Total Probability Theorem

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Q-1 If 60% of boys opted for maths and 40% of girls opted for maths, what is the probability that maths is chosen if half of the population is girls.

Sol →

$$P(\text{maths}) = P(\text{boys})\,P(\text{maths/boys}) + P(\text{girls})\,P(\text{maths/girls})$$

$$= \frac{1}{2}\left(\frac{60}{100}\right) + \frac{1}{2}\left(\frac{40}{100}\right)$$

$$= \frac{1}{2}$$

Q-2 A problem is given to 5 students P, Q, R, S, T, if the probability of solving the problem individually is 1/2, 1/3, 2/3, 1/5, 1/6 respectively, the find the probability that the problem is solved.

Sol →

$$P(A) = P(G)\,P(A/G) + \cdots + P(G')(P\,A/G')$$

$$= \frac{1}{5}\frac{1}{2} + \frac{1}{5}\frac{1}{3} + \frac{1}{5}\frac{2}{3} + \frac{1}{5}\frac{1}{5} + \frac{1}{5}\frac{1}{6}$$

$$= 0.37$$

Q-3 Write the Total probability theorem for n terms and write the theorem where total probability theorem is used?

$$P(A) = P(G_1)P(A/G_1) + P(G_2)P(A/G_2) + P(G_3)P(A/G_3) \ldots$$
$$\ldots + P(G_n)P(A/G_n)$$

It is used in Bayes's theorem.

$$P(E_1/A) = \frac{P(A/G_1) \cdot P(G_1)}{P(A/G_1) \cdot P(G_1) + P(A/G_2) \cdot P(G_2)}$$

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~~Total Probability~~

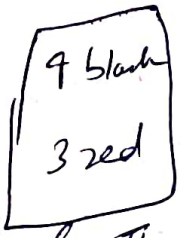
Bayes Theorem

Q - Bag I contains 4 red and 3 black balls
Bag II contains 3 red and 4 black balls.
One ball is drawn and it is found to be red
Find the probability of it was drawn from Bag II.

Ans

Bag I

$P(\text{Bag I}) = 1/2$



Bag II

$P(\text{bag II}) = 1/2$

Let A be the event of drawn a red ball

$P(A|I) = 4/7$

$P(A|II) = 3/7$

$$P(II|A) = \frac{P(II) \cdot P(A|II)}{P(I) \cdot P(A|I) + P(II)\, P(A|II)}$$

$$= \frac{1/2 \cdot 3/7}{1/2\ 4/7 + 1/2\ 3/7}$$

$$P(II|A) = \frac{3}{7}$$

(2) At a certain university 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall what is probability that the student is a women.

Sol →

$$P(M) = 2/5 \qquad P(F) = 3/5$$

$$P\left(\frac{Tall}{M}\right) = 4/100 \qquad P(Tall/F) = 1/100$$

$$P(F/Tall) = \frac{P(T/F)\,P(F)}{P(T/F)\,P(F) + P(T/M)\,P(M)}$$

$$= \frac{\frac{1}{100} \times \frac{3}{5}}{\frac{1}{100} \times \frac{3}{5} + \frac{4}{100} \times \frac{2}{5}} = \frac{3}{11}$$

(3) A Bg-1 has 4 black and 5 white balls bag 2 has 3 black and 2 white balls If a ball is chosen at random is black what is the probability the ball is from bag 1.

A3 $$P = \frac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4}} = \frac{1}{11}$$

Q-4 A factory produces bolts using machine A, B and C of the total output machine A is responsible for 25%, machine B for 35% and machine C for the rest.

Defective bolts : machine A → 5%.
" B → 4%.
" C → 2%.

A bolt is chosen at random is found to be defective what's the probability it came from machine A.

Ans.

$$P = \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} + \frac{4}{100} + \frac{4}{100} \times \frac{2}{100}}$$

$$P = \frac{25}{62}$$

$$P = 0.362$$

Q-5 Three companies A, B & C supply 20%, 45% and 45% of the notebooks to a school. Past experience shows that 5%, 4% and 2% of the notebooks made by the companies are defective. If a notebook is found defective, what is the probability that it was supplied by A

Ans

$$\frac{\frac{20}{100} \times \frac{5}{100}}{\frac{20}{100} \times \frac{5}{100} + \frac{45}{100} \times \frac{4}{100} + \frac{45}{100} \times \frac{2}{100}}$$

$$P = \frac{5}{17}$$

$$P = 0.294$$

Q-6 Box A : 4 red and 5 blue coins

Box B : 6 red and 3 blue coins

If a blue coin is picked at random, what is the probability that it's from A.

Ans

$$P = \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{5}{9} + \frac{1}{2} \times \frac{3}{9}}$$

$$P = 5/8$$

Q-7 Urn B1 : 2 white and 3 black chips
Urn B2 : 3 white and 4 black chips
A chip is randomly drawn and found to be black find probability it's from urn B1

Ans $$P = \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{4}{7}}$$

$$P = \frac{21}{41}$$