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CSCI 3302

Professor Hayes
Homework 1

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1. There are 3 degrees of freedom in the hand-pushed lawnmower. The lawnmower has "standard wheels" so it can move back and forth, and the wheel is able to rotate on the Z axis. That accounts for two degrees of freedom. The third decree is accounted when looking at the pitch axis, the lawnmower is able to be on an uneven surface and still move.

   We are still able to mow the entire lawn because we can rotate the lawnmower. Rotating the lawnmower left or right, by applying more pushing force on one side on the handle, and maybe even a pulling force on the other side, we can cause the lawnmower to turn in one direction or another. Depending how much or how little force is applied, the lawnmower can be caused to turn to face a new direction so it can move forward in the new direction.

2. The maximum degrees of freedom for objects driving on the X-Y plane is 3. An object can move up and down the X or Y axis, accounting for two degrees of freedom. The object is also able to rotate on the Z axis (but not move along the z axis), thus the third degree of freedom.

3. (a) $(\cos 45, -\sin 45, 0)^T$ is equivalent to $(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0)^T$

   $(\sin 45, \cos 45, 0)^T$ is equivalent to $(\frac{1}{\sqrt{2}}), \frac{1}{\sqrt{2}}, 0)^T$

   cosine formula: $\cos\theta = \frac{((\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0)^T \cdot (\frac{1}{\sqrt{2}}), \frac{1}{\sqrt{2}}, 0)^T)}{||(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0)^T|| * ||(\frac{1}{\sqrt{2}}), \frac{1}{\sqrt{2}}, 0)^T||}$

   $\cos\theta = \frac{\frac{1}{2} + \frac{-1}{2} + 0}{1*1}$

   $\cos\theta = \frac{0}{1}$

   $\cos\theta = 0$

   $\theta = \arccos 0$

   $\boxed{\theta = 90^\circ}$

   (b) To form a coordinate system, knowing that the two given vectors are orthogonal, the third vector must also be orthogonal to the two given vectors.

   Let the third vector be $(x, y, z)^T$,

$(\cos 45, -\sin 45, 0)^T$ and $(x, y, z)^T$ are orthogonal, so $\cos\theta = x - y = 0$
Solving for x, $x = y$.
$(\sin 45, \cos 45, 0)^T$ and $(x, y, z)^T$ are orthogonal, so $\cos\theta = x + y = 0$
Using $x = y$ and $x + y = 0$ we can solve x to be 0, y to be 0, and z as any integer c.
Therefore the third vector is $\boxed{(0, 0, c)^T}$

4. (a) ${}^A_BR$
$$\begin{bmatrix} X^AX^B & Y^AX^B & Z^AX^B \\ X^AY^B & Y^AY^B & Z^AY^B \\ X^AZ^B & Y^AZ^B & Z^AZ^B \end{bmatrix}$$
(b) $X^B = [0, 1, 0]^T$ in frame A
$$\begin{bmatrix} X^BX^A & Y^BX^A & Z^BX^A \\ X^BY^A & Y^BY^A & Z^BY^A \\ X^BZ^A & Y^BZ^A & Z^BZ^A \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & X^A & 0 \\ 0 & Y^A & 0 \\ 0 & Z^A & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} X^A & Y^A & Z^A \end{bmatrix}$$
(c) ${}^B_AR$
$$\begin{bmatrix} X^BX^A & Y^BX^A & Z^BX^A \\ X^BY^A & Y^BY^A & Z^BY^A \\ X^BZ^A & Y^BZ^A & Z^BZ^A \end{bmatrix}$$

5. Tricycle, 2 wheels on back, 1 steerable on front.

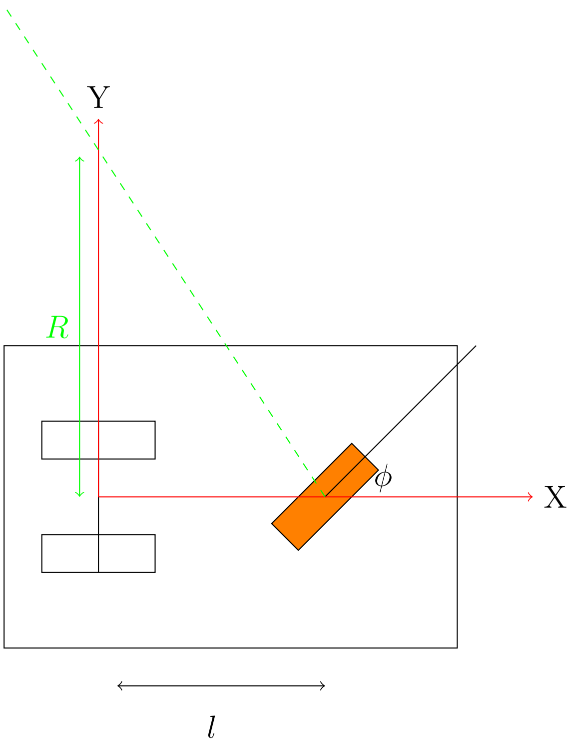
r =radius of front wheel

l = distance between front and rear axle

ϕ = steering wheel angle

ω = angular velocity

Let the center point of the common circle represent, described by all wheels, to be a distance R from the tricycle's center line.



ϕ is given by $\tan\phi = \frac{l}{R}$. Rearranging this, we can find R to be, $R = l\tan\phi$

The coordinate system, (x_r, y_r, θ_r) is centered on the tricycle's rear axis.

The forward kinematics of a mobile robot are given as,

$\dot{x} = \cos\theta\dot{x_r}$

$\dot{y} = -\sin\theta\dot{x_r}$

$\dot{\theta} = \frac{\tan\phi}{l}\dot{x_r}$

Assuming that the tricycle does not skid, then $\dot{y_r} =$; Therefore the steering angle of ϕ only affects the rotation of the tricycle.

$\frac{l}{R} = \tan(\frac{\pi}{2} - \phi)$

$R = l\tan\phi$

6. First, calculaing the rotation of the robot in coordinate frame B with respect to A

$${}^A_BR = \begin{bmatrix} \cos 135 & -\sin 135 & 0 \\ \sin 135 & \cos 135 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using the homogeneous Transformation matrices, we get,

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 8 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 2\sqrt{2} \\ 10 + 6\sqrt{2} \\ 0 \\ 1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 6 - 2.828 \\ 10 + 8.485 \\ 0 \\ 1 \end{bmatrix}$$

So the coordinates of Q in the frame of A is $\boxed{(3.173, 18.485)}$.