Signal & Systems

(1)

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Q-1

(a) $y(t) = \cos(x(t)) + x(t/2)$

Delaying in input

$y_1(t) = \cos(x(t-T_0)) - x(t/2 - T_0)$

Delaying in output

$y_2(t-T_0) = \cos(x(t-T_0)) - x(t/2 - T_0/2)$

Since $y_1 \neq y_2$

This is Time variant

(b) $$y(n) = \frac{x(n)}{2x(n)-1}$$

$$y_1(n) = \frac{x_1(n)}{2x_1(n)-1}, \quad y_2(n) = \frac{x_2(n)}{2x_2(n)-1}$$

let

$x_3(n) = ax_1(n) + bx_2(n)$

$$y_3(n) = \frac{x_3(n)}{2x_3(n)-1}$$

$$y_3(n) = \frac{ax_1(n) - bx_2(n)}{2ax_1(n) + 2bx_2(n) - 1}$$

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(b) continue $y_3(n) \neq a y_1(n) + b y_2(n)$

Hence non-linear

(c) $y(n) = (x(n)+1)^2 + x(n/2)$

$x(n/2)$ depends upon future and past values of n

Hence it is memory

for eg $n = 6$

$y(6) = (x(6)+1)^2 + x(3)$

$x(3) \downarrow$ Past value

(d) ~~$y(t) = x(t)$~~

$y(t) = x(1/t)$

$x(1/t)$ depends on the future value

of $-1<t<1$

Hence, it is non-Causal

for eg $t = 1/2$

$y(1/2) = x(2)$

$x(2) \downarrow$ future value

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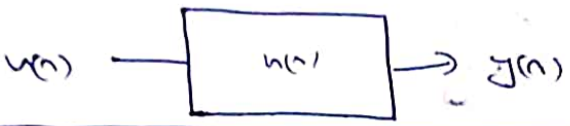
Q-2

$$h[n] = \begin{cases} 3, & n=0 \\ 1, & n=2,3 \\ 0, & \text{else} \end{cases}$$

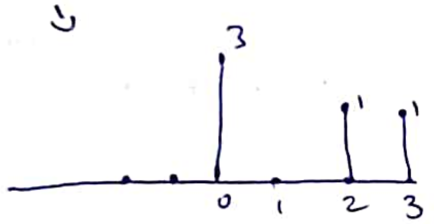
(3)

$$x(n) = \left(\frac{1}{10}\right)^n u(n) \quad -1 \le n \le 5$$

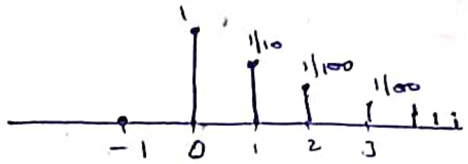


$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

h(k)



x(k)

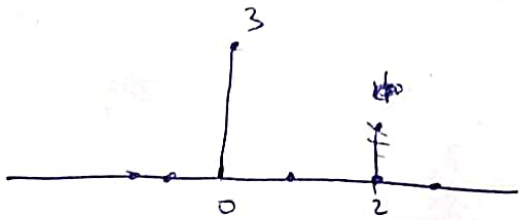


at n=0

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(0-k)$$

h(-k)

multiply ⇒



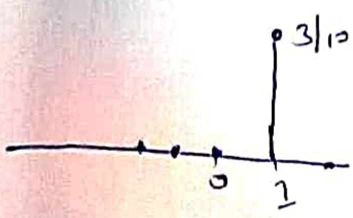
$\boxed{y(0) = 3}$

at n=1

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k) = \boxed{3/10}$$

h(1-k)

multiply ⇒



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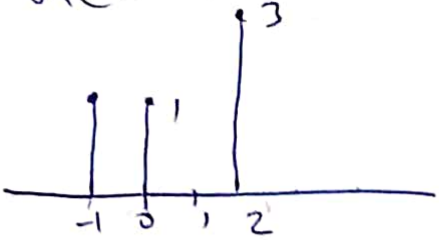
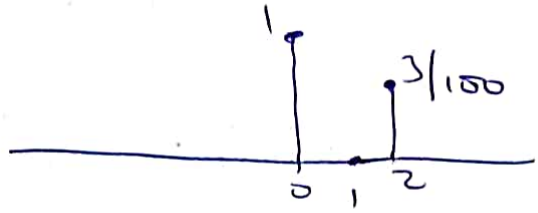
at $n=2$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, y(2-k)$$

$h(2-k)$



$\Rightarrow$

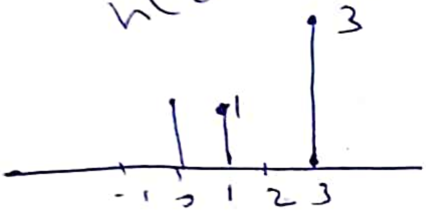


$$\boxed{y(2) = 1 + \frac{3}{100}}$$

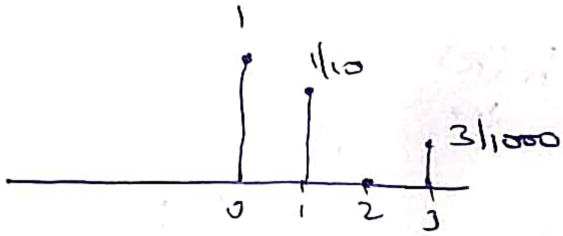
at $n=3$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, y(3-k)$$

$h(3-k)$



$\Rightarrow$

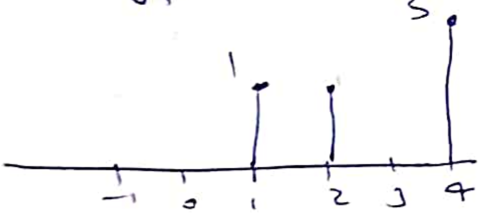


$$\boxed{y(3) = 1 + \frac{1}{10} + \frac{3}{1000}}$$

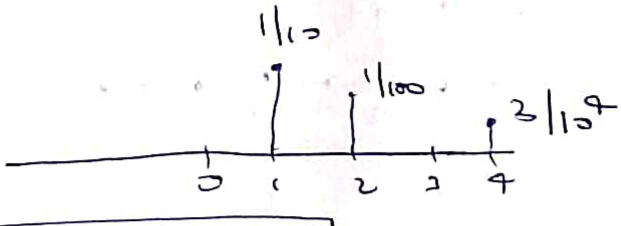
at $n=4$

$$y(4) = \sum_{k=-\infty}^{\infty} u(k)\, y(4-k)$$

$h(4-k)$



$\Rightarrow$



$$\boxed{y(4) = \frac{1}{10} + \frac{1}{100} + \frac{3}{10^4}}$$

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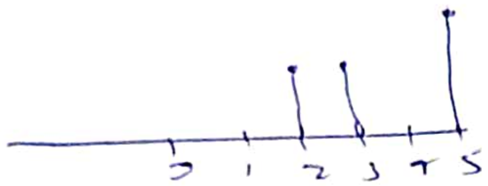
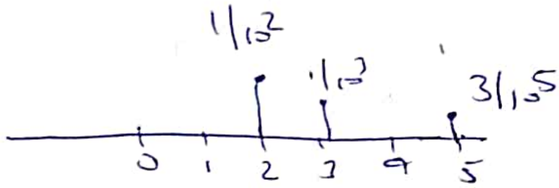
at $n=5$

$$y(5) = \sum_{k=-\infty}^{\infty} h(k)\,y(5-k)$$

$h(5-k)$



$\Rightarrow$



$$\boxed{y(5) = \frac{1}{10^2} + \frac{1}{10^3} + \frac{3}{10^5}}$$

at $n=-1$

$$y(-1) = \sum_{k=-\infty}^{\infty} h(k)\,y(-1-k)$$

$h(-1-k)$

$\Rightarrow$

$$\boxed{y(-1) = 0}$$

Similarly

at $n = 6$ $\quad y(6) = \frac{1}{10^3} + \frac{1}{10^4}$

at $n = 7$ $\quad y(7) = \frac{1}{10^4} + \frac{1}{10^5}$

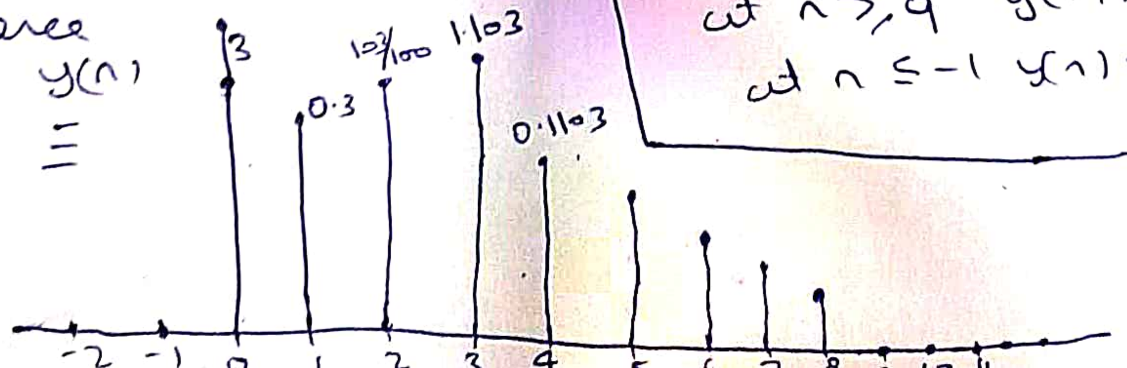
at $n = 8$ $\quad y(8) = \frac{1}{10^5}$

at $n = 9$ $\quad y(9) = 0$

at $n \geq 9$ $\quad y(n) = 0$

at $n \leq -1$ $\quad y(n) = 0$

Hence $y(n) =$



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$y(n) =$

$$\left\{ \ldots, 0, \underset{\uparrow}{3}, 0.3, 1.03, 1.103, 0.1103, 0.01103, 0.0011, 0.00011, 0.00001, 0, \ldots \right\}$$