Problem 1 within project 1

September 7, 2022

We are interested in the solution for

$$-\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} = 100e^{-10x}$$

It is claimed that $u(x) = 1 - (1 - e^{-10})x - e^{-10x}$ is a solution. Inserting this into the lefthand side of the original expression will show that this is indeed a solution if both sides of the equation still match:

$$\begin{aligned} &-\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(1 - (1 - e^{-10})x - e^{-10x}\right) \\ &= -(-100e^{-10x}) = 100e^{-10x} \end{aligned}$$

As such, we've shown that $u(x) = 1 - (1 - e^{-10})x - e^{-10x}$ is an exact solution, meaning that this solution holds $\forall x$.