Assignment - 7

S20200010234

Q-1 Two fair dice - - - - - - numbers?

let A be the event 6 is appeared on dice

let B be the event both dice got different numbers

then $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = \frac{30}{36} = \frac{5}{6}$, $P(A) = \frac{11}{36}$

$P(A \cap B) = \frac{10}{36}$

$P(A|B) = \frac{\frac{10}{36}}{\frac{30}{36}} = \boxed{\frac{1}{3}}$

Q-8 A couple . . . . two is a girl?

$P(A)$ = Both are girls $= \frac{1}{4}$

$P(B)$ = Older is a girl $= \frac{1}{2}$

$$P(A \cap B) = \frac{1}{4}$$

$$P(B) = \frac{1}{2}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}}$$

$$P(A|B) = \frac{1}{2}$$

Q-17

36% owns a dog
22% owns a dog and cat
30% owns a cat

(a) 0.22 = probability of families owns both cat and dog

(b) A → familes owns dog
B → family owns cat

$P(B) = 0.52$

$P(A) = 0.6$

$P(A \cap B) = 0.22$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.22}{0.52} = 0.42$$

Q 30

Box 1	Box 2
1 Black	2 Black
1 white	1 White
1	2

(a) $P(\text{Black}) = P(1) \cdot P(\text{Black}) + P(2) \cdot P(\text{Black})$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{7}{12}$$

(b) ~~P Black~~ A → first box was selected

B → marble is white

~~P(A)~~: $P(B) = 1 - P(\text{Black})$

$$= 1 - \frac{7}{12}$$

$$= 5/12$$

$$P(A \cap B) = \frac{1}{2} \times \frac{1}{2}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{5/12}$$

$$= \frac{3}{5}$$

Q-33 On rainy days ....

A → He is late to work if rains

B → " " " " " it doesn't rain

C → It will rain

(a) $P(A)$ = $P(C)\,P(\bar{A}) + P(\bar{C})\,P(\bar{B})$

$= 0.7 \times 0.7 + 0.3 \times 0.1$

$= 0.49 + 0.03$

$= 0.52$

(b) A → He was early

B → got rained

$P(A) = 0.52$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = 0.7$$

$$P(B|A) = \underline{0.7}$$

~~Q-23 Consider a sample....~~

Q-23

2 White
4 red
Urn I

7 White
1 red
Urn II

(a) $P(W_2) = P(W_2/W_1)\,P(W_1) + P(W_2|R_1)\,P(R_1)$

$$= \frac{2}{3} \times \frac{2}{6} + \frac{4}{6} \times \frac{1}{3} = \frac{4}{9}$$

(b) $$P(W_1|W_2) = \frac{P(W_2/W_1)\,P(W_1)}{P(W_2)} = \frac{\frac{2}{3} \times \frac{2}{6}}{\frac{4}{9}} = \frac{1}{2}$$

Q-26

C - person is color blind

M → male F → female

$P(C|M) = 0.05 \quad P(C|F) = 0.0025$

$P(M) = P(F) = 0.5$

I $\quad P(M|C) = \dfrac{0.05 \times 0.5}{0.05 \times 0.5 + 0.0025 \times 0.5}$

$= \dfrac{20}{21}$

II $\quad P(M) = 2\,P(F) \quad P(F) = \dfrac{1}{3} \quad P(M) = \dfrac{2}{3}$

$P(M|C) = \dfrac{0.05 \times \frac{2}{3}}{0.05 \times \frac{2}{3} + 0.0025 \times \frac{1}{3}}$

$= \dfrac{40}{41}$

(25) p - proportion of population over age 50 in town. a_1 proportion of time under 50 spends in streets,

a_2 proportion of time over 50 spends in streets

method $= \dfrac{100000 \times p \times 2a_1}{a_1 + a_2} \Rightarrow 10^5 p$

if $a_1 = a_2$

Q-27

In 2nd option we are choosing n cars which gives different values. So this is better way of estimating average number of passengers travelling in a car

Q-28

First three balls are picked

S_0 = No used ball = $\frac{{}^9C_3}{{}^{15}C_3}$

S_1 = 1 used ball = $\frac{6 \cdot {}^9C_2}{{}^{15}C_3}$

S_2 = 2 used ball = $\frac{9 \cdot {}^6C_2}{{}^{15}C_3}$

S_3 = 3 used ball = $\frac{{}^6C_3}{{}^{15}C_3}$

2nd three ball not used probability

$$S_0 \frac{{}^6C_3}{{}^{15}C_3} + S_1 \frac{{}^7C_3}{{}^{15}C_3} + S_2 \frac{{}^8C_3}{{}^{15}C_3} + S_3 \frac{{}^9C_3}{{}^{15}C_3}$$

~~Q-3~~

Q-38 Urn A 5 white and 7 black balls
Urn B has 3 white 12 black balls

$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$

W = white B = Black

$P(W|H) = 5/12$ $P(W|T) = \frac{1}{5}$

$$P(T/W) = \frac{\frac{3}{15} \times \frac{1}{2}}{\frac{3}{15} \times \frac{1}{2} + \frac{5}{12} \times \frac{1}{2}} = \frac{\frac{3}{15}}{\frac{3}{15} + \frac{5}{12}}$$

$$= \frac{12}{37}$$

Q-59 (a) $P(a) = P \times P \times P \times P$

(b) $P(b) = (1-P)P^3$

(c) $= P(c) = 1 - P^4$

Q-69 (a) $P(a) = \frac{1}{2} \times P + \frac{1}{2} \times P$

(b) $\frac{1}{2}P + \frac{1}{2}P$

Both strategies are equally likely

Q.43 $B \rightarrow$ Biased $\quad U_nB \rightarrow$ Un Biased

$$P(B|H) = \frac{\frac{1}{3} \times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times 2 + \frac{1}{3}\times 2}$$

$$= \frac{1}{4}$$

Q-15 A - a women has an ectobic pregnancy

B - a women of child bearing age is a smoker

$P(A|B) = 2P(A|B^c) \quad P(S) = 0.32$

$$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{P(A\cap B)}{P(A\cap B) + P(A\cap B^c)}$$

$$= \frac{2P(B)}{2P(B) + P(B^c)} = \frac{2\times 0.32}{2\times 0.32 + 1 - 0.32}$$

$= 0.4848$

Q-16 A-successful delivery

B- Pregnancy ends in C section

$P(A) = 0.98$

$P(B) = 0.15$

$P(A|B) = 0.96$

$$P(A|B^c) = \frac{P(AB^c)}{P(B^c)}$$

$$= \frac{P(A) - P(A|B)\,P(B)}{P(B^c)}$$

$$= \frac{0.98 - 0.96 \times 0.15}{1 - 0.15}$$

$= 0.984$

Q-6

A = 2nd & 3rd card are spade

B = 1st card is spade

$$P(A\cap B) = \frac{13\times12\times11}{52\times51\times50}$$

$$P(A) = \frac{13\times12\times11}{52\times51\times50} + \frac{(52-13)\times13\times12}{52\times51\times50}$$

$$P(B|A) = \frac{11}{11+52-13} = \frac{11}{50}$$

$$= 0.22$$

Q-7 5 white and 7 black balls

$B_1 = 7/12$ $B_2 = 9/14$ $W_3 = 5/16$

$W_4 = 7/18$

$P(B_1, B_2, W_3, W_4)$

$$= \frac{7}{12}\times\frac{9}{14}\times\frac{5}{16}\times\frac{7}{18}$$

$$= \frac{35}{768}$$

$P(\text{2 balls are black}) = {}^4C_2 \times \frac{35}{768}$

Q-7 A king comes from a family of 2 children

2 Children - (B, B) (B, G), (G, B) (G, G)

One of the children is king (boy) only
probably that the other child is his sister
= 2/3

Q-9 Urn A - 2W, 4R
Urn B - 8W, 4R
Urn C - 1W, 3R

let E be the event of selecting a exactly two white balls by selecting and one ball from each of three coins

$$P(E) = P(W, W, R) + P(W, R, W) + P(R, W, W)$$

$$= \frac{2}{6} \times \frac{8}{12} \times \frac{3}{4} + \frac{2}{6} \times \frac{4}{12} \times \frac{1}{4} + \frac{4}{6} \times \frac{8}{12} \times \frac{1}{4}$$

$$= \frac{88}{288} = \frac{11}{36}$$

Q-5 Probability of selecting 4 balls without replacement which has first 2 ball white and last 2 ball black

$$= \frac{{}^6C_2 \, {}^9C_2}{{}^{13}C_4}$$

Q-6 A= first and third drawn balls are white

B = sample exactly three white balls

$$P(B) = \frac{{}^8C_3 \, {}^4C_1}{{}^{12}C_4} \qquad P(B \cap A) = \frac{2}{{}^{12}C_4}$$

$$P(A|B) = \frac{2}{{}^8C_3 \, {}^4C_1}$$

Theoritical exercises

Q-1 $A \subset A \cup B$

$P(A) \subset P(A \cup B) \Rightarrow P(A \cup B) > 0$

$P(A) > 0$

We have $\frac{1}{P(A)} \geq \frac{1}{P(A \cup B)}$

$$\frac{P(A\,B)}{P(A)} \geq \frac{P(AB)}{P(A \cup B)}$$

$$\frac{P((AB)\,A)}{P(A)} \geq \frac{P(AB \cup AB)}{P(A \cup B)}$$

$$P(AB|A) \geq \frac{P(AB)A \cup AB B}{P(A\cup B)}$$

$$\frac{P(AB(A\cup B))}{P(A\cup B)} = P(AB|A\cup B)$$

$$\therefore \quad P(AB|A) \geq P(AB|A\cup B)$$

Q-2 Given that $A \subset B$

(a) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)}$.

(b) $P(A|B^c) = \frac{P(AB^c)}{P(B^c)} = \frac{P(\emptyset)}{P(B^c)} = 0$

(c) $P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(A)}{P(A)} = 1$

(d) $P(B|A^c) = P\left(\frac{BA^c}{A^c}\right) = \frac{P(B) - P(BA)}{P(A^c)}$

$$= \frac{P(B) - P(A)}{1 - P(A)}$$

Q-6

$P(E_1 \cup E_2 \cup E_3 \cup \dots E_n)$

$= 1 - P(\overline{E_1 \cup E_2 \cup E_3 \dots E_n})$

All events are independent

$= 1 - P(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3} \dots \overline{E_n})$

$P(A \cap B) = P(A) \times P(B)$

$= 1 - \prod_{i=1}^{n} (P(\overline{E_i}))$

$= 1 - \prod_{i=1}^{n} [1 - P(E_i)]$

Q-11 Consider the act on falling coin as random variable $= x$

$x = 0, 1, 2, 3$

Coin is ~~to~~ tossed n times and probability of getting head is 'p'.

X follows binomial distribution (n, p)

$1 - {}^{n}C_{0}\, p\, (1-p)^n \geq 0.5$

$1 - (1-p)^n \geq 0.5$

$(1-p)^n \leq 0.5$

$n \log(1-p) \leq \log(0.5)$

$$n \leq \frac{\ln 0.5}{\ln(1-p)}$$

(25) LHS

$$P\left(\frac{E}{FG}\right) P\left(\frac{G}{F}\right) + P\left(\frac{E}{FG^c}\right) P\left(\frac{G^c}{F}\right)$$

Conditional probability

$$\frac{P(EFG)}{P(FG)} \cdot \frac{P(GF)}{P(F)} + \frac{P(EFG^c)}{P(FG^c)} \cdot \frac{P(G^c F)}{P(F)}$$

$$\frac{P(EFG) + P(EFG^c)}{P(F)}$$

$$\frac{P(EFG \cup EFG^c) - P(EFG \cap EFG^c)}{P(F)}$$

$$\frac{P(EF\, G \cup G^c) - P(EF(G \cap G^c))}{P(F)}$$

$G \cup G^c = 1 \qquad G \cap G^c = \phi$

$$\Rightarrow \frac{P(EF)}{P(F)} = \frac{P(E \cap F)}{P(F)} = P\left(\frac{E}{F}\right)$$