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M-2

Mid Sem Exam

Set C-1

Q-1 $P(A) = 0.7$ $P(B) = 0.16$ $P(C) = 0.8$

(a) Probability that pet will be alive when you return

$= P(\bar{A}) \times P(\bar{C}) + P(\bar{B}) \times P(C)$

$= 0.3 \times 0.2 + 0.84 \times 0.8$

$= 0.06 + 0.672$

$= \boxed{0.732}$

(b) Probability that pet will be dead when you return

$= P(A) \times P(\bar{C}) + P(B) \times P(C)$

$= 0.7 \times 0.2 + 0.16 \times 0.8$

$= 0.14 + 0.128$

$= \boxed{0.268}$

(c) Probability that (pet dies) and your friend forgot to take care

$$= \frac{P(A)\,P(\bar{C})}{P(die)} = \frac{0.2 \times 0.7}{0.268} = \boxed{0.522}$$

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Q:2

let A be the prob. that India wins Test series

" B " " " " " " first game

(a) $P\left(\frac{A}{B}\right) = ?$ (b) $P\left(\frac{B}{A}\right) = ?$

Sol) Ind can win the test series if

~~WWWLL~~ if it win 3 games $\rightarrow$ $\left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 \times 10$ $\left(WWWLL \;\; \frac{\underline{5}}{\underline{3}\,\underline{2}}\right)$

if it wins 4 games $\rightarrow$ $\left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) \times 5$ $\left(WWWWL \;\; \frac{\underline{5}}{\underline{4}}\right)$

if it win 5 games $\rightarrow$ $\left(\frac{1}{3}\right)^5 \times 1$ (WWWWW)

$P(B) = \frac{1}{3}$, $P(A) = \frac{51}{3^5}$

$P(A \cap B) = \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 \times 6 + \left(\frac{1}{3}\right)^4 \times \frac{2}{3} \times 4 + \left(\frac{1}{3}\right)^5$

$= \frac{33}{3^5}$

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{33}{3^5}}{\frac{1}{3}}$

$= \frac{33}{3^4} = \boxed{\cancel{0.345}}\ 0.407$

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(b) $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{\frac{32}{3^5}}{\frac{50}{3^5}}$$

$$= \frac{33}{51} = \boxed{0.647}$$

Q-3

A: purchasing a CFL
B: purchasing a halogen
C: purchasing a LED

$P(A) = 0.23$ $\quad$ $P(A \cap C) = 0.1$

$P(B) = 0.31$ $\quad$ $P(A \cap B) = 0.13$

$P(C) = 0.26$ $\quad$ $P(B \cap C) = 0.12$

$P(A \cap B \cap C) = 0.06$

(a) P(None of the bulbs) = $1 - P(A \cup B \cup C)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = 0.23 + 0.31 + 0.26 - 0.13 - 0.12 - 0.1 + 0.06$$

$$P(A \cup B \cup C) = 0.51$$

$\therefore$ P(None of the bulbs) $= 1 - 0.51$
$= 0.49$

Q-5

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(b) P(exactly two bulb)

$= P(A \cup B \cup C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + 2P(A \cap B \cap C)$

$= 0.51 - 0.13 - 0.12 - 0.10 + 2(0.06)$

$= 0.51 - 0.35 + 0.12$

$= 0.28$

(c) P(2 or more bulbs)

$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

$= 0.13 + 0.12 + 0.1 - 0.12$

$= 0.23$

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Q-5

Let A represents the number of games played and B represents the number of games lost

If we won the fifth game, we continue playing untile we lose.

i.e.
After the 4th game we will play until we lose

$\therefore$ (A - 4) is the geometric random variable with parameter $1-p$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore N(A) = N(4 + (A-4)) = 4 + N(A-4)$$
$$= 4 + \frac{1}{1-p} = 4 + \frac{1}{1-1/2}$$
$$= 4+2 = 6$$

(b) $\therefore$ The expected no. of games you play = 6

(a) let C be the no of lost matches in first 4 matches C is binomial distribution with $n=4$ & $(1-p)$

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we know, $B = C+1$

$$N(B) = N(C+1)$$
$$= N(C) + 1 = 4(1-p)+1$$
$$= 4(1 - 1/2) + 1$$
$$= 3.$$

No. of games you lose = 3

Q8 $P(A) = 0.6$ $P(B) = 0.98$

$$P(+ve) = \frac{0.6 \times 0.98}{0.6 \times 0.98 + 0.4 \times 0.02}$$
$$= 0.986$$

$$P(-ve) = \frac{0.4 \times 0.98}{0.4 \times 0.98 + 0.6 \times 0.02}$$

$$= 0.97$$

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Q-6

(a) $\mu = 4, \sigma = 2$

$$P(x > 3) = 1 - P(x \le 3)$$
$$= 1 - P\left(\frac{x-\mu}{\sigma} \le \frac{3-4}{2}\right)$$
$$= 1 - P\left(\frac{x-\mu}{\sigma} < -\frac{1}{2}\right)$$
$$= 1 - \phi(-1/2)$$
$$= 1 - 0.308$$
$$= 0.692$$

(b) $P(3 < x < 5)$

$$P(3 < x < 5) = P\left(\frac{3-4}{2} < \frac{x-4}{2} < \frac{5-4}{2}\right)$$
$$= P\left(\frac{-1}{2} < \frac{x-4}{2} < \frac{1}{2}\right)$$
$$= \phi\left(\frac{1}{2}\right) - \phi(-1/2)$$
$$= 0.691 - 0.308$$
$$= 0.383$$

(c) $$P(x < 3) = P\left(\frac{x-4}{2} < \frac{3-4}{2}\right)$$
$$= \phi(-1/2) = 0.308$$

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(d) $P(X>5)$

$= 1 - P(X<5)$

$= 1 - \phi(1/2)$

$= 1 - 0.691$

$= 0.309$

(e) $P(X<5) = P\left(\frac{X-4}{2} < \frac{5-4}{2}\right)$

$= \phi(1/2) = 0.691$

7(b) $m_X(t) = (1-2t)^{-1/2} \quad : t < 1/2$

Differentiating for first time and so on

$m'_X(t) = -\frac{1}{2}(1-2t)^{-3/2} \times (-2)$

$= (1-2t)^{-3/2}$

$m''_X(t) = -\frac{3}{2}(1-2t)^{-5/2} \times (-2)$

$= 3(1-2t)^{-5/2}$

$m'''_X(t) = 3 \times -5/2 \, (1-2t)^{-7/2} \times (-2)$

$= 15(1-2t)^{-7/2}$

$m''''_X(t) = 15 \times (-7/2)(1-2t)^{-9/2}(-2)$

$= 105(1-2t)^{-9/2}$

Second moment $= 3(1-0)^{-5/2} = 3$

fourth moment $= 105(1-0)^{-9/2} = 105$

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Q-9

$E[x] = 60$

Using Markov

$P(x \geq a) = \frac{E[x]}{a}$

a) $P(x \geq 80) = \frac{60}{80} = 3/4$

b) $\sigma = 4$

Using Chebyshev formula

$\cancel{P(100)}\ P\left(\frac{x-\mu}{\ }\right)$

$P(|x - \mu| \geq k\sigma) \leq 1/k^2$

so $c\,x < 70 = |x - 60| \leq 10$

$P(|x - \mu| \leq 10) \geq 1 - 1/k^2$

$k\sigma = 10$

$k(4) = 10$

$k = 5/2$

$P(|x - \mu| \leq 10) \geq 1 - 1/(5/2)^2$

$\geq 1 - 4/25$

$P(|x - \mu| \leq 10) \geq 21/25$

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(a)

$$P(K) = \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} \times \frac{3}{12} = \frac{1}{91}$$

(b) $$P(M) = \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \frac{12}{1365}$$

(c) P(None of the four are keyboards)

$$= 1 - P(K)$$
$$= 1 - 2/91$$
$$= \frac{89}{91}$$

(d) P(different)

$$= 6 \times \frac{6}{15} \times \frac{7}{14} \times \frac{5}{13}$$
$$= \frac{24}{91}$$