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Set-6

Q-1
(a)

$$\frac{1}{n-1} - \frac{1}{n} = \frac{0.9}{100}$$

$$n(n-1) = \frac{1000}{9}$$

$$n(n-1) = 111$$

$$\boxed{n \approx 11}$$

P of drawing A_1 is $1/n$

Since A_1 is not replaced $P(A_2) = \frac{1}{n-1}$

$$\therefore \frac{1}{n}\left(\frac{1}{n-1}\right) = \frac{0.9}{100}$$

(b)

P(at least one got promoted)

$$= 1 - P(\text{both got promoted})$$

$$= 1 - \frac{{}^{96}C_2}{{}^{100}C_2}$$

$$= 1 - \frac{96 \cdot 95}{100 \cdot 99}$$

$$= \frac{9900 - 9120}{9900}$$

$$= \frac{780}{9900} = \boxed{0.078}$$

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1(C)

Probability of choosing a defective product from C

$$= \frac{\frac{6}{100} \times \frac{25}{100}}{\frac{50}{100} \times \frac{9}{100} + \frac{25}{100} \times \frac{5}{100} + \frac{6}{100} \times \frac{25}{100}}$$

$$= \frac{6}{8 + 5 + 6} = \frac{6}{19}$$

$$= \boxed{0.315}$$

Q-2
(a)

Total cars : 14
Good : 7
Defective Transmission (DT) : 3
Defective Steering : 4

$X \rightarrow DT$
$Y \rightarrow DS$

$$P(Y < 2) = P(Y = 0) + P(Y = 1)$$

$$= \frac{{}^{10}C_2}{{}^{14}C_2} + \frac{{}^{4}C_1 \cdot {}^{10}C_1}{{}^{14}C_2}$$

$$= \frac{10 \cdot 9 + 8 \cdot 10}{14 \cdot 13} = \frac{170}{14 \cdot 13} = \boxed{0.934}$$

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Q-2

(b)

$$Cov(x, y) = E(xy) - E(x)E(y)$$

$$E(xy) = \sum \sum xy \, P(x, y)$$

$$\rho_{xy} = \frac{Cov(x, y)}{\sqrt{Var(x)} \sqrt{Var(y)}}$$

y \ x	0	1	2	3	
0	$\frac{^7C_2}{^{14}C_2}$	$\frac{^3C_1\,^7C_1}{^{14}C_2}$	$\frac{^3C_2}{^{14}C_2}$	0	45/91
1	$\frac{^4C_1\,^7C_1}{^{14}C_2}$	$\frac{^3C_1\,^4C_1}{^{14}C_2}$	0	0	40/91
2	$\frac{^4C_2}{^{14}C_2}$	0	0	0	6/91
3	0	0	0	0	
P(x)	$\frac{55}{91}$	$\frac{33}{91}$	$\frac{3}{91}$		

$$E(x) = 0 \times \frac{55}{91} + \frac{33}{91} \times 1 + \frac{2 \times 3}{91}$$

$$E(x) = \frac{39}{91}$$

$$E(y) = 0 \times \frac{45}{91} + \frac{40}{91} \times 1 + \frac{2 \times 6}{91}$$

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3)(b) $P(X > 80) \leq \frac{E[X]}{80}$ (Using Markov's Inequality)

(i)

$E[X] = 70$

$P(X > 80) \leq \frac{7}{8}$

(ii) $\sigma^2 = 16$, $\mu = 70$

$P(|X - \mu| \leq 20) = ?$

$P(|X - \mu| \geq k) \leq \frac{\sigma^2}{k^2}$

$P(|X - \mu| \leq k) \geq 1 - \frac{\sigma^2}{k^2}$

$k = 20$

$P(|X - 70| \leq 20) \geq 1 - \frac{16}{400}$

$P(|X - 70| \leq 20) \geq \frac{384}{400}$

$P(|X - 70| \leq 20) \geq \frac{24}{25}$

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Q.3
(a) $n = 400$

X is the no of heads
∴ Binomial distribution

$P(180 \le x \le 220) = ?$

$p = 1/2$

$\mu = np = 400 \times 1/2 = 200$

$\sigma^2 = np(1-p)$

$= 400 \times 1/2 \times 1/2 = 100$

$P(180 \le x \le 220)$

$= P\left(\frac{180-200}{\sqrt{100}} \le \frac{x-200}{\sqrt{100}} \le \frac{220-200}{\sqrt{100}}\right)$

$= P\left(\frac{-20}{10} \le Z \le \frac{20}{10}\right)$

$= P(-2 \le Z \le 2)$

$= \phi(2) - \phi(-2)$

$= 0.97725 - 0.02275$

$= \boxed{0.9545}$

Q-4
(b)

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$f(x) = \theta^{-2} x e^{(-x/\theta)}$

$L(\theta) = \prod_{i=1}^{n} f(x_i|\theta) = \prod_{i=1}^{n} \theta^{-2} x e^{-x/\theta}$

$L(\theta) = \theta^{-2n} \prod_{i=1}^{n} x_i e^{-1/\theta \sum_{i=1}^{n} x_i}$

$\log L(\theta) = (-2n)\theta + \log\left(\prod_{i=1}^{n} x_i\right) - \frac{1}{\theta}\sum_{i=1}^{n} x_i$

$\frac{d}{d\theta} L(\theta) = -2n\frac{d\theta}{d\theta} + \frac{d}{d\theta}\left(\log \prod_{i=1}^{n} x_i\right) - \frac{d}{d\theta}\left(\frac{1}{\theta}\right)\sum_{i=1}^{n} x_i = 0$

$\sum_{i=1}^{n} x_i = 6.85$

$0 = -2n + 0 + \frac{6.85}{\theta^2}$

$2n = \frac{6.85}{\theta^2}$

$\theta^2 = \frac{6.85}{2n}$

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$$\frac{d^2}{d\theta^2}(\log L(\theta)) = \frac{d}{d\theta}\left(-2n + \frac{6.85}{\theta^2}\right)$$

$$= 0 - \frac{6.55 \times 2}{\theta^3} < 0$$

Hence it is maximum likelihood estimation

$$\text{MLE of } \theta = \hat{\theta} = \sqrt{\frac{6.85}{2n}}$$

$$= 0.755$$

Q-4 (a)

$$\theta_1 = \frac{2x_1 + 3x_2}{5}$$

$$E(\theta_1) = \frac{2E(x_1)}{5} + \frac{3E(x_2)}{5}$$

$$= \frac{5}{5}\theta$$

$$E(\theta_1) = \theta \rightarrow \text{Unbaised Estimator}$$

$$Var(\theta_1) = Var\left(\frac{2x_1}{5} + \frac{3x_2}{5}\right)$$

$$= \frac{4}{25}Var(x_1) + \frac{9}{25}Var(x_2)$$

$$= \boxed{\frac{13}{25}\sigma^2}$$

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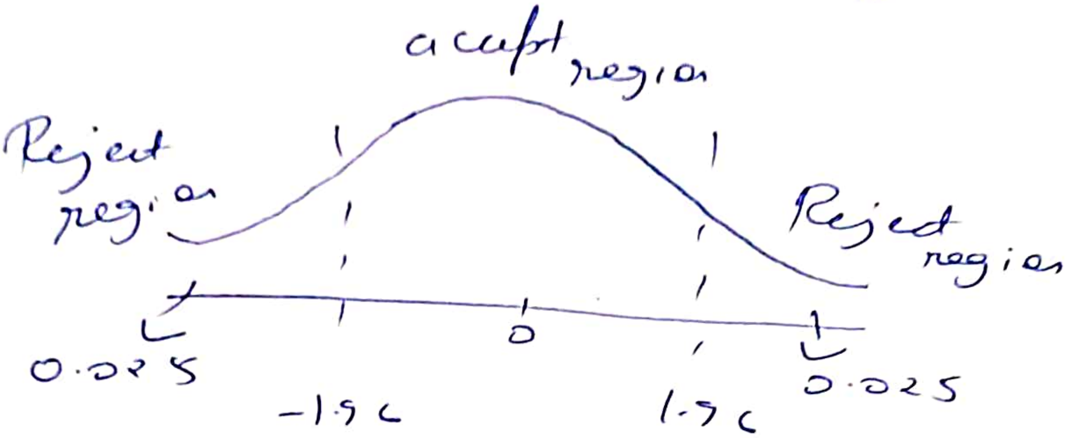
Q-5
(a)

$H_0 = \mu = 3$

$H_1 = \mu \neq 3$ (use two tailed test)

Significance level = 5% = 0.05

$\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.005$



for $\alpha/2 = 0.025$

Z value = 1.96

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 30}{2/\sqrt{100}} = \left(\frac{\bar{x} - 30}{2}\right)10$$

$Z > Z_c$ we will reject hypothesis

$$\left(\frac{\bar{x} - 30}{2}\right)10 > 1.96$$

and if $Z < 1.96$ (we will also reject hypothesis)

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Q.5
(b)

$n = 49, \ \sigma = 0.8, \ \bar{x} = 89, \ s^2 = 0.5625$

$s = \sqrt{0.5625} = 0.75$

$\alpha = 1 - 0.95 = 0.05$

$\mu = \bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$

$= 89 \pm t_{0.025} \times \dfrac{0.75}{\sqrt{49}}$

$= 89 \pm 2.04 \times \dfrac{0.75}{7}$

$\mu = 89 \pm 0.215$

$88.785 \leq \mu \leq 89.215$