1 Mechanics

1.1 Newton's Laws of motion

- (Law of inertia) Every object continues in its state of rest or ofuniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.
- (Law of motion) If a object of mass m is moving with accelerationa, then it is under the influence of a (resultant) force F where
$$F = ma \quad [N] = [kg \cdot m/s^2].$$
- (Law of action and reaction) When two objects interacts, the force F_1 exerted on object 1 by object 2 is equal in magnitude and opposite in directed to the force F_2 exerted on object 2 by object 1, that is
$$F_1 = -F_2.$$

Inertial coordinate systems: Postulate of special relativity. Every law of physics must be such that if it holds in any coordinate system, it holds also in any other coordinate system moving at a constant velocity with respect to the first. You need to experiment to determine which coordinate system are inertial.

By assuming that $\mathbf{F} = \mathbf{F}(t, \mathbf{r}, \mathbf{v}, \mathbf{a})$ we see that the equation of motion is a second order ordinary differential equation
$$\ddot{\mathbf{r}} = \frac{1}{m}\mathbf{F}$$
We always assume $\mathbf{F} = \mathbf{F}(t, \mathbf{r}, \mathbf{v})$.
$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \frac{1}{m}\mathbf{F}(t, \mathbf{x}, \mathbf{v}) \\ &\Updownarrow \\ \begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{pmatrix} &= \begin{pmatrix} \frac{1}{m}\mathbf{F}(t, \mathbf{x}, \mathbf{v}) \end{pmatrix} = f(t, \mathbf{x}, \mathbf{v}) \\ \dot{\mathbf{z}} &= f(t, \mathbf{z}) \end{aligned}$$

1.2 Work and Energy

Assume that a force $\mathbf{F}$ acts on a particle between time t_1 and t_2. The work $W = W(t_1, t_2)$ done on the particle by the force is defined as
$$W = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} \, \mathrm{d}t \quad [Nm] = [kg \cdot m^2/s^2] = [j]$$
with $\mathbf{v}$ the velocity of the particle.

Note that
$$\begin{aligned} m\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{v} \cdot \mathbf{v}) &= m(\dot{\mathbf{v}} \cdot \mathbf{v} + \mathbf{v} \cdot \dot{\mathbf{v}}) = m2\mathbf{v} \cdot \mathbf{v} = 2\mathbf{F} \cdot \mathbf{v} \\ \mathbf{F} \cdot \mathbf{v} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv^2\right). \end{aligned}$$

and

$$v = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \|\mathbf{v}\|$$

for which $W = T_2 - T_1$ with $T_i = T(t_i)$ and

$$T = \frac{1}{2} m v^2$$

called the kinetic energy (of the particle).

The power $P = P(t)$ supplied by the force $\mathbf{F}$ at time t is

$$P = \mathbf{F} \cdot \mathbf{v} \quad [Nm/s] = [j/s]$$

hence

$$W = \int_{t_1}^{t_2} P \,\mathrm{d}t \quad \text{or} \quad P = \frac{\mathrm{d}W}{\mathrm{d}t_2} \quad \text{or} \quad P = \frac{\mathrm{d}T}{\mathrm{d}t}.$$

If the force only depends on position, $\mathbf{F} = \mathbf{F}(\mathbf{r})$, then

$$W = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} \,\mathrm{d}t = \int_C \mathbf{F} \cdot \frac{\mathbf{v}}{v} \,\mathrm{d}s$$

where $\mathrm{d}s$ denote the (infinitesimal) arc-length element and the integral is to be taken along the curve C described by the particle between t_1 and t_2.

Note that

$$\mathbf{v} \,\mathrm{d}t = \frac{\mathrm{d}x}{\mathrm{d}t} \,\mathrm{d}t = \mathrm{d}x,$$

and thus

$$\int_{t_1}^{t_2} \mathbf{F} \cdot \mathrm{d}\mathbf{r} \,\mathrm{d}t.$$

The work is difficult to compute as the position over time must be known, which requires the ability to solve the differential equation.

Conservative force: TO BE WRITTEN

A force $\mathbf{F}$ which is a function of position alone and which can be expressed as minus the gradient of a (real valued) function U is said to be conservative

$$\mathbf{F} = \mathbf{F}(\mathbf{r}) \quad \mathbf{F} = -\boldsymbol{\nabla} U.$$

The function U is said to be a potential energy (or function).

2 Electrical Systems

2.1 Resistive Circuitry – Kirchhoff's Laws

- Schematic: Physical devices and their inter-connection.
- Circuit Diagram: A model of the circuit consisting of ideal components.

Conducters connecting components are not physical wires, but idealised connections with no resistance. When multiple conducters (branches) meet at a connection, the connections are specified. If conducters cross each other without a specified connection, it is understood that they are not connected. Un-connected connectors are used for specifying when there is no connection to anything else at that point (terminals).

Conductance is the reciprocal value of the resistance: $K = \frac{1}{R}$.

Definition 2.1 (Kirchhoff's Current Law)
The algebraic sum of the currents leaving any enclosed volume (node) in a circuit equals zero:

$$\sum_n i_n = 0.$$

Node analysis algorithm:

1. Identify and label a reference node for other voltages in the circuit (ground).
2. Identify and label each other node for which the voltage at that node, relative to the reference node, is unknown.
3. Write Kirchhoff's current law at each such node, using Ohm's law to express the currents in terms of the node voltage and the resistance of the resistors attached to the node.
4. Solve the resulting equations for the node voltages.
5. Use the node voltages and Ohm's law to find any currents of interest.

Each independent voltage source that appears in a circuit reduces the number of unknown node voltages by one.

Definition 2.2 (Kirchhoff's Voltage Law)
The algebraic sum of the voltage drops around any closed path equals zero.

A **loop** is a closed path in or around a circuit. A **mesh** is a loop that does not enclose (encircle) any other loops.

Mesh analysis:

1. Identify each mesh and associate a mesh current with each.

2. Write Kirchhoff's voltage law around each mesh, using Ohm's law to express the voltages across resitors in the mesh in terms of the mesh currents and the resistances of the resistors.
3. Solve the resulting equations for the mesh currents.
4. Use the mesh currents and Ohm's law to find any voltages of interest.
5.

Each independent current source that appears in a cicruit reduces the number of unknown mesh currents by one.

Voltage divider, current divider.

Superposition: $f(x_1,\ x_2) = f(x_1) + f(x_2)$. Hence the circuit is analysed for each with all other sources set to zero. The individual contributions are added at the end.