AALBORG UNIVERSITY
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ModSimII
Mathematical Engineering

System Identification

Mini-Project

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1 | Introduction

The purpose of this mini-project is to obtain a simulation model of a DC-motor. This is achieved by modelling the physical properties of the DC-motor. Furthermore, an experiment is conducted in order to obtain measurements of the input and output of the DC-motor. Thereafter, system identification and least squares approximation are utilised to estimate the parameters of the model. The computations are conducted in Python. Lastly, the output of the constructed simulation model is compared with the measured data to verify the model.

2 | Modelling of DC-motor

A DC-motor consists of electrical and mechanical systems, which can both be modelled. In this mini-project the focus is to model the mechanical system of the DC-motor. The mechanical system is seen in Figure 2.1.

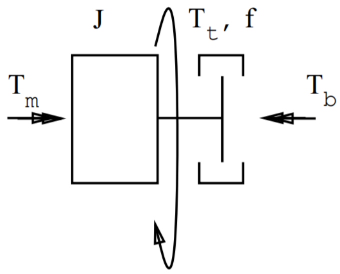

Figure 2.1: A diagram of a DC-motor. [Knudsen, 1993, p. 16]

The model of the mechanical system is based on Knudsen [1993] and is given by

$$J\dot{\omega} = T_m + T_t + T_v + T_b,$$

where J is the moment of inertia of the motor, T_m is the torque of the motor, T_t is the dry friction torque, T_v is the viscous torque, and T_b is the torque load. These expressions are modelled as

$$\begin{aligned} T_m &= K_m i \\ T_t &= -\operatorname{sgn}(\omega)F \\ T_v &= -\alpha\omega \\ T_b &= -mgl\sin(\theta), \end{aligned}$$

where K_m is the motor constant, i is the current, ω is the angular velocity, F is the friction force, α is the viscous friction coefficient, m is the mass of the load, g is the gravitational acceleration, l is the distance between the load and the motor, and θ is the angle of the load relative to the starting position.

The coefficients K_m, F, and α are unknown constants and so is the moment of inertia J. These need to be determined. The model of the system is constructed as an autonomous linear model, $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x}$ will be approximated using least squares. First the angular velocity ω and acceleration $\dot{\omega}$ are discretised:

$$\begin{aligned} \omega &= \frac{\Delta\theta}{\Delta t} \Rightarrow \omega_k = \frac{\theta_{k+1} - \theta_k}{t_s} \\ \dot{\omega} &= \frac{\Delta\omega}{\Delta t} \Rightarrow \dot{\omega}_k = \frac{\omega_{k+1} - \omega_k}{t_s}, \end{aligned}$$

where t_s is the sampling time. This yields the discretised model:

$$J\dot{\omega}_k = K_m i_k - \operatorname{sgn}(\omega_k)F - \alpha\omega_k - mgl\sin(\theta_k).$$

The model is then constructed as an autonomous linear model:

$$\begin{bmatrix} i_0 & -\operatorname{sgn}(\omega_0) & -\omega_0 & -\dot{\omega}_0 \\ i_1 & -\operatorname{sgn}(\omega_1) & -\omega_1 & -\dot{\omega}_1 \\ \vdots & \vdots & \vdots & \vdots \\ i_N & -\operatorname{sgn}(\omega_N) & -\omega_N & -\dot{\omega}_N \end{bmatrix} \begin{pmatrix} K_m \\ F \\ \alpha \\ J \end{pmatrix} = mgl \begin{pmatrix} \sin(\theta_0) \\ \sin(\theta_1) \\ \vdots \\ \sin(\theta_N) \end{pmatrix}. \tag{2.1}$$

If the angle θ is known to a corresponding current at a given time, then the parameters can be estimated by least squares.

3 | Experimental Setup

The purpose of the experiment is to create a setup such that the angle can be measured to corresponding times for different currents, which allows the motor coefficients to be estimated.

The setup consists of a DC-motor connected to a rod that is assumed to be massless and rigid. A load is attached to the end of the rod with mass m. The length of the rod is $l = 0.282$ m and the mass of the load is $m = 0.175$ kg.

By supplying the motor a current the motor starts, and if its torque is greater than the torque of the load, it will start to rotate with an angular velocity ω. If at some point the torque of the load becomes greater than or equal to the torque of the motor (to the given current), then the angle of the motor will stabilise at some equilibrium point. In steady state the load will then be stationary at a constant angle. This is conducted for multiple currents such that the angle of the motor at different currents with the same load can be analysed. A sketch of the experimental setup is seen in Figure 3.1.

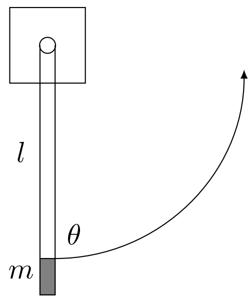

Figure 3.1: Load on a rod connected to a DC-motor.

4 | Data Processing

This chapter analyses the data collected in the conducted experiment. The data is seen in Figure 4.1.

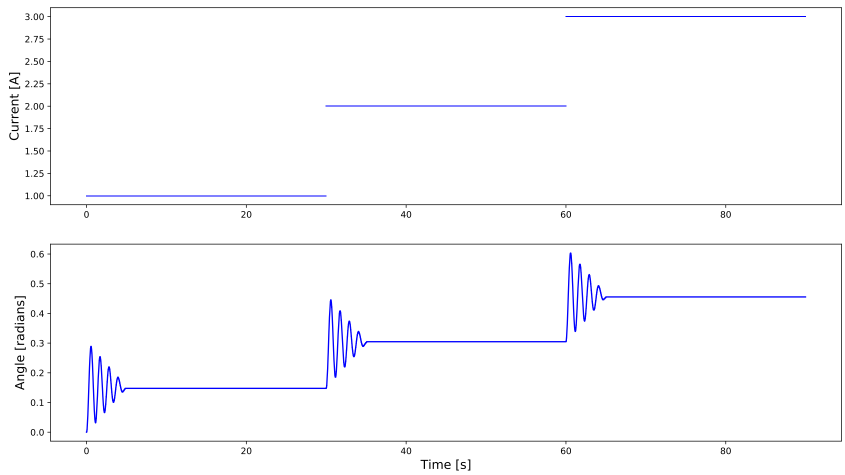

Figure 4.1: The first plot shows the input current. The second plot shows the measured angle.

The simulation model is now determined based on the collected data. This is done by computing the angular velocity as well as the angular acceleration. These are then filtered by an averaging filter of order 60.

Then the least squares approximation is performed on Equation (2.1) via QR factorisation. More specifically, the matrix A is factorised as $A = QR$ and then the vector $Q^*\mathbf{b}$ is computed (note that $Q^* = Q^\intercal$ since Q is real valued). Lastly the linear system $R\mathbf{x} = Q^*\mathbf{b}$ is solved for $\mathbf{x}$ which yields the estimated motor coefficients: $K_m \approx 0.0713$ Nm/A, $F \approx 0.00494$ N, $\alpha \approx -0.170$, $J \approx 0.0200 \text{ kg} \cdot \text{m}^2$. The model is then simulated which is seen in Figure 4.2.

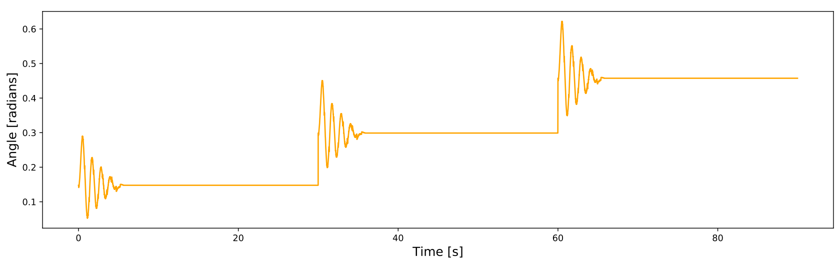

Figure 4.2: The plot shows the simulated model.

5 | Verification of Model

The purpose of this chapter is to compare the output of the constructed simulation model to the measured output data. The comparison is illustrated in Figure 5.1.

The experimentally estimated motor constant is $K_m \approx 0.0713$ Wb and the one specified in the data sheet for the motor is $K_{m.data} = 0.0934$ Wb. Their relative deviation is approximately -23.7%. The large deviation is presumably caused by the assumption that the rod is massless. As seen in Figure 5.1 the output of the simulation model is almost

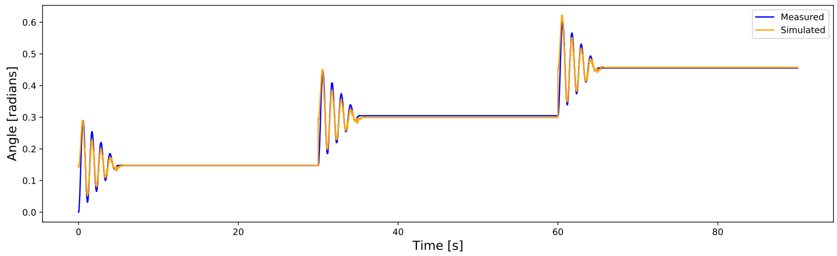

Figure 5.1: The plot shows the measured and simulated angle.

identical to the measured output of the DC-motor. However, some small deviations are seen at the oscillations. This is likely caused by the discretised angular velocity and acceleration since these are computed as the slope of the secant. An averaging filter is applied to minimise the effect of the discretisation. Thus, in order to improve the simulated model, another method for calculating the angular velocity and acceleration could be implemented. Furthermore, alternative filters could be applied instead to improve the model. If the velocity and acceleration could have been measured in the experiment, then the model would likely have been more accurate.

6 | Conclusion

The mechanical system was successfully modelled and data were experimentally collected. This allowed for a least squares approximation that was used to estimate the motor coefficients. The model combined with the motor coefficients allowed for a simulation model of the motor to be constructed, which was compared to the collected data and it showed that the simulation model is rather accurate. In conclusion, a simulation model has been created via system identification, and this model can be used to simulate realistic outputs for the system for arbitrary currents as long as it is within the tolerance of the DC-motor.

Bibliography

Knudsen, T. (1993). Systemidentifikation.