Signal & Systems

Assignment - 1

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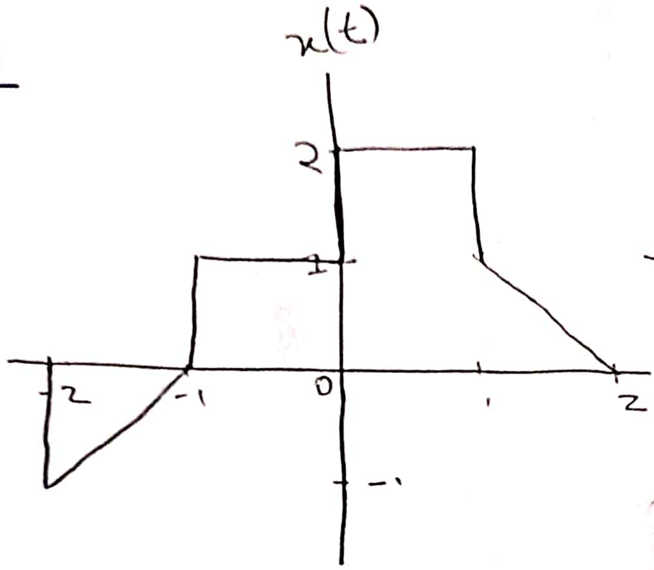
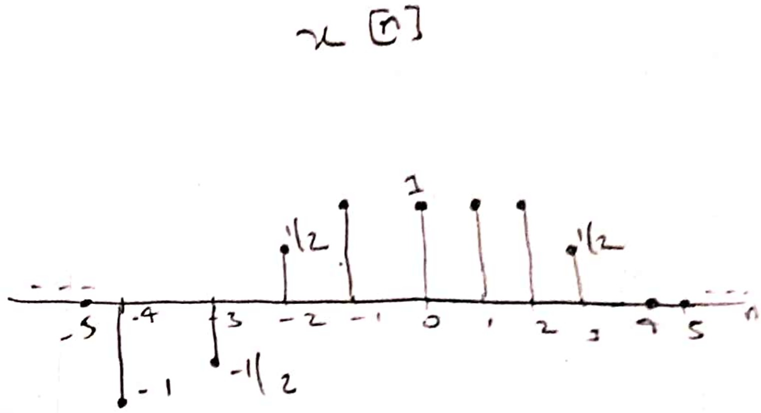
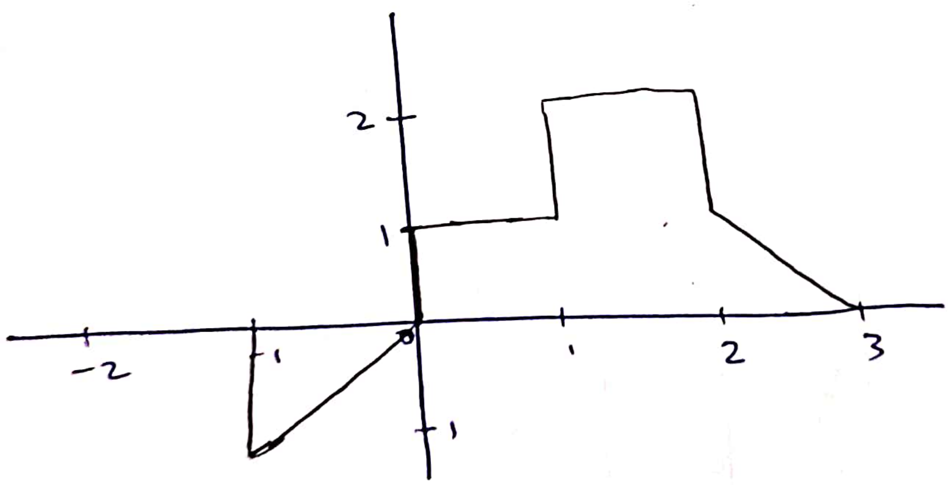
Q-0

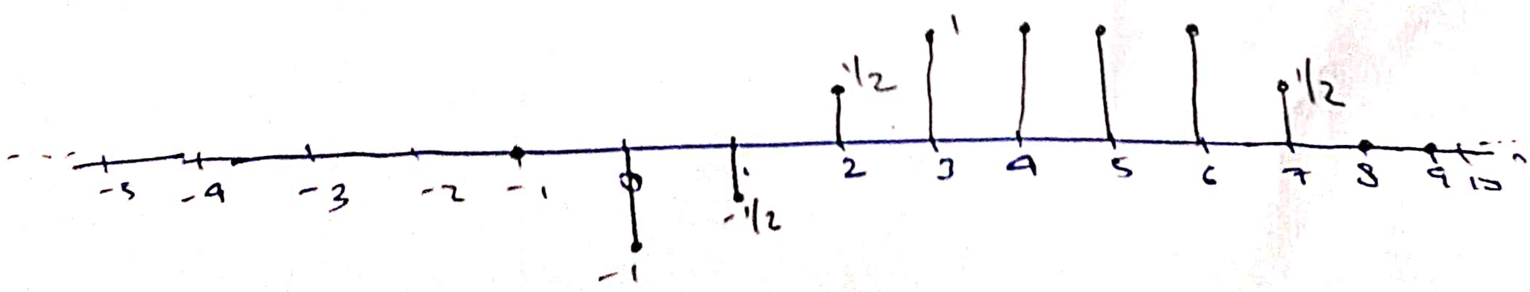




(a) $x(t-1)$

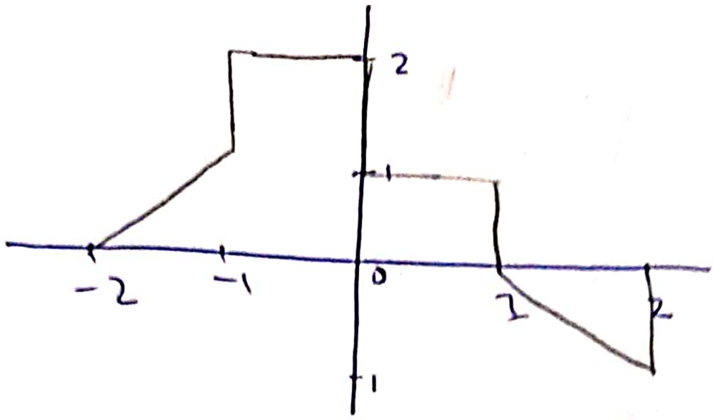


(b) $x[n-4]$

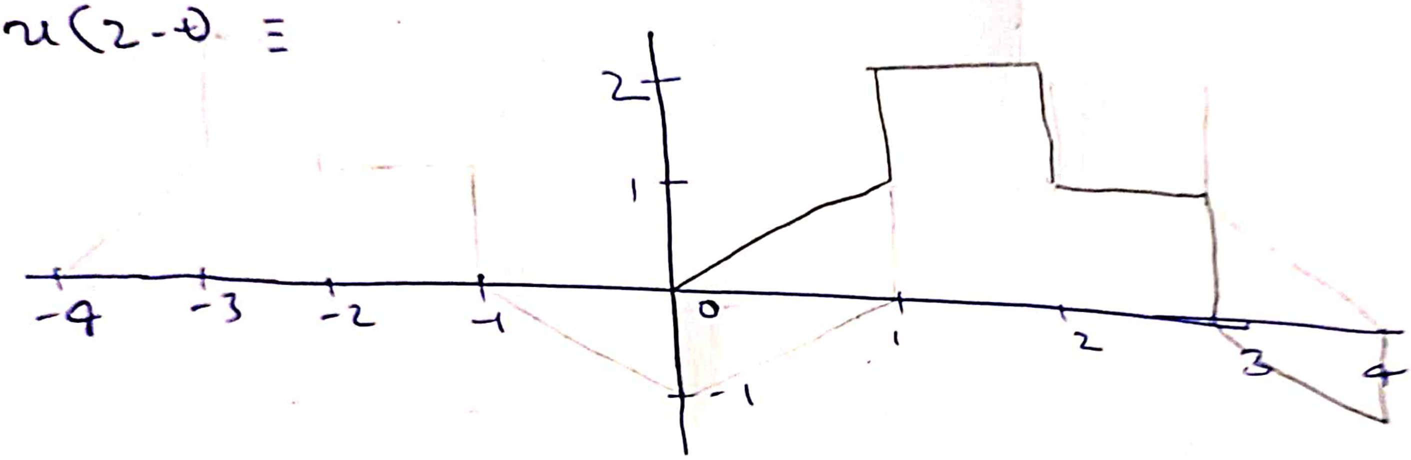


(c) $\mathcal{E}\{x(2-t)$

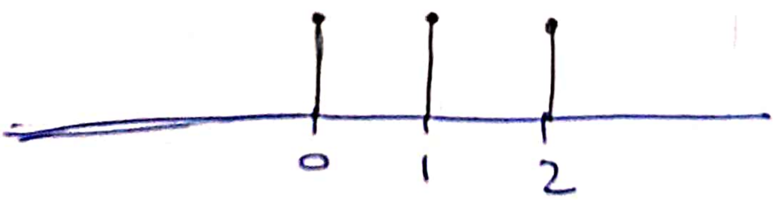
$x(-t) =$



$x(2-t) =$

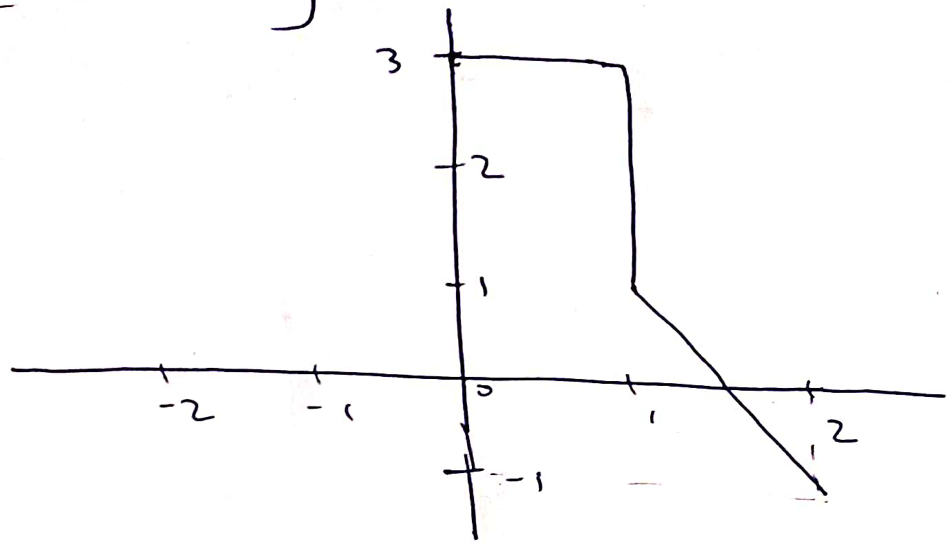


(d) $x[(n-1)^2]$



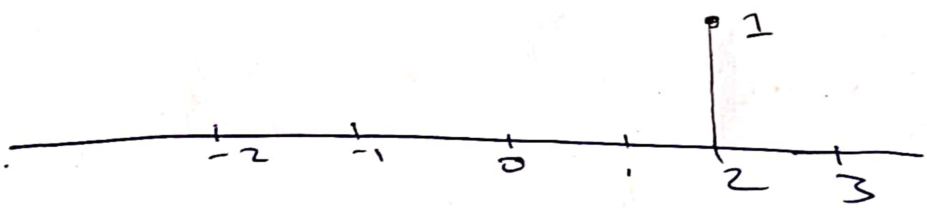
(e) $[x(t) + x(-t)]u(t)$

$u(t)[x(t) + x(-t)] =$



(f) $x(n-2)\,\delta[n-2]$

$=$



End

Q-1

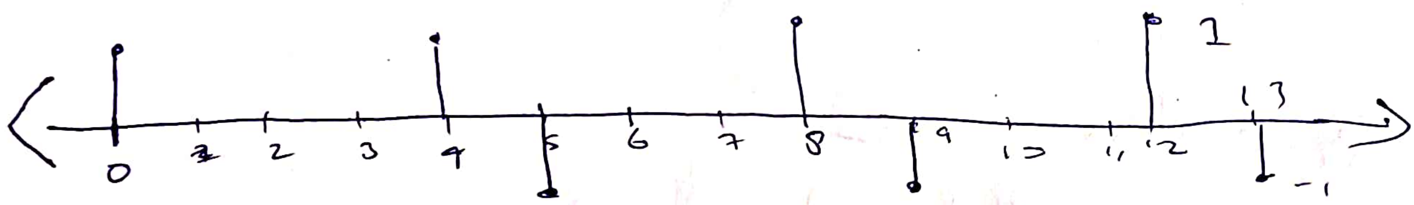
(a) $x(t) = 2e^{i(4t + \frac{\pi}{4})} u(t)$

$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$

$x(t) = \begin{cases} 2e^{i(4t+\frac{\pi}{4})} & t \geq 0 \\ 0 & t < 0 \end{cases}$

Hence ~~a~~ signal is aperiodic

(b) $x[n] = \sum_{n=-\infty}^{\infty} (\delta[n-4k] - \delta[n-(4k+1)])$



It is period signal of 4

Q.

Q-2

(a) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$

let $x(t)$ be periodic of period T

then $x(t) = x(t+T)$

$$3\cos\left(4t + \frac{\pi}{3}\right) = 3\cos\left(4(t+T) + \frac{\pi}{3}\right)$$

$$= 3\left[\cos\left(4t + \frac{\pi}{3}\right) \cdot \cos(4T) - \sin\left(4t+\frac{\pi}{3}\right) \cdot \sin(4T)\right]$$

This can be periodic only if $4T = 2\pi k$

$k = 1$

$$\boxed{T = \frac{\pi}{2}}$$

(b) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$

$x(t) = x(t+T)$

Similar to ~~prev~~ ~~privious~~ previous question

this can be periodic only if

$2T = 2\pi k$

$k = 1$

$$\boxed{T = \pi}$$

(c) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(t-n)} u(2t-n)$

$u(2t-n) = \begin{cases} 1 & t \geq n/2 \\ 0 & t < n/2 \end{cases}$

$x(t+T) = \sum_{n=-\infty}^{\infty} e^{-(t-n+2T)} u(2t-n+2T)$

$x(t)$ is decreasing n is integer and t is rational.

only possible when $x(t) = x(t+T)$

$T = 0$ but T can't be zero, hence

$x(t)$ is aperiodic

(d) $x(n) = \sin\left(\frac{6\pi}{7} n + 1\right)$

$x[n]$ is periodic

$x(n) = x[n+N]$

$\sin\left(\frac{6\pi}{7} n + 1\right) = \sin\left(\frac{6\pi}{7} n + 1 + \frac{6\pi}{7} N\right)$

$= \sin\left(\frac{6\pi}{7} n + 1\right) \cos\left(\frac{6\pi}{7} N\right)$

$+ \cos\left(\frac{6\pi}{7} n + 1\right) + S\left(\frac{6\pi}{7} N\right)$

Only possible when $\frac{6\pi}{7} N = 2\pi k$

$\boxed{N = 7}$ $k = 3$

(e) $x[n] = \sin\left(\frac{\pi}{5}n^2\right)$

$$\frac{\pi}{5}N^2 = 2\pi k$$

$$N^2 = \sqrt{10k}$$

$$k = 10$$

$$\boxed{N = 10}$$

(f) $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{\pi}{4}n\right)\right]$$

Time period of $\cos\left(\frac{3\pi}{4}n\right)$

$$\frac{3\pi N}{4} = 2\pi k$$

$$k = 3$$

$$N = 8$$

$\cos\left(\frac{\pi}{4}n\right)$

$$\frac{\pi}{4}N = 2\pi k$$

$$k = 1$$

$$N = 8$$

Time period of $x[n]$ = lcm of Time period of $x_1[n]$ & $x_2[n]$

$$= 8$$

(g) $x[n] = e^{(-1+j)n}$

$x[n] = x[n+N]$

$e^{(-1+j)n} = e^{(-1+j)(n+N)}$

$e^{(-1+j)n} = e^{(-1+j)n} \times e^{(-1+j)N}$

$1 = e^{(-1+j)N}$

$1 = e^{-N} e^{jN}$

This is only possible when N is zero but

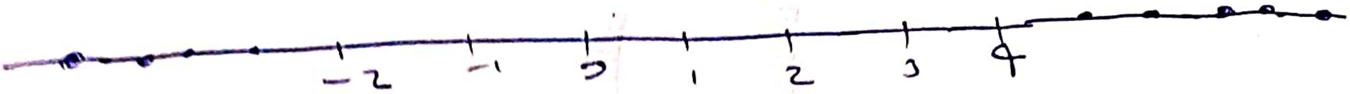
N cannot be zero So

$1 \neq e^{-N} e^{jN}$ for $N \neq 0$

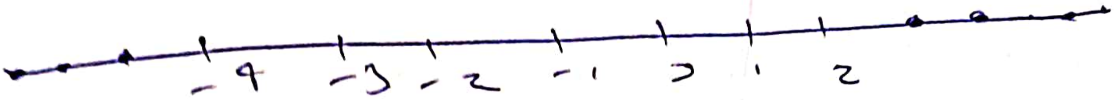
$x[n]$ is aperiodic

Q-3

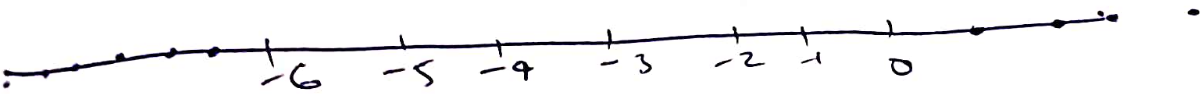
$u[n] = 0$ for $n < -2$
$n > 4$



$u[-n]$

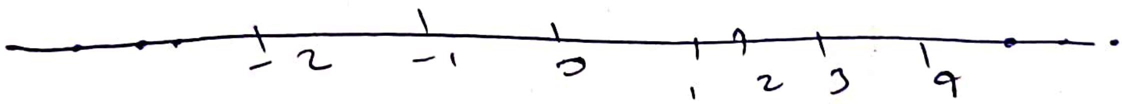


b) $u[-n+2]$



$u[n+2] = 0$ for $u > 0$
$u < -6$

(b) $u[-n+2]$



$u[n+2] = 0$ for $u > 4$
$u < -2$

4-9 $u(t) = 0$ for $t < 3$

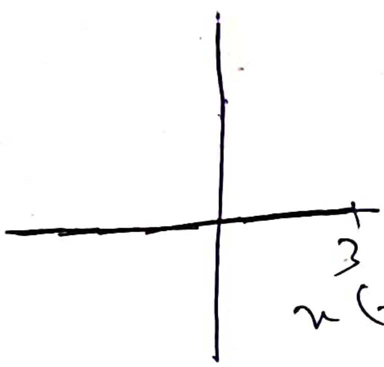
(a) $u(1-t) + u\left(\frac{t}{3}\right)$

$u\left(\frac{t}{3}\right)$ will also be equal to zero for $t < 3$

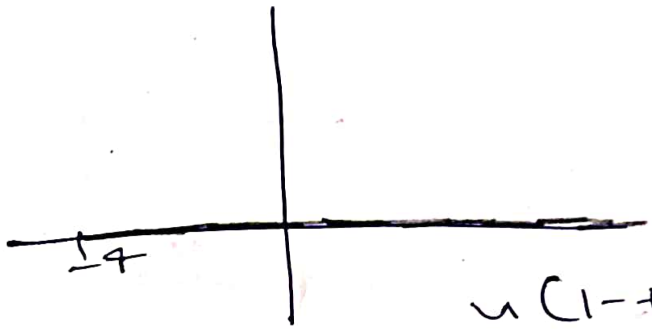
$u(-t) = 0$ for $t > -3$

(b) $u(1-t) = 0$ for $t > -2$

(a) $u(1-t) + u\left(\frac{t}{3}\right) = 0$ for $t > -2$ and $t < 3$



$u(t)$



$u(1-t)$

$u(1-t) + u\left(\frac{t}{3}\right)$

Q-5 $x(t) = 0$ for $t < 3$

$x(t) = e^{-2t}u(t)$

We know that $e^{-2t} > 0$ for all values of t

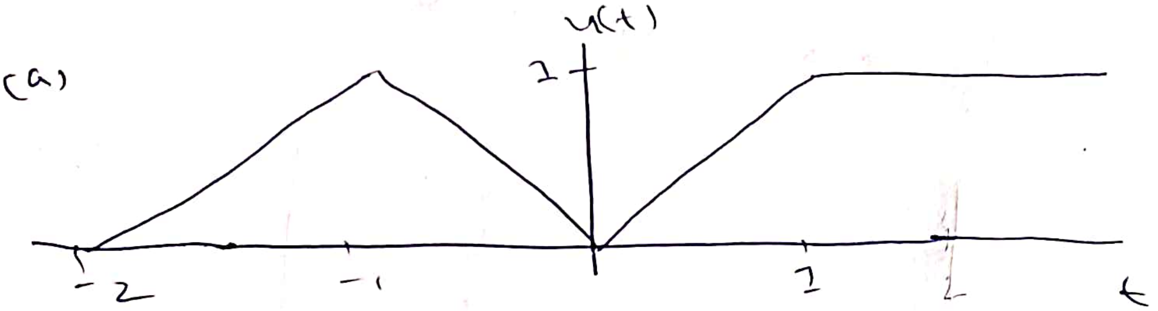
$x(t)$ will only be zero when

$u(t) = 0$

$u(t) = 0$ for $(t < 0)$

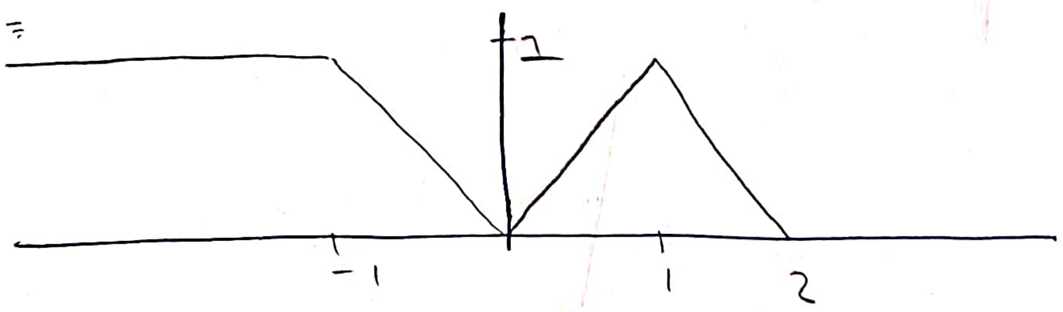
hence $x(t) = 0$ for $t < 0$

Q-7 (a)

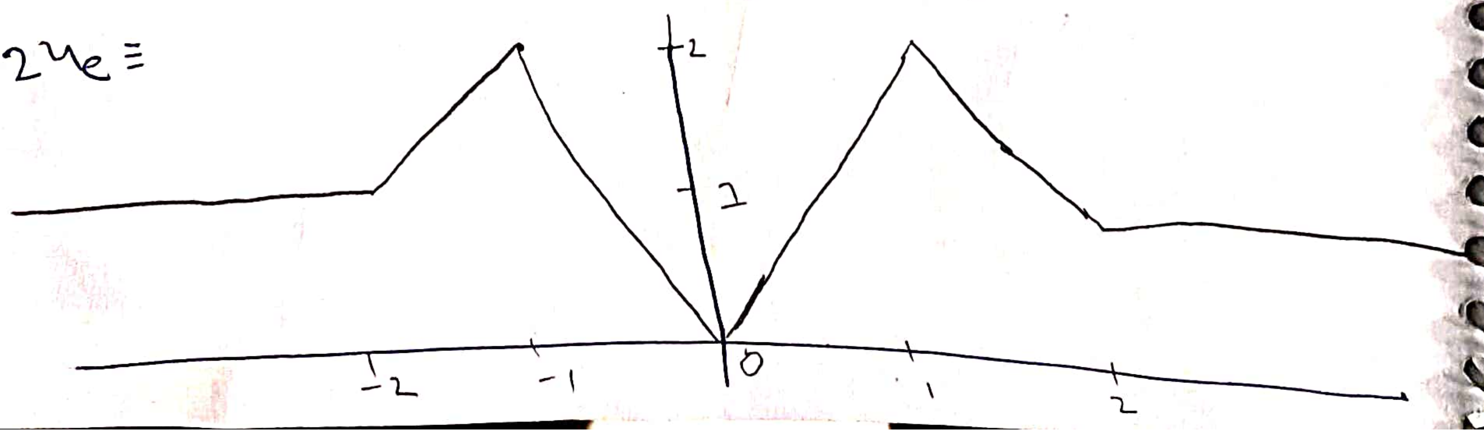


$x_e = x(t) + x(-t)$

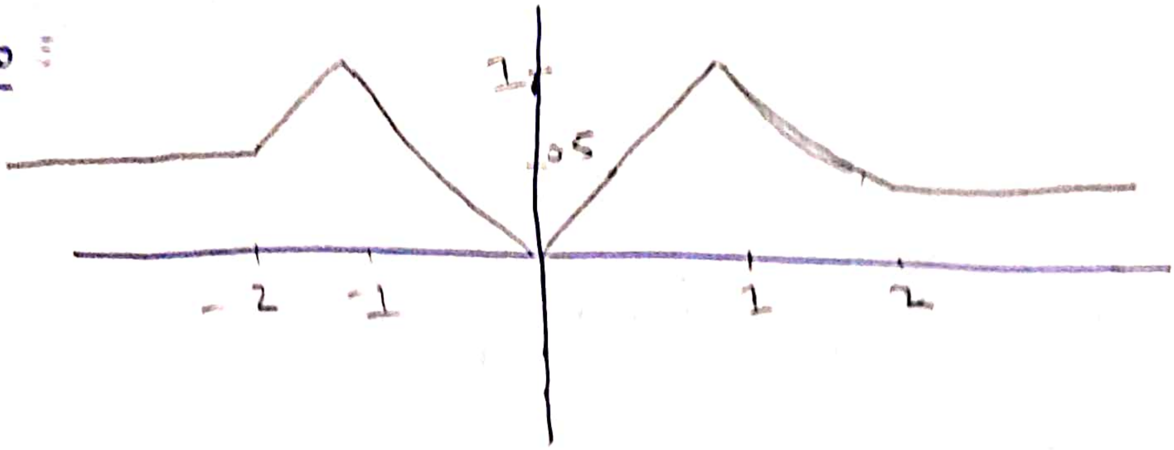
$x(-t) =$



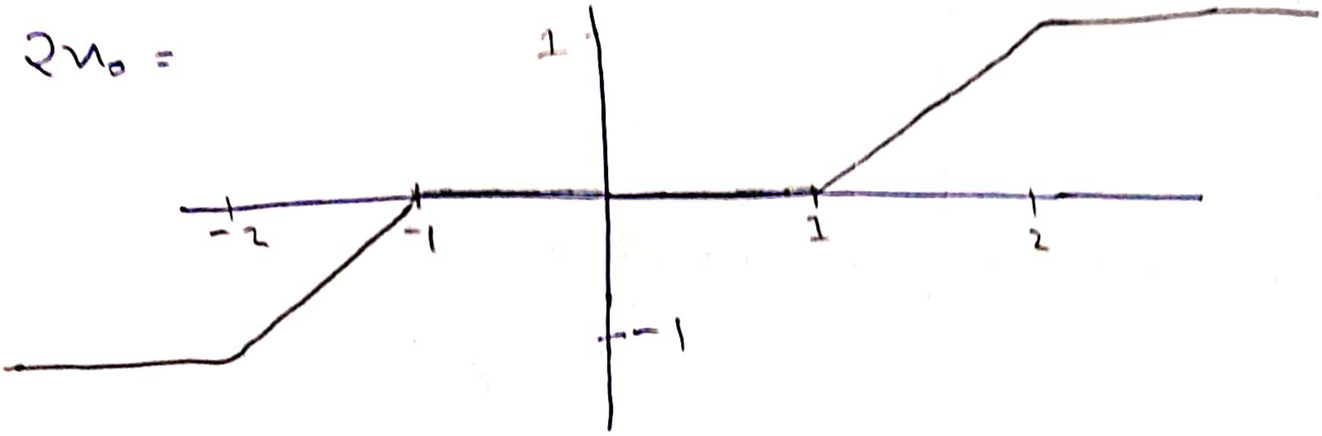
$2x_e \equiv$



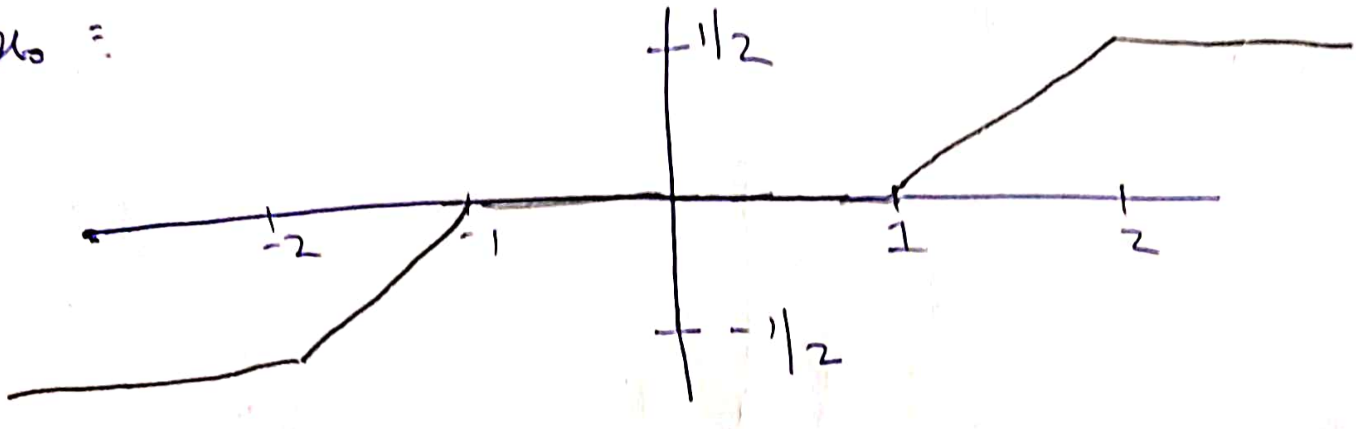
x_e :



$2x_o$ =



x_o :



Q5 (b) $e^{j(2t+\frac{\pi}{4})}$

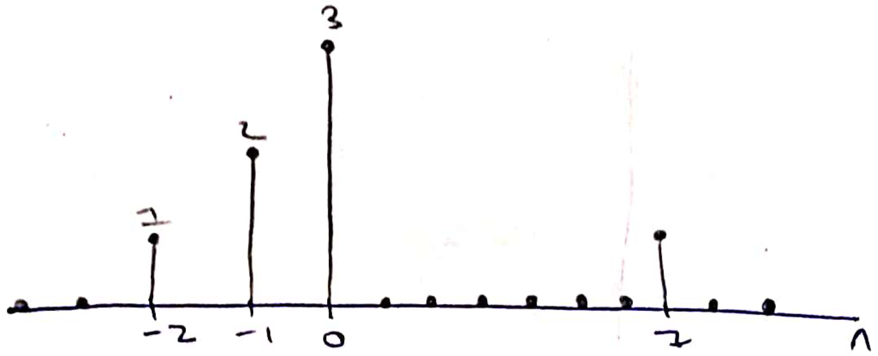
$= \cos(2t + \frac{\pi}{4}) + i \sin(2t + \frac{\pi}{4})$

$\neq 0$

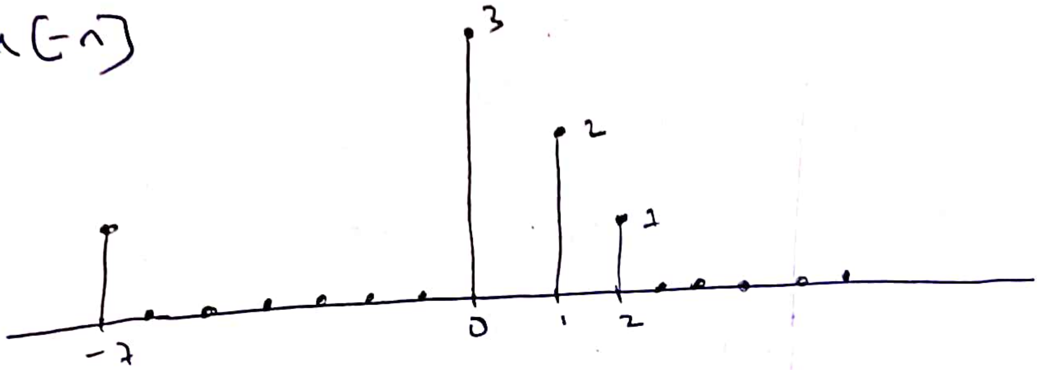
for any value of t

(b)

x[n]



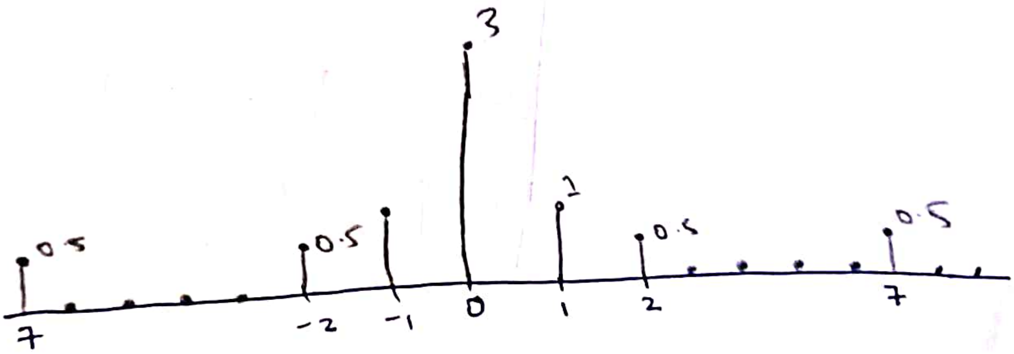
x[-n]



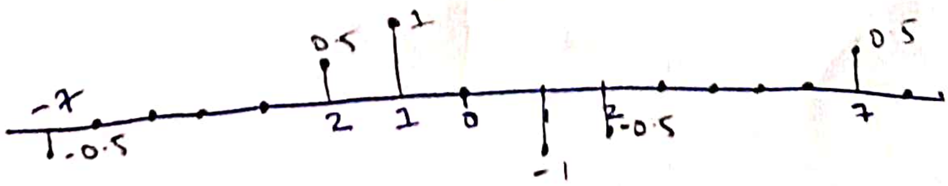
$\frac{1}{2}(x[n] + x[-n]) = x_e$

$\frac{1}{2}(x[n] - x[-n]) = x_o$

x_e



x_o



Q-6

(a) $y(t) = t^2(u-1)$

Considering two inputs $u_1(t)$ and $u_2(t)$

$u_1(t) \longrightarrow t^2 u_1(t-1)$

$u_2(t) \longrightarrow t^2 u_2(t-1)$

let $u_3(t) = a u_1(t) + b u_2(t)$

$u_3 \rightarrow y_3(t) = t^2 u_3(t-1)$

$= t^2\{a u_1(t_1-1) + b u_2(t-1)$

$= a t^2 u_1(t-1) + b u_2(t-1)$

linear

Shifting $u(t)$ by t_0

$x(t-t_0) \rightarrow t^2(t-1-t_0)$ — (I)

Shifting $y(t)$ by t_0

$y(t-t_0) = (t-t_0)^2 u(t-1-t_0)$ — (II)

(I) $\neq$ (II)

Time ~~invariant~~ Variant

(b) $y[n] = x^2[n-2]$

let's consider two inputs $x_1(t)$ & $x_2(t)$

$x_1[n] \Rightarrow x_1^2[n-2]$

$x_2[n] \rightarrow x_2^2[n-2]$

let $x_3[n] = ax_1[n] + bx_2[n]$

$x_3[n] \rightarrow y_3[n] = x_3^2[n-2]$

$= (ax_1[n-2] + bx_2[n-2])^2$

$\neq ay_1 + by_2$

Hence non-linear

Shifting $x[n]$ by n_0

$x[n-n_0] \rightarrow x^2[n-n_0]$ — (I)

Shifting $y[n]$ by n_0

$y[n-n_0] \rightarrow x^2[\cancel{n}-n_0]$ — (II)

(I) = (II) Time Invariant

(c) $y(t) = \sum_{k=-\infty}^{\infty} x(t)\,\delta(t-T_s)$

let's consider two arbitrary input

$x_1(t)$ & $x_2(t)$

$x_1(t) \rightarrow \sum_{k=-\infty}^{\infty} x_1(t)\,\delta(t-kT_s)$

$x_2(t) \rightarrow \sum_{k=-\infty}^{\infty} x_2(t)\,\delta(t-kT_s$

let $x_3(t) = a x_1(t) + b x_2(t)$

$$x_3(t) \rightarrow y_3(t) = \sum_{k=-\infty}^{\infty} x_3(t)\,\delta(t - kT_s)$$

$$= \sum_{k=-\infty}^{\infty} (a x_1(t) + b x_2(t))\,\delta(t - kT_s)$$

$$= \sum_{k=-\infty}^{\infty} a x_1(t)\,\delta(t - kT_s) + \sum_{k=-\infty}^{\infty} b x_2(t)\,\delta(t - kT_s)$$

$$= a y_1 + b y_2$$

Hence linear

Shifting $x(t)$ by t_0

$$x(t - t_0) \rightarrow \sum_{k=-\infty}^{\infty} x(t - t_0)\,\delta(t - kT_s)$$

Shifting $y(t)$ by t_0

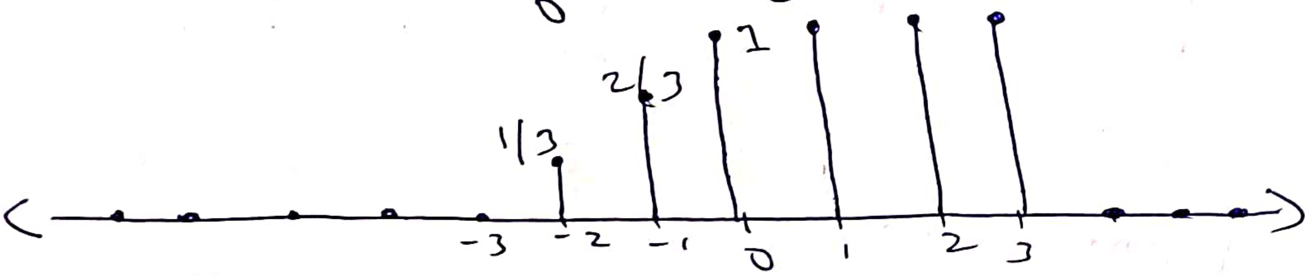
$$y(t - t_0) \rightarrow \sum_{k=-\infty}^{\infty} x(t - t_0)\,\delta(t - t_0 - kT_s)$$

$\text{I} \neq \text{II}$ Time variant

Q. - 17

$$x(n) = \begin{cases} 1 + \frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{else} \end{cases}$$

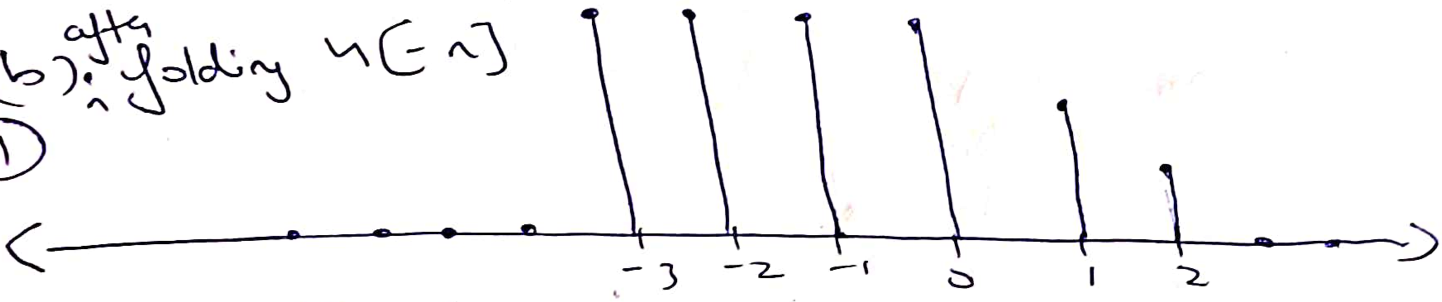
$$= \begin{cases} 0 & n = -3 \\ 1/3 & n = -2 \\ 2/3 & n = -1 \\ 1 & n = 0, 1, 2, 3 \\ 0 & \text{else} \end{cases}$$



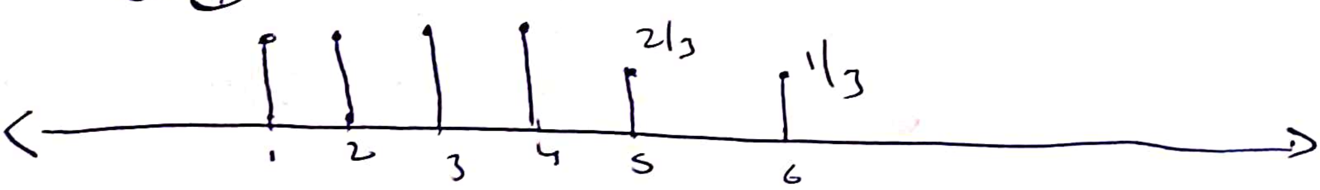
$x(n)$

(b): after folding $x[-n]$

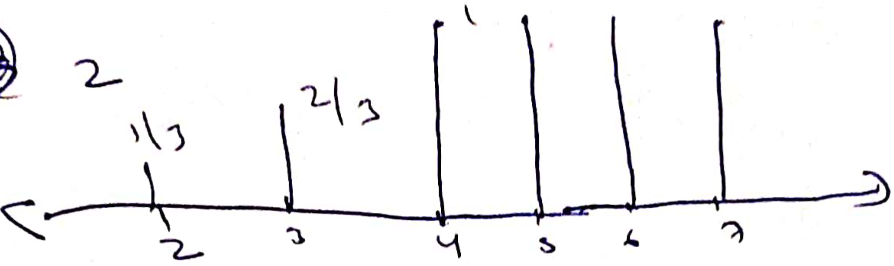
(1)



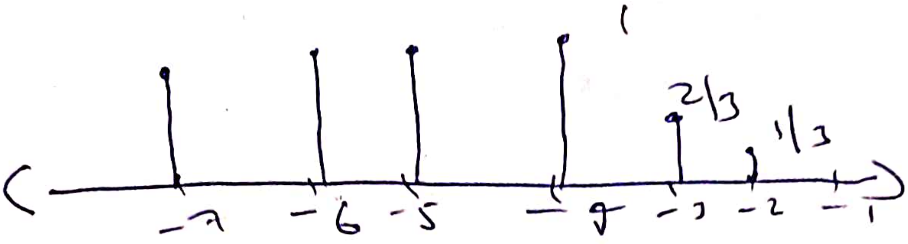
- after delay by 4 samples



(b) 2

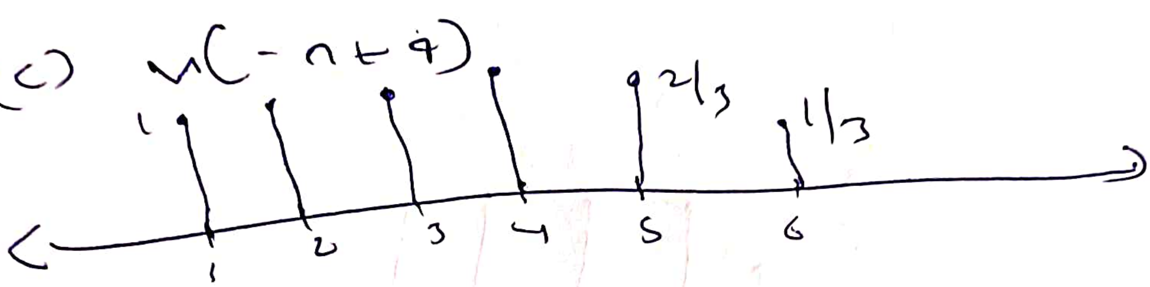


delay by 4 Samples



folding

(c) $u(-n+7)$



(d) From comparing (b) & (c), we observe (b) is correct

Priority:-
1. folding
2. Shifting
3. Scaling

(e) $u[n] = \sum_{k=-\infty}^{\infty} u(k)\,\delta(n-k)$

$$u(n) = \frac{1}{3}\Big(u(n+2) - u(n+1)\Big) + \frac{2}{3}\Big(u(n+1) - u(n)\Big) + \Big(u(n) - u(n-7)\Big)$$

Q-11

$$h[n] = \begin{cases} 1 & 3 \le n \le 8 \\ 0 & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1 & 4 \le n \le 15 \\ 0 & \text{otherwise} \end{cases}$$

$$h[-n] = \begin{cases} 1 & -15 \le n \le -4 \\ 0 & \text{otherwise} \end{cases}$$

$$h[-n+1] = \begin{cases} 1 & -14 \le n \le -3 \\ 0 & \text{otherwise} \end{cases}$$

$$h[-n+2] = \begin{cases} 1 & -13 \le n \le -2 \\ 0 & \text{otherwise} \end{cases}$$

$$h[-n-1] = \begin{cases} 1 & -16 \le n \le -5 \\ 0 & \text{otherwise} \end{cases}$$

$$h[-n-2] = \begin{cases} 1 & -17 \le n \le -6 \\ 0 & \text{otherwise} \end{cases}$$

$x[n]\,h[-n] = \{0$ for every $n\}$, $y(0) = 0$

$x[n]\,h[-n+1] = \{0$ for every n, $y(1) = 0$

$x[n]\,h[-n+2] = \{0$ for every n, $y(2) = 0$

$x[n](-\cancel{h}\,n-1) = \{0$ for every $n\}$ $y(-1) = 0$

$x[n]\,h[-n-2] = \{0$ for every $n\}$ $y(-1) = 0$

$y(n) = \{\ldots 0, 0, 0, 0, 0, 0 \ldots\}$

Q-13) (a) $h(n) = \frac{1}{5} u(n)$

$h[0] = \frac{u[0]}{5}$ → present Input

$h[n] = 1/5\, u(n)$ → " "

$h(-n) = \frac{1}{5}[u-n]$ → " "

Causal System

Amplitude of $h[n]$ is 1/5 or zero of Bounded output

Stable system

(b) $h[n] = (0.8)^n u[n+2]$

$h[0] = (0.8)^0 u(2)$ → future Input

$h[n] = (0.8)^n u[n+2]$ → " "

$h[-n] = (0.8)^{-n} u[-n+2]$ → " "

Amplitude of $h(n)$ is b/w $(0.8)^{-2}$ and 0

Bounded output

Stable System

(c) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n-1]$

$h[0] = (-\frac{1}{2})^0 u[0] + (1.01)^0 u[-1]$ → past Input

$h[n] = \underbrace{(-\frac{1}{2})^n u(n)}_{\text{present input}} + \underbrace{(1.01)^n u[n-1]}_{\text{past input}}$

$h[-n] = \left(-\frac{1}{2}\right)^{-n} u(-n) + (0.1)^{-n} u(n-1)$

present input $\qquad$ past input

Causal System

Amplitude of $h(n)$ is 0 to infinity

unbounded ~~s~~ output

unstable System

Q-15 (a) $\delta(n) = u(n) - u(n-1)$ n is integer

LHS

$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

RHS

$u(n) = \begin{cases} 1 & n \geq 1 \\ 0 & n < 1 \end{cases}$

$u(n-1) = \begin{cases} 1 & n \geq 1 \\ 0 & n < 1 \end{cases}$

$u(n) - u(n-1) = \begin{cases} 0 & n \geq 1 \\ 1 & n = 0 \\ 0 & n < 0 \end{cases}$

LHS = RHS

(b)

$$u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \qquad \delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Case 1 $n < 0$ or negative integer

$u(n) = 0$ for $n < 0$

$\sum_{k=-\infty}^{n} \delta(k) = 0$ as k will be always less then zero, so $\delta(k) = 0$ for every value of k

$\sum_{k=0}^{\infty} \delta(n-k) = 0$ as $n-k$ will be always less than zero so $\delta(n-k) = 0$ for every value of k.

Case 2, $n = 0$

$u(n) = 1$ for $n = 0$

$\sum_{k=-\infty}^{n} \delta(k) = 1$ $\quad \delta(k) = 1$ for $k = 0$

$\delta(k) = 0$ for $k < 0$

$$\sum_{k=0}^{\infty} \delta(n-k) = 1 \qquad \delta(n-k) = 1 \text{ for } k=0$$
$$\delta(n-k) = 0 \text{ for } k \neq 0$$

Case 3 $n > 0$ a positive integer

$$u(n) = 1 \quad n > 0$$

$$\sum_{-\infty}^{n} \delta(k) = 1 \qquad \delta(k) = 1 \quad k = 0$$
$$\delta(k) = 0 \quad k \neq 0$$

$$\sum_{0}^{\infty} \delta(n-k) = 1 \qquad \delta(n-k) = 1 \quad k = n$$
$$\delta(n-k) = 0 \quad k \neq n$$

Q-16

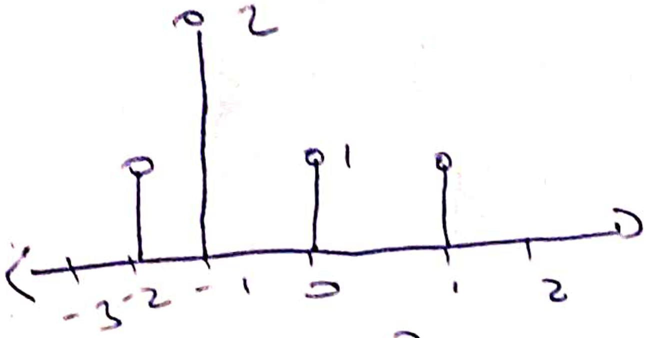
$$u(n) = \begin{cases} 1, & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{else} \end{cases}$$

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$h(n) = \begin{cases} 1 & n = 0 \\ -1 & n = 1 \\ 1 & n = 4 \\ 1 & n = 5 \\ 0 & \text{else} \end{cases}$$

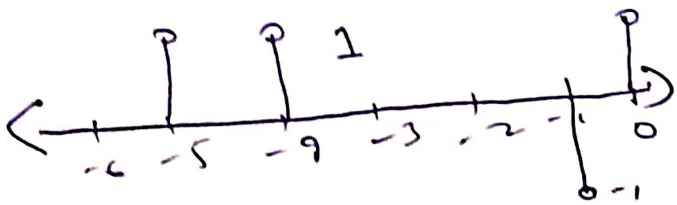
$$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$$

$$y(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

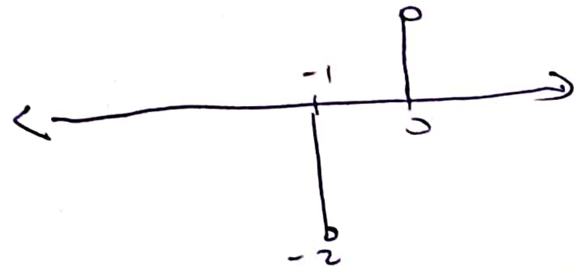


$x(n)$

$h(n)$



$h(-n)$

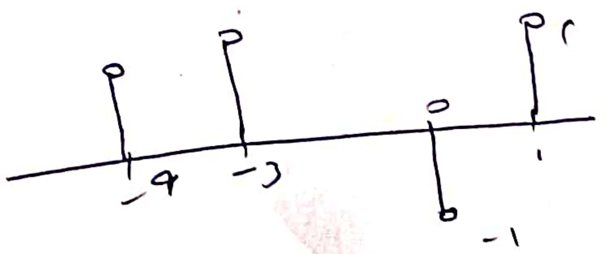


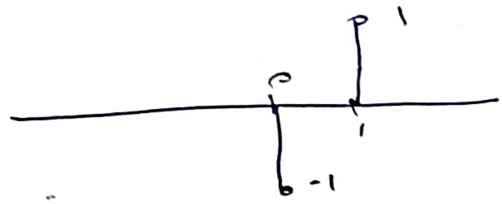
$x(n)\,h(-n)$

$y(0) = -2 + 1 = -1$

$h(n+1)$

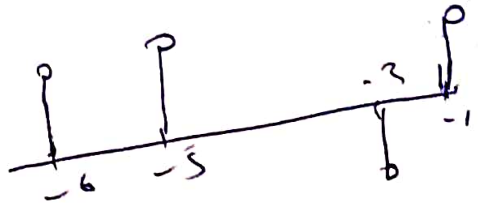
$x(n)\,h(-n+1)$

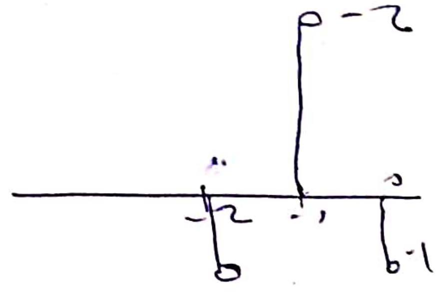




$x(n)\,h(n-1)$

$h(n-1)$

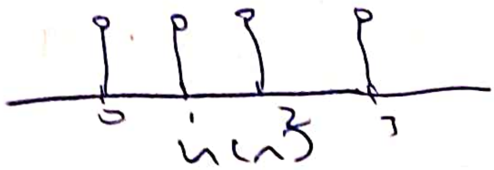


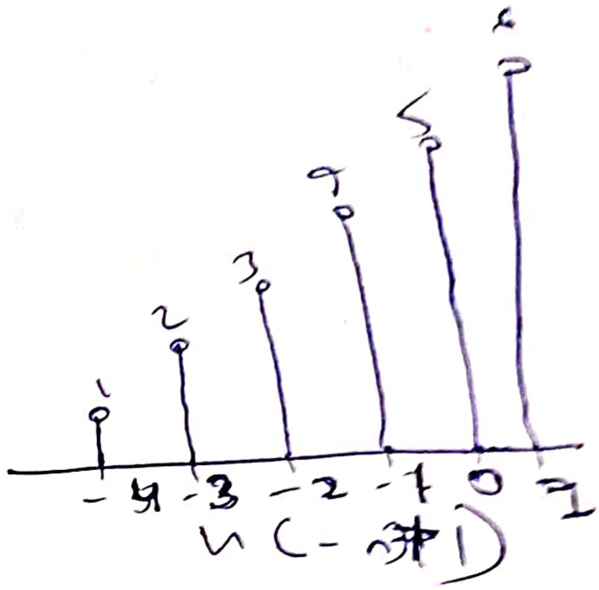


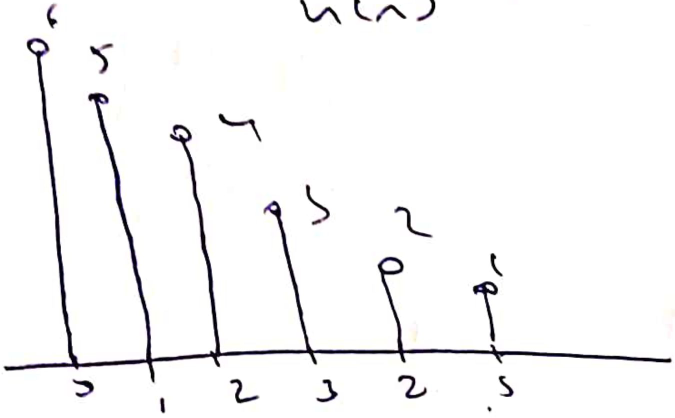
$y(n) = \{ \dots -1, -1, 0 \dots \}$

$y(-1) = -1+2$
$= 1$

(1b)
(b)

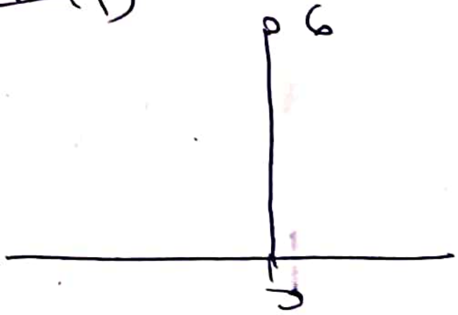






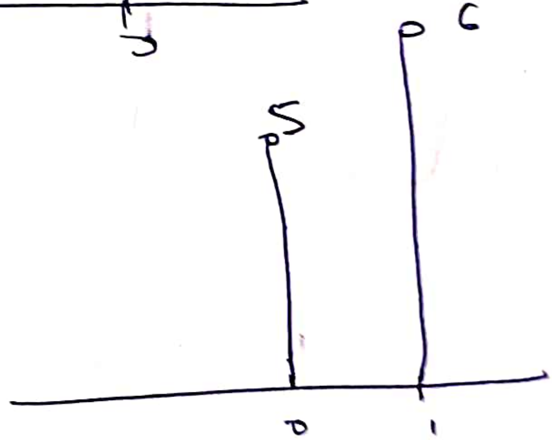
$h(n)$ ~~$h(-n+1)$~~

$x(n)\,h(-n)$



$y(0) = 6$

$x(n)\,h(-n+1)$



$y(1) = 5+6 = 11$

$x(n)\,h(-n-1)$

$y(-1) = 0$

$y(n) = \{0, 6, 11 \dots\}$

Q-10 (a) $x(t) = t\,u(t)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} (t\,u(t))^2 dt$$

$$= \int_{-\infty}^{0} (t \cdot 0)^2 dt + \int_{0}^{\infty} (t \cdot 1)^2 dt$$

$$= \int_{0}^{\infty} t^2 dt = \left[\frac{t^3}{3}\right]_0^{\infty}$$

not finite

So it is not energy signal

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T \to \infty} \left[\frac{1}{T} \int_{-T/2}^{0} (t \cdot 0)^2 dt + \int_{0}^{T/2} (t \cdot 1)^2 dt\right]$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[\int_{0}^{T/2} t^2 dt\right] = \lim_{T \to \infty} \frac{1}{T} (t^3)_0^{T/2} = \lim_{T \to \infty} \frac{T^3}{T \times 8}$$

$$= \lim_{T \to \infty} \frac{T^2}{8} = \infty$$

not finite value so it is not power signal

(b) $x(n) = (-0.5)^n \, u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} ((-0.5)^n)^2 u(n)$$

$$= \sum_{n=-\infty}^{\infty} (-0.5)^{2n} u(n)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n)$$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$\cancel{= \sum_{n=-\infty}^{0} \left(\frac{1}{4}\right)^n \cdot 0 + \left(\frac{1}{4}\right)^?}$$

$$= \sum_{n=-\infty}^{0} \left(\frac{1}{4}\right)^n \cdot 0 + \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \cdot 1$$

$$= \left(\frac{1}{4}\right)^0 + \left(\frac{1}{4}\right)^1 + \left(\frac{1}{4}\right)^2 \cdots \left(\frac{1}{4}\right)^\infty$$

$$= \frac{1}{1 - \frac{1}{4}}$$

$$= \frac{4}{3} \quad \text{finite value}$$

So it is Energy Signal

(1a) $x(n) = \delta(n) + 2\delta[n-1] - \delta(n-3]$

$h(n) = 2\delta(n+1] + 2\delta[n-1]$

$$x(n) = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ -1 & n=3 \\ 0 & \text{else} \end{cases}$$

$$h(n) = \begin{cases} 2 & n=1 \\ 2 & n=-1 \\ 0 & \text{else} \end{cases}$$

(a) $y(n) = x(n) * h(n)$

$$h[-n] = \begin{cases} 2 & n=1 \\ 2 & n=-1 \\ 0 & \text{else} \end{cases}$$

$$h[-n+1] = \begin{cases} 2 & n=2 \\ 2 & n=0 \\ 0 & \text{else} \end{cases}$$

$$h[-n-1] = \begin{cases} 2 & n=0 \\ 2 & n=-2 \\ 0 & \text{else} \end{cases}$$

$$x(n]h[-n) = \begin{cases} 4 & n=1 \\ 0 & \text{else} \end{cases}$$

$$x(n]h[-n+1] = \begin{cases} 2 & n=0 \\ 0 & \text{else} \end{cases}$$

$$x(n)h[-n-1] = \begin{cases} 2 & n=0 \\ 0 & \text{else} \end{cases}$$

(b) $y(n) = u[n+2]\, h(n)$

$$u[n+2] = \begin{cases} 1 & n=-2 \\ 2 & n=-1 \\ -1 & n=1 \\ 0 & \text{else} \end{cases}$$

$$u[n+2]\, h[-n] = \begin{cases} -2 & n=1 \\ 4 & n=-1 \\ 0 & \text{else} \end{cases}$$

$$u[n+2]\, h[-n+1] = \{0\}$$

$$u[n+2]\, h[-n-1] = \begin{cases} 2 & n=-2 \\ 0 & \text{else} \end{cases}$$

$$y(0) = 4 - 2 = 2$$

$$y(n) = \{\ldots\ 2, 2, 0 \ldots\}$$

(c) $y(n) = u(n) * h(n+2)$

$$= u(n)\, h'(n)$$

$$h'(n) = h[n+2]$$

$$h[n+2] = \begin{cases} 2 & n=-1 \\ 2 & n=-3 \\ 0 & \text{else} \end{cases}$$

$$h'[n] = \begin{cases} 2 & n=-1 \\ 2 & n=-3 \\ 0 & \text{else} \end{cases}$$

$$h'[-n+1] = \begin{cases} 2 & n=2 \\ 2 & n=4 \\ 0 & \text{else} \end{cases}$$

$$x(n)\,h[-n] = \begin{cases} 4 & n=1 \\ -2 & n=3 \\ 0 & \text{else} \end{cases}$$

$$x(n)\,h'(-n+1) = \{0\}$$

$$x(n)\,h'[n-1] = \begin{cases} 2 & n=0 \\ 0 & \text{else} \end{cases}$$

$$y(n) = \{ \ldots\ldots 2, 0, 2 \ldots\ldots \}$$