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LAPLACE TRANSFORM

- Why called Laplace Transform ?
- Why Study Laplace Transform ?
- Defination Laplace Transform ?

WHY CALLED LAPLACE TRANSFORM

Laplace : Mathematician Name

Transform: one variable change into another variable (s variable change
into t and t change into s)

Why Study Laplace Transform

Solve initial value problem

Solve Linear Differential Equation (solve in minimum steps)

Defination

The Laplace Transform is a linear operator that switched a function f(t) to f(s).

LAPLACE TRANSFORM FORMULAS

1 L[sinkt] $= \frac{k}{s^2+k^2}$

2 L[coskt] $= \frac{s}{s^2+k^2}$

3 L[sinhkt] $= \frac{k}{s^2-k^2}$

4 L[coskt] $= \frac{s}{s^2-k^2}$

5 $L[t^n] = \frac{n}{s^{n+1}}$

LAPLACE TRANSFORM

PROOF:

F(t)=e^{at}

$$L[e^{at}] = \int_0^{\infty} e^{-st}\, e^{at}\, dt$$

$$L[e^{at}] = \int_0^{\infty} e^{-st+at}\, dt$$

$$L[e^{at}] = \int_0^{\infty} e^{-t(s-a)}\, dt$$

$$L[e^{at}] = \frac{e^{-t(s-a)}}{-(s-a)}\, dt$$

$$L[e^{at}] = -\frac{1}{(s-a)}\,(0-1) \qquad = \frac{1}{s-a}$$

PROPERTIES OF LAPLACE TRANSFORM :

1 Linear Property

2 First Shifting Property

3 Division Property

LINEAR PROPERTY

L[A f(t)+ B g(t)]= A L[f(t)] + B L[g(t)]

F(t)= t^2 **+ 6t – 17**

Solution:

$L[t^2 + 6t - 17] = L[t^2] + 6L[t] - 17\ L[1]$

$$= \frac{2!}{s^{2+1}} + 6\frac{1}{s^2} - 17\frac{1}{s}$$

$$= \frac{2!}{s^{2+1}} + \frac{6}{s^2} - \frac{17}{s}$$

SHIFTING PROPERTY

$$\mathrm{L}[e^{at} f(t)] = F(s-a)$$

EXAMPLE:

$$\mathrm{L}[e^{-t}\, t^3\,]$$

$$\mathrm{L}[\mathrm{t}^3\,] = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^4}$$

$$= \frac{6}{s^4}$$ **Shifting Property**

$$\mathrm{L}[e^{-t}\, t^3\,] = \frac{6}{(s+1)^4}$$

DVISION PROPERTY

$$\mathbf{L}[\frac{\boldsymbol{f(t)}}{\boldsymbol{t}}] = \int_{\mathbf{0}}^{\infty} \boldsymbol{F(u)}$$

EXAMPLE:

$$\mathbf{L}[\frac{\boldsymbol{sint}}{\boldsymbol{t}}]$$

Let f(t)= sint

L[f(t)]= $\frac{1}{s^2+1}$

Now taking laplace tranfrom

L[$\frac{sint}{t}$]= $\int_0^{\infty} \frac{1}{s^2+1} ds$

$=\frac{\pi}{2}\ tan^{-1}s$

Inverse Laplace

Definition:

- The inverse Laplace transform definition comes as the inverse operation of the Laplace transformation and is mathematically is written as:

$$f=L-1(F).$$

LAPLACE INVERSE

In Laplace Transform

T variable change into S variable

In Laplace Inverse

S variable change into T variable

INVERSE LAPLACE TRANSFORM FORMULAS:

1 $L^{-1}[1]$=f(t)

2 $L^{-1}[\frac{1}{s}]$= t

3 $L^{-1}[\frac{1}{s-a}]= e^{at}$

4 $L^{-1}[\frac{1}{s+a}]= e^{-at}$

5 $L^{-1}[\frac{1}{s^2+a^2}]=\frac{1}{a}\sin at$

6 $L^{-1}[\frac{s}{s^2+a^2}]=\cos at$

7 $L^{-1}[\frac{s}{s^2-a^2}]=\cos hat$

Solving ODE’S USING LAPLACE

SOLVING ODES USING LAPLACE:

- The first step is to take the **Laplace** transform of both sides of the original differential equation
- Put the **initial** value
- Taking **Inverse** Laplace

EXAMPLE

$$\boldsymbol{\frac{dy}{dx} \;.\; ky = ce^{kt}} \qquad\qquad \boldsymbol{y(0) = 0}$$

$$\frac{dy}{dx}\;.\;ky = ce^{kt} \ldots\ldots 1$$

Taking Laplace Transform

SY(S) – y(0) – KY(S) = c $\frac{1}{s-k}$

SY(S) – 0 – KY(S) = c $\frac{1}{s-k}$

SY(S) – KY(S) = c $\frac{1}{s-k}$

$Y(S)\ (s - k) = c\,\frac{1}{s-k}$

$Y(S) = \frac{c}{(s-k)^2}$

Taking Inverse Laplace Transform

$y(t) = c\ L^{-1}[\frac{1}{(s-k)^2}]$

$\mathbf{y(t) = c}\ \boldsymbol{e^{kt}}$

SOLVING PDES USING LAPLACE:

SOLVING PDES USING LAPLACE:

1. Taking **Laplace** Transform

2. Putting Initial conditions

3. Find $\boldsymbol{\bar{u}c}$ and $\boldsymbol{\bar{u}}$ **p**

4. Taking Laplace Inverse of **Boundary Coundition** and putting this in $\boldsymbol{\bar{u}}$

5. Taking **Inverse** Laplace

FORMULAS FOR PDES:

1. $L[\frac{\partial u}{\partial T}\ s] = s\bar{u} - \mathrm{u(x,0)}$

2. $L[\frac{\partial^2 u}{\partial t^2}\ ,\ s] = s^2\bar{u} - \mathrm{su(x,0)} - \mathrm{ut(x,0)}$

3. $L[\frac{\partial u}{\partial x}\ s] = \frac{d}{dx}\bar{u}$

4. $L[\frac{\partial^2 u}{\partial x^2}\ ,\ s] = \frac{d^2}{dx^2}\bar{u}$

5. $L[\frac{\partial^2 u}{\partial x \partial t},\ s] = s\frac{d}{dx}\bar{u} - \frac{d}{dx}\mathrm{u(x,0)}$

EXAMPLE:

$$\frac{\partial u}{\partial t} = 2\,\frac{\partial^2 u}{\partial x^2} \qquad u(0,t) = 0\ ,\ u(5,t) = 0\ ,\ \text{u(x,0)=10sin4}\pi x$$

$$\text{L}[\frac{\partial u}{\partial t}] = 2\,L\left[\frac{\partial^2 u}{\partial x^2}\right]$$

Taking Laplace

$$\text{s}\overline{u} - \text{u(x,0)} = 2\frac{d^2\overline{u}}{dx^2}$$

$$2\frac{d^2\overline{u}}{dx^2} - \text{s}\,\overline{u} = -\,\text{u(x,0)}$$

$$\frac{d^2\overline{u}}{dx^2} - \frac{S\overline{u}}{2} = -\,\frac{\text{u(x,0)}}{2}$$

dividing by "2"

$$(D^2 - \frac{s}{2})\overline{u} = -\,\frac{10sin4\pi x}{2}$$

As given u(x,0)=10sin4 πx

Charactertistic equation is

$m^2 - \frac{s}{2} = 0$

$$\mathbf{m} = -\sqrt{\frac{s}{2}},\ +\sqrt{\frac{s}{2}}$$

$$\bar{u}\,\mathrm{c} = \mathrm{c1}e^{\sqrt{\frac{s}{2}}\,\mathbf{x}} + c2e^{-\sqrt{\frac{s}{2}}\,\mathbf{x}}$$

$$\bar{u}\,\mathrm{p} = \frac{-5sin4\pi x}{\left(D^2 - \frac{s}{2}\right)}$$

$$c\mathrm{p} = \frac{1}{\left(-16\pi^2 - \frac{s}{2}\right)} - 5sin4\pi x$$

$$\bar{u}\,\mathrm{p} = \frac{10sin4\pi x}{\left(s + 32\pi^2\right)}$$

$$\bar{u}(x\ ,\ s)=c1e^{\sqrt{\frac{s}{2}}\,x}+c2e^{-\sqrt{\frac{s}{2}}\,x}+\frac{10\sin 4\pi x}{(s+32\pi^2)} \quad \ldots\ldots\ 2$$

Taking Laplace of boundary equation and put in 2 will give us c1= 0 , c2=0

$$\bar{u}(x\ ,\ s)=\frac{10\sin 4\pi x}{(s+32\pi^2)} \ldots\ldots\ 3$$

Taking Laplace inverse of 3

$$U(x\ ,\ t)= 10\sin 4\pi x e^{-32\pi^2 t}$$

THANK YOU