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End Sem Exam

Mathematics - 1

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Q-1

(i) Given A.P

Then = $a + (n-1)d$ {a: first term of A.P
d: common difference}

Case 1

$a_9 = a + (9-1)d$

$a_9 = a + 8d$

$30 = a + 8d$

Case 2

$a_{17} = a + (17-1)d$

$a_{17} = a + 16d$

$50 = a + 16d$

Solving Eq ① & ②

$$\begin{array}{r} a + 8d = 30 \\ a + 16d = 50 \\ - \quad - \\ \hline -8d = -20 \\ d = 2.5 \end{array}$$

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Putting in eq ②

$$a + 16d \times \frac{5}{2} = 50$$

$$a + 40 = 50$$

$$\boxed{a = 10}$$

$\therefore$ first term $a = 10$

Second term $a_2 = 10 + (2-1)(2.5)$
$= 12.5$

third term $a_3 = 10 + (3-1)(2.5)$
$= 15$

(ii) Given expression

$$\left(2x^2 - \frac{1}{x^2}\right)^{12}$$

We know that $T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$

$$T_8 = T_{7+1} = {}^{12}C_7 (2x^2)^{12-7} \left(-1/x^2\right)^7$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times \cancel{\lfloor 7}}{\cancel{\lfloor 7} \times 5 \times 4 \times 3 \times 2} (2x^2)^5 \left(-1/x^2\right)^7$$

$$= 11 \times 9 \times 8 \times 2^5 (\cancel{x^{10}}) \times \left(-1/\cancel{x^{14}}\right)$$

$$= -25,344\, x^{-4}$$

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Q.2 ~~Show~~

$p \to q$
$q \to r$
$r \to s$
$\neg s$
$p \vee t$

$p \to q$ (given) — ①
$q \to r$ (given) — ②
$p \to r$ hypothetical syllogism (from ① & ②) — ③
$r \to s$ (given) — ④
$p \to s$ hypothetical syllogism (from ③ & ④) — ⑤
$\neg s$ (given) — ⑥
$\neg p$ Modus tollens (from ⑤ & ⑥) → 7
$p \vee t$ (given) — ⑧
t Disjunctive Syllogism (from ⑦ & ⑧)

Hence, t is a valid conclusion.

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Q-3

let $a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $c = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

let A, B, C be the orthogonal basis to be found by gram - Schmidt process

$$A = a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$B = b - \frac{A^T b}{A^T A} A$$

$$= \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} - \frac{4}{2}^{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

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$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$$C = C - \frac{A^T C}{A^T A} A - \frac{B^T C}{B^T B} B$$

$$C = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} - \frac{[1\ 0\ 1] \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}}{[1\ 0\ 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} - \frac{[1\ 1\ -1] \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}}{[1\ 1\ -1] \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1/6 \\ -1/3 \\ -1/6 \end{bmatrix}$$

$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $C = \begin{bmatrix} 1/6 \\ -1/3 \\ -1/6 \end{bmatrix}$

let q_1, q_2, q_3 be orthogonal basis

$q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $q_2 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $q_3 = \frac{1/1}{\sqrt{6}} \begin{bmatrix} 1/6 \\ -1/3 \\ -1/6 \end{bmatrix}$

$\therefore$ orthonormal basis =

$$\left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \sqrt{6} \begin{bmatrix} 1/6 \\ -1/3 \\ -1/6 \end{bmatrix} \right.$$

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Q-9

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

$[A - \lambda I] = 0$ for eigen values

$$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} = 0$$

$R_3 = R_3 - 1$

$$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & 1 \\ 2+\lambda & 0 & -2-\lambda \end{bmatrix} = 0$$

$(2+\lambda)(\cancel{8}1-9+\lambda) - (2+\lambda)[(3-\lambda)(1-\lambda) - 1] = 0$

$(\lambda^2-4)(\lambda-5) = 0$

$\lambda = 5, 2, -2$

$\lambda = 2$

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$$\begin{bmatrix} -1 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_1, \quad R_3 \rightarrow R_3 + 3R_1$

$$\begin{bmatrix} -1 & 1 & 3 \\ 0 & 2 & 4 \\ 0 & 4 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} -1 & 1 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

free variables

$-x_1 + x_2 + 3x_3 = 0 \quad — (1)$

$2x_2 + 4x_3 = 0$

$x_2 = -2x_3 \quad — (2)$

from (1) & (2)

$-x_1 + x_3 = 0$

$x_3 = x_1 = 1$

$x_2 = -2$

$$v_2 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

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$\lambda = -2$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1/3$

$$\begin{bmatrix} 3 & 1 & 3 \\ 0 & 19/3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

E.V

$3x_1 + x_2 + 3x_3 = 0$

$19/3 \; x_2 = 0$

$x_2 = 0$

$3x_1 + 0 + 3x_3 = 0$

$x_1 = -x_3$

$x_3 = -1$

$x_1 = 1$

$$v_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$\lambda = 5$

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$$\begin{bmatrix} -4 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad R_3 \to R_3 - R_1$$

$$\begin{bmatrix} -4 & 1 & 3 \\ 1 & -2 & 1 \\ 7 & 0 & -7 \end{bmatrix} \quad R_2 \to R_2 + R_1/4$$

$$\begin{bmatrix} -4 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \quad \begin{aligned} &R_3 \to R_3/7 \\ &R_2 \to R_2/7/4 \\ &R_3 \to R_3 + R_1/4 \end{aligned}$$

$$\begin{bmatrix} -4 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 1/4 & -1/4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3/4$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -4 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1, \; x_2 = 1, \; x_3 = 1$

$$v_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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$$Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix}$$

$$\Sigma = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$Q^T = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix}$$

X

PTO

Q-5

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$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 3 & 1 & 2 \\ 1 & 3 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A \approx \begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

Reduced Row echlon form

$$A \approx \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ \text{pivot column} & \text{free variables} & \text{pivot column} & \text{free variables} \end{array}$$

→ Basis of Row space

= Independent rows of matrix A

$= \{(1, 3, 0, 0), (1, 3, 1, 2)\}$

Dimension of row space = 2

→ Basis of Null space

=

S2 [illegible]
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$$RREF = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x_1 - 3x_2 = 0$

$x_3 - 2x_4 = 0$

free variables $= x_2, x_4$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$X = \begin{bmatrix} 3x_2 \\ x_2 \\ 2x_4 \\ x_4 \end{bmatrix}$$

$$X = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Basis of null spaces

$= \{(3, 1, 0, 0), (0, 0, 2, 1)\}$

Dimension of null spaces $= 2$

Dimension of column space

$=$ no of pivot columns $= 2$

Dimension of left null spaces of A

$= m - R$ ($m \to$ row, $R \to$ Rank)

$= 3 - 2$ (Rank $= d(C(A)) = 2$)

$= 1$

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Q.6

Hasse diagram for the relation

Explanation:- we know that 1 divides every number it is comparable to every element so one is at the bottom & the other elements will be higher than 1

→ 1 divides 2, 2 divides 4, they are comparable

→ 4 and 5 has no comparison 2 and 5 are not comparable, so 5 is higher ~~that~~ than 1 but no comparision b/w 2 & 5 it is same level Q6.2

→ 5 divides 10 ; 4, 10 divides 20

(b) least element for $(P, \preccurlyeq)$
$\{y \in P \mid \forall x \in P, y \preccurlyeq x\}$ (it is unique)

Here least element is 1

(c) greatest element $\{y \in P \mid \forall x \in P\ x \preccurlyeq y\}$ (it is unique)

Here greatest element is 20

(d) which elements are maximal ?
for $(P, \preccurlyeq)$ $\{y \in P \mid \text{No } x \in P\ y \prec x\}$

Here maximal is 20

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(e) which elements are minimal

(P, α)

$\{y \in P \mid \text{No } x \in P, x \alpha y\}$

Here minimal is 1

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Q.7

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 4 & 4 \end{bmatrix}$$

By gaussian elimination

$R_2 \to R_2 - R_1 \quad R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = U$$

$$E_{21} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

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$$L = G_{21}^{-1}\, G_{31}^{-1}\, G_{32}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$U' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$U' = D \cdot U$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = LDU$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Yes A is invertible, because A has all 3 pivots and the rows are linearly ~~that~~ independent

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