Assignment 2

5 October, 2022

Due date: 25 October, by 13:20 in class

Instructions

- If you write your solutions by hand, please ensure your handwriting is legible. We may subtract marks for hard-to-read solutions.
- Submit a hardcopy of your assignment in class.
- **After 13:20, an assignment will be accepted only with an MSAF.**
- Name your MATLAB files **exactly** as specified.
- Do not submit zipped files to Avenue. We will **ignore any compressed file** containing your files.
- Submit **only what is required**.

Problem 1 [4 points] Write the body of the MATLAB function

```
function  B = inverse(A)
```

For an $n \times n$ nonsingular matrix A, it should compute the inverse A^{-1} of A. You can use only the `lu` function of MATLAB. Your `inverse` function must compute A^{-1} in $O(n^3)$ operations.

Then run the script

```
clear all;
n = 700;
m = 4;
for i = 1:m
    A = rand(n,n);
    tic;
    B = inverse(A);
    time(i) = toc;
    Ainv = inv(A);
    error(i) = norm(B-Ainv,inf)/norm(Ainv,inf);
    N(i) = n;
    n = 2*n;
    if (i==1)
        fprintf('n= %4d  time=% .1e                error=%.2e\n', ...
            N(i), time(i), error(i));
    else
        fprintf('n= %4d  time=% .1e  ratio=%5.1f   error=%.2e\n', ...
            N(i), time(i), time(i)/time(i-1),error(i));
    end
end
```

Submit

- hardcopy: `inverse.m` and the output of this script
- Avenue: `inverse.m`

Problem 2 [6 points] Consider the system $Ax = b$, where

$$A = \begin{bmatrix} 0.1 & 0.3 & 0.7 \\ 0.3 & 0.6 & 0.9 \\ 0.6 & 1.5 & 3 \end{bmatrix}$$

and $b = [1.4, 1.8, 6]^T$.

a. [2 points] Show that A is singular.

b. [2 points] If we were to use Gaussian elimination with partial pivoting to solve this system using exact arithmetic, show where the process fails.

c. [1 point] Although A is singular, `cond(A)` does not return `Inf`. Why?

d. [1 point] If we solve $Ax = b$ in Matlab using `A\b`, how accurate is the computed x?

Problem 3 [4 points] The gamma function has the following values: $\Gamma(0.5) = \sqrt{\pi}$, $\Gamma(0.75) = \sqrt{\pi/2}$, $\Gamma(1) = 1$.

a. [2 points] Derive the quadratic interpolation polynomial interpolating these values.

b. [2 points] From this polynomial, determine the value of x for which $\Gamma(x) = 1.5$.

Problem 4 [4 points] Given the three data points $(-1, 1)$, $(0, 0)$, $(1, 1)$, determine the interpolating polynomial of degree two using :

a. [1 point] monomial basis

b. [1 point] Lagrange basis

c. [1 point] Newton basis

[1 point] Show that the three representations give the same polynomial.

Problem 5 [8 points] Consider $f(x) = \sqrt{x}$ on $[0, 2]$.

a. [3 points] Interpolate $f(x)$ at 15 evenly spaced points $x_0 = 0 < x_1 < \cdots < 2 = x_{14}$ in $[0, 2]$. You can use the `polyfit` function; see also `linspace`. Denote the resulting interpolation polynomial by $p(x)$.

   - Plot on the same plot $f(x)$ and $p(x)$ versus x at 100 evenly spaced points in $[0, 2]$.
   - Plot $|f(x) - p(x)|$ versus x at these points; use `semilogy`.
   - Why is the error largest when x is close to 0?

b. [2 points] Instead of `polyfit` use `spline` and produce the plots in a.

c. [2 points] Repeat a. with 15 Chebyshev points.

d. [1 point] Explain the differences in the error plots in a. and c.

Submit the 6 plots in your hardcopy.

Problem 6 [2 points] Suppose you interpolate e^{x^2} at $x = 0, 0.5, 1$ by a polynomial of degree 2. Derive a bound for the error of this interpolation.