Assignment 8

Group Members

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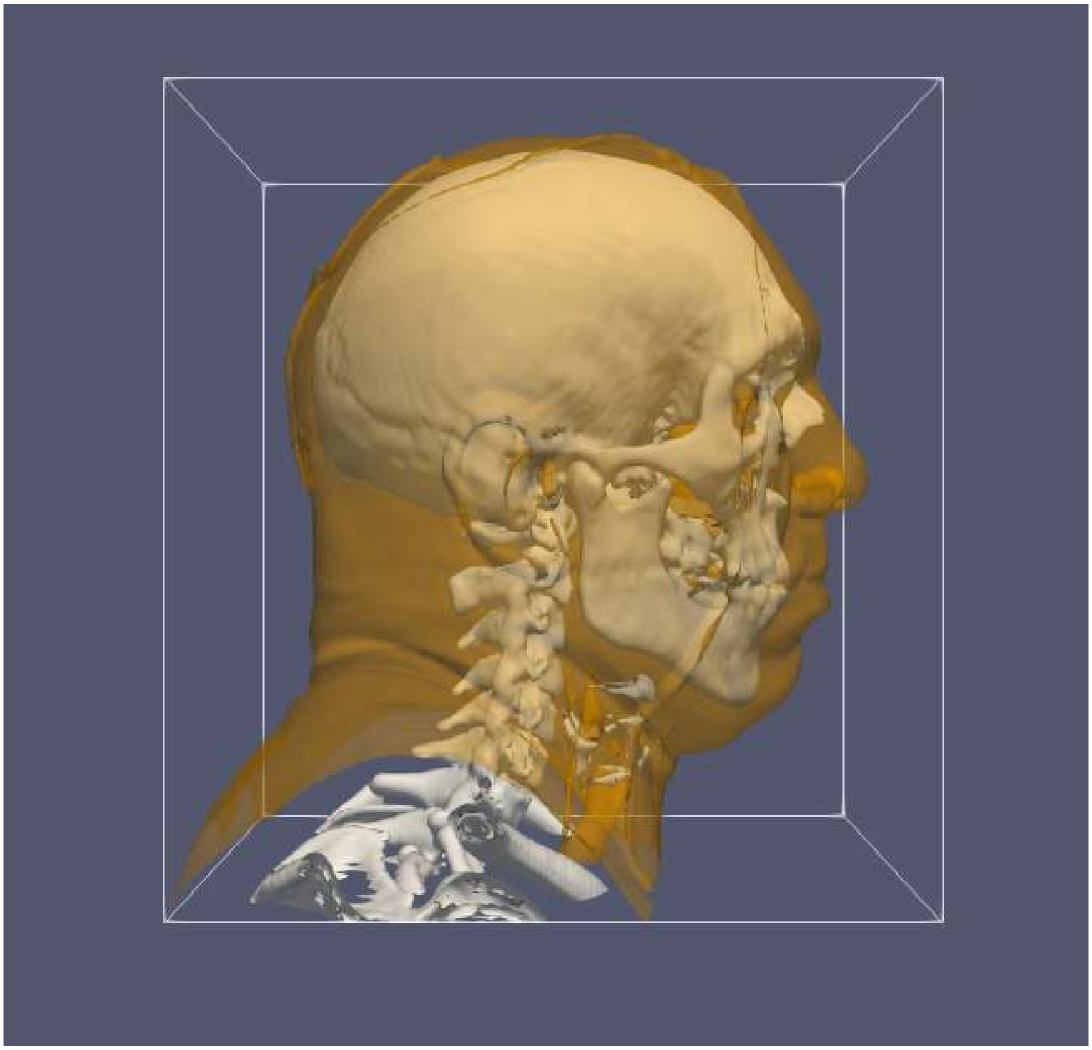
Ex.1: Introduction to ParaView

a)

The grid type of this dataset is Uniform Rectilinear Grid. The type of this dataset is Image and its range is 0 to 269.473 (delta: 269.473), 0 to 269.473 (delta: 269.473) and 0 to 293 (delta: 293) with three axis respectively.

b)

c)



Ex.2: Grids

a)

The numbers needed to store and fully specify a scalar field on a Cartesian grid of size 10 × 10 × 10 (including the data itself) are 1005. These numbers represent: Number of grid points (10 × 10 × 10) 1000, 1 integer to represent the number of dimensions, 3 integers to present axis lenghts, 1 float to present the uniform spacing between grid points. Therefore, a total of 1000 + 1 + 3 + 1 = 1005 numbers need to be stored.

b)

The number of cells in the grid can be calculated by the formula given in slides: Number of cells = (Nx - 1) × (Ny - 1) × (Nz - 1). We know that the grid size is 10 × 10 × 10, the number of cells would be: Number of cells = (10 - 1) × (10 - 1) × (10 - 1) = 9 × 9 × 9 = 729 cells.

c)

For the same grid is to be specified in more general form as a rectilinear grid, 3 1D arrays are required to store the Spacing explicitly. These arrays can be represented as: x_coord[10-1], y_coord[10-1], z_coord[10-1] or x_coord[9], y_coord[9], z_coord[9]. So, 9 numbers for each array which makes 27 numbers in total required to add additionally.

d)

To determine if a rectilinear grid can be represented as a uniform grid, we need to examine whether the spacing between grid points along each axis is uniform or constant. If the spacing remains the same for all grid points along each axis, then the rectilinear grid can be considered as a uniform grid.

Ex.5: Finite Differences

a)

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

re-write it : $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

Taylor expension of $f(x+h)$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \cdots$$

Substituting this into formula

$$f'(x) \approx \frac{f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \cdots - f(x)}{h}$$

$$f'(x) \approx \frac{\cancel{f(x)} + \cancel{h}f'(x) - \cancel{f(x)}}{\cancel{h}} + \frac{\frac{h^2}{2} f''(x) + \cdots}{h}$$

$$f'(x) = f'(x) + O(h)$$

error decreasing linearly as stepsize 'h' decreases.

b)

$$f_c'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}}$$

re-write it : $f'(x) \simeq \dfrac{f(x+h) - f(x-h)}{2h}$

Taylor expension of $f(x+h)$ and $f(x-h)$ around x:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) + \cdots$$

subtract expensions

$$f(x+h) - f(x-h) = 2hf'(x) + 2\frac{h^3}{3!}f'''(x) + \cdots$$

Substituting into formula

$$f'(x) \approx \frac{\cancel{2}\cancel{h}f'(x) + 2\frac{h^{\cancel{3}}}{3!}f'''(x) + \cdots}{\cancel{2}\cancel{h}}$$

$$f'(x) \approx f'(x) + \frac{h^2}{3!}f'''(x) + \cdots$$

$$f'(x) \simeq f'(x) + O(h^2)$$

error term in central finite difference scheme is of order $O(h^2)$, means it decreases quadratically.