INSTITUT FOR ØKONOMI
AARHUS UNIVERSITET

EKSAMENSOPGAVE

Uddannelse og niveau	BA – Økonomi					
Termin	Sommer 2021 ordinær eksamen					
Kursusnavn og eksamenskode(r)	3611: Programming in Quantitative Economics					461111E018
Eksamensform og varighed	WHAI**:** Hjemmeopgave med digital opgaveaflevering i WISEflow evt. efterfulgt af et mundtligt forsvar. (FLOWassign)					3 timer
Dato og tidspunkt	26.05.2021					09.00 – 12.00
Hjælpemidler	Alle	X	Anviste		Ingen	
Anden relevant information						
Det er tilladt at aflevere håndskrevet materiale	Ja		Nej	X	Kommentar:	
Anonym eksamen?	Ja	X	Nej			
Antal sider (inkl. forside)	6 (inkl. forside)					

IMPORTANT!
If you have questions the first hour of the exam, you may contact either Phillip Heiler at pheiler@econ.au.dk or Frederik Okslund at frederik.okslund@econ.au.dk. They will then as soon as possible answer you on Blackboard.

Practical information:

This is an open book exam. You are allowed to use any course material provided throughout the lectures. Unless explicitly stated otherwise you are NOT allowed to use external packages to solve the exam.

To submit your exam, you must upload a .ZIP or similar archive file containing all the source code files necessary to reproduce your results. In addition, the script(s) must contain the required comments to your code and answers. It must be clear which question you are answering in your script(s). You do NOT have to produce a separate report.

In order to obtain points for an exercise your script should produce the correct result directly without any alteration of your code. Any mismatch will result in a reduction of points. If you are convinced that a small bug, that you cannot fix, is causing your code to fail at any point in your script you may be able to salvage a few points by clearly and concisely explaining where it occurs and how you think it should be solved.

Communication during the exam is strictly forbidden. The exam has to be done individually by yourself, strictly without consulting anyone. If some suspicion is raised upon submitted solutions, it will be reported to the Board of Studies of Economics and Management. The form of this exam is based on the "honor system".

This exam contains 2 set of questions. There are 70 points on this exam. The exam will be graded on the Danish seven-point scale.

Good luck!

Problem 1:

30 P

Begin by setting the seed to 134.

1. Use the inversion method to simulate data from a Cauchy distribution with scale-parameter $\sigma = 2$.

   The CDF is given by

   $$F(X) = \frac{1}{\pi} tan^{-1}\left(\frac{X}{\sigma}\right) + \frac{1}{2}$$

   and we wish to simulate $N = 5000$ samples of length $T = 1000$.

   Write a function, `fn_slowCauchy`, which uses a nested `for`-loop to loop over N and T and transforms one $U(0, 1)$ variable at a time. The function must output a $T \times N$ matrix `mX`, and must be populated column-wise. 6 P

2. You now realize that your function is very inefficient, and you decide to write a different one. Your new function, `fn_fastCauchy`, must be vectorized and generate a $T \times N$ matrix `mY` which is identical to `mX`. It is up to you to write it in whichever way you believe is most efficient (while still using the inversion method).

   Verify that `mX` and `mY` are identical with the `all.equal` function. (Hint: remember to re-set the seed before calling each of your functions). 6 P

3. We can estimate the scale-parameter of our Cauchy random variables as

   $$\hat{\sigma} = median(|X|)$$

   Write a function, `fn_estimateScale`, which estimates $\hat{\sigma}$ for each column in `mX`. Save the resulting 5000 estimates in a vector `vSigma`.

   However, your function must **NOT** use the built-in functions `abs` and `median`. You must compute the median of the absolute value of each column without using those two functions. 7 P

4. Create a histogram of the estimates $\hat{\sigma}$ contained in the vector `vSigma`. Set `breaks = 50` and `freq = F`. Let the title of the plot be "Distribution of estimates" and set the label on the x-axis to "Estimates".

   Add a density plot to the histogram you have just created. Assume $\hat{\sigma}$ is Gaussian and use the empirical mean and standard deviation as the parameters. Superimpose the density plot on the histogram with `lines()`. 4 P

5. You wish to find the maximum of a 3^{rd} order polynomial given by

$$g(x) = -0.2x^3 + 3x^2 + 5x - 3$$

The method you have chosen is an iterative scheme which proceeds by the following algorithm

(a) Begin at $x_0 = 0$

(b) Set $x_n = x_{n-1} + \frac{sign(g'(x_{n-1}))}{\sqrt{n}}$

(c) Repeat until $|g'(x_n)| < \varepsilon$ with $\varepsilon = 0.00001$

where g' is the derivative of g wrt. x.

Write a function, `fn_findMax`, that implements the algorithm and returns a `List` with the final value of x_n, the maximum value of $g(\cdot)$ in that point, as well as the number if iterations it took to get there. 7 P

Problem 2: (Constrained) Optimization, C++, and Packaging

40 P

Note: This problem is best solved in order from 1 – 7.

In this problem, you are supposed to work with the `mDATA.r` file provided with the exam. It contains the daily stock returns for the stocks of two danish companies, Danske Bank and Ørsted, from 9th of June 2016 to 9th of March 2021 traded on the Copenhagen Stock Exchange. In the following, we denote $r_{A,t}$ as the return of Danske Bank and $r_{B,t}$ as the return of Ørsted at time period t.

1. Load the data file `mDATA.r` into your R workspace using `readRDS()`. 1 P

You would like to model the joint risk of the two stocks. For that you choose to implement a multivariate constant conditional correlation GARCH (CCC-GARCH) model. In particular you assume that, for all periods $t = 1, \dots, T$, the two stock returns $(r_{A,t}, r_{B,t})$ follow a bivariate normal distribution conditional on the information from all previous periods (denoted as $\mathcal{F}_{t-1}$), i.e

$$f(r_{A,t}, r_{B,t}|\mathcal{F}_{t-1}) = \frac{1}{\sqrt{2\pi \det |\Sigma_t|}} \exp\left(-\frac{1}{2}(r_{A,t}\ r_{B,t})'\Sigma_t^{-1}(r_{A,t}\ r_{B,t}) \right)$$
$$\Sigma_t = \begin{pmatrix} \sigma_{A,t}^2 & \rho\sigma_{A,t}\sigma_{B,t} \\ \rho\sigma_{A,t}\sigma_{B,t} & \sigma_{B,t}^2 \end{pmatrix}$$

with conditional variances each following a GARCH process

$$\sigma_{A,t}^2 = \omega_A + \alpha_A r_{A,t-1}^2 + \beta_A \sigma_{A,t-1}^2$$
$$\sigma_{B,t}^2 = \omega_B + \alpha_B r_{B,t-1}^2 + \beta_B \sigma_{B,t-1}^2.$$

Their joint likelihood function is then given by

$$L(\omega_A, \alpha_A, \beta_A, \omega_B, \alpha_B, \beta_B, \rho) = \prod_{t=1}^{T} f(r_{A,t}, r_{B,t}|\mathcal{F}_{t-1}).$$

2. Write an R function that returns the **average negative log-likelihood function** for the model above for periods $t = 2, 3, \dots, T$. It should take a vector of parameters as first input and a matrix of dimension $[T \times 2]$ as second input. It should return a warning if the matrix has a column dimension different from 2. *Hint: Initialize both GARCH processes at* $t = 1$ *with* $\sigma_{A,1}^2 = \omega_A/(1 - \alpha_A - \beta_A)$ *and* $\sigma_{B,1}^2 = \omega_B/(1 - \alpha_B - \beta_B)$. *You can use* `det()` and `solve()` *to calculate the determinant and the inverse of a matrix in R .* 8 P

3. Use the BFGS algorithm with `optim()` to find the maximizers of the log-likelihood function in 2). Use the following starting values:

$$\omega_A = s_A^2 \times 0.05, \qquad \alpha_A = 0.05, \qquad \beta_A = 0.90,$$
$$\omega_B = s_B^2 \times 0.05, \qquad \alpha_B = 0.05, \qquad \beta_B = 0.90,$$
$$\rho = 0$$

where s_A^2 and s_B^2 are the sample standard deviations of the return series A and B respectively over all time periods. Did your algorithm converge? If not, briefly describe a potential source of the problem. 4 P

4. Write an R function that returns the **average negative log-likelihood function** for a **reparameterized** CCC-GARCH with same input types as in 2). The reparameterization should impose the following constraints on the original parameters of the likelihood function:

$$\begin{aligned} &\omega_A > 0, \qquad \alpha_A \in (0,1), \qquad \beta_A \in (0,1),\\ &\omega_B > 0, \qquad \alpha_B \in (0,1), \qquad \beta_B \in (0,1),\\ &\rho \in (-1,1) \end{aligned}$$

10 P

5. Use the BFGS algorithm with `optim()` to find the maximizers of the reparameterized log-likelihood function. Use starting values that correspond to the starting values of the original parameterization in 3). What are your estimates of the transformed parameters? 4 P

6. Write a `C++` function using `Rcpp` and `RcppArmadillo` (optional) that can be loaded with `sourceCpp()`. It should take a numerical vector input corresponding to the parameters of the reparameterized likelihood function input and return a numerical vector that corresponds to the parameters of the original likelihood function. Apply it to your results in 5). What is your estimate for ρ? 4 P

7. Create an R package that contains the functions from 4.) and 6.) and edit the title description to "This is my exam package". Export the package as a bundled development version. *Remark: If you cannot solve 4.) or 6.) create a package that contains an R and a C++ function with single scalar inputs that always return the number 5.* 4 P