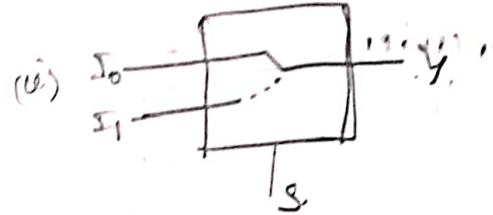
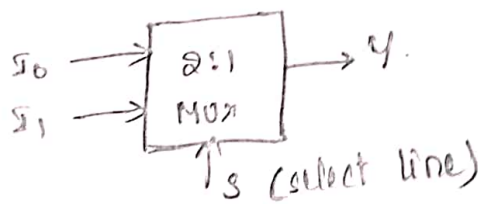


2/1/24 Multiplexers:

Selects binary input from many input lines.

(input: output \Rightarrow 2:1, 4:1, 8:1, 16:1, ...).

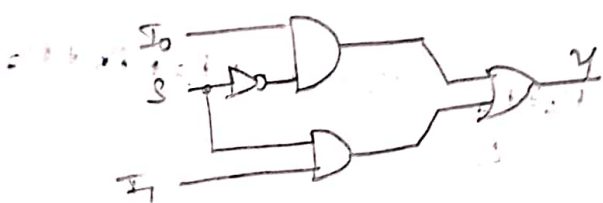
2:1 Mux:



Output 'Y' can be connected to only one input (I_0/I_1) at a time (to depends on select line).

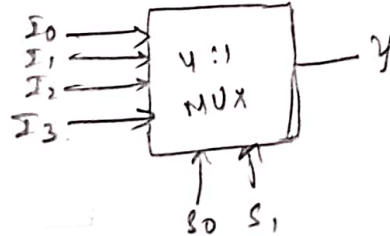
(u) eg: if $S=0$, $Y=I_0$.
if $S=1$, $Y=I_1$

$$\text{Here, } Y = I_0 \bar{S} + I_1 S$$



\Rightarrow 2:1 multiplexer.

4:1 MUX:

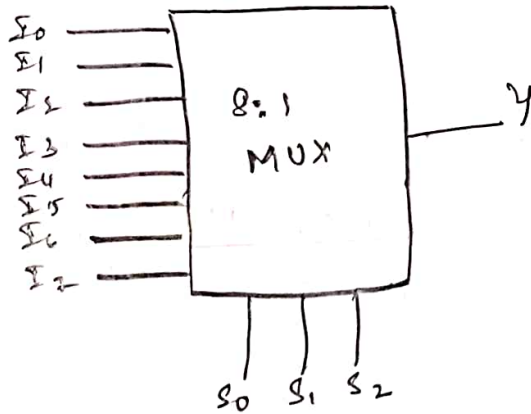


if no. of select lines = n , then, input lines = 2^n .

S_0	S_1	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

$$Y = I_0 \bar{S}_0 \bar{S}_1 + I_1 \bar{S}_0 S_1 + I_2 S_0 \bar{S}_1 + I_3 S_0 S_1$$

8:1 MUX:

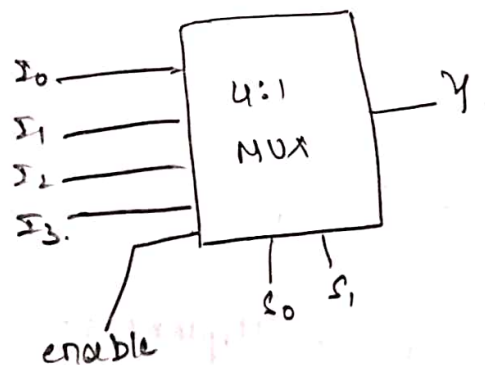


S_0	S_1	S_2	Y
0	0	0	I_0
0	0	1	I_1
0	1	0	I_2
0	1	1	I_3
1	0	0	I_4
1	0	1	I_5
1	1	0	I_6
1	1	1	I_7

$$Y = I_0 \bar{S}_0 \bar{S}_1 \bar{S}_2 + I_1 \bar{S}_0 \bar{S}_1 S_2 + I_2 \bar{S}_0 S_1 \bar{S}_2 + I_3 \bar{S}_0 S_1 S_2 + I_4 S_0 \bar{S}_1 \bar{S}_2 + I_5 S_0 \bar{S}_1 S_2 + I_6 S_0 S_1 \bar{S}_2 + I_7 S_0 S_1 S_2$$

Enable: (eg: for 4:1 MUX)

E	S_0	S_1	Y
0	x	x	0
1	0	0	I_0
1	0	1	I_1
1	1	0	I_2
1	1	1	I_3



We can have enables for 8:1 / 16:1 ... multiplexers.

De-Multiplexers:

(input: output \Rightarrow 1:2, 1:4, 1:8, 1:16 ---)

eg: for 1:4 De-MUX:

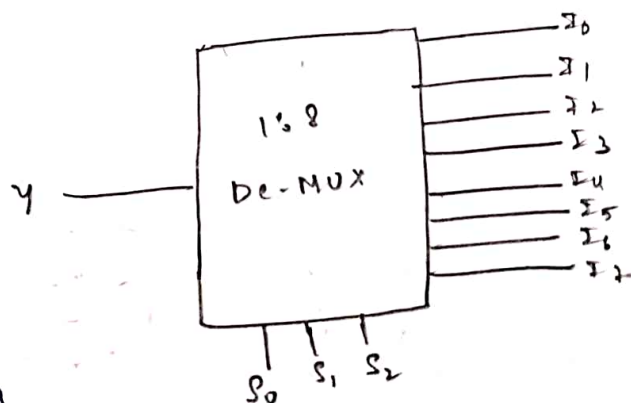


Y	S ₀	S ₁	I ₀	I ₁	I ₂	I ₃
1(0)	0	0	1(1)	x	x	x
1(0)	0	1	x	1(0)	x	x
1(0)	1	0	x	x	1(0)	x
1(0)	1	1	x	x	x	1(0)

input gets connected to any one of the output.
 outputs: $I_0 = Y \bar{S}_0 \bar{S}_1$; $I_1 = Y \bar{S}_0 S_1$; $I_2 = Y S_0 \bar{S}_1$; $I_3 = Y S_0 S_1$

eg: for 1:8 De-MUX:

Y	S ₀	S ₁	S ₂	I ₀	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇
1	0	0	0	1	x	x	x	x	x	x	x
1	0	0	1	x	1	x	x	x	x	x	x
1	0	1	0	x	x	1	x	x	x	x	x
1	0	1	1	x	x	x	1	x	x	x	x
1	1	0	0	x	x	x	x	1	x	x	x
1	1	0	1	x	x	x	x	x	1	x	x
1	1	1	0	x	x	x	x	x	x	1	x
1	1	1	1	x	x	x	x	x	x	x	1



$$I_0 = Y \bar{S}_0 \bar{S}_1 \bar{S}_2$$

$$I_1 = Y \bar{S}_0 \bar{S}_1 S_2$$

$$I_2 = Y \bar{S}_0 S_1 \bar{S}_2$$

$$I_3 = Y \bar{S}_0 S_1 S_2$$

$$I_4 = Y S_0 \bar{S}_1 \bar{S}_2$$

$$I_5 = Y S_0 \bar{S}_1 S_2$$

$$I_6 = Y S_0 S_1 \bar{S}_2$$

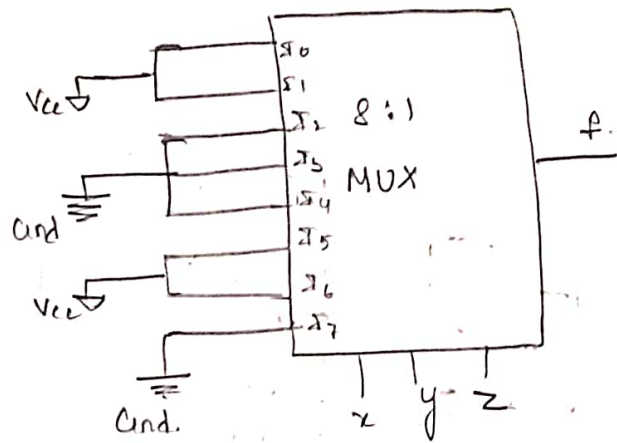
$$I_7 = Y S_0 S_1 S_2$$

Example problems on multiplexers:

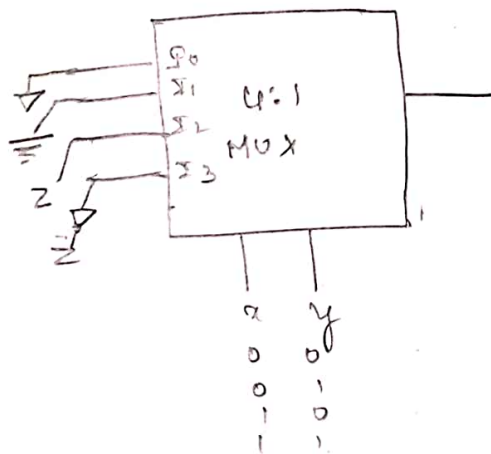
(1) Implement $f(x, y, z) = \pi(2, 3, 4, 7)$ using 2:1 MUX, 4:1 MUX, 8:1 MUX.

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Using 8:1 mux:

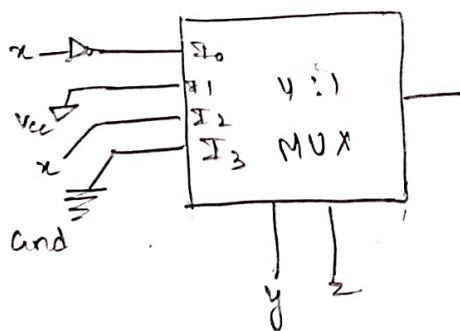


Using 4:1 MUX:



x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Using 4:1 MUX:

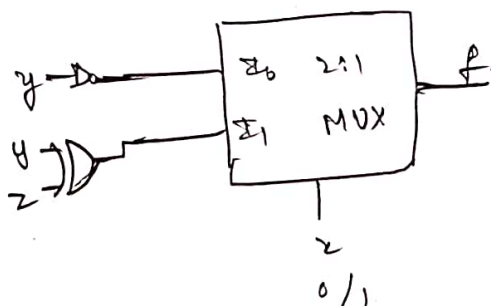


x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

By we can give z & x to selection pins.

Using 2:1 MUX:

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



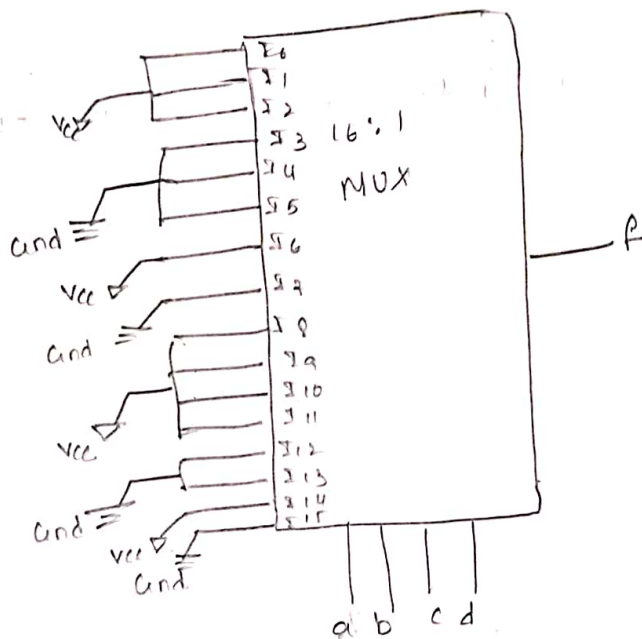
y	z	f
0	0	1
0	1	1
1	0	0
1	1	0

$$\begin{aligned}
 f &= \bar{y} \bar{z} + \bar{y} z \\
 &= \bar{y} (z + \bar{z}) \\
 &= \bar{y} \\
 &\text{(for } z_0)
 \end{aligned}$$

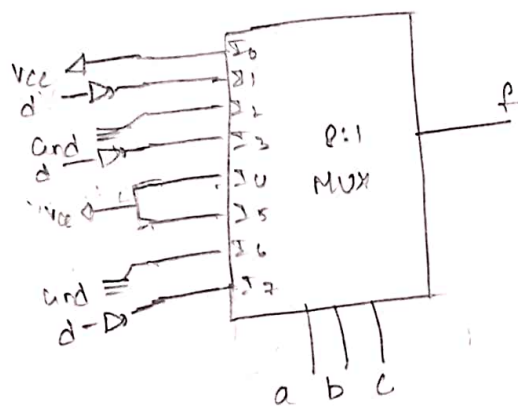
(2) Implement $f(a, b, c, d) = abc + b\bar{c} + c\bar{d}$ using 16:1 MUX.

(3)

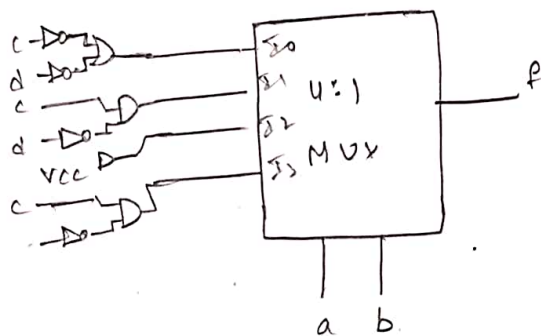
a	b	c	d	f
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0



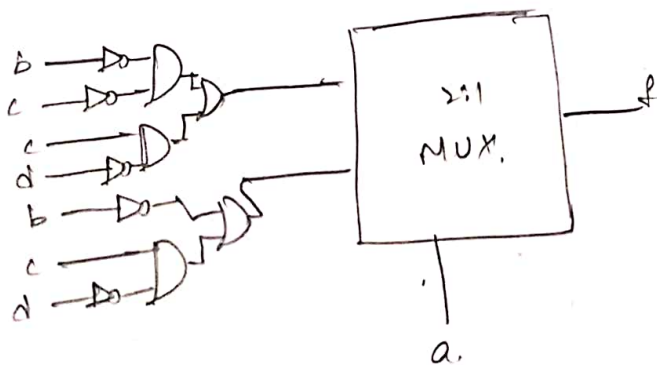
Using 8:1 MUX:



Using 4:1 MUX:

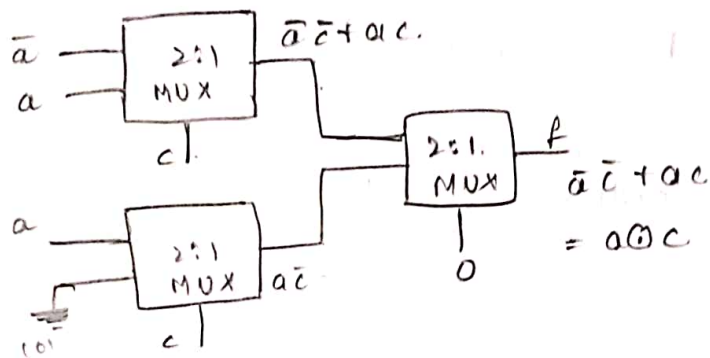


Using 2:1 MUX:

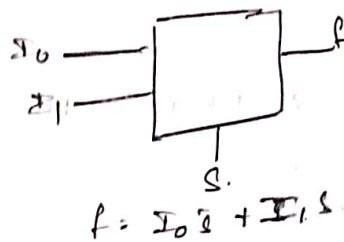


3)

find f .



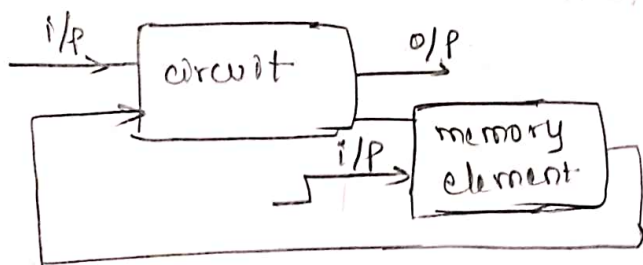
since



2/2/21

Sequential Circuits

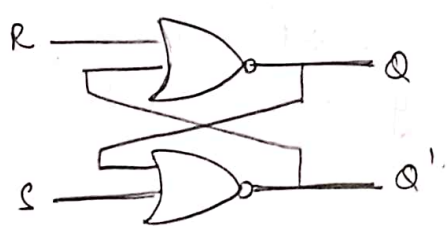
A combinational circuit along with a memory element is called sequential circuits.



SR latch:

There are two types of SR latches: NOR based, NAND based.

NOR based:



(cross coupled circuit)

NOR Gate:

input		output
0	0	1
0	1	0
1	0	0
1	1	0

if any one of the inputs is high the output is 0.

S	R	Q	Q'
0	0	x	x
1	0	1	0
0	0	1	0
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1

0	0	0	1
1	0	1	0
0	0	1	0

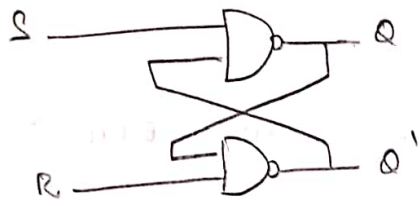
1 1 0 0 (not valid)

($Q = Q'$ which is not possible)

Being in 0 0 state is same as being in memory state for the SR latch. The output don't change when S & R are in 0 0 state.

Here 'i' bit is stored in the output 'Q' (or Q').

SR latch (NAND):

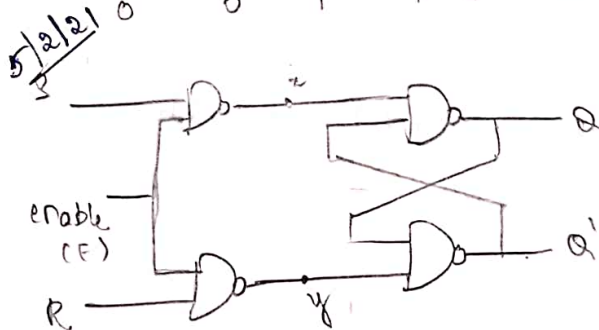


Input		Output
0	0	1
0	1	1
1	0	1
1	1	0

S	R	Q	Q'
1	1	x	x (don't know)
0	1	1	0
1	1	1	0
1	0	0	1
1	1	0	1
1	1	0	1
0	0	1	1 (invalid)

For NAND based SR latch 1 1 is the memory state.

S	R	Q	Q'
1	1	memory	
0	1	1	0
1	0	0	1
0	0	invalid	



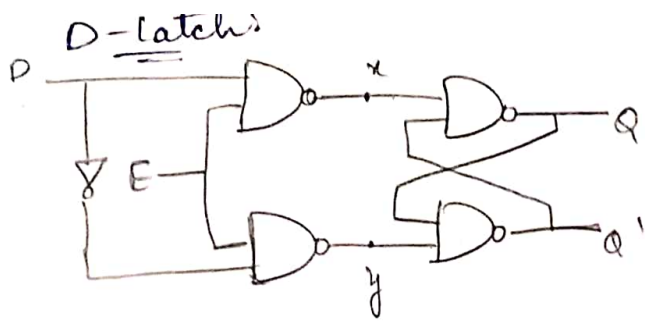
$$x = S \cdot \bar{E} = S + \bar{E}$$

$$y = \bar{R} \cdot \bar{E} = \bar{R} + \bar{E}$$

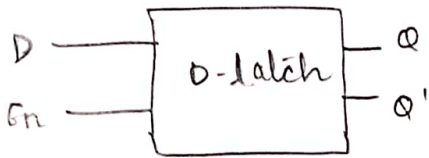
when enable = 0, $x = y = 1$ (in memory state)

when enable = 1, $x = S$, $y = \bar{R}$

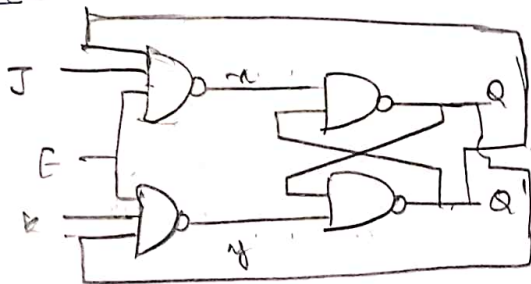
E	S	R	x	y	Q	Q'
1	1	1	0	0	invalid	
1	0	0	1	1	memory	
1	1	0	0	1	1	0
1	0	0	1	1	1	0
1	0	1	1	0	0	1
1	0	0	1	1	0	1



E	D	Q	Q'
0	x	memory state	
1	0	0	1
0	x	0	1
1	1	1	0
0	x	1	0



J-K latch:



$$x = \overline{J \cdot E \cdot Q} = \overline{J} + \overline{E} + \overline{Q}$$

$$y = \overline{K \cdot E \cdot Q'} = \overline{K} + \overline{E} + \overline{Q'}$$

If enable = 0; $x = y = 1$ (in memory state)

If enable = 1; $x = \overline{J} + Q$, $y = \overline{K} + Q'$

inputs

intermediate inputs

outputs

Q	J	K	x	y	Q	Q'
1	1	0	1	1	1	0
1	0	1	1	0	0	1
0	0	0	1	1	0	1
0	1	1	0	1	1	0
1	0	0	1	1	1	0
1	1	1	1	0	0	1
0	1	1	0	1	1	0
1	1	1	1	0	0	1
1	1	1	1	0	0	1
0	1	1	0	1	1	0
0	1	1	0	1	1	0
0	1	1	0	1	1	0
1	1	1	1	0	0	1
1	1	1	1	0	0	1

$J = K = 0 \Rightarrow$ memory state

$J = 1, K = 0 \Rightarrow Q = 1, Q' = 0$

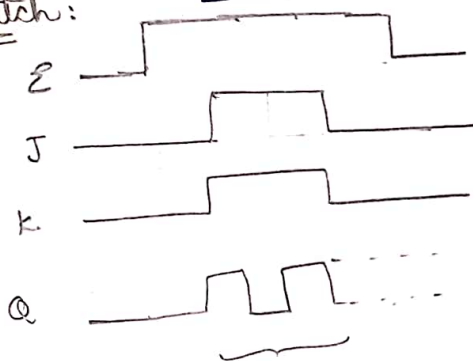
$J = 0, K = 1 \Rightarrow Q = 0, Q' = 1$

$J = K = 1 \Rightarrow Q$ is complement of previous output Q
 \Rightarrow racing condition

Q	J	K	Q	Q'
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

Timing diagrams:

s/2/2/ Jk latch:

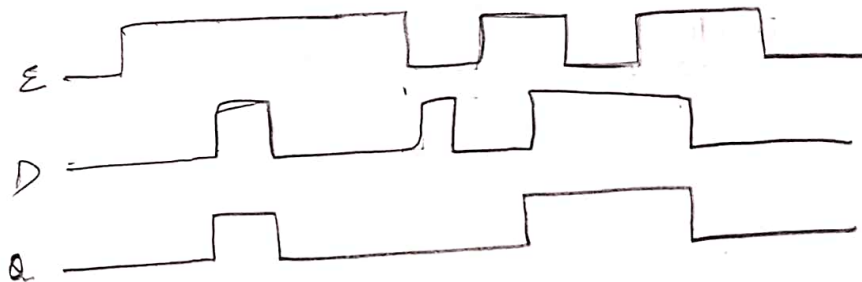
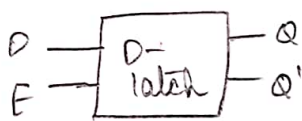


E	J	K	Q	Q'
0	0	0	0	1
1	0	0	0	1
1	1	1	1	0
1	1	1	0	1
1	0	0	1	0
0	0	0	1	0

y axis - amplitude
x axis - time

For D-latch:

→ racing behaviour.



Enable: It is a random signal which can be changed by the user.

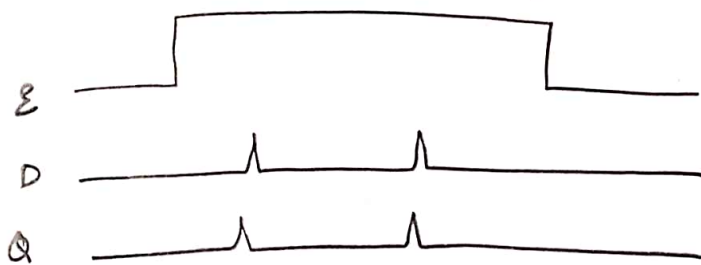


→ Fixed time period ($T = \frac{1}{f}$)
→ clock input

if the enable turns to be the clock input, the D-latch becomes D-Flipflop.



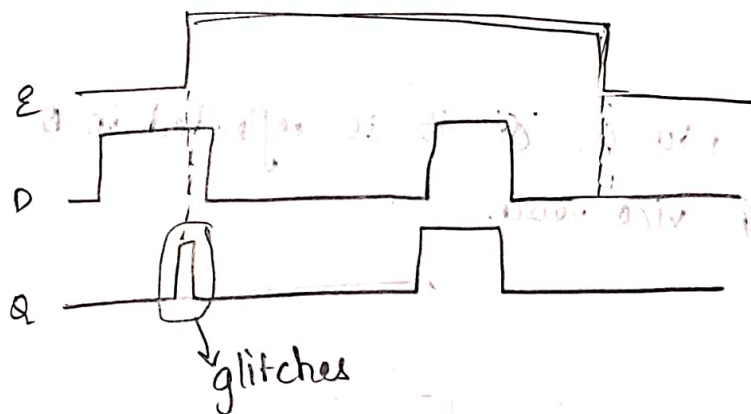
for D-latch: (disadvantage)



(some disturbance in input signal)

(reflected in the output)

latches are level sensitive circuits (depends on i/p or enable signals being high or low). \rightarrow two types — high (en) — low (en)



When Q becomes high for a very small period of time, the jerks produced in Q are called glitches.

So, to avoid these unwanted disturbances and glitches, we use edge-sensitive circuits instead of level-sensitive circuits.

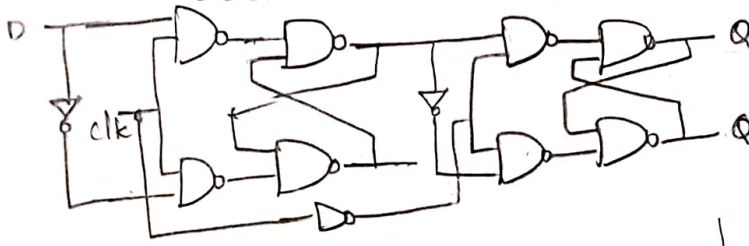
edges can be, high to low or low to high

\rightarrow There are two types of edge sensitive circuits:

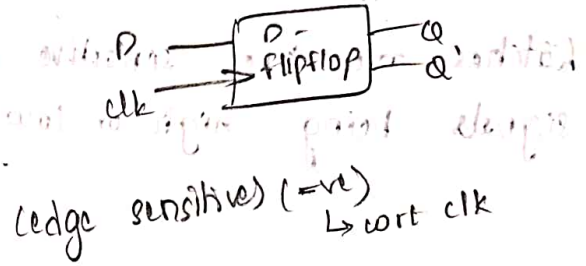
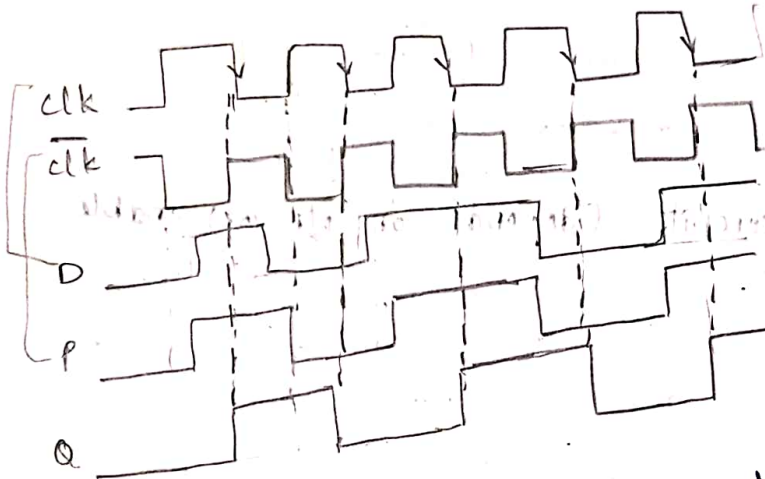
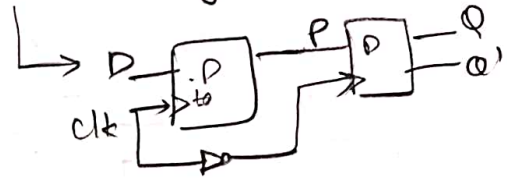
- (1) positive (0 to 1)
 - (2) negative (1 to 0)
- \rightarrow enable

\rightarrow Converting level-sensitive to edge-sensitive

-ve edge sensitive D-flipflop:

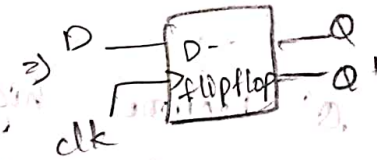
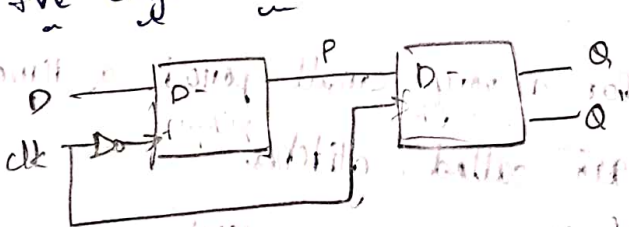


combining two d-flipflops make it edge-sensitive



whenever, clk is falling from 1 to 0, it is reflected in Q if Q is low it becomes high & vice versa

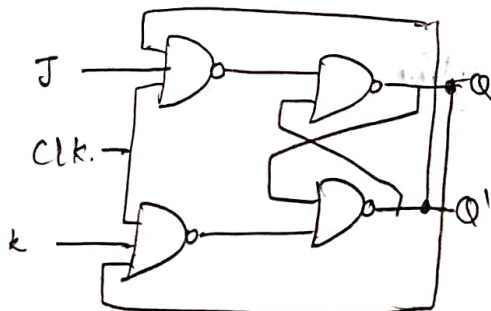
+ve edge sensitive:



If clk is involved, that signal is called 'triggering' signal or 'control' signal. Two types can be called:

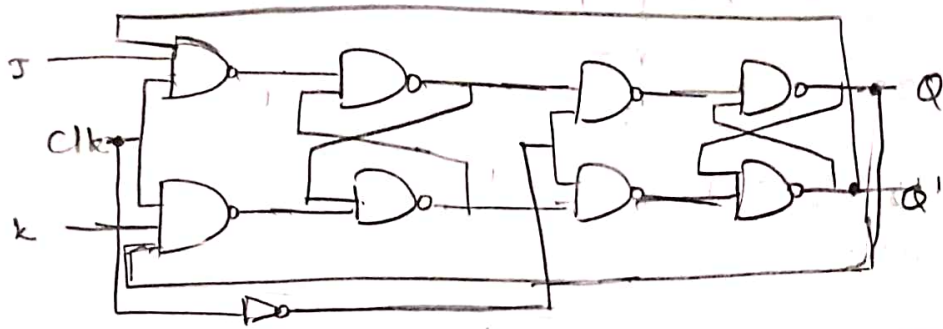
- 1) -ve edge triggering (or) falling edge triggering
- 2) +ve edge triggering (or) rising edge triggering

12/2/21 Difference b/w JK latch & JK flipflop:

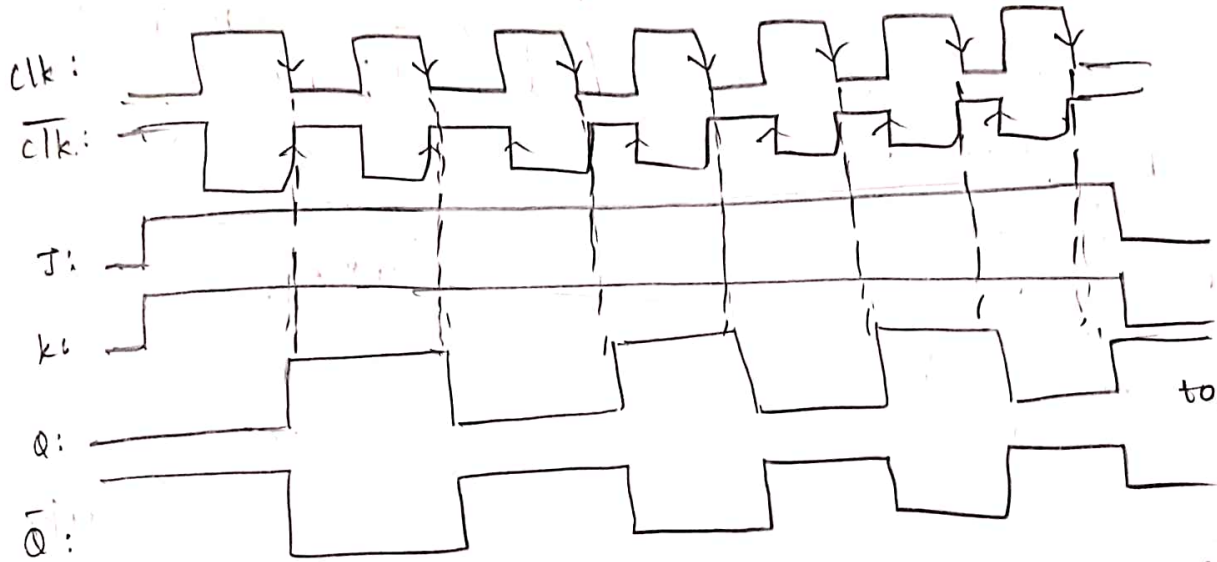


clk = J = K = 1

→ racing condition.

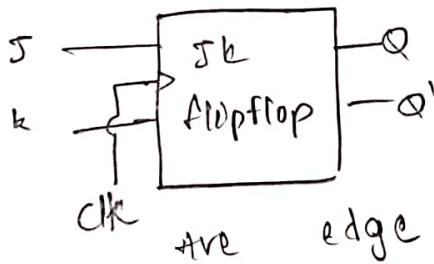


Negative edge
JK flipflop



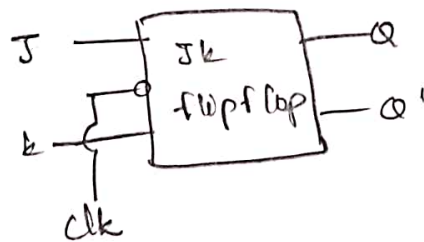
toggling behaviour

when clk goes to 0, $J = k = 1$, Q complements its previous value.
To get a positive edge JK flipflop, just interchange the clock signal
& complement of clock signal inputs (\bar{clk}) to 1st JK flipflop & clk - given to 2nd JK flipflop.



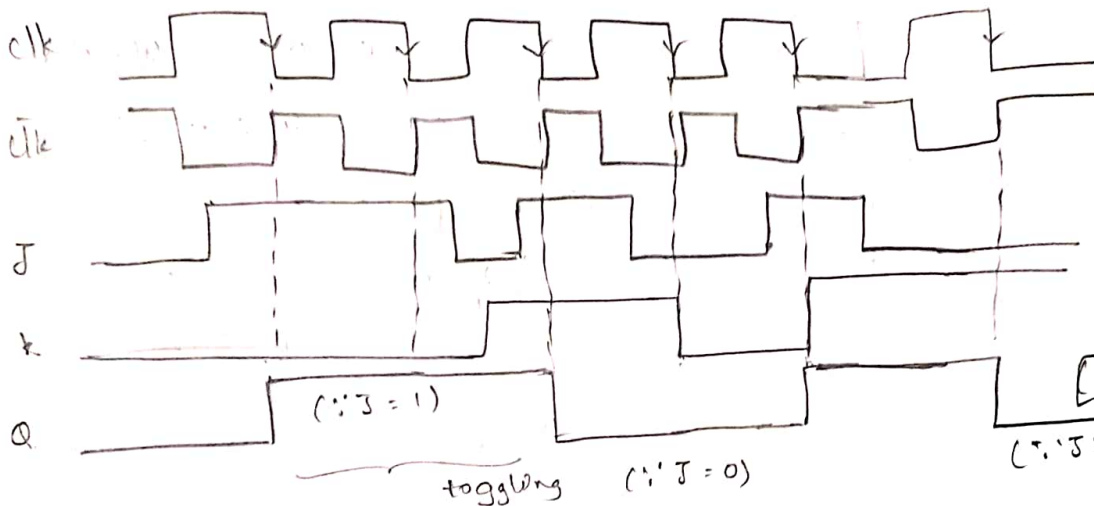
latch

- 1) Enable signal
- 2) level sensitive



flipflop

- 1) clk signal
- 2) edge sensitive / level sensitive



J	K	Q	Q'
1	0	1	0
0	1	0	1
0	0	memory state	
1	1	complement	

(i) if $clk = 2\text{ Hz}$ (eg)

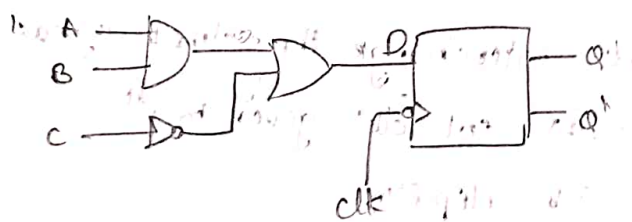
$$Q = 1\text{ Hz}$$

if $Q = clk$

output = 0.5 Hz and so on.

increased toggling

Applications:



A	B	C	D	Q(t+1)	Q'
0	0	0	1	0	1
0	0	1	0	0	1
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0

$$D(t) = AB + \bar{C} \quad (\text{state eqn})$$

The truth table here is called state table.

$Q(t+1)$ = next state

(a) A _____

(b) B _____

(c) C _____



d _____

Q _____



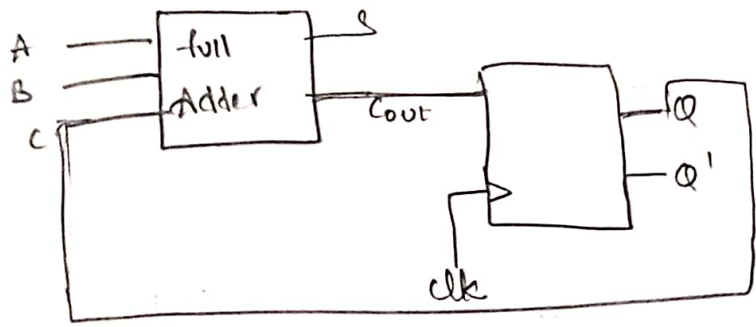
$$D(t) = A \oplus B \oplus Q$$

$Q(t)$ → present state (i.e. op going to ip)

A	B	Q	D(t)	Q(t+1)
0	0	0	0	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

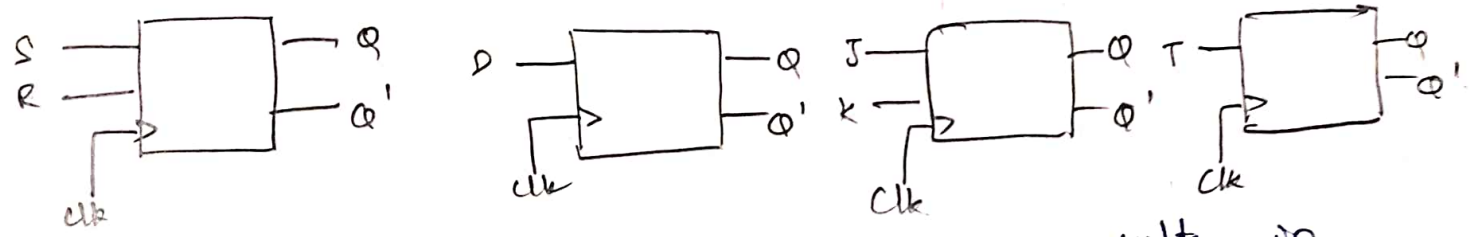
and so on

3



A	B	C	Q	(PS) Cout	(NS) Q(t+1)
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	0
1	0	0	0	1	0
1	0	1	0	0	1
1	1	0	0	0	1
1	1	1	1	1	1

1 and so on.



when j, k combine to be a single input, it results in T-flip flop.

$T=0 (J=k=0) \rightarrow$ circuit is in memory state
 $T=1 (J=k=1) \rightarrow$ output toggles

Counters:

In BCD to 7-segment decoder, the output changes according to the input. Counters can count the numbers itself (0 1 2 3 4 5 ---) without changing inputs.

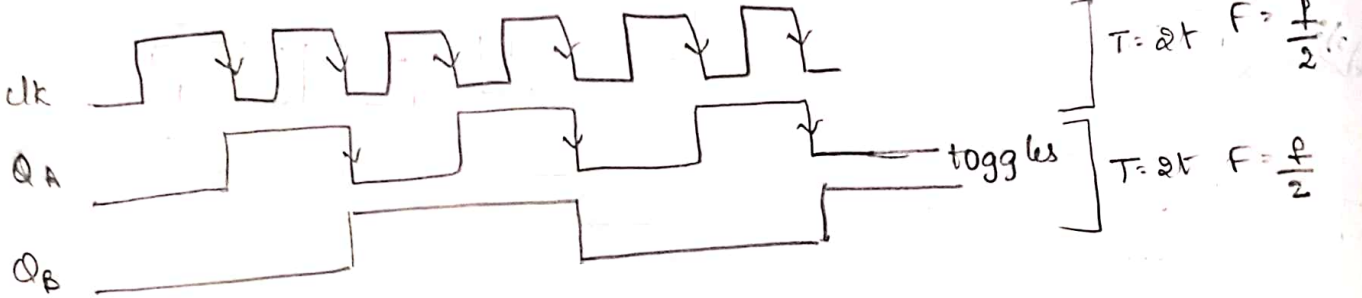
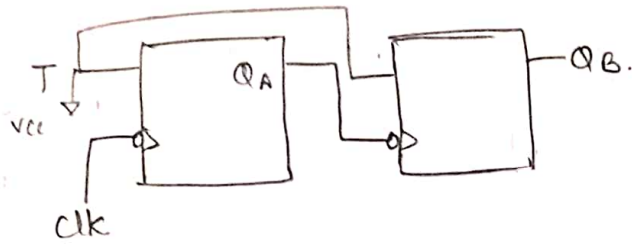
Applications: digital clock (0 \rightarrow 60) (0 \rightarrow 60 min) (0 \rightarrow 12 hrs)
 sec, min, hrs all have different frequencies.

Designing of a counter:

- \rightarrow we use T-flip flops.
- There are two types of counters:
 - (1) asynchronous counter (or) ripple counter
 - (2) synchronous counter.

Asynchronous counters:

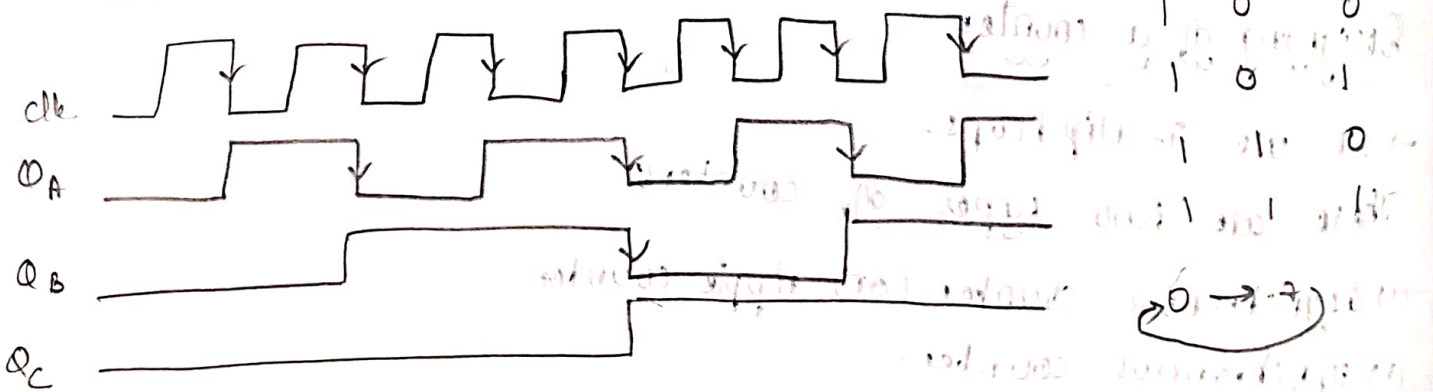
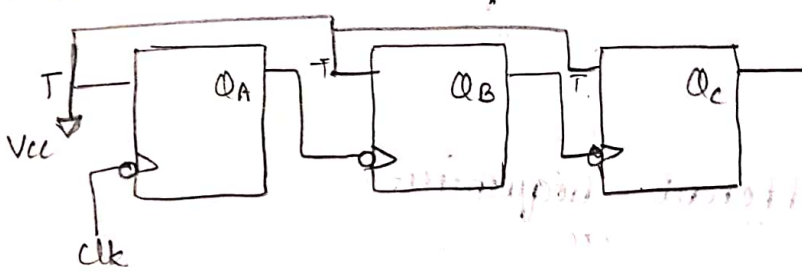
→ 8 flip-flops are used



QA	QB	after 1st clk pulse	after 2nd clk pulse	after 3rd clk pulse
0	0	0	1	0
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1
0	0	0	1	0
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

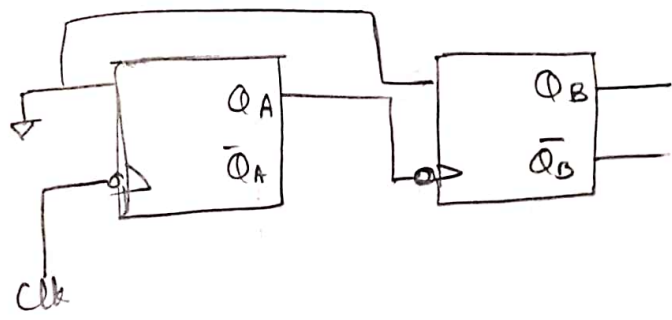
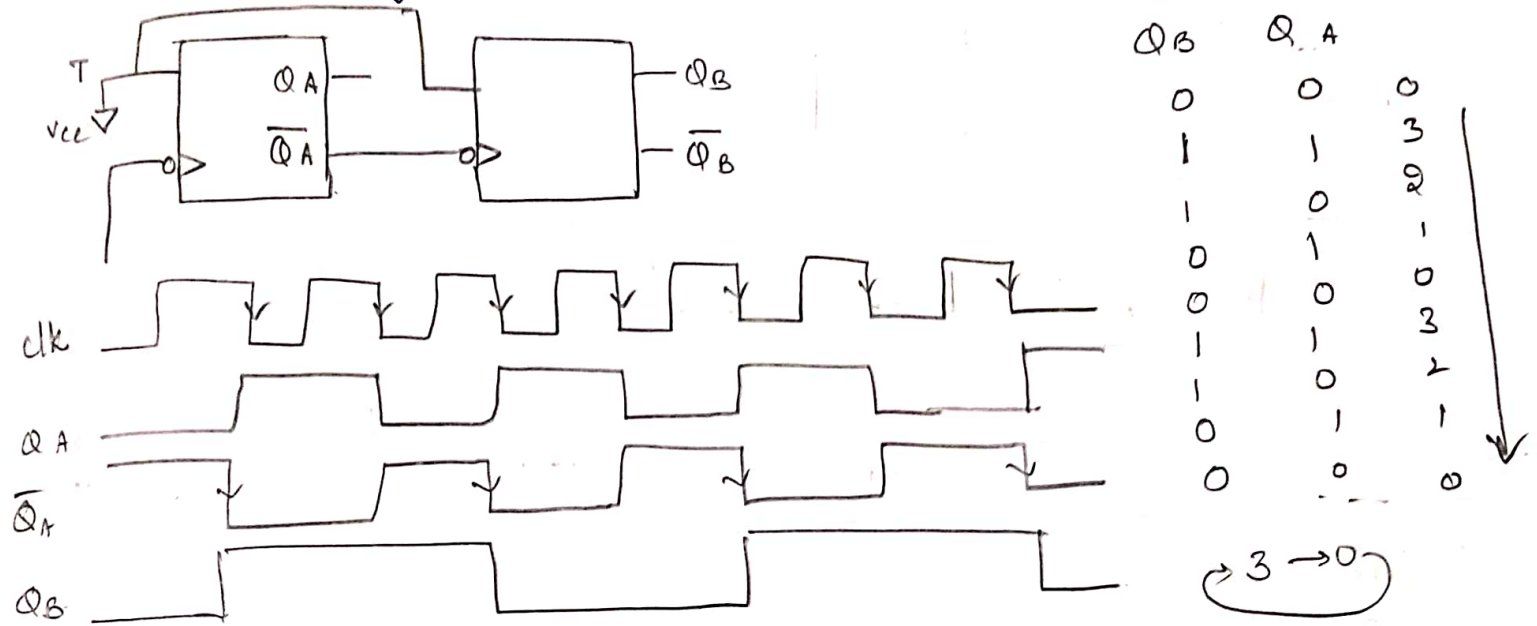
0 → 1 → 2 → 3

If we give the 1st two of 4 inputs in a 7-segment decoder as QB, QA. without changing inputs manually output changes from 1 to 3.



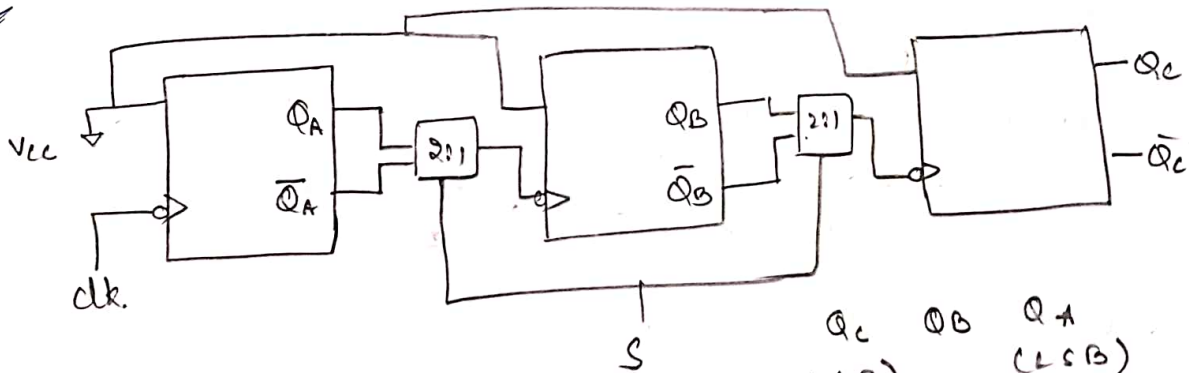
If no. of flipflops = n , count range = 0 to $2^n - 1$.

for down counting:



\bar{Q}_B	\bar{Q}_A	Count
1	1	3
1	0	2
0	1	1
0	0	0
1	1	3

(down counting)



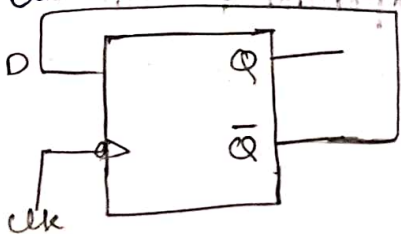
$S=0$; upcounting upto 7. ($Q_C Q_B Q_A$)

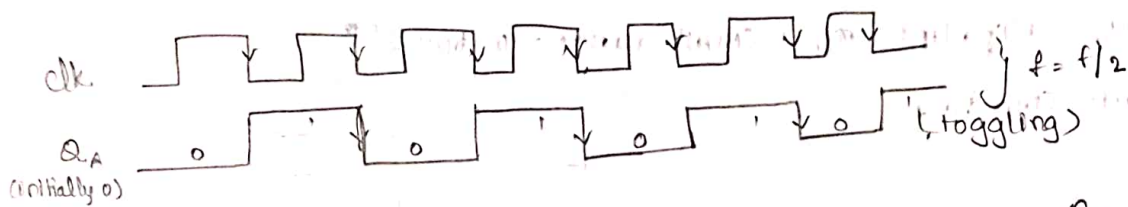
$S=1$; downcounting upto 7. ($\bar{Q}_C \bar{Q}_B \bar{Q}_A$)

$S=0$, down $\rightarrow (\bar{Q}_C \bar{Q}_B \bar{Q}_A)$

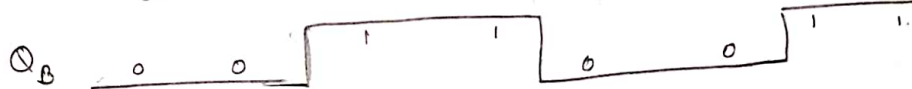
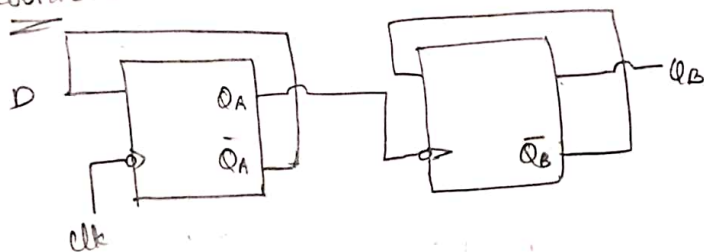
$S=1$, up $\rightarrow (Q_C Q_B Q_A)$

Counter using D-flip flop:





UpCounter:

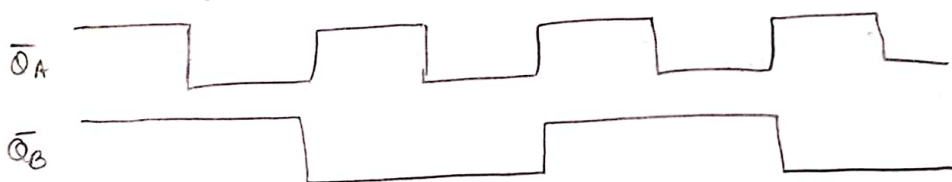


Q_B	Q_A
0	0
0	1
1	0
1	1
0	0
0	1

upcounting $0 \rightarrow 3$

Down counter:

Considering \bar{Q}_A \bar{Q}_B instead of Q_A Q_B .



\bar{Q}_B	\bar{Q}_A
1	1
1	0
0	1
0	0
1	1
1	0

down counting $3 \rightarrow 0$

Negative edge

up

$(J=K=1)$ $Q \rightarrow \text{clk of next flipflop}$

down

$\bar{Q} \rightarrow \text{clk of next flipflop}$
 $Q \rightarrow \text{clk of next flipflop}$

LEB/MSB

output from Q_A & Q_B
 Outputs from \bar{Q}_A & \bar{Q}_B

$(D=\bar{Q})$ $Q \rightarrow \text{clk of next flipflop}$

↓
 output from Q_A & Q_B

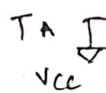
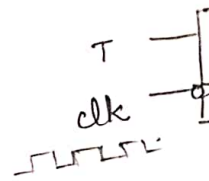
$Q \rightarrow \text{clk of next flipflop}$

↓
 Output from \bar{Q}_A & \bar{Q}_B

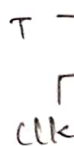
In asynchronous counters, each flipflop is dependent on the other flipflop. (\therefore clk of 1 flipflop depends on output of previous flipflop)

Synchronous

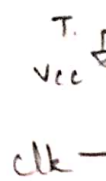
Complexity \rightarrow Using



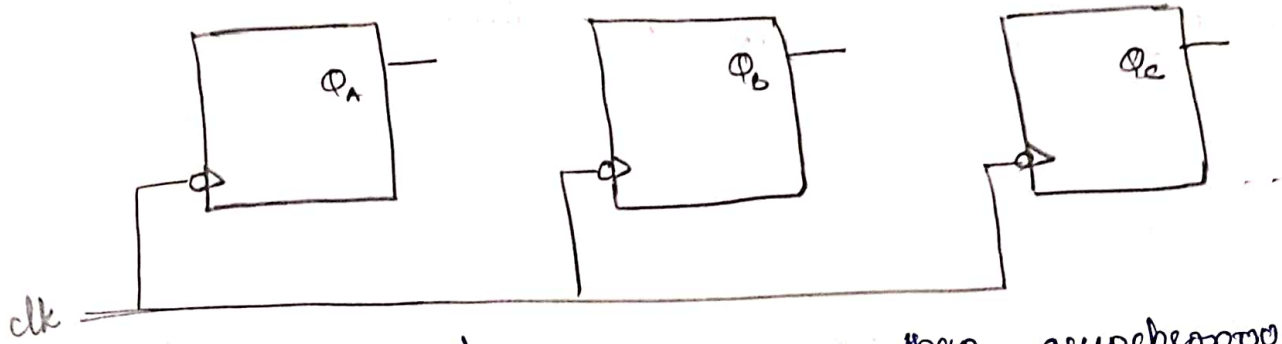
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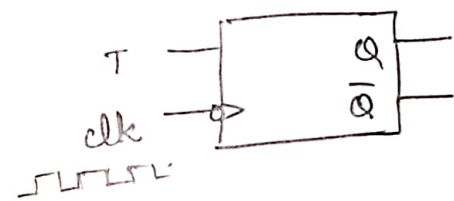
Both
 both
 be



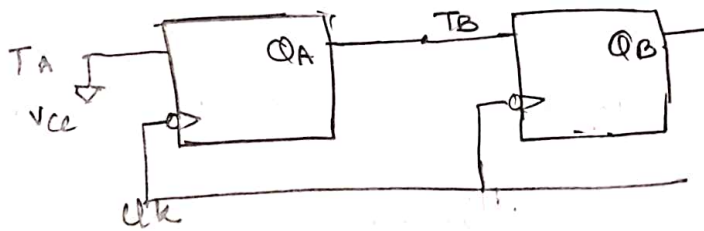
Synchronous counters (faster than asynchronous)
 (clock of ff donot depend on previous output)



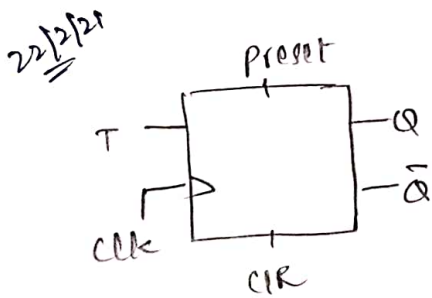
Complexity of synchronous is more than asynchronous.
 → Using T-flip-flop.



T	(PS) present state	(NS) next state
0	0	0
0	1	1
1	0	1
1	1	0

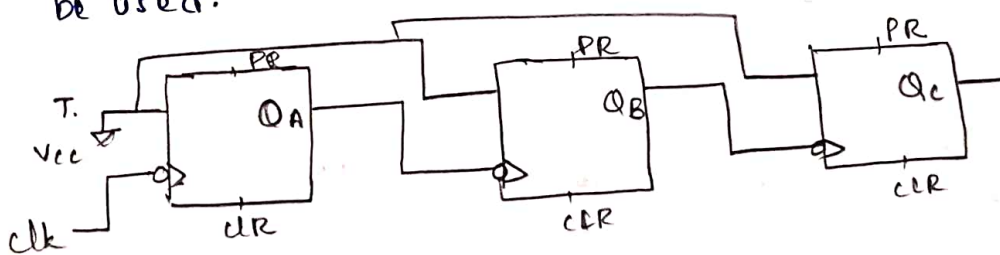


(PS)		(NS)		
QB	QA	QB	QA	TA
0	0 (0)	0	1 (1)	1
0	1 (1)	1	0 (2)	1
1	0 (2)	1	1 (3)	1
1	1 (3)	0	0 (0)	1



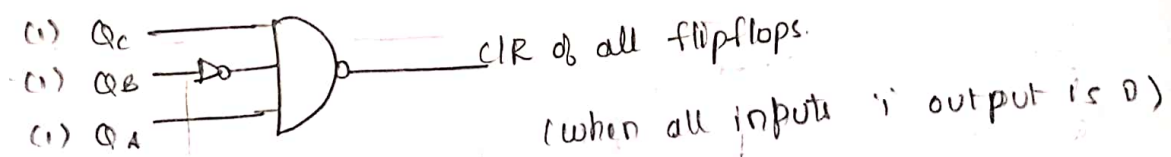
Preset = 0, Q = 1 (regardless of input values)
 Preset = 1, Q depends on input
 Clear = 0, Q = 0 (regardless of inputs)
 Clear = 1, Q depends on input

Both preset & clear are not given simultaneously (a)
 both cannot be used simultaneously, any one of them can be used.

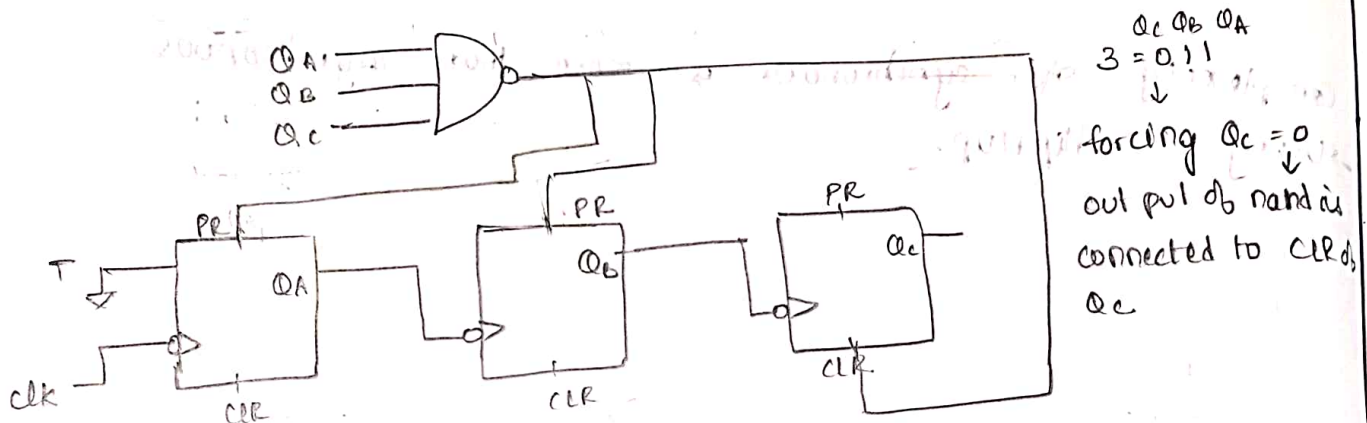


QA	QB	QC	
0	0	0	0
1	0	0	1
0	1	0	2
1	1	0	3
0	0	1	4
1	0	1	5
0	1	1	6
1	1	1	7

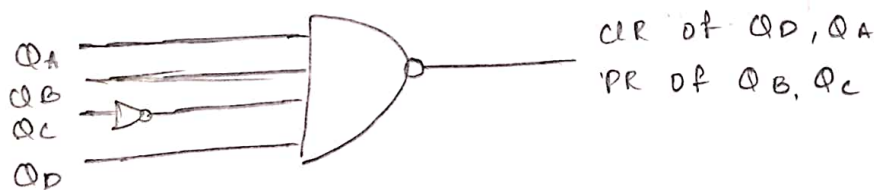
→ To count from 0 to 4, [5 = 101 → all to be 1's]



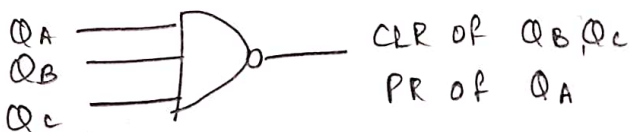
→ To count from 3 to 6 [7 = 111 → all to be 1's for input of 7 only]



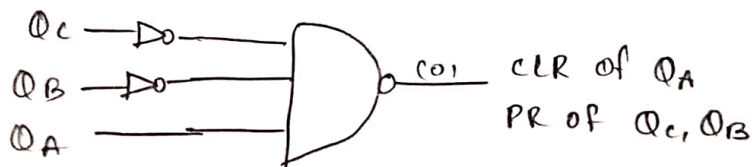
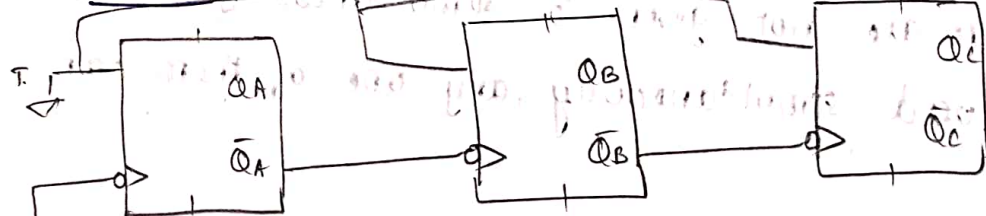
→ To count from 6 to 12 ($13 = 1101$) ($6 = 0110$)



→ To count from 1 to 6 [7 = 111] [1 = 001]



down counter:



Q_C	Q_B	Q_A
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

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Synchronous Counter:

PS [Q(t)]			NS [Q(t+1)]					
Q _C	Q _B	Q _A	Q _C	Q _B	Q _A	T _C	T _B	T _A
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	0	1	1	1	0	0	0	1

$$T_A = 1$$

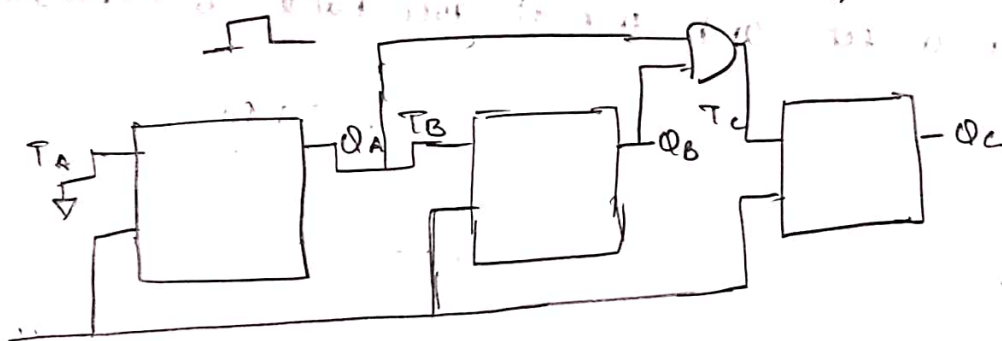
$$T_B = Q_A$$

$$T_C = Q_A \cdot Q_B$$

$$T_C = \bar{Q}_C Q_B Q_A + Q_C Q_B Q_A$$

$$T_C = Q_A Q_B (\bar{Q}_C + Q_C)$$

$$T_C = Q_A Q_B$$



Upcounter from 0 to 7

→ The circuit remains same for -ve or +ve edge flipflop.

Down counter:

PS [Q(t)]			NS [Q(t+1)]					
Q _C	Q _B	Q _A	Q _C	Q _B	Q _A	T _C	T _B	T _A
1	1	1	1	1	0	0	0	1
1	1	0	1	0	1	0	1	1
1	0	1	1	0	0	0	0	1
1	0	0	0	1	1	1	1	1
0	1	1	0	1	0	0	0	1
0	1	0	0	0	1	0	1	1
0	0	1	0	0	0	0	0	1
0	0	0	1	1	1	1	1	1

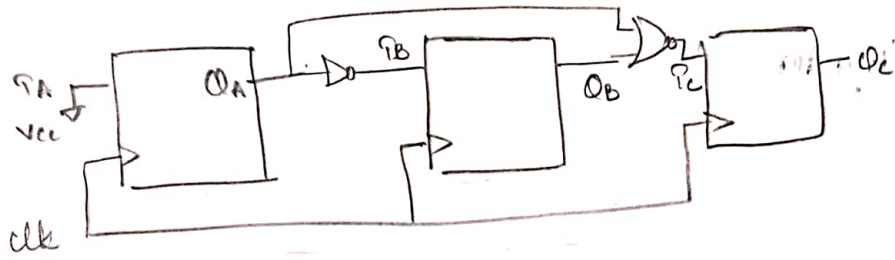
$$T_A = 1$$

$$T_B = \bar{Q}_A$$

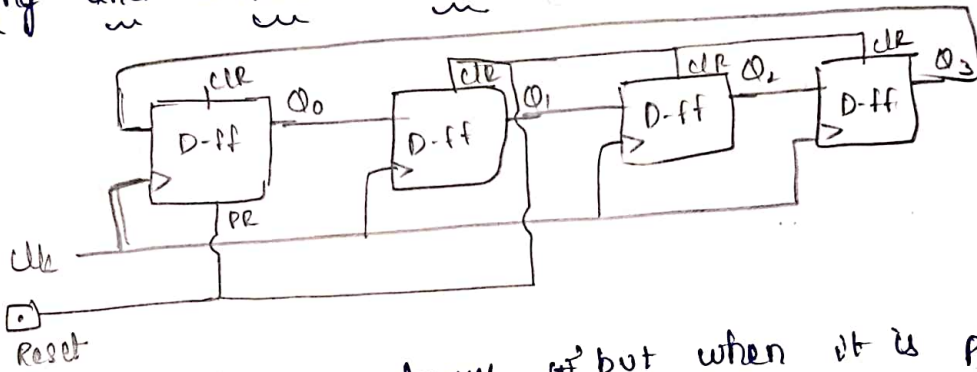
$$T_C = Q_C \bar{Q}_A \bar{Q}_B + \bar{Q}_C \bar{Q}_B \bar{Q}_A$$

$$= \bar{Q}_A \bar{Q}_B$$

$$= \overline{(Q_A + Q_B)}$$



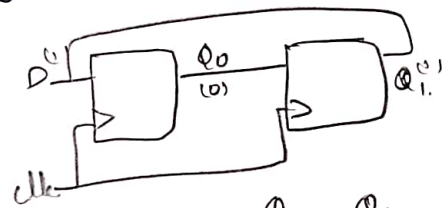
Ring and Johnson Counter:



ring counter.

Reset: It will be 1 always but when it is pushed once it becomes 0 for a sec and then it goes back to normal.

	Q_0	Q_1	Q_2	Q_3
Reset	1	0	0	0
clk	0	1	0	0
clk	0	0	1	0
clk	0	0	0	1
clk	1	0	0	0
clk	0	1	0	0



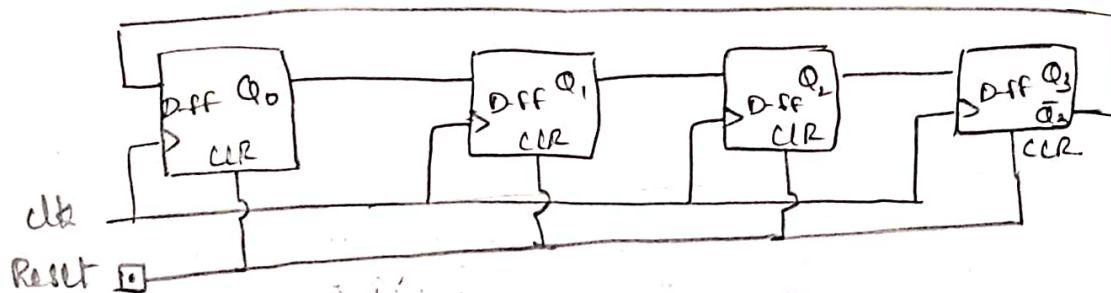
	Q_0	Q_1
clk	0	1
clk	1	0
clk	0	1
clk	1	0

→ for 5 flop flops:

	Q_0	Q_1	Q_2	Q_3	Q_4
clk	1	0	0	0	0
clk	0	1	0	0	0
clk	0	0	1	0	0
clk	0	0	0	1	0
clk	0	0	0	0	1
clk	1	0	0	0	0
clk	0	1	0	0	0

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Johnson Counter:



Reset	clk	Q ₀	Q ₁	Q ₂	Q ₃
	x	0	0	0	0
		1	0	0	0
		1	1	0	0
		1	1	1	0
		1	1	1	1
		0	1	1	1
		0	0	1	1
		0	0	0	1
		0	0	0	0

Synchronous Counters:

→ Divided into two types:

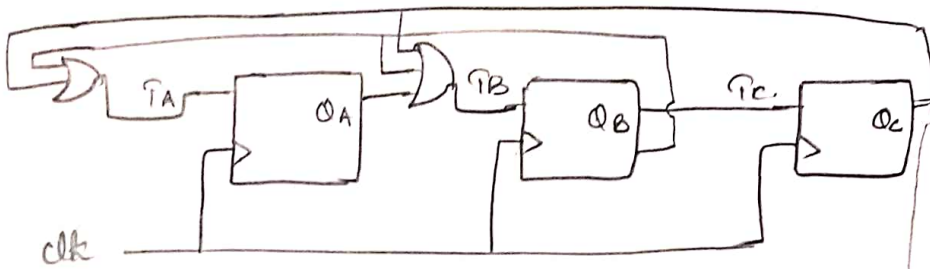
(1) Sequential (0-3, 0-7, ...)

(2) Non-sequential (skipping some no's in b/w).

eg: 0-3-4-5-7

0-1-2-4-7

(PS)			(NS)			(NS)		
Q _C	Q _B	Q _A	Q _C	Q _B	Q _A	Q _C	Q _B	Q _A
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	1	0
0	1	1	x	x	x	x	x	x
1	0	0	1	1	1	0	1	1
1	0	1	x	x	x	x	x	x
1	1	0	x	x	x	x	x	x
1	1	1	0	0	0	1	1	1



$T_B = \overline{Q_B} + Q_C + Q_A$

Q_C

$Q_B \backslash Q_A$	00	01	11	10
0	0	1	X	1
1	1	X	1	X

$T_B = Q_B + Q_C + Q_A$

Q_C

$Q_B \backslash Q_A$	00	01	11	10
0	0	0	X	1
1	0	X	1	X

$T_C = Q_B$

Q_C

$Q_B \backslash Q_A$	00	01	11	10
0	1	1	X	0
1	1	X	1	X

$T_A = \overline{Q_B} + Q_C$

not a 3-bit counter
because it has 4 states
and 3 bits can only have 8 states