Advanced Design Methods (MDO, KBE) AE4233

MDO Practical session 2

Dr.ir. G. La Rocca

Dr.ir. H.A. Visser

Ir. D. Steenhuizen

Dr. A. Elham

Flight Performance & Propulsion Group (FPP)

- *Structuring a Multi-Disciplinary Optimization problem*

Overview

- Example Problem

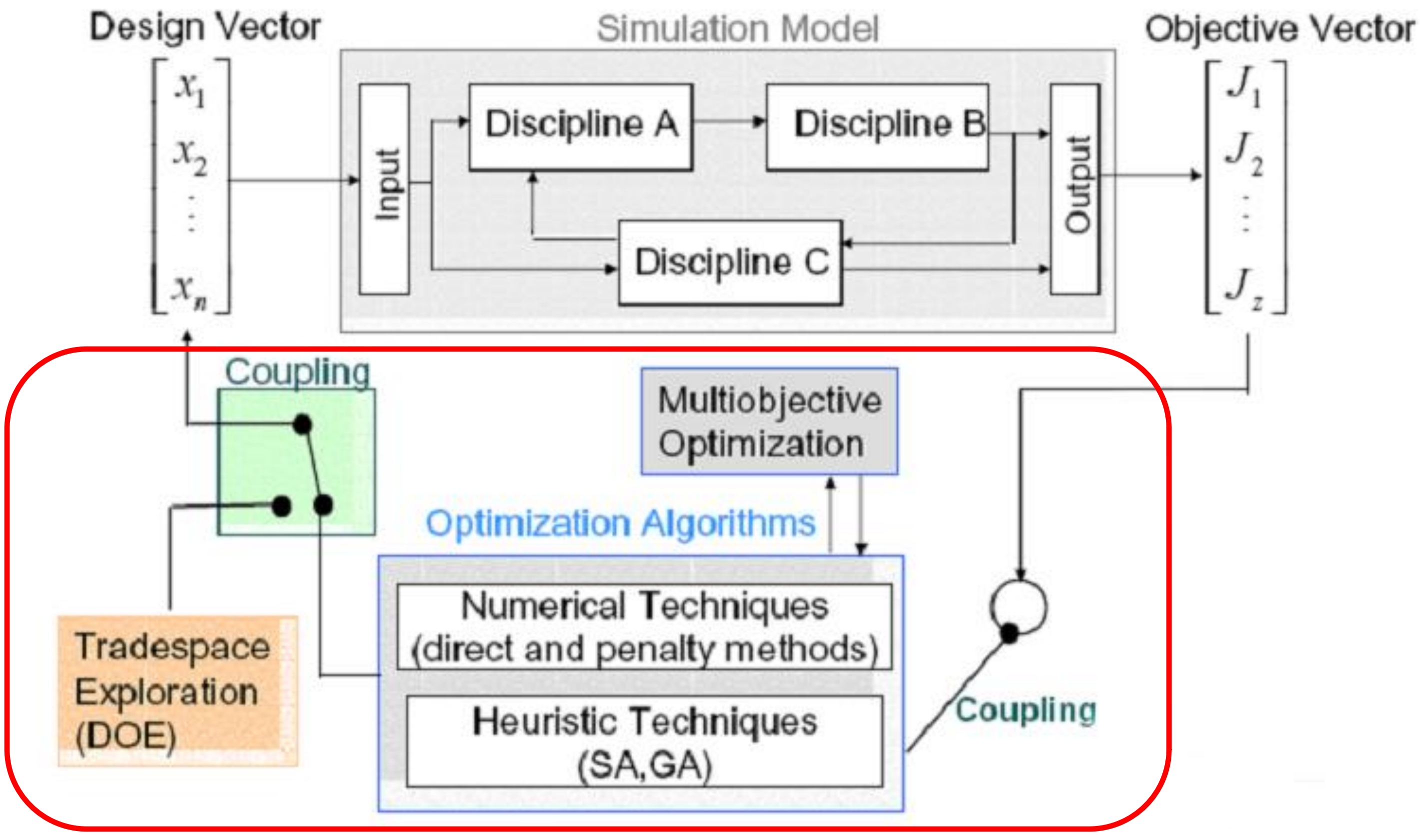
- Multi-Discipline Feasible scheme
- Individual Discipline Feasible scheme

Overview

- ***Example Problem***

- Multi-Discipline Feasible scheme
- Individual Discipline Feasible scheme

Optimization algorithm



Example problem

$$\min_{\mathbf{x}} J = \boxed{x_2^2 + x_3 + y_1 + e^{-y_2}}$$ Discipline 3; objective

$$\boxed{y_1 = x_1^2 + x_2 + x_3 - 0.2 y_2}$$ Discipline 1

$$\boxed{y_2 = \sqrt{y_1} + x_1 + x_3}$$ Discipline 2

$$s.t.$$

$$y_1 / 3.16 - 1 \geq 0$$

$$1 - y_2 / 24 \geq 0$$

$$-10 \leq x_1 \leq 10,\ 0 \leq x_2 \leq 10,\ 0 \leq x_3 \leq 10$$

Source: AIAA 2004-4537

Sub-systems I/O

$$x_1,\ x_2,\ x_3,\ y_2 \longrightarrow \boxed{y_1 = x_1^2 + x_2 + x_3 - 0.2y_2} \longrightarrow y_1$$

$$x_1,\ x_3,\ y_1 \longrightarrow \boxed{y_2 = \sqrt{y_1} + x_1 + x_3} \longrightarrow y_2$$

$$x_2,\ x_3,\ y_1,\ y_2 \longrightarrow \boxed{J = x_2^2 + x_3 + y_1 + e^{-y_2}} \longrightarrow J$$

Sub-systems I/O

$x_1, x_2, x_3, y_2 \longrightarrow \boxed{y_1 = x_1^2 + x_2 + x_3 - 0.2y_2} \longrightarrow y_1$

$x_1, x_3, y_1 \longrightarrow \boxed{y_2 = \sqrt{y_1} + x_1 + x_3} \longrightarrow y_2$

$x_2, x_3, y_1, y_2 \longrightarrow \boxed{J = x_2^2 + x_3 + y_1 + e^{-y_2}} \longrightarrow J$

Interaction between 2 subsystems

Sub-sys 1

$$y_1 = x_1^2 + x_2 + x_3 - 0.2 y_2$$

x_1, x_2, x_3

y_1

y_2

$Y_{1,cons}$

$y_{2,cons}$

Sub-sys 2

$$y_2 = \sqrt{y_1} + x_1 + x_3$$

design vector

consistent y's

Aggregate sub-system

- The aggregate of the two subsystems can be viewed as a black box itself
- This black-box takes as input the **design vector** X
- This aggregate black box is characterised by its **state vector** Y
- For a feasible design to ensue, this black box' state should be consistent, i.e. both sub-systems are in agreement with each other

Recap question

- *What is the mathematical relation that is generally used to evaluate and assert a (sub)-system's level of consistency?*

$$r_i = s_i - s_i^*$$

- *What exactly is the mathematical state that is used to assert consistency of a (sub)-system?*

$$r_i = 0$$

TU Delft

Overview

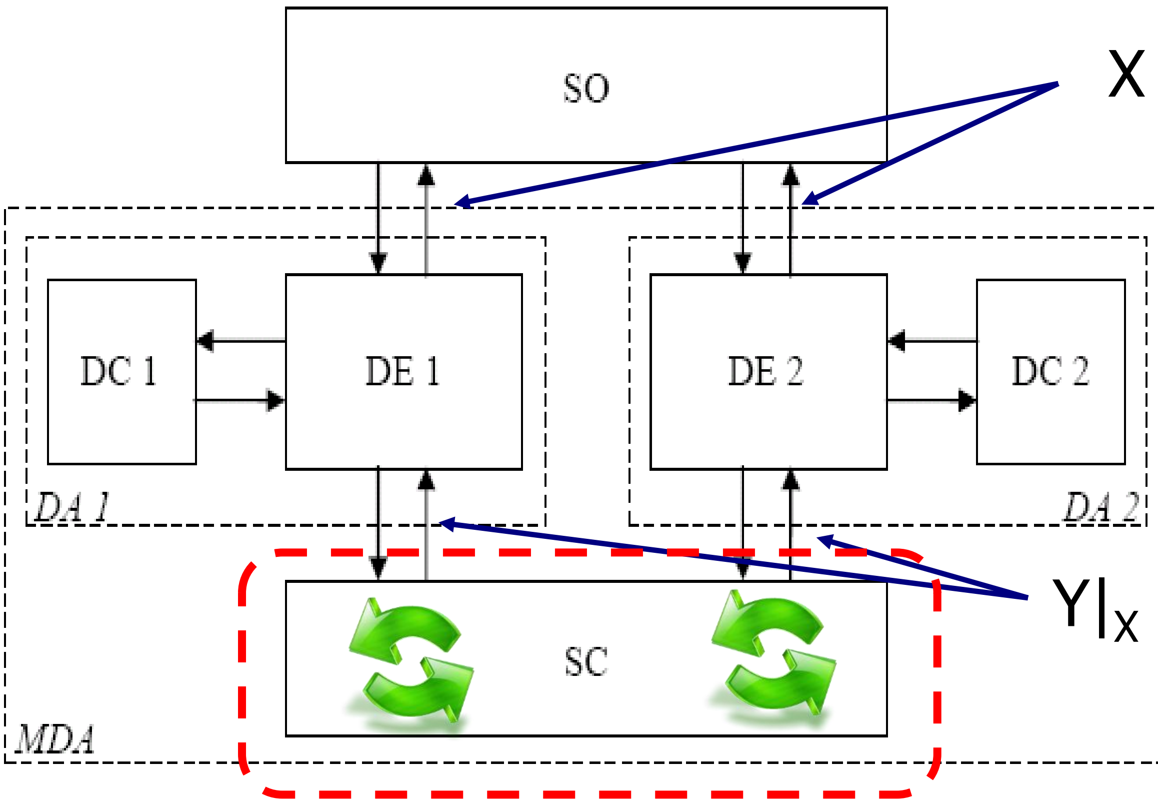
- Example Problem

- ***Multi-Discipline Feasible scheme***
- Individual Discipline Feasible scheme

MDF approach; build system coordinator

- In the MDF approach, a system coordinator is built inside the aggregate black-box.
- This coordinator will iterate the aggregate system for a fixed value of the input X, until consistency is reached for state Y
- The consistent state vector Y_{cons} will be passed forward as output
- Effectively, the aggregate system's solution is rendered as a function of input X: $Y_{cons} = Y_{cons}(X)$

MultiDiscipline Feasible; System Coordinator

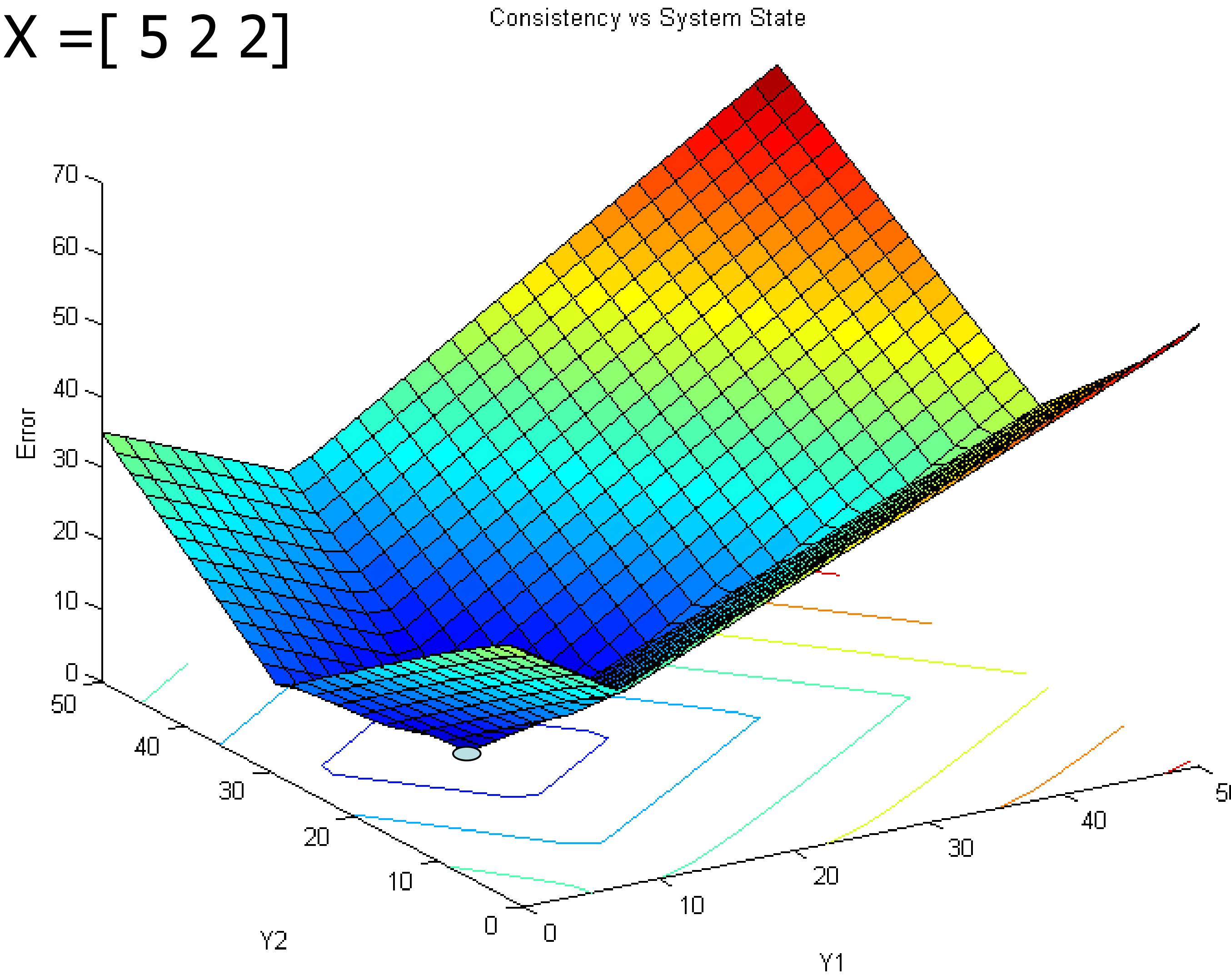


Characteristics of the MDF approach

- In an MDF scheme the system optimizer tries to optimize the aggregate system that is formed by the collection of all sub-systems and its coordinating block(s), in terms of the design vector X
- Notably, for MDF every test-design X (thus, for every iteration) that is evaluated for its objective-value has overall system-level consistency. I.e. every sub-system is in agreement with all others for each specified input X
- Typically, a coordinator-routine is added, in order to achieve this iteration-wise overall consistency
- Responsibility for overall system consistency is assigned to the *system-coordinator*

Example consistency behaviour

X =[5 2 2]



Iterate until consistent

- In order to find the consistent state, an iterative search has to be performed
- Just as for an optimization, some starting point must be given for this process
- Some termination error-tolerance has to be set for this search routine
- Required number of iterations, hence search time, increases with accuracy (i.e. a lower error-tolerance)

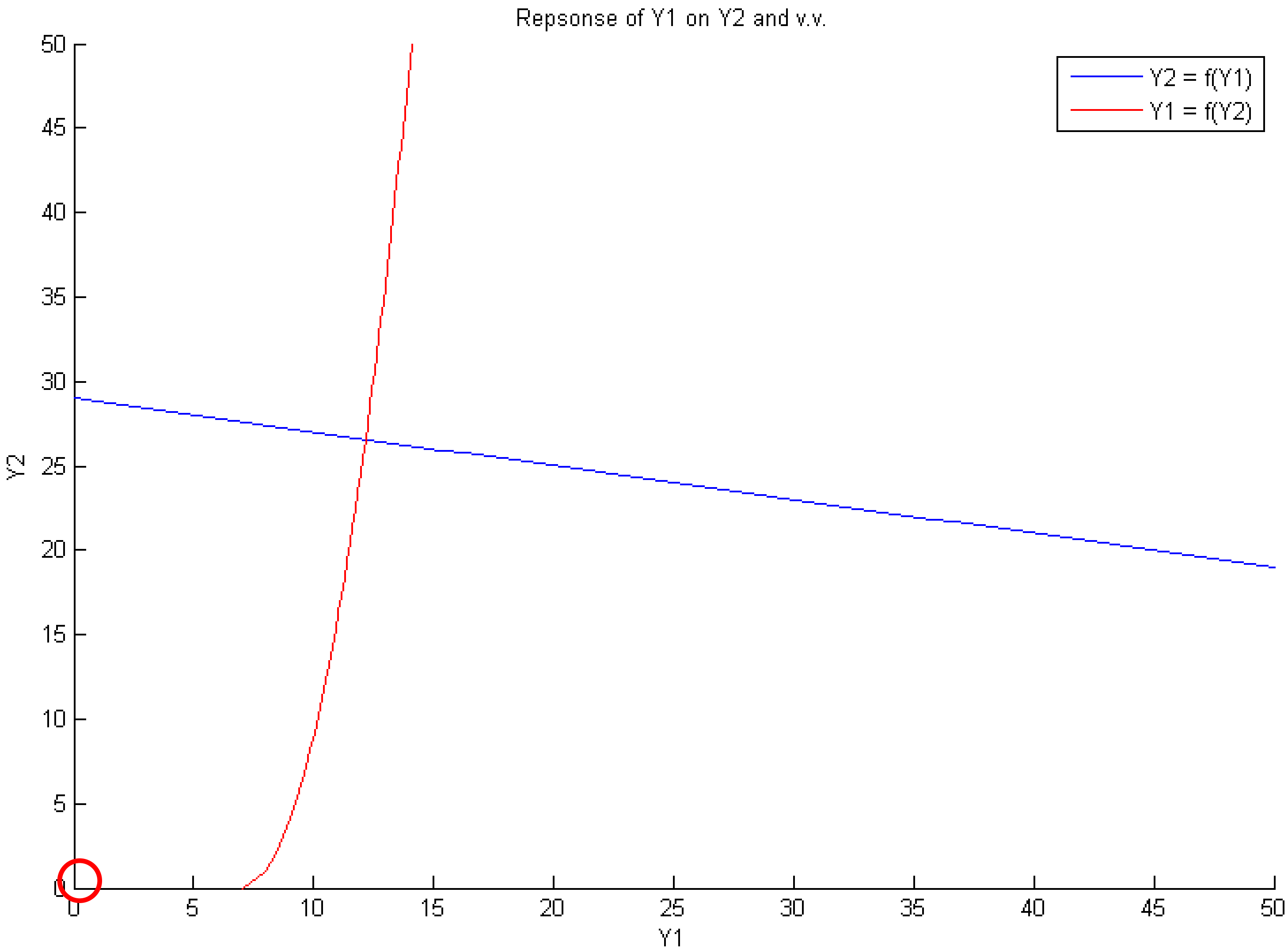
Iterate until consistent

Basic search strategy for this problem:

1. Start with initial state vector
2. Evaluate initial state with each sub-system; use outcome to update state-vector (i.e. the new search point)
3. Evaluate consistency error: $r(X) = |Y_{out} - Y_{in}|$
4. Evaluate updated state-vector with both sub-systems and, again, update its values with the outcome
5. Repeat 3 and 4 until error is lower than desired value

Iterate until consistent

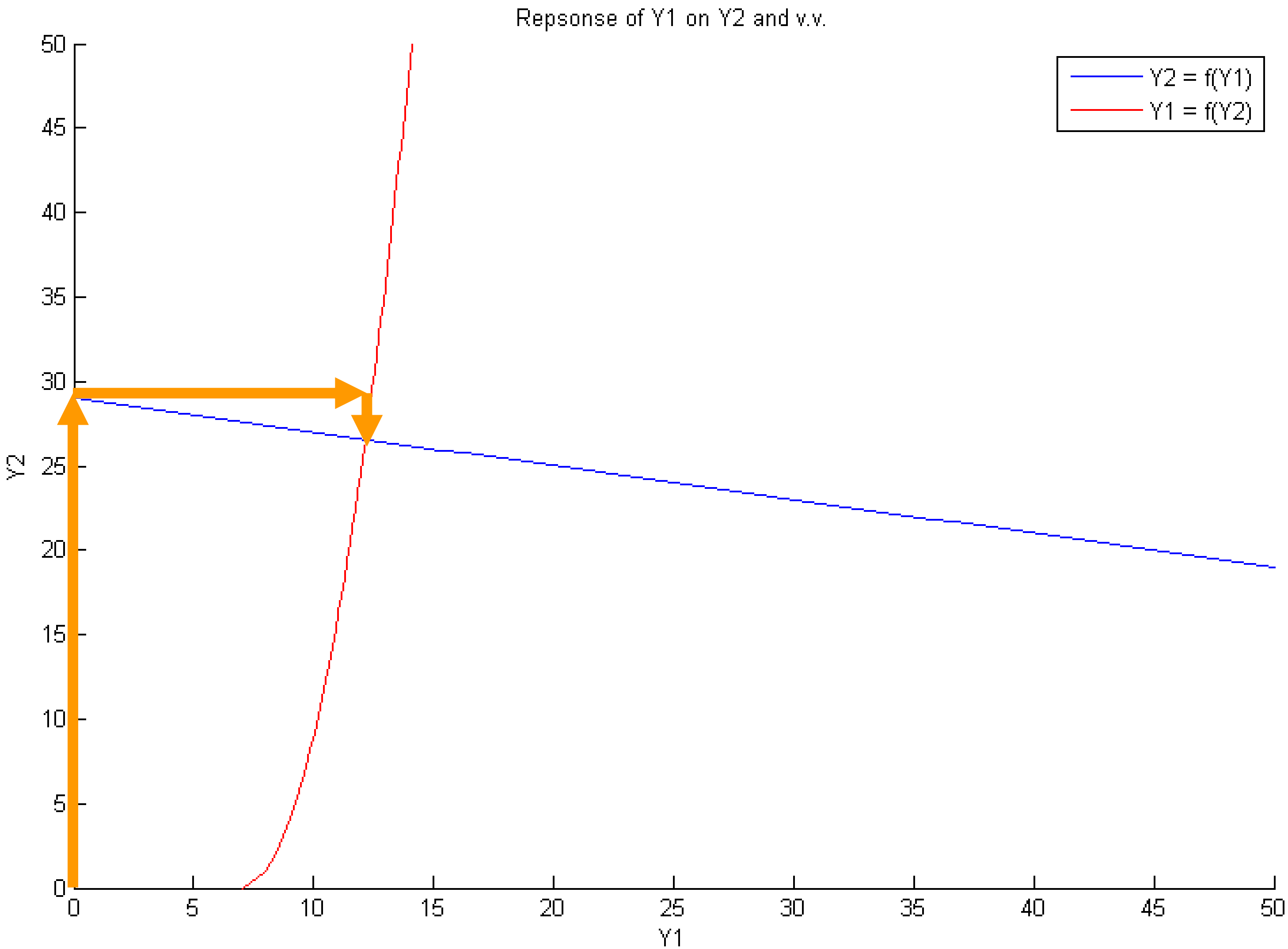
$X = [\ 5\ 2\ 2]$



$Y_{start} = [\ 0\ 0]$

Iterate until consistent

X =[5 2 2]



Y_{start} = [0 0]

TU Delft

Example DSM (MDF)

SO	**x**	**x**	**x**	**x**
y	**SC**	$y_{2,i}$	$y_{1,i}$	**y**
	$y_{1,i+1}$	$y_1 = x_1^2 + x_2 + x_3 - 0.2y_2$		
	$y_{2,i+1}$		$y_2 = \sqrt{y_1} + x_1 + x_3$	
J				$J = x_2^2 + x_3 + y_1 + e^{-y_2}$

TU Delft

Example DSM (MDF)

SO	x	x	x	x
y	SC	$y_{2,i}$	$y_{1,i}$	y
	$y_{1,i+1}$	$y_1 = x_1^2 + x_2 + x_3 - 0.2y_2$		
	$y_{2,i+1}$		$y_2 = \sqrt{y_1} + x_1 + x_3$	
J		LOOP, stop if $y_{j,i+1} - y_{j,i} < \varepsilon$		$J = x_2^2 + x_3 + y_1 + e^{-y_2}$

Assignment

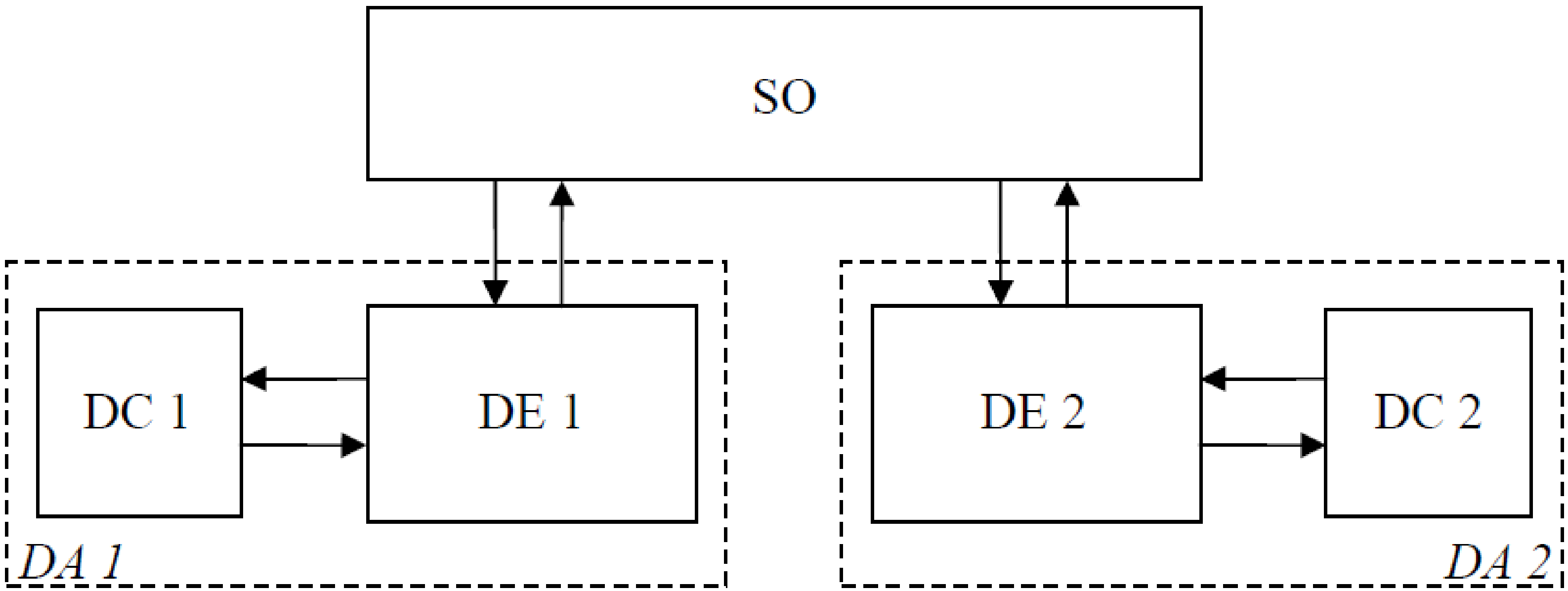
- *Work out the presented example problem in an MDF optimization scheme in MatLab*

Overview

- Example Problem

- Multi-Discipline Feasible scheme
- ***Individual Discipline Feasible scheme***

Individual Discipline Feasible (IDF)



Recap question

- Why is the Individual Discipline Feasible approach called this way?
- How does it differ from the Multi-Discipline Feasible approach in this respect?

Recap question; answer

- Why is the Individual Discipline Feasible approach called this way?

  The IDF approach has assigned the coordination of inter-system consistency to the optimizer. This optimizer will, parallel to searching for the optimum, search for an inter-discipline consistent solution. Therefore, at each iteration of the optimizer, the sub-disciplines are only self-consistent, but not inter-discipline consistent, except at the final optimum

Recap question; answer

- How does it differ from the Multi-Discipline Feasible approach in this respect?

  Because the MDF approach includes a dedicated system coordinator, it is ensured that overall inter-discipline consistency is achieved at each iteration of the optimizer. This results in an overall feasible design solution at each iteration, even before reaching the final optimum

IDF approach; tear coupling relations

- In the IDF approach the couplings relations between individual sub-systems are removed, or "torn"
- The torn state variables are replaced by surrogate variables that are now directly provided by the optimizer
- In order to ensure consistency of the sub-systems, extra constraints are added to the optimization problem

Characteristics of the IDF approach

- By tearing the coupling variables between sub-systems, these sub-systems will no longer be consistent throughout every iteration (each sub-system is consistent by itself, however)
- Responsibility for consistency is assigned to the optimizer
- The optimizer is tasked with finding a final design that is both consistent and optimal

Example

Mathematically;

$$\max R(X) \quad s.t. \quad g_i \le 0$$

Introduce surrogate variables $\Rightarrow$

$$\max R\left(X, Y^*\right) \quad s.t. \quad \boxed{\begin{array}{l} g_i \le 0 \\ Y = Y^* \end{array}}$$

Additional equality constraints

Example DSM (IDF)

SO	$\mathbf{x}, y_2^*$	$\mathbf{x}, y_1^*$	$\mathbf{x}, y_1^*, y_2^*$
$y_1, r_1 = y_1^* - y_1$	$y_1 = x_1^2 + x_2 + x_3 - 0.2y_2^*$		
$y_2, r_2 = y_2^* - y_2$		$y_2 = \sqrt{y_1^*} + x_1 + x_3$	
J			$J = x_2^2 + x_3 + y_1^* + e^{-y_2^*}$

Example DSM (IDF)

SO	$\mathbf{x}, y_2^*$	$\mathbf{x}, y_1^*$	$\mathbf{x}, y_1^*, y_2^*$
$y_1, r_1 = y_1^* - y_1$	$y_1 = x_1^2 + x_2 + x_3 - 0.2y_2^*$		
$y_2, r_2 = y_2^* - y_2$		$y_2 = \sqrt{y_1^*} + x_1 + x_3$	
J			$J = x_2^2 + x_3 + y_1^* + e^{-y_2^*}$

2 additional constraints and more variables to handle,
But no more loop

Assignment

- *Work out the earlier presented example problem in an IDF optimization scheme in MatLab*