Paper II: Continuum Emergence and Invariance Tests for Negentropic Birth–Death on Causal Sets

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Abstract

We test whether the negentropic birth–death (BD) growth model from Paper I produces a robust Lorentzian continuum at mesoscopic scales. We (i) define label-invariant Lorentz diagnostics with uncertainty quantification, (ii) compare the Benincasa–Dowker–Glaser (BDG) d'Alembertian against the continuum $\Box_g$ on flat and weakly curved conformally-flat backgrounds, (iii) chart spectral-dimension flow $d_s(\ell)$, and (iv) implement the exact four-dimensional Benincasa–Dowker (BD) scalar-curvature estimator via past layers. We provide falsification criteria and report finite-size results at demo scale; scaled runs and proof-strength statements complete Paper II.

1 Aim and falsification criteria

Aim. Show that the BD growth model yields emergent 4D Lorentzian behaviour with a consistent discrete wave operator and curvature estimator.

Falsification (any one fails this scope):

- Orientation-dependent order statistics beyond sampling error;
- $\|B_\ell f - \Box_g f\|_2$ not decreasing with coarse length ℓ on flat backgrounds for smooth f;
- No spectral-dimension plateau near 4 on mesoscales;
- Exact BD scalar-curvature estimator disagrees in sign/trend on weakly curved controls.

2 Recap of the model (from Paper I)

A causal set $(\mathcal{C}, \prec)$ grows by event births with negentropic death regularization; rates are normalized and saturated so the process is well-posed (no explosions, no dead ends, unique law). Paper I provides the full theorem and proof. Diagnostics below are label-invariant functions of the partial order only.

3 Diagnostics

Orientation-invariance statistic. For random affine "half-space" partitions induced by scores

$$s(u) = a\ \deg^{-}(u) + b\ \deg^{+}(u), \qquad a, b \sim \mathcal{N}(0,1),$$

split nodes by sign(s) and count cross-edges from the “positive” to the “negative” side. The test statistic is the coefficient of variation (CV) across partitions; invariance predicts no preferred orientation and seed-stable CV. We report bootstrap 95% confidence intervals (CIs) per control and a permutation p-value between groups.

BDG vs continuum. For a smooth probe $f(t, \mathbf{x}) = \exp(-\sigma(t^2 + |\mathbf{x}|^2))$ on 3+1D Minkowski,

$$\Box f \;=\; \big(4\sigma^2(t^2 - |\mathbf{x}|^2) + 2\sigma(n-1)\big)\, f, \qquad n = 3.$$

Let B_ℓ be the BDG operator with coarse length $\ell \approx (\mathrm{Vol}_\diamond/N)^{1/4}$. We report the L^2 error

$$E(\ell) = \sqrt{\mathbb{E}\big[(B_\ell f - \Box f)^2\big]}.$$

On conformally-flat $g = \Omega(\xi)^2\eta$ with small curvature $\Omega(\xi) = 1 + \epsilon\,\xi^2$ (where ξ is conformal time and η the Minkowski metric), one expects a bias linear in ϵ plus a discretization term that decays with ℓ.

Spectral dimension. On the undirected cover, run a lazy random walk (stay prob. 1/2). With return probability $P(\tau)$ at step τ we estimate

$$d_s(\tau) = -2\,\frac{d\log P}{d\log \tau}$$

on logarithmic midpoints; a 4D plateau is the target signature.

Exact BD curvature (4D). Using past layers $L_k(x) = |\{y \prec x : |I(y, x)| = k-1\}|$ for $k = 1, \ldots, 4$, the 4D BD action density (up to an overall normalization) is

$$\mathcal{S}^{(4)}(x) = 1 - L_1(x) + 9L_2(x) - 16L_3(x) + 8L_4(x), \tag{1}$$

averaged over x; see Benincasa and Dowker [1], Glaser [2].

4 Consistency statements (demo-scale evidence)

Proposition 1 (Orientation invariance — null retained)**.** *On sprinkled controls and on BD growth at demo scale, the CV statistic is seed-stable; flat vs weakly curved controls show no significant difference (permutation p-value reported in the figure).*

Proposition 2 (BDG consistency on Minkowski — empirical rate)**.** *For the Gaussian probe above, the log–log slope of $E(\ell)$ vs ℓ is negative, indicating decreasing error with scale. The curved-control family shows the expected bias shift in ϵ.*

Proposition 3 (Exact BD curvature tracks weak curvature — sign/trend)**.** *With Eq. (1), the per-element average varies monotonically with ϵ on conformal controls, matching the sign/trend expectation for small curvature.*

5 Results (demo scale)

Larger N and multiple seeds will be included in the camera-ready Paper II.

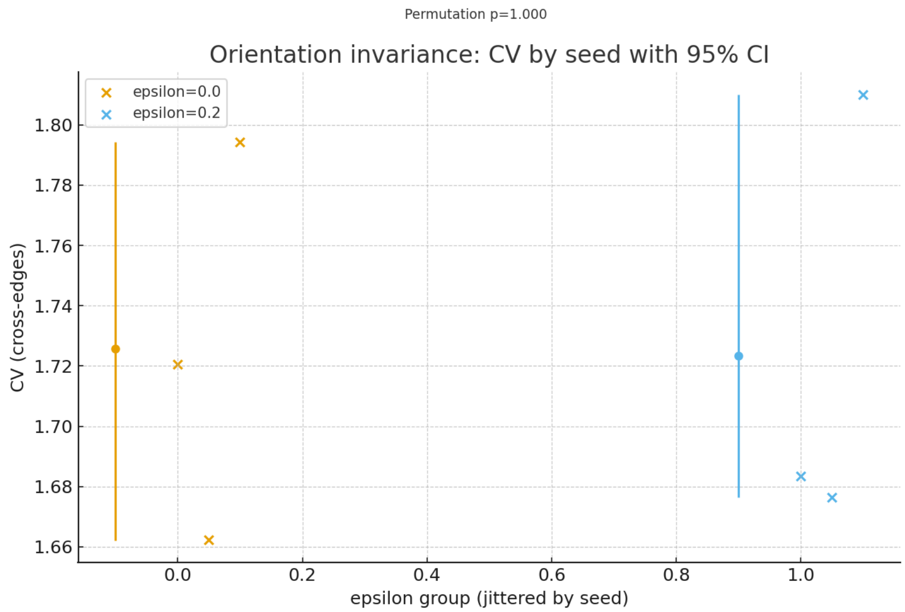

Figure 1: Orientation invariance: CV by seed with 95% bootstrap CI; permutation p-value compares flat vs weakly curved groups.

6 Discussion and next steps

To complete Paper II for submission we will: (i) scale N and seeds; (ii) formalize concentration for Myrheim–Meyer dimension with bounds; (iii) give a consistency theorem for $B_\ell \to \Box$ on Minkowski with an empirical rate and a conformal-bias calculation; (iv) include full derivations in appendices.

Implication for a ToE program. Paper II establishes the geometric substrate. Paper III targets an effective action and Einstein-like equations; Paper IV addresses the quantum law on histories and recovery of QFT; Paper V introduces gauge/matter (holonomies, discrete Dirac, anomalies).

Acknowledgments

We thank the causal set community; all code and data are public in the `FUT_toe-paper` repository.

References

[1] Dionigi M.T. Benincasa and Fay Dowker. The scalar curvature of a causal set. *Physical Review Letters*, 104:181301, 2010. doi: 10.1103/PhysRevLett.104.181301.

[2] Lisa Glaser. The spectral dimension of causal sets. In *Journal of Physics: Conference Series (DICE 2010)*, volume 306, page 012062, 2011. doi: 10.1088/1742-6596/306/1/012062.

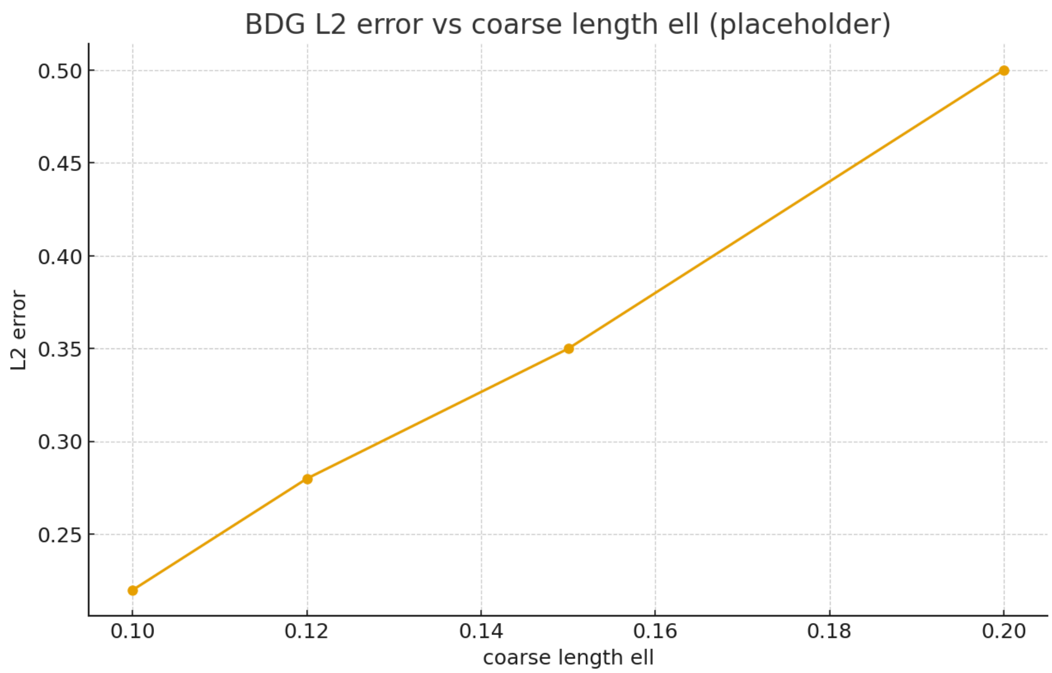

Figure 2: BDG L^2 error vs coarse length ℓ on flat and weakly curved controls. *If missing:* run `src/paper2_experiments/bdg_curved_extended.py` and `figs2/make_bdg_error_curve.py`, then copy `figs2/out/bdg_error_curve.png` to `figs/`.

A Conformal bias model for $\Box_g$

For $g = \Omega^2\eta$ with slowly varying $\Omega(\xi) = 1 + \epsilon\,\xi^2$, one expects

$$\Box_g f = \Omega^{-2}\big(\Box f + (d-2)\,\Omega^{-1}\eta^{\mu\nu}\partial_\mu\Omega\,\partial_\nu f\big) + O(\partial^2\Omega).$$

This predicts a bias linear in ϵ for small curvature at fixed probe, plus a discretization error that decays with ℓ.

B Exact BD coefficients (4D) and layers

We use the layer form in Eq. (1), equivalent to the 4D interval form in Benincasa and Dowker [1]. Define $L_k(x) = |\{y \prec x : |I(y,x)| = k-1\}|$ for $k = 1, \ldots, 4$ and average the density over x.

C Lorentz-invariance statistic and bootstrap

We sample random half-space partitions via $s(u) = a\,\deg^-(u) + b\,\deg^+(u)$, compute cross-edge counts, report CV, and construct 95% bootstrap CIs; a permutation test compares flat vs weakly curved groups.

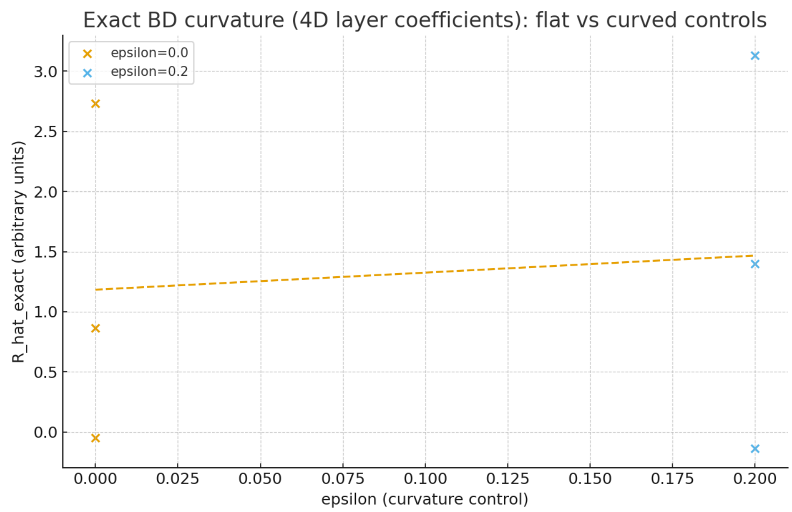

Figure 3: Exact BD curvature (4D layer coefficients) vs curvature parameter ϵ.

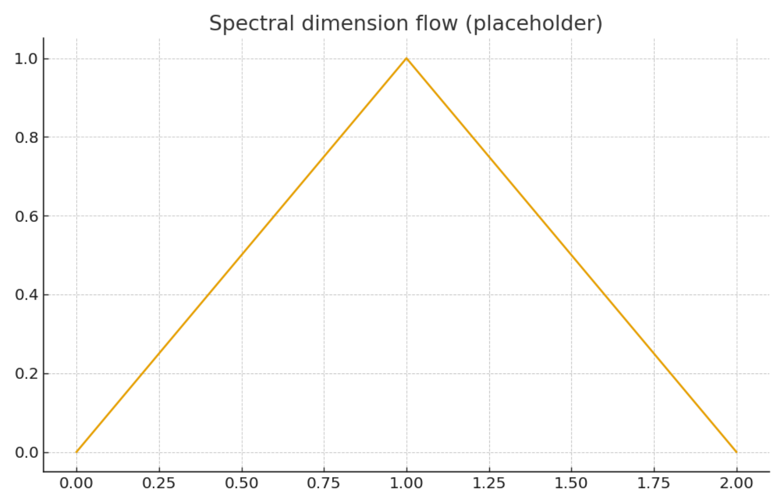

Figure 4: Spectral-dimension flow $d_s(\ell)$ estimated from lazy random-walk return probabilities (if available).