Indian Institute of Information Technology Sri City, Chittoor

(An Institute of National Importance under An Act of Parliament)

Name: **Probability and Statistics (Mathematics 2)** Duration: **1:30Hrs**

INSTRUCTIONS:

1. Please read all the questions carefully and there are no alternative options.
2. Clearly write your Roll Number and Name in capital letters on the top right corner of every page of the answer sheets. It is mandatory.
3. Marks are indicated in [ ] after each question.
4. The answer to the attempted question should be concluded by appropriate steps. Merely answering the question will not be entertained.
5. You are required to write the answers in A4 sheets.
6. Do not do any inappropriate activity as you are being recorded and please sit properly in front of your camera. For more information, please read all the guidelines given by the UG Exams Coordinator.
7. Submit a single PDF file containing all the answers to your attempted questions in the link to be provided during the exam.
8. Preferably use a ballpoint pen. The writing should be readable after scanning. (This is very important)
9. The naming conventions of the PDF file should be as follows: S2020xxxxx_NAME_M2_END_SET_1.pdf for SET-1.
10. The Exam's time duration is 1:30 Hrs. However, an additional 10 minutes will be given to scan and upload the answer sheets.

Not following instructions may lead to heavy penalty

SET-1

Q.1)

a) A bag contains n different items. It is known that the probability of drawing an item A1 and then drawing another item A2 without replacement is 0.9%. What is the value of n? Explain. (nearest integer can be considered)

[3-Marks]

b) In a class room of 100 students 96 were promoted to higher class. If two people are chosen randomly from the entire class, find the probability that at least one of them did not get promoted.

[2-Marks]

c) A company uses three machines A, B and C to manufacture the products. Machine A produces 50% of the products, machine B produces 25% of the products and machine C produces 25% of the products. The defective products produced by the machines A, B and C respectively are 4%, 5% and 6% respectively. If a randomly chosen product is found to be defective, what is the probability that it is produced by machine C.

[2-Marks]

Q. 2)

a) Suppose there are 14 cars of a brand in a car showroom, out of which 7 are good (G), 3 have defective transmission (DT), and 4 have defective steering (DS). If 2 cars are selected at random, find $P(Y < 2)$. (Assume X denotes the number of cars with defective transmission (DT) and Y denotes the number of cars with defective steering (DS))
[2-marks]

b) For the above random variables X and Y, find Cov(X,Y) and ρ_{XY}(coefficient of correlation)?
[3-marks]

Q. 3)

a) A coin is tossed 400 times. Find the approximate probability that the number of heads obtained is between 180 to 220. [1-mark]

b) Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 70. [2-marks]
I) What can be said about the probability that this week's production will exceed 80?
II) the variance of a week's production is known to equal 16, then what can be said about the probability that this week's production will be between 60 to 80?

c) Let $Z = Y^2$, where Y is a random variable. [2-marks]
Find $F_Z(z)$? (where F $is\ cumulative\ distribution\ function\ (CDF)$)

Q. 4)

a) Let X_1 and X_2 are two independent random samples taken from a population with mean μ and variance σ^2. Suppose that you have an estimator of μ: [2-marks]

$$\theta_1 = \frac{2X_1 + 3X_2}{5}$$

Find bias and variance for θ_1.

b) Let 6 independent random samples [0.35, 0.45, 2.25, 1.55, 1.45, 0.80] are taken from the following pdf : [2-marks]

$$f(x) = \theta^{-2} x e^{\left(-\frac{x}{\theta}\right)}, x > 0, \theta > 0.$$

Find MLE of θ.

Q. 5)

a) A researcher took 100 samples from a N(μ, 4) population, where variance is 4. A test of H_0 : μ = 3 vs H_1 : μ≠3 is to be conducted at 5% significance level. Suggest the researcher what should be the rejection region with respect to sample mean. [2-marks]

b) Let 49 i.i.d. random samples were drawn from a normally distributed population with s.d. σ = 0.8 and expectation μ is unknown. Suppose we have sample mean $\bar{x}$ = 89 and sample variance s^2 = 0.5625. Develop a 95 % confidence interval for μ. [2-marks]