Edge Computing Resource Management and Pricing for Mobile Blockchain

Zehui Xiong[1], Shaohan Feng[1], Dusit Niyato[1], Ping Wang[1], and Zhu Han[2]
[1]School of Computer Engineering, Nanyang Technological University (NTU), Singapore
[2]Department of Electrical and Computer Engineering, University of Houston, Texas, USA

***Abstract*—The mining process in blockchain requires solving a proof-of-work puzzle, which is resource expensive to implement in mobile devices due to the high computing power and energy needed. In this paper, we, for the first time, consider edge computing as an enabler for mobile blockchain. In particular, we study edge computing resource management and pricing to support mobile blockchain applications in which the mining process of miners can be offloaded to an edge computing service provider. We formulate a two-stage Stackelberg game to jointly maximize the profit of the edge computing service provider and the individual utilities of the miners. In the first stage, the service provider sets the price of edge computing nodes. In the second stage, the miners decide on the service demand to purchase based on the observed prices. We apply the backward induction to analyze the sub-game perfect equilibrium in each stage for both uniform and discriminatory pricing schemes. For the uniform pricing where the same price is applied to all miners, the existence and uniqueness of Stackelberg equilibrium are validated by identifying the best response strategies of the miners. For the discriminatory pricing where the different prices are applied to different miners, the Stackelberg equilibrium is proved to exist and be unique by capitalizing on the Variational Inequality theory. Further, the real experimental results are employed to justify our proposed model.**

***Index Terms*—edge computing, offloading, mobile blockchain, proof-of-work puzzle, pricing, mining, game theory, Variational Inequality.**

I. Introduction

Electronic trading with digital transactions is becoming popular than ever in e-commerce society, where the consensus is reached through trusted centralized authorities. The introduction of centralized authorities incurs additional cost, i.e., nominal fees which become more excessive when the number of digital transactions becomes large. In 2008, a new peer-to-peer electronic payment system called "*Bitcoin*" was introduced that avoids this additional cost caused by digital transactions [1]. As one popular digital cryptocurrency, Bitcoin can record and store all digital transactions in a decentralized append-only public ledger called "*blockchain*". The Bitcoin is the first application of blockchain technologies. Subsequently, the blockchain technologies have generated remarkable public interests via a distributed network with independence from central authorities. With blockchain, a transaction can take place in a decentralized fashion, which greatly save the cost and improve the efficiency. Since its launch in 2009, Bitcoin economy has experienced an exponential growth, and its capital market now has reached over 70 billion dollars [2]. After the success of Bitcoin, blockchain has been applied in many applications, such as access control systems [3], smart contracts [4], [5], content delivery networks [6], cognitive radio networks [7], and smart grid powered systems [8], [9].

The core issue of the blockchain is a computational process called *mining*, where the transaction records are added into the blockchain via the solution of computational difficult problem, i.e., the proof-of-work puzzle. Confirming and securing the integrity and validity of transactions are processed by a set of participants called "*miners*". The security of blockchain directly relies on the distributed consensus mechanism maintained by these miners [10]. The typical consensus mechanism is introduced as follows. First, the miners consider and bundle a number of transactions that are processed to form a single "*block*". The miner propagates its mined block to the rest of the blockchain network as soon as it solves the puzzle in order to claim the mining reward. Then, this block is verified by the majority of miners in this network, i.e., trying to reach consensus. After the propagated block reaches the consensus, it is successfully added into the globally-accessible distributed public ledger, i.e., blockchain. The miner which mines a block receives the mining reward when the mined block is successfully added to the blockchain. This consensus mechanism guarantees the security and dependability of blockchain systems [11], [12].

However, blockchain has not been adopted widely in mobile applications [13]. This is because blockchain mining needs to solve a proof-of-work puzzle, which is expensive to implement in mobile devices due to the high computing power needed. Thus, deploying blockchain in a mobile environment is truly challenging. In this paper, we consider the edge computing as a network enabler for the mobile blockchain. In particular, we consider the price-based resource management in mobile blockchain, in which an Edge computing Service Provider (ESP) is introduced to support proof-of-work puzzle offloading [14] by using its edge computing nodes. Further, we propose a two-stage game model, i.e., the Stackelberg game. In the first stage, the edge computing service provider sets the price and obtains the revenue from charging the miners for offloading the mining task. In the second stage, the miners decide on the service demand to purchase from edge computing service provider. Specifically, we analyze two pricing schemes, i.e., uniform pricing in which a uniform unit price is applied to all the miners and discriminatory pricing in which different unit prices are assigned to different miners. The uniform pricing is easier to be implemented as the ESP does not need to keep track of information of all miners, and charging same prices which is fair to all miners. However, it may not yield the highest profit compared with discriminatory pricing in which the price can be adjusted for an individual miner [15].

To the best of our knowledge, this is the first work to investigate the mobile blockchain with resource management and pricing using game theory. The main contributions of this work are summarized as follows.

1) We formulate a pricing and service demand problem to analyze the interactions among the ESP and miners. In particular, we adopt the two-stage Stackelberg game to model their interactions to maximize the profit of the ESP and the individual utilities of miners jointly for mobile blockchain applications.
2) Through backward induction, we derive a unique Nash equilibrium point among the miners in the second stage, and investigate the uniform pricing as well as discriminatory pricing for profit maximization of the ESP in the first stage. The Stackelberg equilibrium is derived analytically for both the pricing schemes.
3) In particular, the existence and uniqueness of Stackelberg equilibrium are validated by identifying the best response strategies of the miners under the uniform pricing scheme. Likewise, the Stackelberg equilibrium is proved to exist and be unique by capitalizing on the Variational Inequality theory under discriminatory pricing scheme.
4) We conduct extensive numerical simulations to evaluate the performance of the proposed price-based resource management for mobile blockchain. The results show that the discriminatory pricing helps the ESP to encourage more service demand from the miners and achieve greater profit. Moreover, under uniform pricing, the service provider has an incentive to set the maximum price for the profit maximization.
5) Our work helps to achieve the proof-of-work puzzle offloading and guide the ESP to extract the surplus through charging miners strategically. Further, we perform the real experiment on mobile blockchain mining to validate the proposed analytical model.

The rest of the paper is organized as follows. Section II presents a review of the related work. We describe the system model and formulate the two-stage Stackelberg game in Section III. In Section IV, we analyze the optimal service demand of miners as well as the profit maximization of the ESP using backward induction for both uniform and discriminatory pricing schemes. We present the performance evaluations in Section V. Section VI concludes the paper with summary and future directions.

II. Related Work

Recently, there have been several studies on mining schemes management for blockchain network. In [16], the authors designed a noncooperative game among the miners, i.e., the players. The miner's strategy is to choose the number of transactions to be included in a block. In the model, solving the proof-of-work puzzle for mining is modeled as a Poisson process. The solution of the game is the Nash equilibrium which was derived only for two miners in [16]. Then, the authors in [17] modeled the mining process as a sequential game where the miners compete for mining reward in sequentially among them. In the game model, the miners are assumed to be rational, and they have to choose whether or not to propagate their solution, i.e., the mined block. It is proved in [17] that there exists a multiplicity of Nash equilibrium. Further, it is found that not propagating is an optimal strategy under certain conditions. Similar to that in [17], the authors in [18] formulated the stochastic game for modeling the mining process, where miners decide on which blocks to extend and whether to propagate the mined block. In particular, two game models in which miners play a complete information stochastic game are studied. In the first model, each miner propagates immediately the mined block that it mines. The strategy of each miner is to select an appropriate block to mine. In the second model, the miner selects which block to mine, but it may not propagate its mined block immediately. For both models, it is proved in [18] that when the number of miners is sufficiently small, the Nash equilibrium with respect to mining behaviors exists.

Traditionally, miners mine blocks individually, which we call the solo mining. The advantage of solo mining is that the miner obtains all the reward when it successfully mines the block. Recently, the pool mining is introduced as a alternative way for miners to pool their resources together for mining the block, in order to obtain steady reward [19]. The authors in [20] proposed the cooperative game based blockchain mining scheme, in which the pool mining was examined. In particular, the cooperative game theoretic tools are used to study which pools that the miners want to join and how the miners in the same pool share their reward. The interactions of miners and pool are modeled as a coalitional game. It is proved that there is no stable way to divide the payoff among the miners, i.e., the coalition structure has an empty core. Further, the proposed scheme is applied to the real-world Bitcoin network environment. It is found in [20] that under any reward allocation schemes, some miners always have the incentive to switch to other pools for higher expected reward. Inspired by [20], the authors in [21] further studied optimal pool mining mechanisms, in which the utility model and social welfare of miners are considered. It is demonstrated that the geometric pay pooling strategy [22] achieves the optimal steady-state utility for miners. The results in [21] can also be applied to other forms of mining systems.

In [23], the authors defined a novel model, in which the pools use some of their miners to infiltrate other pools and perform such an attack. The attacked pool shares its reward with the attacker, and so each of its miners in the pool earns less reward. It is proved that the decision to attack or not is the miner's dilemma and an instance of iterative prisoner's dilemma [24]. In [25], the authors developed a novel incentive payment mechanism for pool mining, in which the group bargaining solution is adopted by considering peer-to-peer

Table I
NOTATIONS

Symbol	Definition
$\mathcal{N}$, N	Set of miners, and the total number of miners, respectively
x_i	Edge computing service demand of miner i
$\underline{x}$	The minimum service requirement for all the miners
$\overline{x}$	The maximum service demand for all the miners
$\mathbf{x}$	The service demand profile of all the miners
$\mathbf{x}_{-i}$	The service demand profile of all other miners except miner i
t_i	The size of block mined by miner i, i.e., the number of transactions in its block
α_i	The relative computing or hash power of miner i
u_i	The utility of miner i
p	The unit price set by the edge computing service provider for all the miners under uniform pricing
p_i	The unit price set by the edge computing service provider for miner i under discriminatory pricing
$\overline{p}$	The maximum price constriant
R	Fixed reward of successfully mining a block
r	Variable reward factor
Π	The profit of edge computing service provider
T	The average time taken to mine a block, $\lambda = 1/T$
c	The service cost factor
τ	The block propagation time
P_i	The probability of miner i successfully mining a block or winning the mining reward
$\mathbb{P}_{\text{orphan}}$	The probability that the block is orphaned
z	The propagation delay factor

relationship of miners. In the proposed scheme, miners are grouped as a miner pool based on contribution levels. Therefore, multiple groups with different computing contributions, i.e., mining pools are formulated. As such, the bargaining occurs within individual miners in each mining pool and across multiple pools simultaneously. It is proved in [25] that the unique Nash bargaining solution [26] exists in the proposed scheme. The authors in [27] proposed a new computational power splitting game for the blockchain mining. This game model includes multiple incentivized miners to compete in solving proof-of-work puzzle in exchange for mining reward. Based on the game model, the miner with computation power play a game of solving the puzzle through distributing its computing power into different pools such that its expected reward is maximum. However, it is shown that this game has no pure Nash equilibrium with pure strategies. Some findings in [27] are consistent with those in [23].

Nevertheless, all of the above works studied the mining management problem by using dedicated nodes, without considering the operation of blockchain with mobile devices, i.e., mobile blockchain. Therefore, this motivates us to take a step further to reconsider the mining strategies as well as resource management in mobile environment, thereby opening new opportunities for the development of blockchain in mobile services and applications.

III. SYSTEM MODEL AND GAME FORMULATION

In this section, we first propose the system model of the mobile blockchain network under our consideration. Then, we present the Stackelberg game formulation for the price-based

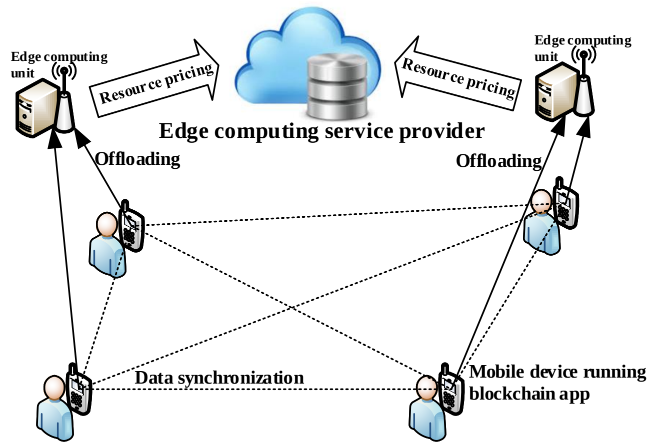

Figure 1. System model.

edge computing resource management for mobile blockchain applications.

A. Mobile Blockchain

Blockchain can be employed for mobile applications to support peer-to-peer (P2P) secure data services, e.g., P2P file transfer and P2P direct payment [28], [29]. To create a chain of blocks, the mining needs to be done. The mining process is used to confirm and secure transactions to be stored in a block. This mining process is organized as a speed game among the miners with different computing powers. The problem is a proof-of-work puzzle, which is expensive to solve and takes high computing power, time, and energy. In brief, this proof-of-work puzzle includes considering a set of transactions that are present in the network, solving a mathematical problem[1] that depends on this set and propagating the result to the blockchain network for this solution to be checked and reach consensus. Once all these steps are done successfully, the set of transactions proposed by the miner forms a block that is appended to the current blockchain. The first miner which successfully obtains the solution of the puzzle and reach the consensus is considered to be the winner to which a certain reward[2] is given. This is referred to as the speed game among the miners. The earlier studies, e.g., in [29], have found that the puzzle cannot be handled efficiently using mobile devices. In the following, we introduce using the edge computing concept for offloading the mobile blockchain processing, e.g., mining process. The more details of blockchain networks and the mining process can be found in [12].

B. Chain Mining with Edge Computing

We consider a mobile blockchain application, e.g., as presented in [29]. There is a group of N mobile users, i.e., the miners, the set of which is denoted by $\mathcal{N} = \{1, \ldots, N\}$. Each

[1] Solving the problem needs hashing the blockchain information by the miners, and thus we use the hash power and computing power interchangeably throughout the paper.

[2] The detailed discussion of reward mechanism can be found in [30].

mobile user runs mobile blockchain applications to record the transactions performed in the group. There is an Edge computing Service Provider (ESP) deploying the edge computing units/nodes for the miners. The aforementioned proof-of-work puzzle can be offloaded to a nearby edge computing unit. In particular, the edge computing units offer computing resources to mobile users, i.e., the miners which will be priced by the ESP. Figure. 1. shows the system model of the mobile blockchain network under our consideration. Note that we assume the link between the mobile nodes and edge computing units are always secured which can be achieved by some security solutions.

The ESP, i.e., the seller, sells the edge computing services, and the miner, i.e., the buyers, access and consume this service from the nearby edge computing unit. Each miner $i \in \mathcal{N}$ determines their individual service demand, denoted by x_i. Additionally, we consider $x_i \in [\underline{x}, \overline{x}]$, in which $\underline{x}$ is the minimum service demand, e.g., for blockchain data synchronization, and $\overline{x}$ is the maximum service demand governed by the ESP. Note that each miner has no incentive to unboundedly increase its service demand due to its financial burden. Then, let $\mathbf{x} \triangleq (x_1, \ldots, x_N)$ and $\mathbf{x}_{-i}$ represent the service demand profile of all the miners and all other miners except miner i, respectively. As such, the miner $i \in \mathcal{N}$ with the service demand x_i has a relative computing power (hash power) α_i with respect to the total hash power of the network, which is defined as follows:

$$\alpha_i(x_i, \mathbf{x}_{-i}) = \frac{x_i}{\sum_{j \in \mathcal{N}} x_j}, \alpha_i > 0, \tag{1}$$

such that $\sum_{j \in \mathcal{N}} \alpha_j = 1$.

In the mobile blockchain network, miners compete against each other in order to be the first one to solve the proof-of-work puzzle and receive the reward from the speed game accordingly. The occurrence of solving the puzzle can be modeled as a random variable following a Poisson process with mean $\lambda = \frac{1}{600\,\mathrm{sec}}$ [16]. Note that our model is general that can be applied with other values of λ easily. The set of transactions to be included in a block chosen by miner i is denoted as t_i. Once the miner successfully solves the puzzle, the miner needs to propagate its solution to the whole mobile blockchain network and its solution needs to reach consensus. Because there is no centralized authority to verify the validate a newly mined block, a mechanism for reaching network consensus must be employed. In this mechanism, the verification needs to be processed by other miners before the new mined block is appended to the current blockchain.

The first miner to successfully mine a block that reaches consensus earns the reward. The reward consists of a fixed reward denoted by R, and a variable reward which is defined as rt_i, where r denotes a given variable reward factor and t_i denotes the number of transactions included in the block mined by miner i [16], [31]. Additionally, the process of solving the puzzle incurs an associated cost, i.e., the payment from miner i to the ESP, p_i. The objective of the miners is to maximize their individual expected utility, and for miner i, it is defined as follows:

$$u_i = (R + rt_i) P_i\left(\alpha_i(x_i, \mathbf{x}_{-i}), t_i\right) - p_i x_i, \tag{2}$$

where $P\left(\alpha_i(x_i, \mathbf{x}_{-i}), t_i\right)$ is the probability that miner i successfully mines the block and its solutions reach consensus, i.e., miner i wins the mining reward.

The process of successfully mining a block consists of two steps, i.e., the mining step and the propagation step. In the mining step, the probability that miner i mines the block is directly proportional to its relative computing power α_i. Furthermore, there are diminishing chances of wining if one miner chooses to propagate a block that propagates slowly to other miners in the propagation step. In other words, even though one miner may find the first valid block, if its mined block is large, then this block will be likely to be discarded because of long latency, which is called orphaning [16]. Considering this fact, the probability of successful mining by miner i is discounted by the chances that the block is orphaned, $\mathbb{P}_{\mathrm{orphan}}(t_i)$, which is expressed by

$$P_i(\alpha_i(x_i, \mathbf{x}_{-i}), t_i) = \alpha_i(1 - \mathbb{P}_{\mathrm{orphan}}(t_i)). \tag{3}$$

Using the fact that block mining times follow the Poisson distribution aforementioned, the orphaning probability is approximated as [32], [33]:

$$\mathbb{P}_{\mathrm{orphan}}(t_i) = 1 - e^{-\lambda \tau(t_i)}, \tag{4}$$

where $\tau(t_i)$ is the block propagation time, which is a function of the block size. In other words, the propagation time needed for a block to reach consensus is dependent on its size t_i, i.e., the number of transactions in it [16], [34]. Thus, the bigger the block is, the more time needed to propagate the block to the whole mobile blockchain network [35]. Same as [16], we assume this time function is linear, i.e., $\tau(t_i) = z \times t_i$ with $z > 0$ represents a given delay factor. Note that this linear approximation is acceptable according to the numerical results from [16], [36]. Additionally, it would be more appropriate to add a constant term in this function [35], but apparently this constant term has no effect on our subsequent analytical results. Thus, the probability that the miner i successfully mines a block and its solution reaches consensus is expressed as follows:

$$P_i(\alpha_i(x_i, \mathbf{x}_{-i}), t_i) = \alpha_i e^{-\lambda z t_i}, \tag{5}$$

where $\alpha_i(x_i, \mathbf{x}_{-i})$ is given in (1).

C. Two-Stage Stackelberg Game Formulation

The interaction between the ESP and miners can be modeled as a two-stage Stackelberg game, as illustrated in Fig. 2. The ESP, i.e., the leader, sets the price in the upper Stage I. The miners, i.e., the followers, decide on their optimal computing service demand for offloading in the lower Stage II, being aware of the price set by the ESP. By using backward induction, we formulate the optimization problems for the leader and followers as follows.

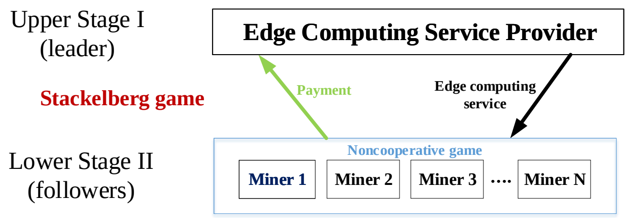

Figure 2. Two-stage Stackelberg game model of the interactions among the ESP and miners in the mobile blockchain.

1) Miners' mining strategies in Stage II: Given the pricing of the ESP and other miners' strategies, the miner i determines its computing service demand for its hash power maximizing the expected utility which is given as:

$$u_i(x_i, \mathbf{x}_{-i}, p_i) = (R + rt_i)\frac{x_i}{\sum_{j\in\mathcal{N}} x_j} e^{-\lambda z t_i} - p_i x_i, \tag{6}$$

where p_i is the price per unit for service demand of miner i. The miner sub-game problem can be written as follows:

Problem 1. (Miner i sub-game):

$$\begin{aligned} \underset{x_i}{\text{maximize}} \quad & u_i(x_i, \mathbf{x}_{-i}, p_i) \\ \text{subject to} \quad & x_i \in [\underline{x}, \overline{x}]. \end{aligned} \tag{7}$$

2) ESP's pricing strategies in Stage I: The profit of the ESP is the revenue obtained from charging the miners for computing service minus the service cost. The service cost is directly related to the time that the miner takes to mine a block, the cost of electricity, c, and the other cost that is a function of the service demand x_i. Therefore, the ESP decides the pricing within the strategy space $\{\mathbf{p} = [p_i]_{i\in\mathcal{N}} : 0 \leq p_i \leq \overline{p}\}$ to maximize its profit which is represented as:

$$\Pi(\mathbf{p}, \mathbf{x}) = \sum_{i\in\mathcal{N}} p_i x_i - \sum_{i\in\mathcal{N}} cTx_i. \tag{8}$$

Note that practically the price is bounded by maximum price constraint that is denoted by $\overline{p}$. Then, the profit maximization problem of the ESP is formulated as follows.

Problem 2. (ESP sub-game):

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \Pi(\mathbf{p}, \mathbf{x}) \\ \text{subject to} \quad & 0 \leq p_i \leq \overline{p}. \end{aligned} \tag{9}$$

Problem 1 and Problem 2 together form the Stackelberg game, and the objective of this game is to find the Stackelberg equilibrium. The Stackelberg equilibrium is a point where the payoff of the leader is maximized given that the followers adopt their best responses, i.e., the Nash eqilibrium [37]. In our problem, the Stackelberg equilibrium can be written as follows.

Definition 1. *Let $\mathbf{x}^*$ and $\mathbf{p}^*$ denote the optimal service demand vector of all the miners and optimal unit price vector of edge computing service, respectively. Then, the point $(\mathbf{x}^*, \mathbf{p}^*)$ is the Stackelberg equilibrium if the following conditions,*

$$\Pi(\mathbf{p}^*, \mathbf{x}^*) \geq \Pi(\mathbf{p}, \mathbf{x}^*) \tag{10}$$

and

$$u_i(x_i^*, \mathbf{x}_{-i}^*, \mathbf{p}^*) \geq u_i(x_i, \mathbf{x}_{-i}^*, \mathbf{p}^*), \forall x_i \geq 0, \forall i \tag{11}$$

are satisfied, where $\mathbf{x}_{-i}^*$ is the best response service demand vector for all the miners except miner i.

Note that the same or different prices can be applied to the miners, which we refer to them as the uniform and discriminatory pricing schemes, respectively. In the following, we investigate these two pricing schemes for resource management in mobile blockchain.

IV. Equilibrium Analysis for Edge Computing Resource Management

In this section, we propose the uniform pricing and discriminatory pricing schemes for resource management in mobile blockchain. We then analyze the optimal service demand of miners as well as the profit maximization of the ESP under both pricing schemes.

A. Uniform Pricing Scheme

We first consider the uniform pricing scheme, in which the ESP charges all the miners the same unit price for their computing service demand, i.e., $p_i = p, \forall i$. Given the payoff functions defined in Section III, we use backward induction to analyze the Stackelberg game.

1) Stage II: Miners' Demand Game: Given the price p decided by the ESP, in Stage II, the miners compete with each other to maximize their own utility by choosing their individual service demand, which forms the noncooperative Miners' Demand Game (MDG) $\mathcal{G}^u = \{\mathcal{N}, \{x_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$, where $\mathcal{N}$ is the set of miners, $\{x_i\}_{i\in\mathcal{N}}$ is the strategy set, and u_i is the utility, i.e., payoff, function of miner i. Specifically, each miner $i \in \mathcal{N}$ selects its strategy to maximize its utility function $u_i(x_i, \mathbf{x}_{-i}, p)$. We next analyze the existence and uniqueness of the Nash equilibrium in the MDG.

Definition 2. *A demand vector $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$ is the Nash equilibrium of the MDG $\mathcal{G}^u = \{\mathcal{N}, \{x_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$, if, for every miner $i \in \mathcal{N}$, $u_i(x_i^*, \mathbf{x}_{-i}^*, p) \geq u_i(x_i{}', \mathbf{x}_{-i}^*, p)$ for all $x_i{}' \in [\underline{x}, \overline{x}]$, where $u_i(x_i, \mathbf{x}_{-i})$ is the resulting utility of the miner i, given the other miners' demand $\mathbf{x}_{-i}$.*

Theorem 1. *A Nash equilibrium exists in MDG $\mathcal{G}^u = \{\mathcal{N}, \{x_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$.*

Proof. Firstly, the strategy space for each miner is defined to be $[\underline{x}, \overline{x}]$, which is a non-empty, convex, compact subset of the Euclidean space. From (6), u_i is apparently continuous in $[\underline{x}, \overline{x}]$. Then, we take the first order and second order derivatives of (6) with respect to x_i to prove its concavity, which can be written as follows:

$$\frac{\partial u_i}{\partial x_i} = (R + rt_i)e^{-\lambda z t_i}\frac{\partial \alpha_i}{\partial x_i} - p, \tag{13}$$

and

$$\frac{\partial^2 u_i}{\partial {x_i}^2} = (R + rt_i)e^{-\lambda z t_i}\frac{\partial^2 \alpha_i}{\partial {x_i}^2} < 0, \tag{14}$$

$$x_i{}^* = \mathscr{F}_i(\mathbf{x}) = \begin{cases} \underline{x}, & \sqrt{\frac{(R+rt_i)\sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x_j < \underline{x} \\ \sqrt{\frac{(R+rt_i)\sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x_j, & \underline{x} \le \sqrt{\frac{(R+rt_i)\sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x_j \le \overline{x} \\ \overline{x}, & \sqrt{\frac{(R+rt_i)\sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x_j > \overline{x} \end{cases}. \tag{12}$$

where

$$\frac{\partial \alpha_i}{\partial x_i} = \frac{\sum_{i\neq j} x_j}{\left(\sum_{i\in\mathcal{N}} x_j\right)^2} > 0, \tag{15}$$

and

$$\frac{\partial^2 \alpha_i}{\partial x_i{}^2} = -2\frac{\sum_{i\neq j} x_j}{\left(\sum_{i\in\mathcal{N}} x_j\right)^3} < 0. \tag{16}$$

Therefore, we have proved that u_i is strictly concave with respect to x_i. Accordingly, the Nash equilibrium exists in this noncooperative MDG $\mathcal{G}^u$ [37]. The proof is now completed. □

Further, based on the first order derivative condition, we have

$$\frac{\partial u_i}{\partial x_i} = (R + rt_i)e^{-\lambda z t_i}\frac{\partial \alpha_i}{\partial x_i} - p = 0, \tag{17}$$

and we obtain the best response function of miner i by solving (17), as shown in (12).

Theorem 2. *The uniqueness of the Nash equilibrium in the noncooperative MDG is guaranteed given the following condition*

$$\frac{2(N-1)e^{-\lambda z t_i}}{R + rt_i} < \sum_{i\in\mathcal{N}} \frac{e^{-\lambda z t_i}}{R + rt_i} \tag{18}$$

is satisfied.

Proof. Let $\mathbf{x}^*$ denote the Nash equilibrium of the MDG. By definition, the Nash equilibrium needs to satisfy $\mathbf{x} = \mathscr{F}(\mathbf{x})$, in which $\mathscr{F}(\mathbf{x}) = (\mathscr{F}_1(\mathbf{x}), \mathscr{F}_2(\mathbf{x}), \ldots, \mathscr{F}_N(\mathbf{x}))$. In particular, $\mathscr{F}_i(\mathbf{x})$ is the best response function of miner i, given the demand strategies of other miners. The uniqueness of the Nash equilibrium can be proved by showing that the best response function of miner i, i.e., as given in (12), is the standard function [37].

Definition 3. *A function $\mathscr{F}(\mathbf{x})$ is a standard function when the following properties are guaranteed [37]:*
(1) *Positivity: $\mathscr{F}(\mathbf{x}) > \mathbf{0}$;*
(2) *Monotonicity: If $\mathbf{x} \le \mathbf{x}'$, then $\mathscr{F}(\mathbf{x}) \le \mathscr{F}(\mathbf{x}')$;*
(3) *Scalability: For all $\lambda > 1$, $\lambda\mathscr{F}(\mathbf{x}) > \mathscr{F}(\lambda\mathbf{x})$.*

Firstly, for the positivity, under the condition in (18), we have (from Lemma 1)

$$\sum_{i\neq j} x_j < \frac{R + rt_i}{pe^{-\lambda z t_i}} < \frac{R + rt_i}{4pe^{-\lambda z t_i}}, \tag{19}$$

then we can conclude that

$$\sum_{i\neq j} x_j < \sqrt{\frac{R + rt_i \sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}}. \tag{20}$$

Thus, we can prove that

$$\mathscr{F}_i(\mathbf{x}) = \sqrt{\frac{R + rt_i \sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x_j > 0, \tag{21}$$

which is the positivity condition. Secondly, we prove the monotonicity of (12). Let $\mathbf{x}' > \mathbf{x}$, we can further simplify the expression of $\mathcal{F}_i(\mathbf{x}') - \mathcal{F}_i(\mathbf{x})$, which is shown in (22). In particular, we have $\sqrt{\sum_{i\neq j} x'_j} - \sqrt{\sum_{i\neq j} x_j} > 0$, and we can easily verify that

$$\sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}}} - \sqrt{\sum_{i\neq j} x'_j} - \sqrt{\sum_{i\neq j} x_j} \in \left(\sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}}} - 2\sqrt{\sum_{i\neq j} x'_j}, \sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}}} - 2\sqrt{\sum_{i\neq j} x_j}\right). \tag{24}$$

Under the condition in (32), we can prove that

$$\sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}}} - 2\sqrt{\sum_{i\neq j} x_j} > 0, \forall x_j. \tag{25}$$

Thus, the best response function of miner i in (12) is always positive.

At last, as for scalability, we need to prove that $\lambda\mathscr{F}(x) > \mathscr{F}(\lambda x)$, for $\lambda > 1$. The steps of proving the positivity of $\lambda\mathcal{F}(x) - \mathcal{F}(\lambda x)$ are shown in (23). Therefore, $\lambda\mathscr{F}(x) > \mathscr{F}(\lambda x)$ is always satisfied for $\lambda > 1$. Until now, we have proved that the best response function in (12) satisfies three properties described in Definition 2. Therefore, the Nash equilibrium of MDG $\mathcal{G}^u = \{\mathcal{N}, \{x_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$ is unique. The proof is now completed. □

$$\begin{aligned}\mathcal{F}_i(\mathbf{x}') - \mathcal{F}_i(\mathbf{x}) &= \sqrt{\frac{R + rt_i \sum_{i\neq j} x'_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x'_j - \sqrt{\frac{(R + rt_i) \sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} x_j \\ &= \sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}}} \left(\sqrt{\sum_{i\neq j} x'_j} - \sqrt{\sum_{i\neq j} x_j} \right) - \left(\sum_{i\neq j} x'_j - \sum_{i\neq j} x_j \right) \\ &= \left(\sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}}} - \sqrt{\sum_{i\neq j} x'_j} - \sqrt{\sum_{i\neq j} x_j} \right) \left(\sqrt{\sum_{i\neq j} x'_j} - \sqrt{\sum_{i\neq j} x_j} \right). \end{aligned} \tag{22}$$

$$\begin{aligned}\lambda\mathcal{F}_i(\mathbf{x}) - \mathcal{F}_i(\lambda\mathbf{x}) &= \lambda\sqrt{\frac{(R + rt_i) \sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} - \lambda\sum_{i\neq j} x_j - \sqrt{\frac{(R + rt_i) \sum_{i\neq j} \lambda x_j}{pe^{-\lambda z t_i}}} - \sum_{i\neq j} \lambda x_j \\ &= \left(\lambda - \sqrt{\lambda}\right) \sqrt{\frac{(R + rt_i) \sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}} > 0, \forall \lambda > 1. \end{aligned} \tag{23}$$

Theorem 3. *The unique Nash equilibrium for miner i in the MDG is given by*

$$x_i{}^* = \frac{N-1}{\sum_{j\in\mathcal{N}} \frac{pe^{-\lambda z t_j}}{R+rt_j}} - \left(\frac{N-1}{\sum_{j\in\mathcal{N}} \frac{pe^{-\lambda z t_j}}{R+rt_j}} \right)^2 \frac{pe^{-\lambda z t_i}}{R + rt_i}, \forall i, \tag{26}$$

provided that the condition in (18) holds.

Proof. According to (13), for each miner i, we have the mathematical expression

$$\frac{\sum_{i\neq j} x_j}{\left(\sum_{j\in\mathcal{N}} x_j\right)^2} = \frac{pe^{-\lambda z t_i}}{R + rt_i}. \tag{27}$$

Then, we calculate the summation of this expression for all the miners as follows:

$$\frac{(N-1) \sum_{j\in\mathcal{N}} x_j}{\left(\sum_{j\in\mathcal{N}} x_j\right)^2} = \sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R + rt_i}, \tag{28}$$

which means $\frac{(N-1)}{\sum_{j\in\mathcal{N}} x_j} = \sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R+rt_i}$. Thus, we have

$$\sum_{j\in\mathcal{N}} x_j = \frac{N-1}{\sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R+rt_i}}. \tag{29}$$

Recall from (12), according to the first order derivative condition, we have

$$\sum_{j\in\mathcal{N}} x_j = \sqrt{\frac{(R + rt_i) \sum_{i\neq j} x_j}{pe^{-\lambda z t_i}}}. \tag{30}$$

By substituting (30) into (29), we have

$$\frac{N-1}{\sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R+rt_i}} = \sqrt{\frac{R + rt_i}{pe^{-\lambda z t_i}} \left(\frac{N-1}{\sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R+rt_i}} - x_i \right)}. \tag{31}$$

After squaring both sides, we have $\left(\frac{N-1}{\sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R+rt_i}} \right)^2 = \frac{R+rt_i}{pe^{-\lambda z t_i}} \left(\frac{N-1}{\sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R+rt_i}} - x_i \right)$. With simple transformations, we obtain the Nash equilibrium for miner i as shown in (26). □

Lemma 1. *Given*

$$\frac{2(N-1)e^{-\lambda z t_i}}{R + rt_i} < \sum_{i\in\mathcal{N}} \frac{e^{-\lambda z t_i}}{R + rt_i}, \tag{32}$$

the following condition

$$\sum_{i\neq j} x_j < \frac{R + rt_i}{4pe^{-\lambda z t_i}} \tag{33}$$

is satisfied.

Proof. According to (26) and (29), we can obtain

$$\sum_{j\neq i} x_j = \left(\frac{N-1}{\sum_{j\in\mathcal{N}} \frac{pe^{-\lambda z t_j}}{R+rt_j}} \right)^2 \frac{pe^{-\lambda z t_i}}{R + rt_i}. \tag{34}$$

After substituting (32) into (34), we have

$$\frac{2(N-1)pe^{-\lambda z t_i}}{R + rt_i} < \sum_{i\in\mathcal{N}} \frac{pe^{-\lambda z t_i}}{R + rt_i}, \tag{35}$$

which means that the condition in (32) needs to be ensured. On the contrary, if the condition in (32) holds, then, the condition in (34) is satisfied. The proof is now completed. □

Generally, we can use the best-response dynamics for obtaining the Nash equilibrium of the N-player noncooperative game in Stage II [37]. In the following, we analyze the profit maximization of the ESP in Stage I under uniform pricing.

2) Stage I: ESP's Profit Maximization: Based on the Nash equilibrium of the computing service demand in the MDG $\mathcal{G}^u = \{\mathcal{N}, \{x_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$ in Stage II, the leader of the Stackelberg game, i.e., the ESP, can optimize its pricing strategy in Stage I to maximize its profit defined in (8). Thus, the optimal pricing can be formulated as an optimization problem. By substituting (26) into (8), the profit maximization of the ESP is simplified as follows:

$$\begin{aligned} &\underset{p>0}{\text{maximize}} \quad \Pi(p) = (p - cT)\frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{pe^{-\lambda z t_j}}{R+rt_j}} \\ &\text{subject to} \quad 0 \le p \le \overline{p}. \end{aligned} \tag{36}$$

Theorem 4. *Under uniform pricing, the ESP achieves the globally optimal profit, i.e., profit maximization, under the unique optimal price.*

Proof. From (36), we have

$$\Pi(p) = \frac{p-cT}{p}\frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{e^{-\lambda z t_j}}{R+rt_j}}. \tag{37}$$

The first and second derivatives of profit $\Pi(p)$ with respect to price p are given as follows:

$$\frac{d\Pi(p)}{dp} = \frac{cT}{p^2}\frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{e^{-\lambda z t_j}}{R+rt_j}} \tag{38}$$

and

$$\frac{d^2\Pi(p)}{dp^2} = -\frac{2cT}{p^2}\frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{e^{-\lambda z t_j}}{R+rt_j}} < 0. \tag{39}$$

Due to the negativity of (39), the strict concavity of the objective function is ensured. Thus, the ESP is able to achieve the maximum profit with the unique optimal price. The proof is now completed. □

Note that the profit maximization defined in (36) is a convex optimization problem, and thus it can be solved by standard convex optimization algorithms, e.g., gradient assisted binary search. Under uniform pricing, we have proved that the Nash equilibrium in Stage II is unique and the optimal price in Stage I is also unique. Thus, we can conclude that the Stackelberg equilibrium is unique and accordingly the best-response dynamics algorithm can achieve this unique Stackelberg equilibrium [37].

B. Discriminatory Pricing Scheme

Then, we consider the discriminatory pricing scheme, in which the ESP is able to set different unit prices of service demand for different miners. Again, we use the backward induction to analyze the optimal service demand of miners and the profit maximization of the ESP.

1) Stage II: Miners' Demand Game: Under discriminatory pricing scheme, the strategy space of the ESP becomes $\{\mathbf{p} = [p_i]_{i\in\mathcal{N}} : 0 \le p_i \le \overline{p}\}$. Recall that we prove the existence and uniqueness of MDG $\mathcal{G}^u = \{\mathcal{N}, \{x_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$, given the fixed price from the ESP. Thus, under discriminatory pricing, the existence and uniqueness of the MDG can be still guaranteed. With minor change from Theorem 3, we have the following theorem immediately.

Theorem 5. *Under uniform pricing, the unique Nash equilibrium demand of miner i can be obtained as follows:*

$$x_i{}^* = \frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{p_j e^{-\lambda z t_j}}{R+rt_j}} - \left(\frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{p_j e^{-\lambda z t_j}}{R+rt_j}}\right)^2 \frac{p_i e^{-\lambda z t_i}}{R+rt_i}, \forall i, \tag{40}$$

if the following condition

$$\frac{2(N-1)p_i e^{-\lambda z t_i}}{R+rt_i} < \sum_{j\in\mathcal{N}} \frac{p_j e^{-\lambda z t_j}}{R+rt_j} \tag{41}$$

holds.

Proof. The steps of proof are similar to those in the case of uniform pricing as shown in Section IV-A1, and thus we omit them for brevity. □

We next analyze the profit maximization of the ESP in Stage I under discriminatory pricing to further investigate the Stackelberg equilibrium.

2) Stage I: ESP's Profit Maximization: Similar to that in Section IV-A2, we analyze the profit maximization with the analytical result from Theorem 5, i.e., the Nash equilibrium of the computing service demand in Stage II. After substituting (40) into (8), we have the following optimization,

$$\begin{aligned} &\underset{\mathbf{p}>\mathbf{0}}{\text{maximize}} \quad \Pi(\mathbf{p}) = \sum_{i\in\mathcal{N}}\left(p_i - cT\frac{N-1}{\sum\limits_{j\in\mathcal{N}} \frac{p_j e^{-\lambda z t_j}}{R+rt_j}}\right) \\ &\text{subject to} \quad 0 \le p_i \le \overline{p}, \forall i. \end{aligned} \tag{42}$$

Theorem 6. $\Pi(\mathbf{p})$ *is concave on each* p_i*, when* $\sum\limits_{i\neq j}(a_i + a_j)\left(1 - \frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right) \le 0$*, and decreasing on each* p_i *when* $\sum\limits_{i\neq j}(a_i + a_j)\left(1 - \frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right) > 0$*, provided that the following condition*

$$\frac{p_i}{a_i} \ge \frac{\sum\limits_{j\in\mathcal{N}} \frac{p_j}{a_j}}{(N-1)^2} \tag{45}$$

$$g(\mathbf{p}) = \frac{\sum\limits_{j\neq h}\left(a_h\left(\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}} - \left(\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\right)^2\frac{p_h}{a_h}\right)\left(\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}} - \left(\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\right)^2\frac{p_j}{a_j}\right)\right)}{\left(\frac{N-1}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right)^2}$$

$$= \sum_{j\neq h}\left(a_h\left(1-\frac{p_h}{a_h}\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\right)\left(1-\frac{p_j}{a_j}\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\right)\right). \tag{43}$$

$$\frac{\partial g(\mathbf{p})}{\partial p_i} = \sum_{j\neq i}\left((a_i+a_j)\left(\frac{-\frac{N-1}{a_i}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right) + \frac{\frac{N-1}{a_i}\frac{p_j}{a_j}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\right). \tag{44}$$

---

is satisfied, where $a_i = (R+rt_i)e^{-\lambda z t_i}$.

Proof. We firstly decompose the objective function in (42) into two parts, namely, $\sum\limits_i cTx_i^*$ and $\sum\limits_i p_i x_i^*$. Then, we analyze the properties of each part. We define

$$f(\mathbf{p}) = -cTx_i^* = -cT\frac{N-1}{\sum\limits_{j\in\mathcal{N}}\frac{p_j e^{-\lambda z t_j}}{R+rt_j}}. \tag{46}$$

Let $a_j = (R+rt_j)e^{-\lambda z t_j}$, and we have $f(\mathbf{p}) = \frac{-cT(N-1)}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}$. Then, we obtain the first and the second partial derivatives of (46) with respect to p_i as follows.

$$\frac{\partial f(\mathbf{p})}{\partial p_i} = \frac{(N-1)cT}{a_i\left(\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}\right)^2}, \tag{47}$$

$$\frac{\partial^2 f(\mathbf{p})}{\partial {p_i}^2} = \frac{-2(N-1)cT}{{a_i}^2\left(\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}\right)^3}. \tag{48}$$

Further, we have

$$\frac{\partial f(\mathbf{p})}{\partial p_i p_j} = \frac{-2(N-1)cT}{a_i a_j\left(\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}\right)^3}. \tag{49}$$

Thus, we can obtain the Hessian matrix of $f(\mathbf{p})$, which is expressed as:

$$\nabla^2 f(\mathbf{p}) = \frac{-2(N-1)cT}{\left(\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}\right)^3}\begin{bmatrix} \frac{1}{{a_1}^2} & \frac{1}{a_1a_2} & \cdots & \frac{1}{a_1a_N} \\ \frac{1}{a_2a_1} & \frac{1}{{a_2}^2} & \cdots & \frac{1}{a_2a_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_Na_1} & \frac{1}{a_Na_2} & \cdots & \frac{1}{{a_N}^2} \end{bmatrix}. \tag{50}$$

For each $i \in \mathcal{N}$, we have $\frac{1}{{a_i}^2} > 0$. Thus, the diagonal elements of the Hessian matrix are all larger than zero, and the principle minors are equal to zero. Therefore, the Hessian matrix of $f(\mathbf{p})$ is semi-negative definite.

Then, we analyze the properties of $\sum\limits_i p_i x_i^*$. We first define

$$g(\mathbf{p}) = \sum_{i\in\mathcal{N}} p_i {x_i}^* = \frac{\sum\limits_{j\neq i} a_i x_i x_j}{\left(\sum\limits_{j\neq i} x_j\right)^2}. \tag{51}$$

By substituting (40) into (51), we can obtain the final expression for $g(\mathbf{p})$, which can be rewritten as in (43). Then, we derive the first order and the second partial derivatives of (43) with respect to p_i as shown in (44) and (53). Since we have $x_i = \frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}} - \frac{p_i}{a_i}\left(\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\right)^2 = \frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right) > 0$, $1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i} > 0$. When $\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right) \leq 0$, it is observed that $\frac{\partial^2 g(\mathbf{p})}{\partial {p_i}^2} < 0$, i.e., $g(\mathbf{p})$ is concave on each p_i. Now we prove that $\Pi(\mathbf{p})$ is a monotonically decreasing function with respect to p_i, when $\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right) > 0$. The steps are shown in (52), where $p_{\min} = \min\{p_1, p_2, \ldots, p_N\}$. Practically, $p_{\min} > cT$. Thus, with some manipulations, we can prove $\frac{\partial \Pi}{\partial p_i} < 0$ when $\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right) > 0$, if the condition in (45) holds. The proof is now completed. □

Theorem 7. *Under discriminatory pricing, the ESP achieves the profit maximization by finding the unique optimal pricing vector.*

Proof. From Theorem 6, we know that $\Pi(\mathbf{p})$ is concave on each p_i, when $\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right) \leq 0$, and

$$
\begin{aligned}
\frac{\partial \Pi(\mathbf{p})}{\partial p_i} &= \sum_{j\neq i}\left((a_i+a_j)\left(\frac{\frac{N-1}{a_i}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)+\frac{\frac{N-1}{a_i a_j}p_j}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\right)+\frac{\frac{N-1}{a_i}cT}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\\
&= \frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(\sum_{j\neq i}\left((a_i+a_j)\left(-\sum_{h\neq i}\frac{p_h}{a_h}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)+\frac{p_j}{a_j}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\right)+cT\right)\\
&\leq \frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(\sum_{j\neq i}\left((a_i+a_j)\left(-\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)+\frac{p_j}{a_j}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\right)+cT\right)\\
&= \frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(\sum_{j\neq i}\left((a_i+a_j)\left(\underbrace{-\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)}_{>0}-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\frac{p_j}{a_j}\right)\right)+cT\right)\\
&= \underbrace{-\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\sum_{j\neq i}\left((a_i+a_j)\left(\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}\left(1-\frac{N}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)\right)\right)}_{<0}+\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(cT-\sum_{j\neq i}\left((a_i+a_j)\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\frac{p_j}{a_j}\right)\right)\\
&= -\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\sum_{j\neq i}\left((a_i+a_j)\left(-\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}\left(1-\frac{N}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)\right)\right)+\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(cT-\sum_{j\neq i}\left(\underbrace{\frac{a_i+a_j}{a_j}}_{<1}\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i p_j}{a_i}\right)\right)\\
&\leq -\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\sum_{j\neq i}\left((a_i+a_j)\left(-\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}\left(1-\frac{N}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)\right)\right)+\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(cT-p_{\min}\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{N-1p_i}{a_i}\right)\\
&= \underbrace{-\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\sum_{j\neq i}\left((a_i+a_j)\left(-\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}\left(1-\frac{N}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)\right)\right)}_{<0}+\underbrace{\frac{\frac{N-1}{a_i}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\left(cT-p_{\min}\frac{(N-1)^2}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)}_{<0}<0. \qquad (52)
\end{aligned}
$$

decreasing on each p_i when $\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right)>0$. In other words, when $\Pi(\mathbf{p})$ is concave on p_i, p_i needs to be smaller than a certain threshold, and $\Pi(\mathbf{p})$ is decreasing on p_i when p_i is larger than this threshold. Then, it can be concluded that if the price is higher than the threshold, the miner is not willing to purchase the computing service from the ESP. Therefore, we know that the optimal value of profit of the ESP, i.e., $\Pi^*(\mathbf{p})$ is achieved in the concave parts when $\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right)\leq 0$. Clearly, the maximization of profit $\Pi(\mathbf{p})$ is achieved either in the boundary of domain area or in the local maximization point. Since we know that the optimal value of profit, i.e., $\Pi^*(\mathbf{p})$ is achieved in the interior area, and thus $\mathbf{p}^*$ exists. In the following, we prove that there exists at most one optimal solution by using Variational Inequality theory [38], from which the uniqueness of the optimal solution, i.e., the Stackelberg equilibrium, follows.

Let the set $\mathcal{K}=\left\{\mathbf{p}=[p_1,\ldots,p_N]^\top\middle|\sum\limits_{i\neq j}(a_i+a_j)\left(1-\frac{N\frac{p_j}{a_j}}{\sum\limits_{j\in\mathcal{N}}\frac{p_j}{a_j}}\right)\leq 0,\forall i\in\mathcal{N}\right\}$. The constraint can be rewritten as follows:

$$
\sum_{i\neq j}\left((a_i+a_j)\left(\sum_{h\in\mathcal{N}}\frac{p_h}{a_h}-N\frac{p_j}{a_j}\right)\right)\leq 0. \qquad (56)
$$

Thus, we redefine the set $\mathcal{K}$ as $\left\{\mathbf{p}=[p_1,\ldots,p_N]^\top\middle|\sum\limits_{i\neq j}\left((a_i+a_j)\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}-N\frac{p_j}{a_j}\right)\right)\leq 0,\forall i\in\mathcal{N}\right\}$. Then,

$$
\begin{aligned}
\frac{\partial^2 g(\mathbf{p})}{\partial {p_i}^2} &= \sum_{j\neq i}\left((a_i+a_j)\left(\frac{2\frac{N-1}{{a_i}^2}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)-2\left(\frac{\frac{N-1}{a_i}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\frac{\frac{N-1}{a_i}\frac{p_j}{a_j}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^2}\right)-\frac{2\frac{N-1}{{a_i}^2}\frac{p_j}{a_j}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\right)\\
&= \sum_{j\neq i}\left((a_i+a_j)\left(\frac{2\frac{N-1}{{a_i}^2}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\left(1-2\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)-\frac{2\frac{N-1}{{a_i}^2}\frac{p_j}{a_j}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\right).
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
\frac{\partial^2 g(\mathbf{p})}{\partial {p_i}^2} &= \frac{2\frac{N-1}{{a_i}^2}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\sum_{j\neq i}\left((a_i+a_j)\left(1-2\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)\right)-\frac{2\frac{N-1}{{a_i}^2}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\sum_{j\neq i}\left((a_i+a_j)\frac{p_j}{a_j}\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)\right)\\
&\leq \frac{2\frac{N-1}{{a_i}^2}\sum\limits_{h\neq i}\frac{p_h}{a_h}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\underbrace{\sum_{j\neq i}\left((a_i+a_j)\left(1-\frac{N}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_j}{a_j}\right)\right)}_{\leq 0}-\frac{2\frac{N-1}{{a_i}^2}}{\left(\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}\right)^3}\sum_{j\neq i}\left((a_i+a_j)\frac{p_j}{a_j}\underbrace{\left(1-\frac{N-1}{\sum\limits_{h\in\mathcal{N}}\frac{p_h}{a_h}}\frac{p_i}{a_i}\right)}_{\geq 0}\right).
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
&\sum_{i\neq j}\left((a_i+a_j)\left(\sum_{h\in\mathcal{N}}\frac{\lambda p_h^{'}+(1-\lambda)p_h^{''}}{a_h}-N\frac{\lambda p_j^{'}+(1-\lambda)p_j^{''}}{a_j}\right)\right)\\
=\ &\sum_{i\neq j}\left((a_i+a_j)\left(\lambda\sum_{h\in\mathcal{N}}\frac{p_h^{'}}{a_h}-(1-\lambda)\sum_{h\in\mathcal{N}}\frac{p_h^{''}}{a_h}-\lambda N\frac{p_j^{'}}{a_j}-(1-\lambda)N\frac{p_j^{''}}{a_j}\right)\right)\\
=\ &\lambda\sum_{i\neq j}\left((a_i+a_j)\left(\sum_{h\in\mathcal{N}}\frac{p_h^{'}}{a_h}-N\frac{p_j^{''}}{a_j}\right)\right)+(1-\lambda)\sum_{i\neq j}\left((a_i+a_j)\left((1-\lambda)\sum_{h\in\mathcal{N}}\frac{p_h^{'}}{a_h}-N\frac{p_j^{''}}{a_j}\right)\right)\leq 0.
\end{aligned}
\tag{55}
$$

---

we formulate an equivalent problem to (42) as follows:

$$
\begin{aligned}
&\underset{\mathbf{p}>\mathbf{0}}{\text{minimize}} \quad -\Pi(\mathbf{p})\\
&\text{subject to} \quad \mathbf{p}\in\mathcal{K}.
\end{aligned}
\tag{57}
$$

Let $F(\mathbf{p})=\nabla\left(-\Pi(\mathbf{p})\right)=-[\nabla_{p_i}\Pi]_{i\in\mathcal{N}}^{\top}$. Accordingly, the optimization problem in (57) is equivalent to find a point set $\mathbf{p}^*\in\mathcal{K}$, such that $(\mathbf{p}-\mathbf{p}^*)F(\mathbf{p}^*)\geq 0, \forall\mathbf{p}\in\mathcal{K}$, which is the Variational Inequality (VI) problem: $\mathrm{VI}(\mathcal{K},F)$.

Definition 4. *If F is strictly monotone on $\mathcal{K}$, then VI$(\mathcal{K},F)$ has at most one solution, where $\mathcal{K}\in\mathbb{R}^N$ is a convex closed set, and the mapping $F:\mathcal{K}\mapsto\mathbb{R}^N$ is continuous [38].*

Let $\lambda\in(0,1)$, $\mathbf{p}',\mathbf{p}''\in\mathcal{K}$, it can be concluded that $\lambda\mathbf{p}'+(1-\lambda)\mathbf{p}'\in\mathcal{K}$, which is shown in (55). Accordingly, $\mathcal{K}$ is a convex and closed set. To prove that the mapping $F:\mathcal{K}\mapsto\mathbb{R}^N$ is strictly monotone on $\mathcal{K}$, we check the positivity of $(\mathbf{p}'-\mathbf{p}'')^{\top}(F(\mathbf{p}')-F(\mathbf{p}'')), \forall\mathbf{p}',\mathbf{p}''\in\mathcal{K}$ and $\mathbf{p}'\neq\mathbf{p}''$. We know

$$
\begin{aligned}
&(\mathbf{p}'-\mathbf{p}'')^{\top}(F(\mathbf{p}')-F(\mathbf{p}''))=\\
&\quad\sum_{i\in\mathcal{N}}\left((p_i'-p_i'')\left(-\left.\nabla_{p_i}\Pi\right|_{p_i=p_i'}+\left.\nabla_{p_i}\Pi\right|_{p_i=p_i''}\right)\right),
\end{aligned}
\tag{58}
$$

and from Theorem 6, we have

$$
\frac{\partial^2\Pi(\mathbf{p})}{\partial {p_i}^2}=\frac{\partial^2(f(\mathbf{p})+g(\mathbf{p}))}{\partial {p_i}^2}<0. \tag{59}
$$

Thus, $\nabla_{p_i}\Pi$ is decreasing on each p_i, and $-\nabla_{p_i}\Pi$ is increasing on each p_i. It can be concluded that

$$
-\left.\nabla_{p_i}\Pi\right|_{p_i=p_i'}+\left.\nabla_{p_i}\Pi\right|_{p_i=p_i''}=\begin{cases}\geq 0, p_i'\geq p_i''\\ <0, p_i'<p_i''.\end{cases} \tag{60}
$$

Then, we have

$$
\left((p_i'-p_i'')\left(-\left.\nabla_{p_i}\Pi\right|_{p_i=p_i'}+\left.\nabla_{p_i}\Pi\right|_{p_i=p_i''}\right)\right)\geq 0, \forall i\in\mathcal{N}, \tag{61}
$$

and we know $\mathbf{p}'\neq\mathbf{p}''$, and accordingly there exists at least one $j\in\mathcal{N}$ which satisfies the constraint in (61). Therefore, we have proved that F is strictly monotone on $\mathcal{K}$ and continuous. Until now, we have proved that $\mathrm{VI}(\mathcal{K},F)$ has at most one solution according to Definition 4 in [38]. Thus, the equivalent problem admits at most one optimal solution. Since we know the existence of a single optimal solution, and thus the uniqueness of the optimal solution is validated. The proof is now completed. $\square$

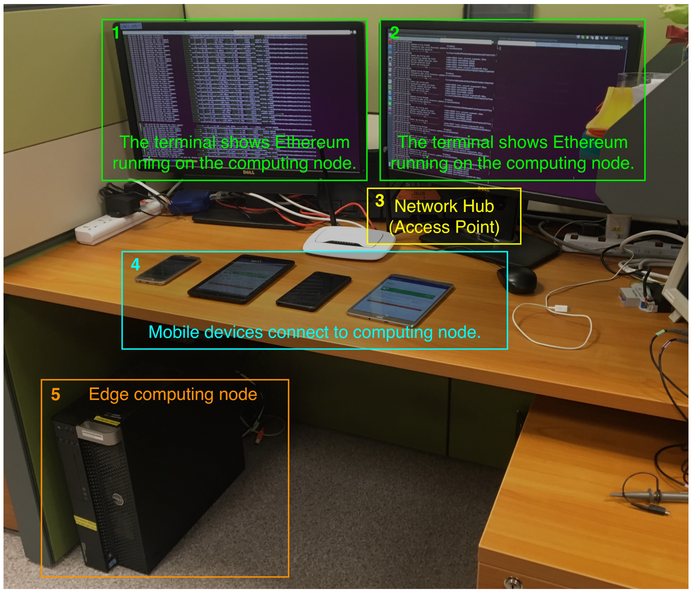

Figure 3. Real mobile blockchain mining experimental setup with Ethereum which is a popular open ledger.

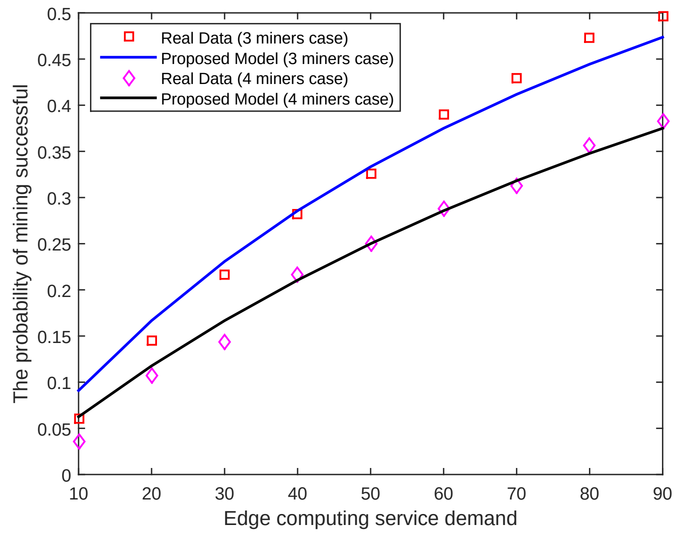

Figure 4. The comparison of real experiment results with our proposed model.

Similar to that in Section IV-A, we can apply the low-complexity gradient based searching algorithm to achieve the maximized profit $\Pi(\mathbf{p})$ of the ESP. In particular, we adopt Algorithm 1 to obtain the unique Stackelberg equilibrium, under which the ESP achieves the profit maximization according to Theorem 7. The basic description is explained as follows: for the given prices imposed by the ESP, the followers' sub-game is solved first. After substituting the best responses of the followers' sub-game into the leader sub-game, the optimal prices can be obtained by a gradient-based algorithm. The similar algorithm can be used for uniform pricing as well.

Algorithm 1 Gradient iterative algorithm to find Stackelberg equilibrium under discriminatory pricing

1: **Initialization:**
Select initial input $\mathbf{p} = [p_i]_{i\in\mathcal{N}}$ where $p_i \in [0, \overline{p}]$, $k \leftarrow 1$, precision threshold ε;
2: **repeat**
3: Each miner i decides its computing service demand $x_i^{[k]}$ based on (12);
4: ESP updates the prices using a gradient assisted searching algorithm, i.e.,

$$\mathbf{p}(t+1) = \mathbf{p}(t) + \mu\nabla\Pi(\mathbf{p}(t)), \tag{62}$$

where μ is the step size of the price update and $\mu\nabla\Pi(\mathbf{p}(t))$ is the gradient with $\frac{\partial\Pi(\mathbf{p}(t))}{\partial\mathbf{p}(t)}$. The price information is sent to all miners;
5: $k \leftarrow k+1$;
6: **until** $\frac{\left\|\mathbf{p}^{[k]}-\mathbf{p}^{[k-1]}\right\|_1}{\left\|\mathbf{p}^{[k-1]}\right\|_1} < \varepsilon$
7: **Output:** optimal demand $\mathbf{x}^{*[k]}$ and optimal price $\mathbf{p}^{*[k]}$.

V. Performance Evaluation

In this section, we first perform the real experiment on mobile blockchain mining to validate the proposed utility function of the miner. Then, we conduct the extensive numerical simulations to evaluate the performance of our proposed price-based resource management for mobile blockchain.

A. Environmental Setup

We first set up the real mobile blockchain mining experiment, as illustrated in Fig. 3. The experiment is performed on a workstation with Intel Xeon CPU E5-1630, and android devices (mobile node) installing a mobile blockchain client application. The mobile blockchain client application is implemented by the Android Studio and Software Development Kits (SDK) tools [39]. All transactions are created by the mobile blockchain client application. Each miner's working environment has one CPU core as its processor. The miner's processor and its CPU utilization rate are generated and managed by the Docker platform [40]. The mobile device of each miner has installed Ubuntu 16.04 LTS (Xenial Xerus) and Go-Ethereum [41] as the operation system and the blockchain framework, respectively.

In Fig. 3, from Box 1 and 2, the screen of computer terminal shows that the Ethereum is running on the host, i.e., edge computing node (Box 5). The mobile devices in Box 4 are connected to the edge computing node through network hub (Box 3) using mobile blockchain client application. The basic steps can be implemented as follows. The mobile users, i.e., miners use the Android device to connect to the edge computing node through network hub, i.e., access point. Then, the miners can request the service from edge node, and mine the block with the assistance of Ethereum service provided accordingly.

We create 1000 blocks employing Node.js [42] and use the mobile device to mine these blocks in the experiment. We

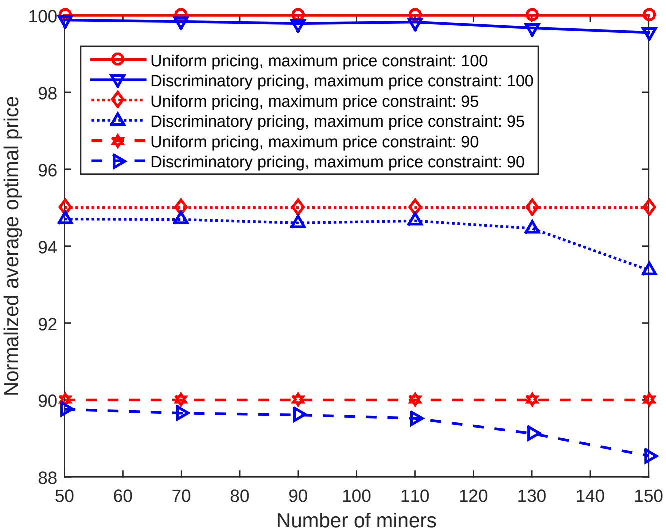

Figure 5. Normalized average optimal price versus the number of miners.

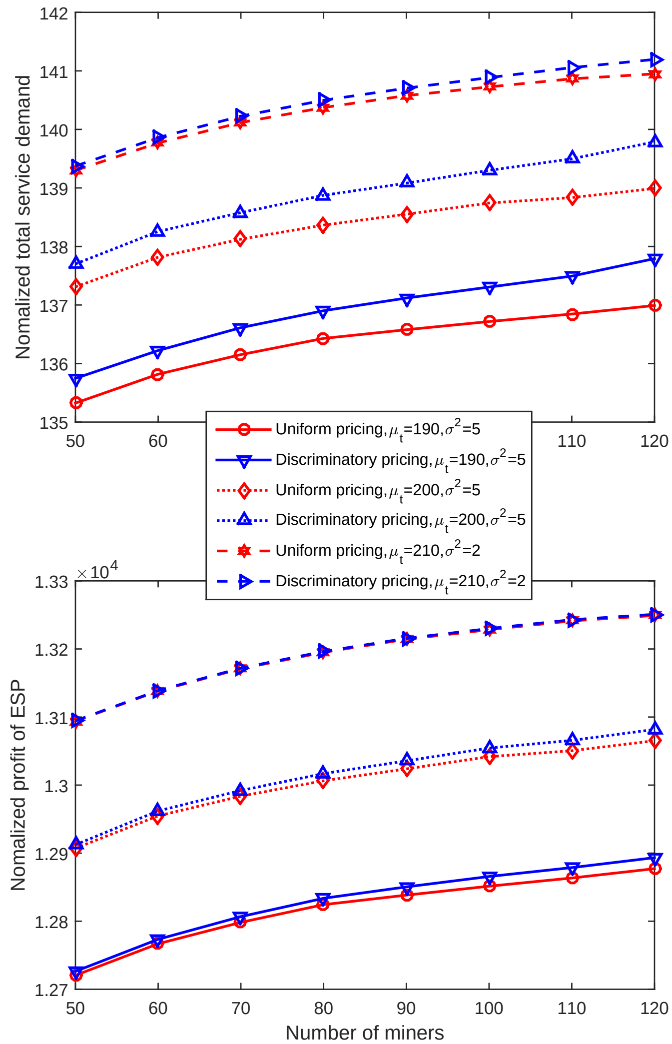

Figure 6. Normalized total service demand of miners and the profit of the ESP versus the number of miners.

consider two cases with three miners and four miners. In the three-miner case, we first fix the other two miners' service demand (CPU utilization) at 40 and 60, and then vary one miner's service demand. In the four-miner case, we first fix other three miners' service demand as 40, 50 and 60, and then vary one miner's service demand. For our experiment, the number of transactions in each mined block is 10, i.e., the size of block is the same. The comparison of the real experimental results and our proposed analytical model is shown in Fig. 4. As expected, there is not much difference between the real results and our analytical model. This is because the probability that the miner successfully mines the block is directly proportional to its relative computing power when the block size are identical. Note that the delay effects are negligible. In the sequel, we present the numerical results to evaluate the performance of the proposed price-based resource management for mobile blockchain applications.

B. Numerical Results

To illustrate the impacts of different parameters from the proposed model on the performance, we consider a group of N miners, i.e., mobile users in a blockchain network. We assume the size of a block mined by miner i follows the normal distribution $\mathcal{N}(\mu_t, \sigma^2)$. The default parameter values are set as follows: $\underline{x} = 10^{-2}$, $\overline{x} = 100$, $\overline{p} = 100$, $\mu_t = 200$, $\sigma^2 = 5$, $R = 10^4$, $r = 20$, $z = 5 \times 10^{-3}$, $c = 10^{-3}$ and $N = 100$. Further, we employ the 'fix' function in MATLAB to round each t_i to the nearest integer toward zero. Note that some of these parameters are varied according to the evaluation scenarios. We evaluate the performance of uniform pricing and discriminatory pricing in the following.

1) Investigation on total service demand of miners and the profit of the ESP:

a) The comparison of uniform pricing and discriminatory pricing: We first address the comparison of uniform pricing and discriminatory pricing schemes. Figure. 5 demonstrates the comparison of the normalized average optimal price under two proposed pricing schemes. It is worth noting that the optimal price under uniform pricing is the same as the maximum price, which can be explained by (38). Specifically, the expression in (38) is always positive, and thus the profit of the ESP increases with the increase of price. This means that the maximum price is the optimal value for profit maximization of the ESP under uniform pricing. Thus, we have the following conclusion: the ESP intends to set the maximum possible value as optimal price under uniform pricing. This conclusion is still useful even when the ESP does not have the complete information about the miners.

Further, we find that the average optimal price of discriminatory pricing is slightly lower than that of uniform pricing. The intuition is that, under under discriminatory pricing, the ESP can set different unit prices of service demand for different miners. For the details of operation of discriminatory

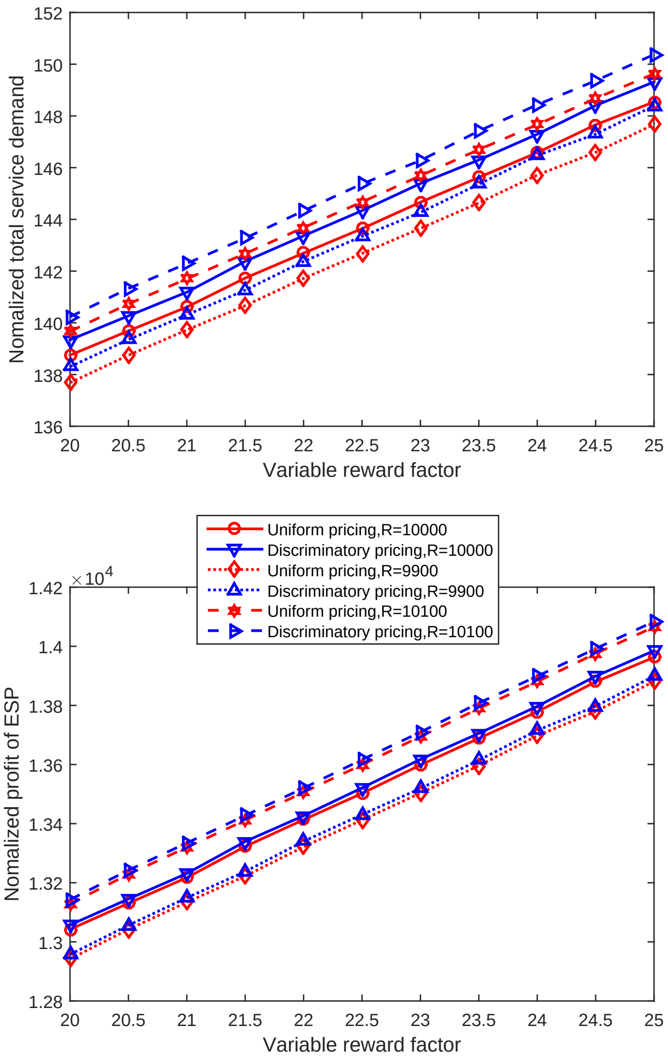

Figure 7. Normalized total service demand of miners and the profit of the ESP versus the variable reward factor.

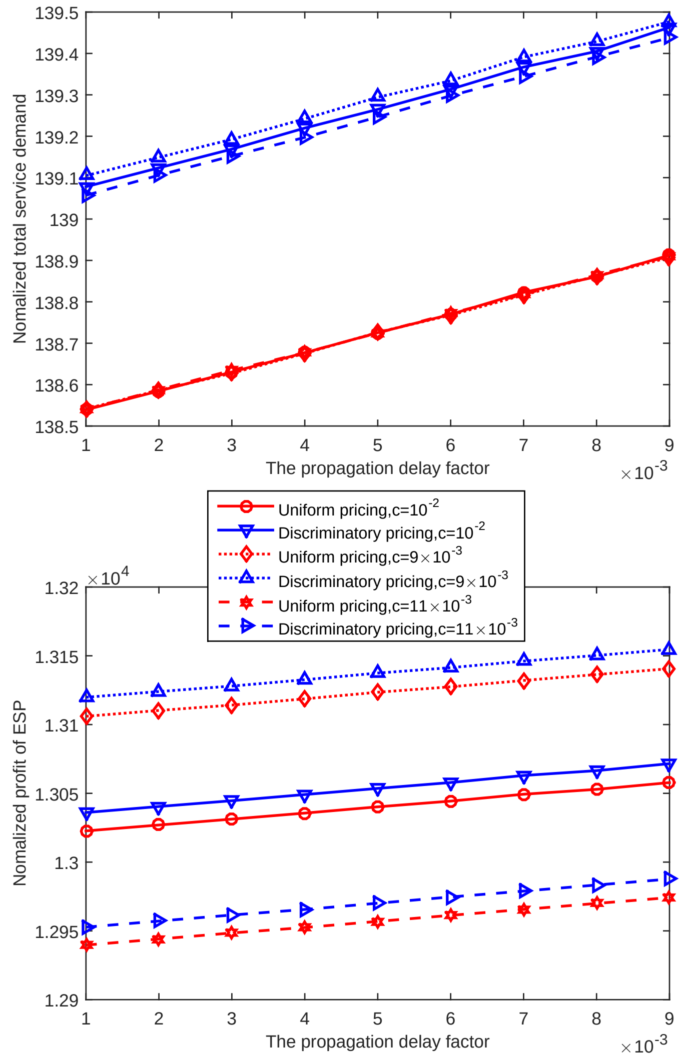

Figure 8. Normalized total service demand of miners and the profit of the ESP versus the propagation delay factor.

pricing, we conduct the case study in Section V-B2. In this case, the ESP can significantly encourage the higher total service demand from miners and achieve greater profit gain under discriminatory pricing, which is also consistent with the following results. As shown in Figs. 6-8, in all cases, the total service demand from miners and the profit of the ESP under the uniform pricing scheme is slightly smaller than that under the discriminatory pricing scheme.

From Fig. 6, we find that when σ^2 decreases, the results under uniform pricing scheme is close to that under discriminatory pricing. This is because the heterogeneity of miners in blockchain is reduced as σ^2 decreases. We may consider one symmetric case, where the miners are homogeneous with the same size of blocks to mine, i.e., $\sigma^2 = 0$. In this case, the discriminatory pricing scheme yields the same results as those of the uniform pricing scheme.

b) The impacts of the number of miners: We next evaluate the impacts brought by the number of miners, and the results are shown in Fig. 6. From Fig. 6, we find that the total service demand of miners and the profit of the ESP increase with the increase of the number of miners in mobile blockchain. This is due to the fact that having more miners will intensify the competition among the miners, which potentially motivates them to have higher service demand. Further, the coming miners have their service demand, and thus the total service demand from miners is increased. In turn, the ESP extracts more surplus from miners and thereby has greater profit gain. Additionally, it is observed that the rate of service demand increment decreases as the number of miners increases. This is from the fact that the incentive of miners to increase their service demand is weakened because the probability of their successful mining is reduced when the number of miners is increasing. Comparing different results, it is also observed that the total service demand of miners and the profit of the ESP increase as μ_t increases. This is because when μ_t increases, i.e., the average size of one block becomes larger, the variable reward for each miner also increases. The potential incentive of miners to increase their service demand is improved, and accordingly the total service demand of miners increases. Consequently, the ESP achieves greater profit gain.

c) The impacts of reward for successful mining: Then, we investigate the impacts of variable reward and fixed reward on miners and the ESP, which are shown in Fig. 7. It is observed that with the increase of variable reward factor, both the total service demand of miners and the profit of the ESP increase. This is from the fact that the increased variable reward enhances the motivation of miners for higher service demand, and the total service demand is enhanced accordingly. As a result, the ESP achieves greater profit gain. Further, by comparing curves with different value of fixed reward, we find that as the fixed reward increases, the total service demand of miners and the profit of the ESP also increase. Similarly, this is because the increased fixed reward induces greater incentive of miners, which in turn improves the total service demand of miners and the profit of the ESP.

d) The impacts of propagation delay: At last, we examine the impact of propagation delay on miners and the ESP, as illustrated in Fig. 8. It is observed that as the propagation delay factor increases, the total service demand and the profit of the ESP increase. This is because when the propagation delay effects are strong, the miners with larger mined block need to have higher service demand to reduce the propagation delay of their propagated solutions. At the same time, a miner with smaller mined block is also incentivized from the demand competition with the other miners. Therefore, the total service demand increases, which in turn improves the profit of the ESP. Additionally, we observe that as the value of service cost factor increases, the total service demand decreases under discriminatory pricing and remains unchanged under uniform pricing. On the contrary, the profit of the ESP increases in both schemes. Recall from Fig. 5, the reason is that the optimal price under uniform pricing remains unchanged from varying the value of service cost factor, and thus the service demand remains unchanged under uniform pricing. Correspondingly, the ESP achieves greater profit gain from the lower cost under uniform pricing. However, under discriminatory pricing, when the service cost decreases, the ESP has an incentive to set lower price for some miners to encourage higher total service demand. On the contrary, when the value of service cost factor increases, the ESP has no incentive to set lower price for these miners, since the higher total service demand results in higher cost for the ESP. Therefore, as the value of service cost factor decreases, the total service demand and the profit of ESP increase.

2) Investigation on optimal price under uniform and discriminatory pricing schemes: Then, to explore the impacts of discriminatory pricing on each specific miner, we investigate the optimal price and resulting individual edge computing service demand from miners. We conduct a case study for three-miner mining with the following parameters: $t_1 = 100$, $t_2 = 200$, $t_3 = 300$, $\underline{x} = 10^{-2}$, $\overline{x} = 100$, $\overline{p} = 100$, $R = 10^4$, $r = 20$, $z = 5 \times 10^{-3}$, and $c = 10^{-3}$.

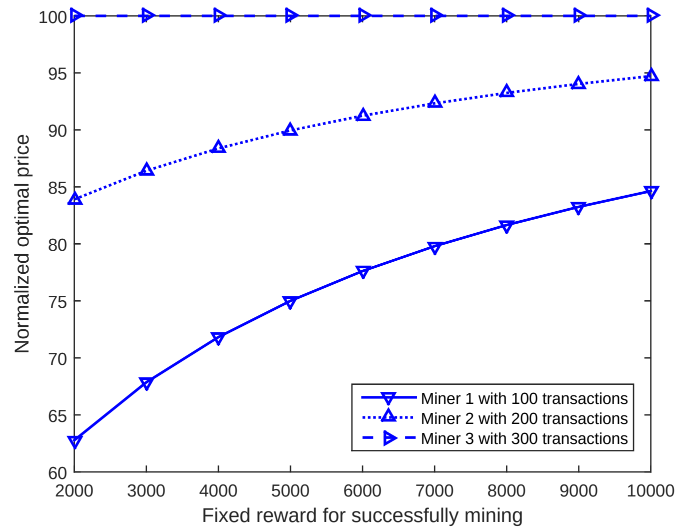

Figure 9. Normalized optimal price versus the fixed reward for mining successfully under discriminatory pricing.

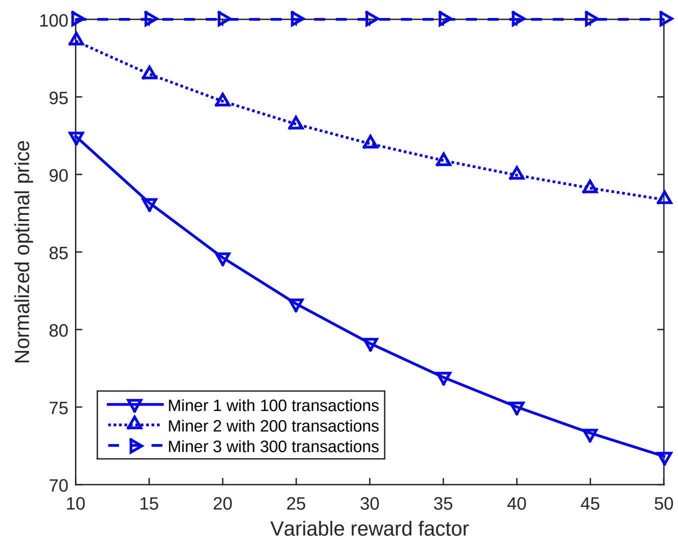

Figure 10. Normalized optimal price versus the variable reward factor under discriminatory pricing.

As expected, we observe from Figs. 9 and 10 that the optimal price charging to the miners with the smaller block is lower, e.g., miners 1 and 2. This is because the variable reward of miners 1 and 2 for successful mining is smaller than that of miner 3. Thus, the miners 1 and 2 have no incentive to pay a high price for their service demand as miner 3. In this case, the ESP can greatly improve the individual service demand of miners 1 and 2 by setting lower prices to attract them, as illustrated in Figs. 11 and 12. Due to the competition from other two miners, the miner 3 also has the potential incentive to increase its service demand. However, due to the high service unit price, as a result, the miner 3 reduces its service demand for saving cost. Nevertheless, the increase of service demand

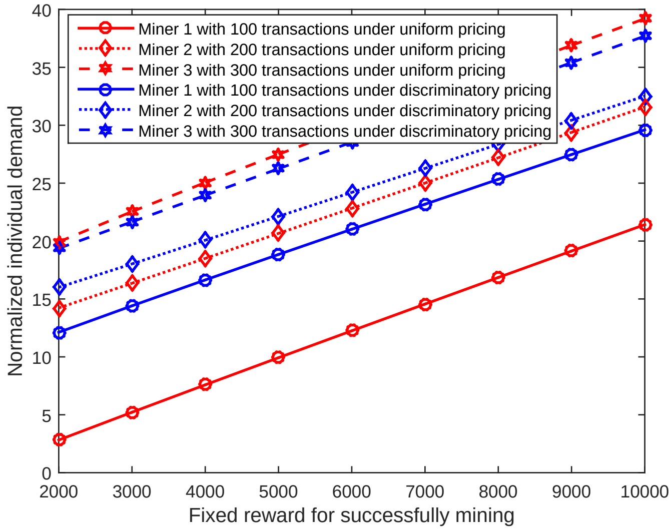

Figure 11. Normalized individual demand versus the fixed reward for mining successful.

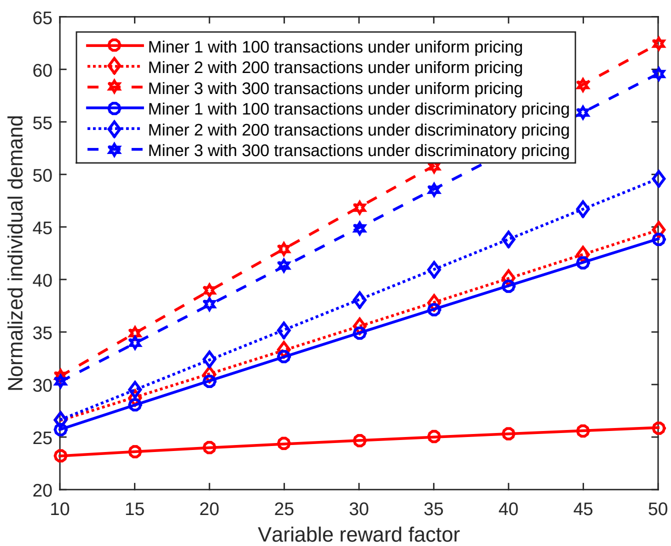

Figure 12. Normalized individual demand versus the variable reward factor.

from miners 1 and 2 are greater. Therefore, the total service demand and the profit of the ESP are still improved under discriminatory pricing compared with uniform pricing.

Further, from Fig. 9, we observe that the optimal prices for miners 1 and 2 increase with the increase of fixed reward. This is because as the fixed reward increases, the incentives of miners 1 and 2 to have higher service demand is greater. In this case, the ESP is able to raise the price and charge more for higher revenue, and thus achieves greater profit. Therefore, for each miner, the individual service demand increases as the fixed reward increases, as shown in Fig. 11. Additionally, we observe from Fig. 10 that the optimal prices for miners 1 and 2 decrease as the variable reward factor increases. This is because when the variable reward factor increases, the incentive of each miner to have higher service demand is greater. However, the incentives of the miners with smaller block to mine, i.e., the miners 1 and 2 are still not much as that of miner 3, and become smaller than that of miner 3 as the variable reward factor increases. Therefore, the ESP intends to set the lower price for miners 1 and 2 which may induce more individual service demand as shown in Fig. 12.

VI. Conclusion

In this paper, we have investigated the price-based edge computing resource management, which for the first time, to support offloading mobile blockchain process. In particular, we have adopted the two-stage Stackelberg game model to jointly study the profit maximization of edge computing service provider and the utility maximization of miners. Through backward induction, we have derived the unique Nash equilibrium point of the game among the miners. The optimal resource management schemes including the uniform and discriminatory pricing for the edge computing service provider have been presented and examined. Further, the existence and uniqueness of the Stackelberg equilibrium have been proved analytically for both pricing schemes. We have performed the real experiment to validate the proposed analytical model. Additionally, we have conducted the numerical simulations to evaluate the network performance, which help the edge computing service provider to achieve optimal resource management and gain the highest profit.

For the future work, we will further investigate the pooling schemes in mobile blockchain, where the miners in the same pool can collaborate to mine the block and thereby earn the reward that are appropriately split among all pool miners [20], [21]. Another natural extension is to consider the oligopoly market with multiple edge computing service providers, where different service providers compete with each other for selling computing services to miners.

References

[1] "Bitcoin: A peer-to-peer electronic cash system," Available at: http://bitcoin.org/bitcoin.pdf, 2008.

[2] "Crypto-currency market capitalizations," Available at: https://coinmarketcap.com, 2017.

[3] D. D. F. Maesa, P. Mori and L. Ricci, "Blockchain based access control," in *IFIP International Conference on Distributed Applications and Interoperable Systems*, Neuchatel, Switzerland, June 2017.

[4] P. Rimba, A. B. Tran, I. Weber, M. Staples, A. Ponomarev and X. Xu, "Comparing blockchain and cloud services for business process execution," in *IEEE International Conference on Software Architecture (ICSA)*, Gothenburg, Sweden, April 2017.

[5] K. Christidis and M. Devetsikiotis, "Blockchains and smart contracts for the internet of things," *IEEE Access*, vol. 4, pp. 2292–2303, 2016.

[6] N. Herbaut and N. Negru, "A model for collaborative blockchain-based video delivery relying on advanced network services chains," *IEEE Communications Magazine*, vol. 55, no. 9, pp. 70–76, 2017.

[7] K. Kotobi and S. G. Bilén, "Blockchain-enabled spectrum access in cognitive radio networks," in *Wireless Telecommunications Symposium (WTS)*, Chicago, IL, USA, April 2017.

[8] J. Kang, R. Yu, X. Huang, S. Maharjan, Y. Zhang and E. Hossain, "Enabling localized peer-to-peer electricity trading among plug-in hybrid electric vehicles using consortium blockchains," *IEEE Transactions on Industrial Informatics*, 2017.

[9] N. Z. Aitzhan and D. Svetinovic, "Security and privacy in decentralized energy trading through multi-signatures, blockchain and anonymous messaging streams," *IEEE Transactions on Dependable and Secure Computing*, 2016.

[10] N. Bozic, G. Pujolle and S. Secci, "A tutorial on blockchain and applications to secure network control-planes," in *Smart Cloud Networks & Systems (SCNS)*, Dubai, UAE, December 2016.

[11] R. Pass and E. Shi, "Fruitchains: A fair blockchain," in *Proceedings of the ACM Symposium on Principles of Distributed Computing*, Egham, United Kingdom, July 2017.

[12] M. Conti, C. Lal, S. Ruj et al., "A survey on security and privacy issues of bitcoin," *arXiv preprint arXiv:1706.00916*, 2017.

[13] "Global m-commerce market 2016-2020," Available at: https://www.technavio.com/report/global-media-and-entertainment-servicesglobal-m-commerce-market-2016-2020, 2016.

[14] Y. Zhang, L. Liu, Y. Gu, D. Niyato, M. Pan and Z. Han, "Offloading in software defined network at edge with information asymmetry: A contract theoretical approach," *Journal of Signal Processing Systems*, vol. 83, no. 2, pp. 241–253, May 2016.

[15] J.-J. Laffont, P. Rey and J. Tirole, "Network competition: II. price discrimination," *The RAND Journal of Economics*, pp. 38–56, 1998.

[16] N. Houy, "The bitcoin mining game," *Browser Download This Paper, Available at SSRN: https://ssrn.com/abstract=2407834*, March 2014.

[17] J. Beccuti, C. Jaag et al., "The bitcoin mining game: On the optimality of honesty in proof-of-work consensus mechanism," Tech. Rep., 2017.

[18] A. Kiayias, E. Koutsoupias, M. Kyropoulou and Y. Tselekounis, "Blockchain mining games," in *Proceedings of the ACM Conference on Economics and Computation (EC)*, Maastricht, Netherlands, July 2016.

[19] L. Wang and Y. Liu, "Exploring miner evolution in bitcoin network," in *International Conference on Passive and Active Network Measurement*. Springer, 2015, pp. 290–302.

[20] Y. Lewenberg, Y. Bachrach, Y. Sompolinsky, A. Zohar and J. S. Rosenschein, "Bitcoin mining pools: A cooperative game theoretic analysis," in *Proceedings of the ACM International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, Istanbul, Turkey, May 2015.

[21] B. A. Fisch, R. Pass and A. Shelat, "Socially optimal mining pools," *arXiv preprint arXiv:1703.03846*, 2017.

[22] M. Rosenfeld, "Analysis of bitcoin pooled mining reward systems," *arXiv preprint arXiv:1112.4980*, 2011.

[23] I. Eyal, "The miner's dilemma," in *IEEE Symposium on Security and Privacy (SP)*, San Jose, CA, USA, May 2015.

[24] R. Axelrod et al., "The evolution of strategies in the iterated prisoner's dilemma," *The dynamics of norms*, pp. 1–16, 1987.

[25] S. Kim, "Group bargaining based bitcoin mining scheme using incentive payment process," *Transactions on Emerging Telecommunications Technologies*, vol. 27, no. 11, pp. 1486–1495, 2016.

[26] K. Binmore, A. Rubinstein and A. Wolinsky, "The nash bargaining solution in economic modelling," *The RAND Journal of Economics*, pp. 176–188, 1986.

[27] L. Luu, R. Saha, I. Parameshwaran, P. Saxena and A. Hobor, "On power splitting games in distributed computation: The case of bitcoin pooled mining," in *IEEE the 28th Computer Security Foundations Symposium (CSF)*, Verona, Italy, July 2015.

[28] F. Tschorsch and B. Scheuermann, "Bitcoin and beyond: A technical survey on decentralized digital currencies," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 3, pp. 2084–2123, 2016.

[29] K. Suankaewmanee, D. T. Hoang, D. Niyato, S. Sawadsitang, P. Wang and Z. Han, "Performance analysis and application of mobile blockchain," in *International Conference on Computing, Networking and Communications (ICNC)*, Maui, Hawaii, USA, March 2018.

[30] "Comparison of existing bitcoin pools and their reward strategies," Available at: https://en.bitcoin.it/wiki/Comparisonofminingpools, 2017.

[31] N. Houy, "The economics of bitcoin transaction fees," *Available at SSRN: https://ssrn.com/abstract=2400519*, February 2014.

[32] R. Peter, "A transaction fee market exists without a block size limit," 2015.

[33] "Orphan probablity approximation," https://gist.github.com/gavinandresen/5044482.

[34] E. Senmarti Robla, *Analysis of Reward Strategy and Transaction Selection in Bitcoin Block Generation*, Ph.D. thesis, University of Washington, 2015.

[35] C. Decker and R. Wattenhofer, "Information propagation in the bitcoin network," in *IEEE 13th International Conference on Peer-to-Peer Computing (P2P)*, Trento, Italy, September 2013.

[36] S. Andrew, "An examination of single-transaction blocks and their effect on network throughput and block size," 2015.

[37] Z. Han, D. Niyato, W. Saad, T. Baar and A. Hjrungnes, *Game theory in wireless and communication networks: theory, models, and applications*, Cambridge University Press, 2012.

[38] G. Scutari, D. P. Palomar, F. Facchinei and J.-s. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 35–49, 2010.

[39] "Android studio," https://developer.android.com/studio/index.html.

[40] "Docker," https://www.docker.com/community-edition.

[41] "Go-ethereum," https://ethereum.github.io/go-ethereum/.

[42] "Node.js," https://nodejs.org/en/.