SS EXPERIMENT LAB 9

TITLE: Laplace transform

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OBSERVATION: In this lab, I learned how to evaluate the Laplace transform of a signal.

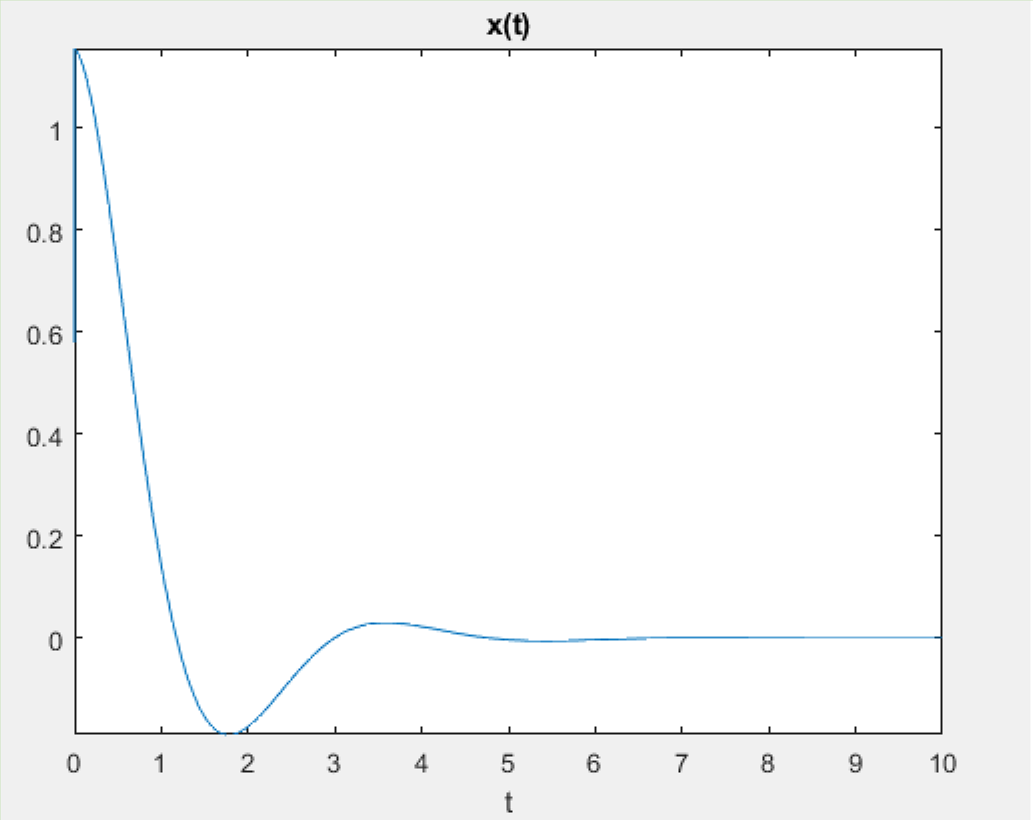
1. Write a MATLAB script to evaluate the Laplace transform of the following signals and then verify the obtained result with analytical method. Next, indicates the poles and zeros of the X(s).

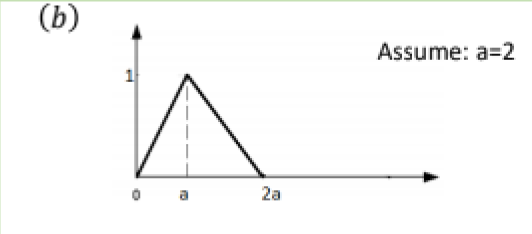
$(a)\ x(t) = \frac{1}{\sqrt{3}}[\sin(\sqrt{3}t)+2\cos(\sqrt{3}t)]\ e^{-t}u(t)$

```
syms t
xt=(1/sqrt(3))*(sin((sqrt(3))*t)+2*cos((sqrt(3))*t))*exp(-t)*heaviside(t);
fplot(xt,[0,10]);
xlabel('t');
title('x(t)');
xs=laplace(xt);
display(xs);
num=[0 2 2+sqrt(3)];
den=[1 2 4];
z=roots(num);
p=roots(den);
```

Output:

```
xs =

(3^(1/2)*((2*(s + 1))/((s + 1)^2 + 3) + 3^(1/2)/((s + 1)^2 + 3)))/3
```

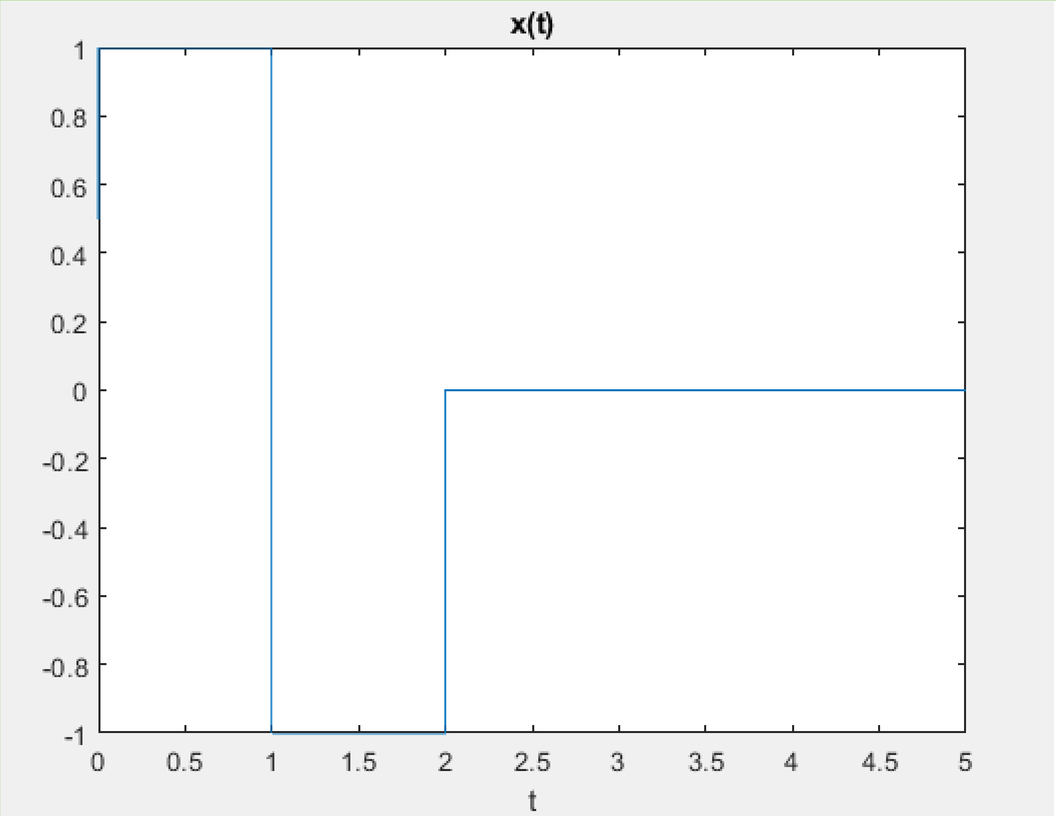

x(t)
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
t


(b)
Assume: a=2
1
0
a
2a

```
syms t
xt=heaviside(t)-2*heaviside(t-1)+heaviside(t-2);
fplot(xt,[0,5]);
xlabel('t');
title('x(t)');
xs=laplace(xt);
display(xs);
num=[1 -2 -1];
den=[0 1 0];
z=roots(num);
p=roots(den);
```

Output:

```
xs =

exp(-2*s)/s - (2*exp(-s))/s + 1/s
```

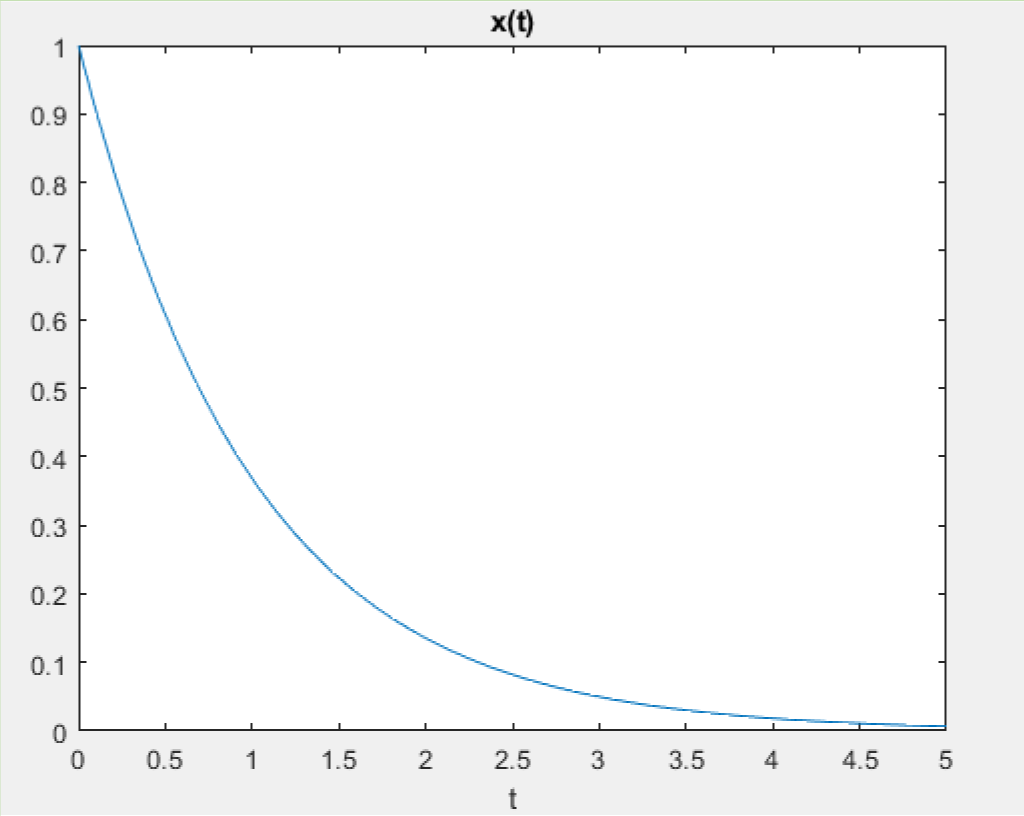


(c) $x(t) = u(t) - 2u(t-1) + u(t-2)$

```
syms t
xt=exp(-abs(t))*heaviside(t+1);
fplot(xt,[0,5]);
xlabel('t');
title('x(t)');
xs=laplace(xt);
display(xs);
num= [0 1];
den = [1 1];
z=roots(num);
p=roots(den);
```

Output:

```
xs =

1/(s + 1)
```

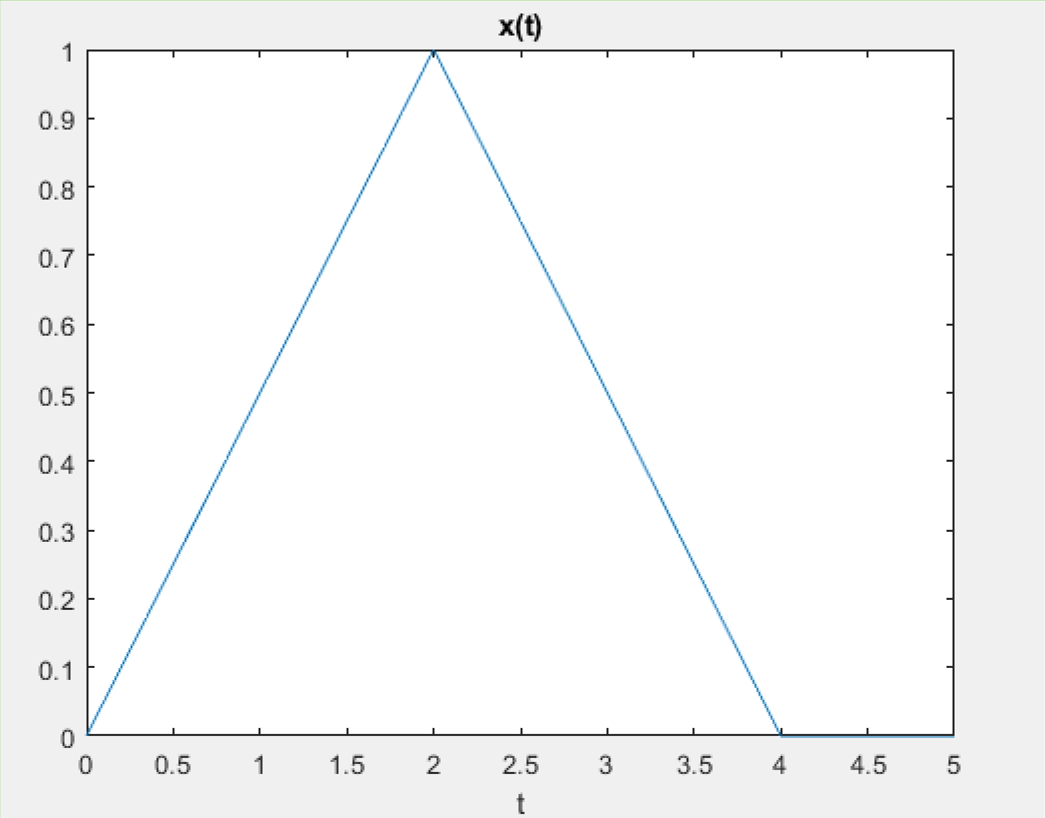


(d) $x(t) = e^{-|t|}u(t+1)$

```
syms t
a=2;
xt=(1/a)*t*(heaviside(t)-heaviside(t-a))+(1/a)*(2*a-t)*(heaviside(t-a)-heaviside(t-2*a));
fplot(xt,[0,5]);
xlabel('t');
title('x(t)');
xs=laplace(xt);
display(xs);
num= [0 1];
den = [1 1];
z=roots(num);
p=roots(den);
```

Output:

```
xs =

1/(2*s^2) - exp(-2*s)/(2*s^2) - exp(-2*s)/s + (exp(-4*s)*(2*s*exp(2*s) - exp(2*s) + 1))/(2*s^2)
```



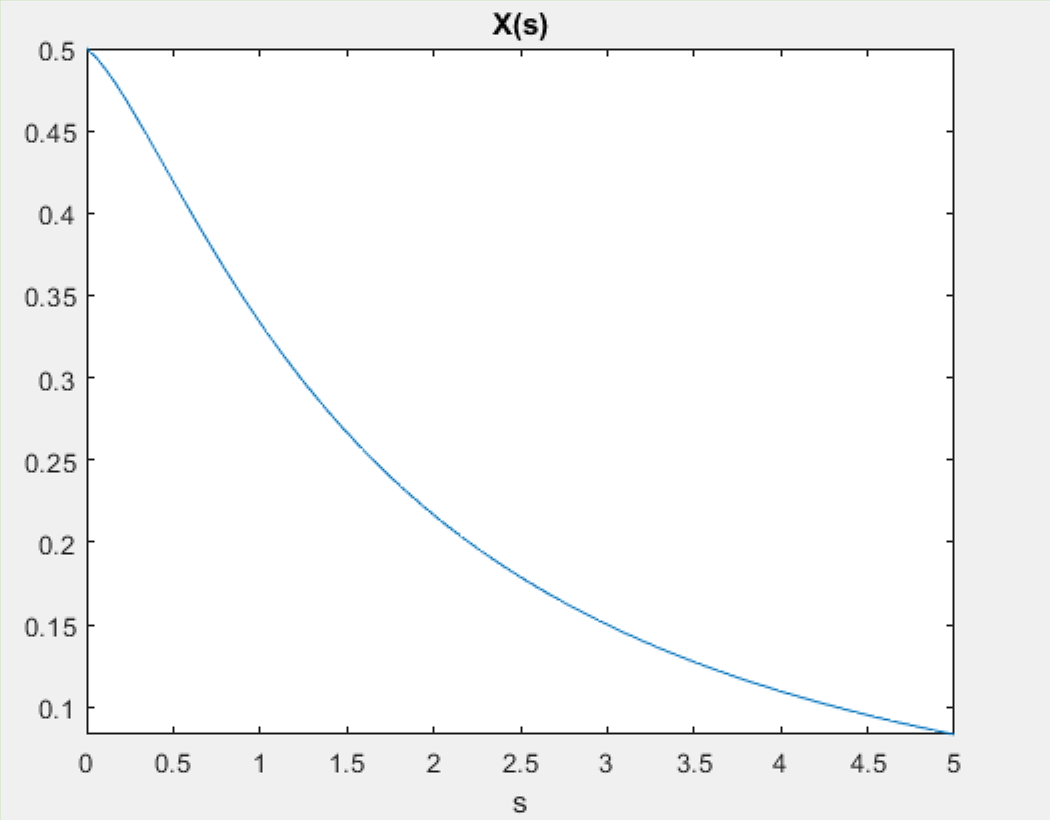
2. Write a MATLAB script to evaluate the inverse Laplace transform of the following signals and then verify the obtained result with analytical method

$$(a)\ X(s) = \frac{5s+3}{(s+1)(s+2)(s+3)}$$

```
syms s
xs=(5*s+3)/((s+1)*(s+2)*(s+3));
fplot(xs,[0,5]);
xlabel('s');
title('X(s)');
xt=ilaplace(xs);
display(xt);
num=[5 3];
den=[1 6 11 6];
z=roots(num);
p=roots(den);
display(z);
display(p);
```

Output:

```
xt =

7*exp(-2*t) - exp(-t) - 6*exp(-3*t)


z =

   -0.6000


p =

   -3.0000
   -2.0000
   -1.0000
```

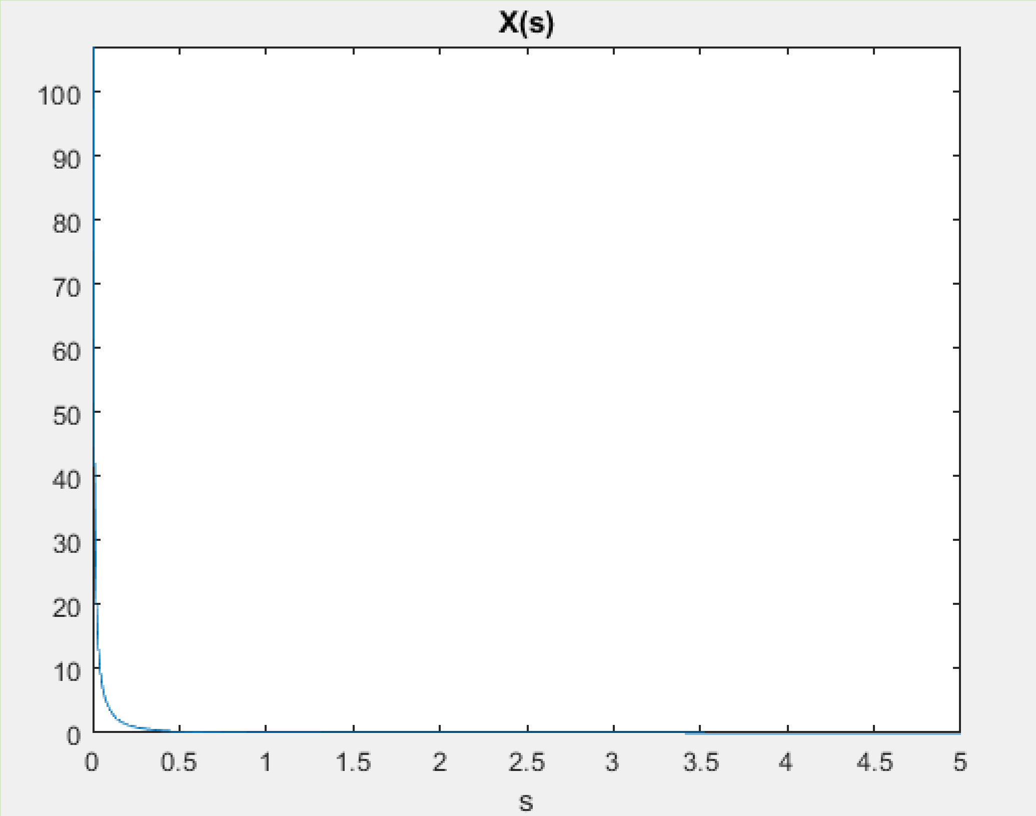


$$(b)\ X(s) = \frac{1}{s(s+1)^3\,(s+2)}$$

```
syms s
xs=1/(s*(s+2)*(s+1)^3);
fplot(xs,[0,5]);
xlabel('s');
title('X(s)');
xt=ilaplace(xs);
display(xt);
num=[1];
den=[1 5 9 7 2 0];
z=roots(num);
p=roots(den);
display(z);
display(p);
```

Output:

```
xt =

exp(-2*t)/2 - exp(-t) - (t^2*exp(-t))/2 + 1/2


z =

  0×1 empty double column vector


p =

   0.0000 + 0.0000i
  -2.0000 + 0.0000i
  -1.0000 + 0.0000i
  -1.0000 - 0.0000i
  -1.0000 + 0.0000i
```

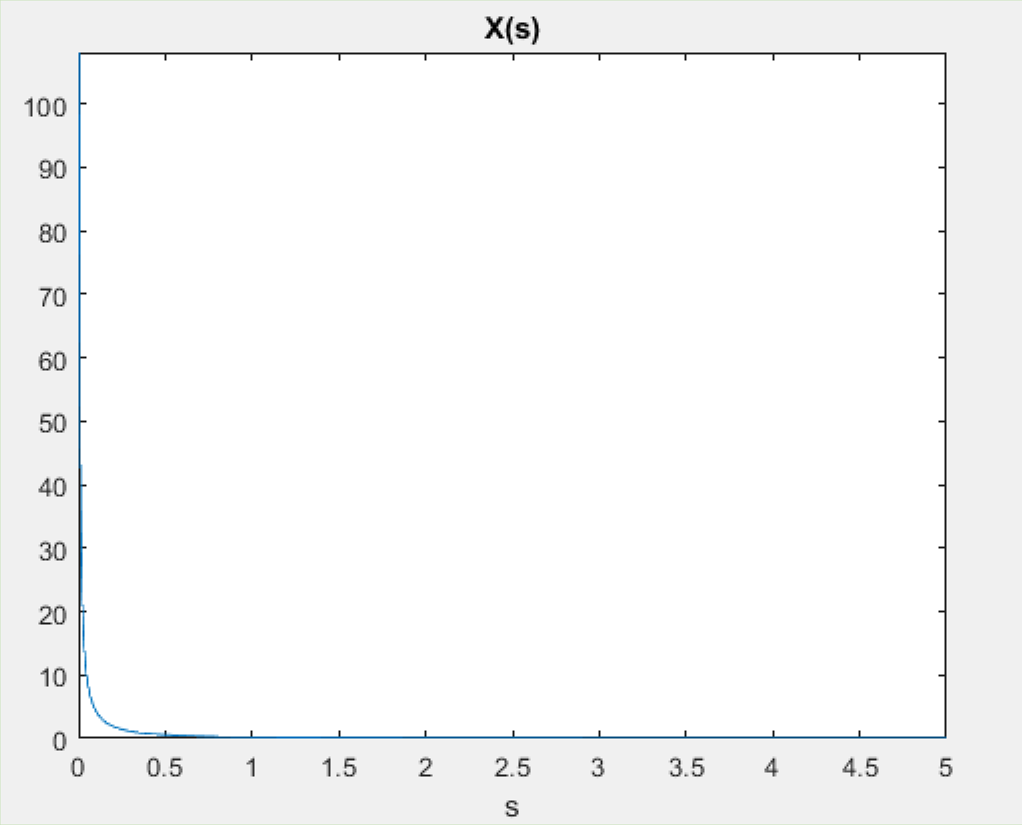


$$(c)\ X(s) = \frac{(1 - se^{-s})}{s(s+2)}$$

```
syms s
xs=(1-s*exp(-s))/(s*(s+2));
fplot(xs,[0,5]);
xlabel('s');
title('X(s)');
xt=ilaplace(xs);
display(xt);
den=[1 2 0];
p=roots(den);
display(p);
```

Output:

```
xt =

1/2 - heaviside(t - 1)*exp(2 - 2*t) - exp(-2*t)/2


p =

     0
    -2
```

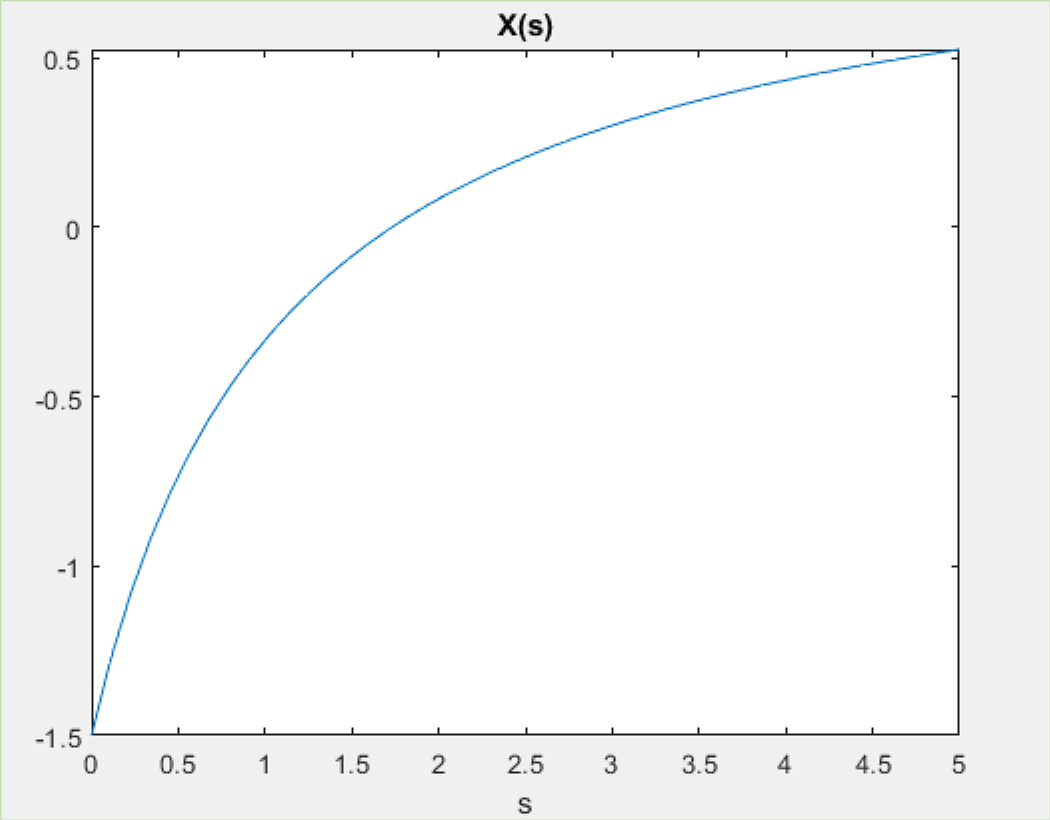


(d) $X(s) = \frac{s^2-3}{(s+1)(s+2)}$

```
syms s
xs=(s^2-3)/((s+1)*(s+2));
fplot(xs,[0,5]);
xlabel('s');
title('X(s)');
xt=ilaplace(xs);
display(xt);
num=[1 0 -3];
den=[1 3 2];
z=roots(num);
p=roots(den);
display(z);
display(p);
```

Output:

```
xt =

dirac(t) - exp(-2*t) - 2*exp(-t)


z =

    1.7321
   -1.7321


p =

    -2
    -1
```



Analytical method:

1.

a) $x(t) = \frac{1}{\sqrt{3}}\left[\sin\sqrt{3}t + 2\cos\sqrt{3}t\right] e^{-t} u(t)$

$$x(t) = \frac{1}{\sqrt{3}}\left[\frac{e^{i\sqrt{3}t} - e^{-i\sqrt{3}t}}{2i} + \frac{2e^{i\sqrt{3}t} + 2i e^{-i\sqrt{3}t}}{2i}\right] e^{-t} u(t)$$

$$= \frac{1}{\sqrt{3}}\left[\frac{(1+2i)e^{i\sqrt{3}t} + (-1+2i)e^{-i\sqrt{3}t}}{2i}\right] e^{-t} u(t)$$

$$x(t) = \frac{u(t)}{2\sqrt{3}\,i}\left((1+2i)\,e^{t(-1+i\sqrt{3})} + (-1+2i)\,e^{t(-1-i\sqrt{3})}\right)$$

$$x(t) = \frac{(1+2i)\,i}{2\sqrt{3}\,i\cdot i}\left[e^{-(1-i\sqrt{3})t} u(t)\right] + \frac{(-1+2i)\,i}{2\sqrt{3}\,i\,i}\left[e^{-(1+i\sqrt{3})t} u(t)\right]$$

$$= \frac{2-i}{2\sqrt{3}}\left(e^{-(1-i\sqrt{3})t} u(t)\right] + \frac{(i+2)}{2\sqrt{3}}\left(e^{-(1+i\sqrt{3})t} u(t)\right]$$

$$x(t) = e^{-at} u(t) \xrightarrow{L.T} X(s) = \frac{1}{s+a}$$

$$= \frac{2-i}{2\sqrt{3}} \times \frac{1}{(s+1-i\sqrt{3})} + \frac{i+2}{2\sqrt{3}} \times \frac{1}{(s+1+i\sqrt{3})}$$

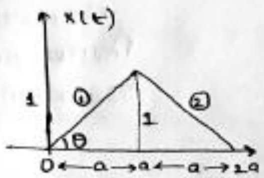
$$= \frac{2-i}{2\sqrt{3}s + 2\sqrt{3} - 16} + \frac{2+i}{2\sqrt{3}(s+1) + 6i}$$

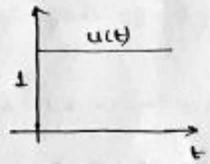
$$= \frac{(2-i)(2\sqrt{3}(s+1)+6i) + (2+i)(2\sqrt{3}(s+1)-6i)}{(2\sqrt{3}(s+1))^2 - (6i)^2}$$

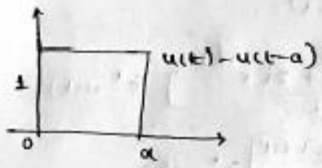
$$= \frac{2\sqrt{3}s + 2\sqrt{3} + 3}{6(s+1)^2 + 18}$$

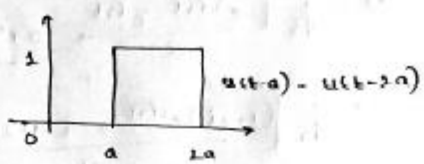
$$\boxed{\therefore \text{Zeroes} = -1.86 \qquad \text{Poles} = -1+\sqrt{3}i,\ -1-\sqrt{3}i}$$

b)









① $\rightarrow$ $x_1(t) = \frac{1}{a} \times t \, (u(t) - u(t-a))$

② $\rightarrow$ $x_2(t) =$ $(a,1)$ and $(2a,0)$

$$(y-1)(a) = (-1)(x-a)$$

$$y = \frac{1}{a}(a-x) + \frac{a}{a}$$

$$y = \frac{1}{a}(2a-x)$$

$$x_2(t) = \frac{1}{a}(2a-t)$$

$$x(t) = x_1(t) + x_2(t)$$

$$x(t) = \frac{t}{a}\left(u(t) - u(t-a)\right) + \frac{(2a-t)}{a}\left(u(t-a) - u(t-2a)\right)$$

$$x(t) = u(t) \xrightarrow{LT} x(s) = \frac{1}{s}$$

$$u(t-t_0) \xrightarrow{LT} U(s)\, e^{-st_0}$$

$$-t\,u(t) \longrightarrow \frac{d}{ds} x(s) = \frac{-1}{s^2}$$

$$t(u(t)) \longrightarrow \frac{1}{s^2}$$

(2)

$x(t) = \frac{1}{a} t\, u(t) - \frac{1}{a} t\, u(t-a) + 2u(t-a) - 2u(t-2a) - \frac{1}{a} t\, u(t-a) + \frac{1}{a} t\, u(t-2a)$

$= \frac{1}{a} t\, u(t) - \frac{2}{a} t\, u(t-a) + \frac{1}{a} t\, u(t-2a) + 2u(t-a) - 2u(t-2a)$

$LT[x(t)] = X(s)$

$\Rightarrow \frac{1}{a}\frac{1}{s^2} - \frac{2}{a}\frac{1}{(s-a)^2} + \frac{1}{a}\frac{1}{(s-2a)^2} + \frac{2}{s-a} - \frac{2}{(s-2a)}$

$X(s) = \frac{1}{as^2} - \frac{2}{a(s-a)^2} + \frac{1}{a(s-2a)^2} + \frac{2}{s-a} - \frac{2}{(s-2a)}$

$= \frac{1}{as^2} + \frac{1}{s-a}\left[\frac{-2}{a(s-a)} + 2\right] + \frac{1}{(s-2a)}\left[\frac{1}{a(s-2a)} - 2\right]$

$\boxed{\therefore \text{ Poles} = 0, \text{ Zeroes} = 0}$

c) $x(t) = u(t) - 2u(t-1) + u(t-2)$

$x(t) = u(t) \xrightarrow{LT} X(s) = \frac{1}{s}$

$LT[x(t)] = X(s)$

$\Rightarrow \frac{1}{s} - \frac{2}{s-1} + \frac{1}{s-2}$

$X(s) = \frac{1}{s} - \frac{2}{s-1} + \frac{1}{s-2}$

$\boxed{\therefore \text{ Poles} = 0, \text{ Zeroes} = 0}$

d) $x(t) = e^{-|t|} u(t+1)$

$LT[x(t)] = X(s)$

$$= \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} e^{-|t| - st} u(t+1) x(t) dt$$

$$= \int_{-1}^{\infty} e^{-|t| - st} dt$$

$$= \int_{-1}^{0} e^{-t(-s+1)} dt + \int_{0}^{\infty} e^{t(1-s)} dt$$

$$= \frac{e^{-t(s+1)}}{s+1}\Big|_{0}^{1} - \frac{e^{-t(s+1)}}{s+1}\Big|_{0}^{\infty}$$

$$= \frac{-e^{(s+1)}}{s+1} + \frac{1}{s+1} + 0 + \frac{1}{s+1}$$

$$= \frac{2 - e^{s+1}}{s+1}$$

$$\boxed{\therefore \text{Poles} = -1, \ \text{Zeroes} = 0}$$

2)

a) $X(s) = \frac{5s+3}{(s+1)(s+2)(s+3)}$

$$X(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$\boxed{\begin{array}{l} \text{Zeroes} = -3/5 \\ \text{Poles} = -1, -2, -3 \end{array}}$$

$A = -1$; $B = 7$; $C = -6$

$$x(t) = -e^{-t} u(t) + 7 e^{-2t} u(t) - 6 e^{-3t} u(t)$$

b) $x(s) = \dfrac{1}{s(s+1)^3(s+2)}$

$\boxed{\text{Zeroes} = 0 \quad \text{Poles} = 0, -1, -1, -1, -2}$

$x(s) = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{(s+1)^2} + \dfrac{D}{(s+1)^3} + \dfrac{E}{s+2}$

$A = \frac{1}{2}$; $B = -1$; $C = 0$; $D = -1$, $E = \frac{1}{2}$

$x(t) = \dfrac{u(t)}{2} - e^{-t}u(t) - \dfrac{t^2}{2} e^{-t}u(t) + \dfrac{e^{-2t}u(t)}{2}$

c) $x(s) = \dfrac{1 - s e^{-s}}{s(s+2)}$

$\boxed{\text{Poles} = 0, -2}$

$x(s) = \dfrac{1}{s(s+2)} - \dfrac{s e^{-s}}{s(s+2)}$

$= \dfrac{1}{2s} - \dfrac{1}{2(s+2)} - \dfrac{e^{-s}}{s+2}$

$u(t) \xrightarrow{LT} 1/s$

$e^{s_0 t} x(t) \xrightarrow{LT} x(s - s_0)$

∴ $x(t) = \dfrac{u(t)}{2} - \dfrac{e^{-2t}u(t)}{2} - e^{-2(t-1)}u(t-1)$

$\boxed{\because e^{-2(t-1)}u(t-1) \rightarrow \dfrac{e^{-s}}{s+2}}$

d) $x(s) = \dfrac{s^2 - 3}{(s+1)(s+2)}$

$x(s) = 1 - \dfrac{2}{s+1} - \dfrac{1}{s+2}$

$x(t) = \delta(t) - 2e^{-t}u(t) - e^{-2t}u(t)$

Zeroes $= \sqrt{3}, -\sqrt{3}$; Poles $= -1, 2$