Project 2 Cosmology

Candidate Nr: 24

May 14, 2023

Task a

We are to show that:

$$\frac{\mathrm{d}Y_{\mathrm{n}}}{\mathrm{d}(\ln T)} = -\frac{1}{H}\left(Y_{\mathrm{p}}\Gamma_{\mathrm{p}\to\mathrm{n}} - Y_{\mathrm{n}}\Gamma_{\mathrm{n}\to\mathrm{p}}\right), \tag{1}$$

and that:

$$\frac{\mathrm{d}Y_{\mathrm{p}}}{\mathrm{d}(\ln T)} = -\frac{1}{H}\left(Y_{\mathrm{n}}\Gamma_{\mathrm{n}\to\mathrm{p}} - Y_{\mathrm{p}}\Gamma_{\mathrm{p}\to\mathrm{n}}\right) \tag{2}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, $Y_{\mathrm{n}} = \frac{n_{\mathrm{n}}}{n_{\mathrm{b}}}$ and $Y_{\mathrm{p}} = \frac{n_{\mathrm{p}}}{n_{\mathrm{n}}}$ are relative number densities, where $n_{\mathrm{b}} = n_{\mathrm{b}0}a^{-3}$ is the total baryon number density, n_{n} is the total number density for free neutrons, $\Gamma_{\mathrm{p}\to\mathrm{n}}$ and $\Gamma_{\mathrm{n}\to\mathrm{p}}$ are decay rates, and finally $T = \frac{T_0}{a}$ is the photon temperature.
Firstly we solve $\frac{\mathrm{d}\ln T}{\mathrm{d}t}$ for $\mathrm{d}t$:

$$\begin{aligned}
\ln T &= \ln T_0 - \ln a,\\
\frac{\mathrm{d}\ln T}{\mathrm{d}t} &= -H,\\
\Rightarrow \mathrm{d}\ln T &= -H\mathrm{d}t.
\end{aligned} \tag{3}$$

solving for $\frac{\mathrm{d}Y_{\mathrm{n}}}{\mathrm{d}\ln T}$

We insert for Eq. 3 into $\frac{\mathrm{d}Y_{\mathrm{n}}}{\mathrm{d}(\ln T)}$:

$$\begin{aligned}
\frac{\mathrm{d}Y_{\mathrm{n}}}{\mathrm{d}(\ln T)} &= -\frac{1}{H}\frac{\mathrm{d}Y_{\mathrm{n}}}{\mathrm{d}t},\\
&= -\frac{1}{H}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{n_{\mathrm{n}}}{n_{\mathrm{b}}}\right),\\
&= -\frac{1}{H}\left(\frac{\frac{\mathrm{d}n_{\mathrm{b}}}{\mathrm{d}t}n_{\mathrm{b}}}{n_{\mathrm{b}}^2} - \frac{n_{\mathrm{n}}\frac{\mathrm{d}n_{\mathrm{n}}}{\mathrm{d}t}}{n_{\mathrm{b}}^2}\right),\\
&= \frac{n_{\mathrm{n}}}{n_{\mathrm{b}}^2}\frac{1}{H}\frac{\mathrm{d}n_{\mathrm{b}}}{\mathrm{d}t} - \frac{1}{H}\frac{1}{n_{\mathrm{b}}}\frac{\mathrm{d}n_{\mathrm{n}}}{\mathrm{d}t}
\end{aligned} \tag{4}$$

We solve for $\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t}$:

$$\begin{aligned}\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{n_\mathrm{b0}}{a^3},\\ &= -3n_\mathrm{b0}\frac{\dot{a}}{a^4},\\ &= -3Hn_\mathrm{b}.\end{aligned} \tag{5}$$

We know that $\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t} + 3Hn_\mathrm{n} = n_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}} - n_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}}$, and we solve for $\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t}$:

$$\begin{aligned}&\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t} + 3Hn_\mathrm{n} = n_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}} - n_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}},\\ \Rightarrow\ &\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t} = n_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}} - n_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}} - 3Hn_\mathrm{n}.\end{aligned} \tag{6}$$

We insert for $\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t}$ and $\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t}$ into Eq. 4:

$$\begin{aligned}\frac{\mathrm{d}Y_\mathrm{n}}{\mathrm{d}(\ln T)} &= \frac{n_\mathrm{n}}{n_\mathrm{b}^2}\frac{1}{H}\left(-3Hn_\mathrm{b}\right) - \frac{1}{H}\frac{1}{n_\mathrm{b}}\left(n_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}} - n_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}} - 3Hn_\mathrm{n}\right),\\ &= -3\frac{n_\mathrm{n}}{n_\mathrm{b}} + 3\frac{n_\mathrm{n}}{n_\mathrm{b}} - \frac{1}{H}\left(\frac{n_\mathrm{p}}{n_\mathrm{b}}\Gamma_{\mathrm{p}\to\mathrm{n}} - \frac{n_\mathrm{n}}{n_\mathrm{b}}\Gamma_{\mathrm{n}\to\mathrm{p}}\right),\\ &= -\frac{1}{H}\left(Y_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}} - Y_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}}\right).\end{aligned} \tag{7}$$

Solving for $\frac{\mathrm{d}Y_\mathrm{p}}{\mathrm{d}\ln T}$

We insert for Eq. 3 into $\frac{\mathrm{d}Y_\mathrm{p}}{\mathrm{d}\ln T}$, exactly as we did in Eq. 4, except swap n for p:

$$\frac{\mathrm{d}Y_\mathrm{p}}{\mathrm{d}\ln T} = \cdots = \frac{n_\mathrm{p}}{n_\mathrm{b}^2}\frac{1}{H}\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t} - \frac{1}{H}\frac{1}{n_\mathrm{b}}\frac{\mathrm{d}n_\mathrm{p}}{\mathrm{d}t}. \tag{8}$$

We know that $\frac{\mathrm{d}n_\mathrm{p}}{\mathrm{d}t} + 3Hn_\mathrm{p} = n_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}} - n_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}}$, and we solve for $\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t}$:

$$\frac{\mathrm{d}n_\mathrm{p}}{\mathrm{d}t} = n_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}} - n_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}} - 3Hn_\mathrm{p}. \tag{9}$$

We insert for $\frac{\mathrm{d}n_\mathrm{p}}{\mathrm{d}t}$ and $\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t}$ into Eq 8, again nearly exactly as we already did in Eq. 7, merely swapping out $\frac{\mathrm{d}n_\mathrm{n}}{\mathrm{d}t}$ for $\frac{\mathrm{d}n_\mathrm{p}}{\mathrm{d}t}$:

$$\begin{aligned}\frac{\mathrm{d}Y_\mathrm{p}}{\mathrm{d}\ln T} &= \frac{n_\mathrm{p}}{n_\mathrm{b}^2}\frac{1}{H}\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t} - \frac{1}{H}\frac{1}{n_\mathrm{b}}\frac{\mathrm{d}n_\mathrm{p}}{\mathrm{d}t},\\ &= 3\frac{n_\mathrm{p}}{n_\mathrm{b}} - 3\frac{n_\mathrm{p}}{n_\mathrm{b}} - \frac{1}{H}\left(Y_\mathrm{n}\Gamma_{\mathrm{n}\to\mathrm{p}} - Y_\mathrm{p}\Gamma_{\mathrm{p}\to\mathrm{n}}\right).\end{aligned} \tag{10}$$

As such we've shown what we've been requested to show.

Bonus Question

We are to make an order-of-magnitude estimate for the total baryon mass density ρ_{b0}, at the time of Bing Bang Nucleosynthesis (BBN), where $T \sim 10^9$K. We know that $\rho_{\mathrm{b0}} = \Omega_{\mathrm{b0}}\rho_{\mathrm{c0}}$.

Task b

We are to show that $T_\nu = \left(\frac{4}{11}\right)^{1/3} T$ holds, where T_ν is the neutrino temperature, and T is the photon temperature.
we assume that entropy S is conserved prior and post to BBN. the entropy density S relates to s such that $S = sa^{-3}$, as such sa^3 is also conserved. we express s:

$$s = \frac{2\pi^2}{45} k_{\mathrm{B}} g_{*s} \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3, \tag{11}$$

where coefficients g_{*s} and T^3 are the only non-constant. as such, $g_{*\mathrm{s}}a^3T^3$ is constant. This means that the following holds:

$$g_{*\mathrm{s}}(aT_{\mathrm{before}})^3 = g_{*\mathrm{s}}(aT_{\mathrm{after}})^3 \tag{12}$$

$$g_{*\mathrm{s}} = \sum_{i=\mathrm{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=\mathrm{fermions}} g_i \left(\frac{T_i}{T}\right)^3, \tag{13}$$

where g_i is the internal degrees of freedom for some particle i, and T_i is its temperature. Before BBN we have particles photons, positrons and electrons. The positron and electron each have half spin, with two spinstates, and two internal degrees of freedom as such. the photon is a boson, which has 3 internal degrees of freedom, but the photon is massless which removes 1, as such it has 2 degrees of freedom. We also assmue thermal equilibrium prior to BBN, such that $T = T_{\mathrm{photon}} = T_{\mathrm{positron}} = T_{\mathrm{electron}}$. As such:

$$\begin{aligned} g_{*\mathrm{s}} &= 2 + \frac{7}{8}(2+2), \\ &= \frac{11}{2}. \end{aligned} \tag{14}$$

Post BBN there is only photons, as the positrons and electrons are assumed annihilated:

$$g_{*\mathrm{s}} = 2. \tag{15}$$

Inserting for these in 12:

$$\begin{aligned} &\frac{11}{2}(aT_{\mathrm{before}})^3 = 2(aT_{\mathrm{after}})^3, \\ &\Rightarrow \left(\frac{11}{4}\right)^{1/3} T_{\mathrm{before}} = T_{\mathrm{after}}. \end{aligned} \tag{16}$$

Photons and neutrinos were thermally coupled prior to BBN, such that $T_{\text{before}} = T_\nu$, but are decoupled after. we know that $(aT_\nu)_{\text{after}} = (aT_\nu)_{\text{before}}$, as such, we use $T \equiv T_{\text{after}}$, and we find that $T_\nu = \left(\frac{4}{11}\right)^{1/3} T$.

Task c

We are to show the following expression for the fraction of energy in the form of radiation in our universe today:

$$\Omega_{\text{r}0} = \frac{8\pi^3 G(k_{\text{B}}T_0)^4}{45H_0^2\hbar^3 c^5}\left(1 + N_{\text{eff}}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right), \tag{17}$$

defined such that $\Omega_{\text{r}0} = \frac{\rho_{\text{r}0}}{\rho_{\text{c}0}}$, where $\rho_{\text{r}0}$ is the energy density for radiation, and $\rho_{\text{c}0}$ is the critical energy density of the universe today. It has the following expression:

$$\rho_{\text{c}0} = \frac{3H_0^2}{8\pi G}, \tag{18}$$

where H_0 is the hubble parameter today, and G is the gravitational constant. The total energy density in the early universe in terms of photon temperature T, including only the contributions from ultrarelativistic particles is expressed:

$$\rho c^2 = \frac{\pi^2 (k_{\text{B}}T)^4}{30(\hbar c^3)}\left(\sum_{i=\text{bosons}} g_i\left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=\text{fermions}} g_i\left(\frac{T_i}{T}\right)^4\right), \tag{19}$$

where g_i is the internal degrees of freedom for particle species i. $g_{\text{photon}} = 2$ due to being massless, but otherwise $g_i = 1 + 2S$, and for the neutrinos which are all fermions with $S = \frac{1}{2}$, their g_i's are equal to two as well, and we account for N_{eff} numbers of them. recall that we showed $T_\nu = \left(\frac{4}{11}\right)^{1/3} T$. As such:

$$\begin{aligned}\rho c^2 &= \frac{\pi^2 (k_{\text{B}}T)^4}{30(\hbar c)^3}\left(2 + \frac{7}{8}2N_{\text{eff}}\left(\frac{4}{11}\right)^{4/3}\right),\\ &= \frac{\pi^2 (k_{\text{B}}T)^4}{15(\hbar c)^3}\left(1 + \frac{7}{8}N_{\text{eff}}\left(\frac{4}{11}\right)^{4/3}\right)\end{aligned} \tag{20}$$

For the time in which we are considering, the universe is radiation dominated, as such we do $\rho = \rho_c$. To find the total energy density for radiation today, we simply insert for $T = T_0$:

$$\rho_{\text{r}0} = \frac{\pi^2 (k_{\text{B}}T_0)^4}{15\hbar^3 c^5}\left(1 + \frac{7}{8}N_{\text{eff}}\left(\frac{4}{11}\right)^{4/3}\right) \tag{21}$$

Solving for $\Omega_{\text{r}0}$:

$$\Omega_{\text{r}0} = \frac{8\pi^3 G(k_{\text{B}}T_0)^4}{45H_0^2\hbar^3 c^5}\left(1 + N_{\text{eff}}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right). \tag{22}$$

As such, we've shown what must be shown.

Task d

We are to integrate the following expression to obtain $a(t)$ and $t(T)$:

$$H = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_{\mathrm{r}0}}a^{-2}, \tag{23}$$

As such, integrating, and solving for a(t):

$$\begin{aligned}
&H = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_{\mathrm{r}0}}a^{-2}, \\
&\Rightarrow \dot{a}a = H_0\sqrt{\Omega_{\mathrm{r}0}}, \\
&\Rightarrow \int a\frac{\mathrm{d}a}{\mathrm{d}t}\mathrm{d}t = \sqrt{\Omega_{\mathrm{r}0}}\int \mathrm{d}t, \\
&\Rightarrow \int a\mathrm{d}a = \sqrt{\Omega_{\mathrm{r}0}}(t+c), \\
&\Rightarrow \frac{a^2}{2} + c = \sqrt{\Omega_{\mathrm{r}0}}(t+c), \\
&\Rightarrow a = \sqrt{2\sqrt{\Omega_{\mathrm{r}0}}t + C}
\end{aligned} \tag{24}$$

we know that $a(0) = 0$, which implies $C = 0$, as such:

$$a = \sqrt{2\sqrt{\Omega_{\mathrm{r}0}}t} \tag{25}$$

Photon temperature T relates to a such that $T = T_0/a$, where T_0 [k] is the photon temperature today. We solve for $t(t)$:

$$\begin{aligned}
&T^2 = \frac{T_0^2}{a^2}, \\
&\Rightarrow a^2 = \frac{T_0^2}{T^2}, \\
&\Rightarrow 2\sqrt{\Omega_{\mathrm{r}0}}t = \frac{T_0^2}{T^2}, \\
&\Rightarrow t = \frac{1}{2\sqrt{\Omega_{\mathrm{r}0}}}\frac{T_0^2}{T^2}
\end{aligned} \tag{26}$$

times for when the universe had photon temperatures of $T \in [10^10, 10^9, 10^8]$K are found in Table 1.

Temperature (K)	Age
10^{10}	1.8s
10^9	2.96min
10^8	4.94hrs

Table 1: Times at which the temperature had certain photon temperatures.

Task e

We are to derive the following equations:

$$Y_{\rm n} = \left(1 + \exp\{(m_{\rm n} - m_{\rm p})c^2/k_{\rm B}T_i\}\right)^{-1}, \\ Y_{\rm p} = 1 - Y_{\rm n} \tag{27}$$

assuming that the baryonic mass $\rho_{\rm b}$ at initial temperature T_i is in neutrons and protons, and that they are in thermal equilibrium. As such, we express the numer density for baryons $n_{\rm b}$ as the sum of the number densities for protons and neutrons, such that $n_{\rm b} = n_{\rm n} + n_{\rm p}$, and express $Y_{\rm n}$ in terms of it:

$$\begin{aligned} Y_{\rm n} &= \frac{n_{\rm n}}{n_{\rm b}}, \\ &= \frac{n_{\rm n}}{n_{\rm n} + n_{\rm p}}, \\ &= \left(1 + \frac{n_{\rm p}}{n_{\rm n}}\right)^{-1}. \end{aligned} \tag{28}$$

Recall that $n_{\rm n} = n_{\rm n0}a^{-3}$, and that $n_{\rm n} = n_{\rm n0}a^{-3}$, i.e. the number densities change with the change of volume caused by expansion. Dividing one by another shows the following:

$$\frac{n_{\rm n}}{n_{\rm p}} = \frac{n_{\rm n0}}{n_{\rm p0}}. \tag{29}$$

adwuhauh

$$\frac{n_{\rm n0}}{n_{\rm p0}} = \left(\frac{m_{\rm p}}{m_{\rm n}}\right)^{3/2} \exp\{-(m_{\rm n} - m_{\rm p})c^2/k_{\rm B}T_i\} \tag{30}$$

As such, solve for $n_{\rm p}/n_{\rm n}$, and plug into Eq. 27:

$$\tag{31}$$

Task f

We are looking to solve Eq. 1 and 2, for the relative number densities of protons Y_n and neutrons Y_n, relative to the number density for baryons, starting at initial temperature $T_i = 100 \cdot 10^9$ Kelvin, with initial conditions solved for in last exercice. We solve with scipy's solve_ivp() Raudau method. The result is found in Fig. 1

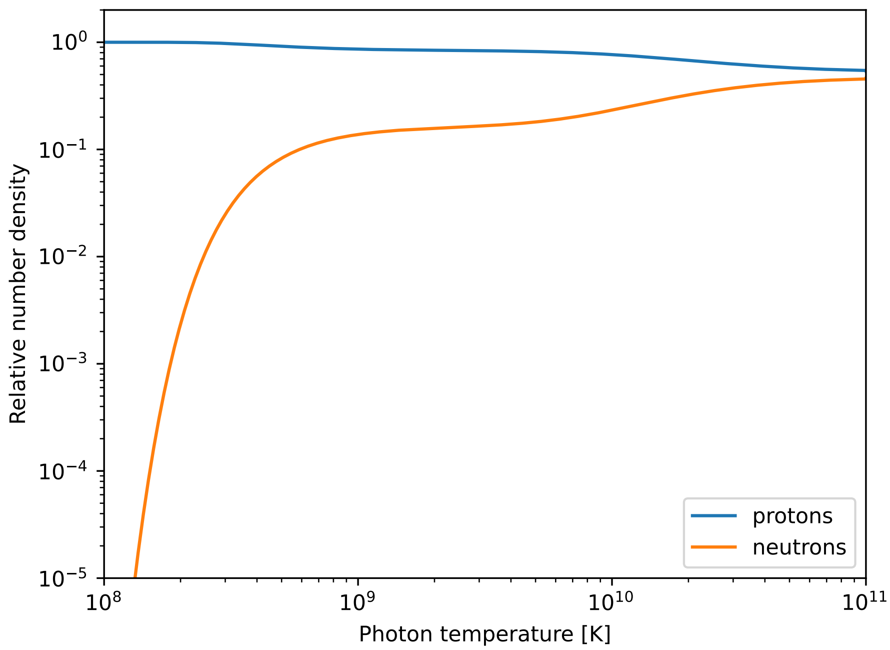

Figure 1: Number densities for protons and neutrons, relative tot he baryon density, as functions of temperature.

Task g

We are asked to show the following:

$$\frac{\mathrm{d}Y_i}{\mathrm{d}\ln T} = -\frac{1}{H}\left(\sum_{i\neq j}\left(Y_j\Gamma_{j\to i} - Y_i\Gamma_{i\to j}\right) + \sum_{jkl}\left(Y_kY_l\Gamma_{kl\to ij} - Y_iY_j\Gamma_{ij\to kl}\right)\right). \tag{32}$$

We have the expression for the general Boltzmann equation for a particle i that interacts with any number of other particles j, both through decays and two-body reactions:

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} + 3Hn_i = \sum_{i\neq j}\left(n_j\Gamma_{j\to i} - n_i\Gamma_{i\to j}\right) + \sum_{jkl}\left(n_kn_l\gamma_{kl\to ij} - n_in_j\gamma_{ij\to kl}\right). \tag{33}$$

We divided by n_b on both sides, and solve the left hand side for $\frac{\mathrm{d}Y_i}{\mathrm{d}\ln T}$.

$$\begin{aligned}
&\frac{1}{n_\mathrm{b}}\frac{\mathrm{d}n_i}{\mathrm{d}t} + \frac{1}{n_\mathrm{b}}3Hn_i = \frac{1}{n_\mathrm{b}}\left(\sum_{i\neq j}\left(n_j\Gamma_{j\to i} - n_i\Gamma_{i\to j}\right) + \sum_{jkl}\left(n_kn_l\gamma_{kl\to ij} - n_in_j\gamma_{ij\to kl}\right)\right),\\
&\Rightarrow \frac{n_\mathrm{b}}{n_\mathrm{b}^2}\frac{\mathrm{d}n_i}{\mathrm{d}t} - \frac{n_i}{n_\mathrm{b}^2}\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t} = \left(\sum_{i\neq j}\left(\frac{n_j}{n_\mathrm{b}}\Gamma_{j\to i} - \frac{n_i}{n_\mathrm{b}}\Gamma_{i\to j}\right) + \sum_{jkl}\left(\frac{n_k}{n_\mathrm{b}}\frac{n_l}{n_\mathrm{b}}n_\mathrm{b}\gamma_{kl\to ij} - \frac{n_i}{n_\mathrm{b}}\frac{n_j}{n_\mathrm{b}}n_\mathrm{b}\gamma_{ij\to kl}\right)\right),\\
&\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{n_i}{n_\mathrm{b}}\right) = \sum_{i\neq j}\left(Y_j\Gamma_{j\to i} - Y_i\Gamma_{i\to j}\right) + \sum_{jkl}\left(Y_kY_l\Gamma_{kl\to ij} - Y_iY_j\Gamma_{ij\to kl}\right),\\
&\Rightarrow \frac{\mathrm{d}Y_i}{\mathrm{d}\ln T} = -\frac{1}{H}\left(\sum_{i\neq j}\left(Y_j\Gamma_{j\to i} - Y_i\Gamma_{i\to j}\right) + \sum_{jkl}\left(Y_kY_l\Gamma_{kl\to ij} - Y_iY_j\Gamma_{ij\to kl}\right)\right)
\end{aligned} \tag{34}$$

As such, we've shown what is requested. We used that $\frac{\mathrm{d}}{\mathrm{d}\ln T} = -\frac{1}{H}\frac{\mathrm{d}}{\mathrm{d}t}$, and that $3H = -\frac{1}{n_\mathrm{b}}\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t}$.

Task h

We repeat the procedure of task f, in addition to solving for the the number density for deuterium relative to the number density for baryons: Y_D. maintaining mostly the same approach as in task f, the governing equations are different. As specified in task G, the system now has the following governing equations for

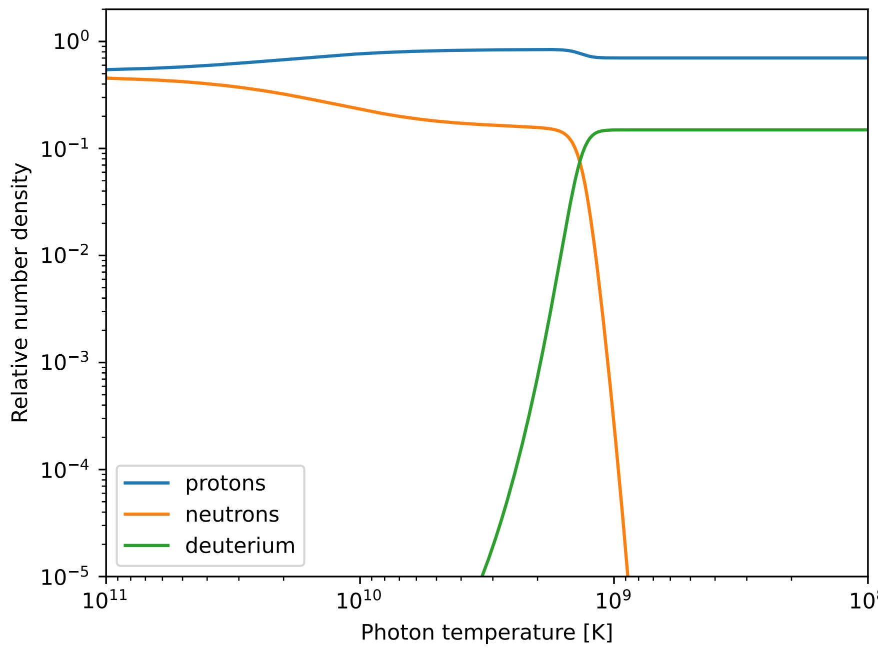

Figure 2: Number densities for protons, neutrons, and deuterium, relative to the baryon density, as functions of temperature.

the abundances:

$$\begin{aligned}
\frac{\mathrm{d}Y_p}{\mathrm{d}\ln T} &= -\frac{1}{H}\left(Y_n\Gamma_{n\to p} - Y_p\Gamma_{p\to n} + Y_D\lambda(D) - [np]Y_nY_p\right), \\
\frac{\mathrm{d}Y_n}{\mathrm{d}\ln T} &= -\frac{1}{H}\left(Y_p\Gamma_{p\to n} - Y_n\Gamma_{n\to p} + Y_D\lambda(D) - [np]Y_nY_p\right), \\
\frac{\mathrm{d}Y_D}{\mathrm{d}\ln T} &= -\frac{1}{H}\left([np]Y_nY_p - Y_D\lambda(D)\right). \qquad (35)
\end{aligned}$$

where $[np] \equiv \Gamma_{np\to D}$, and $\lambda(D) \equiv \Gamma_{D\to np}$. Additionaly the initial value for $Y_D = 0$, and Y_n and Y_p remain the same.

Results are found in Fig. 2'

Task i

Again, we are looking to solve differentials on the form expressed task g, although this time we include tritium, helium 3, helium 4, lithium 7, and beryllium 7. We modify the differential equations accordingly according to Eq. 32,

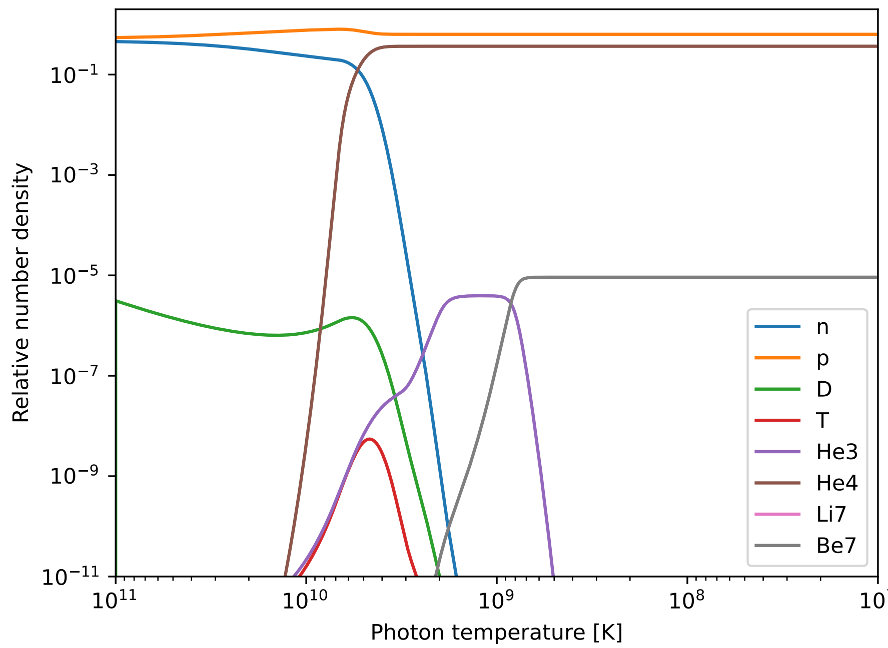

Figure 3: Number densities for Protons, neutrons, deuterium, tritium, helium 3, helium 4, lithium 7, and beryllium 7, relative to the baryon density, as functions of temperature [K].

and initial conditions identical to those in task h, with 0 for all other initial abundances not specified. Results are shown in Fig. 3. The figure shows that we're not too far away from successfully solving the Boltzmann equations, yet ultimately failed. The results deviate from expected by lithium-7 abundance so scarce that it doesn't show up when the figure is limited to $yin[10^{-11}, 2]$, helium 3 abundance drops off unexpectedly, when it should've maintained its approximate abundance where the curve flattens out for a bit. Tritium also peaks way too low, and drops off again, and it remains unclear if it flattens out like it should. Deuterium starts off **WAY** too abundant, and completely dissapears, while it shouldve achived constant abundance. The model gets the helium 4 and proton abundances reasonably correct, however!

Task j

I have insufficiently allocated time to make it this far numerically, so i will describe what steps i would've taken. Firstly assuming a succesful model from task i, i would create some array for Ω_{b0}, in which i effectively just solve task i for each element in the array, which would span $[0.01, 1]$. the resulting Y_i, which now are arrays of floating point numbers, not just a single floating point numbers, can now compute the relic number abundances. N number of Ω_{b0} is N different models, the best of which is found by the chi squared, i.e. minimizing some $\chi^2(\mathbf{p})$ for parameters $\mathbf{p}$:

$$\chi^2(\mathbf{p}) = \sum_i \frac{(d_i(\mathbf{p}) - d_i)^2}{\sigma_i^2}, \tag{36}$$

where $d_i(\mathbf{p})$ is some model, (i.e. a solution to some Ω_{b0}, d_i is the observed data provided in the task, and σ_i^2 is the error of the observed data. the index i indicates a particular quantity measured or computed. The principle of chi squared is that whichever list of parameters minimizes it the most is the best fitting one. Given that some significant fraction of energy mass density Ω_{m0} is in the form of dark, matter, $\omega_{b0} < \Omega_{m0}$ would infer that dark matter can not consist solely of baryonic matter. if they are close in value, this would further infer that most baryonic matter is dark. My confidence in the latter statement is weak, as i am not sure if baryonic mass can even be dark matter, given that all of the relative number abundances interacted with in this project interact with EM forces.

Task k

As with task j, i wouldve solved task i for all $N_{\text{eff}} = 1, 2, \ldots, 5$, individually, leaving 5 models. To find the best one i would've used the chi squared scheme to find the best fitted model. It is my understanding that the standard model predicts 3 neutrinos species exist.