Assignment - 2

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Mutually exclusive Events

① The probabilities of teams A, B & C winning a football competition are $1/3, 1/5$ & $1/9$ respectively. Calculate the probability that either A or B will win.

Sol, $P(A \text{ or } B \text{ will win}) = P(A) + P(B)$

$$= 1/3 + 1/5$$
$$= 8/15$$

② The probabilities of three mutually exclusive events $1/4, \frac{1}{6}, 2/3$. Prove whether the given statement is correct or not.

Sol, $X \cap Y = 0, \ Y \cap Z = 0, X \cap Z = 0,$
for mutually exclusive

$\therefore P(X \cup Y \cup Z) = P(X) + P(Y) + P(Z)$

$$= 1/4 + 1/6 + 2/3$$
$$= 13/12 > 1$$

Probability > 1
$\therefore$ Statement is incorrect

Q-3 It is known that the probability of obtaining zero defectives in a sample of 40 times is 0.34 whilst the probability of obtaining 1 defective item in the sample is 0.46. What is probability of obtaining not more then 1 defective item in a sample?

$$P(G \cap G) = P(E_1) + P(E_2)$$
$$= 0.34 + 0.46$$
$$= 0.80$$

Q-4 If the independent probabilities that three people A, B and C will be alive in 30 years time are 0.4, 0.3, 0.2 respectively, Calculate the probability that in 30 year's time all will be alive

$$P = P(A) \times P(B) \times P(C)$$
$$= 0.4 \times 0.3 \times 0.2$$
$$= 0.024$$

Q-95

If $P(x) = 1/3$ and $P(y) = 2/3$. Examine whether x and y are mutually exclusive

$$P(x) + P(y) = 1/3 + 2/3 = 1$$
$$P(x \cup y) = P(x) + P(y) + P(x \cap y)$$
$$1 = 1 + P(x \cap y)$$
$$P(x \cap y) = 0$$

∴ x & y are mutually exclusive

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Mutually Non exclusive events

Q-1   Two coins are tossed. find the probability getting atleast one head or atleast one tail.

$$S = \{ HH, HT, TH, TT \}$$

$P(G) = P(\text{atleast one head} = 3/4$
$P(G) = P(\text{atleast one tail} = 3/4$
$$P(G \cup G) = P(G) + P(G) - P(G \cap G)$$
$$= \frac{3}{9} + \frac{3}{4} + \frac{1}{9}$$
$$= 1$$

Q-2 Two cards are drawn at random from a deck of cards. And the probability that both the cards are of red colour or both are queen.

Sol⟩

$$P(Red) = \frac{26C_2}{52C_2}$$

$$P(Queen) = \frac{4C_2}{52C_2}$$

$$P(Red \cap Queen) = \frac{2C_2}{52C_2}$$

$$P(Red \cup Q) = \frac{26C_2}{52C_2} + \frac{4C_2}{52C_2} - \frac{2C_2}{52C_2}$$

$$= \frac{26 \times 25 \times 4 \times 3 - 1 \times 2}{52 \times 51}$$

$$= \frac{55}{221}$$

Q-3

In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or a a student?

$P(G) = P(\text{Choosing a girl}) = 13/30$

$P(G_2) = P(\text{Choosing a student}) = 9/30$

$P(G \cap G_2) = 5/30$

$P(G \cup G_2) = P(G) + P(G_2) - P(G \cap G_2)$

$$= 13/30 + 9/30 - 5/30$$

$$= 17/30$$

Q-7 What is the probability of getting a diamond or queen from a well shuffled deck of 52 cards.

Sol: Probability of getting a diamond

$P(x) = 13/52 = 1/4$

Probability of getting a queen

$P(y) = 4/52 = 1/13$

Probability of getting a diamond queen

$= P(x \cap y) = 1/52$

$P(x \cup y) = \dfrac{1}{4} + \dfrac{1}{13} - \dfrac{1}{52} \qquad = 4/13$

Q-5   A pair of dice is rolled. find the
probability of getting an even number on
first die, or a total of 8.

$$A = \{ (2,1) - - - - - (2,6),$$
$$(4,1) - - - - - (4,6),$$
$$(6,1) - - - - - (6,6)\}$$

$$B = \{ (2,6), (6,2), (3,5), (5,3), (4,4)\}$$

$$A \cap B = \{ (2,6), (6,2), (4,4)\}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{5}{36} - \frac{1}{12}$$

$$= \frac{5}{9}$$

Complimentary Events

Q-1 A fair die is thrown. What is the probability that the score is not a factor of 6?

$$P(a\ factor\ of\ 6) = \frac{4}{6} = \frac{2}{3}$$

$$P(Not\ a\ factor\ of\ 6) = 1 - \frac{2}{3} = \frac{1}{3}$$

Q-2 Two cards are chosen at random from a pack of 52 playing cards. What is the probability that at least one of them is a heart?

$$P(The\ first\ card\ chosen\ is\ not\ a\ heart) = \frac{3}{4}$$

$$P(Second\ card\ chosen\ is\ also\ not\ a\ heart) = \frac{38}{51}$$

$$P(neither\ of\ them\ is\ heart) = \frac{3}{4} \times \frac{38}{51}$$

$$= \frac{19}{34}$$

$$P(at\ least\ one\ of\ them\ is\ a\ heart) = 1 - \frac{19}{34} = \frac{15}{34}$$

Q-3 Two fair coins are tossed. What is the probability at least one coin lands head up?

P(both coins land tails up) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P(at least one coin land heads up)

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Q-4 The letters of the alphabet are written on 26 cards. Two cards are chosen at a random (without replacement). What is the probability that at least one them is consonant.

P(first card is not a consonant)

P(second card is not a consonant) $= \frac{5}{26}$

$= \frac{5}{25}$

P(neither of them is a consonant)

$$= \frac{5}{26} \times \frac{4}{25} = \frac{20}{650} = \frac{2}{65}$$

P(at least one of them is a consonant) $= 1 - \frac{2}{65} = \frac{63}{65}$

Q- Two cards are chosen at random from a pack of 52 playing cards. what is probability that at least one of them is a picture card?

$$P(\text{first card is not picture card}) = \frac{42}{52} = \frac{12}{13}$$

$$P(\text{Second Card is not picture Card})$$
$$= \frac{39}{51} = \frac{13}{17}$$

$$P(\text{Neither of them is a picture Card})$$
$$= \frac{12}{13} \times \frac{13}{17} = \frac{12}{17}$$

$$P(\text{at least one of them is a picture Card}) = 1 - \frac{12}{17} = \frac{5}{17}$$

with replacement

Q-1 Two marbles are drawn at random and with replacement from a box containing 2 red, 3 green, 4 blue marbles.

find probability that two red marbles are drawn.

Total marbles = 9

probability that two marbles are drawn of red colour

$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$

Q-2 In a chest there are two types of blocks, 32 are of diorite and 64 are of andersite. what is the probability to pick 2 block of each colour with replacement.

$$probability = \frac{32}{96} \times \frac{64}{96}$$

Q-3 In a box, there are 8 pens, 3 red, 3 blue 4 black, what is the probability to pick 2 pens of black colour with replacement.

probability to pick 2 black pens

$$= \frac{2}{8} \times \frac{2}{8} = \frac{4}{64} = \frac{1}{16}$$

Q-4 A bag contains 10 fruits 5 oranges, 3 apple 2 bananas. What is the probability to pick 2 apples without replacement.

probability to pick 2 apples without

$$= \frac{3}{10} \times \frac{3}{10} \times = \frac{9}{100}$$

Q-5 A bag contains 15 balls, 5 tennis balls, 10 cricket balls. What is the probability to choose 7 cricket balls without replacement.

probability to pick 7 cricket balls

$$= \left(\frac{10}{15}\right)^7 \text{ Ans}$$

Sampling without deplacenet

Q-1 A bag contains 4 red balls and 6 green balls 4 balls are drawn at random from the bag without replacemet. Calculate the probability that all are green.

Ans we have $^6C_4$ ways to choose 4 green balls and $^{12}C_4$ ways to choose 4 balls

So probability that all are green

$$= \frac{^6C_4}{^{12}C_4} = 1/19$$

Q-2 In a bag of marbles there are 3 green and 2 red. What is the probability that at least one marble of each colour is drawn marble.

Soln We have $3C_1$ ways to choose 2 green and $2C_1$ ways to choose 1 red

$$P_{rob} = \frac{3C_1 \times 2C_1}{5C_2} = \frac{7}{5}$$

Q.3 In a box there are 8 pens, 3 blues
2 black and 3 red. What is the
probability to pick 3 pens. 1 pen of
each.

Soln we have $3c_1$ ways to
pick blue pen

$2c_1$ ways to pick black pen

$3c_1$ ways to pick red pen.

$$Probability = \frac{3c_1 \times 2c_1 \times 3c_1}{8c_3}$$

Q.4 A bag contains 10 fruits. 4 apples,
3 mangoes, 3 watermelons. What is
the probability to choose 5 fruits. It
must contains 1 watermelon.

$$prob = \frac{3c_1 \times 7c_9}{12c_5}$$

Q-5

In a player's minecraft inventory, there are 8 dyes, 3 red colour, 4 of blue colour and 1 of green colour. If he has 3 beds what is the prob that they are blue colour?

$$Prob = \frac{4C_3}{8C_3}$$