1 Problem 3

1.1 Definitions

a few definitions will be made:

$$\begin{aligned}
x &\equiv x_i \\
x+h &\equiv x_{i+1} \\
u(x_i) &\equiv u_i \\
f(x_i) &\equiv f_i
\end{aligned}$$

1.2 Taylor series

The taylor expansion is defined for a function $u(x)$ around the region $x = a$:

$$u(x) = \sum_{n=0}^{\infty} \frac{u^{(n)}(a)}{n!}(x-a)^n$$

alternatively it can be expressed

$$u(x) = u(a) + u'(a)(x+a) + O(h) \tag{1}$$

Where $O(h)$ is the error function. for $a = x - h$, 1 becomes

$$\begin{aligned}
&u(x) = u(x-h) + u'(x-h)(x-(x-h)) + O(h) \\
&= u(x-h) + u'(x-h)h + O(h) \\
&\Rightarrow u_i = u'_{i-1}h + u_{i-1} + O(h) \\
&\Rightarrow u'_{i-1} = \frac{u_i - u_{i-1}}{h} + O(h) \\
&\Rightarrow u_i = \frac{u_{i+1} - u_i}{h} + O(h)
\end{aligned}$$

Alternatively for $a = x + h$

$$\begin{aligned}
&u(x) = u(x+h) - u'(x+h)h + O(h) \\
&\Rightarrow u_i = u_{i+1} - u'_{i+1}h + O(h) \\
&\Rightarrow u'_{i+1} = \frac{u_{i+1} - u_i}{h} \\
&\Rightarrow u'_i = \frac{u_i - u_{i-1}}{h}
\end{aligned}$$

1.3 Solving for second derivative of u

$$u(x) = u() \tag{2}$$

A Deriving first derivative for u, from taylor expansion of $u(x)$ around $x = x + h$

$$eh$$