

Simplify Boolean Expressions

$$\begin{aligned}
 \textcircled{1} \quad & [A\bar{B}(C+BD) + \bar{A}\bar{B}]C \\
 & = [A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B}]C \quad (\text{Using distributive law}) \\
 & = [A\bar{B}C + \bar{A}\bar{B}]C \quad (A\bar{B}B = 0) \\
 & = A\bar{B}CC + \bar{A}\bar{B}C \quad (\text{Using distributive law}) \\
 & = A\bar{B}C + \bar{A}\bar{B}C \quad (A\bar{C}C = C) \\
 & = \bar{B}C(A + \bar{A}) \quad (\text{Taking } \bar{B}C \text{ common}) \\
 & = \boxed{\bar{B}C} \quad (A + \bar{A} = 1)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad & [AB(C + \bar{B}D) + \bar{A}\bar{B}]CD \\
 & = [ABC + AB\bar{B}D + \bar{A}\bar{B}]CD \quad (\text{Using distributive law}) \\
 & = [ABC + AB(\bar{B} + \bar{D}) + \bar{A}\bar{B}]CD \\
 & = [ABC + AB\bar{B} + AB\bar{D} + \bar{A}\bar{B}]CD \quad (\text{Using De Morgan's second law}) \\
 & \quad \quad \quad (\text{Using distributive law}) \\
 & = [ABC + AB\bar{D} + \bar{A}\bar{B}]CD \quad (\text{as } B\bar{B} = 0) \\
 & = ABCD + AB\bar{D}D + \bar{A}\bar{B}CD \quad (\text{distributive law}) \\
 & = ABCD + \bar{A}\bar{B}CD \quad (\text{as } \bar{D}D = 0 \text{ and } C \cdot C = C) \\
 & = CD(AB + \bar{A}\bar{B}) \quad (CD \text{ common}) \\
 & = \boxed{CD} \quad (AB + \bar{A}\bar{B} = 1)
 \end{aligned}$$

Shivatal

$$\begin{aligned}
 (3) \quad & \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC \\
 &= A + A(BC) + \bar{A}\bar{B}(C + \bar{C}) + \bar{A}\bar{B}\bar{C} \text{ (taking } BC \text{ \& } \bar{A}\bar{B} \text{ common)} \\
 &= BC + A\bar{B} + \bar{A}\bar{B}\bar{C} \text{ (} A + \bar{A} = 1 \text{ \& } C + \bar{C} = 1 \text{)} \\
 &= BC + \bar{B}(A + \bar{A}\bar{C}) \text{ (taking } \bar{B} \text{ common)} \\
 &= BC + \bar{B}(A + \bar{C}) \text{ (as } A + \bar{A}\bar{B} = A + \bar{B} \\
 &\quad \therefore A + \bar{A}\bar{C} = A + \bar{C} \text{)} \\
 &= \boxed{BC + A\bar{B} + \bar{B}\bar{C}} \text{ (distributive law)}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & \overline{AB + AC} + \bar{A}\bar{B}C \\
 &= \bar{A}\bar{B} \cdot \bar{A}\bar{C} + \bar{A}\bar{B}C \text{ (using de Morgan's law)} \\
 &= (\bar{A} + \bar{B})(\bar{A} + \bar{C}) + \bar{A}\bar{B}C \text{ (using de Morgan's law)} \\
 &= \bar{A} \cdot \bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C \text{ (distributive law)} \\
 &= \bar{A} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}(1 + C) \text{ (taking } \bar{A}\bar{B} \text{ common)} \\
 &= \bar{A} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B} \text{ (as } 1 + C = 1 \text{)} \\
 &= \bar{A}(1 + \bar{C} + \bar{B}) + \bar{B}\bar{C} \text{ (taking } \bar{A} \text{ common)} \\
 &= \boxed{\bar{A} + \bar{B}\bar{C}} \text{ (as } 1 + \bar{C} + \bar{B} = 1 \text{)}
 \end{aligned}$$

$$(5) \quad \overline{A}B + \overline{A}C + \overline{A}B\overline{C}$$

$$= \overline{A} + B + \overline{A} + C + \overline{A}B\overline{C} \quad (\text{Using de Morgan's second law})$$

$$= \overline{A} + B + C + \overline{A}B\overline{C} \quad (\text{as } \overline{A} + \overline{A} = \overline{A})$$

$$= \overline{A} + B + C (1 + \overline{A}B) \quad (\text{taking } C \text{ common})$$

$$= \overline{A} + B + C \quad (\text{as } 1 + \overline{A}B = 1)$$

$$= \boxed{\overline{A}B\overline{C}} \quad (\text{Using de Morgan's first law})$$

$$(6) \quad A + AB + A\overline{B}C$$

$$= A(1 + B + \overline{B}C) \quad (\text{taking } A \text{ common})$$

$$= \boxed{A} \quad (\text{as } 1 + B + \overline{B}C = 1)$$

$$(7) \quad (\overline{A} + B)C + AB$$

$$= \overline{A}C + BC + ABC \quad (\text{distributive law})$$

$$= \overline{A}C + BC(1 + A) \quad (BC \text{ common})$$

$$= \overline{A}C + \overline{A}C + BC \quad (\text{as } 1 + A = 1)$$

$$= \boxed{(\overline{A} + B)C} \quad (\text{taking } C \text{ common})$$

$$\textcircled{8} \quad ABC(DD + CDE) + \bar{A}C$$

$$= ABCBD + A\bar{B}C\bar{C}DE + A\bar{C} \quad (\text{distribute})$$

$$(1 + 0 + 0) = A\bar{B}CDE + A\bar{C} \quad (\text{as } 0\bar{0} = 0)$$

$$= \boxed{A(\bar{B}CDE + \bar{C})} \quad \text{taking } A \text{ common}$$