Signal & Systems

Set-D

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Q-1 $(n+2)(1/2)^n u(n) \rightarrow \left(\frac{1}{4}\right)^n u(n)$

$x(n)$ $\qquad\qquad$ $y(n)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \frac{1}{1-\frac{1}{2}e^{-j\omega}}\left(2 + \frac{\frac{e^{-j\omega}}{2}}{1-e^{-j\omega}}\right)$$

$$= \frac{2 - \frac{e^{-j\omega}}{2}}{\left(1-\frac{e^{-j\omega}}{2}\right)^2}$$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y(n) e^{-j\omega n} = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(n) = h(n) * h(n)$$

$$Y(\omega) = X(\omega) \times h(\omega)$$

$$h(\omega) = \frac{Y(\omega)}{X(\omega)} = \left(\frac{1}{1-1/4\, e^{-j\omega}}\right)\left(\frac{\left(1-\frac{e^{-j\omega}}{2}\right)^2}{1-\frac{e^{-j\omega}}{2}}\right)$$

$$y_2(t) = ?$$

$$y_2(t) = \delta(n) - (1/2)^n u(n)$$

$$y_2(\omega) = 1 - \frac{1}{1-\frac{e^{-j\omega}}{2}} = \frac{-\frac{e^{-j\omega}}{2}}{1-\frac{e^{-j\omega}}{2}}$$

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$$y_2(n) = x_2(n) * h(n)$$

$$Y_2(\omega) = X_2(\omega) H(\omega)$$

$$X_2(\omega) = \frac{Y_2(\omega)}{H(\omega)}$$

$$= \frac{-\frac{e^{-j\omega}}{2}}{1 - \frac{e^{-j\omega}}{2}} \times \frac{\left(1 - \frac{e^{-j\omega}}{4}\right)\left(2 - \frac{e^{-j\omega}}{2}\right)}{\left(1 - \frac{e^{-j\omega}}{2}\right)^2}$$

after the inversing ~~inversing~~ $X_2(\omega)$ we will get $x_2(t)$

Q-2

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$$x(t) = e^{-at} u(t)$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= -\frac{1}{a+j\omega} \left[-1 \right.$$

$$= \frac{1}{a+j\omega}$$

$$e^{-at} u(t) = \frac{1}{\pi} \int_0^{\omega_{70}} |X(\omega)|^2 d\omega = 0.7 E_x$$

$$= \frac{1}{\pi} \int_0^{\omega_{70}} \left| \frac{1}{a+j\omega} \right|^2 d\omega$$

$$= \frac{1}{\pi} \int_0^{\omega_{70}} \frac{1}{a^2+\omega^2} d\omega$$

$$= \frac{1}{a\pi} \left[\tan^{-1}\left(\frac{\omega}{a}\right) \right]_0^{\omega_{70}} = 0.7 E_x$$

$$\frac{1}{a\pi} \tan^{-1}\left(\frac{\omega_{70}}{a}\right) = 0.7 E_x$$

$$\boxed{\omega_{70} = a \tan(0.7 E_x \times a\pi)}$$

Q-3

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$x(n) = \{4, 2, 1, \underset{\uparrow}{-1}, 1, 2, 4\}$

$$\boxed{\sum_{n=-\infty}^{\infty} [x(n)]^2 = \frac{1}{2\pi} \int_{2\pi} [X(\omega)]^2 d\omega}$$

$\cancel{q_x = x(n) e^{-jn}}$

$$X(\omega) = \sum_{-3}^{3} x[n] e^{-j\omega n}$$

$$= 4e^{-3\omega j} + 2e^{-2j\omega} + 1e^{-j\omega} - 1e^{-j\omega(0)} + e^{j\omega} + 2e^{2j\omega} + 4e^{3\omega j}$$

$$= 4(2\cos 3\omega) + 2(2\cos 2\omega) + 2\cos\omega - 1$$

$$= 8\cos 3\omega + 4\cos 2\omega + 2\cos\omega - 1$$

Ch. 5

$$\sum_{n=-\infty}^{\infty} [x(n)]^2 = 4^2 + 2^2 + 1^2 + (-1)^2 + 1^2 + 2^2 + 4^2$$

$$= 16 + 4 + 1 + 1 + 1 + 4 + 16$$

$$= 43$$

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R.H.S

$$= \frac{1}{2\pi} \int_0^{2\pi} |X(\omega)|^2 \, d\omega$$

$$= \frac{1}{2\pi} \int_0^{2\pi} (8\cos 3\omega + 4\cos 2\omega + 2\cos\omega - 1)^2 \, d\omega$$

after solving

R.H.S = 43

R.H.S = L.H.S

Hence proved