Assignment - 3

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Q-2 In an exp . . . .

What is $\left(\bigcup_1^{\infty} E_n\right)$?

Sol: Sample space:

$$S = \{(6), (u_1, 6), (u_1, u_2, 6), (u_1, u_2, u_3, 6) \ldots \ldots\} \ \forall\, u_i \in \{1,2,3,4,5,6\}$$

Sample Space of E_n

$$E_n = \{(u_1, u_2, u_3, u_4, u_5 \ldots u_{n-1}, 6)\} \ \forall\, u_i = \{1,2,3,4,5\}$$

$$\left(\bigcup_1^{\infty} E_n\right)^c = (E_1 \cup E_2 \cup E_3 \ldots \cup E_{\infty})^c$$
$$= (E_1^c \cap E_2^c \cap E_3^c \ldots E_{\infty}^c)$$
$$= \left(\bigcap_1^{\infty} E_n^c\right)$$

It is the Event where 6 never appears

$$\left(\bigcup_1^{\infty} E_n\right)^c = (u_1, u_2, u_3 \ldots u_{\infty}\} \ \forall\, (1,2,3,4,5)$$

(4) A, B and C . . . .

. . . soon

(i) The Sample space contains

(n-1) tails before a head shows up

or there is a case 0000. . .

where head doesn't shows up

A wins if 1^{st}, 4^{th}, 7^{th} chance . . .

B wins if 2^{nd}, 5^{th}, 8^{th} chance . . .

C wins if 3^{rd}, 6^{th}, 9^{th} chance

(b) (i) $S_A = \{1, 0001, 0000001 \ldots\}$

(ii) $S_B = \{01, 00001, 00000001 \ldots\}$

(iii) $(A \cup B)^c = \{001, 000001, 000000001 \ldots\}$

or S_C

(6) (i) $S = \{(1,9)(1,7)(1,5), (0,9), (0,7)(0,5)\}$

(ii) $A = \{(1,5)(0,5)\}$

(iii) $B = \{(0,9), (0,7), (0,5)\}$

(iii) $B^c \cup A = \{(0,5)(1,9), (1,7), (1,5)\}$

(8) $P(A) = 0.3$ $\quad P(B) = 0.5$

$P(A \cup B) = 0.8$ $\quad P(A \cap B) = 0$

(i) either A or B occurs $P(A \cup B)$

$$= P(A) + P(B) - 2P(A \cap B)$$
$$= 0.3 + 0.5 - 0$$
$$= 0.8$$

(ii) A occurs but B does not $= P(A) - P(A \cap B)$

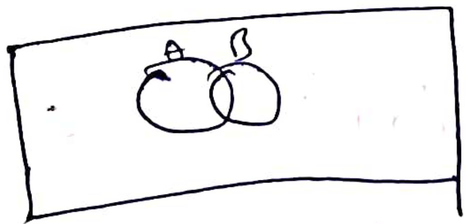
$$= 0.3$$

(iii) Both A and B occur $= P(A \cap B) = 0$

(10) A: Studed wears ring

B: Student wears necklace

$P(A^c \cap B^c) = 6/10$ $\quad P(A) = \frac{2}{10}$ $\quad P(B) = \frac{3}{10}$



$$1 - P(A \cup B) = 6/10$$
$$P(A \cup B) = \frac{4}{10}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \frac{2}{10} + \frac{3}{10} - \frac{4}{10}$$
$$= \frac{1}{10}$$

(a) a ring or a necklace

$$P(A \cup B) = \frac{4}{10} = 0.4$$

(b) a ring and a necklace

$$P(A \cap B) = \frac{1}{10} = 0.1$$

(12) A : Student takes Spanish

B: Student takes French

C: Student takes German

$P(A) = \frac{28}{100}$ $P(B) = \frac{26}{100}$ $P(C) = \frac{16}{100}$

$P(A \cap B) = \frac{12}{100}$ $P(A \cap C) = \frac{4}{100}$ $P(B \cap C) = \frac{6}{100}$

$P(A \cap B \cap C) = 2/100$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = \frac{28}{100} + \frac{26}{100} + \frac{16}{100} - \frac{12}{100} - \frac{4}{100} - \frac{6}{100} + \frac{2}{100}$$

$$P(A \cup B \cup C) = \frac{50}{100}$$

(a) Probability that student is in none of the language classes

$= 1 - P(A \cup B \cup C)$

$= 1 - 1/2 = 0.5$

(b) Probability that student is taking exactly one class

$= P(A \cup B \cup C) - P(A \cap B) - P(B \cap C) - P(A \cap C)$

$+ 2[P(A \cap B \cap C)]$

$= \frac{50}{100} - \frac{12}{100} - \frac{4}{100} - \frac{6}{100} + 2 \cdot \frac{2}{100}$

$= 0.32$

(c) If 2 students are chosen, atleast one is taking a class

$= {}^2C_1 (1 - P(A \cup B \cup C)) P(A \cup B \cup C) + P(A \cup B \cup C) \cdot P(A \cup B \cup C)$

$= \left(2 \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$

$= \frac{3}{4} = 0.75$

(14) A: professionals B: married
C: college graduates

$P(A) = \frac{312}{1000}$ $P(B) = \frac{470}{1000}$ $P(C) = \frac{525}{1000}$

$P(A \cap C) = \frac{42}{1000}$ $P(B \cap C) = \frac{147}{1000}$ $P(A \cap B) = \frac{86}{1000}$

$P(A \cap B \cap C) = \frac{25}{1000}$

$P(A \cup B \cup C) = \frac{312}{1000} + \frac{470}{1000} + \frac{525}{1000} - \frac{86}{1000} - \frac{147}{1000}$
$- \frac{42}{1000} + \frac{25}{1000}$

$= \frac{1060}{1000}$

but $P(A \cup B \cup C)$ can't be greater than 1.

Thus numbers reported in the study are incorrect.

(15) (a) P(no two alike) $= \frac{6}{6} \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6}$

$= 0.0926$

(b) P(one pair) $= {}^5C_2 \left(\frac{6}{6} \times \frac{1}{6}\right) \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6}$

$= 0.463$

(c) P(two pair) $= \frac{{}^5C_2 \cdot {}^3C_2 \cdot {}^6C_1 \cdot {}^5C_1 \cdot 4}{6^4} \times \frac{1}{2}$

$= \frac{10 \cdot 3 \cdot 6 \cdot 5}{6 \cdot 6 \cdot 6 \cdot 6 \times 2} \cancel{\times} \frac{4}{6}$

$= 0.2315$

(d) P(three alike) $= \frac{{}^5C_3 \cdot {}^6C_1}{6^3} \cdot \frac{5}{6} \cdot \frac{4}{6}$

$= 0.154$

(e) P(full house) $= \frac{{}^5C_3 \, {}^2C_2 \, {}^5C_1}{6^5} = 0.386$

(f) P(four alike) $= \frac{{}^5C_4 \cdot {}^6C_1}{6^4} \cdot \frac{5}{6}$

$= 0.0193$

(g) P(five alike) $= \frac{{}^5C_5 \, {}^6C_1}{6^5} = 0.0008$

(19) $P = {}^2C_1 \times 4/52 \times 16/51$

$= 2 \times 1/13 \times 16/51$

$P = 0.482$

(20) A : player gets a black jack

B: dealer gets a black jack

We need to find $P(\bar{A} \cap \bar{B})$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} \cdot \frac{16}{51} + \frac{4}{52}\frac{16}{51} - \frac{4}{52}\frac{16}{51}\frac{3}{50}\frac{13}{49}$

$= 0.04782$

$P(\bar{A} \cap \bar{B}) = 1 - 0.04782$

$= 0.93218$

(22) The ordering remains same if we get k heads first and the rest all tails (n-k).

$P = \frac{n+1}{2^n}$

Q.24

$i=2 \Rightarrow P = 1/36$

$i=3 \rightarrow P = 2/36 = 1/18$

$i=4 \rightarrow P = 3/36 = 1/12$

$i=5 \rightarrow P = 5/36$

$i=6 \rightarrow P = \frac{5}{36}$

$i=7 \rightarrow P = \frac{1}{6}$

$i=8 \rightarrow P = \frac{4}{36} = \frac{1}{9}$

$i=9 \rightarrow P = \frac{4}{36} = \frac{1}{9}$

$i=10 \rightarrow P = \frac{3}{36} = 1/12$

$i=11 \rightarrow P = 2/36 = 1/18$

$i=12 \rightarrow P = 1/36$

(2c) $P(E) = \sum_{i=2}^{12} P(G_i)$

$P(G_i) = P\left(\bigcup_{n=2}^{\infty} E_{i,n}\right)$

$= \sum_{n=2}^{\infty} P(G_{i,n})$

$= \sum_{n=2}^{\infty} \frac{P_i \times (30 - P_i)^{n-2}}{36^n}$

$$= \frac{P_i^2}{36^2} \sum_{n=2}^{\infty} \left(\frac{30 - P_i}{36}\right)^{n-2}$$

$$= \left(\frac{P_i}{36}\right)^2 \sum_{k=0}^{\infty} \left(\frac{30 - P_i}{36}\right)^k$$

Sum of infinite ~~GP~~ $= \frac{a}{1-r}$

$$= \left(\frac{P_i}{36}\right)^2 \left(\frac{1}{1 - \left(\frac{30 - P_i}{36}\right)}\right)$$

$$= \frac{P_i^2}{36(6 + P_i)}$$

now for $i = \{4, 5, 6, 8, 9, 10\}$

also $P(E_4) = P(E_{10})$

$P(E_5) = P(E_9)$

$P(E_6) = P(E_8)$

$$P(E) = \frac{1}{36}\left[2 \cdot \frac{3^2}{6+3} + 2\left(\frac{4^2}{6+4}\right) + 2\left(\frac{5^2}{6+5}\right)\right] + \frac{1}{6} + \frac{1}{18}$$

$$P(E) = \frac{1}{36}\left(\frac{536}{55}\right) + \frac{2}{9}$$

$$P(E) = 0.49$$

(28) Sampling without replacement

(a) $P = \frac{^5C_3}{^{19}C_3} + \frac{^6C_3}{^{19}C_3} + \frac{^8C_3}{^{19}C_3}$

(b) $P = \frac{^5C_1 \cdot {^6C_1} \cdot {^8C_1}}{^{19}C_3}$

(ii) Sampling without replacement

(a) $P = \left(\frac{5}{19}\right)^3 + \left(\frac{6}{19}\right)^3 + \left(\frac{8}{19}\right)^3$

(b) $P = \frac{5 \cdot 6 \cdot 8 \cdot 3!}{(19)^3}$

(30) (a) $P = \frac{^7C_3 \cdot {^8C_3} \cdot \lfloor 3}{^8C_4 \cdot {^9C_4} \cdot \lfloor 4}$

$P = \frac{1}{18}$

(b) $P = \frac{^7C_3 \cdot {^8C_3}}{^8C_4 \cdot {^9C_4}} - \frac{1}{18}$

$P = \frac{3}{18} = \frac{1}{6}$

(c) $P = \frac{^7C_3 \cdot {^8C_4} + {^7C_4} \cdot {^8C_3}}{^8C_4 \cdot {^9C_4}}$

$P = \frac{1}{2}$

(32) $P = \frac{9(b+9-1)!}{(b+9)!}$

$P = \frac{9}{b+9}$

(34) $P = \frac{{}^{32}C_{13}}{{}^{52}C_{13}} = 5.47 \times 10^{-4}$

(36) (i) $P = \frac{{}^{4}C_{2}}{{}^{52}C_{2}}$

$= 0.045$

(ii) $P = \frac{52}{52} \cdot \frac{3}{51}$

$= \frac{1}{17} = 0.0588$

(38) $\frac{1}{2} = \frac{3}{n} \times \frac{2}{n-1}$

$n(n-1) = 12$

$(n-4)(n+3) = 0$

$n = 4$

(40) • $i = 1$

$P = {}^{4}C_{1}\left(\frac{1}{4}\right)^{4}$

$P = \frac{1}{64}$

• $i = 2$

$P = \frac{{}^{4}C_{2}(4 + {}^{4}C_{2} + 4)}{(4)^{4}} = \frac{84}{256}$

$P = \frac{21}{64}$

$\cdot\ i = 3$

$$P = \frac{{}^4C_3 \times {}^3C_1 \ \frac{4!}{2!}}{4^4} = \frac{36}{64}$$

$$P = \frac{9}{16}$$

$i = 4$

$$P = \frac{4!}{4^4} = \frac{3}{32}$$

(42) P(not getting double six) $= \frac{35}{36}$

P(getting atleast one double six in 'n' throws) $= 1 - \left(\frac{35}{36}\right)^n$

now $\frac{1}{2} \leq 1 - \left(\frac{35}{36}\right)^n$

$1 \leq 2 - 2\left(\frac{35}{36}\right)^n$

$\left(\frac{35}{36}\right)^n \leq \frac{1}{2}$

$(0.97)^n \leq 0.5$

$n = 23$

(44) (i) $P = \frac{{}^3C_1 \cdot 2 \cdot 13}{15}$

$P = 0.3$

(ii) $P = \frac{{}^3C_2 \cdot (2)^3}{15}$ $P = 0.2$

(iii) $P = \frac{2 \cdot 13}{15}$

$P = 0.1$

(46) $P\{\text{all different}\} = \dfrac{12\cdot 11\cdot 10\cdots(12-n+1)}{12^n}$

$\therefore$ P{atleast 2 people share 2 birthday month} $= 1 - \dfrac{\dfrac{\lfloor 12}{\lfloor 12-n}}{(12)^n}$

$$\frac{1}{2} \le 1 - \frac{\dfrac{\lfloor 12}{\lfloor 12-n}}{12^n}$$

for $n=3$, $1 - \dfrac{12\cdot 11\cdot 10}{(12)^3} = \dfrac{34}{144} \le 1/2$

for $n=4$, $1 - \dfrac{12\times 11\times 10\times 9}{(12)^4} = \dfrac{41}{96} \le 1/2$

for $n=5$, $1 - \dfrac{12\times 11\times 10\times 9\times 8}{(12)^5} = \dfrac{89}{144} \ge 1/2$

$\therefore \boxed{n=5}$

(48) $$\frac{{}^{12}C_4 \cdot {}^{8}C_4 \cdot \dfrac{\lfloor 20}{(\lfloor 3)^4(\lfloor 2)^4}}{(12)^{20}}$$

(52)

(i) $P = \dfrac{20}{20}\cdot\dfrac{18}{19}\cdot\dfrac{16}{18}\cdot\dfrac{14}{17}\cdot\dfrac{12}{16}\cdot\dfrac{10}{15}\cdot\dfrac{8}{14}\cdot\dfrac{6}{13}$

(ii) $$P = \frac{{}^{10}C_1 \cdot {}^{9}C_6 \cdot \dfrac{\lfloor 8}{\lfloor 2}(2)^6}{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15\cdot 14\cdot 13}$$

(50) $\dfrac{{}^{13}C_5 \cdot {}^{39}C_8 \cdot {}^{8}C_8 \cdot {}^{31}C_5}{{}^{52}C_{13} \cdot {}^{39}C_{13}}$

$\dfrac{P(5 \text{ spade}) \cdot P(8 \text{ non spade}) \quad P(8 \text{ spades}) \cdot P(5 \text{ other})}{\text{~~other~~ total}}$

(54) $P = P(\text{void of one suit}) - P(\text{void of 2 suits})$
$+ P(\text{void of 3 suits}) - P(\text{void of 4 suits})$

$P = {}^4C_1 \, {}^{39}C_{13} - \dfrac{{}^4C_2 \, {}^{26}C_{13}}{{}^{52}C_{13}} + {}^4C_3 \dfrac{{}^{3}C_{13}}{{}^{52}C_{13}}$

$P = \dfrac{4 \cdot {}^{39}C_{13} - 6 \cdot {}^{26}C_{13} + 4}{{}^{52}C_{13}}$

(56) Player B

(i) If player A chooses spinner (a) then B can choose (c)

(ii) If player A chooses spinner (b) then B can choose (a)

(iii) If player A chooses (c) then B can choose (b)

Case (i) possible combos:
(9, 7), (9, 6) (9, 2), (5, 7), (5, 6), (5, 2),
(1, 7), (1, 6), (1, 2)

player B wins $5/9^{th}$ of the time

$\therefore$ So his chances are more

Similarly in all 3 cases.

Player B has a winning probability

of $5/9$.