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$$w_1 = 0.8$$

$$w_2 = 0.4$$

$$b = -4.0$$

1. Compute Probabilities $z = w_1x_1 + w_2x_2 + b$ then $\hat{y} = \sigma(z)$

Customer A ($x_1 = 1, x_2 = 4, y = 0$)

$$z = 0.8(1) + 0.4(4) + (-4.0) = \boxed{-1.6}$$

$$\hat{y} = \frac{1}{1 + e^{-(-1.6)}} = \boxed{0.168}$$

Customer B ($x_1 = 2, x_2 = 3, y = 0$)

$$z = 0.8(2) + 0.4(3) + (-4.0) = \boxed{-1.2}$$

$$\hat{y} = \frac{1}{1 + e^{-(-1.2)}} = \boxed{0.231}$$

Customer C ($x_1 = 3, x_2 = 7, y = 1$)

$$z = 0.8(3) + 0.4(7) + (-4.0) = \boxed{1.2}$$

$$\hat{y} = \frac{1}{1 + e^{-1.2}} = \boxed{0.769}$$

Customer D ($x_1 = 5, x_2 = 2, y = 1$)

$$z = 0.8(5) + 0.4(2) + (-4.0) = \boxed{0.8}$$

$$\hat{y} = \frac{1}{1 + e^{-0.8}} = \boxed{0.690}$$

Customer E ($x_1 = 6, x_2 = 6, y = 1$)

$$z = 0.8(6) + 0.4(6) + (-4.0) = \boxed{3.2}$$

$$\hat{y} = \frac{1}{1 + e^{-3.2}} = \boxed{0.961}$$

Customer	Time on site (x_1)	Pages viewed (x_2)	Purchase (y)	\hat{y}
A	1	4	0	0.168
B	2	3	0	0.231
C	3	7	1	0.769
D	5	2	1	0.690
E	6	6	1	0.961

2. Compute Average Loss

Customer A ($y=0$, $\hat{y}=0.168$)

$$\text{loss}_A = -(0 \times \ln(0.168) + (1-0) \times \ln(1-0.168))$$

$$= -(0 + \ln(0.832))$$

$$= -(0.184)$$

$$= \boxed{0.184}$$

Customer B ($y=0$, $\hat{y}=0.231$)

$$\text{loss}_B = -(0 \times \ln(0.231) + (1-0) \times \ln(1-0.231))$$

$$= -(0 + \ln(0.769))$$

$$= -(-0.262)$$

$$= \boxed{0.262}$$

Customer C ($y=1$, $\hat{y}=0.769$)

$$\text{loss}_C = -(1 \times \ln(0.769) + (1-1) \times \ln(1-0.769))$$

$$= -(\ln(0.769) + 0)$$

$$= -(-0.262)$$

$$= \boxed{0.262}$$

Customer D ($y=1$, $\hat{y}=0.690$)

$$\text{loss}_D = -(1 \times \ln(0.690) + (1-1) \times \ln(1-0.690))$$

$$= -(\ln(0.690) + 0)$$

$$= -(-0.371)$$

$$= \boxed{0.371}$$

Customer E ($y=1$, $\hat{y}=0.961$)

$$\text{loss}_E = -(1 \times \ln(0.961) + (1-1) \times \ln(1-0.961))$$

$$= -(\ln(0.961) + 0)$$

$$= -(-0.039)$$

$$= \boxed{0.039}$$

Customer	Time on site (x_1)	Pages viewed (x_2)	(y)	\hat{y}	Loss
A	1	4	0	0.168	0.184
B	2	3	0	0.231	0.262
C	3	7	1	0.769	0.262
D	5	2	1	0.690	0.371
E	6	6	1	0.961	0.039

$$\text{Average Loss} = \frac{0.184 + 0.262 + 0.262 + 0.371 + 0.039}{5} = \boxed{0.2236}$$

3. Update the slope and intercept using Gradient Descent
learning rate $\eta = 0.1$

Compute $(\hat{y}_i - y_i)$ for each customer

$$A: 0.168 - 0 = \boxed{0.168}$$

$$B: 0.231 - 0 = \boxed{0.231}$$

$$C: 0.769 - 1 = \boxed{-0.231}$$

$$D: 0.690 - 1 = \boxed{-0.310}$$

$$E: 0.961 - 1 = \boxed{-0.039}$$

Compute Gradients

$$\begin{aligned} \frac{\partial L}{\partial w_1} &= \frac{1}{5} ((0.168 \times 1) + (0.231 \times 2) + (-0.231 \times 3) + (-0.310 \times 5) + (-0.039 \times 6)) \\ &= \frac{1}{5} (0.168 + 0.462 - 0.693 - 1.550 - 0.234) \\ &= \frac{1}{5} (-1.847) \\ &= \boxed{-0.3694} \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial w_2} &= \frac{1}{5} ((0.168 \times 4) + (0.231 \times 3) + (-0.231 \times 7) + (-0.310 \times 2) + (-0.039 \times 6)) \\ &= \frac{1}{5} (0.672 + 0.693 - 1.617 - 0.620 - 0.234) \\ &= \frac{1}{5} (-1.106) \\ &= \boxed{-0.2212} \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial b} &= \frac{1}{5} (0.168 + 0.231 - 0.231 - 0.310 - 0.039) \\ &= \frac{1}{5} (-0.181) \\ &= \boxed{-0.0362} \end{aligned}$$

Update Parameters ($n = 0.1$)

$$\text{new } m_1 = 0.8 - 0.1 \times (-0.3694)$$

$$= 0.8 + 0.03694$$

$$= \boxed{0.83694}$$

$$\bullet \text{ new } m_1 = 0.83694$$

$$\bullet \text{ new } m_2 = 0.42212$$

$$\bullet \text{ new } b = -3.99638$$

$$\text{new } m_2 = 0.4 - 0.1 \times (-0.2212)$$

$$= 0.4 + 0.02212$$

$$= \boxed{0.42212}$$

$$\text{new } b = -4.0 - 0.1 \times (-0.0362)$$

$$= -4.0 + 0.00362$$

$$= \boxed{-3.99638}$$

4. Compute new probabilities using the new slopes and intercept

Customer A ($x_1 = 1, x_2 = 4$)

$$z = 0.83694(1) + 0.42212(4) + (-3.99638)$$

$$= \boxed{-1.47096}$$

$$\hat{y} = \frac{1}{1 + e^{-(-1.47096)}}$$

$$= \boxed{0.187}$$

Customer B ($x_1 = 2, x_2 = 3$)

$$z = 0.83694(2) + 0.42212(3) + (-3.99638)$$

$$= \boxed{-1.05614}$$

$$\hat{y} = \frac{1}{1 + e^{-(-1.05614)}}$$

$$= \boxed{0.258}$$

Customer C ($x_1 = 3, x_2 = 7$)

$$z = 0.83694(3) + 0.42212(7) + (-3.99638)$$

$$= \boxed{1.46928}$$

$$\hat{y} = \frac{1}{1 + e^{-1.46928}}$$

$$= \boxed{0.813}$$

Customer D ($x_1 = 5, x_2 = 2$)

$$z = 0.83694(5) + 0.42212(2) + (-3.99638)$$

$$= \boxed{1.03256}$$

$$\hat{y} = \frac{1}{1 + e^{-1.03256}}$$

$$= \boxed{0.737}$$

Customer E ($x_1 = 6, x_2 = 6$)

$$z = 0.83694(6) + 0.42212(6) + (-3.99638)$$

$$= 3.55798$$

$$\hat{y} = \frac{1}{1 + e^{-3.55798}}$$

$$= 0.972$$

New Predicted Probabilities

Customer	x_1	x_2	y	new \hat{y}
A	1	4	0	0.187
B	2	3	0	0.258
C	3	7	1	0.813
D	5	2	1	0.737
E	6	6	1	0.972

5. Compute new Average Loss

Customer A ($y = 0, \hat{y} = 0.187$)

$$\text{loss}_A = -(0 \times \ln(0.187) + (1-0) \times \ln(1-0.187))$$

$$= -\ln(0.813) \Rightarrow -(-0.207)$$

$$= 0.207$$

Customer B ($y = 0, \hat{y} = 0.258$)

$$\text{loss}_B = -(0 \times \ln(0.258) + (1-0) \times \ln(1-0.258))$$

$$= -\ln(0.742) \Rightarrow -(-0.298)$$

$$= 0.298$$

Customer C ($y = 1, \hat{y} = 0.813$)

$$\text{loss}_C = -(1 \times \ln(0.813) + (1-1) \times \ln(1-0.813))$$

$$= -\ln(0.813) \Rightarrow -(-0.207)$$

$$= 0.207$$

Customer D ($y = 1, \hat{y} = 0.737$)

$$\text{loss}_D = -(1 \times \ln(0.737) + (1-1) \times \ln(1-0.737))$$

$$= -\ln(0.737) \Rightarrow -(-0.305)$$

$$= 0.305$$

Customer E ($y = 1, \hat{y} = 0.972$)

$$\text{loss}_E = -(1 \times \ln(0.972) + (1-1) \times \ln(1-0.972)) \Rightarrow -\ln(0.972)$$

$$= -(-0.028)$$

$$= 0.028$$

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$$\text{NewLoss}_{\text{avg}} = \frac{0.207 + 0.298 + 0.207 + 0.305 + 0.028}{5} = \boxed{0.209}$$

Customer	X_1	X_2	Y	\hat{Y}	new loss
A	1	9	0	0.187	0.207
B	2	3	0	0.258	0.298
C	3	7	1	0.813	0.207
D	5	2	1	0.737	0.305
E	6	6	1	0.972	0.028
					0.209