## Exercise 6.4

**(1)**

```
let f x = 1 in f f end
```

$$\text{(P8)}\ \frac{\text{(P1)}\ \dfrac{\rho \vdash 1 : \text{int}}{\rho[x \mapsto t_x, f \mapsto t_x \mapsto t_r] \vdash 1 : t_r} \qquad \dfrac{\text{(P3)}\ \dfrac{\rho[f \mapsto \forall\alpha.\,\alpha \mapsto \text{int}](f) = \forall\alpha.\,\alpha \mapsto \text{int}}{\rho[f \mapsto \forall\alpha.\,\alpha \mapsto \text{int}] \vdash f : \forall\alpha.\,\alpha \to \text{int}} \quad \dfrac{\rho[f \mapsto \forall\alpha.\,\alpha \mapsto \text{int}](f) = \forall\alpha.\,\alpha \mapsto \text{int}}{\rho[f \mapsto \forall\alpha.\,\alpha \mapsto \text{int}] \vdash f : \forall\alpha.\,\alpha \to \text{int}}}{\rho[f \mapsto \forall\alpha_1 \ldots \alpha_n.\,t_x \mapsto t_r] \vdash f\ f :\ t_r}}{\rho \vdash \text{let } f\ x = 1 \text{ in } f\ f \text{ end} : {'a} \to \text{int}}$$

Polymorphic because $x$ is $\alpha$.

**(2)**

```
let f x = if x < 10 then 42 else f(x+1) in f 20 end
```

**Condition:**

$$\text{(P5)}\ \frac{\text{(P3)}\ \dfrac{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}](n) = \text{int}}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash n : \text{int}} \quad \text{(P1)}\ \dfrac{}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash 10 : \text{int}}}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash x < 10 : \text{bool}}$$

**Body derivation:**

$$\text{(Condition)}\quad \text{(P1)}\ \frac{}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash 42 : \text{int}} \qquad \text{(P3)}\ \frac{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}](f) = \text{int} \to \text{int}}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash f : \text{int} \to \text{int}} \qquad \text{(P4)}\ \frac{\text{(P3)}\ \dfrac{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}](x) = \text{int}}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash x : \text{int}} \quad \text{(P1)}\ \dfrac{}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash 1 : \text{int}}}{\rho[x \mapsto \text{int}, f \mapsto \text{int} \mapsto \text{int}] \vdash n + 1 : \text{int}}$$

$$\rho[x \mapsto t_x, f \mapsto t_x \mapsto t_r] \vdash \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) : t_r$$

**Result:**

$$\text{(P8)}\ \frac{\text{(Body derivation)}\ \dfrac{\dfrac{\rho[f \mapsto \text{int} \mapsto \text{int}](f) = \text{int} \mapsto \text{int}}{\rho[f \mapsto \text{int} \mapsto \text{int}] \vdash f : \text{int} \to \text{int}} \quad \text{(P1)}\ \dfrac{}{\rho \vdash 20 :\ \text{int}}}{\rho[f \mapsto \forall\alpha_1 \ldots \alpha_n.\,t_x \mapsto t_r] \vdash f\ 20 : t_r}}{\rho \vdash \text{let } f\ x = \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) \text{ in } f\ 20 \text{ end} : \text{int} \to \text{int}}$$