Indian Institute of Information Technology Sri City, Chittoor (An Institute of National Importance under An Act of Parliament)

Name: **Probability and Statistics (M-2)** Duration: **1:30Hrs**

INSTRUCTIONS:

1. Please read all the questions carefully and there are no alternative options.
2. There are ten questions in total and each question carries 1.5 marks. If a question has multiple parts, the marks will be shared equally.
3. The answer to the attempted question should be concluded by appropriate steps. Merely answering the question will not entertain.
4. Do not do any inappropriate activity as you are being recorded and please sit properly in front of your camera. For more information, please read all the guidelines given by the UG Exams Coordinator.
5. Submit a single PDF file containing all the answers to your attempted questions in the Google Classroom.
6. The naming conventions of the PDF file should be as follows: S2020xxxxx_SET_C_1.pdf for SET-C-1.
7. The Exam's time duration is 1:30 Hrs. However, additional 10 minutes will be given to scan and upload the answer sheets.

Not following instructions leads to heavy penalty

SET-C-1

Q.1 You have a sick pet and are on vacation. Therefore, you ask your friend to take care of the pet. Without care, it will die with probability 0.7; With care, it will die with a 0.16 probability. You are 80% confident that your friend will take care of the pet.

a) What is the probability that the pet will be alive when you return ?.
b) What is the probability that the pet will die when you return ?.
c) If the pet dies upon your return, what is the probability that your friend forgot to take care of the pet ?.

Q.2 A five-match Test series is played between India and Australia at the Adelaide Oval, Adelaide. Any team that wins three games is declared the winner of the series. Suppose the Indian team wins each game independently with probability ⅓. Find out the conditional probability that Team India wins

a) the Test series reported (given) that it wins the first game;
b) the first game given that it wins the Test series.

Q.3 A person visiting the electronics store will purchase a compact fluorescent (CFL) with probability 0.23, a halogen with probability 0.31, and a light-emitting diode (LED) with probability

0.26. The person will purchase both a compact fluorescent (CFL) and a light-emitting diode (LED) with probability 0.10, both a compact fluorescent (CFL) and a halogen with probability .13, and both a light-emitting diode (LED) and halogen with probability 0.12. The person will purchase all three bulbs with probability 0.06. What is the probability that a person purchases

a) None of these bulbs?
b) Exactly 1 of these bulbs?
c) Two or more of these bulbs?

4. Let us assume that

$$P\{Y=p\}=r,\ P\{Y=q\}=1-r$$

a) Show that Y-q/p-q is a Bernoulli random variable.
b) Find Var(Y)

Q.5 Each game you play is a win with probability 1/2. You plan to play 5 games, but if you win the fifth game, then you will keep on playing until you lose.

a) Find the expected number of games that you lose.
b) Find the expected number of games that you play.

Q.6 If X is a normal random variable with parameters $\mu=4$ and $\sigma=2$, compute

a)$P\{X>3\}$
b)$P\{3<X<5\}$
c)$P\{X<3\}$
d)$P\{X>5\}$
e)$P\{X<5\}$

Q.7

a) The lecturer has 60 exams that are to be graded in sequence. The times required to grade the 60 exams are independent, with a common distribution (variance =25 minutes and mean=22 minutes). Approximate the probability that the lecturer will grade at least 28 of the exams in the first 550 minutes of work.

b) Suppose X has the MGF

$$m_X(t) = (1-2t)^{-\frac{1}{2}} \text{ for } t < \frac{1}{2}$$

Find both the second and fourth moments of X?.

Q.8 Due to the rapid spread of coronavirus disease (Covid-19), it is believed that 60% of the population has been infected in a city of Andhra Pradesh (AP). RT-PCR testing is done to find out exactly how many people have been infected. However, the RT-PCR test is 98% accuracy.

a) If a person's test is positive, what is the probability that they are actually infected with Covid-19.
b) If a person's test is negative, what is the probability that they are actually not infected with Covid-19

Q.9. Suppose it is known that the number of cars produced by a certain company during a week is a random variable with an average (mean) of 60.

a) What can be said about the probability that this week's production will exceed 80?
b) If the standard deviation of a week's production is known to equal 4, then what can be said about the probability that this week's production will be between 50 and 70?

Q.10 There are a total of 15 input devices including 6 keyboards, 4 mouse and 5 light pens. If all of them are arranged randomly in a row, then

a) What is the probability that the first 4 devices are keyboards?
b) What is the probability that the first 3 devices are mouses?
c) What is the probability that none of the first 4 devices are keyboards?
d) What is the probability that the first three devices are of different types ?
e) What is the probability that all the light pens are together?