

HW 1 Math 421 Winter 2021, due Monday January 25

Problem 1.

- How many north-east grid paths are there from $(1, 2)$ to $(4, 5)$?
- How many north-east grid paths are there from $(0, 0)$ to $(6, 4)$, that don't go through $(1, 2)$ and $(3, 3)$? (Here, a path that goes through $(1, 2)$ but not $(3, 3)$ is allowed, A north-east path that goes through $(3, 3)$ but not $(1, 2)$ is also allowed. Only the paths that go through both $(1, 2)$ and $(3, 3)$ are not allowed.)

Problem 2 Suppose that a traveler is planning a visit to Canada. They would like to spend exactly 3 days in each of the following cities (consecutive days are allowed): Calgary, Edmonton, Toronto, Vancouver, Ottawa, and Montreal. They must also end their trip at the same city that they started at. How many different schedules are possible? Example of a schedule: Calgary day 1, 3 days in Edmonton, 1 day in Calgary, followed by 3 days in Toronto, etc., and end the last day in Calgary. So a total of $3 * 6 = 18$ days are spent in a particular schedule.

Problem 3 How many 4 digit positive integers are divisible by 10 and have distinct digits?

Problem 4 Suppose that there are 13 students (all distinct) taking a combinatorics final exam. The students all get different scores. Among the students are Jim, Sarah, and Zeratul. It is known that Jim got a lower score than Sarah, and Sarah got a lower score than Zeratul. Given this extra information, how many possible rankings of students by their final exam score are possible? (Scores are listed from lowest to highest)

Problem 5 Suppose that there are 8 people going to a dinner. The 8 people will be seated at 2 circular tables so that 4 people are assigned to each table. Two seating arrangements s_1, s_2 are equal if for each person p , the left neighbor of p in s_1 is equal to the left neighbor of p in s_2 . How many different seating arrangements are possible?

Example: if there are 2 people and 2 circular tables, then there is only 1 way to assign the 2 people evenly to two tables (understand why it is not equal to 2 under our rules). If there are 4 people, then there are 3 possible seating arrangements.