HW1 陳[illegible][illegible] 311540001

1) $$p(t \mid x, \mathbf{x}, \mathbf{t}) = \int_{-\infty}^{\infty} p(t \mid x, w, \beta)\, p(w, \mathbf{x}, \mathbf{t})\, dw$$

∘ $p(w \mid \mathbf{x}, \mathbf{t}) \propto p(\mathbf{t} \mid \mathbf{x}, w)\, p(w \mid \alpha)$

In addition

$$p(t \mid x, w) = \mathcal{N}(t \mid w^T \Phi(x), \beta^{-1} I)$$
$$= \mathcal{N}(t \mid w^T A + b, L^{-1})$$

$\Rightarrow A = \Phi(x)^T,\ b = 0,\ L = \beta I)$

∘ $p(w \mid \alpha) = \mathcal{N}(w \mid 0, \alpha^{-1} I) = \mathcal{N}(w \mid \mu, \Lambda^{-1})$

$\Rightarrow \mu = 0,\ \Lambda = \alpha I$

∘ $p(w \mid \mathbf{x}, \mathbf{t}) = \mathcal{N}(w \mid \Sigma\{A^T L (w - b) + \Lambda \mu\}, \Sigma)$

which $\Sigma = (\alpha I + A^T L A)^{-1}$

$\Rightarrow \mathcal{N}(w \mid S(\Phi^T(x) \beta t), S)$ where $S = (\alpha I + \Phi(x) \beta \Phi(x)^T)^{-1}$

$$\left.\begin{aligned} p(t \mid w, x) &= \mathcal{N}(t \mid w^T \Phi(x), \beta^{-1}) \\ &= \mathcal{N}(t \mid w^T A + b, L^{-1}) \end{aligned}\right\} \Rightarrow \begin{aligned} &A = \Phi(x),\ b = 0 \\ &L = \beta I \end{aligned}$$

$$\left.\begin{aligned} p(w \mid x, t) &= \mathcal{N}(w \mid S(\beta \Phi(x) t), S) \\ &= p(w \mid \mu, \Lambda^{-1}) \end{aligned}\right\} \Rightarrow \begin{aligned} &\mu = S(\beta \Phi(x) t) \\ &\Lambda^{-1} = S \end{aligned}$$

$$\Rightarrow p(t \mid x, \mathbf{x}, \mathbf{t}) = \mathcal{N}(t \mid A\mu + b, L^{-1} + A \Lambda^{-1} A^T)$$
$$= \mathcal{N}(t \mid \beta \Phi(x)^T S\, \Phi(x) t, \beta^{-1} + \Phi(x)^T S \Phi(x))$$