no internal randomness of the coalition. Instead the common public randomness from the random oracle H is used.

 \mathcal{S} takes as input 1^{κ} , messages $(x_{i,j})_{i=u+1,\ldots,n,j=1,\ldots,\lambda}$ and the protocol outputs $(X_j)_{j=1,...,\lambda} = (f_{\Sigma}((x_{i,j})_{i=1,...,n}))_{j=1,...,\lambda}.$

1. Choose messages $(x'_{i,j})_{i=1,\dots,u,j=1,\dots,\lambda}$ with each $x'_{i,j} \in [0,2^l-1]$, such that for all $j = 1, \ldots, \lambda$:

$$\sum_{i=1}^{u} x'_{i,j} = X_j - \sum_{i=u+1}^{n} x_{i,j}.$$

2. Compute $c'_{i,j} = \mathsf{PSAEnc}_{s_i}(t_j, x'_{i,j})$ for $i = 1, \dots, u$ and $c_{i,j} = \mathsf{PSAEnc}_{s_i}(t_j, x_{i,j})$ for $i = u + 1, \dots, n, \ j = 1, \dots, \lambda$ for the keys $s_1, \dots, s_n \leftarrow_R \mathbb{Z}_{pq}$ with $s_0 \equiv -\sum_{i=1}^n s_i \bmod pq$. \mathcal{S} outputs $(x_{i,j})_{i=u+1,\dots,n,j=1,\dots,\lambda}, \bot, (c'_{i,j})_{i=1,\dots,u,j=1,\dots,\lambda}$.

The computational indistinguishability of the output distribution of $\mathcal S$ from $\left\{\mathsf{view}^{\varSigma}((x_{i,j})_{i=1,\ldots,n,j=1,\ldots,\lambda},\kappa)\right\}_{(x_{i,j})_{i=1,\ldots,n,j=1,\ldots,\lambda},\kappa} \text{ immediately follows from the AO1 security of } \varSigma, \text{ which holds by the DDH assumption.}$

Lemma 5 The protocol Υ in Figure 3 securely computes the accepting functionality⁵ f_{Υ} .

Proof. Assume the honest verifier is corrupted and the prover knows the discrete logarithm of either R or $S = R \cdot g^{-1}$ base h, where g, h, are public. We construct a simulator S for the

$$\mathsf{view}^{\Upsilon}(t, R, S, \kappa) = (R, S, v, a_1, a_2, (v_1, w_1), (v_2, w_2))$$

of the verifier, such that $v = v_1 + v_2$ and $h^{w_1} = a_1 R^{v_1}, h^{w_2} = a_2 S^{v_2}$ and t is the input of the prover, as follows.

S takes as input 1^{κ} , the verifier's input (R,S) and the protocol output b=1.

- 1. Choose random values $v'_1, v'_2, w'_1, w'_2 \leftarrow_R \mathbb{Z}_q$.

2. Set $v' = v'_1 + v'_2$. 3. Set $a'_1 = h^{w'_1} R^{-v'_1}$ and $a'_2 = h^{w'_2} S^{-v'_2}$. S outputs $R, S, v', a'_1, a'_2, (v'_1, w'_1), (v'_2, w'_2)$.

The value v' is random, since v'_1, v'_2 are random. Then the distribution of the output of S is indistinguishable from $\left\{\mathsf{view}^{\Upsilon}(t,R,S,\kappa)\right\}_{t,R,S,\kappa}$ by the special honest-verifier zero-knowledge property of Υ , which holds perfectly

Lemma 6 Under the DDH assumption, the algorithms DIEEnc, DIECom and DIEUnvPay securely compute the functionality f_{β} in the random oracle model.

Proof. In order to compute $A_i = \sum_{j=0}^{\lambda} \beta_j x_{i,j}$ and $B_i = \sum_{j=1}^{\lambda} \beta_j r_{i,j}$, the algorithm DIEUnvPay (executed by the system administrator) takes as input the values $\beta_0, \ldots, \beta_{\lambda}$ and the encryptions $c_{i,j}, \tilde{c}_{i,j}$ of $x_{i,j}, r_{i,j}$ respectively generated by the algorithms DIEEnc and DIECom (executed by investor N_i) for $j = (0), 1, \ldots, \lambda$. Since the encryptions $c_{i,j}, \tilde{c}_{i,j}$ are both generated by the protocol Σ in Figure 1, the statement follows from the AO1 security of Σ as in the proof of Lemma 4.

Lemma 7 Under the DDH assumption, Investoin satisfies Property 1 of Definition 5 in the random oracle model.

⁵ I.e. the case b = 1. In the case b = 0, by the completeness property, the prover has tried to convince the verifier about a wrong statement. Thus, we can assume that the prover was intending some malicious behaviour. In this case, we do not care about preserving the privacy of this prover.

Proof. Let $\Omega^{f_{\Sigma},f_{\Upsilon},f_{\beta}}$ be the protocol Ω , where executions of the aforementioned protocols are replaced by calls to a trusted party computing the according functionalities $f_{\Sigma}, f_{\Upsilon}, f_{\beta}$. Assume the coalition is corrupted. We first describe $\Omega^{f_{\Sigma},f_{\Upsilon},f_{\beta}}$. Thereby we assume that the key generation in DIESet is completed as a pre-computation. The other algorithms work as follows.

- For all $j = 0, ..., \lambda$, the input $(x_{i,j})_{i=1,...,u}$ of the uncorrupted group of investors and the input $(x_{i,j})_{i=u+1,...,n}$ of the coalition is sent to the trusted party which returns $X_j = \sum_{i=1}^n x_{i,j} = f_{\Sigma}(x_{1,j},...,x_{n,j})$ to the system administrator. The system administrator verifies that $X_0 = 0$, otherwise it aborts.
- For all $i=1,\ldots,u,\ j=1,\ldots,\lambda$, investor N_i chooses $r_{i,j}\leftarrow_R [0,m]$ and sends $com_{i,j}=\mathsf{Com}_{pk}(x_{i,j},r_{i,j})$ to the system administrator (where Com is as in Figure 2).
- For all $i=1,\ldots,u,\ j=1,\ldots,\lambda,\ k=0,\ldots,l-1$, investor N_i chooses $x_{i,j}^{(k)}\in\{0,1\},r_{i,j}^{(k)}\leftarrow_R[0,m]$ with $x_{i,j}\equiv\sum_{k=0}^{l-1}x_{i,j}^{(k)}\cdot 2^k$ mod pq and $r_{i,j}\equiv\sum_{k=0}^{l-1}r_{i,j}^{(k)}\cdot 2^k$ mod pq and sends $com_{i,j}^{(k)}=\operatorname{Com}_{pk}(x_{i,j}^{(k)},r_{i,j}^{(k)})$ to the system administrator. The system administrator verifies that $com_{i,j}\equiv\prod_{k=0}^{l-1}(com_{i,j}^{(k)})^{2^k}$ mod p^2 for all i,j and sends the $com_{i,j}^{(k)}$ to the trusted party. Moreover, the investors send the $x_{i,j}^{(k)},r_{i,j}^{(k)}$ to the trusted party which returns

$$f_{\Upsilon}(r_{i,j}^{(k)}, com_{i,j}^{(k)}) = b_{T,i,j,k}$$

- for all $i = 1, ..., u, j = 1, ..., \lambda, k = 0, ..., l 1$ to the system administrator.
- For i = 1, ..., u, investor N_i sends $x_{i,j}, r_{i,j}, j = 1, ..., \lambda$ to the trusted party and the system administrator sends $\boldsymbol{\beta} = (\beta_0, ..., \beta_{\lambda})$ to the trusted party which returns

$$(A_i, B_i) = \left(\sum_{j=1}^{\lambda} \beta_j x_{i,j}, \sum_{j=1}^{\lambda} \beta_j r_{i,j}\right) = f_{\beta}(0, x_{i,1}, r_{i,1}, \dots, x_{i,\lambda}, r_{i,\lambda})$$

for all $i = 1, \dots, u$ to the system administrator which verifies that

$$\mathsf{Unv}_{pk}\left(\prod_{i=1}^{\lambda}com_{i,j}^{\beta_{j}},A_{i},B_{i}\right)=1.$$

Then the uncorrupted investors N_i , i = 1, ..., u, send $C_i = \sum_{j=0}^{\lambda} x_{i,j}$ and $D_i = \sum_{j=1}^{\lambda} r_{i,j}$ to the system administrator which computes

$$b_{P,i} = \mathsf{Unv}_{pk}\left(\prod_{i=1}^{\lambda}com_{i,j}, C_i, D_i\right), b_{P,i} \in \{0,1\}.$$

– The uncorrupted investors N_i , $i=1,\ldots,u$, send $E_i=\sum_{j=0}^{\lambda}\alpha_jx_{i,j}$ and $F_i=\sum_{j=1}^{\lambda}\alpha_jr_{i,j}$ to the system administrator which computes

$$b_{R,i} = \mathsf{Unv}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}^{\alpha_j}, E_i, F_i \right), b_{R,i} \in \{0,1\}.$$

⁶ The system administrator, which is part of the coalition, and the investors simply feed the trusted party with their according inputs, outputs and received messages to compute the respective functionalities. Since we deal with modular sequential composition, the trusted party cannot be invoked to compute the functionalities simultaneously. Rather a functionality is executed after the output of the previous functionality was distributed to the parties.

This is exactly the protocol in Figure 5 where sub-protocols are exchanged by calls to a trusted party computing the ideal functionalities $f_{\Sigma}, f_{\Upsilon}, f_{\beta}$ and the system administrator knows all secret values of the corrupted investors. We show that $\Omega^{f_{\Sigma},f_{\Upsilon},f_{\beta}}$ performs a secure computation. In the following, we construct a simulator \mathcal{S} for the view $\Omega^{f_{\Sigma},f_{\Upsilon},f_{\beta}}((x_{i,j})_{i=1,...,n,j=0,...,\lambda},\kappa)=(y,r,m)$ of the coaltion with

$$y = ((x_{i,j})_{i=u+1,\dots,n,j=0,\dots,\lambda}, (x_{i,j}^{(k)})_{i=u+1,\dots,n,j=1,\dots,\lambda,k=0,\dots,l-1}),$$

$$r = ((r_{i,j})_{i=u+1,\dots,n,j=1,\dots,\lambda}, (r_{i,j}^{(k)})_{i=u+1,\dots,n,j=1,\dots,\lambda,k=0,\dots,l-1}),$$

$$m = ((com_{i,j})_{i=1,\dots,u,j=1,\dots,\lambda}, (com_{i,j}^{(k)})_{i=1,\dots,u,j=1,\dots,\lambda,k=0,\dots,l-1}, (C_i, D_i, E_i, F_i)_{i=1,\dots,u}).$$

 \mathcal{S} takes as input 1^{κ} , y and the protocol outputs X_j for $j=0,\ldots,\lambda$, $\{b_{T,i,j,k}=1: i=1,\ldots,u, j=1,\ldots,\lambda, k=0,\ldots,l-1\}, A_i, B_i \text{ for } i=1,\ldots,u.$

- 1. For $i=1,\ldots,u,\ j=1,\ldots,\lambda,$ set $x_{i,0}'=0$ and choose $x_{i,j}'\in[0,2^l-1],$ such that $\sum_{i=1}^{u} x'_{i,j} = X_j - \sum_{i=u+1}^{n} x_{i,j}$ for all $j = 1, ..., \lambda$, $\sum_{j=1}^{\lambda} \beta_j x'_{i,j} = A_i$ for all i = 1, ..., u. Choose $(x'_{i,j}^{(0)}, ..., x'_{i,j}^{(l-1)})$ with $x'_{i,j} \equiv \sum_{k=0}^{l-1} x'_{i,j}^{(k)} \cdot 2^k \mod pq$ for all $i = 1, ..., u, j = 1, ..., \lambda$.
- 2. For $i = 1, ..., n, j = 1, ..., \lambda$, choose $r'_{i,j} \leftarrow_R [0, 2^l 1]$ and $(r'^{(0)}_{i,j}, ..., r'^{(l-1)}_{i,j})$ with $r'_{i,j} \equiv \sum_{k=0}^{l-1} r'^{(k)}_{i,j} \cdot 2^k \mod pq$ and $\sum_{j=1}^{\lambda} \beta_j r'_{i,j} = B_i$ for all $i = 1, ..., n, j = 1, ..., \lambda$.
- 3. Compute $com'_{i,j} = \mathsf{Com}_{pk}(x'_{i,j}, r'_{i,j})$ and $com'_{i,j}^{(k)} = \mathsf{Com}_{pk}(x'_{i,j}^{(k)}, r'_{i,j}^{(k)})$ for all $i = 1, \ldots, u, \ j = 1, \ldots, \lambda, \ k = 0, \ldots, l 1.$ 4. For $i = 1, \ldots, u$, compute $C'_i = \sum_{j=1}^{\lambda} x'_{i,j}, \ D'_i = \sum_{j=1}^{\lambda} r'_{i,j}, \ E'_i = \sum_{j=1}^{\lambda} \alpha_j x'_{i,j},$
- $F_i' = \sum_{i=1}^{\lambda} \alpha_i r_{i,i}'.$

S outputs (y', r', m') with

$$\begin{split} y' = & ((x'_{i,j})_{i=u+1,...,n,j=0,...,\lambda}, (x'_{i,j})_{i=u+1,...,n,j=1,...,\lambda,k=0,...,l-1}), \\ r' = & ((r'_{i,j})_{i=u+1,...,n,j=1,...,\lambda}, (r'_{i,j})_{i=u+1,...,n,j=1,...,\lambda,k=0,...,l-1}), \\ m' = & ((com'_{i,j})_{i=1,...,u,j=1,...,\lambda}, (com'^{(k)}_{i,j})_{i=1,...,u,j=1,...,\lambda,k=0,...,l-1}, (C'_i, D'_i, E'_i, F'_i)_{i=1,...,u}) \end{split}$$

and the output of S is indistinguishable from the view of the coalition by the perfect hiding property of Γ . The statement of the lemma follows from Lemma 4, Lemma 5, Lemma 6, and Theorem 1.

Security against malicious investors.

Lemma 8 Under the dlog assumption, Investoin satisfies Property 2 of Definition 5 with overwhelming probability in the random oracle model.

Proof. For any $i \in [n]$, assume that $c_{i,j} \leftarrow \mathsf{DIEEnc}_{sk_i}(t_j, x_{i,j})$ for all $j = 1, \ldots, \lambda$, $\mathsf{DIEUnvPay}_{pk}\left(\prod_{i=1}^{\lambda}com_{i,j},C_{i},D_{i}\right) = 1, \ \mathsf{DIEUnvRet}_{pk}\left(\prod_{j=1}^{\lambda}com_{i,j}^{\alpha_{j}},E_{i},F_{i}\right) = 1.$ Then by the rules of the protocol also $\mathsf{Unv}_{pk}\left(\prod_{j=1}^{\lambda}com_{i,j}^{\beta_{j}},A_{i},B_{i}\right)=1$ for some $\beta_0, \ldots, \beta_{\lambda} \in [-q', q']$ (unknown to N_i) with $q' < q/(m\lambda)$, where we have defined

$$A_i = \left(\left(\prod_{j=0}^{\lambda} c_{i,j}^{\beta_j} \mod p^2 \right) - 1 \right) / p \text{ and } B_i = \left(\left(\prod_{j=1}^{\lambda} \tilde{c}_{i,j}^{\beta_j} \mod p^2 \right) - 1 \right) / p$$

with $c_{i,j} \equiv (1 + p \cdot x_{i,j}) \cdot H(t_j)^{s_i} \mod p^2$ and $\tilde{c}_{i,j} \equiv (1 + p \cdot r_{i,j}) \cdot H(\tilde{t}_j)^{\tilde{s}_i} \mod p^2$ for all $j = 1, \ldots, \lambda$ and for a random oracle $H : T \cup \tilde{T} \to \mathcal{QR}_{p^2}$. Now assume that either $C_i \neq \sum_{j=1}^{\lambda} x_{i,j}$ or $E_i \neq \sum_{j=1}^{\lambda} \alpha_j x_{i,j}$. Then by the homomorphy and computational binding of the Pedersen commitment scheme Γ , which holds by the dlog assumption, there is at least one $j' \in [\lambda]$, such that $com_{i,j'}$ is not the commitment of $x_{i,j'}$ but of some $x'_{i,j'} \neq x_{i,j'}$. We define the mapping

$$f_{\beta_0,\dots,\beta_\lambda}:[0,m]^\lambda\to[-q,q], f_{\beta_0,\dots,\beta_\lambda}(x_0,\dots,x_\lambda)=\sum_{j=0}^\lambda\beta_jx_j.$$

Then, by Equation (1), $f_{\beta_0,\ldots,\beta_\lambda}(x_{i,0},\ldots,x_{i,\lambda})=A_i$. Moreover, $f_{\beta_0,\ldots,\beta_\lambda}$ is not efficiently computable by N_i , since it does not know $\beta_0,\ldots,\beta_\lambda\in[-q',q']$, if q' is super-polynomial in κ (from Equation (1), N_i can compute the β_j only with negligible probability, since H is a random oracle). Therefore with overwhelming probability, for any choice of $(x'_{i,0},\ldots,x'_{i,\lambda})$ by N_i , such that $com_{i,j}$ is valid for $x'_{i,j}$ for all $j=1,\ldots,\lambda$, the equation $f_{\beta_0,\ldots,\beta_\lambda}(x'_{i,0},\ldots,x'_{i,\lambda})=A_i$ is only true, if $x'_{i,j}=x_{i,j}$ for all $j=0,\ldots,\lambda$. This is a contradiction to the existence of at least one j' with $x'_{i,j'}\neq x_{i,j'}$. By the binding of Γ , the case $f_{\beta_0,\ldots,\beta_\lambda}(x'_{i,0},\ldots,x'_{i,\lambda})\neq A_i$ is a contradiction to $\mathsf{Unv}_{pk}\left(\prod_{j=1}^\lambda com_{i,j}^{\beta_j},A_i,B_i\right)=1$.

Lemma 9 Under the dlog assumption, Investcoin satisfies Property 3 of Definition 5 with overwhelming probability.

Proof. We have to show that for all $i=1,\ldots,n,j=1,\ldots,\lambda$ and $pk=(h_1,h_2)$, with overwhelming probabilty, $\mathsf{DIETes}_{pk}(x_{i,j})=1$ iff $x_{i,j}\geqslant 0$. The Range test in Figure 4 performs l times the protocol from Figure 3. I.e. for every bit $x_{i,j}^{(k)}$ of $x_{i,j}$, the Range test performs an Extended Schnorr protocol (Figure 3) for proving the knowledge of 1 out of 2 secrets: either the prover knows the opening $(0,r_{i,j}^{(k)})$ to the Pedersen commitment $com_{i,j}^{(k)}=h_2^{r_{i,j}^{(k)}}$ or the opening $(1,r_{i,j}^{(k)})$ to $com_{i,j}^{(k)}=h_1\cdot h_2^{r_{i,j}^{(k)}}$, where $k=0,\ldots,l-1$.

Since the Pedersen Commitment is homomorphic, the commitment $com_{i,j}$ for $x_{i,j}$ is congruent $\prod_{k=0}^{l-1} (com_{i,j}^{(k)})^{2^k}$. Then the direction " \Leftarrow " immediately follows from the completeness property of the protocol in Figure 3.

Assume a malicious prover tries to prove that some committed value $x_{i,j} \notin [0, 2^l - 1]$ is in $[0, 2^l - 1]$. Then there exists at least one $k' \ge l$ such that $x_{i,j} = x_{i,j}^{(k')} \cdot 2^{k'} + \sum_{k=0}^{l-1} x_{i,j}^{(k)} \cdot 2^k$ with $x_{i,j}^{(k')}, x_{i,j}^{(k)} \in \{0,1\}$ for all $k = 0, \ldots, l-1$ (since we reduce modulo pq). Since the verifier has to receive commitments to exactly l bits,

the prover needs to know the discrete logarithm of at least one $h_2^{r_{i,j}^{(k'')}}$ or $h_1 \cdot h_2^{r_{i,j}^{(k'')}}$ which is not among $com_{i,j}^{(k)}$, $k=0,\ldots,l-1,k'$. The prover can compute it only with negligible probability by the dlog assumption. Thus, the direction " \Rightarrow " follows from the special soundness of the protocol from Figure 3.

Lemma 10 Under the dlog assumption, Investcoin satisfies Property 4 of Definition 5 with overwhelming probability in the random oracle model.

Proof. Let $T = \{t_1, \ldots, t_{\lambda}\}$ be the set of public parameters used for encryption and let pk be the public key for the commitment. For all $i \in [n]$, such that investor N_i is not in the coalition with the system administrator, i.e. $i \in U$, let sk_i be the secret key of investor N_i . We construct \mathcal{D}_i as follows. On input

$$1^{\kappa}, sk_0, c_{i,0}, c_{i,1}^*, com_{i,1}^*, \tilde{c}_{i,1}^*, \dots, c_{i,\lambda}^*, com_{i,\lambda}^*, \tilde{c}_{i,\lambda}^*,$$

it has to decide whether $(c_{i,j}^*,com_{i,j}^*,\tilde{c}_{i,j}^*)=(c_{i,j},com_{i,j},\tilde{c}_{i,j})$ for all $j=1,\ldots,\lambda$ or $(c_{i,j}^*,com_{i,j}^*,\tilde{c}_{i,j}^*)=(c_{i,j}',com_{i,j}',\tilde{c}_{i,j}')$ for all $j=1,\ldots,\lambda$, where $c_{i,j}\leftarrow \mathsf{PSAEnc}_{s_i}(t_j,x_{i,j}),com_{i,j}\leftarrow \mathsf{Com}_{pk}(x_{i,j},r_{i,j}),\tilde{c}_{i,j}\leftarrow \mathsf{PSAEnc}_{\tilde{s}_i}(\tilde{t}_j,r_{i,j}),c_{i,j}'\leftarrow \mathsf{PSAEnc}_{\tilde{s}_i'}(\tilde{t}_j',x_{i,j}'),c_{i,j}'\leftarrow \mathsf{PSAEnc}_{\tilde{s}_i'}(\tilde{t}_j',r_{i,j}'),c_{i,j}'\leftarrow \mathsf{PSAEnc}_{\tilde{s}_i'}(\tilde{t}_j',r_{i,j}')$ for all $j=1,\ldots,\lambda$, s.t. $(pk^{(j)},s_i^{(j)},\tilde{s}_i^{(j)},t_j',\tilde{t}_j')\neq (pk,s_i,\tilde{s}_i,t_j,\tilde{t}_j)$ for at least one $j\in[\lambda]$, i.e. there is a difference in at least one entry for at least one tuple. \mathcal{D}_i uses its input to compute

$$A_i = \left(\left(c_{i,0}^{\beta_0} \cdot \prod_{j=1}^{\lambda} c_{i,j}^{*^{\beta_j}} \mod p^2 \right) - 1 \right) / p \text{ and}$$

$$B_i = \left(\left(\prod_{j=1}^{\lambda} \tilde{c}_{i,j}^{*^{\beta_j}} \mod p^2 \right) - 1 \right) / p.$$

Then it computes and outputs $b = \mathsf{Unv}_{pk} \left(\prod_{j=1}^{\lambda} com_{i,j}^{*^{\beta_j}}, A_i, B_i \right)$. For the first case, i.e. if $(c_{i,j}^*, com_{i,j}^*, \tilde{c}_{i,j}^*) = (c_{i,j}, com_{i,j}, \tilde{c}_{i,j})$ for all $j = 1, \ldots, \lambda$, we have b = 1 by construction. We have to show that in the second case, i.e. if $(c_{i,j}^*, com_{i,j}^*, \tilde{c}_{i,j}^*) = (c_{i,j}', com_{i,j}', \tilde{c}_{i,j}')$ for all $j = 1, \ldots, \lambda$, with overwhelming probability b = 0. Thus, \mathcal{D}_i will distinguish the cases on its input.

As a pre-computation, based on the key generation protocol from Section 3.3, for all i' = 1, ..., n, investor $N_{i'}$ has published

$$T_{i,i'} = \mathsf{PSAEnc}_{s_{i,i'}}(t_0,0)$$

on a black board for all the key shares $s_{1,i'}, \ldots, s_{n,i'}$ received during the key generation and the system administrator has verified that

$$\mathsf{PSADec}_{-\sum_{i=1}^{n} s_{i,i'}}(t_0, T_{1,i'}, \dots, T_{n,i'}) = 0,$$

i.e. all verifications are true (if not, the system aborts and the process is repeated with all but the excluded cheating investors) as described above. Thus,

$$c_{i,0} = \mathsf{PSAEnc}_{s_i}(t_0,0) = \prod_{i'=1}^n T_{i,i'}$$

is fixed, i.e. $c_{i,0}$ is generated with the correct parameters from the first case. Now consider the second case, where $(c_{i,j}^*, com_{i,j}^*, \tilde{c}_{i,j}^*) = (c_{i,j}', com_{i,j}', \tilde{c}_{i,j}')$ for all $j = 1 \dots, \lambda$. Recall that the system administrator's secret values $\beta_0, \dots, \beta_\lambda$ are not efficiently computable by the investors (who do not collaborate with the system administrator). Assume b = 1. Then

$$h_1^{A_i} h_2^{B_i} = \prod_{j=1}^{\lambda} com_{i,j}^{*^{\beta_j}},$$

where (h_1, h_2) is the public key for the Pedersen commitment which is used for verification by the system administrator in both cases. By the computational binding of the Pedersen commitment, which holds under the dlog assumption, N_i must have known A_i , B_i for computing a valid commitment $\prod_{j=1}^{\lambda} com_{i,j}^{*\beta_j}$ (note that in the first case, this is not necessary, since all ciphers and commitments were generated with consistent parameters). This is not possible, except with negligible probability, since from N_i 's point of view, $\beta_0, \ldots, \beta_{\lambda}$ are unknown.

References

- 1. Emmanuel A. Abbe, Amir E. Khandani, and Andrew W. Lo. Privacy-preserving methods for sharing financial risk exposures. *Am. Economic Review*, 102(3):65–70, 2012.
- Fabrice Benhamouda, Marc Joye, and Benoît Libert. A new framework for privacypreserving aggregation of time-series data. ACM Transactions on Information and System Security, 18(3), 2016.
- 3. Avrim Blum, Jamie Morgenstern, Ankit Sharma, and Adam Smith. Privacy-preserving public information for sequential games. In *Proc. of ITCS '15*, pages 173–180, 2015.
- 4. Manuel Blum. Coin flipping by telephone. In Proc. of CRYPTO '81, pages 11-15, 1981.
- 5. Fabrice Boudot. Efficient proofs that a committed number lies in an interval. In *Proc.* of EUROCRYPT '00, pages 431–444, 2000.
- Gilles Brassard, David Chaum, and Claude Crépeau. Minimum disclosure proofs of knowledge. J. Comput. Syst. Sci., 37(2):156–189, 1988.
- Jan Camenisch, Rafik Chaabouni, and Abhi Shelat. Efficient protocols for set membership and range proofs. In Proc. of ASIACRYPT '08, pages 234–252, 2008.
- 8. Ran Canetti. Security and Composition of Multiparty Cryptographic Protocols. *Journal of Cryptology*, 13:143–202, 2000.
- 9. Financial Crisis Inquiry Comission. The Financial Crisis Inquiry Report: Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States, 2011.
- Ronald Cramer, Ivan Damgård, and Berry Schoenmakers. Proofs of partial knowledge and simplified design of witness hiding protocols. In *Proc. of CRYPTO '94*, pages 174–187, 1994.
- 11. Amos Fiat and Adi Shamir. How to prove yourself: Practical solutions to identification and signature problems. In *Proc. of CRYPTO '86*, pages 186–194, 1987.
- Mark Flood, Jonathan Katz, Stephen Ong, and Adam Smith. Cryptography and the economics of supervisory information: Balancing transparency and confidentiality. Federal Reserve Bank of Cleveland, Working Paper no. 13-11, 2013.
- 13. Oded Goldreich. Foundations of Cryptography: Volume 2, Basic Applications. Cambridge University Press, 2004.
- Nicola Jentzsch. The Economics and Regulation of Financial Privacy A Comparative Analysis of the United States and Europe. 2001.
- 15. Marc Joye and Benoît Libert. A scalable scheme for privacy-preserving aggregation of time-series data. In *Proc. of FC '13*, pages 111–125. 2013.
- Ian Miers, Christina Garman, Matthew Green, and Aviel D. Rubin. Zerocoin: Anonymous distributed e-cash from bitcoin. In Proc. of SP '13, pages 397–411, 2013.
- 17. Satoshi Nakamoto. Bitcoin: A peer-to-peer electronic cash system.
- 18. Michael Nofer. The Value of Social Media for Predicting Stock Returns Preconditions, Instruments and Performance Analysis. PhD thesis, Techn. Univ. Darmstadt, 2014.
- 19. Torben P. Pedersen. Non-interactive and information-theoretic secure verifiable secret sharing. In *Proc. of CRYPTO '91*, pages 129–140, 1991.
- Kun Peng, Colin Boyd, Ed Dawson, and Eiji Okamoto. A novel range test. pages 247–258, 2006.
- Kun Peng and Ed Dawson. A range test secure in the active adversary model. In Proc. of ACSW '07, pages 159–162, 2007.
- Claus-Peter Schnorr. Efficient identification and signatures for smart cards. In Proc. of CRYPTO '89, pages 239–252, 1989.
- Elaine Shi, T.-H. Hubert Chan, Eleanor G. Rieffel, Richard Chow, and Dawn Song. Privacy-preserving aggregation of time-series data. In Proc. of NDSS '11, 2011.
- 24. Filipp Valovich. On the hardness of the learning with errors problem with a discrete reproducible error distribution. CoRR abs/1605.02051, 2016.
- 25. Filipp Valovich and Francesco Aldà. Private stream aggregation revisited. CoRR abs/1507.08071, 2015.

A Preservation of Market liquidity

We show that it is still possible to privately exchange any part of an investment between any pair of investors within Investcoin, i.e. market liquidity is unaffected. Assume that an investment round is over but the returns are not yet executed, i.e. the system administrator already received $c_{i,j}$, $(com_{i,j}, \tilde{c}_{i,j})$, C_i , D_i for all $i=1,\ldots,n$ and $j=1,\ldots,\lambda$, but not E_i,F_i . Assume further that for some $i,i'\in\{1,\ldots,n\}$, investors N_i and $N_{i'}$ confidentially agree on a transfer of amount $x_{(i,i'),j}$ (i.e. a part of N_i 's investment in project P_j) from investor N_i to investor $N_{i'}$. This fact needs to be confirmed by the protocol in order to guarantee the correct returns from project P_j to investors N_i and $N_{i'}$. Therefore the commitments to the invested amounts $x_{i,j}$ and $x_{i',j}$ respectively need to be updated. For the update, N_i and $N_{i'}$ agree on a value $r_{(i,i'),j} \leftarrow_R [0,m]$ via secure channel. This value should be known only to N_i and $N_{i'}$. Then N_i and $N_{i'}$ respectively compute

$$com'_{i,j} \leftarrow \mathsf{Com}_{pk}(x_{(i,i'),j}, r_{(i,i'),j}), \\ com'_{i',j} \leftarrow \mathsf{Com}_{pk}(x_{(i,i'),j}, r_{(i,i'),j})$$

and send their commitments to the system administrator which verifies that $com'_{i,j} = com'_{i',j}$. Then the system administrator updates

$$com_{i,j}$$
 by $com_{i,j} \cdot (com'_{i,j})^{-1}$,

which is possible since the Com algorithm is injective, and

$$com_{i',j}$$
 by $com_{i',j} \cdot com'_{i',j}$.

As desired, the updated values commit to $x_{i,j} - x_{(i,i'),j}$ and to $x_{i',j} + x_{(i,i'),j}$ respectively. Moreover, N_i updates the return values (E_i, F_i) by

$$(E_i - \alpha_j \cdot x_{(i,i'),j}, F_i - \alpha_j \cdot r_{(i,i'),j})$$

and $N_{i'}$ updates $(E_{i'}, F_{i'})$ by

$$(E_{i'} + \alpha_i \cdot x_{(i,i'),i}, F_{i'} + \alpha_i \cdot r_{(i,i'),i}).$$

The correctness of the update is guaranteed by Property 2 and the confidentiality of the amount $x_{(i,i'),j}$ (i.e. only N_i and $N_{i'}$ know $x_{(i,i'),j}$) is guaranteed by Property 1 of Definition 5 which are satisfied by Lemma 8 and Lemma 7 respectively.

Note that in general, this procedure allows a short sale for N_i when $x_{(i,i'),j} > x_{i,j}$ or for $N_{i'}$ when $x_{(i,i'),j} < 0$ and $|x_{(i,i'),j}| > x_{i',j}$ (over the integers). If this behaviour is not desired, it may also be necessary to perform a Range test for the updated commitments $com_{i,j} \cdot (com'_{i,j})^{-1}$ (between the system administrator and N_i) and $com_{i',j} \cdot com'_{i',j}$ (between the system administrator and $N_{i'}$) to ensure that they still commit to amounts ≥ 0 .