Directions for Numeric Entry questions

Enter your answer in the answer box(es) below the question.

- Your answer may be an integer, a decimal, or a fraction, and it may be negative.
- If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.
- Enter the exact answer unless the question asks you to round your answer.

SET 1. Discrete Questions: Easy

Quantitative Comparison

For Questions 1 to 6, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.
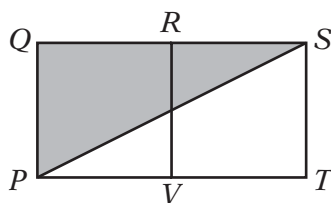
Ⓐ   Quantity A is greater.
Ⓑ   Quantity B is greater.
Ⓒ   The two quantities are equal.
Ⓓ   The relationship cannot be determined from the information given.

A symbol that appears more than once in a question has the same meaning throughout the question.

Emma spent $75 buying a used bicycle and $27 repairing it. Then she sold the bicycle for 40 percent more than the total amount she spent buying and repairing it.

	Quantity A	Quantity B	
1.	The price at which Emma sold the bicycle	$140	Ⓐ Ⓑ Ⓒ Ⓓ

In the figure above, squares *PQRV* and *VRST* have sides of length 6.

Quantity A	Quantity B	
2. The area of the shaded region	36	Ⓐ Ⓑ Ⓒ Ⓓ

In 2009 the property tax on each home in Town *X* was *p* percent of the assessed value of the home, where *p* is a constant. The property tax in 2009 on a home in Town *X* that had an assessed value of $125,000 was $2,500.
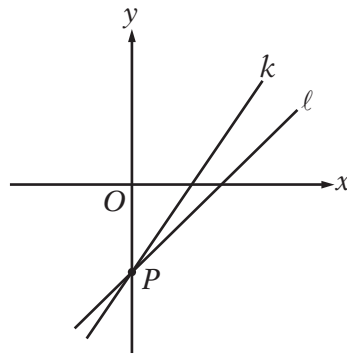
Quantity A	Quantity B	
3. The property tax in 2009 on a home in Town *X* that had an assessed value of $160,000	$3,000	Ⓐ Ⓑ Ⓒ Ⓓ

$$x + y = -1$$

Quantity A	Quantity B	
4. x	y	Ⓐ Ⓑ Ⓒ Ⓓ

r, *s*, and *t* are three consecutive odd integers such that $r < s < t$.

Quantity A	Quantity B	
5. $r + s + 1$	$s + t - 1$	Ⓐ Ⓑ Ⓒ Ⓓ

Quantity A	Quantity B				
6. The slope of line k	The slope of line ℓ	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Multiple-choice Questions—Select One Answer Choice

For Questions 7 to 11, select a single answer choice.
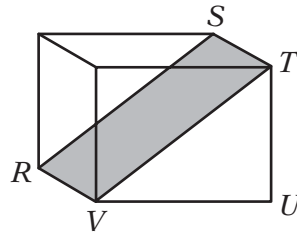

7. In the figure above, what is the value of $\dfrac{x+y+z}{45}$?

Ⓐ 2

Ⓑ 3

Ⓒ 4

Ⓓ 5

Ⓔ 6

8. A certain store sells two types of pens: one type for $2 per pen and the other type for $3 per pen. If a customer can spend up to $25 to buy pens at the store and there is no sales tax, what is the greatest number of pens the customer can buy?

Ⓐ 9

Ⓑ 10

Ⓒ 11

Ⓓ 12

Ⓔ 20

9. If $y = 3x$ and $z = 2y$, what is $x + y + z$ in terms of x ?

  Ⓐ $10x$

  Ⓑ $9x$

  Ⓒ $8x$

  Ⓓ $6x$

  Ⓔ $5x$

10. A certain shipping service charges an insurance fee of $0.75 when shipping any package with contents worth $25.00 or less and an insurance fee of $1.00 when shipping any package with contents worth over $25.00. If Dan uses the shipping company to ship three packages with contents worth $18.25, $25.00, and $127.50, respectively, what is the total insurance fee that the company charges Dan to ship the three packages?

  Ⓐ $1.75

  Ⓑ $2.25

  Ⓒ $2.50

  Ⓓ $2.75

  Ⓔ $3.00

11. If 55 percent of the people who purchase a certain product are female, what is the ratio of the number of females who purchase the product to the number of males who purchase the product?

  Ⓐ 11 to 9

  Ⓑ 10 to 9

  Ⓒ   9 to 10

  Ⓓ   9 to 11

  Ⓔ   5 to 9

Numeric Entry

> **For Questions 12 and 13, enter your answer in the answer box(es) below the question.**
>
> ● **Your answer may be an integer, a decimal, or a fraction, and it may be negative.**
> ● **If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.**
> ● **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.**
> ● **Enter the exact answer unless the question asks you to round your answer.**

12. In the rectangular solid above, $TU = 3$, $UV = 4$, and $VR = 2$. What is the area of the shaded rectangular region?

13. A list of numbers has a mean of 8 and a standard deviation of 2.5. If x is a number in the list that is 2 standard deviations above the mean, what is the value of x ?

$x =$

Multiple-choice Questions—Select One or More Answer Choices

For Question 14, select all the answer choices that apply.
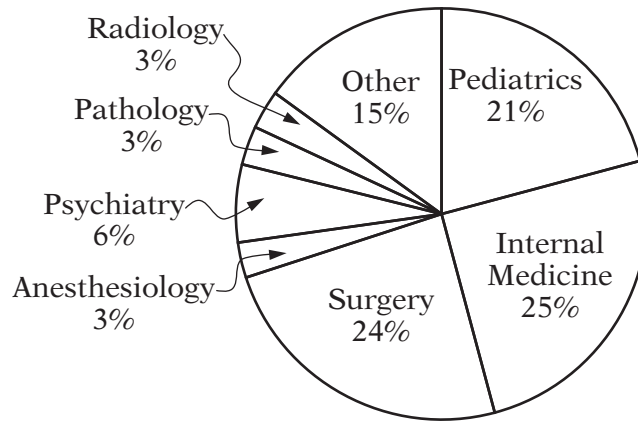
14. The circle graph above shows the distribution of 200,000 physicians by specialty. Which of the following sectors of the circle graph represent more than 40,000 physicians?

Indicate all such sectors.

A Pediatrics
B Internal Medicine
C Surgery
D Anesthesiology
E Psychiatry

SET 2. Discrete Questions: Medium

Quantitative Comparison

For Questions 1 to 5, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.

Ⓐ   Quantity A is greater.
Ⓑ   Quantity B is greater.
Ⓒ   The two quantities are equal.
Ⓓ   The relationship cannot be determined from the information given.

A symbol that appears more than once in a question has the same meaning throughout the question.

Machine R, working alone at a constant rate, produces x units of a product in 30 minutes, and machine S, working alone at a constant rate, produces x units of the product in 48 minutes, where x is a positive integer.

Quantity A	Quantity B

1. The number of units of the product that machine R, working alone at its constant rate, produces in 3 hours   The number of units of the product that machine S, working alone at its constant rate, produces in 4 hours   Ⓐ   Ⓑ   Ⓒ   Ⓓ

Frequency Distribution for List X

Number	1	2	3	5
Frequency	10	20	18	12

Frequency Distribution for List Y

Number	6	7	8	9
Frequency	24	17	10	9

List X and list Y each contain 60 numbers. Frequency distributions for each list are given above. The average (arithmetic mean) of the numbers in list X is 2.7, and the average of the numbers in list Y is 7.1. List Z contains 120 numbers: the 60 numbers in list X and the 60 numbers in list Y.

Quantity A	Quantity B

2. The average of the 120 numbers in list Z The median of the 120 numbers in list Z Ⓐ   Ⓑ   Ⓒ   Ⓓ

In the figure above, the diameter of the circle is 10.

Quantity A	Quantity B				
3. The area of quadrilateral *ABCD*	40	Ⓐ	Ⓑ	Ⓒ	Ⓓ

$$x^2 y > 0$$
$$xy^2 < 0$$

Quantity A	Quantity B				
4. x	y	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Among the 9,000 people attending a football game at College *C*, there were *x* students from College *C* and *y* students who were <u>not</u> from College *C*.

Quantity A	Quantity B				
5. The number of people attending the game who were <u>not</u> students	$9,000 - x - y$	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Multiple-choice Questions—Select One Answer Choice

For Questions 6 to 10, select a single answer choice.

6. If $x \neq 0$, which of the following is equivalent to $\dfrac{x(x^2)^3}{x^2}$?

    Ⓐ x^2
    Ⓑ x^3
    Ⓒ x^4
    Ⓓ x^5
    Ⓔ x^6

7. The figure above shows the graph of the function f in the xy-plane. What is the value of $f(f(-1))$?

    Ⓐ −2

    Ⓑ −1

    Ⓒ   0

    Ⓓ   1

    Ⓔ   2

8. If $\dfrac{d - 3n}{7n - d} = 1$, which of the following statements describes d in terms of n?

    Ⓐ d is 4 less than n.

    Ⓑ d is 4 more than n.

    Ⓒ d is $\dfrac{3}{7}$ of n.

    Ⓓ d is 2 times n.

    Ⓔ d is 5 times n.

9. By weight, liquid A makes up 8 percent of solution R and 18 percent of solution S. If 3 grams of solution R are mixed with 7 grams of solution S, then liquid A accounts for what percent of the weight of the resulting solution?

    Ⓐ 10%

    Ⓑ 13%

    Ⓒ 15%

    Ⓓ 19%

    Ⓔ 26%

10. Of the 700 members of a certain organization, 120 are lawyers. Two members of the organization will be selected at random. Which of the following is closest to the probability that <u>neither</u> of the members selected will be a lawyer?

    Ⓐ 0.5

    Ⓑ 0.6

    Ⓒ 0.7

    Ⓓ 0.8

    Ⓔ 0.9

Numeric Entry

For Questions 11 and 12, enter your answer in the answer box(es) below the question.

- Your answer may be an integer, a decimal, or a fraction, and it may be negative.
- If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.
- Enter the exact answer unless the question asks you to round your answer.

Garden   Walkway

11. The figure above represents a rectangular garden with a walkway around it. The garden is 18 feet long and 12 feet wide. The walkway is uniformly 3 feet wide, and its edges meet at right angles. What is the area of the walkway?

   [_____] square feet

12. Line k lies in the xy-plane. The x-intercept of line k is -4, and line k passes through the midpoint of the line segment whose endpoints are $(2, 9)$ and $(2, 0)$. What is the slope of line k ?

   Give your answer as a fraction.

   $$\frac{[\quad]}{[\quad]}$$

Multiple-choice Questions—Select One or More Answer Choices

For Questions 13 and 14, select all the answer choices that apply.

13. If the lengths of two sides of a triangle are 5 and 9, respectively, which of the following could be the length of the third side of the triangle?

    Indicate <u>all</u> such lengths.

    A 3
    B 5
    C 8
    D 15
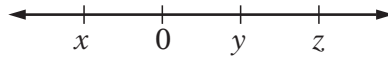
14. On the number line shown above, the tick marks are equally spaced. Which of the following statements about the numbers x, y, and z must be true?

    Indicate <u>all</u> such statements.

    A $xyz < 0$
    B $x + z = y$
    C $z(y - x) > 0$

SET 3. Discrete Questions: Hard

Quantitative Comparison

For Questions 1 to 6, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

A symbol that appears more than once in a question has the same meaning throughout the question.

In the figure above, *ABCD* is a parallelogram.

	Quantity A	Quantity B	
1.	The area of *ABCD*	24	Ⓐ Ⓑ Ⓒ Ⓓ

In the course of an experiment, 95 measurements were recorded, and all of the measurements were integers. The 95 measurements were then grouped into 7 measurement intervals. The graph above shows the frequency distribution of the 95 measurements by measurement interval.

	Quantity A	Quantity B	
2.	The average (arithmetic mean) of the 95 measurements	The median of the 95 measurements	Ⓐ Ⓑ Ⓒ Ⓓ

---

x is an integer greater than 1.

	Quantity A	Quantity B	
3.	3^{x+1}	4^x	Ⓐ Ⓑ Ⓒ Ⓓ

A, B, and C are three rectangles. The length and width of rectangle A are 10 percent greater and 10 percent less, respectively, than the length and width of rectangle C. The length and width of rectangle B are 20 percent greater and 20 percent less, respectively, than the length and width of rectangle C.

	Quantity A	Quantity B	
	Quantity A	Quantity B	
4.	The area of rectangle A	The area of rectangle B	Ⓐ Ⓑ Ⓒ Ⓓ

The random variable X is normally distributed. The values 650 and 850 are at the 60th and 90th percentiles of the distribution of X, respectively.

	Quantity A	Quantity B	
5.	The value at the 75th percentile of the distribution of X	750	Ⓐ Ⓑ Ⓒ Ⓓ

Set S consists of all positive integers less than 81 that are <u>not</u> equal to the square of an integer.

	Quantity A	Quantity B	
6.	The number of integers in set S	72	Ⓐ Ⓑ Ⓒ Ⓓ

Multiple-choice Questions—Select One Answer Choice

For Questions 7 to 12, select a single answer choice.

7. A manager is forming a 6-person team to work on a certain project. From the 11 candidates available for the team, the manager has already chosen 3 to be on the team. In selecting the other 3 team members, how many different combinations of 3 of the remaining candidates does the manager have to choose from?

    (A) 6
    (B) 24
    (C) 56
    (D) 120
    (E) 462

8. Which of the following could be the graph of all values of x that satisfy the inequality $2 - 5x \le -\dfrac{6x - 5}{3}$?

    (A)
    (B)
    (C)
    (D)
    (E)
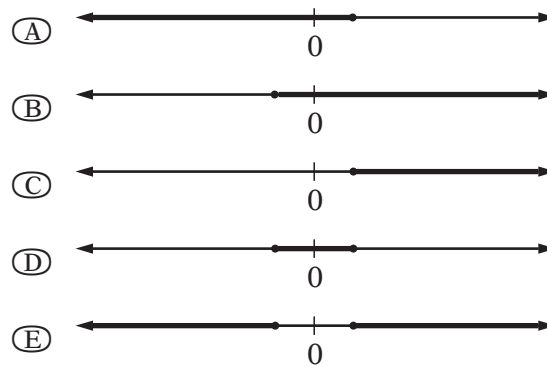
9. If $1 + x + x^2 + x^3 = 60$, then the average (arithmetic mean) of x, x^2, x^3, and x^4 is equal to which of the following?
    (A) $12x$
    (B) $15x$
    (C) $20x$
    (D) $30x$
    (E) $60x$

10. Parallelogram *OPQR* lies in the *xy*-plane, as shown in the figure above. The coordinates of point *P* are $(2, 4)$ and the coordinates of point *Q* are $(8, 6)$. What are the coordinates of point *R* ?

    (A) $(3, 2)$

    (B) $(3, 3)$

    (C) $(4, 4)$

    (D) $(5, 2)$

    (E) $(6, 2)$

11. The relationship between the area *A* of a circle and its circumference *C* is given by the formula $A = kC^2$, where *k* is a constant. What is the value of *k* ?
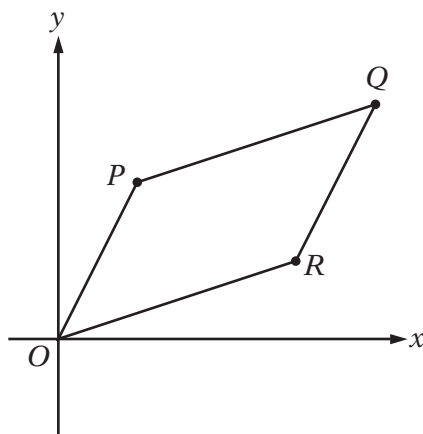
    (A) $\dfrac{1}{4\pi}$

    (B) $\dfrac{1}{2\pi}$

    (C) $\dfrac{1}{4}$

    (D) 2π

    (E) $4\pi^2$

12. The sequence of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$ is defined by $a_n = \dfrac{1}{n} - \dfrac{1}{n+2}$ for each integer $n \geq 1$. What is the sum of the first 20 terms of this sequence?

    (A) $\left(1 + \dfrac{1}{2}\right) - \dfrac{1}{20}$

    (B) $\left(1 + \dfrac{1}{2}\right) - \left(\dfrac{1}{21} + \dfrac{1}{22}\right)$

    (C) $1 - \left(\dfrac{1}{20} + \dfrac{1}{22}\right)$

    (D) $1 - \dfrac{1}{22}$

    (E) $\dfrac{1}{20} - \dfrac{1}{22}$

Numeric Entry

For Question 13, enter your answer in the answer box(es) below the question.

- **Your answer may be an integer, a decimal, or a fraction, and it may be negative.**
- **If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.**
- **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.**
- **Enter the exact answer unless the question asks you to round your answer.**

Y	Frequency
$\dfrac{1}{2}$	2
$\dfrac{3}{4}$	7
$\dfrac{5}{4}$	8
$\dfrac{3}{2}$	8
$\dfrac{7}{4}$	9

13. The table above shows the frequency distribution of the values of a variable Y. What is the mean of the distribution?

Give your answer to the <u>nearest 0.01</u>.

[   ]

Multiple-choice Questions—Select One or More Answer Choices

For Questions 14 and 15, select all the answer choices that apply.

14. Let S be the set of all positive integers n such that n^2 is a multiple of both 24 and 108. Which of the following integers are divisors of every integer n in S ?

Indicate <u>all</u> such integers.

- [A] 12
- [B] 24
- [C] 36
- [D] 72

15. The range of the heights of the female students in a certain class is 13.2 inches, and the range of the heights of the male students in the class is 15.4 inches.

Which of the following statements <u>individually</u> provide(s) sufficient additional information to determine the range of the heights of all the students in the class?

Indicate <u>all</u> such statements.

[A] The tallest male student in the class is 5.8 inches taller than the tallest female student in the class.

[B] The median height of the male students in the class is 1.1 inches greater than the median height of the female students in the class.

[C] The average (arithmetic mean) height of the male students in the class is 4.6 inches greater than the average height of the female students in the class.

SET 4. Data Interpretation Sets

For Questions 1 to 7, select a single answer choice unless otherwise directed.

Questions 1 to 3 are based on the following data.

PERCENT OF FEMALE FACULTY AND PERCENT OF MALE FACULTY
AT UNIVERSITY *X*, BY FIELD
Total female faculty: 200
Total male faculty: 250

Medium Question

1. There are 275 students in the field of engineering at University *X*.
   Approximately what is the ratio of the number of students in engineering to
   the number of faculty in engineering?

   Ⓐ   8 to 1

   Ⓑ   10 to 1

   Ⓒ   12 to 1

   Ⓓ   14 to 1

   Ⓔ   20 to 1

Medium Question

2. Approximately what percent of the faculty in humanities are male?

   Ⓐ   35%

   Ⓑ   38%

   Ⓒ   41%

   Ⓓ   45%

   Ⓔ   51%

For Question 3, use the directions for Numeric Entry questions.

Hard Question

3. For the biological sciences and health sciences faculty combined, $\frac{1}{3}$ of the female and $\frac{2}{9}$ of the male faculty members are tenured professors. What fraction of all the faculty members in those two fields combined are tenured professors?

$$\frac{\boxed{}}{\boxed{}}$$

Questions 4 to 7 are based on the following data.

VALUE OF IMPORTS TO AND EXPORTS FROM COUNTRY *T,* 2000–2009
(in United States dollars)
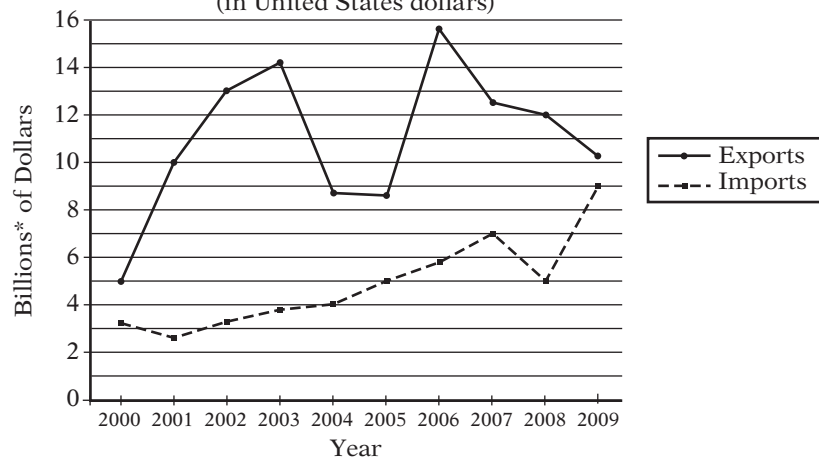
*1 billion = 1,000,000,000

For Question 4, select all the answer choices that apply.

Easy Question

4. For which of the eight years from 2001 to 2008 did exports exceed imports by more than $5 billion?

    Indicate <u>all</u> such years.

    [A] 2001
    [B] 2002
    [C] 2003
    [D] 2004
    [E] 2005
    [F] 2006
    [G] 2007
    [H] 2008

Medium Question

5. Which of the following is closest to the average (arithmetic mean) of the 9 changes in the value of imports between consecutive years from 2000 to 2009 ?

    (A) $260 million

    (B) $320 million

    (C) $400 million

    (D) $480 million

    (E) $640 million

Medium Question

6. In 2008 the value of exports was approximately what percent greater than the value of imports?

    (A)   40%

    (B)   60%

    (C)   70%

    (D) 120%

    (E) 140%

Hard Question

7. If it were discovered that the value of imports shown for 2007 was incorrect and should have been $5 billion instead, then the average (arithmetic mean) value of imports per year for the 10 years shown would have been approximately how much less?

    (A) $200 million

    (B) $50 million

    (C) $20 million
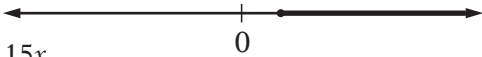
    (D) $7 million

    (E) $5 million

ANSWER KEY

SET 1. Discrete Questions: Easy

1. **Choice A**: Quantity A is greater.
2. **Choice C**: The two quantities are equal.
3. **Choice A**: Quantity A is greater.
4. **Choice D**: The relationship cannot be determined from the information given.
5. **Choice B**: Quantity B is greater.
6. **Choice A**: Quantity A is greater.
7. **Choice C**: 4
8. **Choice D**: 12
9. **Choice A**: $10x$
10. **Choice C**: $2.50
11. **Choice A**: 11 to 9
12. **10**
13. **13**
14. **Choice A**: Pediatrics
    AND
    Choice B: Internal Medicine
    AND
    Choice C: Surgery

SET 2. Discrete Questions: Medium

1. **Choice A**: Quantity A is greater.
2. **Choice B**: Quantity B is greater.
3. **Choice D**: The relationship cannot be determined from the information given.
4. **Choice B**: Quantity B is greater.
5. **Choice C**: The two quantities are equal.
6. **Choice D**: x^5
7. **Choice D**: 1
8. **Choice E**: d is 5 times n.
9. **Choice C**: 15%
10. **Choice C**: 0.7
11. **216**
12. $\frac{3}{4}$ (or any equivalent fraction)
13. **Choice B**: 5
    AND
    Choice C: 8
14. **Choice A**: $xyz < 0$
    AND
    Choice B: $x + z = y$
    AND
    Choice C: $z(y - x) > 0$

SET 3. Discrete Questions: Hard

1. **Choice B**: Quantity B is greater.
2. **Choice A**: Quantity A is greater.
3. **Choice D**: The relationship cannot be determined from the information given.
4. **Choice A**: Quantity A is greater.
5. **Choice B**: Quantity B is greater.
6. **Choice C**: The two quantities are equal.
7. **Choice C**: 56
8. **Choice C**:
9. **Choice B**: $15x$
10. **Choice E**: $(6, 2)$
11. **Choice A**: $\dfrac{1}{4\pi}$
12. **Choice B**: $\left(1 + \dfrac{1}{2}\right) - \left(\dfrac{1}{21} + \dfrac{1}{22}\right)$
13. **1.29**
14. **Choice A**: 12
    AND
    Choice C: 36
15. **Choice A**: The tallest male student in the class is 5.8 inches taller than the tallest female student in the class.

SET 4. Data Interpretation Sets

1. **Choice A**: 8 to 1
2. **Choice E**: 51%
3. $\dfrac{24}{87}$ (or any equivalent fraction)
4. **Choice A**: 2001
    AND
    Choice B: 2002
    AND
    Choice C: 2003
    AND
    Choice F: 2006
    AND
    Choice G: 2007
    AND
    Choice H: 2008
5. **Choice E**: $640 million
6. **Choice E**: 140%
7. **Choice A**: $200 million

Answers and Explanations

For the practice questions in this chapter, use the following directions.

General Directions

For each question, indicate the best answer, using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, **are not necessarily** drawn to scale. That is, you should **not** assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities by sight or by measurement.

Coordinate systems, such as *xy*-planes and number lines, **are** drawn to scale; therefore, you can read, estimate, or compare quantities in such figures by sight or by measurement.

Graphical data presentations, such as bar graphs, circle graphs, and line graphs, **are** drawn to scale; therefore, you can read, estimate, or compare data values by sight or by measurement.

Directions for Quantitative Comparison questions

Compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

A symbol that appears more than once in a question has the same meaning throughout the question.

Directions for Numeric Entry questions

Enter your answer in the answer box(es) below the question.

- Your answer may be an integer, a decimal, or a fraction, and it may be negative.
- If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.
- Enter the exact answer unless the question asks you to round your answer.

SET 1. Discrete Questions: Easy

Quantitative Comparison

For Questions 1 to 6, use the directions for Quantitative Comparison questions.

Emma spent $75 buying a used bicycle and $27 repairing it. Then she sold the bicycle for 40 percent more than the total amount she spent buying and repairing it.

Quantity A	Quantity B

1. The price at which Emma sold the bicycle $140   Ⓐ   Ⓑ   Ⓒ   Ⓓ

Explanation

In this question you are asked to compare the price at which Emma sold the bicycle with $140. From the information given, you can conclude that Emma spent a total of $75 + 27 = 102$ dollars buying and repairing the bicycle and that she sold it for 40 percent more than the $102 she spent buying and repairing it. If you notice that 140 is 40 percent more than 100, you can conclude that 40 percent more than 102 is greater than 40 percent more than 100, and therefore, Quantity A is greater than Quantity B. The correct answer is Choice A. (If you solve the problem in this way, you do not have to calculate the value of Quantity A.)

Another way to solve the problem is by explicitly calculating the value of Quantity A and comparing the result with $140 directly. Since 40 percent of 102 is $(0.4)(102) = 40.8$, it follows that Quantity A, the price at which Emma sold the bicycle, is $102.00 + 40.80 = 142.80$ dollars. Thus Quantity A, $142.80, is greater than Quantity B, $140, and the correct answer is **Choice A**.
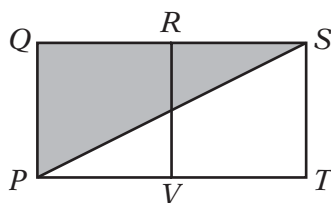
In the figure above, squares *PQRV* and *VRST* have sides of length 6.

Quantity A	Quantity B	
2. The area of the shaded region	36	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare the area of the shaded region with 36. You are given that both *PQRV* and *VRST* are squares with sides of length 6. Therefore, you can conclude that the length of *QS* is 12, and the area of the shaded right triangle *PQS* is $\frac{1}{2}$(12)(6), or 36. Thus Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

In 2009 the property tax on each home in Town *X* was *p* percent of the assessed value of the home, where *p* is a constant. The property tax in 2009 on a home in Town *X* that had an assessed value of $125,000 was $2,500.

Quantity A	Quantity B	
3. The property tax in 2009 on a home in Town *X* that had an assessed value of $160,000	$3,000	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Before making the comparison in this problem, you need to analyze the information given to see what it tells you about the value of Quantity A, the property tax in 2009 on a home in Town *X* that had an assessed value of $160,000. One way of doing this is to determine the value of the constant *p* and then use that value to calculate the tax on the home that had an assessed value of $160,000.

Since it is given that a home that had an assessed value of $125,000 had a property tax of $2,500, you can conclude that *p* is equal to $\frac{2,500}{125,000}$, or 2%. Once you know that the property tax is 2% of the assessed value, you can determine that tax on the home that had an assessed value of $160,000 was 2% of 160,000, or 3,200. The correct answer is **Choice A**.

Another way to calculate the property tax on a home with an assessed value of $160,000 is by setting up a proportion. Because the tax rate is the same for each home in Town X, you can let the variable x represent the tax for the home assessed at $160,000 and solve for x as follows.

$$\frac{x}{160{,}000} = \frac{2{,}500}{125{,}000}$$
$$125{,}000x = (160{,}000)(2{,}500)$$
$$x = \frac{(160{,}000)(2{,}500)}{125{,}000}$$
$$x = 3{,}200$$

The correct answer is **Choice A**.

$$x + y = -1$$

Quantity A	Quantity B	
4. x	y	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

One way to approach this question is to plug in values for one of the variables and determine the corresponding value for the other variable.

One way to plug in: Plug in easy values. For example, you can plug in $x = 0$ and find that the corresponding value of y is -1; then you can plug in $y = 0$ and find that the corresponding value of x is -1. Since in the first case x is greater than y and in the second case y is greater than x, the correct answer is **Choice D**, the relationship cannot be determined from the information given.

A second way to plug in: If you prefer to always plug in values of x to determine corresponding values of y, you can begin by writing the equation $x + y = -1$ as $y = -x - 1$. Writing it in this form makes it easier to find the corresponding values of y.

You can start by plugging in the value $x = 0$. For this value of x, the corresponding value of y is $y = -1$, and therefore, x is greater than y. If you continue plugging in a variety of values of x, some negative and some positive, you will see that sometimes x is greater than y and sometimes y is greater than x.

If you inspect the equation $y = -x - 1$, you can conclude that since there is a negative sign in front of the x but not in front of the y, for each value of x that is greater than 0, the corresponding value of y is less than 0; therefore, for each $x > 0$, x is greater than y.

What about negative values of x? A quick inspection of the equation $y = -x - 1$ allows you to conclude that if $x < -1$, then $y > 0$, so y is greater than x.

So for some values of x and y that satisfy the equation, x is greater than y; and for other values, y is greater than x. Therefore, the relationship between the two quantities x and y cannot be determined from the information given, and the correct answer is **Choice D**.

r, s, and t are three consecutive odd integers such that $r < s < t$.

Quantity A	Quantity B	
5. $r + s + 1$	$s + t - 1$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

You are given that three numbers, r, s, and t, are consecutive odd integers and that $r < s < t$. This means that if you express the three consecutive odd integers in terms of r, they are r, $r + 2$, and $r + 4$.

One way to approach this problem is to set up a placeholder relationship between the two quantities and simplify it to see what conclusions you can draw.

Simplification 1: Begin simplifying by expressing s and t in terms of r. The steps in this simplification can be done as follows.

$$r + s + 1 \boxed{?} s + t - 1$$
$$r + (r + 2) + 1 \boxed{?} (r + 2) + (r + 4) - 1$$
$$2r + 3 \boxed{?} 2r + 5$$
$$3 \boxed{?} 5$$

In the last step of the simplification, you can easily see that $3 < 5$. If you follow the simplification steps in reverse, you can see that the placeholder in each step remains unchanged, so you can conclude that Quantity B is greater than Quantity A, and the correct answer is **Choice B**.

Simplification 2: Since the number s appears in both quantities, you can begin the simplification by subtracting s from both sides of the relationship and then express t in terms of r. The steps in this simplification can be done as follows.

$$r + s + 1 \boxed{?} s + t - 1$$
$$r + 1 \boxed{?} t - 1$$
$$r + 1 \boxed{?} (r + 4) - 1$$
$$r + 1 \boxed{?} r + 3$$
$$1 \boxed{?} 3$$

In the last step of the simplification, you can easily see that $1 < 3$. If you follow the simplification steps in reverse, you can see that the placeholder in each step remains unchanged, so you can conclude that Quantity B is greater than Quantity A, and the correct answer is **Choice B**.

Note that in this solution, the fact that r is odd is not used; what is used is the fact that the consecutive odd integers differ by 2.
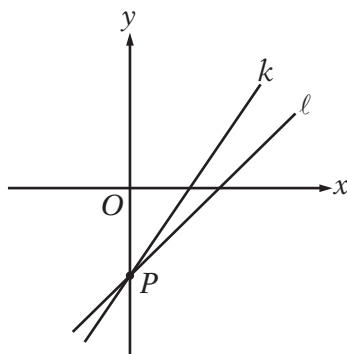
| Quantity A	Quantity B	
6. The slope of line k	The slope of line ℓ	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Note that the slope of each of the lines is positive, since each line rises as it goes to the right. Since the slopes of both lines are positive and line k rises faster (or is steeper) than line ℓ, line k has the greater slope, and the correct answer is **Choice A**.

You can also use the definition of the slope to arrive at the correct answer. Slope can be defined as the ratio of "rise" to "run" between any two points on a line, where the rise is the vertical distance between the points and the run is the horizontal distance, and the slope is respectively positive or negative depending on whether the line rises or falls when viewed from left to right. Because both lines pass through point P on the y-axis, they have the same rise from P to the x-axis. However, line ℓ intersects the x-axis at a greater value than line k. Thus, the run of line ℓ from the y-axis to the x-intercept is greater than the run of line k. When the slope is expressed as a ratio, both lines have the same numerator (rise), but line ℓ has a greater denominator (run). The greater denominator results in a lesser fraction and a lesser slope for line ℓ. Therefore, the correct answer is **Choice A**.

Multiple-choice Questions—Select One Answer Choice

For Questions 7 to 11, select a single answer choice.

7. In the figure above, what is the value of $\dfrac{x+y+z}{45}$?

      Ⓐ 2

      Ⓑ 3

      Ⓒ 4

      Ⓓ 5
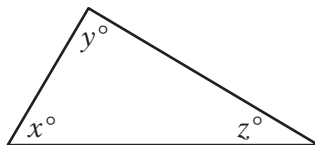
      Ⓔ 6

Explanation

The sum of the measures, in degrees, of the three interior angles of any triangle is 180°. As shown in the figure, the three angles of the triangle have measures of $x°$, $y°$, and $z°$, so $x + y + z = 180$. Therefore, $\dfrac{x+y+z}{45} = \dfrac{180}{45} = 4$, and the correct answer is **Choice C**.

8. A certain store sells two types of pens: one type for $2 per pen and the other type for $3 per pen. If a customer can spend up to $25 to buy pens at the store and there is no sales tax, what is the greatest number of pens the customer can buy?

    (A) 9

    (B) 10

    (C) 11

    (D) 12

    (E) 20

Explanation

It is fairly clear that the greatest number of pens that can be bought for $25 will consist mostly, if not entirely, of $2 pens. In fact, it is reasonable to begin by looking at how many of the $2 pens the customer can buy if the customer does not buy any $3 pens. It is easy to see that the customer could buy 12 of the $2 pens, with $1 left over.

   If the customer bought 11 of the $2 pens, there would be $3 left over with which to buy a $3 pen. In this case, the customer could still buy 12 pens.

   If the customer bought 10 of the $2 pens, there would be $5 left over. Only 1 of the $3 pens could be bought with the $5, so in this case, the customer could buy only 11 pens.

   As the number of $2 pens decreases, the total number of pens that the customer can buy with $25 decreases as well. Thus the greatest number of pens the customer can buy with $25 is 12. The correct answer is **Choice D**.

9. If $y = 3x$ and $z = 2y$, what is $x + y + z$ in terms of x ?

    (A) $10x$

    (B) $9x$

    (C) $8x$

    (D) $6x$

    (E) $5x$

Explanation

It is not necessary to find the individual values of x, y, and z to answer the question. You are asked to rewrite the expression $x + y + z$ as an equivalent expression in terms of x. This means that you need to use the information provided about y and z to express them in terms of the variable x. The variable y is already given in terms of x; that is, $y = 3x$; and because $z = 2y$, it follows that $z = (2)(3x) = 6x$. Using substitution, you can rewrite the expression as follows.

$$x + y + z = x + (3x) + (6x)$$
$$= (1 + 3 + 6)x$$
$$= 10x$$

The correct answer is **Choice A**.

10. A certain shipping service charges an insurance fee of $0.75 when shipping any package with contents worth $25.00 or less and an insurance fee of $1.00 when shipping any package with contents worth over $25.00. If Dan uses the shipping company to ship three packages with contents worth $18.25, $25.00, and $127.50, respectively, what is the total insurance fee that the company charges Dan to ship the three packages?

   Ⓐ $1.75

   Ⓑ $2.25

   Ⓒ $2.50

   Ⓓ $2.75

   Ⓔ $3.00

Explanation

Note that two of the packages being shipped have contents that are worth $25.00 or less. Therefore, each of them has an insurance fee of $0.75, for a total of $1.50. The third package has contents worth over $25.00, and it has an insurance fee of $1.00. Therefore, the total insurance fee for the three packages is $1.50 + $1.00 = $2.50, and the correct answer is **Choice C**.

11. If 55 percent of the people who purchase a certain product are female, what is the ratio of the number of females who purchase the product to the number of males who purchase the product?

   Ⓐ   11 to 9

   Ⓑ   10 to 9

   Ⓒ   9 to 10

   Ⓓ   9 to 11

   Ⓔ   5 to 9

Explanation

Note that because 55 percent of the people who purchase the product are females, it follows that 45 percent of the people who purchase the product are males. Therefore, the ratio of the number of females who purchase the product to the number of males who purchase the product is 55 to 45, or 11 to 9, and the correct answer is **Choice A**.

Numeric Entry

For Questions 12 and 13, use the directions for Numeric Entry questions.
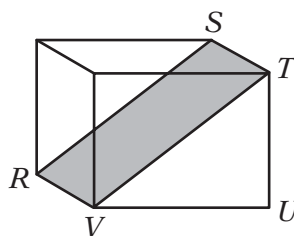
12. In the rectangular solid above, $TU = 3$, $UV = 4$, and $VR = 2$. What is the area of the shaded rectangular region?

Explanation

To find the area of the shaded rectangular region, you need to multiply the length of the rectangular region by its width. In this question you are given the lengths of three edges: $TU = 3$, $UV = 4$, and $VR = 2$. Note that VR is the length of the shaded rectangle. To find the width of the shaded rectangle, you need to find either RS or VT. Note that VT lies on the front face of the rectangular solid. It is the hypotenuse of right triangle VUT. You know that $UV = 4$ and $TU = 3$, so by the Pythagorean theorem you can conclude that $VT = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. Therefore, the area of the shaded rectangular region is $(5)(2) = 10$. The correct answer is **10**.

13. A list of numbers has a mean of 8 and a standard deviation of 2.5. If x is a number in the list that is 2 standard deviations above the mean, what is the value of x ?

$$x = \boxed{}$$

Explanation

You are given that x is 2 standard deviations above the mean, 8. Because the standard deviation of the numbers in the list is 2.5, it follows that x is $(2)(2.5)$, or 5 units above the mean 8. Therefore, $x = 8 + 5 = 13$, and the correct answer is **13**.

Multiple-choice Questions—Select One or More Answer Choices

For Question 14, select all the answer choices that apply.
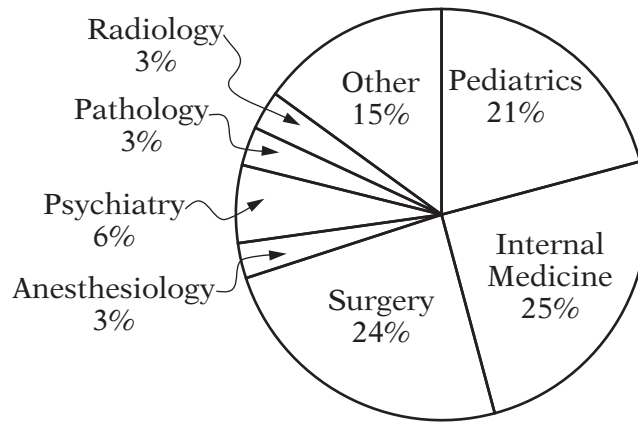
14. The circle graph above shows the distribution of 200,000 physicians by specialty. Which of the following sectors of the circle graph represent more than 40,000 physicians?

   Indicate all such sectors.

   A   Pediatrics
   B   Internal Medicine
   C   Surgery
   D   Anesthesiology
   E   Psychiatry

Explanation

One approach to solve this problem is to find out what percent of 200,000 is 40,000 and then compare this percent with the percents given in the circle graph. Because $\frac{40,000}{200,000} = 0.2$, it follows that 40,000 is 20% of 200,000, and any specialty that has more than 20% of the distribution has more than 40,000 physicians. This is true for the specialties of pediatrics, internal medicine, and surgery. The correct answer consists of **Choices A, B, and C**.

SET 2. Discrete Questions: Medium

Quantitative Comparison

For Questions 1 to 5, use the directions for Quantitative Comparison questions.

Machine R, working alone at a constant rate, produces x units of a product in 30 minutes, and machine S, working alone at a constant rate, produces x units of the product in 48 minutes, where x is a positive integer.

Quantity A	Quantity B

1. The number of units of the product that machine R, working alone at its constant rate, produces in 3 hours   The number of units of the product that machine S, working alone at its constant rate, produces in 4 hours   Ⓐ   Ⓑ   Ⓒ   Ⓓ

Explanation

In this question you are given that machine R, working alone at its constant rate, produces x units of a product in 30 minutes. Since it is easy to see that 3 hours is 6 times 30 minutes, you can conclude that Quantity A is $6x$.

You can compare $6x$ with Quantity B in two ways.

One: In the additional information centered above the quantities, you are given that machine S, working alone at its constant rate, produces x units of the product in 48 minutes, so you can conclude that machine S can produce $6x$ units of the product in $(6)(48)$ minutes, or 4.8 hours. So in 4 hours, machine S produces less than $6x$ units, and Quantity B is less than $6x$.

Two: First, convert 48 minutes to $\frac{4}{5}$ hour, then find the number of 48-minute periods there are in 4 hours by computing $\frac{4}{\left(\frac{4}{5}\right)} = (4)\left(\frac{5}{4}\right) = 5$. Thus, Quantity B is $5x$.

Either way, Quantity A is greater than Quantity B, and the correct answer is **Choice A**.

Frequency Distribution for List X

Number	1	2	3	5
Frequency	10	20	18	12

Frequency Distribution for List Y

Number	6	7	8	9
Frequency	24	17	10	9

List X and list Y each contain 60 numbers. Frequency distributions for each list are given above. The average (arithmetic mean) of the numbers in list X is 2.7, and the average of the numbers in list Y is 7.1. List Z contains 120 numbers: the 60 numbers in list X and the 60 numbers in list Y.

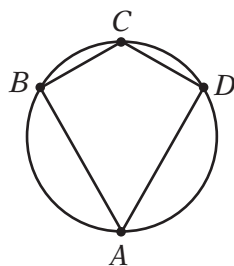| Quantity A	Quantity B	
2. The average of the 120 numbers in list Z	The median of the 120 numbers in list Z	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this problem you are asked to compare the average with the median of the 120 numbers in list Z. Since list Z consists of the numbers in lists X and Y combined, it is reasonable to try to use the information about lists X and Y to calculate the average and the median of the numbers in list Z.

To determine the average of the 120 numbers in list Z, you can use the information given about the individual averages of the numbers in lists X and Y. Because lists X and Y each contain 60 numbers, the average of the numbers in list Z is the average of the individual averages of the numbers in lists X and Y. Thus, the average of the numbers in list Z is $\frac{2.7 + 7.1}{2}$, or 4.9.

To determine the median of the 120 numbers in list Z, first note that list Z contains an even number of numbers, so the median of the numbers in list Z is the average of the middle two numbers when the numbers are listed in increasing order. If you look at the numbers in the two lists, you will see that the 60 numbers in list X are all less than or equal to 5, and the 60 numbers in list Y are all greater than or equal to 6. Thus, the two middle numbers in list Z are 5 and 6, and the average of these numbers is $\frac{5 + 6}{2}$, or 5.5. Therefore, the median of the numbers in list Z is 5.5, and this is greater than the average of 4.9. The correct answer is **Choice B**.

In the figure above, the diameter of the circle is 10.

| Quantity A	Quantity B	
3. The area of quadrilateral *ABCD*	40	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

You are given that the circle has a diameter of 10, and from the figure you can assume that points *A*, *B*, *C*, and *D* lie on the circle in the order shown. However, because figures are not necessarily drawn to scale, you cannot assume anything else about the positions of points *A*, *B*, *C*, and *D* on the circle. Therefore, to get an idea of how various possible positions of these four points could affect the area of quadrilateral *ABCD*, it is a good idea to see how the figure can vary but still have points *A*, *B*, *C*, and *D* in the same order as in the figure above.

One way that you might vary the figure is to evenly space the four points along the circle, as shown below.
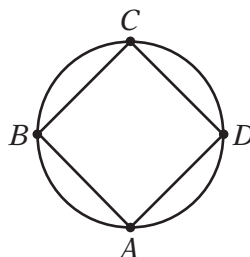
Another way is to draw points *A* and *C* opposite each other, with points *B* and *D* close to point *C*, as shown below.
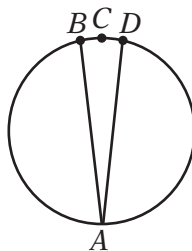
From these figures you can draw some basic conclusions about the area of *ABCD*.

If points *A* and *C* are opposite each other, with points *B* and *D* very close to point *C*, the area of quadrilateral *ABCD* is very close to 0. Clearly, the area can be less than 40 (Quantity B).

If points *A*, *B*, *C*, and *D* are evenly spaced, the area is not close to 0. How does the area compare with 40? To calculate the area of *ABCD*, draw the

diameters *AC* and *BD* in the figure. The two diameters are perpendicular bisectors of each other, so they divide *ABCD* into four right triangles, as shown.
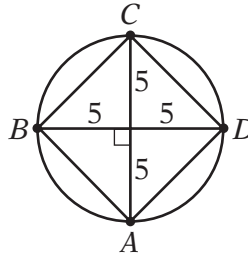
The area of each of the right triangles is $\left(\frac{1}{2}\right)(5)(5)$, or 12.5. Thus, the area of *ABCD* is (4)(12.5), or 50.

Since the area of the quadrilateral in the first figure is less than 40 and the area of the quadrilateral in the second figure is greater than 40, the relationship cannot be determined from the information given. The correct answer is **Choice D**.

$$x^2y > 0$$
$$xy^2 < 0$$

	Quantity A	Quantity B	
4.	x	y	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

You are given that $x^2y > 0$, which means that the product of the two numbers x^2 and y is positive. Recall that the product of two numbers is positive only if both numbers are positive or both numbers are negative. The square of a number is always greater than or equal to 0. In this case, x^2 cannot equal 0 because the product x^2y is not 0. Thus, x^2 is positive and it follows that y is also positive.

You are also given that $xy^2 < 0$, which means that the product of the two numbers x and y^2 is negative. The product of two numbers is negative only if one of the numbers is negative and the other number is positive. In this case, y^2 cannot be negative because it is the square of a number, and it cannot be 0 because the product x^2y is not 0. Thus, y^2 is positive and so x must be negative.

Because x is negative and y is positive, y must be greater than x, and the correct answer is **Choice B**.

Among the 9,000 people attending a football game at College *C*, there were *x* students from College *C* and *y* students who were <u>not</u> from College *C*.

	Quantity A	Quantity B	
5.	The number of people attending the game who were <u>not</u> students	$9,000 - x - y$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are not told whether all of the 9,000 people attending the game were students. Let z be the number of people attending the game who were not students. The people attending the game can be broken down into three groups: students from College C, students not from College C, and people who were not students. This can be expressed algebraically as $9{,}000 = x + y + z$, where x represents the number of students from College C attending the game and y represents the number of students attending the game who were not from College C. Therefore, $9{,}000 - x - y = z$ is the number of people attending the game who were not students. The correct answer is **Choice C**.

Multiple-choice Questions—Select One Answer Choice

For Questions 6 to 10, select a single answer choice.

6.   If $x \neq 0$, which of the following is equivalent to $\dfrac{x(x^2)^3}{x^2}$?

    (A) x^2

    (B) x^3

    (C) x^4

    (D) x^5

    (E) x^6

Explanation

To simplify $\dfrac{x(x^2)^3}{x^2}$, it can be helpful to write $(x^2)^3$ as $(x^2)(x^2)(x^2)$ in the given expression; that is, $\dfrac{x(x^2)^3}{x^2} = \dfrac{x(x^2)(x^2)(x^2)}{x^2}$. Because $x \neq 0$, both numerator and denominator can be divided by x^2, and the expression simplifies to $x(x^2)(x^2)$, which, by the rules of exponents, is equal to x^5.

    Another way to simplify the expression using the rules of exponents directly is as follows.

$$\frac{x(x^2)^3}{x^2} = \frac{x(x^6)}{x^2} = \frac{x^7}{x^2} = x^5$$

The correct answer is **Choice D**.
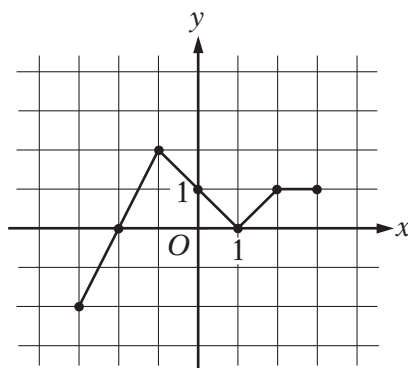
7. The figure above shows the graph of the function f in the xy-plane. What is the value of $f(f(-1))$?

    Ⓐ −2

    Ⓑ −1

    Ⓒ   0

    Ⓓ   1

    Ⓔ   2

Explanation

Note that to find $f(f(-1))$, you must apply the function f twice, first to find the value of $f(-1)$ and then to find the value of $f(f(-1))$. To find the value of $f(-1)$, find the point on the graph of the function f whose x-coordinate is $x = -1$. This point has y-coordinate $y = 2$. Therefore, the value of $f(-1)$ is 2, and $f(f(-1)) = f(2)$. Next you need to find the value of $f(2)$. To find the value of $f(2)$, find the point on the graph whose x-coordinate is $x = 2$. This point has y-coordinate $y = 1$. Therefore, $f(2) = 1$, and because $f(f(-1)) = f(2)$, you can conclude that $f(f(-1)) = 1$. The correct answer is **Choice D**.

8. If $\dfrac{d - 3n}{7n - d} = 1$, which of the following statements describes d in terms of n?

    Ⓐ d is 4 less than n.

    Ⓑ d is 4 more than n.

    Ⓒ d is $\dfrac{3}{7}$ of n.

    Ⓓ d is 2 times n.

    Ⓔ d is 5 times n.

Explanation

To describe d in terms of n, you need to solve the equation $\dfrac{d - 3n}{7n - d} = 1$ for d. To simplify the equation, you can begin by multiplying both sides by $7n - d$ and then proceed as follows.

$$(7n - d)\left(\frac{d - 3n}{7n - d}\right) = (7n - d)(1)$$
$$d - 3n = 7n - d$$
$$d = 10n - d$$
$$2d = 10n$$
$$d = 5n$$

The correct answer is **Choice E**.

9. By weight, liquid *A* makes up 8 percent of solution *R* and 18 percent of solution *S*. If 3 grams of solution *R* are mixed with 7 grams of solution *S*, then liquid *A* accounts for what percent of the weight of the resulting solution?

    Ⓐ 10%
    Ⓑ 13%
    Ⓒ 15%
    Ⓓ 19%
    Ⓔ 26%

Explanation

Liquid *A* makes up 8 percent of the weight of solution *R* and 18 percent of the weight of solution *S*. Therefore, 3 grams of solution *R* contain (0.08)(3), or 0.24 gram of liquid *A*, and 7 grams of solution *S* contain (0.18)(7), or 1.26 grams of liquid *A*. When the two solutions are mixed, the resulting solution weighs $3 + 7$, or 10 grams and contains $0.24 + 1.26$, or 1.5 grams of liquid *A*. This means that liquid *A* makes up $\frac{1.5}{10}$, or $\frac{15}{100}$, or 15 percent of the weight of the resulting solution. The correct answer is **Choice C**.

10. Of the 700 members of a certain organization, 120 are lawyers. Two members of the organization will be selected at random. Which of the following is closest to the probability that <u>neither</u> of the members selected will be a lawyer?

    Ⓐ 0.5
    Ⓑ 0.6
    Ⓒ 0.7
    Ⓓ 0.8
    Ⓔ 0.9

Explanation

The probability that neither of the members selected will be a lawyer is equal to the fraction

$$\frac{\text{the number of ways 2 members who are not lawyers can be selected}}{\text{the number of ways 2 members can be selected}}$$

where the order of selection does not matter.

Since there are 120 members who are lawyers, there must be $700 - 120$, or 580 members who are not lawyers. There are 580 ways of selecting a first member who is not a lawyer and 579 ways of selecting a second member who is not a lawyer. Multiplying these two numbers gives the number of ways to select 2 members who are not lawyers. However, in the (580)(579) ways, each group of 2 members who are not lawyers is counted twice. You can see this by considering 2 members, *A* and *B*. The 2 members can be chosen in 2 ways: *A* first, followed by *B*, and *B* first, followed by *A*. To adjust for double counting, you need to divide (580)(579) by 2.

Similarly, the number of ways 2 members can be selected from among the 700 members is (700)(699) divided by 2. Thus, the desired probability is

$$\frac{\dfrac{(580)(579)}{2}}{\dfrac{(700)(699)}{2}} = \frac{(580)(579)}{(700)(699)}$$

Since the answer choices are all tenths, you need to approximate the value of this fraction to the nearest tenth. There are several ways to do this approximation. One way is to use your calculator to convert the fraction to a decimal and round the decimal to the nearest tenth.

Another way is to approximate the value of the fraction as follows.

$$\frac{(580)(579)}{(700)(699)} \approx \frac{(600)(600)}{(700)(700)} = \left(\frac{6}{7}\right)^2 = \frac{36}{49} \approx \frac{36}{50} = 0.72$$

Either way, the answer choice that is closest to the value of the fraction is 0.7. The correct answer is **Choice C**.

Another approach to this problem is to consider the random selections as two separate but successive events. The probability of selecting a first member who is not a lawyer is $\frac{580}{700}$, because there are 580 members out of the 700 members who are not lawyers. For the second selection, there are only 699 members left to select from, because one member has already been selected. If the first member selected is not a lawyer, then there are only 579 members left who are not lawyers. So the probability of selecting a second member who is not a lawyer, given the condition that the first member selected was not a lawyer, is $\frac{579}{699}$. The probability that both members selected will not be lawyers is the product of the two probabilities, or $\left(\frac{580}{700}\right)\left(\frac{579}{699}\right)$, which is approximated above as 0.72. The correct answer is **Choice C**.

Numeric Entry

For Questions 11 and 12, use the directions for Numeric Entry questions.
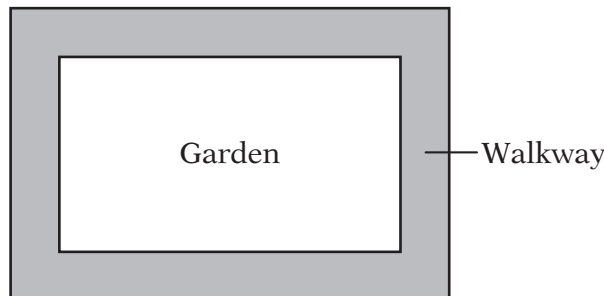
11. The figure above represents a rectangular garden with a walkway around it. The garden is 18 feet long and 12 feet wide. The walkway is uniformly 3 feet wide, and its edges meet at right angles. What is the area of the walkway?

	square feet

Explanation

You can see from the figure that the shaded region is the region between the two rectangles. Looking at the shaded region in this way suggests that the area of the walkway can be calculated as the difference between the area of the larger rectangle and the area of the smaller rectangle.

The region represented by the smaller rectangle is the garden. Since the garden is 18 feet long and 12 feet wide, its area is (18)(12), or 216 square feet.

The region represented by the larger rectangle is the garden and the walkway combined. The length of the region is the length of the garden plus twice the width of the walkway, or $18 + (2)(3) = 24$ feet. The width of the region is the width of the garden plus twice the width of the walkway, or $12 + (2)(3) = 18$ feet. Therefore, the area of the region represented by the larger rectangle is (24)(18), or 432 square feet, and the area of the walkway is $432 - 216$, or 216 square feet.

Another way to approach this problem is to think of the walkway as being composed of four rectangles and four squares, as shown in the figure below.

Garden — Walkway
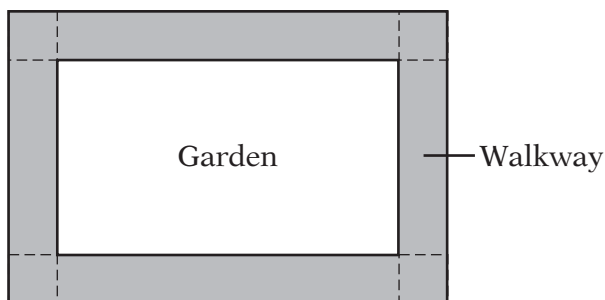
Each of the four squares is 3 feet long and 3 feet wide. The two rectangles running along the length of the garden are 18 feet long and 3 feet wide, and the two rectangles running along the width of the garden are 12 feet long and 3 feet wide. Thus, the area of the walkway is

$$4(3)(3) + 2(18)(3) + 2(12)(3) = 36 + 108 + 72 = 216 \text{ square feet}$$

The correct answer is **216**.

12. Line k lies in the xy-plane. The x-intercept of line k is -4, and line k passes through the midpoint of the line segment whose endpoints are $(2, 9)$ and $(2, 0)$. What is the slope of line k ?

    Give your answer as a fraction.

Explanation

You can calculate the slope of a line if you know the coordinates of two points on the line. In this question you are given information about two points on line k, namely,

- the point at which line k crosses the x-axis has x-coordinate -4;
- the midpoint of the line segment with endpoints at $(2, 9)$ and $(2, 0)$ is on line k.

The coordinates of the first point are $(-4, 0)$, since the x-coordinate is -4 and the y-coordinate of every point on the x-axis is 0. For the second point, the midpoint of the line segment is halfway between the endpoints $(2, 9)$ and $(2, 0)$. Thus, the midpoint has x-coordinate 2 and y-coordinate $\frac{9}{2}$, the number halfway between 9 and 0. Based on the coordinates $(-4, 0)$ and $\left(2, \frac{9}{2}\right)$, the slope of line k is

$$\frac{\frac{9}{2} - 0}{2 - (-4)} = \frac{\frac{9}{2}}{6} = \frac{3}{4}$$

The correct answer is $\frac{3}{4}$ (or any equivalent fraction).

Multiple-choice Questions—Select One or More Answer Choices

For Questions 13 and 14, select all the answer choices that apply.

13. If the lengths of two sides of a triangle are 5 and 9, respectively, which of the following could be the length of the third side of the triangle?

    Indicate <u>all</u> such lengths.

    | A | 3 |
    | B | 5 |
    | C | 8 |
    | D | 15 |

Explanation
A good way to approach this problem is to think about how much the length of the third side of a triangle with two fixed side lengths can vary. If you think about it a bit, you will see that the smaller the interior angle between the two sides of the triangle is, the smaller the length of the third side is; and the larger the interior angle between the two sides of the triangle is, the larger the length of the third side is. This suggests drawing two triangles, one in which the angle between the two sides is close to 0 degrees and one in which the angle between the two sides is close to 180 degrees, like the triangles below.

In the triangle in which the angle between the sides of length 5 and 9 is small, you can see that the length of the third side is a bit greater than $9 - 5$, or 4. If it were equal to 4, the triangle would degenerate into a line segment.

In the triangle in which the angle between the sides of length 5 and 9 is large, you can see that the length of the third side is a bit less than $9 + 5$, or 14. If it were equal to 14, the triangle would degenerate into a line segment.

Therefore, the length of the third side of the triangle must be greater than 4 and less than 14. Furthermore, it is intuitive that any length between these two numbers can be achieved by some triangle. The correct answer consists of **Choices B and C**.
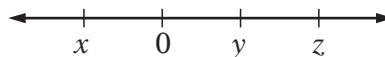
$$\xleftarrow{\quad\begin{array}{ccccc} x & 0 & y & z \end{array}\quad}$$

14. On the number line shown above, the tick marks are equally spaced. Which of the following statements about the numbers x, y, and z must be true?

   Indicate all such statements.

   A $xyz < 0$
   B $x + z = y$
   C $z(y - x) > 0$

Explanation

You can see from their positions on the number line that x is less than 0 and both y and z are greater than 0. Because the tick marks are equally spaced, you can also see that $x = -y$ and $z = 2y$. You need to evaluate each answer choice separately to determine whether it must be true.

Choice A says that the product of the three numbers x, y, and z is less than 0. Recall that the product of three numbers is negative under either of the following two conditions.

- All three numbers are negative.
- One of the numbers is negative and the other two numbers are positive.

Choice A must be true, since x is negative and y and z are positive.

Choice B is the equation $x + z = y$. To see whether the equation must be true, it is a good idea to express two of the variables in terms of the third (that is, to "get rid of" two of the variables). The equations $x = -y$ and $z = 2y$ give x and z in terms of y, so the equation $x + z = y$ can be rewritten, substituting $-y$ for x and $2y$ for z, as $-y + 2y = y$. In this form you can quickly conclude that the equation must be true.

Choice C says that the product of the two numbers z and $y - x$ is greater than 0. Recall that the product of two numbers is positive under either of the following two conditions.

- Both numbers are positive.
- Both numbers are negative.

Since you already know that z is positive, you can conclude that the product $z(y - x)$ will be positive if $y - x$ is positive. By adding x to both sides of the inequality $y - x > 0$, you can see that it is equivalent to the inequality $y > x$, which is clearly true from the number line. Since $y - x$ is positive, the product $z(y - x)$ must be positive.

Therefore, the correct answer consists of **Choices A, B, and C**.

SET 3. Discrete Questions: Hard

Quantitative Comparison

For Questions 1 to 6, use the directions for Quantitative Comparison questions.
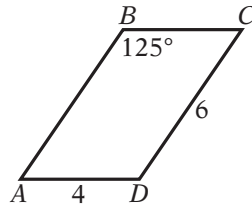
In the figure above, *ABCD* is a parallelogram.

	Quantity A	Quantity B	
1.	The area of *ABCD*	24	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare the area of a parallelogram with an area of 24, given two side lengths and the measure of one interior angle of the parallelogram. Since the measure of the interior angle given is 125°, you can conclude that the parallelogram is not a rectangle.

Recall that the area of a parallelogram is found by multiplying the length of a base by the height corresponding to the base. It is helpful to draw the vertical height from vertex *C* to base *AD* of the parallelogram, as shown in the figure below.
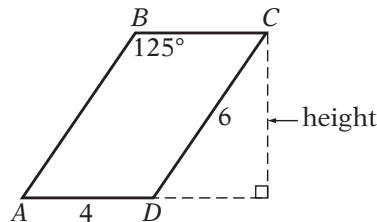
Note that the newly drawn height is a leg in a newly formed right triangle. The hypotenuse of the triangle is a side of the parallelogram and has length 6. Thus, the leg of the triangle, which is the height of the parallelogram, must be less than the hypotenuse 6. The area of the parallelogram is equal to the length of base *AD*, which is 4, times the height, which is less than 6. Since the product of 4 and a number less than 6 must be less than 24, the area of the parallelogram must be less than 24. Quantity B is greater than Quantity A, and the correct answer is **Choice B.**
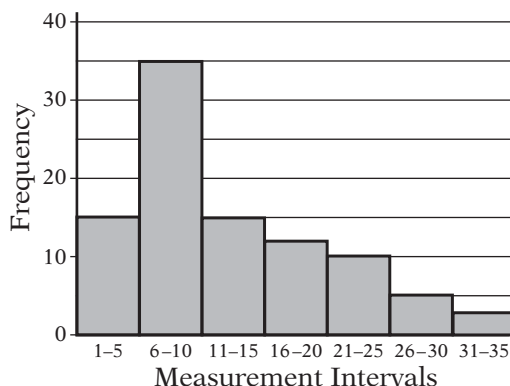
In the course of an experiment, 95 measurements were recorded, and all of the measurements were integers. The 95 measurements were then grouped into 7 measurement intervals. The graph above shows the frequency distribution of the 95 measurements by measurement interval.

Quantity A	Quantity B				
2.	The average (arithmetic mean) of the 95 measurements	The median of the 95 measurements	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

From the histogram, you can observe that

- all of the measurement intervals are the same size,
- the distribution has a peak at the measurement interval 6–10, and
- more of the measurement intervals are to the right of the peak than are to the left of the peak.

Since in the histogram the 95 measurements have been grouped into intervals, you cannot calculate the exact value of either the average or the median; you must compare them without being able to determine the exact value of either one.

The median of the 95 measurements is the middle measurement when the measurements are listed in increasing order. The middle measurement is the 48th measurement. From the histogram, you can see that the measurement interval 1–5 contains the first 15 measurements, and the measurement interval 6–10 contains the next 35 measurements (that is, measurements 16 through 50). Therefore, the median is in the measurement interval 6–10 and could be 6, 7, 8, 9, or 10.

Estimating the average of the 95 measurements is more complicated.

Since you are asked to compare the average and the median, not necessarily to calculate them, you may ask yourself if you can tell whether the average is greater than or less than the median. Note that visually the measurements in the first three measurement intervals are symmetric around the measurement interval 6–10, so you would expect the average of the measurements in just these three measurement intervals to lie in the 6–10 measurement interval. The 30 measurements in the remaining four measurement intervals are all greater than 10, some significantly greater than 10. Therefore, the average of the 95 measurements is greater than the average of the measurements in the first three measurement in-

tervals, probably greater than 10. At this point it seems likely that the average of the 95 measurements is greater than the median of the 95 measurements. It turns out that this is true.

To actually show that the average must be greater than 10, you can make the average as small as possible and see if the smallest possible average is greater than 10. To make the average as small as possible, assume that all of the measurements in each interval are as small as possible. That is to say, all 15 measurements in the measurement interval 1–5 are equal to 1, all 35 measurements in the measurement interval 6–10 are equal to 6, etc. Under this assumption, the average of the 95 measurements is

$$\frac{(1)(15) + (6)(35) + (11)(15) + (16)(12) + (21)(10) + (26)(5) + (31)(3)}{95} = \frac{1{,}015}{95}$$

The value of the smallest possible average, $\frac{1{,}015}{95}$, is greater than 10.

Therefore, since the average of the 95 measurements is greater than 10 and the median is in the measurement interval 6–10, it follows that the average is greater than the median, and the correct answer is **Choice A**.

x is an integer greater than 1.

	Quantity A	Quantity B	
3.	3^{x+1}	4^x	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation
One way to approach this question is to plug in numbers for the variables and see what the relationship between the two quantities is for each of the numbers you plug in.

If you plug in $x = 2$, you see that Quantity A is $3^{x+1} = 3^3$, or 27, and Quantity B is $4^x = 4^2$, or 16. In this case, Quantity A is greater than Quantity B.

If you plug in $x = 3$, you see that Quantity A is $3^{x+1} = 3^4$, or 81, and Quantity B is $4^x = 4^3$, or 64. In this case, Quantity A is greater than Quantity B.

If you plug in $x = 4$, you see that Quantity A is $3^{x+1} = 3^5$, or 243, and Quantity B is $4^x = 4^4$, or 256. In this case, Quantity B is greater than Quantity A. Since for $x = 2$ and for $x = 3$, Quantity A is greater than Quantity B, and for $x = 4$, Quantity B is greater than Quantity A, it follows that the relationship between the two quantities cannot be determined. The correct answer is **Choice D**.

Since both quantities are algebraic expressions, another way to approach this problem is to set up a placeholder relationship between the two quantities and simplify it to see what conclusions you can draw.

$$3^{x+1} \; \boxed{?} \; 4^x$$
$$3(3^x) \; \boxed{?} \; 4^x$$
$$\frac{3(3^x)}{3^x} \; \boxed{?} \; \frac{4^x}{3^x}$$
$$3 \; \boxed{?} \; \left(\frac{4}{3}\right)^x$$

For any value of x, the value of 3^x is positive, so dividing by 3^x does not change any inequality that could be put in the placeholder. Since each step in this simplification is reversible, this reduces the problem to comparing 3 with $\left(\frac{4}{3}\right)^x$.

You can see that because $\frac{4}{3}$ is greater than 1, the value of $\left(\frac{4}{3}\right)^x$ becomes greater as x becomes larger. In particular, it is greater than 3 for large enough values of x. For the smallest value of x, $x = 2$, the relationship is $\left(\frac{4}{3}\right)^2 = \frac{16}{9} < 3$.

Since for $x = 2$, Quantity A is greater than Quantity B and for large values of x, Quantity B is greater than Quantity A, it follows that the relationship between the two quantities cannot be determined. The correct answer is **Choice D**.

A, B, and C are three rectangles. The length and width of rectangle A are 10 percent greater and 10 percent less, respectively, than the length and width of rectangle C. The length and width of rectangle B are 20 percent greater and 20 percent less, respectively, than the length and width of rectangle C.

	Quantity A	Quantity B	
4.	The area of rectangle A	The area of rectangle B	Ⓐ　Ⓑ　Ⓒ　Ⓓ

Explanation
In this question you are asked to compare the area of rectangle A and the area of rectangle B. Since the information given relates the dimensions of both rectangle A and rectangle B to the corresponding dimensions of rectangle C, you can try to use the relationships to make the desired comparison.

If ℓ represents the length of rectangle C and w represents its width, then the length and width of rectangles A and B can be translated into algebraic expressions as follows.

- The length of rectangle A is 10 percent greater than the length of rectangle C, or 1.1ℓ.
- The width of rectangle A is 10 percent less than the width of rectangle C, or $0.9w$.
- The length of rectangle B is 20 percent greater than the length of rectangle C, or 1.2ℓ.
- The width of rectangle B is 20 percent less than the width of rectangle C, or $0.8w$.

In terms of ℓ and w, the area of rectangle A is $(1.1\ell)(0.9w)$, or $0.99\ell w$.
In terms of ℓ and w, the area of rectangle B is $(1.2\ell)(0.8w)$, or $0.96\ell w$.
Since $0.99\ell w$ is greater than $0.96\ell w$, Quantity A is greater than Quantity B, and the correct answer is **Choice A**.

The random variable X is normally distributed. The values 650 and 850 are at the 60th and 90th percentiles of the distribution of X, respectively.

	Quantity A	Quantity B	
5.	The value at the 75th percentile of the distribution of X	750	Ⓐ　Ⓑ　Ⓒ　Ⓓ

Explanation

You are given that the distribution of random variable X is normal and that the values 650 and 850 are at the 60th and 90th percentiles of the distribution, respectively.

Both of the values 650 and 850 are greater than the mean of the distribution. If you draw a rough sketch of the graph of the normal distribution, the sketch could look something like the one below. Note that it is not necessary to know the exact location of 650 and 850, just that both values are above the mean.

650   850
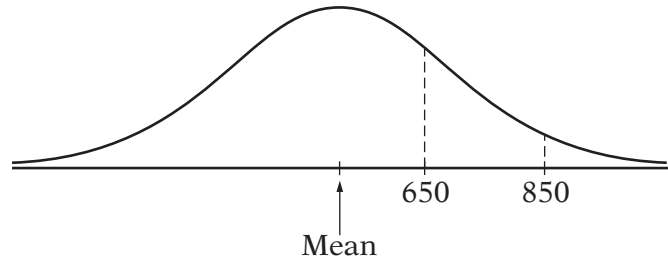
Mean

To say that the value 650 is at the 60th percentile of the distribution means, graphically, that 60 percent of the area between the normal curve and the horizontal axis lies to the left of the vertical line segment at 650. To say that 850 is at the 90th percentile of the distribution means that 90 percent of the area between the normal curve and the horizontal axis lies to the left of the vertical line segment at 850.

The value 750 is halfway between 650 and 850. However, because the curve is decreasing in that interval, the area between 650 and 750 is greater than the area between 750 and 850. Since the value at the 75th percentile should divide in half the <u>area</u> between the value at the 60th percentile (650) and the value at the 90th percentile (850), this value is closer to 650 than to 850. Thus you can conclude that Quantity A, the value at the 75th percentile of the distribution of X, is less than Quantity B. The correct answer is **Choice B**.

Set S consists of all positive integers less than 81 that are <u>not</u> equal to the square of an integer.

Quantity A	Quantity B	
6. The number of integers in set S	72	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Set S consists of all integers from 1 to 80, except those that are equal to the square of an integer. So, Quantity A, the number of integers in set S, is equal to the number of positive integers that are less than 81 minus the number of positive integers less than 81 that <u>are</u> equal to the square of an integer.

Clearly, there are 80 positive integers that are less than 81.

One way to determine the number of positive integers less than 81 that are squares of integers is by noticing that 81 is equal to 9^2 and concluding that the squares of the integers from 1 to 8 are all positive integers that are less than 81.

You can also draw this conclusion by squaring each of the positive integers, beginning with 1, until you get to an integer n such that n^2 is greater than or equal to 81. Either way, there are 8 positive integers less than 81 that are squares of integers.

Therefore, the number of integers in set S is $80 - 8$, or 72, which is equal to Quantity B. So Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

Multiple-choice Questions—Select One Answer Choice

> **For Questions 7 to 12, select a single answer choice.**

7. A manager is forming a 6-person team to work on a certain project. From the 11 candidates available for the team, the manager has already chosen 3 to be on the team. In selecting the other 3 team members, how many different combinations of 3 of the remaining candidates does the manager have to choose from?

    Ⓐ 6
    Ⓑ 24
    Ⓒ 56
    Ⓓ 120
    Ⓔ 462

Explanation

To determine the number of different combinations of 3 of the remaining candidates that the manager has to choose from, you first have to know the number of remaining candidates. Since you know that the manager has already chosen 3 of the 11 candidates to be on the team, it is easy to see that there are 8 remaining candidates. Now you need to count how many different combinations of 3 objects can be chosen from a group of 8 objects.

If you remember the combinations formula, you know that the number of combinations is $\dfrac{8!}{3!(8-3)!}$ (which is denoted symbolically as $\binom{8}{3}$ or $_8C_3$). You can then calculate the number of different combinations of 3 of the remaining candidates as follows.

$$\frac{8!}{3!(8-3)!} = \frac{(8)(7)(6)(5!)}{(3!)(5!)} = \frac{(8)(7)(6)}{6} = 56$$

The correct answer is **Choice C**.

8. Which of the following could be the graph of all values of x that satisfy the inequality $2 - 5x \le -\dfrac{6x-5}{3}$?

    Ⓐ

    0

    Ⓑ

    0

    Ⓒ

    0

    Ⓓ

    0

    Ⓔ

    0
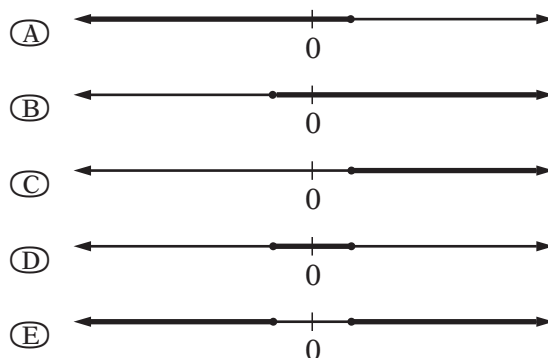
Explanation

To determine which of the graphs is the correct answer, you first need to determine all values of x that satisfy the inequality. To do that you need to simplify the inequality until you isolate x.

You can begin by multiplying both sides of the inequality by 3 to obtain $(3)(2 − 5x) \leq −(6x − 5)$. Note that when you multiply by 3, the right-hand side of the inequality becomes $−(6x − 5)$, <u>not</u> $−6x − 5$.

The rest of the simplification is as follows.

$$(3)(2 − 5x) \leq −6x + 5$$
$$6 − 15x \leq −6x + 5$$
$$−15x \leq −6x − 1$$
$$−9x \leq −1$$
$$x \geq \frac{1}{9}$$

Note that when an inequality is multiplied (or divided) by a negative number, the direction of the inequality reverses.

The graphs in the answer choices are number lines on which only the number 0 is indicated. Therefore, you do not need to locate $\frac{1}{9}$ on the number line; it is enough to know that $\frac{1}{9}$ is a positive number. Choice C is the only choice in which the shaded part of the line is equal to or greater than a positive number. Therefore, the correct answer is **Choice C**.

9. If $1 + x + x^2 + x^3 = 60$, then the average (arithmetic mean) of x, x^2, x^3, and x^4 is equal to which of the following?

(A) $12x$

(B) $15x$

(C) $20x$

(D) $30x$

(E) $60x$

Explanation

A quick inspection of the answer choices shows that it is not necessary to solve the equation $1 + x + x^2 + x^3 = 60$ for x to answer this question. You are being asked to express the average of the four quantities x, x^2, x^3, and x^4 in terms of x. To express this average in terms of x, you need to add the 4 quantities and divide the result by 4; that is, $\frac{x + x^2 + x^3 + x^4}{4}$.

The only information given in the question is that the sum of the 4 quantities, $1 + x + x^2 + x^3$, is 60, so you need to think of a way to use this information to simplify the expression $\frac{x + x^2 + x^3 + x^4}{4}$.

Note that the numerator of the fraction is a sum of 4 quantities, each of which has an x term raised to a power. Thus, the expression in the numerator can be

factored as $x + x^2 + x^3 + x^4 = x(1 + x + x^2 + x^3)$. By using the information in the question, you can make the following simplification.

$$\frac{x + x^2 + x^3 + x^4}{4} = \frac{x(1 + x + x^2 + x^3)}{4} = \frac{x(60)}{4} = 15x$$

Therefore, the correct answer is **Choice B**.
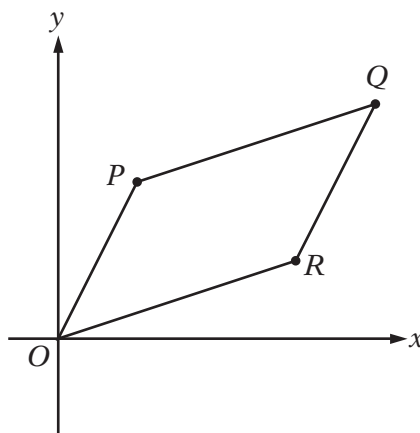
10. Parallelogram $OPQR$ lies in the xy-plane, as shown in the figure above. The coordinates of point P are $(2, 4)$ and the coordinates of point Q are $(8, 6)$. What are the coordinates of point R ?

Ⓐ $(3, 2)$
Ⓑ $(3, 3)$
Ⓒ $(4, 4)$
Ⓓ $(5, 2)$
Ⓔ $(6, 2)$

Explanation
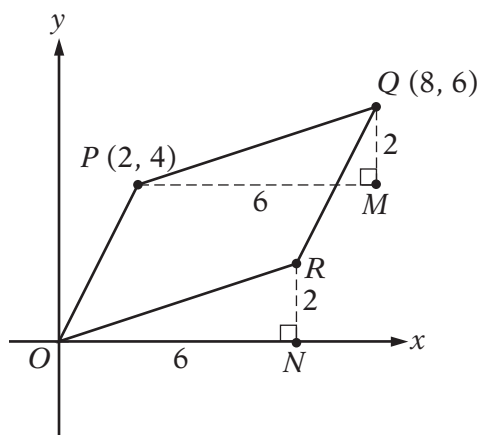
Since $OPQR$ is a parallelogram, line segments PQ and OR have the same length and the same slope. Therefore, in the figure above, PQM and ORN are congruent right triangles. From the coordinates of P and Q, the lengths of the legs

of triangle PQM are $PM = 8 - 2 = 6$ and $QM = 6 - 4 = 2$. Thus, the lengths of the legs ON and RN of triangle ORN are also 6 and 2, respectively. So the coordinates of point R are $(6, 2)$. The correct answer is **Choice E**.

11. The relationship between the area A of a circle and its circumference C is given by the formula $A = kC^2$, where k is a constant. What is the value of k ?

Ⓐ $\dfrac{1}{4\pi}$

Ⓑ $\dfrac{1}{2\pi}$

Ⓒ $\dfrac{1}{4}$

Ⓓ 2π

Ⓔ $4\pi^2$

Explanation

One way to approach this problem is to realize that the value of the constant k is the same for all circles. Therefore, you can pick a specific circle and substitute the circumference and the area of that particular circle into the formula and calculate the value of k.

Say, for example, that you pick a circle with radius 1. The area of the circle is π and the circumference of the circle is 2π. Inserting these values into the formula gives $\pi = k(2\pi)^2$. Solving this equation for k gives $k = \dfrac{1}{4\pi}$, and the correct answer is

Choice A.

Another way to approach the problem is to express A and C in terms of a common variable and then solve the resulting equation for k. Recall the commonly used formulas for the area and the circumference of a circle: $A = \pi r^2$ and $C = 2\pi r$. Note that in these formulas, both A and C are expressed in terms of the radius r. So, in the formula $A = kC^2$, you can substitute expressions for A and C in terms of r.

Substituting πr^2 for A and $2\pi r$ for C gives $\pi r^2 = k(2\pi r)^2$.

Now you can determine the value of k by solving the equation for k as follows.

$$\pi r^2 = k(2\pi r)^2$$
$$\pi r^2 = k(4\pi^2 r^2)$$
$$\pi = k(4\pi^2)$$
$$\frac{1}{4\pi} = k$$

The correct answer is **Choice A**.

12. The sequence of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$ is defined by $a_n = \dfrac{1}{n} - \dfrac{1}{n+2}$ for each integer $n \geq 1$. What is the sum of the first 20 terms of this sequence?

(A) $\left(1 + \dfrac{1}{2}\right) - \dfrac{1}{20}$

(B) $\left(1 + \dfrac{1}{2}\right) - \left(\dfrac{1}{21} + \dfrac{1}{22}\right)$

(C) $1 - \left(\dfrac{1}{20} + \dfrac{1}{22}\right)$

(D) $1 - \dfrac{1}{22}$

(E) $\dfrac{1}{20} - \dfrac{1}{22}$

Explanation

This question asks for the sum of the first 20 terms of the sequence. Obviously, it would be very time-consuming to write out the first 20 terms of the sequence and add them together, so it is reasonable to try to find a more efficient way to calculate the sum. Questions involving sequences can often be answered by looking for a pattern. Scanning the answer choices and noting that they contain fractions with denominators 2, 20, 21, and 22, and nothing in between, seems to confirm that looking for a pattern is a good approach to try.

To look for a pattern, begin by adding the first two terms of the sequence.

$$\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) = \left(\frac{1}{1} + \frac{1}{2}\right) - \left(\frac{1}{3} + \frac{1}{4}\right)$$

Now, if you add the first three terms of the sequence, you get

$$\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right)$$

Note that you can simplify the sum by canceling the fraction $\frac{1}{3}$; that is, the sum of positive $\frac{1}{3}$ and negative $\frac{1}{3}$ is 0.

$$\left(\frac{1}{1} - \frac{1}{\cancel{3}}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{\cancel{3}} - \frac{1}{5}\right) = \left(\frac{1}{1} + \frac{1}{2}\right) - \left(\frac{1}{4} + \frac{1}{5}\right)$$

If you add the first four terms, you get

$$\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right)$$

Again, you can simplify the sum by canceling. This time, you can cancel the fractions $\frac{1}{3}$ and $\frac{1}{4}$.

$$\left(\frac{1}{1} - \frac{1}{\cancel{3}}\right) + \left(\frac{1}{2} - \frac{1}{\cancel{4}}\right) + \left(\frac{1}{\cancel{3}} - \frac{1}{5}\right) + \left(\frac{1}{\cancel{4}} - \frac{1}{6}\right) = \left(\frac{1}{1} + \frac{1}{2}\right) - \left(\frac{1}{5} + \frac{1}{6}\right)$$

If you write out the next two sums and simplify them, you will see that they are

$$\left(\frac{1}{1}+\frac{1}{2}\right)-\left(\frac{1}{6}+\frac{1}{7}\right) \text{ and } \left(\frac{1}{1}+\frac{1}{2}\right)-\left(\frac{1}{7}+\frac{1}{8}\right)$$

Working with the sums makes it clear that this pattern continues to hold as you add more and more terms of the sequence together and that a formula for the sum of the first k terms of the sequence is

$$\left(\frac{1}{1}+\frac{1}{2}\right)-\left(\frac{1}{k+1}+\frac{1}{k+2}\right)$$

Therefore, the sum of the first 20 terms of the sequence is equal to

$$\left(\frac{1}{1}+\frac{1}{2}\right)-\left(\frac{1}{20+1}+\frac{1}{20+2}\right)=\left(1+\frac{1}{2}\right)-\left(\frac{1}{21}+\frac{1}{22}\right)$$

The correct answer is **Choice B**.

Numeric Entry

For Question 13, use the directions for Numeric Entry questions.

Y	Frequency
$\frac{1}{2}$	2
$\frac{3}{4}$	7
$\frac{5}{4}$	8
$\frac{3}{2}$	8
$\frac{7}{4}$	9

13. The table above shows the frequency distribution of the values of a variable Y. What is the mean of the distribution?

   Give your answer to the <u>nearest 0.01</u>.

   [   ]

Explanation

The mean of distribution of the variable Y is the sum of all the values of Y divided by the number of values of Y. However, before you begin the summing process, you need to understand how the information is presented in the question. Information about the variable is given in a table, where any repetitions of values have been summarized in the column labeled "Frequency." Reading from the

table, you can see that the value $\frac{1}{2}$ occurs twice, the value $\frac{3}{4}$ occurs seven times, and so on. To sum all the values of Y, you could add the value $\frac{1}{2}$ twice, add the value $\frac{3}{4}$ seven times, and continue the addition process in this manner. It is easier, however, to multiply the values by their corresponding frequencies and then sum the individual products, as shown below.

$$(2)\left(\frac{1}{2}\right) + (7)\left(\frac{3}{4}\right) + (8)\left(\frac{5}{4}\right) + (8)\left(\frac{3}{2}\right) + (9)\left(\frac{7}{4}\right) = \frac{4}{4} + \frac{21}{4} + \frac{40}{4} + \frac{48}{4} + \frac{63}{4}$$
$$= \frac{176}{4}$$
$$= 44$$

To find the average, you need to divide the sum, 44, by the number of values of Y. The number of values can be found by looking at the column of frequencies in the table. The sum of the numbers in this column, $2 + 7 + 8 + 8 + 9$, or 34, is the number of values of Y. Thus, the mean of the distribution is $\frac{44}{34}$, which, as a decimal, equals 1.2941…. Rounded to the nearest 0.01, the correct answer is **1.29**.

Multiple-choice Questions — Select One or More Answer Choices

For Questions 14 and 15, select all the answer choices that apply.

14. Let S be the set of all positive integers n such that n^2 is a multiple of both 24 and 108. Which of the following integers are divisors of every integer n in S ?

    Indicate all such integers.

    A 12
    B 24
    C 36
    D 72

Explanation
To determine which of the integers in the answer choices is a divisor of every positive integer n in S, you must first understand the integers that are in S. Note that in this question you are given information about n^2, not about n itself. Therefore, you must use the information about n^2 to derive information about n.

The fact that n^2 is a multiple of both 24 and 108 implies that n^2 is a multiple of the least common multiple of 24 and 108. To determine the least common multiple of 24 and 108, factor 24 and 108 into prime factors as $(2^3)(3)$ and $(2^2)(3^3)$, respectively. Because these are prime factorizations, you can conclude that the least common multiple of 24 and 108 is $(2^3)(3^3)$.

Knowing that n^2 must be a multiple of $(2^3)(3^3)$ does not mean that every multiple of $(2^3)(3^3)$ is a possible value of n^2, because n^2 must be the square of an

integer. The prime factorization of a square number must contain only even exponents. Thus, the least multiple of $(2^3)(3^3)$ that is a square is $(2^4)(3^4)$. This is the least possible value of n^2, and so the least possible value of n is $(2^2)(3^2)$, or 36. Furthermore, since every value of n^2 is a multiple of $(2^4)(3^4)$, the values of n are the positive multiples of 36; that is, $S = \{36, 72, 108, 144, 180, \ldots\}$.

The question asks for integers that are divisors of every integer n in S, that is, divisors of every positive multiple of 36. Since Choice A, 12, is a divisor of 36, it is also a divisor of every multiple of 36. The same is true for Choice C, 36. Choices B and D, 24 and 72, are not divisors of 36, so they are not divisors of every integer in S. The correct answer consists of **Choices A and C**.

15. The range of the heights of the female students in a certain class is 13.2 inches, and the range of the heights of the male students in the class is 15.4 inches.

    Which of the following statements <u>individually</u> provide(s) sufficient additional information to determine the range of the heights of all the students in the class?

    Indicate <u>all</u> such statements.

    A The tallest male student in the class is 5.8 inches taller than the tallest female student in the class.
    B The median height of the male students in the class is 1.1 inches greater than the median height of the female students in the class.
    C The average (arithmetic mean) height of the male students in the class is 4.6 inches greater than the average height of the female students in the class.

Explanation

Choice A tells you that the tallest male student is 5.8 inches taller than the tallest female student. You can combine this information with the given information about the male and female height ranges to place four students—the shortest male, the shortest female, the tallest male, and the tallest female—in relative order according to height, as shown in the figure below.
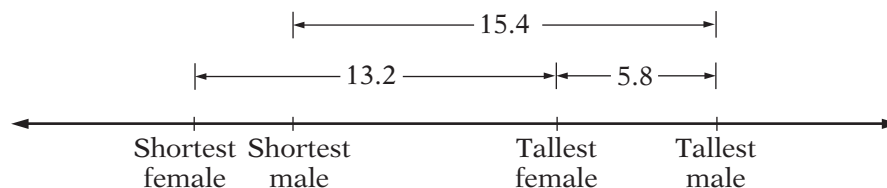
You can see from the figure that the tallest student must be a male and the shortest student must be a female. You can also see the difference in height between those two students, which is the range of the heights of the entire class. Therefore, Choice A provides sufficient additional information to determine the range.

Choice B provides information about one of the centers of the data—the median; it does not say anything about how spread out the data are around that center. You are given that the median height of the males is 1.1 inches greater than that of the females. First note that it is possible for two different sets of

data to have the same median but have very different ranges. Choice B gives the difference between the medians of the male heights and the female heights, without giving the actual medians. However, even if you knew the medians, the fact that the ranges can vary widely indicates that the range of the heights of the entire class can also vary widely.

It is possible to construct examples of heights of students that satisfy all of the information in the question and in Choice B but have different ranges for the heights of the entire class. Here are two such examples, each of which has only three females and three males. Although the examples are small, they illustrate the fact that the range of the heights of the entire class can vary. In both examples, the range of female heights is 13.2, the range of male heights is 15.4, and the difference between the median heights is 1.1 inches.

Example 1
Female heights: 50.0   56.6   63.2   which have a median of 56.6
Male heights: 50.0   57.7   65.4   which have a median of 57.7
Range of heights of entire class: 15.4

Example 2
Female heights: 50.0   56.6   63.2   which have a median of 56.6
Male heights: 51.0   57.7   66.4   which have a median of 57.7
Range of heights of entire class: 16.4

Therefore, Choice B does not provide sufficient additional information to determine the range of the heights of the entire class.

Choice C provides information about another center of the data—the average. You are given that the average height of the males is 4.6 inches greater than that of the females. However, like Choice B, the statement gives no information about how spread out the data are around that center. Again, it is possible for two different sets of data to have the same average but have very different ranges. Examples similar to the two examples above can be constructed that satisfy all of the information in the question and in Choice C but have different ranges for the heights of the entire class. Therefore, Choice C does not provide sufficient additional information to determine the range of the heights of the entire class.

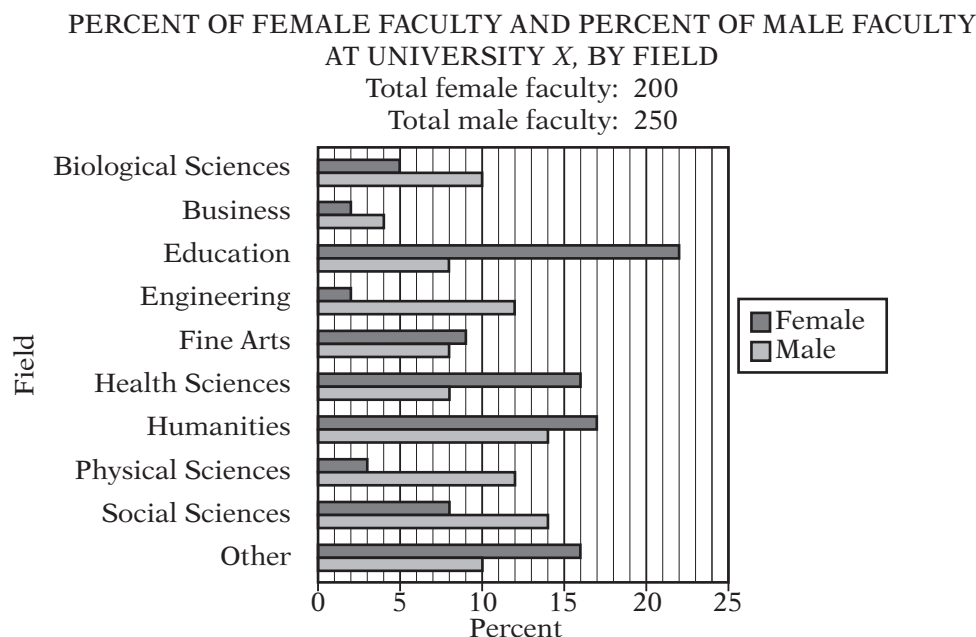The correct answer consists of **Choice A**.

SET 4. Data Interpretation Sets

For Questions 1 to 7, select a single answer choice unless otherwise directed.

Questions 1 to 3 are based on the following data.

PERCENT OF FEMALE FACULTY AND PERCENT OF MALE FACULTY
AT UNIVERSITY *X*, BY FIELD
Total female faculty: 200
Total male faculty: 250

Medium Question

1. There are 275 students in the field of engineering at University *X*.
    Approximately what is the ratio of the number of students in engineering to
    the number of faculty in engineering?

    Ⓐ   8 to 1
    Ⓑ   10 to 1
    Ⓒ   12 to 1
    Ⓓ   14 to 1
    Ⓔ   20 to 1

Explanation

According to the graph, 2 percent of the female faculty and 12 percent of the
male faculty are in the engineering field. To determine the total number of
faculty members in engineering, you need to add 2 percent of 200, which is 4, to
12 percent of 250, which is 30, to get 34. Thus, the ratio of the numbers of
students to faculty in engineering is 275 to 34, which is approximately equal to
280 to 35, or 8 to 1. The correct answer is **Choice A**.

Medium Question

2. Approximately what percent of the faculty in humanities are male?

Ⓐ 35%

Ⓑ 38%

Ⓒ 41%

Ⓓ 45%

Ⓔ 51%

Explanation

You need to determine the numbers of female and male faculty in the humanities field. According to the graph, 17 percent of the 200 females, or 34, and 14 percent of the 250 males, or 35, are in humanities. Thus, the fraction of humanities faculty who are male is $\frac{35}{34 + 35} = \frac{35}{69}$, or approximately 0.507. As a percent, the answer choice that is closest to 0.507 is 51 percent. The correct answer is **Choice E**.

For Question 3, use the directions for Numeric Entry questions.

Hard Question

3. For the biological sciences and health sciences faculty combined, $\frac{1}{3}$ of the female and $\frac{2}{9}$ of the male faculty members are tenured professors. What fraction of all the faculty members in those two fields combined are tenured professors?

$$\frac{\boxed{}}{\boxed{}}$$

Explanation

You need to determine the number of female faculty and the number of male faculty in the combined group. According to the graph, 5 percent of the female faculty, or 10, and 10 percent of the male faculty, or 25, are in the biological sciences. Similarly, 16 percent of the female faculty, or 32, and 8 percent of the male faculty, or 20, are in the health sciences. When you combine the groups, you get a total of 42 females (10 + 32) and 45 males (25 + 20), which is a total of 87 faculty. The tenured faculty are $\frac{1}{3}$ of the 42 females, or 14 females, and $\frac{2}{9}$ of the 45 males, or 10 males. Thus, there are 24 tenured faculty, and the fraction that are tenured professors is $\frac{24}{87}$. The correct answer is $\mathbf{\frac{24}{87}}$ (or any equivalent fraction).

VALUE OF IMPORTS TO AND EXPORTS FROM COUNTRY *T,* 2000–2009
(in United States dollars)
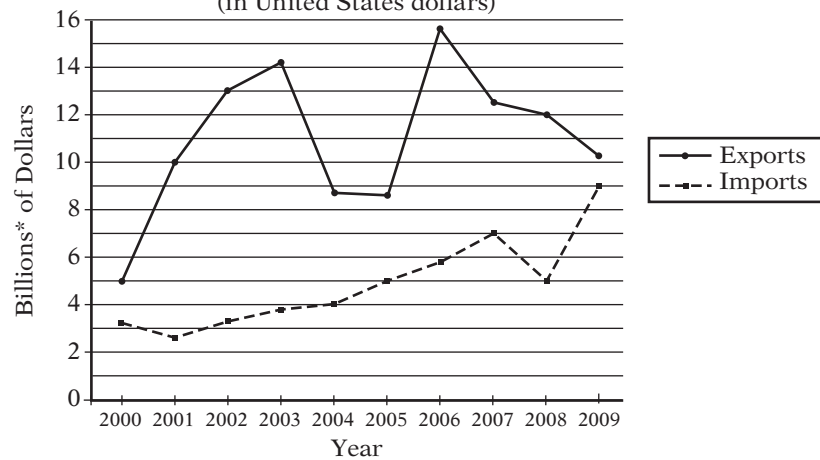
*1 billion = 1,000,000,000

For Question 4, select all the answer choices that apply.

Easy Question

4.   For which of the eight years from 2001 to 2008 did exports exceed imports
      by more than $5 billion?

      Indicate <u>all</u> such years.

      A   2001
      B   2002
      C   2003
      D   2004
      E   2005
      F   2006
      G   2007
      H   2008

Explanation
Note that for all years shown, the dollar value of exports is greater than the
dollar value of imports. For each year, the difference between the dollar value of
exports and the dollar value of imports can be read directly from the graph. The
difference was more than $5 billion for each of the years 2001, 2002, 2003, 2006,
2007, and 2008. The correct answer consists of **Choices A, B, C, F, G, and H**.

Medium Question

5. Which of the following is closest to the average (arithmetic mean) of the 9 changes in the value of imports between consecutive years from 2000 to 2009 ?

    (A) $260 million

    (B) $320 million

    (C) $400 million

    (D) $480 million

    (E) $640 million

Explanation

The average of the 9 changes in the value of imports between consecutive years can be represented as follows, where the function $v(year)$ represents the value of imports for the indicated year.

$$\frac{(v(2001) - v(2000)) + (v(2002) - v(2001)) + (v(2003) - v(2002)) + \cdots + (v(2009) - v(2008))}{9}$$

Note that in the numerator of the fraction, each term, with the exception of $v(2000)$ and $v(2009)$, appears first as positive and then again as negative. The positive and negative pairs sum to 0, and the fraction simplifies to $\frac{v(2009) - v(2000)}{9}$.

Reading the values from the graph, you can approximate the value of the simplified fraction as $\frac{9.0 - 3.2}{9} = \frac{5.8}{9} \approx 0.644$ billion dollars. The answer choice that is closest to $0.644 billion is $640 million. The correct answer is **Choice E**.

Medium Question

6. In 2008 the value of exports was approximately what percent greater than the value of imports?

    (A)   40%

    (B)   60%

    (C)   70%

    (D) 120%

    (E) 140%

Explanation

The difference between the value of exports and the value of imports expressed as a percent of the value of imports is

$$\left(\frac{(\text{value of exports}) - (\text{value of imports})}{\text{value of imports}}\right)(100\%)$$

In 2008 the value of imports was approximately $5 billion and the value of exports was approximately $12 billion, so the value of the fraction is approximately $\frac{12 - 5}{5}$, or $\frac{7}{5}$.

Since the fraction is greater than 1, expressing it as a percent will give a percent greater than 100. The fraction is equal to 1.4, or 140 percent. The correct answer is **Choice E**.

Hard Question

7.   If it were discovered that the value of imports shown for 2007 was incorrect and should have been $5 billion instead, then the average (arithmetic mean) value of imports per year for the 10 years shown would have been approximately how much less?

    Ⓐ $200 million

    Ⓑ $50 million

    Ⓒ $20 million

    Ⓓ $7 million

    Ⓔ $5 million

Explanation

To answer this question, you do not need to compute either of the two 10-year averages referred to in the question; you just need to calculate the difference between the two averages.

The average value of imports for the 10 years shown in the graph is found by adding the 10 values and then dividing the sum by 10. The value of imports in 2007 is $7 billion. If that amount were $5 billion instead, then the sum of the values would be $2 billion less. If the sum were $2 billion less than what it was, then the average would decrease by 2 billion divided by 10, or $\frac{2,000,000,000}{10} = 200,000,000$. The average would therefore be $200 million less, and the correct answer is **Choice A**.

A more algebraic approach to the problem is to let S represent the sum, in billions, of the 10 values of imports in the graph. The average of the 10 values is $\frac{S}{10}$. Note that $S - 2$ represents the sum, in billions, of the 10 values adjusted for the $2 billion correction for 2007. The average of the adjusted sum is $\frac{S-2}{10}$. The difference between the two averages is

$$\frac{S}{10} - \frac{S-2}{10} = \frac{S-(S-2)}{10}$$
$$= \frac{S-S+2}{10}$$
$$= \frac{2}{10}$$

The difference is 0.2 billion dollars, or $200 million. The correct answer is **Choice A**.

O is the center of the circle above.

	Quantity A	Quantity B	
1.	x	5	(A) (B) (C) (D)

Runner *A* ran $\frac{4}{5}$ kilometer and Runner *B* ran 800 meters.

	Quantity A	Quantity B	
2.	The distance that *A* ran	The distance that *B* ran	(A) (B) (C) (D)

$$x < y < z$$

	Quantity A	Quantity B	
3.	$\dfrac{x + y + z}{3}$	y	(A) (B) (C) (D)

	Quantity A	Quantity B	
4.	$\dfrac{x}{y}$	1	(A) (B) (C) (D)

(A) **Quantity A is greater.**

(B) **Quantity B is greater.**

(C) **The two quantities are equal.**

(D) **The relationship cannot be determined from the information given.**

$$0 < x < y < 1$$

Quantity A	Quantity B				
5. $1 - y$	$y - x$	(A)	(B)	(C)	(D)

---

p is the probability that event E will occur, and s is the probability that event E will <u>not</u> occur.

Quantity A	Quantity B				
6. $p + s$	ps	(A)	(B)	(C)	(D)

X is the set of all integers n that satisfy the inequality $2 \le |n| \le 5$.

Quantity A	Quantity B				
7. The absolute value of the greatest integer in X	The absolute value of the least integer in X	(A)	(B)	(C)	(D)

---

x and m are positive numbers, and m is a multiple of 3.

Quantity A	Quantity B				
8. $\dfrac{x^m}{x^3}$	$x^{m/3}$	(A)	(B)	(C)	(D)

A random variable Y is normally distributed with a mean of 200 and a standard deviation of 10.

Quantity A	Quantity B				
9. The probability of the event that the value of Y is greater than 220	$\dfrac{1}{6}$	(A)	(B)	(C)	(D)

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Questions 10 to 25 have several different formats. Unless otherwise directed, select a single answer choice. For Numeric Entry questions, follow the instructions below.

Numeric Entry Questions
Enter your answer in the answer box(es) below the question.

- Your answer may be an integer, a decimal, or a fraction, and it may be negative.
- If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.
- Enter the exact answer unless the question asks you to round your answer.

10. The ratio of $\frac{1}{3}$ to $\frac{3}{8}$ is equal to the ratio of

   (A) 1 to 8
   (B) 8 to 1
   (C) 8 to 3
   (D) 8 to 9
   (E) 9 to 8

11. A reading list for a humanities course consists of 10 books, of which 4 are biographies and the rest are novels. Each student is required to read a selection of 4 books from the list, including 2 or more biographies. How many selections of 4 books satisfy the requirements?

   (A)   90
   (B) 115
   (C) 130
   (D) 144
   (E) 195

For the following question, enter your answer in the box.

12. In a graduating class of 236 students, 142 took algebra and 121 took chemistry. What is the greatest possible number of students that could have taken both algebra and chemistry?

   ☐ students
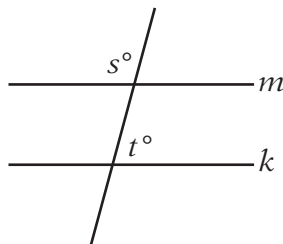
13. In the figure above, if $m \parallel k$ and $s = t + 30$, then $t =$

   Ⓐ 30
   Ⓑ 60
   Ⓒ 75
   Ⓓ 80
   Ⓔ 105

14. If $2x = 3y = 4z = 20$, then $12xyz =$

   Ⓐ 16,000
   Ⓑ 8,000
   Ⓒ 4,000
   Ⓓ 800
   Ⓔ 10

For the following question, select all the answer choices that apply.

15. The total amount that Mary paid for a book was equal to the price of the book plus a sales tax that was 4 percent of the price of the book. Mary paid for the book with a $10 bill and received the correct change, which was less than $3.00. Which of the following statements must be true?

   Indicate <u>all</u> such statements.

   A The price of the book was less than $9.50.
   B The price of the book was greater than $6.90.
   C The sales tax was less than $0.45.

16. If $\dfrac{1}{(2^{11})(5^{17})}$ is expressed as a terminating decimal, how many nonzero digits will the decimal have?

   Ⓐ One
   Ⓑ Two
   Ⓒ Four
   Ⓓ Six
   Ⓔ Eleven

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Questions 17 to 20 are based on the following data.

VARIATION IN THE AMOUNT OF CAFFEINE IN COMMON BEVERAGES AND DRUGS*

Amount of Caffeine (milligrams)	0 25 50 75 100 125 150 175 200
Coffee — Decaffeinated coffee	
Percolated coffee	
Drip-brewed coffee	
Instant coffee	
Other beverages — Brewed tea	
Instant tea	
Cocoa	
Caffeinated soft drinks	
Drugs — Weight-loss drugs, diuretics, and stimulants	
Pain relievers	
Cold/allergy remedies	
Source: Food and Drug Administration	

*Based on 5-ounce cups of coffee, tea, and cocoa; 12-ounce cups of soft drinks; and single doses of drugs.

17. The least amount of caffeine in a 5-ounce cup of drip-brewed coffee exceeds the greatest amount of caffeine in a 5-ounce cup of cocoa by approximately how many milligrams?

    Ⓐ 160
    Ⓑ   80
    Ⓒ   60
    Ⓓ   40
    Ⓔ   20

For the following question, enter your answer in the box.

18. For how many of the 11 categories of beverages and drugs listed in the graph can the amount of caffeine in the given serving size be less than 50 milligrams?

    [   ] categories

19. Approximately what is the minimum amount of caffeine, in milligrams, consumed per day by a person who daily drinks two 10-ounce mugs of percolated coffee and one 12-ounce cup of a caffeinated soft drink?

    Ⓐ 230
    Ⓑ 190
    Ⓒ 140
    Ⓓ 110
    Ⓔ   70

20. Which of the following shows the four types of coffee listed in order according to the range of the amounts of caffeine in a 5-ounce cup, from the least range to the greatest range?

    Ⓐ Decaffeinated, instant, percolated, drip-brewed

    Ⓑ Decaffeinated, instant, drip-brewed, percolated

    Ⓒ Instant, decaffeinated, drip-brewed, percolated

    Ⓓ Instant, drip-brewed, decaffeinated, percolated

    Ⓔ Instant, percolated, drip-brewed, decaffeinated

ENERGY USED PER METER VERSUS SPEED
DURING RUNNING AND WALKING

21. If s is a speed, in miles per hour, at which the energy used per meter during running is twice the energy used per meter during walking, then, according to the graph above, s is between

    Ⓐ 2.5 and 3.0

    Ⓑ 3.0 and 3.5

    Ⓒ 3.5 and 4.0

    Ⓓ 4.0 and 4.5

    Ⓔ 4.5 and 5.0

22. If $n = 2^3$, then $n^n =$

    Ⓐ 2^6

    Ⓑ 2^{11}

    Ⓒ 2^{18}

    Ⓓ 2^{24}

    Ⓔ 2^{27}

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For the following question, select all the answer choices that apply.

The length of AB is $10\sqrt{3}$.

23. Which of the following statements <u>individually</u> provide(s) sufficient additional information to determine the area of triangle ABC above?

    Indicate <u>all</u> such statements.

    A. DBC is an equilateral triangle.
    B. ABD is an isosceles triangle.
    C. The length of BC is equal to the length of AD.
    D. The length of BC is 10.
    E. The length of AD is 10.

For the following question, enter your answer in the box.

$$a_1, a_2, a_3, \ldots, a_n, \ldots$$

24. In the sequence above, each term after the first term is equal to the preceding term plus the constant c. If $a_1 + a_3 + a_5 = 27$, what is the value of $a_2 + a_4$?

    $a_2 + a_4 = \boxed{}$

25. A desert outpost has a water supply that is sufficient to last 21 days for 15 people. At the same average rate of water consumption per person, how many days would the water supply last for 9 people?

    (A) 28.0
    (B) 32.5
    (C) 35.0
    (D) 37.5
    (E) 42.0

STOP. This is the end of Section 5.

SECTION 6
Quantitative Reasoning
Time—40 minutes
25 Questions

For each question, indicate the best answer, using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, **are not necessarily** drawn to scale. That is, you should **not** assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities by sight or by measurement.

Coordinate systems, such as *xy*-planes and number lines, **are** drawn to scale; therefore, you can read, estimate, or compare quantities in such figures by sight or by measurement.

Graphical data presentations, such as bar graphs, circle graphs, and line graphs, **are** drawn to scale; therefore, you can read, estimate, or compare data values by sight or by measurement.

For each of Questions 1 to 9, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	Quantity B	Correct Answer
Example 1:	(2)(6)	2+6	⬤Ⓐ Ⓑ Ⓒ Ⓓ

- -

	Quantity A	Quantity B	Correct Answer
Example 2:	*PS*	*SR*	Ⓐ Ⓑ Ⓒ ⬤Ⓓ

(since equal lengths cannot
be assumed, even though
PS and *SR* appear equal)

Ⓐ **Quantity A is greater.**

Ⓑ **Quantity B is greater.**

Ⓒ **The two quantities are equal.**

Ⓓ **The relationship cannot be determined from the information given.**

Country	Value of 1 United States Dollar
Argentina	0.93 peso
Kenya	32.08 shillings

	Quantity A	Quantity B	

1. The dollar value of 1 Argentine peso according to the table above | The dollar value of 1 Kenyan shilling according to the table above   Ⓐ Ⓑ Ⓒ Ⓓ

---

k is a digit in the decimal 1.3k5, and 1.3k5 is less than 1.33.
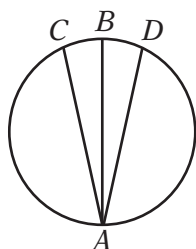
	Quantity A	Quantity B

2. k 1   Ⓐ Ⓑ Ⓒ Ⓓ

AB is a diameter of the circle above.

	Quantity A	Quantity B

3.   The length of *AB*   The average (arithmetic mean) of the lengths of *AC* and *AD*   Ⓐ Ⓑ Ⓒ Ⓓ

$$st=\sqrt{10}$$

	Quantity A	Quantity B

4. s^2 $\dfrac{10}{t^2}$ Ⓐ Ⓑ Ⓒ Ⓓ

Ⓐ	**Quantity A is greater.**
Ⓑ	**Quantity B is greater.**
Ⓒ	**The two quantities are equal.**
Ⓓ	**The relationship cannot be determined from the information given.**

Three consecutive integers have a sum of −84.

Quantity A	Quantity B	
5. The least of the three integers	−28	Ⓐ Ⓑ Ⓒ Ⓓ

In the *xy*-plane, the equation of line *k* is $3x - 2y = 0$.

Quantity A	Quantity B	
6. The *x*-intercept of line *k*	The *y*-intercept of line *k*	Ⓐ Ⓑ Ⓒ Ⓓ

n is a positive integer that is divisible by 6.

Quantity A	Quantity B	
7. The remainder when *n* is divided by 12	The remainder when *n* is divided by 18	Ⓐ Ⓑ Ⓒ Ⓓ

$$\frac{1 - x}{x - 1} = \frac{1}{x}$$

Quantity A	Quantity B	
8. x	$-\dfrac{1}{2}$	Ⓐ Ⓑ Ⓒ Ⓓ

In a set of 24 positive integers, 12 of the integers are less than 50. The rest are greater than 50.

Quantity A	Quantity B	
9. The median of the 24 integers	50	Ⓐ Ⓑ Ⓒ Ⓓ

GO ON TO THE NEXT PAGE ➘

Questions 10 to 25 have several different formats. Unless otherwise directed, select a single answer choice. For Numeric Entry questions, follow the instructions below.

Numeric Entry Questions
Enter your answer in the answer box(es) below the question.

- Your answer may be an integer, a decimal, or a fraction, and it may be negative.
- If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.
- Enter the exact answer unless the question asks you to round your answer.

10. The fabric needed to make 3 curtains sells for $8.00 per yard and can be purchased only by the full yard. If the length of fabric required for each curtain is 1.6 yards and all of the fabric is purchased as a single length, what is the total cost of the fabric that needs to be purchased for the 3 curtains?

Ⓐ $40.00
Ⓑ $38.40
Ⓒ $24.00
Ⓓ $16.00
Ⓔ $12.80

For the following question, select all the answer choices that apply.

11. In the xy-plane, line k is a line that does <u>not</u> pass through the origin.

Which of the following statements <u>individually</u> provide(s) sufficient additional information to determine whether the slope of line k is negative?

Indicate <u>all</u> such statements.

☐A The x-intercept of line k is twice the y-intercept of line k.
☐B The product of the x-intercept and the y-intercept of line k is positive.
☐C Line k passes through the points (a, b) and (r, s), where $(a - r)(b - s) < 0$.

	Distance from Centerville (miles)
Freight train	$-10t + 115$
Passenger train	$-20t + 150$

12. The expressions in the table above give the distance from Centerville to each of two trains t hours after 12:00 noon. At what time after 12:00 noon will the trains be equidistant from Centerville?

    (A) 1:30
    (B) 3:30
    (C) 5:10
    (D) 8:50
    (E) 11:30

13. The company at which Mark is employed has 80 employees, each of whom has a different salary. Mark's salary of $43,700 is the second-highest salary in the first quartile of the 80 salaries. If the company were to hire 8 new employees at salaries that are less than the lowest of the 80 salaries, what would Mark's salary be with respect to the quartiles of the 88 salaries at the company, assuming no other changes in the salaries?

    (A) The fourth-highest salary in the first quartile
    (B) The highest salary in the first quartile
    (C) The second-lowest salary in the second quartile
    (D) The third-lowest salary in the second quartile
    (E) The fifth-lowest salary in the second quartile

For the following question, enter your answer in the box.

14. In the xy-plane, the point with coordinates $(-6, -7)$ is the center of circle C. The point with coordinates $(-6, 5)$ lies inside C, and the point with coordinates $(8, -7)$ lies outside C. If m is the radius of C and m is an integer, what is the value of m ?

    $m =$ [   ]

15. If $-\dfrac{m}{19}$ is an even integer, which of the following must be true?

    (A) m is a negative number.
    (B) m is a positive number.
    (C) m is a prime number.
    (D) m is an odd integer.
    (E) m is an even integer.

For the following question, select all the answer choices that apply.

16. The integer v is greater than 1. If v is the square of an integer, which of the following numbers must also be the square of an integer?

    Indicate <u>all</u> such numbers.

    A $81v$
    B $25v+10\sqrt{v}+1$
    C $4v^2+4\sqrt{v}+1$

Questions 17 to 20 are based on the following data.

DISTANCE TRAVELED BY A CAR ACCORDING TO THE CAR'S SPEED
WHEN THE DRIVER IS SIGNALED TO STOP
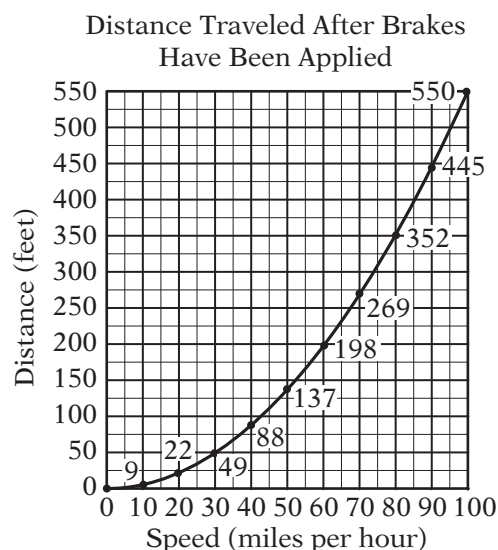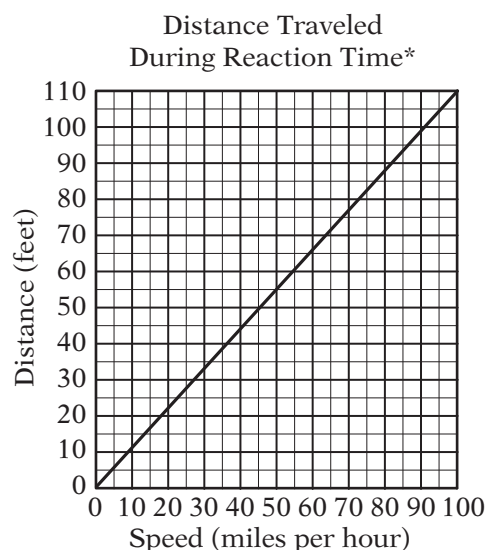
Distance Traveled
During Reaction Time*

Distance Traveled After Brakes
Have Been Applied

*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

<u>Note:</u> Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.

17. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

    Ⓐ 43
    Ⓑ 47
    Ⓒ 51
    Ⓓ 55
    Ⓔ 59

18. Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

    (A) 130
    (B) 110
    (C) 90
    (D) 70
    (E) 40

19. Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

    (A) 50
    (B) 55
    (C) 60
    (D) 65
    (E) 70

20. The total stopping distance for the car traveling at 60 miles per hour is approximately what percent greater than the total stopping distance for the car traveling at 50 miles per hour?

    (A) 22%
    (B) 30%
    (C) 38%
    (D) 45%
    (E) 52%

21. What is the least positive integer that is <u>not</u> a factor of 25! and is <u>not</u> a prime number?

    (A) 26
    (B) 28
    (C) 36
    (D) 56
    (E) 58

22. If $0 < a < 1 < b$, which of the following is true about the reciprocals of a and b ?

    (A) $1 < \dfrac{1}{a} < \dfrac{1}{b}$

    (B) $\dfrac{1}{a} < 1 < \dfrac{1}{b}$

    (C) $\dfrac{1}{a} < \dfrac{1}{b} < 1$

    (D) $\dfrac{1}{b} < 1 < \dfrac{1}{a}$

    (E) $\dfrac{1}{b} < \dfrac{1}{a} < 1$
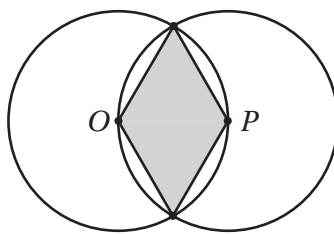
23. In the figure above, *O* and *P* are the centers of the two circles. If each circle has radius *r*, what is the area of the shaded region?

    (A) $\dfrac{\sqrt{2}}{2}r^2$

    (B) $\dfrac{\sqrt{3}}{2}r^2$

    (C) $\sqrt{2}r^2$

    (D) $\sqrt{3}r^2$

    (E) $2\sqrt{3}r^2$

For the following question, enter your answer in the boxes.

24. Of the 20 lightbulbs in a box, 2 are defective. An inspector will select 2 lightbulbs simultaneously and at random from the box. What is the probability that neither of the lightbulbs selected will be defective?

    Give your answer as a fraction.

    $$\frac{}{}$$

25. What is the perimeter, in meters, of a rectangular playground 24 meters wide that has the same area as a rectangular playground 64 meters long and 48 meters wide?
    (A) 112
    (B) 152
    (C) 224
    (D) 256
    (E) 304

STOP. This is the end of Section 6.

Answer Key

Question Number	P+	Correct Answer
23	33	**Choice A:** Some of them offered a religious explanation for their immigration.
24	94	**Sentence 5:** When he moves beyond the principal actors, he finds that religious explanations were less frequently offered, and he concludes that most people immigrated because they were recruited by promises of material improvement.
25	64	**Choice A:** summarizing the findings of an investigation

Section 5. Quantitative Reasoning

Question Number	P+	Correct Answer
1	65	**Choice B:** Quantity B is greater.
2	83	**Choice C:** The two quantities are equal.
3	62	**Choice D:** The relationship cannot be determined from the information given.
4	48	**Choice A:** Quantity A is greater.
5	47	**Choice D:** The relationship cannot be determined from the information given.
6	36	**Choice A:** Quantity A is greater.
7	26	**Choice C:** The two quantities are equal.
8	34	**Choice D:** The relationship cannot be determined from the information given.
9	46	**Choice B:** Quantity B is greater.
10	62	**Choice D:** 8 to 9
11	26	**Choice B:** 115
12	51	**121**
13	52	**Choice C:** 75
14	60	**Choice C:** 4,000
15	18	**Choice C:** The sales tax was less than $0.45.
16	19	**Choice B:** Two
17	69	**Choice D:** 40
18	63	**9**
19	41	**Choice B:** 190
20	60	**Choice B:** Decaffeinated, instant, drip-brewed, percolated
21	70	**Choice A:** 2.5 and 3.0
22	35	**Choice D:** 2^{24}
23	17	**Choice A:** *DBC* is an equilateral triangle. AND **Choice D:** The length of *BC* is 10.
24	51	**18**
25	34	**Choice C:** 35.0

Answer Key

Section 6. Quantitative Reasoning

Question Number	P+	Correct Answer
1	66	**Choice A:** Quantity A is greater.
2	73	**Choice D:** The relationship cannot be determined from the information given.
3	52	**Choice A:** Quantity A is greater.
4	46	**Choice C:** The two quantities are equal.
5	48	**Choice B:** Quantity B is greater.
6	33	**Choice C:** The two quantities are equal.
7	40	**Choice D:** The relationship cannot be determined from the information given.
8	26	**Choice B:** Quantity B is greater.
9	31	**Choice D:** The relationship cannot be determined from the information given.
10	67	**Choice A:** $40.00
11	10	**Choice A:** The x-intercept of line k is twice the y-intercept of line k. AND **Choice B:** The product of the x-intercept and the y-intercept of line k is positive. AND **Choice C:** Line k passes through the points (a, b) and (r, s), where $(a - r)(b - s) < 0$.
12	49	**Choice B:** 3:30
13	27	**Choice E:** The fifth-lowest salary in the second quartile
14	38	**13**
15	40	**Choice E:** m is an even integer.
16	21	**Choice A:** $81v$ AND **Choice B:** $25v + 10\sqrt{v} + 1$
17	79	**Choice B:** 47
18	44	**Choice A:** 130
19	41	**Choice A:** 50
20	30	**Choice C:** 38%
21	11	**Choice E:** 58
22	58	**Choice D:** $\dfrac{1}{b} < 1 < \dfrac{1}{a}$
23	24	**Choice B:** $\dfrac{\sqrt{3}}{2}r^2$
24	15	$\dfrac{153}{190}$ (or any equivalent fraction)
25	30	**Choice E:** 304

SECTION 5
Quantitative Reasoning
25 Questions with Explanations

Ⓐ **Quantity A is greater.**

Ⓑ **Quantity B is greater.**

Ⓒ **The two quantities are equal.**

Ⓓ **The relationship cannot be determined from the information given.**

O is the center of the circle above.

Quantity A	Quantity B

1. x 5   Ⓐ Ⓑ Ⓒ Ⓓ
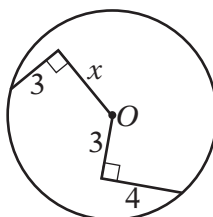
Explanation

In this question you are asked to compare x with 5, where x is the length of a line segment from the center of the circle to a point inside the circle. In a circle the easiest line segments to deal with are the radius and the diameter. Looking at the figure in the question, you can see that you can draw two radii, each of which "completes" a right triangle, as shown in the figure below.
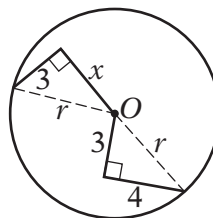
Since in one of the triangles the lengths of both legs are known, you can use that triangle to determine the length of the radius of the circle. The triangle has legs of length 3 and 4. If the length of the radius is r, then, using the Pythagorean theorem, you can see that

$$r^2 = 3^2 + 4^2 \text{ or}$$
$$r^2 = 9 + 16 \text{ or}$$
$$r^2 = 25, \text{ and thus, } r = 5$$

Since the length of the radius of the circle is 5 and the line segment of length x is clearly shorter than the radius, you know that $x < 5$, and the correct answer is **Choice B.**

You could also see that the two triangles are congruent, and so $x = 4$, again yielding **Choice B**.

Runner A ran $\frac{4}{5}$ kilometer and Runner B ran 800 meters.

Quantity A	Quantity B				
2. The distance that A ran	The distance that B ran	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Explanation

In this question you are asked to compare two measurements, one given in kilometers and the other in meters. It would be easier to compare these measurements if they were both given in meters or both given in kilometers.

If you choose to convert the distance that Runner B ran from meters to kilometers, you need to use the conversion 1 meter is equal to $\frac{1}{1,000}$ kilometer. Since B ran 800 meters, it follows that B ran $(800)\left(\frac{1}{1,000}\right)$, or $\frac{4}{5}$ kilometer, which is the same distance that A ran.

If you choose to convert the distance that Runner A ran from kilometers to meters, you need to use the conversion 1 kilometer is equal to 1,000 meters. Since A ran $\frac{4}{5}$ kilometer, it follows that A ran $\left(\frac{4}{5}\right)(1,000)$, or 800 meters, which is the same distance that B ran. Either way, A and B ran the same distance, and the correct answer is **Choice C**.

$$x < y < z$$

Quantity A	Quantity B				
3. $\dfrac{x + y + z}{3}$	y	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Explanation

In this question you are given that $x < y < z$, and you are asked to compare $\frac{x + y + z}{3}$ with y.

Two approaches that you could use to solve this problem are:

1: Search for a mathematical relationship between the two quantities.
2: Plug in numbers for the variables.

Approach 1: Note that $\frac{x + y + z}{3}$ is the average of the three numbers x, y, and z and that y is the median. Is the average of 3 numbers always equal to the median? The average could equal the median, but in general they do not have to be equal. Therefore, the correct answer is **Choice D**.

Approach 2: When you plug in numbers for the variables, it is a good idea to consider what kind of numbers are appropriate to plug in and to choose numbers that are easy to work with, if possible.

Since $\frac{x + y + z}{3}$ is the average of the three numbers x, y, and z and you are comparing it to the median, you may want to try plugging in numbers that are evenly spaced and plugging in numbers that are not evenly spaced.

You can plug in numbers that are both evenly spaced and easy to work with. For example, you can plug in $x = 1$, $y = 2$, and $z = 3$. In this case, $\frac{x + y + z}{3} = \frac{1 + 2 + 3}{3} = \frac{6}{3} = 2$, and so $\frac{x + y + z}{3} = y$.

You can also plug in numbers that not are not evenly spaced and are easy to work with. For example, you can plug in $x = 3$, $y = 6$, and $z = 12$. In this case, $\frac{x + y + z}{3} = \frac{3 + 6 + 12}{3} = \frac{21}{3} = 7$, and $\frac{x + y + z}{3} > y$. Since in the first case, $\frac{x + y + z}{3}$ is equal to y and in the second case, it is greater than y, the relationship between the two quantities $\frac{x + y + z}{3}$ and y cannot be determined from the information given. The correct answer is **Choice D**.
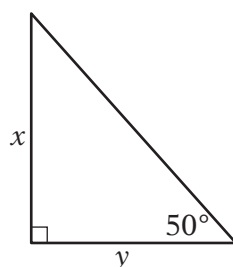
	Quantity A		Quantity B	
4.	$\dfrac{x}{y}$		1	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

One way you can solve this problem is by using the following fact:

- If *ABC* is a triangle and the measure of angle *A* is greater than the measure of angle *B*, then the side opposite angle *A* is longer than the side opposite angle *B*.

Since the third angle of the triangle measures 40°, you can use the fact above to conclude that the side opposite the 50° angle is longer than the side opposite the 40° angle. So $x > y$, and $\frac{x}{y} > 1$, which yields **Choice A**.

You can also solve this problem without using the fact above. Instead, you can use the strategy of adapting solutions to related problems to determine the relationship between x and y.

Note that the angles in the triangle differ only a little from the angles in a $45° - 45° - 90°$ triangle. How do the lengths of the legs of a $45° - 45° - 90°$ triangle compare to the lengths of the legs of the triangle in the question? To make the comparison, add a line segment to the triangle so that the line segment cuts the 50° angle in two parts, making a 45° angle with the horizontal base, as shown in the following figure:
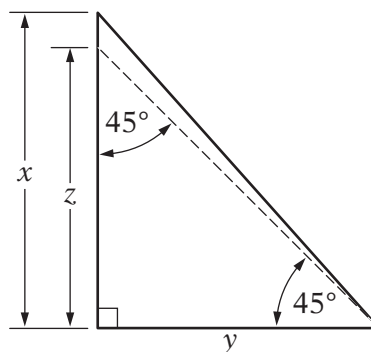
The $45° - 45° - 90°$ triangle has two $45°$ angles, so $z = y$, and $\frac{z}{y} = 1$. Since $\frac{z}{y} = 1$ and $x > z$, it follows that $\frac{x}{y} > 1$. The correct answer is **Choice A**.

$$0 < x < y < 1$$

Quantity A	Quantity B	

5. $\qquad$ $1 - y$ $\qquad\qquad\qquad$ $y - x$ $\qquad$ Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Two approaches that you could use to solve this problem are:

    1: Translate from algebra to a number line.
    2: Plug in values for the variables.

Approach 1: The following figure represents the information given in the problem on a number line.
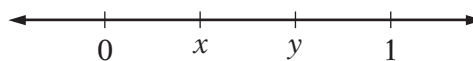
On the number line, $1 - y$ is the distance between 1 and y, and $y - x$ is the distance between y and x. If y is exactly halfway between x and 1, then $1 - y$ is equal to $y - x$; and if y is <u>not</u> halfway between x and 1, then $1 - y$ is <u>not</u> equal to $y - x$. But y can be <u>any</u> number between x and 1, so the correct answer is **Choice D**.

Approach 2: Since this problem involves subtraction, it is a good idea to choose values for x and y that are close to each other as well as values that are far apart. For example, if $x = 0.4$ and $y = 0.5$, then $1 - y = 0.5$ and $y - x = 0.1$; and if $x = 0.1$ and $y = 0.9$, then $1 - y = 0.1$ and $y - x = 0.8$. This shows that the relationship cannot be determined, and the correct answer is **Choice D**.

p is the probability that event E will occur, and s is the probability that event E will <u>not</u> occur.

	Quantity A	Quantity B	
6.	$p + s$	ps	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Since event E will either occur or not occur, it follows that $p + s = 1$, and the value of Quantity A is always 1. Since Quantity B is the product of the two probabilities p and s, you need to look at its value for the cases $p = 1$, $p = 0$, and $0 < p < 1$.

If $p = 1$, then $s = 0$; similarly, if $p = 0$, then $s = 1$. In both cases, ps is equal to 0.

If $0 < p < 1$, both p and s are positive and less than 1, so ps is positive and less than 1. Since Quantity A is equal to 1 and Quantity B is less than 1, the correct answer is **Choice A**.

X is the set of all integers n that satisfy the inequality $2 \le |n| \le 5$.

	Quantity A	Quantity B	
7.	The absolute value of the greatest integer in X	The absolute value of the least integer in X	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

When comparing these quantities, it is important to remember that a nonzero number and its negative have the same absolute value. For example, $|-2| = |2| = 2$. Keeping this in mind, you can see that the positive integers 2, 3, 4, and 5 and the negative integers -2, -3, -4, and -5 all satisfy the inequalities $2 \le |n| \le 5$, and that these are the only such integers. Thus, the set X consists of the integers -5, -4, -3, -2, 2, 3, 4, and 5. The greatest of these integers is 5, and its absolute value is 5. The least of these integers is -5, and its absolute value is also 5. Therefore, Quantity A is equal to Quantity B. The correct answer is **Choice C**.

x and m are positive numbers, and m is a multiple of 3.

	Quantity A	Quantity B	
8.	$\dfrac{x^m}{x^3}$	$x^{m/3}$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Since $\dfrac{x^m}{x^3} = x^{m-3}$, you need to compare x^{m-3} with $x^{m/3}$. Since the base in both expressions is the same, a good strategy to use to solve this problem is to plug in numbers for m in both expressions and compare them.

You know that m is a multiple of 3, so the least positive integer you can plug in for m is 3.

If $m = 3$, then $x^{m-3} = 1$ and $x^{m/3} = x$. Since x can be any real number, its relationship to 1 cannot be determined from the information given. This example is sufficient to show that the relationship between $\dfrac{x^m}{x^3}$ and $x^{m/3}$ cannot be determined from the information given. The correct answer is **Choice D**.

A random variable Y is normally distributed with a mean of 200 and a standard deviation of 10.

Quantity A	Quantity B	
9. The probability of the event that the value of Y is greater than 220	$\dfrac{1}{6}$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

This problem involves a normal distribution with mean 200 and standard deviation 10. Thus, the value of 210 is 1 standard deviation above the mean, and the value of 220 is 2 standard deviations above the mean. To compare Quantity A with Quantity B, it is not necessary to exactly determine the probability of the event that the value of Y is greater than 220. Remember that in any normal distribution, almost all of the data values, or about 95% of them, fall within 2 standard deviations on either side of the mean. This means that less than 5% of the values in this distribution will be greater than 220. Thus, the probability of the event that the value of Y is greater than 220 must be less than 5%, or $\dfrac{1}{20}$, and this is certainly less than $\dfrac{1}{6}$. The correct answer is **Choice B**.

Another approach to this problem is to draw a normal curve, or "bell-shaped curve," that represents the probability distribution of the random variable Y, as shown in the figure below.
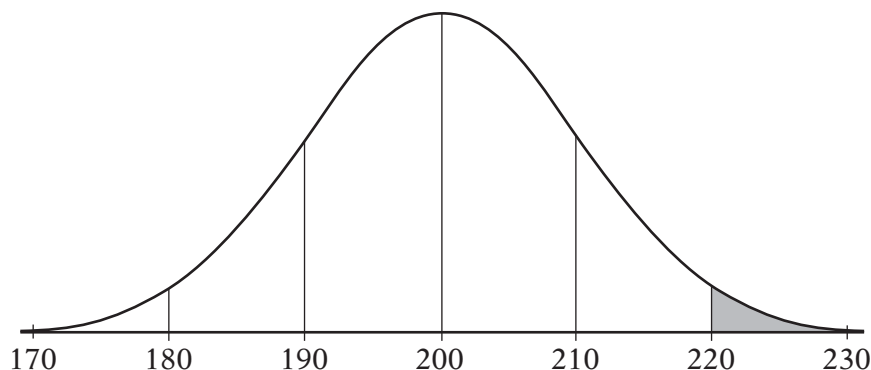
The curve is symmetric about the mean 200. The values of 210, 220, and 230 are equally spaced to the right of 200 and represent 1, 2, and 3 standard deviations, respectively, above the mean. Similarly, the values of 190, 180, and 170 are 1, 2, and 3 standard deviations, respectively, below the mean. Quantity A, the probability of the event that the value of Y is greater than 220, is equal to the area of the shaded region as a fraction of the total area under the curve.

From the figure, you can see that the area under the normal curve has been divided into 6 regions and that these regions are not equal in area. The

shaded region is one of the two smallest of the 6 regions, so its area must be less than $\frac{1}{6}$ of the total area under the curve. The correct answer is **Choice B**.

10. The ratio of $\frac{1}{3}$ to $\frac{3}{8}$ is equal to the ratio of

    (A) 1 to 8

    (B) 8 to 1

    (C) 8 to 3

    (D) 8 to 9

    (E) 9 to 8

Explanation

Multiplying both parts of a ratio by the same number produces an equivalent ratio. While you could multiply both fractions in the ratio by any number, 24 is a good number to choose because it is the least common multiple of 3 and 8.

Thus, multiplying both $\frac{1}{3}$ and $\frac{3}{8}$ by 24, you get that the ratio of $\frac{1}{3}$ to $\frac{3}{8}$ is equal to the ratio of 8 to 9. The correct answer is **Choice D**.

An alternate approach to this problem is to express the ratio of $\frac{1}{3}$ to $\frac{3}{8}$ as the fraction $\dfrac{\frac{1}{3}}{\frac{3}{8}}$. This fraction is equivalent to $\left(\frac{1}{3}\right)\left(\frac{8}{3}\right)$, or $\frac{8}{9}$. The correct answer is **Choice D**.

11. A reading list for a humanities course consists of 10 books, of which 4 are biographies and the rest are novels. Each student is required to read a selection of 4 books from the list, including 2 or more biographies. How many selections of 4 books satisfy the requirements?

    (A) 90

    (B) 115

    (C) 130

    (D) 144

    (E) 195

Explanation

The requirement to select 4 books, including 2 or more biographies, means that you have to consider three cases. A student can choose 4 biographies and no novels, or 3 biographies and 1 novel, or 2 biographies and 2 novels.

    Case 1: Choose 4 biographies. This case is easy, as there is only 1 way to choose all four biographies and no novels.

    In the other two cases, you have to find the number of ways of choosing the biographies and the number of ways of choosing the novels and then multiply these two numbers.

Case 2: Choose 3 biographies and 1 novel. First, you need to find the number of ways of choosing 3 biographies out of 4. If you think of this as <u>not</u> choosing 1 out of the 4, you see that there are 4 choices. The number of ways of choosing 1 novel out of the 6 novels is 6. Therefore, the total number of choices is $(4)(6) = 24$.

Case 3: Choose 2 biographies and 2 novels. First, you need to find the number of ways of choosing 2 biographies out of 4. This number is sometimes called "4 choose 2" or the number of combinations of 4 objects taken 2 at a time. If you remember the combinations formula, you know that the number of combinations is $\frac{4!}{2!(4-2)!}$ (which is denoted symbolically as $\binom{4}{2}$ or $_4C_2$). The value of $\frac{4!}{2!(4-2)!}$ is $\frac{(4)(3)(2!)}{(2)(2!)} = \frac{(4)(3)}{2} = 6$. Thus, there are 6 ways to choose 2 biographies out of 4. Similarly, the number of ways to choose 2 novels out of 6 is $\frac{6!}{2!4!} = \frac{(6)(5)}{2} = 15$. Thus, the total number of ways to choose 2 biographies and 2 novels is $(6)(15) = 90$.

Adding the number of ways to choose the books for each of the three cases, you get a total of $1 + 24 + 90 = 115$. The correct answer is **Choice B**.

For the following question, enter your answer in the box.

12. In a graduating class of 236 students, 142 took algebra and 121 took chemistry. What is the greatest possible number of students that could have taken both algebra and chemistry?

[   ] students

Explanation

This is the type of problem for which drawing a Venn diagram is usually helpful. The Venn diagram below is one you could draw to represent the information given in the question.

Total Students: 236

Algebra
142
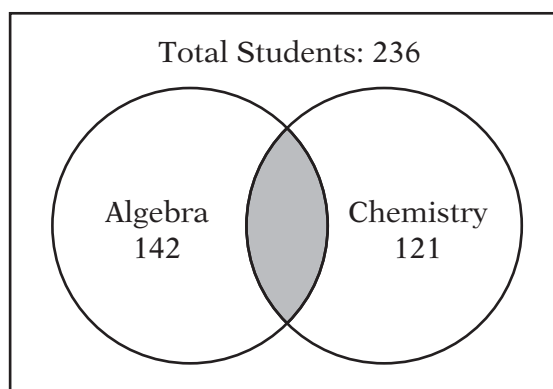
Chemistry
121

Note that the algebra and chemistry numbers given do not separate out the number of students who took both algebra and chemistry, and that this question asks for the greatest possible number of such students. It is a good idea, therefore, to redraw the Venn diagram with the number of students who took

both algebra and chemistry separated out. The revised Venn diagram looks like the one below.
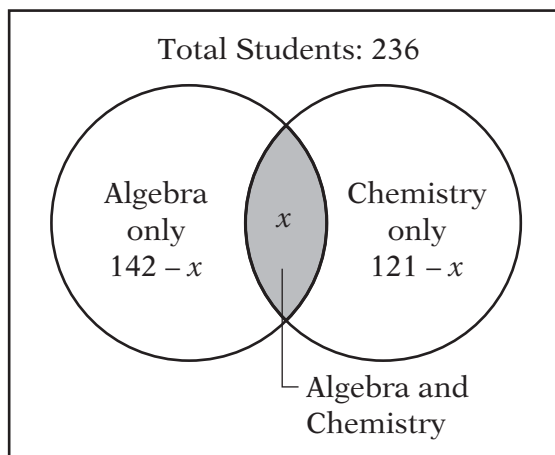
To solve this problem you want the greatest possible value of x. It is clear from the diagram that x cannot be greater than 142 nor greater than 121, otherwise $142 - x$ or $121 - x$ would be negative. Hence, x must be less than or equal to 121. Since there is no information to exclude $x = 121$, the correct answer is **121.**
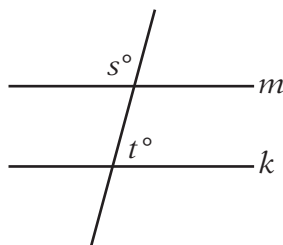
13. In the figure above, if $m \parallel k$ and $s = t + 30$, then $t =$

    (A)   30

    (B)   60

    (C)   75

    (D)   80

    (E)   105

Explanation

When trying to solve a geometric problem, it is often helpful to add any known information to the figure. Since corresponding angles have equal measures, you could place two more angle measures on the figure, as shown below.
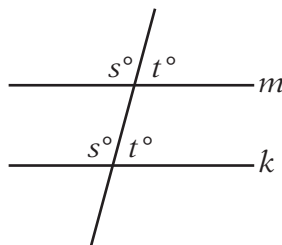
Now, from the figure, you can see that $s + t = 180$. Therefore, since it is given that $s = t + 30$, you can substitute $t + 30$ for s into the equation $s + t = 180$ and get that $(t + 30) + t = 180$, which can be simplified as follows.

$$(t + 30) + t = 180$$
$$2t = 150$$
$$t = 75$$

The correct answer is **Choice C**.

14. If $2x = 3y = 4z = 20$, then $12xyz =$

    (A) 16,000
    (B) 8,000
    (C) 4,000
    (D)   800
    (E)   10

Explanation

One approach you can use to solve this problem is to find the value of all three variables.

$$2x = 20, \text{ or } x = 10$$
$$3y = 20, \text{ or } y = \frac{20}{3}$$
$$4z = 20, \text{ or } z = 5$$

So $12xyz = 12(10)\left(\frac{20}{3}\right)(5) = 4{,}000$, and the correct answer is **Choice C**.

Another approach you can use to solve this problem is to notice that $12xyz = \dfrac{(2x)(3y)(4z)}{2} = \dfrac{(20)(20)(20)}{2} = 4{,}000$. Therefore, the correct answer is **Choice C**.

For the following question, select all the answer choices that apply.

15. The total amount that Mary paid for a book was equal to the price of the book plus a sales tax that was 4 percent of the price of the book. Mary paid for the book with a $10 bill and received the correct change, which was less than $3.00. Which of the following statements must be true?

    Indicate <u>all</u> such statements.

    [A] The price of the book was less than $9.50.
    [B] The price of the book was greater than $6.90.
    [C] The sales tax was less than $0.45.

Explanation

For this problem you may find it helpful to translate the given information into an algebraic expression. Since the price of the book is unknown, you can call it x dollars, and then the total amount that Mary paid is x dollars plus 4% of x dollars, or $1.04x$ dollars. The problem states that Mary received some change from a $10 bill, so $1.04x$ dollars must be less than $10. Since the change was less

than $3.00, the total amount Mary paid for the book must have been greater than $7.00. You can express this information algebraically by the inequality

$$7.00 < 1.04x < 10.00$$

Solving the inequality for x by dividing by 1.04, and rounding, you get

$$6.73 < x < 9.62$$

So you see that x, the price of the book, must be between $6.73 and $9.62. With this information, you can quickly examine the first two statements. Choice A is not necessarily true because the price could be as high as $9.61, and Choice B is not necessarily true because the price could be as low as $6.74.

To examine Choice C, you could compute the tax for the greatest possible price, which would be 4% of 9.61, or $(0.04)(9.61) = 0.38$. Since this greatest possible tax is less than $0.45, Choice C must be true.

You can also quickly see that Choice C must be true if you note that 4% of $10.00 would only be $0.40, and since the price must be less than $10.00, the tax must be less than $0.40. The correct answer consists of **Choice C**.

16. If $\dfrac{1}{(2^{11})(5^{17})}$ is expressed as a terminating decimal, how many nonzero digits will the decimal have?

(A) One

(B) Two

(C) Four

(D) Six

(E) Eleven

Explanation
To convert the fraction to a decimal, it is helpful to first write the fraction in powers of 10. Using the rules of exponents, you can write the following.

$$\frac{1}{(2^{11})(5^{17})} = \frac{1}{(2^{11})(5^{11+pl6})}$$

$$= \frac{1}{(2^{11})(5^{11})(5^{6})}$$

$$= \frac{1}{(10^{11})(5^{6})}$$

$$= \left(\frac{1}{5}\right)^{6}(10^{-11})$$

$$= (0.2)^{6}(10^{-11})$$

$$= ((2)(10)^{-1})^{6}(10^{-11})$$

$$= (2^{6})(10^{-6})(10^{-11})$$

$$= (2^{6})(10^{-17})$$

$$= (64)(10^{-17})$$

So the decimal has two nonzero digits, 6 and 4. The correct answer is **Choice B**.

VARIATION IN THE AMOUNT OF CAFFEINE IN COMMON BEVERAGES AND DRUGS*

Amount of Caffeine (milligrams)	0	25	50	75	100	125	150	175	200

		Caffeine range
Coffee	Decaffeinated coffee	about 0–5
	Percolated coffee	about 40–170
	Drip-brewed coffee	about 60–180
	Instant coffee	about 30–120
Other beverages	Brewed tea	about 25–110
	Instant tea	about 30–50
	Cocoa	about 0–20
	Caffeinated soft drinks	about 30–60
Drugs	Weight-loss drugs, diuretics, and stimulants	about 130–200
	Pain relievers	about 30–75
	Cold/allergy remedies	about 10–30

Source: Food and Drug Administration
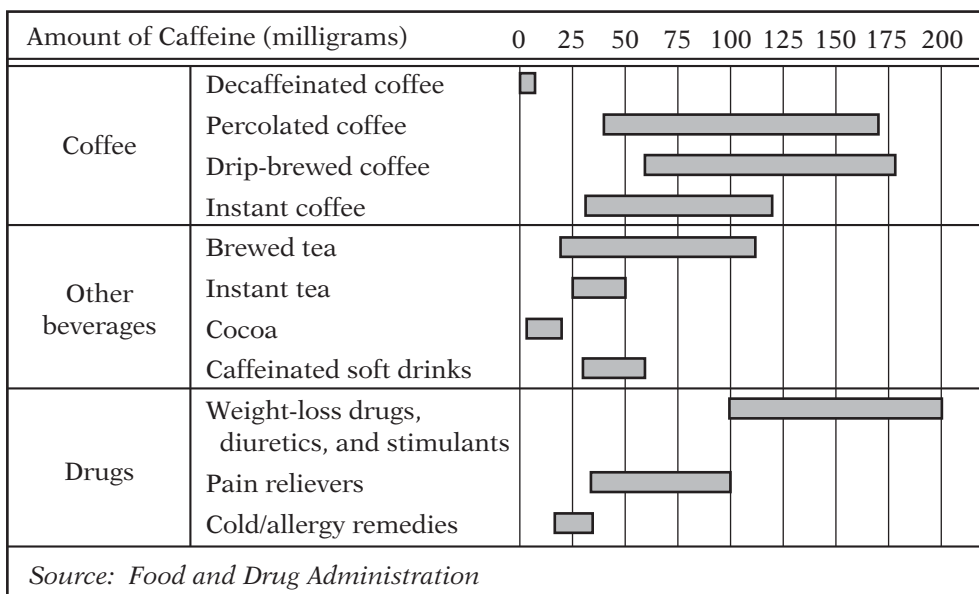
*Based on 5-ounce cups of coffee, tea, and cocoa; 12-ounce cups of soft drinks; and single doses of drugs.

17. The least amount of caffeine in a 5-ounce cup of drip-brewed coffee exceeds the greatest amount of caffeine in a 5-ounce cup of cocoa by approximately how many milligrams?

 (A) 160
 (B) 80
 (C) 60
 (D) 40
 (E) 20

Explanation

Each horizontal bar in the bar graph shows the possible number of milligrams of caffeine in each of the common beverages and drugs. The least possible amount of caffeine in a 5-ounce cup of drip-brewed coffee is about 60 milligrams, and the greatest possible amount of caffeine in a 5-ounce cup of cocoa is about 20 milligrams. So, the difference is approximately $60 - 20$, or 40 milligrams. The correct answer is **Choice D**.

 To check your answer, it is useful to try to solve the problem using another method as well to see if you get the same answer. To solve this problem in another way, note that the distance between each pair of adjacent vertical grid lines represents 25 milligrams of caffeine, and the distance between the high end of the cocoa bar and the low end of the drip-brewed coffee bar is a little more than the distance between a pair of adjacent grid lines. Therefore, the answer is between 25 and 50. Among the choices, only Choice D is between 25 and 50, so the correct answer is **Choice D**.

For the following question, enter your answer in the box.

18. For how many of the 11 categories of beverages and drugs listed in the graph can the amount of caffeine in the given serving size be less than 50 milligrams?

☐ categories

Explanation

In the graph, the left edge of each bar tells you what is the least possible amount of caffeine in the corresponding beverage or drug. A beverage or drug can have less than 50 milligrams of caffeine if the left edge of its bar lies to the left of the vertical line corresponding to 50 milligrams of caffeine. From the graph, you see that there are 9 bars for which this is true. There are only 2 bars that lie entirely to the right of the 50-milligram line—the bar for drip-brewed coffee and the bar for weight-loss drugs, diuretics, and stimulants. So there are 9 categories of beverages and drugs that can have less than 50 milligrams of caffeine in the given serving size. The correct answer is **9**.

19. Approximately what is the minimum amount of caffeine, in milligrams, consumed per day by a person who daily drinks two 10-ounce mugs of percolated coffee and one 12-ounce cup of a caffeinated soft drink?

Ⓐ 230
Ⓑ 190
Ⓒ 140
Ⓓ 110
Ⓔ 70

Explanation

According to the bar graph, the minimum amount of caffeine in a 5-ounce cup of percolated coffee is approximately 40 milligrams. Therefore, the minimum amount of caffeine in two 10-ounce cups of percolated coffee, which is the same as the minimum amount of caffeine in four 5-ounce cups, is approximately (40)(4), or 160 milligrams. The minimum amount of caffeine in a 12-ounce caffeinated soft drink is approximately 30 milligrams. So, the minimum amount of caffeine in two 10-ounce mugs of percolated coffee and one 12-ounce caffeinated soft drink is approximately 160 + 30, or 190 milligrams. The correct answer is **Choice B**.

20. Which of the following shows the four types of coffee listed in order according to the range of the amounts of caffeine in a 5-ounce cup, from the least range to the greatest range?

Ⓐ Decaffeinated, instant, percolated, drip-brewed
Ⓑ Decaffeinated, instant, drip-brewed, percolated
Ⓒ Instant, decaffeinated, drip-brewed, percolated
Ⓓ Instant, drip-brewed, decaffeinated, percolated
Ⓔ Instant, percolated, drip-brewed, decaffeinated

Explanation

For each of the four types of coffee, the range of the amounts of caffeine is the greatest possible amount minus the least possible amount. In the graph, this difference is represented by the length of the corresponding bar, so you can order the four types of coffee according to the lengths of their corresponding bars, from shortest to longest. From the graph, you can see that the order is decaffeinated coffee, instant coffee, drip-brewed coffee, percolated coffee. The correct answer is **Choice B**.
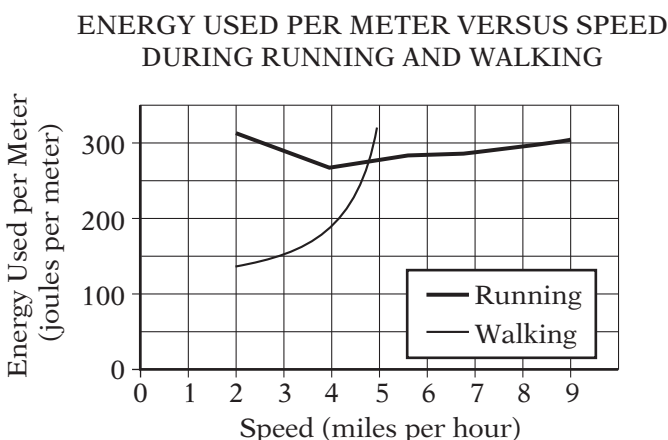
ENERGY USED PER METER VERSUS SPEED
DURING RUNNING AND WALKING

21. If s is a speed, in miles per hour, at which the energy used per meter during running is twice the energy used per meter during walking, then, according to the graph above, s is between

   Ⓐ 2.5 and 3.0

   Ⓑ 3.0 and 3.5

   Ⓒ 3.5 and 4.0

   Ⓓ 4.0 and 4.5

   Ⓔ 4.5 and 5.0

Explanation

This question is about the speed at which the energy used per meter during running is twice that used per meter during walking. Graphically, this is the speed for which the running energy is twice as high as the walking energy. Looking at the graph, you can see that for speeds greater than or equal to 3.0 miles per hour, the running energy is less than twice the walking energy, so the desired speed must be less than 3.0. In fact, the desired speed is between 2.0 (the lowest speed on the graph) and 3.0. Looking at the answer choices, you see that there is only one answer choice that is between 2.0 and 3.0; namely, Choice A, which says the desired speed is between 2.5 and 3.0. The correct answer is **Choice A**.
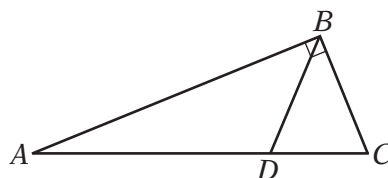
22. If $n = 2^3$, then $n^n =$

   (A) 2^6

   (B) 2^{11}

   (C) 2^{18}

   (D) 2^{24}

   (E) 2^{27}

Explanation

When answering a question in which you are asked to calculate the value of an expression, it is often helpful to look at the answer choices first to see what form they are in. In this question the answer choices are all in the form 2 raised to a power, so you should try to achieve that form. It is given that $n = 2^3 = 8$. Therefore, $n^n = (2^3)^8 = 2^{24}$. The correct answer is **Choice D**.

For the following question, select all the answer choices that apply.

The length of AB is $10\sqrt{3}$.

23. Which of the following statements <u>individually</u> provide(s) sufficient additional information to determine the area of triangle ABC above?

   Indicate <u>all</u> such statements.

   [A] DBC is an equilateral triangle.

   [B] ABD is an isosceles triangle.

   [C] The length of BC is equal to the length of AD.

   [D] The length of BC is 10.

   [E] The length of AD is 10.

Explanation

From the figure you know that ABC is a right triangle with its right angle at vertex B. You also know that point D is on the hypotenuse AC. You are given that the length of AB is $10\sqrt{3}$. However, because the figure is not necessarily drawn to scale, you don't know the lengths of AD, DC, and BC. In particular, you don't know where D is on AC.

   The area of a triangle is $\frac{1}{2}$(base)(height). Thus, the area of right triangle ABC is equal to $\frac{1}{2}$ of the length of AB times the length of BC. You already know that the length of AB is $10\sqrt{3}$. Any additional information that would allow you to calculate the length of BC would be sufficient to find the area of triangle ABC. You need to consider each of the five statements individually, as follows.

   Statement A: DBC is an equilateral triangle. This statement implies that angle DCB is a 60° angle; and therefore, triangle ABC is a 30°−60°−90° triangle. Thus

the length of *BC* can be determined, and this statement provides sufficient additional information to determine the area of triangle *ABC*.

Statement B: ABD is an isosceles triangle. There is more than one way in which triangle *ABD* can be isosceles. Below are two redrawn figures showing triangle *ABD* as isosceles. In the figure on the left, the length of *AD* is equal to the length of *DB*; and in the figure on the right, the length of *AB* is equal to the length of *AD*.
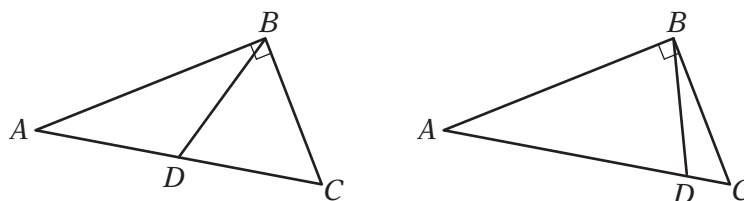
Either of the figures could have been drawn with the length of *BC* even longer. So, statement B does not provide sufficient additional information to determine the area of triangle *ABC*.

Statement C: The length of *BC* is equal to the length of *AD*. You have no way of finding the length of *AD* without making other assumptions, so statement C does not provide sufficient additional information to determine the area of triangle *ABC*.

Statement D: The length of *BC* is 10. The length of *BC* is known, so the area of triangle *ABC* can be found. Statement D provides sufficient additional information to determine the area of triangle *ABC*.

Statement E: The length of *AD* is 10. The relationship between *AD* and *BC* is not known, so statement E does not provide sufficient additional information to determine the area of triangle *ABC*.

Statements A and D individually provide sufficient additional information to determine the area of triangle *ABC*. Therefore, the correct answer consists of **Choices A and D**.

For the following question, enter your answer in the box.

$$a_1, a_2, a_3, \ldots, a_n, \ldots$$

24. In the sequence above, each term after the first term is equal to the preceding term plus the constant c. If $a_1 + a_3 + a_5 = 27$, what is the value of $a_2 + a_4$?

$$a_2 + a_4 = \boxed{}$$

Explanation
Note that answering this question requires information only about the first five terms of the sequence. So it is a good idea to work with the relationships among these five terms to see what is happening.

Since you are given that in this sequence each term after a_1 is c greater than the previous term, you can rewrite the first five terms of the sequence in terms of a_1 and c as follows.

$$a_2 = a_1 + c$$
$$a_3 = a_2 + c = a_1 + 2c$$
$$a_4 = a_1 + 3c$$
$$a_5 = a_1 + 4c$$

From the question, you know that $a_1 + a_3 + a_5 = 27$, and from the equations above, $a_1 + a_3 + a_5 = a_1 + (a_1 + 2c) + (a_1 + 4c) = 3a_1 + 6c$. So you can conclude that $3a_1 + 6c = 27$, or $a_1 + 2c = 9$.

To find $a_2 + a_4$, you can express a_2 and a_4 in terms of a_1 and c and simplify as follows.

$$a_2 + a_4 = (a_1 + c) + (a_1 + 3c)$$
$$= 2a_1 + 4c$$
$$= 2(a_1 + 2c)$$

But $a_1 + 2c = 9$, so $a_2 + a_4 = 2(9) = 18$. The correct answer is **18**.

25. A desert outpost has a water supply that is sufficient to last 21 days for 15 people. At the same average rate of water consumption per person, how many days would the water supply last for 9 people?

Ⓐ 28.0

Ⓑ 32.5

Ⓒ 35.0

Ⓓ 37.5

Ⓔ 42.0

Explanation

The water supply is enough for 15 people to survive 21 days. Assuming the same average rate of water consumption per person, 1 person would have enough water to last for $(15)(21) = 315$ days. Therefore, 9 people would have enough water for $\frac{1}{9}$ of the 315 days, or 35 days. The correct answer is **Choice C**.

Another approach to solving this problem is to recognize that the water supply would last $\frac{15}{9}$ as many days for 9 people as it would for 15 people. Therefore, since the water supply would last 21 days for 15 people, it would last $\left(\frac{15}{9}\right)(21)$, or 35 days for 9 people. The correct answer is **Choice C**.

SECTION 6
Quantitative Reasoning
25 Questions with Explanations

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

Country	Value of 1 United States Dollar
Argentina	0.93 peso
Kenya	32.08 shillings

Quantity A	Quantity B				
1. The dollar value of 1 Argentine peso according to the table above	The dollar value of 1 Kenyan shilling according to the table above	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Explanation

When you are answering Quantitative Comparison questions, it is a good time-saving idea to see whether you can determine the relative sizes of the two quantities being compared without doing any calculations.

In the table accompanying this question, both the value of the Argentine peso and the value of the Kenyan shilling are compared to the United States dollar.

Without doing any calculations, you can see from the information given that 1 United States dollar is worth a little less than 1 Argentine peso, so 1 peso is worth more than 1 United States dollar. On the other hand, 1 United States dollar is worth 32.08 Kenyan shillings, so 1 Kenyan shilling is worth only a <u>small fraction</u> of 1 United States dollar. The correct answer is **Choice A**.

> k is a digit in the decimal $1.3k5$, and $1.3k5$ is less than 1.33.

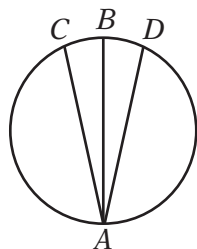| Quantity A	Quantity B				
2. k	1	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Explanation

In this question, you are given that k is a digit in the decimal $1.3k5$ and that $1.3k5$ is less than 1.33. So you can see that $1.30 < 1.3k5 < 1.33$. Therefore, $1.3k5$ must equal 1.305 or 1.315 or 1.325, and the digit k must be 0, 1, or 2. The correct answer is **Choice D**.

AB is a diameter of the circle above.

Quantity A	Quantity B	
3. The length of *AB*	The average (arithmetic mean) of the lengths of *AC* and *AD*	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Recall that in a circle, any diameter is longer than any other chord that is not a diameter. You are given that *AB* is a diameter of the circle. It follows that *AC* and *AD* are chords that are not diameters, since there is only one diameter with endpoint *A*. Hence, *AB* is longer than both *AC* and *AD*. Note that the average of two numbers is always less than or equal to the greater of the two numbers. Therefore, the average of the lengths of *AC* and *AD*, which is Quantity B, must be less than the length of *AB*, which is Quantity A. The correct answer is **Choice A**.

$$st=\sqrt{10}$$

Quantity A	Quantity B	
4. s^2	$\dfrac{10}{t^2}$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare s^2 with $\dfrac{10}{t^2}$. Since it is given that $st = \sqrt{10}$, it follows that $(st)^2 = \left(\sqrt{10}\right)^2$, and $s^2t^2 = 10$. Dividing both sides of the equation $s^2t^2 = 10$ by t^2, you get $s^2 = \dfrac{10}{t^2}$. The correct answer is **Choice C**.

   You can look at this problem in another way. You can use the fact that $st = \sqrt{10}$ to express Quantity A in terms of *t*. Since $st = \sqrt{10}$, it follows that $s = \dfrac{\sqrt{10}}{t}$, and Quantity A is equal to $\left(\dfrac{\sqrt{10}}{t}\right)^2 = \dfrac{10}{t^2}$, which is the same as Quantity B. The correct answer is **Choice C**.

Three consecutive integers have a sum of -84.

	Quantity A	Quantity B	
5.	The least of the three integers	-28	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Two approaches you could use to solve this problem are:

1: Translate from words to algebra.
2: Determine a mathematical relationship between the two quantities.

Approach 1: You can represent the least of the three consecutive integers by x, and then the three integers would be represented by x, $x + 1$, and $x + 2$. It is given that the sum of the three integers is -84, so $x + (x + 1) + (x + 2) = -84$. You can solve this equation for x as follows.

$$x + (x + 1) + (x + 2) = -84$$
$$3x + 3 = -84$$
$$3x = -87$$
$$x = -29$$

Since the least of the three integers, -29, is less than -28, the correct answer is **Choice B**.

Approach 2: You could ask yourself what would happen if the least of the three consecutive integers was -28. The three consecutive integers would then be -28, -27, and -26, and their sum would be -81. But you were given that the sum of the three consecutive integers is -84, which is less than -81. Therefore, -28 is greater than the least of the three consecutive integers, and the correct answer is **Choice B**.

In the *xy*-plane, the equation of line k is $3x - 2y = 0$.

	Quantity A	Quantity B	
6.	The *x*-intercept of line k	The *y*-intercept of line k	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Two approaches you could use to solve this problem are:

1: Reason algebraically.
2: Reason geometrically.

Approach 1: To solve this problem algebraically, note that given the equation of a line in the *xy*-plane, the *x*-intercept of the line is the value of x when y equals 0, and the *y*-intercept of the line is the value of y when x equals 0. The equation of line k is $3x - 2y = 0$. If $y = 0$, then $x = 0$; and if $x = 0$, then $y = 0$. Therefore, both the *x*-intercept and *y*-intercept of the line are equal to 0, which means that the line passes through the origin. The correct answer is **Choice C**.

Approach 2: To solve this problem geometrically, graph the line with equation $3x - 2y = 0$ in the *xy*-plane. Since two points determine a straight line, you can do this by plotting two points on the line and drawing the line they determine. The points (0,0) and (2,3) lie on the line, and the graph of the line in the *xy*-plane is shown in the following figure.
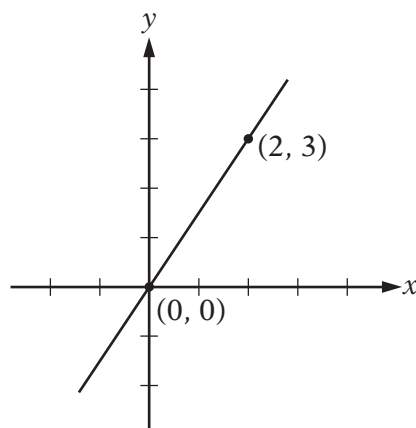
As you can see, the line passes through the origin, and so it crosses both the *x*-axis and the *y*-axis at (0,0). The correct answer is **Choice C**.

n is a positive integer that is divisible by 6.

Quantity A	Quantity B	
7. The remainder when *n* is divided by 12	The remainder when *n* is divided by 18	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

One way to compare the two quantities is to plug in a few values of *n*. If you plug in *n* = 36, you find that both the remainder when *n* is divided by 12 and the remainder when *n* is divided by 18 are equal to 0, so Quantity A is equal to Quantity B. However, if you plug in *n* = 18, you find that the remainder when *n* is divided by 12 is 6 and the remainder when *n* is divided by 18 is 0, so Quantity B is greater than Quantity A. Therefore, the correct answer is **Choice D**.

Another way to compare the two quantities is to find all of the possible values of Quantity A and Quantity B. The positive integers that are divisible by 6 are 6, 12, 18, 24, 30, 36, etc. When dividing each of these integers by 12, you get a remainder of either 0 or 6, so Quantity A is either 0 or 6. When dividing each of these integers by 18, you get a remainder of either 0 or 6 or 12, so Quantity B is either 0 or 6 or 12. Note that when the value of Quantity B is 12, the value of Quantity A, 0 or 6, is less than the value of Quantity B; but when the value of Quantity B is 0, the value of Quantity A is greater than or equal to the value of Quantity B. Thus, the correct answer is **Choice D**.

$$\frac{1-x}{x-1} = \frac{1}{x}$$

Quantity A	Quantity B	
8. x	$-\frac{1}{2}$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

One approach you could use to solve this problem is to solve the equation $\frac{1-x}{x-1} = \frac{1}{x}$ for *x*. Since fractions are defined only when the denominator is not

equal to 0, the denominators of both of the fractions in the equation are nonzero. Therefore, $x \neq 0$ and $x \neq 1$.

To solve the equation for x, begin by multiplying both sides of the equation by the common denominator $x(x + 1)$ to get $x(1 - x) = (x - 1)(1)$. Then proceed as follows.

$$x(1 - x) = (x - 1)(1)$$
$$x - x^2 = x - 1$$
$$x^2 = 1$$

Since $x^2 = 1$ and $x \neq 1$, it follows that $x = -1$.

Quantity A is equal to -1 and Quantity B is equal to $-\dfrac{1}{2}$. Therefore, Quantity B is greater, and the correct answer is **Choice B**.

Another approach is to notice that for all values of $x \neq 1$, the value of $\dfrac{1 - x}{x - 1}$ is equal to -1. You can try plugging in a few numbers for x to see that this is true. For example, if you plug in $x = 7$, you get $\dfrac{7 - 1}{1 - 7} = \dfrac{6}{-6} = -1$.

You can also show that for all values of $x \neq 1$, the value of $\dfrac{1 - x}{x - 1}$ is equal to -1 algebraically by rewriting $1 - x$ as $-(x - 1)$. Thus, $\dfrac{1 - x}{x - 1} = \dfrac{-(x - 1)}{(x - 1)} = -1$.

Because the left side of the equation $\dfrac{1 - x}{x - 1} = \dfrac{1}{x}$ is equal to -1, it follows that $-1 = \dfrac{1}{x}$, and so $x = -1$. Therefore, Quantity A is equal to -1, which is less than Quantity B, $-\dfrac{1}{2}$, and the correct answer is **Choice B**.

In a set of 24 positive integers, 12 of the integers are less than 50. The rest are greater than 50.

Quantity A	Quantity B
9. The median of the 24 integers	50

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In general, the median of a set of n positive integers, where n is even, is obtained by ordering the integers from least to greatest and then calculating the average (arithmetic mean) of the two middle integers. For this set of 24 integers, you do not know the values of the two middle integers; you know only that half of the integers are less than 50 and the other half are greater than 50. If the two middle integers in the list are 49 and 51, the median is 50; and if the two middle integers are 49 and 53, the median is 51. Thus the relationship cannot be determined from the information given, and the correct answer is **Choice D**.

10. The fabric needed to make 3 curtains sells for $8.00 per yard and can be purchased only by the full yard. If the length of fabric required for each curtain is 1.6 yards and all of the fabric is purchased as a single length, what is the total cost of the fabric that needs to be purchased for the 3 curtains?

    Ⓐ $40.00
    Ⓑ $38.40
    Ⓒ $24.00
    Ⓓ $16.00
    Ⓔ $12.80

Explanation

Since 1.6 yards of fabric are required for each curtain, it follows that (3)(1.6), or 4.8, yards of fabric are required to make the 3 curtains. The fabric can be purchased only by the full yard, so 5 yards of fabric would need to be purchased. Since the fabric sells for $8.00 per yard, the total cost of the fabric is $40.00. The correct answer is **Choice A**.

For the following question, select all the answer choices that apply.

11. In the xy-plane, line k is a line that does <u>not</u> pass through the origin. Which of the following statements <u>individually</u> provide(s) sufficient additional information to determine whether the slope of line k is negative?

    Indicate <u>all</u> such statements.

    ☐ A   The x-intercept of line k is twice the y-intercept of line k.
    ☐ B   The product of the x-intercept and the y-intercept of line k is positive.
    ☐ C   Line k passes through the points (a, b) and (r, s), where $(a - r)(b - s) < 0$.

Explanation

For questions involving a coordinate system, it is often helpful to draw a figure to visualize the problem situation. If you draw some lines with negative slopes in the xy-plane, such as those in the figure below, you see that for each line that does not pass through the origin, the x- and y-intercepts are either both positive or both negative. Conversely, you can see that if the x- and y-intercepts of a line have the same sign then the slope of the line is negative.
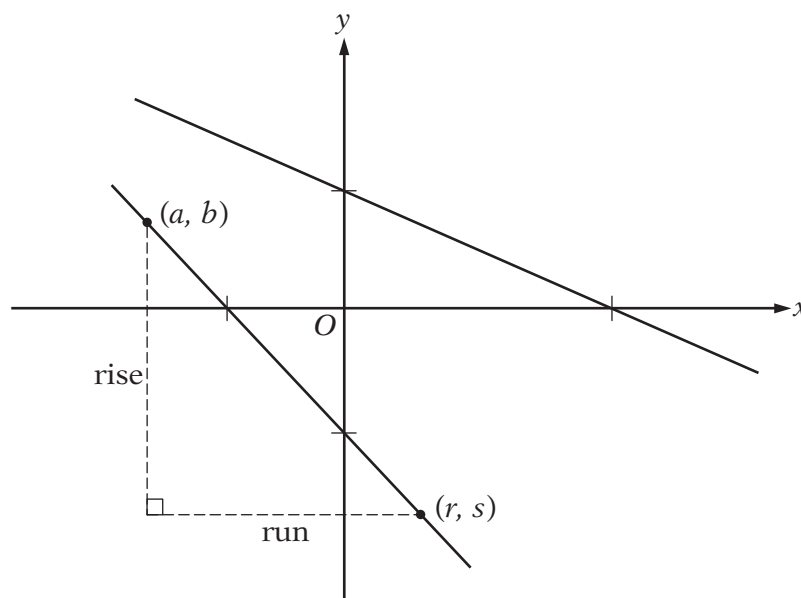
You can use this fact to examine the information given in the first two statements. Remember that you need to evaluate each statement by itself.

Choice A states that the *x*-intercept is twice the *y*-intercept, so you can conclude that both intercepts have the same sign, and thus the slope of line *k* is negative. So the information in Choice A is sufficient to determine that the slope of line *k* is negative.

Choice B states that the product of the *x*-intercept and the *y*-intercept is positive. You know that the product of two numbers is positive if both factors have the same sign. So this information is also sufficient to determine that the slope of line *k* is negative.

To evaluate Choice C, it is helpful to recall the definition of the slope of a line passing through two given points. You may remember it as "rise over run." If the two points are (a, b) and (r, s), then the slope is $\frac{b - s}{a - r}$.

Choice C states that the product of the quantities $(a - r)$ and $(b - s)$ is negative. Note that these are the denominator and the numerator, respectively, of $\frac{b - s}{a - r}$, the slope of line *k*. So you can conclude that $(a - r)$ and $(b - s)$ have opposite signs and the slope of line *k* is negative. The information in Choice C is sufficient to determine that the slope of line *k* is negative.

So each of the three statements individually provides sufficient information to determine whether the slope of line *k* is negative. The correct answer consists of **Choices A, B, and C**.

	Distance from Centerville (miles)
Freight train	$-10t + 115$
Passenger train	$-20t + 150$

12. The expressions in the table above give the distance from Centerville to each of two trains t hours after 12:00 noon. At what time after 12:00 noon will the trains be equidistant from Centerville?

   (A) 1:30
   (B) 3:30
   (C) 5:10
   (D) 8:50
   (E) 11:30

Explanation

The distance between the freight train and Centerville at t hours past noon is $-10t + 115$. The distance between the passenger train and Centerville at t hours past noon is $-20t + 150$. To find out at what time the distances will be the same you need to equate the two expressions and solve for t as follows.

$$-10t + 115 = -20t + 150$$
$$10t + 115 = 150$$
$$10t = 35$$
$$t = 3.5$$

Therefore, the two trains will be the same distance from Centerville at 3.5 hours past noon, or at 3:30. The correct answer is **Choice B**.

13. The company at which Mark is employed has 80 employees, each of whom has a different salary. Mark's salary of $43,700 is the second-highest salary in the first quartile of the 80 salaries. If the company were to hire 8 new employees at salaries that are less than the lowest of the 80 salaries, what would Mark's salary be with respect to the quartiles of the 88 salaries at the company, assuming no other changes in the salaries?

   (A) The fourth-highest salary in the first quartile
   (B) The highest salary in the first quartile
   (C) The second-lowest salary in the second quartile
   (D) The third-lowest salary in the second quartile
   (E) The fifth-lowest salary in the second quartile

Explanation

In this question you are told that Mark's salary is the second-highest in the first quartile. From this you can conclude that the word *quartile* refers to one of the four groups that are created by listing the data in increasing order and then dividing the data into four groups of equal size. When the salaries of the 80 employees are listed in order, the 20 lowest salaries (that is, the salaries in the first quartile) are the first 20 salaries in the list. Since Mark's salary is the

second-highest in the first quartile, 18 salaries in that quartile are lower than his, and one salary in that quartile is higher than his. After the salaries of the 8 new employees are added, there are 26 salaries that are lower than Mark's. The lowest 22 of those would be in the first quartile of the 88 salaries, and the remaining 4 (salaries 23 to 26) would be in the second quartile, followed by Mark's salary. This puts Mark at the fifth-lowest salary in the second quartile. The correct answer is **Choice E**.

Another way to approach this problem is to think of all 80 salaries numbered in order from least to greatest, the lowest salary at the number 1 position and the greatest salary at the number 80 position. There are 20 positions in each quartile, and Mark's salary is at position 19. The diagram below shows the salary positions and the quartile into which each position falls. Note that position 19, where Mark's salary appears, is second-highest in the first quartile.

First quartile	Second quartile	Third quartile	Fourth quartile
1	21	41	61
2	22	42	62
3	23	43	63
.	.	.	.
.	.	.	.
18	38	58	78
19 ← Mark's	39	59	79
20 salary	40	60	80

To see what Mark's position is with respect to the quartiles of the 88 salaries, you need add the 8 new salaries to the list, renumber the list from 1 to 88, and put 22 salaries in each quartile. Because the 8 new salaries are less than the original 80 salaries, they must be listed in positions 1 through 8, and all salaries in the original list must move up by 8 positions in the renumbered list. In particular, Mark's salary moves from position 19 to position 27. The diagram below shows the renumbered list. Note that Mark's salary is in position 27, the fifth position in the second quartile.

First quartile	Second quartile	Third quartile	Fourth quartile
1 ⎫	23	45	67
2 ⎬ New	24	46	68
. ⎪ salaries	25	47	69
8 ⎭	26	48	70
9 ← Salary at	27 ← Mark's	49	71
. position 1 of	. salary	.	.
20 original list	42	64	86
21	43	65	87
22	44	66	88

Since Mark's salary is in the fifth position in the second quartile and the salaries are listed in order from least to greatest, Mark's salary would be the fifth-lowest in the second quartile. The correct answer is **Choice E**.

**For the following question, enter your answer in the box.**

14. In the xy-plane, the point with coordinates $(-6, -7)$ is the center of circle C. The point with coordinates $(-6, 5)$ lies inside C, and the point with coordinates $(8, -7)$ lies outside C. If m is the radius of C and m is an integer, what is the value of m ?

$$m = \boxed{}$$

Explanation

A strategy that is often helpful in working with geometry problems is drawing a figure that represents the given information as accurately as possible.

In this question you are given that the point with coordinates $(-6, -7)$ is the center of circle C, the point with coordinates $(-6, 5)$ lies inside circle C, and the point with coordinates $(8, -7)$ lies outside circle C, so you could draw the following figure.
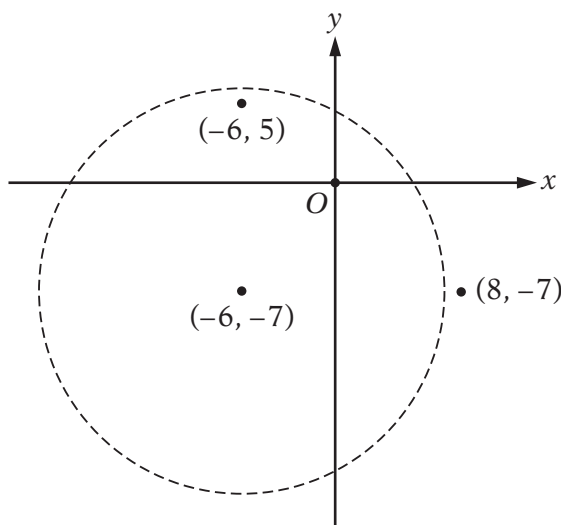
From the figure, you can see that the distance between $(-6, -7)$ and $(-6, 5)$ is $7 + 5$, or 12, and the radius of C must be greater than 12. You can also see that the distance between $(-6, -7)$ and $(8, -7)$ is $6 + 8$, or 14, and the radius of C must be less than 14. Therefore, since the radius is an integer greater than 12 and less than 14, it must be 13. The correct answer is **13**.

15. If $-\dfrac{m}{19}$ is an even integer, which of the following must be true?

   (A) m is a negative number.
   (B) m is a positive number.
   (C) m is a prime number.
   (D) m is an odd integer.
   (E) m is an even integer.

Explanation

An even integer is a multiple of 2. If $-\dfrac{m}{19}$ is an even integer, it must equal 2

times some integer k. This means that $-\dfrac{m}{19} = 2k$, or $m = -19(2k) = 2(-19k)$,

which is a multiple of 2. Thus m must be an even integer, and the correct answer
is **Choice E**. You can see that none of the other choices can be the correct
answer by evaluating them as follows.

Ⓐ m does not have to be a negative number for $-\dfrac{m}{19}$ to be even. For

example, if $m = 38$, then $-\dfrac{m}{19} = -2$, which is an even number.

Ⓑ m does not have to be a positive number for $-\dfrac{m}{19}$ to be even. For

example, if $m = -38$, then $-\dfrac{m}{19} = 2$, which is an even number.

Ⓒ The number used in (A), $m = 38$, shows that m does not have to be a
prime number. In fact, because m is the product of at least two prime
numbers (2 and 19), m cannot be a prime number.

Ⓓ Since m must be an even integer, m cannot be an odd integer.

For the following question, select all the answer choices that apply.

16. The integer v is greater than 1. If v is the square of an integer, which of the
following numbers must also be the square of an integer?

Indicate <u>all</u> such numbers.

A $81v$
B $25v + 10\sqrt{v} + 1$
C $4v^2 + 4\sqrt{v} + 1$

Explanation

If v is the square of an integer, then $\sqrt{v}$ is an integer. You can use this fact,
together with the fact that the product and the sum of integers are also integers,
to examine the first two choices.

Choice A: The square root of $81v$ is $9\sqrt{v}$, which is an integer. So $81v$ is the
square of an integer.

Choice B: $25v + 10\sqrt{v} + 1 = (5\sqrt{v} + 1)^2$ and $5\sqrt{v} + 1$ is an integer. So
$25v + 10\sqrt{v} + 1$ is the square of an integer.

Choice C: Since there is no obvious way to factor the given expression, you
may suspect that it is not the square of an integer. To show that a given
statement is not true, it is sufficient to find one counterexample. In this case,
you need to find one value of v such that v is the square of an integer
but $4v^2 + 4\sqrt{v} + 1$ is not the square of an integer. If $v = 4$, then
$4v^2 + 4\sqrt{v} + 1 = 64 + 8 + 1 = 73$, which is not the square of an integer. This proves
that $4v^2 + 4\sqrt{v} + 1$ does not have to be the square of an integer.

The correct answer consists of **Choices A and B**.

DISTANCE TRAVELED BY A CAR ACCORDING TO THE CAR'S SPEED WHEN THE DRIVER IS SIGNALED TO STOP

Distance Traveled During Reaction Time*

Distance Traveled After Brakes Have Been Applied

*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

Note: Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.

17. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

Ⓐ 43

Ⓑ 47

Ⓒ 51

Ⓓ 55

Ⓔ 59

Explanation

The data accompanying questions 17 to 20 consists of two graphs. It is a good idea to look at the graphs before you try to answer the questions, so you can become familiar with the information contained in the graphs. Then, as you read each question, you should think about which of the graphs contains the information you need to solve the problem. It could be that all the information you need to solve the problem is contained in one of the graphs, or it could be that you need to get information from both of the graphs.

The graph on the left shows the relationship between the speed of the automobile and the distance it traveled during the reaction time. Therefore, the answer to this question is found using this graph by reading the speed, in miles per hour, corresponding to a distance of 52 feet. A distance of 52 feet is a little above the distance of 50 feet on the vertical axis of the graph. On the graph, the

speed corresponding to a distance of 52 feet is a little less than 50 miles per hour. The correct answer is **Choice B**.

18. Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

Ⓐ 130

Ⓑ 110

Ⓒ   90

Ⓓ   70

Ⓔ   40

Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. At a speed of 40 miles per hour, the distance traveled during reaction time is a little less than 45 feet, and the distance traveled after the brakes have been applied is 88 feet. Since 45 + 88 = 133, the correct answer is **Choice A**.

19. Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

Ⓐ 50

Ⓑ 55

Ⓒ 60

Ⓓ 65

Ⓔ 70

Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. A good strategy for solving this problem is to calculate the total stopping distance for the speeds given in the options. For a speed of 50 miles per hour, the distance traveled during reaction time is about 55 feet, and the distance traveled after the brakes have been applied is 137 feet; therefore, the total stopping distance is about 55 + 137, or 192 feet. For a speed of 55 miles per hour, the distance traveled during reaction time is about 60 feet, and the distance traveled after the brakes have been applied is about 170 feet; therefore, the total stopping distance is about 60 + 170, or 230 feet. Since the speeds in the remaining choices are greater than 55 miles per hour and both types of stopping distances increase as the speed increases, it follows that the total stopping distances for all the remaining choices are greater than 200 feet. The correct answer is **Choice A**.

20. The total stopping distance for the car traveling at 60 miles per hour is approximately what percent greater than the total stopping distance for the car traveling at 50 miles per hour?

Ⓐ 22%

Ⓑ 30%

Ⓒ 38%

Ⓓ 45%

Ⓔ 52%

Explanation

To solve this problem you need to find the total stopping distance at 50 miles per hour and at 60 miles per hour, find their difference, and then express the difference as a percent of the shorter total stopping distance. You need to use both graphs to find the total stopping distances. At 50 miles per hour, the total stopping distance is approximately 55 + 137 = 192 feet; and at 60 miles per hour it is approximately 66 + 198 = 264 feet. The difference of 72 feet as a percent of 192 feet is $\frac{72}{192} = 0.375$, or approximately 38%. The correct answer is **Choice C**.

21. What is the least positive integer that is <u>not</u> a factor of 25! and is <u>not</u> a prime number?

Ⓐ 26

Ⓑ 28

Ⓒ 36

Ⓓ 56

Ⓔ 58

Explanation

Note that 25! is equal to the product of all positive integers from 1 to 25, inclusive. Thus, every positive integer less than or equal to 25 is a factor of 25!. Also, any integer greater than 25 that can be expressed as the product of different positive integers less than 25 is a factor of 25!. In view of this, it's reasonable to consider the next few integers greater than 25, including answer choices A and B.

Choice A, 26, is equal to (2)(13). Both 2 and 13 are factors of 25!, so 26 is also a factor of 25!. The same is true for 27, or (3)(9), and for Choice B, 28, or (4)(7). However, the next integer, 29, is a prime number greater than 25, and as such, it has no positive factors (other than 1) that are less than or equal to 25. Therefore, 29 is the least positive integer that is <u>not</u> a factor of 25!. However, the question asks for an integer that is <u>not</u> a prime number, so 29 is not the answer.

At this point, you could consider 30, 31, 32, etc., but it is quicker to look at the rest of the choices. Choice C, 36, is equal to (4)(9). Both 4 and 9 are factors of 25!, so 36 is also a factor of 25!. Choice D, 56, is equal to (4)(14). Both 4 and 14 are factors of 25!, so 56 is also a factor of 25!. Choice E, 58, is equal to (2)(29). Although 2 is a factor of 25!, the prime number 29, as noted earlier, is not a factor of 25!, and therefore 58 is not a factor of 25!. The correct answer must be **Choice E**.

The explanation above uses a process of elimination to arrive at Choice E, which is sometimes the most efficient way to find the correct answer. However, one can also show directly that the correct answer is 58. For if a positive integer n is <u>not</u> a factor of 25!, then one of the following must be true:

(i) n is a prime number greater than 25, like 29 or 31, or a multiple of such a prime number, like 58 or 62;

(ii) n is so great a multiple of some prime number less than 25, that it must be greater than 58.

To see that (i) or (ii) is true, recall that every integer greater than 1 has a unique prime factorization, and consider the prime factorization of 25!. The prime factors of 25! are 2, 3, 5, 7, 11, 13, 17, 19, and 23, some of which occur more than once in the product 25!. For example, there are 8 positive multiples of 3 less than 25, namely 3, 6, 9, 12, 15, 18, 21, and 24. The prime number 3 occurs once in each of these multiples, except for 9 and 18, in which it occurs twice. Thus, the factor 3 occurs 10 times in the prime factorization of 25!. The same reasoning can be used to find the number of times that each of the prime factors occur, yielding the prime factorization $25! = (2^{22})(3^{10})(5^6)(7^3)(11^2)(13)(17)(19)(23)$. Any integer whose prime factorization is a combination of one or more of the factors in the prime factorization of 25!, perhaps with lesser exponents, is a factor of 25!. Equivalently, if the positive integer n is not a factor of 25!, then, restating (i) and (ii) above, the prime factorization of n must

(i) include a prime number greater than 25; or

(ii) have a greater exponent for one of the prime numbers in the prime factorization of 25!.

For (ii), the least possibilities are 2^{23}, 3^{11}, 5^7, 7^4, 11^3, 13^2, 17^2, 19^2, and 23^2. Clearly, all of these are greater than 58. The least possibility for (i) that is not a prime number is 58, and the least possibility for (ii) is greater than 58, so **58** is the correct answer.

22. If $0 < a < 1 < b$, which of the following is true about the reciprocals of a and b ?

(A) $1 < \dfrac{1}{a} < \dfrac{1}{b}$

(B) $\dfrac{1}{a} < 1 < \dfrac{1}{b}$

(C) $\dfrac{1}{a} < \dfrac{1}{b} < 1$

(D) $\dfrac{1}{b} < 1 < \dfrac{1}{a}$

(E) $\dfrac{1}{b} < \dfrac{1}{a} < 1$

Explanation

To answer this question, you must first look at the answer choices. Note that all of the choices are possible orderings of the quantities $\dfrac{1}{a}, \dfrac{1}{b}$, and 1 from least to greatest. So to answer the question, you must put the three quantities in order

from least to greatest. The inequality $0 < a < 1 < b$ tells you that $0 < a < 1$ and that $b > 1$. Since a is a value between 0 and 1, the value of $\frac{1}{a}$ must be greater than 1. Since b is greater than 1, the value of $\frac{1}{b}$ must be less than 1. So you know that $\frac{1}{a} > 1$ and that $\frac{1}{b} < 1$, or combined in one expression, $\frac{1}{b} < 1 < \frac{1}{a}$, and the correct answer is **Choice D**.
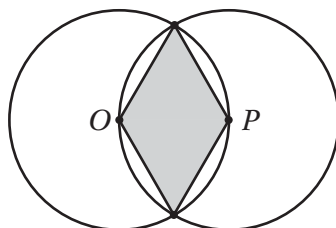
23. In the figure above, O and P are the centers of the two circles. If each circle has radius r, what is the area of the shaded region?

   Ⓐ $\dfrac{\sqrt{2}}{2}r^2$

   Ⓑ $\dfrac{\sqrt{3}}{2}r^2$

   Ⓒ $\sqrt{2}r^2$

   Ⓓ $\sqrt{3}r^2$

   Ⓔ $2\sqrt{3}r^2$

Explanation

If a geometric problem contains a figure, it can be helpful to draw additional lines and add information given in the text of the problem to the figure. For circles, the helpful additional lines are often radii or diameters. In this case, drawing radius OP will divide the shaded region into two triangles, as shown in the figure below.
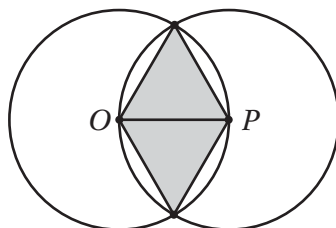
    The two circles have the same radius, r. Therefore, in each of the triangles, all three sides have length r, and each of the triangles is equilateral. If you remember from geometry that the height of an equilateral triangle with sides of length r is $\frac{\sqrt{3}}{2}r$, you could use that fact in solving the problem. However,

if you do not remember what the height is, you can use the following figure to help you find the height.
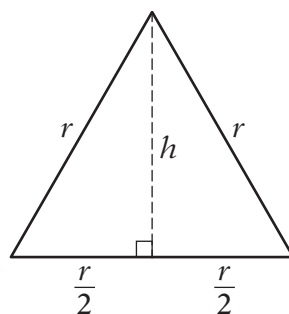
Using the Pythagorean theorem, you get

$$\left(\frac{r}{2}\right)^2 + h^2 = r^2$$

$$\frac{r^2}{4} + h^2 = r^2$$

$$h^2 = \frac{3}{4}r^2$$

$$h = \frac{\sqrt{3}}{2}r$$

So the area of the equilateral triangle is $\frac{1}{2}$(base)(height) $= \frac{1}{2}(r)\left(\frac{\sqrt{3}}{2}r\right) = \frac{\sqrt{3}}{4}r^2$.

Since the shaded region consists of 2 equilateral triangles with sides of length r, the area of the shaded region is $(2)\left(\frac{\sqrt{3}}{4}r^2\right) = \frac{\sqrt{3}}{2}r^2$, and the correct answer is

Choice B.

For the following question, enter your answer in the boxes.

24. Of the 20 lightbulbs in a box, 2 are defective. An inspector will select 2 lightbulbs simultaneously and at random from the box. What is the probability that neither of the lightbulbs selected will be defective?

Give your answer as a fraction.

Explanation
The desired probability corresponds to the fraction

$$\frac{\text{the number of ways that 2 lightbulbs, both of which are not defective, can be chosen}}{\text{the number of ways that 2 lightbulbs can be chosen}}$$

In order to calculate the desired probability, you need to calculate the values of the numerator and the denominator of this fraction.

In the box there are 20 lightbulbs, 18 of which are not defective. The numerator of the fraction is the number of ways that 2 lightbulbs can be chosen from the 18 that are not defective, also known as the number of combinations of 18 objects taken 2 at a time.

If you remember the combinations formula, you know that the number of combinations is $\frac{18!}{2!(18-2)!}$ (which is denoted symbolically as $\binom{18}{2}$ or $_{18}C_2$). Simplifying, you get

$$\frac{18!}{2!16!} = \frac{(18)(17)(16!)}{(2)(16!)} = \frac{(18)(17)}{2} = 153$$

Similarly, the denominator of the fraction is the number of ways that 2 lightbulbs can be chosen from the 20 in the box, which is $\binom{20}{2} = \frac{20!}{2!18!} = \frac{(20)(19)(18!)}{(2)(18!)} = \frac{(20)(19)}{2} = 190$. Therefore, the probability that neither of the lightbulbs selected will be defective is $\frac{153}{190}$. The correct answer is $\frac{\mathbf{153}}{\mathbf{190}}$ (or any equivalent fraction).

Another approach is to look at the selection of the two lightbulbs separately. The problem states that lightbulbs are selected simultaneously. However, the timing of the selection only ensures that the same lightbulb is not chosen twice. This is equivalent to choosing one lightbulb first and then choosing a second lightbulb without replacing the first. The probability that the first lightbulb selected will not be defective is $\frac{18}{20}$. If the first lightbulb selected is not defective, there will be 19 lightbulbs left to choose from, 17 of which are not defective. Thus, the probability that the second lightbulb selected will not be defective is $\frac{17}{19}$. The probability that both lightbulbs selected will not be defective is the product of these two probabilities. Thus, the desired probability is $\left(\frac{18}{20}\right)\left(\frac{17}{19}\right) = \frac{153}{190}$. The correct answer is $\frac{\mathbf{153}}{\mathbf{190}}$ (or any equivalent fraction).

25. What is the perimeter, in meters, of a rectangular playground 24 meters wide that has the same area as a rectangular playground 64 meters long and 48 meters wide?

   Ⓐ 112
   Ⓑ 152
   Ⓒ 224
   Ⓓ 256
   Ⓔ 304

Explanation

The area of the rectangular playground that is 64 meters long and 48 meters wide is $(64)(48) = 3{,}072$ square meters. The second playground, which has the same area, is 24 meters wide and $\frac{3{,}072}{24} = 128$ meters long. Therefore, the perimeter of the second playground is $(2)(24) + (2)(128) = 304$ meters. The correct answer is **Choice E**.

SECTION 5
Quantitative Reasoning
Time—40 minutes
25 Questions

For each question, indicate the best answer, using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, **are not necessarily** drawn to scale. That is, you should **not** assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities by sight or by measurement.

Coordinate systems, such as *xy*-planes and number lines, **are** drawn to scale; therefore, you can read, estimate, or compare quantities in such figures by sight or by measurement.

Graphical data presentations, such as bar graphs, circle graphs, and line graphs, **are** drawn to scale; therefore, you can read, estimate, or compare data values by sight or by measurement.

For each of Questions 1 to 9, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.

Ⓐ   **Quantity A is greater.**
Ⓑ   **Quantity B is greater.**
Ⓒ   **The two quantities are equal.**
Ⓓ   **The relationship cannot be determined from the information given.**

A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	Quantity B	Correct Answer
Example 1:	(2)(6)	2 + 6	Ⓐ Ⓑ Ⓒ Ⓓ

	Quantity A	Quantity B	Correct Answer
Example 2:	*PS*	*SR*	Ⓐ Ⓑ Ⓒ Ⓓ

(since equal lengths cannot be assumed, even though *PS* and *SR* appear equal)

(A) **Quantity A is greater.**
(B) **Quantity B is greater.**
(C) **The two quantities are equal.**
(D) **The relationship cannot be determined from the information given.**

Quantity A	Quantity B

1. $\dfrac{3^{-1}}{4^{-1}}$ $\dfrac{4}{3}$ (A) (B) (C) (D)

$$x < 1 \text{ and } x \neq 0$$

Quantity A	Quantity B

2. $x^2 + 1$ $x^3 + 1$ (A) (B) (C) (D)

$$x > 0$$

Quantity A	Quantity B

3. 0.5% of x $\dfrac{1}{2}x$ (A) (B) (C) (D)

The median income of a group of College C graduates six months after graduation was $3,000 higher than the median income of a group of College D graduates six months after graduation.

Quantity A	Quantity B

4. The 75th percentile of the incomes of the group of College C graduates six months after graduation   The 75th percentile of the incomes of the group of College D graduates six months after graduation   (A) (B) (C) (D)

GO ON TO THE NEXT PAGE ↘

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

The graph above shows the frequency distribution of 50 integer values varying from 1 to 6.

Quantity A	Quantity B

5. The average (arithmetic   The median of the 50   Ⓐ   Ⓑ   Ⓒ   Ⓓ
    mean) of the 50 values   values

Quantity A	Quantity B

6. The area of triangle *PQR*   The area of triangle *PSR*   Ⓐ   Ⓑ   Ⓒ   Ⓓ

Quantity A	Quantity B

7. The sum of the odd   The sum of the even   Ⓐ   Ⓑ   Ⓒ   Ⓓ
    integers from 1 to 199   integers from 2 to 198

s and *t* are positive integers, and $32^s = 2^t$.

Quantity A	Quantity B

8. $\dfrac{s}{t}$ $\dfrac{1}{5}$ Ⓐ   Ⓑ   Ⓒ   Ⓓ

(A) **Quantity A is greater.**

(B) **Quantity B is greater.**

(C) **The two quantities are equal.**

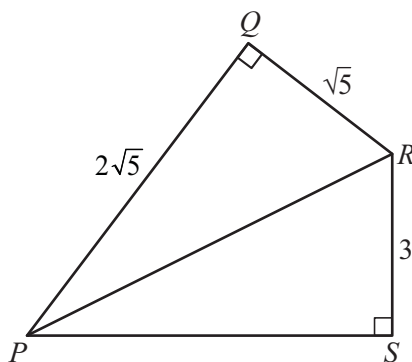(D) **The relationship cannot be determined from the information given.**

In a quality-control test, 50 boxes—each containing 30 machine parts—were examined for defective parts. The number of defective parts was recorded for each box, and the average (arithmetic mean) of the 50 recorded numbers of defective parts per box was 1.12. Only one error was made in recording the 50 numbers: "1" defective part in a certain box was incorrectly recorded as "10".

Quantity A	Quantity B

9. The actual average number   0.94   (A)   (B)   (C)   (D)
   of defective parts per box

Questions 10 to 25 have several different formats. Unless otherwise directed, select a single answer choice. For Numeric Entry questions, follow the instructions below.

Numeric Entry Questions
Enter your answer in the answer box(es) below the question.

- **Your answer may be an integer, a decimal, or a fraction, and it may be negative.**
- **If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.**
- **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.**
- **Enter the exact answer unless the question asks you to round your answer.**

---

10. In year Y, the population of Colorado was approximately half that of New Jersey, and the land area of Colorado was approximately 14 times that of New Jersey. The population density (number of persons per unit of land area) of Colorado in year Y was approximately how many times the population density of New Jersey?

    (A) $\frac{1}{28}$

    (B) $\frac{1}{14}$

    (C) $\frac{1}{7}$

    (D) $\frac{1}{4}$

    (E) $\frac{1}{2}$

For the following question, enter your answer in the box.
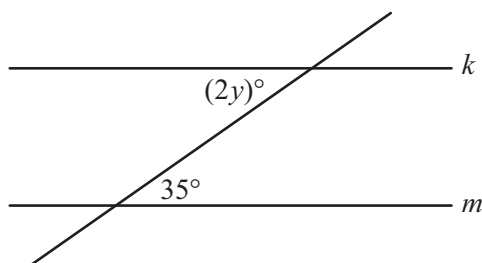
11. In the figure above, line k is parallel to line m. What is the value of y ?

$$y = \boxed{}$$

12. The numbers in data set S have a standard deviation of 5. If a new data set is formed by adding 3 to each number in S, what is the standard deviation of the numbers in the new data set?

   Ⓐ 2
   Ⓑ 3
   Ⓒ 5
   Ⓓ 8
   Ⓔ 15

13. If $\dfrac{2y - 3}{y} = \dfrac{3 - y}{2}$, which of the following could be the value of y ?

   Ⓐ   4
   Ⓑ   1
   Ⓒ −1
   Ⓓ −3
   Ⓔ −5

For the following question, select all the answer choices that apply.

14. List K consists of the numbers −10, −5, 0, 5, and 10. Which of the following lists of numbers have the same range as the numbers in list K ?
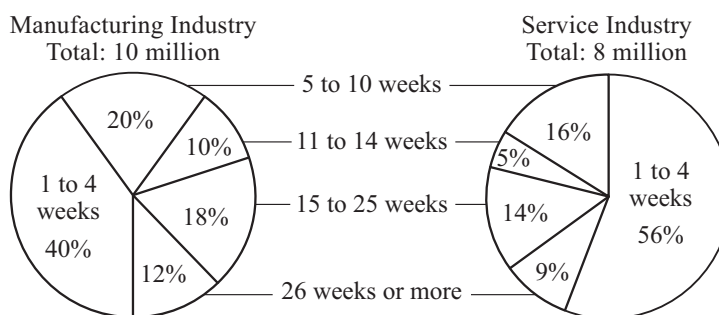
   Indicate <u>all</u> such lists.

   Ⓐ −15, −1, 0, 1, 15
   Ⓑ −7, −4, −2, 1, 13
   Ⓒ 0, 1, 2, 5, 8, 10
   Ⓓ 2, 3, 5, 15, 19, 22
   Ⓔ 4, 5, 6, 24

15. Aisha's income in 2004 was 20 percent greater than her income in 2003. What is the ratio of Aisha's income in 2004 to her income in 2003 ?

Ⓐ 1 to 5

Ⓑ 5 to 6

Ⓒ 6 to 5

Ⓓ 5 to 1

Ⓔ 20 to 1

16. Jacob's weekly take-home pay is n dollars. Each week he uses $\frac{4n}{5}$ dollars for expenses and saves the rest. At those rates, how many weeks will it take Jacob to save $500, in terms of n ?

Ⓐ $\frac{500}{n}$

Ⓑ $\frac{2{,}500}{n}$

Ⓒ $\frac{n}{625}$

Ⓓ $\frac{n}{2{,}500}$

Ⓔ $625n$

Questions 17 to 20 are based on the following data.

LENGTH OF UNEMPLOYMENT FOR WORKERS IN REGION X FOR TWO INDUSTRIES, 2003

Manufacturing Industry
Total: 10 million

Service Industry
Total: 8 million

Note: The circle graphs show the distributions of workers who were unemployed for at least 1 week in 2003, by length of unemployment, rounded to the nearest week.

17. In the circle graphs, the degree measure of the central angle of the sector representing the number of workers unemployed for 11 to 14 weeks is how much greater in the manufacturing industry graph than in the service industry graph?

Ⓐ 5°

Ⓑ 10°

Ⓒ 15°

Ⓓ 18°

Ⓔ 20°

GO ON TO THE NEXT PAGE ↘

18. Which of the following could be the median length of unemployment, in weeks, for manufacturing industry workers who were unemployed for at least 1 week?

    Ⓐ   4
    Ⓑ   8
    Ⓒ   12
    Ⓓ   16
    Ⓔ   20

19. If one of the workers in the manufacturing and service industries who were unemployed for at least 1 week will be randomly selected, what is the probability that the person selected will be a service industry worker who was unemployed for 26 weeks or more?

    Ⓐ   0.04
    Ⓑ   0.09
    Ⓒ   0.21
    Ⓓ   0.40
    Ⓔ   0.90

20. The ratio of the number of manufacturing industry workers who were unemployed for 5 to 10 weeks to the number of service industry workers who were unemployed for 5 to 10 weeks is closest to which of the following?

    Ⓐ   5 to 4
    Ⓑ   6 to 5
    Ⓒ   3 to 2
    Ⓓ   5 to 2
    Ⓔ   7 to 6

For the following question, select all the answer choices that apply.

21. If $|t+3| > 5$, which of the following could be the value of t ?

    Indicate <u>all</u> such values.

    A   −9
    B   −6
    C   −2
    D   0
    E   2
    F   3

22. The operation $\otimes$ is defined for all integers x and y as $x \otimes y = xy - y$. If x and y are positive integers, which of the following CANNOT be zero?

    (A) $x \otimes y$
    (B) $y \otimes x$
    (C) $(x - 1) \otimes y$
    (D) $(x + 1) \otimes y$
    (E) $x \otimes (y - 1)$

23. P, Q, and R are three points in a plane, and R does not lie on line PQ. Which of the following is true about the set of all points in the plane that are the same distance from all three points?

    (A) It contains no points.
    (B) It contains one point.
    (C) It contains two points.
    (D) It is a line.
    (E) It is a circle.

24. If $x < y < 0$, which of the following inequalities must be true?

    (A) $y + 1 < x$
    (B) $y - 1 < x$
    (C) $xy^2 < x$
    (D) $xy < y^2$
    (E) $xy < x^2$

For the following question, enter your answer in the box.

25. What is the length of a diagonal of a rectangle that has width 5 and perimeter 34 ?

    ┌─────────────┐
    │   │
    └─────────────┘

STOP. This is the end of Section 5.

SECTION 6
Quantitative Reasoning
Time—40 minutes
25 Questions

For each question, indicate the best answer, using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, **are not necessarily** drawn to scale. That is, you should **not** assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities by sight or by measurement.

Coordinate systems, such as *xy*-planes and number lines, **are** drawn to scale; therefore, you can read, estimate, or compare quantities in such figures by sight or by measurement.
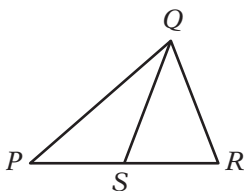
Graphical data presentations, such as bar graphs, circle graphs, and line graphs, **are** drawn to scale; therefore, you can read, estimate, or compare data values by sight or by measurement.

For each of Questions 1 to 9, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices and fill in the corresponding oval to the right of the question.

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	Quantity B	Correct Answer
Example 1:	(2)(6)	2+6	Ⓐ Ⓑ Ⓒ Ⓓ

	Quantity A	Quantity B	Correct Answer
Example 2:	*PS*	*SR*	Ⓐ Ⓑ Ⓒ Ⓓ

(since equal lengths cannot be assumed, even though *PS* and *SR* appear equal)

(A) **Quantity A is greater.**

(B) **Quantity B is greater.**

(C) **The two quantities are equal.**

(D) **The relationship cannot be determined from the information given.**

A circle is inscribed in a square with sides of length 5.

Quantity A	Quantity B				
1. The circumference of the circle	15	(A)	(B)	(C)	(D)

$$2u + v = 14$$
$$uv = 0$$

Quantity A	Quantity B				
2. u	v	(A)	(B)	(C)	(D)

Quantity A	Quantity B				
3. $950^{2,000}$	$10^{6,000}$	(A)	(B)	(C)	(D)

Set A consists of 40 integers, and set B consists of 150 integers. The number of integers that are in both set A and set B is 20.

Quantity A	Quantity B				
4. The total number of integers that are in set A or set B, or both	170	(A)	(B)	(C)	(D)

x is a negative integer.

	Quantity A	Quantity B	
5.	2^x	3^{x+1}	Ⓐ Ⓑ Ⓒ Ⓓ

$(x + 3)(y - 4) = 0$

	Quantity A	Quantity B	
6.	xy	-12	Ⓐ Ⓑ Ⓒ Ⓓ

Geoff used $630 to buy a new guitar. This amount was 15 percent of his earnings last summer.

	Quantity A	Quantity B	
7.	The amount of Geoff's earnings last summer <u>not</u> used to buy the new guitar	$3,570	Ⓐ Ⓑ Ⓒ Ⓓ

Set S consists of 5 objects.

	Quantity A	Quantity B	
8.	The number of subsets of set S that consist of 1 object	The number of subsets of set S that consist of 4 objects	Ⓐ Ⓑ Ⓒ Ⓓ

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

Quantity A	Quantity B

9. The length of line segment *AC*   3   Ⓐ   Ⓑ   Ⓒ   Ⓓ

Questions 10 to 25 have several different formats. Unless otherwise directed, select a single answer choice. For Numeric Entry questions, follow the instructions below.

Numeric Entry Questions
Enter your answer in the answer box(es) below the question.

• **Your answer may be an integer, a decimal, or a fraction, and it may be negative.**
• **If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.**
• **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.**
• **Enter the exact answer unless the question asks you to round your answer.**

10.   What is the slope of line *k* in the *xy*-plane above?

    Ⓐ   −5
    Ⓑ   −1
    Ⓒ   0
    Ⓓ   1
    Ⓔ   5

$$b - 3, \ b - 1, \ b + 2, \ b + 3, \ b + 4$$

11. The median of the five terms listed above is 5, where b is a constant. What is the average (arithmetic mean) of the five terms?

Ⓐ 3

Ⓑ 4

Ⓒ 5

Ⓓ 6

Ⓔ 7

For the following question, enter your answer in the box.
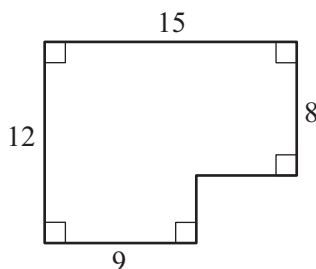
12. What is the area of the region shown above?

13. During a one-year study, biologists observed the number of fish in a certain pond as well as the percent of the fish that were catfish. At the beginning of the year, there were 300 fish in the pond, of which 15 percent were catfish; and at the end of the year, there were 400 fish in the pond, of which 10 percent were catfish. From the beginning of the year to the end of the year, the number of catfish in the pond

Ⓐ decreased by more than 5%

Ⓑ decreased by 5%

Ⓒ did not change

Ⓓ increased by 5%

Ⓔ increased by more than 5%

For the following question, enter your answer in the box.

14. On a radio tower, a red light flashes every 6 seconds and a blue light flashes every 10 seconds. If both lights flash together at a certain time, how many seconds later will both lights flash together the next time?

seconds

For the following question, select all the answer choices that apply.

15. If $a < b < 0$, which of the following numbers must be positive?

    Indicate <u>all</u> such numbers.

    A $a - b$
    B $a^2 - b^2$
    C ab
    D a^2b
    E $a^2b + ab^2$

16. A flat rectangular picture, represented by the unshaded region in the figure above, is mounted in a flat rectangular frame, represented by the shaded region. The frame is 1 inch wide on all sides. For what value of x, in inches, is the area of the frame equal to the area of the picture?

    Ⓐ 4
    Ⓑ 5
    Ⓒ 6
    Ⓓ 7
    Ⓔ 8

Questions 17 to 20 are based on the following data.

PERCENT OF THE 300 PEOPLE IN GROUP 1 AND THE 400 PEOPLE
IN GROUP 2 WHO HAVE SELECTED AILMENTS

Respiratory Ailment	Percent of People in Group 1 Who Have Ailment	Percent of People in Group 2 Who Have Ailment
Allergic sensitivity to endotoxins	14%	21%
Asthma (allergic)	3%	4%
Asthma (nonallergic)	2%	3%
Hay fever	4%	10%
Sneezing and itchy eyes	8%	11%
Wheezing (allergic)	5%	6%
Wheezing (nonallergic)	2%	5%

17. The number of people in group 2 who have hay fever is how much greater than the number of people in group 1 who have hay fever?

    Ⓐ 37
    Ⓑ 35
    Ⓒ 32
    Ⓓ 28
    Ⓔ 24

18. For the seven ailments, what is the median of the numbers of people in group 2 who have the ailments?

    Ⓐ 20
    Ⓑ 22
    Ⓒ 24
    Ⓓ 26
    Ⓔ 28

19. The number of people in group 1 who have the ailment wheezing (allergic) is what percent greater than the number of people in group 1 who have the ailment wheezing (nonallergic)?

    Ⓐ 50%
    Ⓑ 75%
    Ⓒ 150%
    Ⓓ 200%
    Ⓔ 300%

For the following question, enter your answer in the boxes.

20. What is the ratio of the number of people in group 2 with the ailment sneezing and itchy eyes to the total number of people in both groups with the ailment sneezing and itchy eyes?

    Give your answer as a fraction.

$$\boxed{} \over \boxed{}$$

21. Of the people in a certain survey, 58 percent were at most 40 years old and 70 percent were at most 60 years old. If 252 of the people in the survey were more than 40 years old and at most 60 years old, what was the total number of people in the survey?

    (A) 1,900
    (B) 2,100
    (C) 2,400
    (D) 2,700
    (E) 3,000

22. If $x > 0$, which of the following is equal to 1.25 percent of x ?

    (A) $\dfrac{x}{80}$

    (B) $\dfrac{x}{8}$

    (C) $\dfrac{x}{4}$

    (D) $\dfrac{5x}{8}$

    (E) $\dfrac{3x}{4}$

23. Alice earns d dollars and has t percent of what she earns deducted for taxes. How much of what she earns does Alice have left after taxes?

    (A) $d(1 - 100t)$ dollars
    (B) $d(1 - 10t)$ dollars
    (C) $d(1 - t)$ dollars
    (D) $d(1 - 0.1t)$ dollars
    (E) $d(1 - 0.01t)$ dollars

For the following question, select all the answer choices that apply.

24. A student made a conjecture that for any integer n, the integer $4n + 3$ is a prime number. Which of the following values of n could be used to disprove the student's conjecture?

    Indicate <u>all</u> such values.

    A 1
    B 3
    C 4
    D 6
    E 7

25. Eight points are equally spaced on a circle. If 4 of the 8 points are to be chosen at random, what is the probability that a quadrilateral having the 4 points chosen as vertices will be a square?

    (A) $\frac{1}{70}$

    (B) $\frac{1}{35}$

    (C) $\frac{1}{7}$

    (D) $\frac{1}{4}$

    (E) $\frac{1}{2}$

STOP. This is the end of Section 6.

Answer Key

Section 5. Quantitative Reasoning

Question Number	P+	Correct Answer
1	63	**Choice C:** The two quantities are equal.
2	80	**Choice A:** Quantity A is greater.
3	63	**Choice B:** Quantity B is greater.
4	45	**Choice D:** The relationship cannot be determined from the information given.
5	67	**Choice B:** Quantity B is greater.
6	73	**Choice B:** Quantity B is greater.
7	48	**Choice A:** Quantity A is greater.
8	59	**Choice C:** The two quantities are equal.
9	35	**Choice C:** The two quantities are equal.
10	60	**Choice A:** $\dfrac{1}{28}$
11	84	**17.5**
12	54	**Choice C:** 5
13	76	**Choice D:** -3
14	65	**Choice B:** $-7, -4, -2, 1, 13$ AND **Choice D:** 2, 3, 5, 15, 19, 22 AND **Choice E:** 4, 5, 6, 24
15	66	**Choice C:** 6 to 5
16	51	**Choice B:** $\dfrac{2,500}{n}$
17	51	**Choice D:** 18°
18	49	**Choice B:** 8
19	51	**Choice A:** 0.04
20	45	**Choice C:** 3 to 2
21	69	**Choice A:** -9 AND **Choice F:** 3
22	42	**Choice D:** $(x + 1) \otimes y$
23	41	**Choice B:** It contains one point.
24	40	**Choice E:** $xy < x^2$
25	62	**13**

Answer Key

Section 6. Quantitative Reasoning

Question Number	P+	Correct Answer
1	73	**Choice A:** Quantity A is greater.
2	69	**Choice D:** The relationship cannot be determined from the information given.
3	64	**Choice B:** Quantity B is greater.
4	41	**Choice C:** The two quantities are equal.
5	36	**Choice D:** The relationship cannot be determined from the information given.
6	42	**Choice D:** The relationship cannot be determined from the information given.
7	72	**Choice C:** The two quantities are equal.
8	28	**Choice C:** The two quantities are equal.
9	36	**Choice B:** Quantity B is greater.
10	69	**Choice D:** 1
11	69	**Choice B:** 4
12	78	**156**
13	66	**Choice A:** decreased by more than 5%
14	72	**30**
15	58	**Choice B:** $a^2 - b^2$ AND **Choice C:** ab
16	46	**Choice C:** 6
17	90	**Choice D:** 28
18	82	**Choice C:** 24
19	56	**Choice C:** 150%
20	66	$\dfrac{44}{68}$ (or any equivalent fraction)
21	58	**Choice B:** 2,100
22	45	**Choice A:** $\dfrac{x}{80}$
23	37	**Choice E:** $d(1 - 0.01t)$ dollars
24	53	**Choice B:** 03 AND **Choice D:** 6
25	32	**Choice B:** $\dfrac{1}{35}$

SECTION 5
Quantitative Reasoning
25 Questions with Explanations

Ⓐ **Quantity A is greater.**
Ⓑ **Quantity B is greater.**
Ⓒ **The two quantities are equal.**
Ⓓ **The relationship cannot be determined from the information given.**

Quantity A	Quantity B	
1. $\dfrac{3^{-1}}{4^{-1}}$	$\dfrac{4}{3}$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare $\dfrac{3^{-1}}{4^{-1}}$ with $\dfrac{4}{3}$. Recall that if a is a

nonzero number, then $a^{-1} = \dfrac{1}{a}$ and $\dfrac{1}{a^{-1}} = a$. Using these rules of exponents, you

can see that

$$\frac{3^{-1}}{4^{-1}} = (3^{-1})\left(\frac{1}{4^{-1}}\right) = \left(\frac{1}{3}\right)(4) = \frac{4}{3}$$

Thus, $\dfrac{3^{-1}}{4^{-1}} = \dfrac{4}{3}$, and the correct answer is **Choice C**.

$$x < 1 \text{ and } x \neq 0$$

Quantity A	Quantity B	
2. $x^2 + 1$	$x^3 + 1$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that $x < 1$ and $x \neq 0$, and you are asked to compare $x^2 + 1$ with $x^3 + 1$. One way to approach this problem is to set up a comparison between the two quantities using a placeholder symbol to represent the relationship between them as follows.

$$x^2 + 1 \;\boxed{?}\; x^3 + 1$$

Then simplify the comparison.
    Step 1: Subtract 1 from both sides to get

$$x^2 \;\boxed{?}\; x^3$$

    Step 2: Since $x \neq 0$, you can divide both sides by the positive quantity x^2 to get

$$1 \;\boxed{?}\; x$$

Since you are given that $x < 1$, or $1 > x$, you can conclude that the placeholder ? in the simplified comparison 1 ? x represents *greater than* ($>$). Note that the strategy of simplifying the comparison requires you to consider whether the steps in the simplification are reversible. This is because you must arrive at a conclusion about the initial comparison, not the simplified comparison. If you follow the simplification steps in reverse, you can see that the placeholder in each step remains unchanged: $1 > x$ implies $x^2 > x^3$ because multiplying by the positive number x^2 retains the inequality *greater than* ($>$). Also, $x^2 > x^3$ implies $x^2 + 1 > x^3 + 1$. Therefore, Quantity A is greater than Quantity B, and the correct answer is **Choice A**.

$$x > 0$$

Quantity A	Quantity B	
3. 0.5% of x	$\dfrac{1}{2}x$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that $x > 0$, and you are asked to compare 0.5% of x with $\dfrac{1}{2}x$. Recall that $\dfrac{1}{2}$ of a number is the same as 50% of the number.

Therefore, $\dfrac{1}{2}x$ is equal to 50% of x. Since $x > 0$, it follows that 50% of x is greater than 0.5% of x. The correct answer is **Choice B**.

You could also make the comparison by rewriting 0.5% of x as a fraction of x and then comparing the result with $\dfrac{1}{2}x$. Rewrite 0.5% of x as a fraction of x as follows.

$$\frac{0.5}{100}x = \frac{5}{1,000}x$$

Since $x > 0$, it follows that $\dfrac{5}{1,000}x$ is less than $\dfrac{1}{2}x$, and Quantity B is greater than Quantity A. The correct answer is **Choice B**.

The median income of a group of College C graduates six months after graduation was $3,000 higher than the median income of a group of College D graduates six months after graduation.

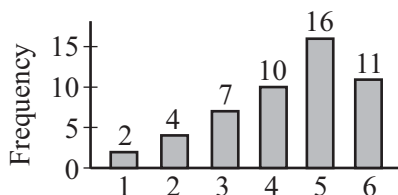| Quantity A	Quantity B	
4. The 75th percentile of the incomes of the group of College C graduates six months after graduation	The 75th percentile of the incomes of the group of College D graduates six months after graduation	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the 75th percentiles of the incomes of two groups of college graduates six months after graduation. The only

information you are given is that the median income of the group in Quantity A is $3,000 greater than the median income of the group in Quantity B.

Recall that the median of a group of numbers is the middle number (or the average of the two middle numbers) when the numbers are listed from least to greatest. The median is also equal to the 50th percentile. The median does not indicate anything about the spread of the numbers in the group. In particular, for each group of incomes, you do not know how much greater than the median the 75th percentile of the group of incomes is, nor do you know the relationship between the 75th percentiles of the two groups. Since the relationship between Quantity A and Quantity B cannot be determined, the correct answer is **Choice D**.

The graph above shows the frequency distribution of 50 integer values varying from 1 to 6.

Quantity A	Quantity B	
5. The average (arithmetic mean) of the 50 values	The median of the 50 values	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation
In this question, you are given a graph of the frequency distribution of 50 integer values and are asked to compare the average (arithmetic mean) with the median of the distribution.

In general, the median of a group of n values, where n is even, is obtained by ordering the values from least to greatest and then calculating the average (arithmetic mean) of the two middle values. So, for the 50 values shown in the graph, the median is the average of the 25th and 26th values, both of which are equal to 5. Therefore, the median of the 50 values is 5.

Once you know that the median of the 50 values is 5, the comparison simplifies to comparing the average of the 50 values with 5. You can make this comparison without actually calculating the average by noting from the graph that of the 50 values,

11 values are 1 unit above 5,
16 values are equal to 5,
10 values are 1 unit below 5, and
13 values are more than 1 unit below 5.

Since the part of the distribution that is below 5 contains 23 values—13 of which are more than 1 unit below 5—and the part of the distribution that is above 5 contains 11 values—none of which is more than 1 unit above 5—the average (arithmetic mean) of the 50 values must be less than 5. The correct answer is **Choice B**.

Alternatively, you can calculate the average of the 50 values as follows.

$$\frac{(1)(2) + (2)(4) + (3)(7) + (4)(10) + (5)(16) + (6)(11)}{50} = \frac{217}{50}$$

Thus the average of the 50 values, $\frac{217}{50}$, or 4.34, is less than the median of the 50 values, 5. The correct answer is **Choice B**.
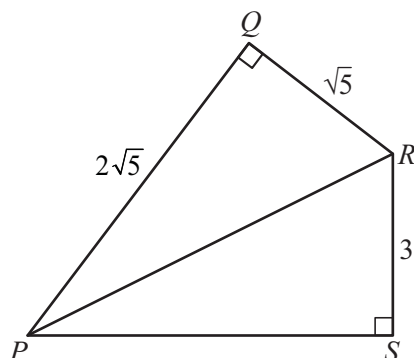
Quantity A	Quantity B	Ⓐ Ⓑ Ⓒ Ⓓ

6. The area of triangle *PQR*   The area of triangle *PSR*

Explanation

In this question, you are asked to compare the area of triangle *PQR* with the area of triangle *PSR*. Note that both triangles are right triangles and that line segment *PR* is the hypotenuse of both triangles. Recall that the area of a triangle is equal to one-half the product of a base and the height corresponding to the base. Also, for any right triangle, the lengths of the two legs of the triangle are a base and the corresponding height.

    The area of triangle PQR: In the figure, it is given that the length of leg *PQ* is $2\sqrt{5}$ and the length of leg *QR* is $\sqrt{5}$. Therefore, you can conclude that the area of triangle *PQR* is $\frac{1}{2}\left(2\sqrt{5}\right)\left(\sqrt{5}\right)$, or 5.

    The area of triangle PSR: To calculate the area of triangle *PSR*, you need to know the lengths of the legs *PS* and *RS*. From the figure, you know that the length of *RS* is 3, but you do not know the length of *PS*. How can you determine the length of *PS* ? If, in addition to the length of *RS*, you knew the length of hypotenuse *PR*, you could use the Pythagorean theorem to determine the length of *PS*. So, to find the length of *PS*, you first need to find the length of hypotenuse *PR*.

    Recall that *PR* is also the hypotenuse of triangle *PQR*. The lengths of legs *PQ* and *QR* of triangle *PQR* are $2\sqrt{5}$ and $\sqrt{5}$, respectively. By the Pythagorean theorem,

$$
\begin{aligned}
(PR)^2 &= (PQ)^2 + (QR)^2 \\
&= \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2 \\
&= 20 + 5 \\
&= 25
\end{aligned}
$$

Thus, the length of *PR* is $\sqrt{25}$, or 5.

Returning to triangle *PSR*, you now know that the length of hypotenuse *PR* is 5 and the length of leg *RS* is 3. Therefore, by the Pythagorean theorem,

$$3^2 + (PS)^2 = 5^2$$
$$9 + (PS)^2 = 25$$
$$(PS)^2 = 25 - 9$$
$$(PS)^2 = 16$$

and the length of *PS* is 4.

Since legs *PS* and *RS* have lengths 4 and 3, respectively, the area of triangle *PSR* is $\frac{1}{2}(4)(3)$, or 6. Recall that you have already determined that the area of triangle *PQR* is 5. So Quantity B, the area of triangle *PSR*, is greater than Quantity A, the area of triangle *PQR*, and the correct answer is **Choice B**.

Quantity A	Quantity B	
7. The sum of the odd integers from 1 to 199	The sum of the even integers from 2 to 198	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the sum of the odd integers from 1 to 199 with the sum of the even integers from 2 to 198. Both of these sums involve many integers. How many integers are in each sum? Note that there are 200 integers from 1 to 200, where 100 of them are even and 100 of them are odd. The 100 odd integers are precisely the odd integers in Quantity A, whereas the 100 even integers include one more integer, 200, than the even integers in Quantity B. So Quantity A is the sum of 100 integers and Quantity B is the sum of 99 integers.

It would be very time-consuming to write out all the terms in each sum and add them together. Therefore, it is reasonable to find a more efficient way to calculate the sums or to find a way to compare the sums without actually calculating them. To find a more efficient way to calculate the two sums, it is often useful to look for ways to rearrange the terms in the sum so that they can be added more easily. You can begin by writing a few terms from the beginning and the end of the sum.

For the sum of the 100 odd integers from 1 to 199, you could write

$$1 + 3 + 5 + \ldots + 195 + 197 + 199$$

You can pair the odd integers in the sum and add the two integers in each pair as follows.

$$1 + 3 + 5 + \ldots + 195 + 197 + 199$$
$$200$$
$$200$$
$$200$$

Note that the sum of the integers in each of the three pairs shown is 200. You can continue pairing terms in the sum in this way until all 100 terms have been rearranged in 50 pairs, where the sum of each pair is 200. It follows that

$$1 + 3 + 5 + \ldots + 195 + 197 + 199 = (1 + 199) + (3 + 197) + (5 + 195) + \ldots + (99 + 101)$$
$$= 50(200)$$
$$= 10{,}000$$

Now consider the sum of the 99 even integers from 2 to 198. For this sum, you could write

$$2 + 4 + 6 + \ldots + 194 + 196 + 198$$

In this sum, note that

the sum of the 1st and 99th terms is $2 + 198 = 200$
the sum of the 2nd and 98th terms is $4 + 196 = 200$

You can continue pairing terms in this way until 98 of the 99 terms in the sum have been rearranged into 49 pairs and the 50th term is unpaired. Note that the unpaired term is 100 (the 50th positive even integer). It follows that

$$2 + 4 + \ldots + 98 + 100 + 102 + \ldots + 196 + 198 = (2 + 198) + (4 + 196) + \ldots + (98 + 102) + 100$$
$$= 49(200) + 100$$
$$= 9{,}900$$

Therefore, Quantity A, 10,000, is greater than Quantity B, 9,900, and the correct answer is **Choice A**.

Alternatively, you can try to compare the two sums without actually calculating them. Recall that Quantity A is the sum of the 100 odd integers from 1 to 199, and Quantity B is the sum of the 99 even integers from 2 to 198. Write each sum with the terms in increasing order, as follows, pairing the nth term in Quantity B with the nth term in Quantity A and noting that there is no term in Quantity B that is paired with the 100th term, 199, in Quantity A.

Quantity A: $1 + 3 + 5 + \ldots + 193 + 195 + 197 + 199$
Quantity B: $2 + 4 + 6 + \ldots + 194 + 196 + 198$

Note that each of the 99 terms in Quantity B is 1 more than its paired term in Quantity A, so Quantity B is 99 more than the sum of all the terms in Quantity A excluding the last term, 199. Consequently, Quantity A is $199 - 99$ more than Quantity B—that is, 100 more than Quantity B—and the correct answer is **Choice A**.

s and t are positive integers, and $32^s = 2^t$.

Quantity A   Quantity B

8. $\dfrac{s}{t}$ $\dfrac{1}{5}$ Ⓐ   Ⓑ   Ⓒ   Ⓓ

Explanation
In this question, you are given that s and t are positive integers and that $32^s = 2^t$, and you are asked to compare $\dfrac{s}{t}$ with $\dfrac{1}{5}$. Since the expression $\dfrac{s}{t}$ involves the variables s and t, you need to look for a relationship between s and t using the equation $32^s = 2^t$.

If the two bases in this equation were equal, then the exponents would be equal. However, one of the bases is 32 and the other is 2. This suggests making the two bases equal by rewriting 32 as a power of 2 if it is possible to do so. In fact, $32 = 2^5$. Therefore, $32^s = (2^5)^s = 2^{5s}$, and the equation $32^s = 2^t$ can be rewritten as $2^{5s} = 2^t$. In the rewritten equation, the bases are equal, so you can conclude that $5s = t$.

Since $5s = t$, it follows that $\frac{s}{t} = \frac{1}{5}$. Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

> In a quality-control test, 50 boxes—each containing 30 machine parts—were examined for defective parts. The number of defective parts was recorded for each box, and the average (arithmetic mean) of the 50 recorded numbers of defective parts per box was 1.12. Only one error was made in recording the 50 numbers: "1" defective part in a certain box was incorrectly recorded as "10".

Quantity A	Quantity B	
9. The actual average number of defective parts per box	0.94	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that the number of defective parts in each of 50 boxes was recorded and that the average of the 50 recorded numbers was 1.12. You are also given that an error was made in recording one of the 50 numbers—the number 10 was recorded instead of the number 1—so the actual number of defective parts in this box is 9 less than the recorded number. Then you are asked to compare the actual average number of defective parts per box with 0.94.

To determine the actual average number of defective parts per box, first note that the sum of the 50 recorded numbers equals the average of the 50 recorded numbers times 50—that is, (1.12)(50), or 56.

Now you know that for 49 of the 50 boxes, the actual number of defective parts is equal to the recorded number; and for one box, the actual number is 9 less than the recorded number. From this you can conclude that the sum of the 50 actual numbers is equal to the sum of the 50 recorded numbers minus 9. So the sum of the actual numbers of defective parts is $56 - 9$, or 47.

Therefore, the actual average number of defective parts per box is $\frac{47}{50}$, or 0.94. Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

10. In year Y, the population of Colorado was approximately half that of New Jersey, and the land area of Colorado was approximately 14 times that of New Jersey. The population density (number of persons per unit of land area) of Colorado in year Y was approximately how many times the population density of New Jersey?

Ⓐ $\frac{1}{28}$

Ⓑ $\frac{1}{14}$

Ⓒ $\frac{1}{7}$

Ⓓ $\frac{1}{4}$

Ⓔ $\frac{1}{2}$

Explanation

The information given in the question can be rewritten algebraically as follows.

$$\text{population of Colorado} \approx \left(\frac{1}{2}\right) \times (\text{population of New Jersey})$$

$$\text{land area of Colorado} \approx (14) \times (\text{land area of New Jersey})$$

Using the information given and the fact that population density is the number of persons per unit of land area, you can express the population density of Colorado in terms of the population density of New Jersey as follows.

$$\text{population density of Colorado} = \frac{\text{population of Colorado}}{\text{land area of Colorado}}$$

$$\approx \frac{\left(\frac{1}{2}\right) \times (\text{population of New Jersey})}{(14) \times (\text{land area of New Jersey})}$$

$$\approx \left(\frac{1}{2}\right) \times \left(\frac{1}{14}\right) \times \left(\frac{\text{population of New Jersey}}{\text{land area of New Jersey}}\right)$$

$$\approx \left(\frac{1}{28}\right) \times (\text{population density of New Jersey})$$

Thus, the population density of Colorado was approximately $\frac{1}{28}$ times the population density of New Jersey. The correct answer is **Choice A**.

For the following question, enter your answer in the box.
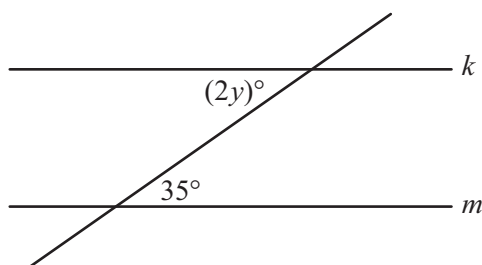
11.   In the figure above, line k is parallel to line m. What is the value of y ?

$$y = \boxed{}$$

Explanation

In the figure, the angles with measures $(2y)°$ and $35°$ are between parallel lines k and m, and they are on opposite sides of the line that crosses k and m. Therefore, you can conclude that these two angles are congruent. So $2y = 35$, and $y = \frac{35}{2} = 17.5$. The correct answer is **17.5**.

12.   The numbers in data set *S* have a standard deviation of 5. If a new data set is formed by adding 3 to each number in *S*, what is the standard deviation of the numbers in the new data set?

  (A)   2
  (B)   3
  (C)   5
  (D)   8
  (E)   15

Explanation

Recall that the standard deviation of the numbers in a data set is a measure of the spread of the numbers about the mean of the numbers. The new data set is formed by adding the <u>same</u> number, 3, to <u>each</u> number in data set *S*. Thus, the mean of the numbers in the new data set is 3 more than the mean of the numbers in *S*, but the spread of the numbers in the new data set about the mean of the numbers in the new data set is the same as the spread of the numbers in *S* about the mean of the numbers in *S*. Because the standard deviation of the numbers in *S* is 5, the standard deviation of the numbers in the new data set is also 5. The correct answer is **Choice C**.

13.   If $\dfrac{2y - 3}{y} = \dfrac{3 - y}{2}$, which of the following could be the value of *y* ?

  (A)   4
  (B)   1
  (C)   −1
  (D)   −3
  (E)   −5

Explanation

One approach to answer the question is to solve the equation for *y* as follows.

$$2(2y - 3) = y(3 - y)$$
$$4y - 6 = 3y - y^2$$
$$y^2 + y - 6 = 0$$
$$(y + 3)(y - 2) = 0$$

Since a product equals 0 only if at least one of the factors equals 0,

$$y + 3 = 0 \ \text{ or } \ y - 2 = 0$$
$$y = -3 \ \text{ or } \ y = 2$$

Thus, there are two values of *y* that satisfy the equation, −3 and 2. The value −3 is Choice D, and the value 2 is not among the answer choices. The correct answer is **Choice D**.

Another approach is to determine, for each answer choice, whether the equation holds. To do this, you can substitute the answer choice for y in the equation $\dfrac{2y-3}{y} = \dfrac{3-y}{2}$, replace the equals sign in the equation by the placeholder symbol $\overset{?}{=}$, and then simplify to see whether the two expressions are in fact equal.

For Choice A, substituting $y = 4$ in the equation $\dfrac{2y-3}{y} = \dfrac{3-y}{2}$ and replacing the equals sign $=$ with the placeholder symbol $\overset{?}{=}$ yields the relationship $\dfrac{2(4)-3}{4} \overset{?}{=} \dfrac{3-4}{2}$, which can be simplified as follows.

$$\frac{2(4)-3}{4} \overset{?}{=} \frac{3-4}{2}$$
$$\frac{8-3}{4} \overset{?}{=} \frac{-1}{2}$$
$$\frac{5}{4} \overset{?}{=} -\frac{1}{2}$$

Since $\dfrac{5}{4} \neq -\dfrac{1}{2}$, you can conclude that the placeholder symbol does not represent equality, and therefore the equation does not hold for $y = 4$.

If you continue evaluating the answer choices, you will find that the correct answer is **Choice D**, -3. To see that the equation $\dfrac{2y-3}{y} = \dfrac{3-y}{2}$ is true when $y = -3$, substitute $y = -3$ in the equation and replace the equals sign $=$ with the placeholder symbol $\overset{?}{=}$. This yields the relationship $\dfrac{2(-3)-3}{-3} \overset{?}{=} \dfrac{3-(-3)}{2}$. This relationship can be simplified as follows.

$$\frac{2(-3)-3}{-3} \overset{?}{=} \frac{3-(-3)}{2}$$
$$\frac{-6-3}{-3} \overset{?}{=} \frac{3+3}{2}$$
$$\frac{-9}{-3} \overset{?}{=} \frac{6}{2}$$
$$3 \overset{?}{=} 3$$

Since $3 = 3$, you can conclude that the placeholder symbol represents equality, and therefore the equation holds for $y = -3$. The correct answer is **Choice D**.

For the following question, select all the answer choices that apply.

14. List K consists of the numbers $-10, -5, 0, 5$, and 10. Which of the following lists of numbers have the same range as the numbers in list K ?

Indicate <u>all</u> such lists.

A $-15, -1, 0, 1, 15$
B $-7, -4, -2, 1, 13$
C $0, 1, 2, 5, 8, 10$
D $2, 3, 5, 15, 19, 22$
E $4, 5, 6, 24$

Explanation

Recall that the range of a list of numbers is defined as the difference between the greatest number and the least number in the list. The greatest number in list K is 10 and the least number is -10. Therefore, the range of the numbers in list K is $10 - (-10) = 10 + 10 = 20$. So, to answer the question, you need to consider each list of numbers given in the choices and determine whether that list of numbers has a range of 20.

Note that in each of the choices, the numbers are listed in order from least to greatest. Therefore, you need to look only at the first number and last number in each list to determine which lists have a range of 20. The ranges can be calculated quickly as follows.

Choice A: The greatest number is 15 and the least number is -15; therefore, the range is $15 - (-15) = 15 + 15 = 30$.

Choice B: The greatest number is 13 and the least number is -7; therefore, the range is $13 - (-7) = 13 + 7 = 20$.

Choice C: The greatest number is 10 and the least number is 0; therefore, the range is $10 - 0 = 10$.

Choice D: The greatest number is 22 and the least number is 2; therefore, the range is $22 - 2 = 20$.

Choice E: The greatest number is 24 and the least number is 4; therefore, the range is $24 - 4 = 20$.

In each of Choices B, D, and E, the range is 20. The correct answer consists of **Choices B, D, and E**.

15. Aisha's income in 2004 was 20 percent greater than her income in 2003. What is the ratio of Aisha's income in 2004 to her income in 2003 ?

(A)   1 to 5

(B)   5 to 6

(C)   6 to 5

(D)   5 to 1

(E)   20 to 1

Explanation

Because Aisha's income in 2004 was 20% greater than her income in 2003, her income in 2004 was equal to

(100% of her income in 2003) + (20% of her income in 2003)

which is 120% of her income in 2003. Therefore, the ratio of her income in 2004 to her income in 2003 is 120 to 100, which is equivalent to 6 to 5. The correct answer is **Choice C**.

Alternatively, to say that Aisha's income in 2004 was 20% greater than her income in 2003 is the same as saying that her income increased by $\frac{1}{5}$. Therefore, her income in 2004 was $\frac{6}{5}$ of her income in 2003, and the ratio of her income in 2004 to her income in 2003 is 6 to 5. The correct answer is **Choice C**.

16. Jacob's weekly take-home pay is n dollars. Each week he uses $\dfrac{4n}{5}$ dollars for expenses and saves the rest. At those rates, how many weeks will it take Jacob to save $500, in terms of n ?

   (A) $\dfrac{500}{n}$

   (B) $\dfrac{2,500}{n}$

   (C) $\dfrac{n}{625}$

   (D) $\dfrac{n}{2,500}$

   (E) $625n$

Explanation

It may be helpful to consider how you would determine the number of weeks it would take Jacob to save $500 if you knew how much he saved each week. For example, suppose Jacob saved $25 each week. At that rate, it is easy to see that it would take him 500 ÷ 25, or 20, weeks to save $500. Using this example, you can see that the number of weeks it will take Jacob to save $500 is equal to 500 divided by the amount he saves each week.
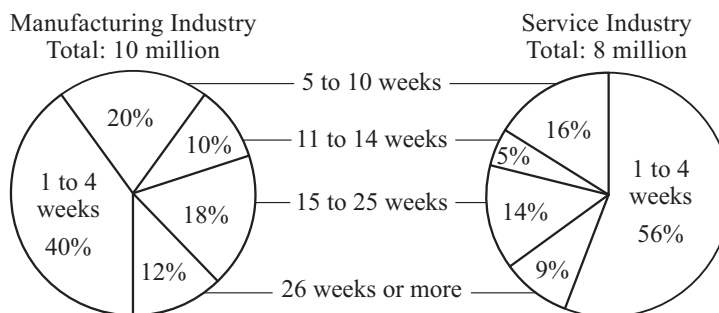
Now use the information given in the question to determine an algebraic expression representing the amount Jacob saved each week. In the question, you are given that Jacob's weekly expenses are $\dfrac{4n}{5}$ dollars. Therefore, the amount he saves each week is equal to his weekly take-home pay minus his weekly expenses, or $n - \dfrac{4n}{5} = \dfrac{5n}{5} - \dfrac{4n}{5} = \dfrac{5n-4n}{5} = \dfrac{n}{5}$ dollars.

Recall that you had already concluded that the number of weeks it will take Jacob to save $500 is equal to 500 divided by the amount he saves each week. So the number of weeks it will take Jacob to save $500 is

$500 \div \dfrac{n}{5} = 500 \times \dfrac{5}{n} = \dfrac{2,500}{n}$. The correct answer is **Choice B**.

Questions 17 to 20 are based on the following data.

LENGTH OF UNEMPLOYMENT FOR WORKERS IN REGION X FOR TWO INDUSTRIES, 2003

Manufacturing Industry
Total: 10 million

Service Industry
Total: 8 million

5 to 10 weeks

11 to 14 weeks

15 to 25 weeks

26 weeks or more

Manufacturing Industry: 20%, 10%, 18%, 12%, 1 to 4 weeks 40%

Service Industry: 16%, 5%, 14%, 9%, 1 to 4 weeks 56%

Note: The circle graphs show the distributions of workers who were unemployed for at least 1 week in 2003, by length of unemployment, rounded to the nearest week.

17. In the circle graphs, the degree measure of the central angle of the sector representing the number of workers unemployed for 11 to 14 weeks is how much greater in the manufacturing industry graph than in the service industry graph?

    (A) 5°
    (B) 10°
    (C) 15°
    (D) 18°
    (E) 20°

Explanation

Recall that in a circle graph, the degree measure of the central angle of a sector representing n percent of the data is equal to n percent of 360°.

   The degree measure of the sector representing the number of workers unemployed for 11 to 14 weeks is 10% of 360°, or 36°, for the manufacturing industry graph and is 5% of 360°, or 18°, for the service industry graph. Since $36° - 18° = 18°$, the measure of the central angle of that sector in the manufacturing industry graph is 18° greater than the measure of the central angle of the corresponding sector in the service industry graph. The correct answer is **Choice D**.

18. Which of the following could be the median length of unemployment, in weeks, for manufacturing industry workers who were unemployed for at least 1 week?

    (A) 4
    (B) 8
    (C) 12
    (D) 16
    (E) 20

Explanation

Note that the sectors in the manufacturing industry circle graph separate the unemployed manufacturing industry workers into five groups by length of unemployment; also, the percent of workers within each of the five groups is given. Also note that there are 10 million lengths of unemployment, one length for each of the 10 million workers. Since the lengths are rounded to whole numbers of weeks, most of the 10 million lengths must be repetitions.

   The median length of unemployment is the average of the two middle lengths when the lengths are listed in order from least to greatest; that is, the median is the number at which 50% of the lengths have been listed.

   To find the median length, first note that the group with the shortest unemployment lengths, 1 to 4 weeks, accounts for the first 40% of the lengths in the ordered list. Then, because the group with the next longer lengths, 5 to 10 weeks, accounts for the next 20% of the lengths in the list, the number at which 50% of the lengths have been listed is in this group. So the median length is in the 5-to-10 week interval. Among the answer choices, the only choice that is in the 5-to-10 week interval is Choice B, 8. The correct answer is **Choice B**.

19. If one of the workers in the manufacturing and service industries who were unemployed for at least 1 week will be randomly selected, what is the probability that the person selected will be a service industry worker who was unemployed for 26 weeks or more?

   Ⓐ 0.04
   Ⓑ 0.09
   Ⓒ 0.21
   Ⓓ 0.40
   Ⓔ 0.90

Explanation

The probability that the person selected will be a service industry worker who was unemployed for 26 weeks or more is equal to the following fraction.

$$\frac{\text{the number of service industry workers who were unemployed 26 weeks or more}}{\text{the number of workers in the two industries who were unemployed at least 1 week}}$$

The number of workers in the two industries who were unemployed for at least 1 week is the sum of the total numbers of workers represented by the two graphs, or 10 million + 8 million = 18 million.

   According to the graph for the service industry, the number of workers who were unemployed for 26 weeks or more is 9% of 8 million, or 0.72 million.

Therefore, the desired probability is $\frac{0.72\text{ million}}{18\text{ million}} = \frac{72}{1,800} = 0.04$. The correct answer is **Choice A**.

20. The ratio of the number of manufacturing industry workers who were unemployed for 5 to 10 weeks to the number of service industry workers who were unemployed for 5 to 10 weeks is closest to which of the following?

   Ⓐ 5 to 4
   Ⓑ 6 to 5
   Ⓒ 3 to 2
   Ⓓ 5 to 2
   Ⓔ 7 to 6

Explanation

According to the graphs, the number of manufacturing industry workers who were unemployed for 5 to 10 weeks is 20% of 10 million, or 2 million; and the number of service industry workers who were unemployed for 5 to 10 weeks is 16% of 8 million, or 1.28 million. Thus, the ratio of the two numbers is 2 to 1.28.

   To answer the question, you must now determine which of the answer choices is closest to the ratio 2 to 1.28. A good way to compare ratios is to express each ratio as a fraction and then as a decimal, and then to compare the decimals. The ratio 2 to 1.28 can be expressed as the fraction $\frac{2}{1.28}$, which is equal to the decimal 1.5625.

Now look at the answer choices. As you go through the answer choices, keep in mind that you are trying to determine which is closest to 1.5625.

Choice A: 5 to 4 can be expressed as $\frac{5}{4}$, which is equal to 1.25.

Choice B: 6 to 5 can be expressed as $\frac{6}{5}$, which is equal to 1.2.

Choice C: 3 to 2 can be expressed as $\frac{3}{2}$, which is equal to 1.5. Note that this ratio is close to 1.5625.

Choice D: 5 to 2 can be expressed as $\frac{5}{2}$, which is equal to 2.5.

Choice E: 7 to 6 can be expressed as $\frac{7}{6}$, which is approximately equal to 1.17.

Of the five choices, the ratio in Choice C is closest to 1.5625. The correct answer is **Choice C**.

For the following question, select all the answer choices that apply.

21.   If $|t + 3| > 5$, which of the following could be the value of t ?

Indicate all such values.

A -9
B -6
C -2
D 0
E 2
F 3

Explanation
One way to approach this question is to substitute each of the answer choices into the inequality and determine which ones satisfy the inequality. If you do this, you will see that Choice A, -9, and Choice F, 3, satisfy the inequality, but the other answer choices do not. The correct answer consists of **Choices A and F**.

An algebraic approach to the question is to note that the inequality $|t + 3| > 5$ is satisfied whenever $t + 3 > 5$ or $t + 3 < -5$, that is, whenever $t > 2$ or $t < -8$. Therefore, all values of t greater than 2 or less than -8 satisfy the inequality $|t + 3| > 5$. The only answer choices that meet those conditions are -9, Choice A, and 3, Choice F. The correct answer consists of **Choices A and F**.

22. The operation $\otimes$ is defined for all integers x and y as $x \otimes y = xy - y$. If x and y are positive integers, which of the following CANNOT be zero?

    (A) $x \otimes y$

    (B) $y \otimes x$

    (C) $(x - 1) \otimes y$

    (D) $(x + 1) \otimes y$

    (E) $x \otimes (y - 1)$

Explanation

In the formula $x \otimes y = xy - y$, the variables x and y are placeholders that can be replaced by integers or by expressions representing integers. Here are two examples.

If x is replaced by 3 and y is replaced by 4, then the formula gives

$$3 \otimes 4 = (3)(4) - 4 = 12 - 4 = 8$$

If x is replaced by $x - 1$ and y is replaced by 2, then the formula gives

$$(x - 1) \otimes 2 = ((x - 1)(2)) - 2 = 2x - 2 - 2 = 2x - 4$$

Scanning the answer choices, you can see that all of them are of the form

"first expression" $\otimes$ "second expression"

For each answer choice, you must determine whether the answer choice can be equal to 0 for some positive integers x and y. Are there positive integers x and y for which the answer choice is equal to 0 ? If not, then that answer choice is the correct answer.

    Choice A: $x \otimes y$. Using the formula, try to find positive integers x and y for which $x \otimes y = 0$, that is, for which $xy - y = 0$. To solve this equation, note that factoring y out of the left-hand side of the equation $xy - y = 0$ gives the equation $(x - 1)y = 0$. So now you must find positive integers x and y such that the product of the two numbers $x - 1$ and y is 0. Since the product of two numbers is 0 only if at least one of the numbers is 0, it follows that the product of $x - 1$ and y will be 0 if $x = 1$, no matter what the value of y is. For example, if $x = 1$ and $y = 2$, then $x \otimes y = 1 \otimes 2 = (1)(2) - 2 = 0$, and both x and y are positive integers. Therefore, Choice A is not correct, since there are positive integers x and y for which $x \otimes y = 0$.

    Choice B: $y \otimes x$. This is similar to Choice A, except the x and y are interchanged. Therefore, you might try the example in Choice A but with the values of x and y interchanged: $y = 1$ and $x = 2$. Using the formula, $y \otimes x = yx - x = (1)(2) - 2 = 0$. Therefore, Choice B is not correct, since there are positive integers x and y for which $y \otimes x = 0$.

    Choice C: $(x - 1) \otimes y$. Using the formula, try to find positive integers x and y for which $(x - 1) \otimes y = 0$, that is, for which $(x - 1)y - y = 0$. Factoring y out of the left-hand side of the equation $(x - 1)y - y = 0$ yields $(x - 1 - 1)y = (x - 2)y = 0$. Here the product of the two numbers $x - 2$ and y is 0. So the product will be 0 if $x = 2$, no matter what the value of y is. For example, if $x = 2$ and $y = 10$, then $(x - 1) \otimes y = (2 - 1) \otimes 10 = 1 \otimes 10 = (1)(10) - 10 = 0$, and both x and y are positive

integers. Therefore, Choice C is not correct, since there are positive integers x and y for which $(x - 1) \otimes y = 0$.

Choice D: $(x + 1) \otimes y$. Using the formula, try to find positive integers x and y for which $(x + 1) \otimes y = 0$, that is, for which $(x + 1)y - y = 0$. Factoring y out of the left-hand side of the equation $(x + 1)y - y = 0$ yields $(x + 1 - 1)y = xy = 0$. Here the product of x and y is 0, so $x = 0$ or $y = 0$. Since both x and y must be positive but 0 is not positive, it follows that there are no positive integers x and y for which $(x + 1) \otimes y = 0$. The correct answer is **Choice D**.

Choice E: $x \otimes (y - 1)$ cannot be correct, since Choice D is correct, but Choice E is considered here for completeness. Using the formula, try to find positive integers x and y for which $x \otimes (y - 1) = 0$, that is, for which $x(y - 1) - (y - 1) = 0$. Factoring $y - 1$ out of the left-hand side of the equation $x(y - 1) - (y - 1) = 0$ yields $(x - 1)(y - 1) = 0$. Here the product of the two numbers $x - 1$ and $y - 1$ is 0. So the product will be 0 if $x = 1$ or $y = 1$, no matter what the value of the other variable is. For example, if $x = 20$ and $y = 1$, then $x \otimes (y - 1) = 20 \otimes (1 - 1) = 20 \otimes 0 = (20)(0) - 0 = 0$, and both x and y are positive integers. Therefore, Choice E is not correct, since there are positive integers x and y for which $x \otimes (y - 1) = 0$.

23. *P, Q,* and *R* are three points in a plane, and *R* does not lie on line *PQ*.
     Which of the following is true about the set of all points in the plane that
     are the same distance from all three points?

     Ⓐ It contains no points.

     Ⓑ It contains one point.

     Ⓒ It contains two points.

     Ⓓ It is a line.

     Ⓔ It is a circle.

Explanation

First consider just two of the three points, say *P* and *Q*, and the set of points in the plane that are the same distance from them. Clearly the midpoint of line segment *PQ* is such a point. Are there others? You may recall from geometry that the points on the line that bisects *PQ* and is perpendicular to *PQ* are all the points that are equidistant from *P* and *Q*. Similarly, the points in the plane that lie on the perpendicular bisector of line segment *PR* are all the points that are equidistant from points *P* and *R*.

Because *R* does not lie on line *PQ*, line segments *PQ* and *PR* do not lie on the same line, and so their respective perpendicular bisectors are not parallel. Therefore, you can conclude that the two perpendicular bisectors intersect at a point. The point of intersection is on both perpendicular bisectors, so it is equidistant from *P* and *Q* as well as from *P* and *R*. Therefore, the point of intersection is equidistant from all three points. Are there any other points that are equidistant from *P, Q,* and *R* ? If there were, they would be on both perpendicular bisectors, but in fact only one point lies on both lines. The correct answer is **Choice B**.

24. If $x < y < 0$, which of the following inequalities must be true?

      (A) $y + 1 < x$

      (B) $y - 1 < x$

      (C) $xy^2 < x$

      (D) $xy < y^2$

      (E) $xy < x^2$

Explanation

The conditions stated in the question, $x < y < 0$, tell you that x and y are negative numbers and that y is greater than x. Keep this in mind as you evaluate each of the inequalities in the answer choices, to see whether the inequality must be true.

    Choice A: $y + 1 < x$. According to the conditions given in the question, y is greater than x. Since y is greater than x and $y + 1$ is greater than y, it follows that $y + 1$ is greater than x. So it cannot be true that $y + 1 < x$. Therefore, Choice A is not the correct answer.

    Choice B: $y - 1 < x$. While it is true that both x and $y - 1$ are less than y, it may not be true that $y - 1 < x$. Consider what happens if $y = -2$ and $x = -7$. In this case, the inequality $y - 1 < x$ becomes $-3 < -7$, which is false. Therefore, Choice B is not the correct answer.

    Choice C: $xy^2 < x$. Note that y^2 is positive and x is negative, so xy^2 is negative. Is the negative number xy^2 less than the negative number x ? It depends on whether $y^2 > 1$ or $y^2 < 1$. Consider what happens if $x = -4$ and $y = -\frac{1}{2}$, where $y^2 < 1$. In this case, the inequality $xy^2 < x$ becomes $-1 < -4$, which is false. Therefore, Choice C is not the correct answer.

    Choice D: $xy < y^2$. Since y is a negative number, multiplying both sides of the inequality $x < y$ by y reverses the inequality, resulting in the inequality $xy > y^2$. So it cannot be true that $xy < y^2$. Therefore, Choice D is not the correct answer.

    Since Choices A through D have been eliminated, the correct answer is *Choice E.* You can show that the inequality in Choice E, $xy < x^2$, must be true as follows: Multiply both sides of the given inequality $x < y$ by x to obtain the inequality $x^2 > xy$, reversing the direction of the inequality because x is negative. Therefore, the inequality $xy < x^2$ must be true, and the correct answer is **Choice E**.

For the following question, enter your answer in the box.

25. What is the length of a diagonal of a rectangle that has width 5 and perimeter 34 ?

                            [   ]

Explanation

In this question, you are given that a rectangle has width 5 and perimeter 34, and you are asked to find the length of a diagonal of the rectangle. Let L and W represent the length and width of the rectangle, respectively, and let D represent the length of a diagonal. Note that you are not given L but you are given that
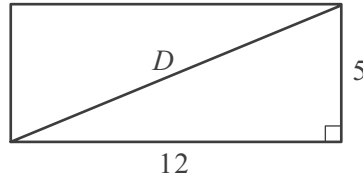
$W = 5$ and that the perimeter is 34. Because the perimeter is equal to $L + L + W + W$, or $2(L + W)$, you can determine L as follows.

$$2(L + 5) = 34$$
$$L + 5 = 17$$
$$L = 12$$

The following figure shows a rectangle of length 12, width 5, and diagonal of length D.

From the figure, you can see that the diagonal is the hypotenuse of a right triangle with legs of length 5 and 12. Therefore, by the Pythagorean theorem,

$$5^2 + 12^2 = D^2$$
$$25 + 144 = D^2$$
$$169 = D^2$$
$$13 = D$$

The length of the diagonal is 13, so the correct answer is **13**.

SECTION 6
Quantitative Reasoning
25 Questions with Explanations

Ⓐ	**Quantity A is greater.**
Ⓑ	**Quantity B is greater.**
Ⓒ	**The two quantities are equal.**
Ⓓ	**The relationship cannot be determined from the information given.**

A circle is inscribed in a square with sides of length 5.

Quantity A	Quantity B	
1. The circumference of the circle	15	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that a circle is inscribed in a square with sides of length 5 and are asked to compare the circumference of the circle with 15. Since the circle is inscribed in the square, the diameter of the circle is equal to the length of a side of the square, or 5. Thus, the circumference of the circle is 5π. Because π is greater than 3, it follows that 5π is greater than 15. Therefore, Quantity A is greater than Quantity B, and the correct answer is **Choice A**.

$$2u + v = 14$$
$$uv = 0$$

Quantity A	Quantity B	
2. u	v	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare u with v, given that $2u + v = 14$ and $uv = 0$.

Consider the equation $uv = 0$. Since a product can equal 0 only if at least one of the factors in the product equals 0, you know that $u = 0$ or $v = 0$, or both. But since you are also given that $2u + v = 14$, it follows that u and v cannot both equal 0.

Knowing that either $u = 0$ or $v = 0$, you can substitute 0 into the equation $2u + v = 14$ for either u or v in order to determine the relationship between u and v if it is possible to do so from the information given. If $u = 0$, then $2u + v = 14$ simplifies to $v = 14$. In this case, u is less than v. However, if $v = 0$, then $2u + v = 14$ simplifies to $2u = 14$, or $u = 7$. And in this case, u is greater than v.

In the first case, $u < v$, and in the second case, $u > v$. Therefore, the relationship between the two quantities u and v cannot be determined from the information given, and the correct answer is **Choice D**.

Quantity A	Quantity B	
3. $950^{2,000}$	$10^{6,000}$	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the quantity $950^{2,000}$ with the quantity $10^{6,000}$. Note that both quantities are written in the form "base to a power." If the bases were equal, you would be able to compare the quantities by comparing the powers. Because powers of 10 are easier to work with than powers of 950, it is reasonable to try to compare the quantities by rewriting the quantity $950^{2,000}$ as a power of 10. Unfortunately, there is no obvious way to do that. However, if you can approximate 950 by a power of 10, you may then be able to use the approximation to compare the quantity $950^{2,000}$ with the quantity $10^{6,000}$.

   Note that 950 is close to, but a little less than, 1,000, or 10^3. Raising both sides of the inequality $950 < 10^3$ to the power 2,000 gives the inequality $950^{2,000} < (10^3)^{2,000}$. Since $(10^3)^{2,000} = 10^{6,000}$, you can conclude that $950^{2,000} < 10^{6,000}$. Thus, Quantity A is less than Quantity B, and the correct answer is **Choice B**.

   Set A consists of 40 integers, and set B consists of 150 integers. The number of integers that are in both set A and set B is 20.

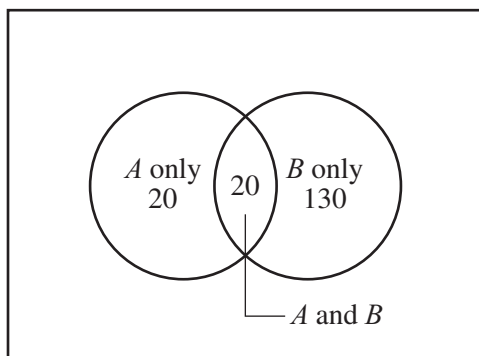| Quantity A	Quantity B	
4. The total number of integers that are in set A or set B, or both	170	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that the number of integers in set A is 40, the number of integers in set B is 150, and the number of integers that are in both A and B is 20. You are asked to compare the total number of integers that are in set A or set B, or both, with 170.

   This is the type of question for which a Venn diagram is usually helpful to represent the information given. The following Venn diagram is a representation of the integers in sets A and B.
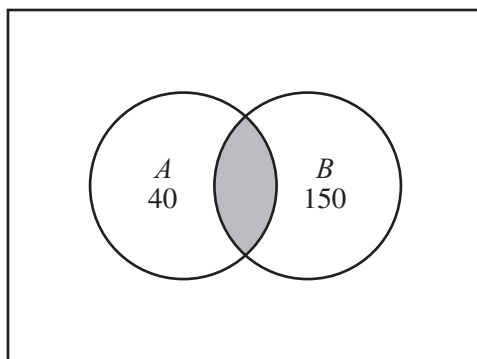
Note that there is no number in the shaded region of the diagram—the region representing the integers in both A and B. In fact, the number of integers in both A and B is included in both the number of integers in A and the number of

integers in B. It is a good idea, therefore, to redraw the Venn diagram so that the numbers are separated into three categories: the integers in A only, the integers in B only, and the integers in both A and B. The revised Venn diagram follows.

Observe that summing the numbers of integers in set A only, set B only, and both A and B yields the total number of integers that are in set A or set B, or both. Therefore, Quantity A is $20 + 130 + 20$, or 170, and the correct answer is **Choice C**.

Another approach is to realize that if you listed the integers in set A and the integers in set B, you would have listed the integers that are in both A and B twice and all of the other integers once. So the total number of integers in set A or set B, or both, is equal to

$$(\text{number in set } A) + (\text{number in set } B) - (\text{number in both sets})$$

Thus, the number of integers in set A or set B, or both, is $40 + 150 - 20$, or 170, and the correct answer is **Choice C**.

x is a negative integer.

Quantity A	Quantity B	
5. $\qquad$ 2^x	3^{x+1}	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare 2^x with 3^{x+1}, given that x is a negative integer. One way to approach this problem is to plug a value of x in both expressions and compare the results.

You are given that x is a negative integer, so the greatest integer you can plug in for x is -1.

For $x = -1$, it follows that $2^x = 2^{-1} = \frac{1}{2}$ and $3^{x+1} = 3^{-1+1} = 3^0 = 1$.

In this case, 2^x is less than 3^{x+1}. However, to conclude that Quantity B is greater, it is not sufficient for 2^x to be less than 3^{x+1} for one particular value of x; the relationship would need to be true for <u>all</u> negative integer values of x. To analyze this relationship further, plug in another value of x, for example, -2.

For $x = -2$, it follows that $2^x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ and $3^{x+1} = 3^{-2+1} = 3^{-1} = \frac{1}{3}$.

Again, 2^x is less than 3^{x+1}, but note that these values are closer together than the previous values of 2^x and 3^{x+1}. It appears that the relationship between the quantities may differ for smaller values of x, so now try plugging in -3 for x.

For $x = -3$, it follows that $2^x = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $3^{x+1} = 3^{-3+1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

In this case, 2^x is greater than 3^{x+1}.

Since 2^x is less than 3^{x+1} for $x = -1$ and 2^x is greater than 3^{x+1} for $x = -3$, the relationship between these two quantities cannot be determined from the information given. The correct answer is **Choice D**.

Since both quantities are algebraic expressions, another way to approach the comparison is to set up a placeholder relationship, denoted by ?, between the two quantities and then to simplify to see what conclusions you can draw. As you simplify and draw conclusions, keep in mind that x is a negative integer.

$$2^x \; ? \; 3^{x+1}$$

$$2^x \; ? \; 3(3^x)$$

$$\frac{2^x}{3^x} \; ? \; 3$$

$$\left(\frac{2}{3}\right)^x \; ? \; 3$$

For any value of x (including negative integer values of x), the value of 3^x is positive, so dividing by 3^x does not affect any inequality that might be represented by the placeholder. Since each step in this simplification is reversible, the simplification reduces the problem to comparing $\left(\frac{2}{3}\right)^x$ with 3, given that x is a negative integer. Note that $\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^n$, where $n = -x$; so the problem can be reduced further to comparing $\left(\frac{3}{2}\right)^n$ with 3, given that n is a positive integer.

Because $\frac{3}{2}$ is greater than 1, the value of $\left(\frac{3}{2}\right)^n$ becomes greater as n becomes larger. For small values of n, $\left(\frac{3}{2}\right)^n$ is less than 3, but for large values of n, $\left(\frac{3}{2}\right)^n$ is greater than 3. Therefore, the relationship between Quantity A and Quantity B cannot be determined from the given information, and the correct answer is **Choice D**.

$$(x + 3)(y - 4) = 0$$

	Quantity A	Quantity B	
6.	xy	-12	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that $(x + 3)(y - 4) = 0$, and you are asked to compare the product xy with -12. Since $(x + 3)(y - 4) = 0$ and the product of two numbers can equal 0 only if at least one of the numbers in the product equals 0, you can conclude that $x = -3$ or $y = 4$, or both.

Consider the case $x = -3$. When $x = -3$, you can choose any number as the value of y and the equation $(x + 3)(y - 4) = 0$ will be satisfied. Depending on the particular value of y you choose, the product xy may be greater than, less than, or equal to -12. For example, if $y = 1$, then $xy = -3$ is greater than -12; and if $y = 10$, then $xy = -30$ is less than -12.

Since different examples for the value of *y* yield different relationships between Quantities A and B, the relationship cannot be determined from the information given. The correct answer is **Choice D**.

> Geoff used $630 to buy a new guitar. This amount was 15 percent of his earnings last summer.

	Quantity A	Quantity B	
7.	The amount of Geoff's earnings last summer <u>not</u> used to buy the new guitar	$3,570	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the amount of Geoff's earnings last summer <u>not</u> used to buy a new guitar with the amount $3,570. You are given that Geoff used 15% of his earnings last summer, or $630, to buy the new guitar. So the relationship between Geoff's earnings last summer and the amount he spent to buy the guitar can be expressed by the equation

$$(0.15) \times (\text{Geoff's earnings last summer}) = \$630$$

Therefore, you can conclude that Geoff's earnings last summer totaled $\frac{\$630}{0.15}$, or $4,200. Since Geoff earned $4,200 last summer and spent $630 of his earnings to buy the guitar, the amount he did <u>not</u> spend to buy the guitar was $4,200 − $630, or $3,570. Therefore, Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

> Set *S* consists of 5 objects.

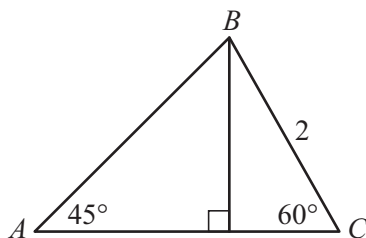|	Quantity A	Quantity B	
8.	The number of subsets of set *S* that consist of 1 object	The number of subsets of set *S* that consist of 4 objects	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that set *S* consists of 5 objects and are asked to compare the number of subsets of set *S* that consist of 1 object with the number of subsets of set *S* that consist of 4 objects. Recall that a set *R* is a subset of set *S* if all of the members of *R* are also members of *S*.

Note that if you select 1 object from set *S*, there are 4 objects in *S* that you have not selected; that is to say, each subset of *S* with 1 object corresponds to a subset of *S* with 4 objects. Therefore, the number of subsets of *S* with 1 object is equal to the number of subsets of *S* with 4 objects. Since Quantity A is equal to Quantity B, the correct answer is **Choice C**.

Another approach to solving this problem is to consider a particular set of 5 objects and determine all of the subsets consisting of 1 object and all of the subsets consisting of 4 objects. For example, let $S = \{a, b, c, d, e\}$. There are 5 subsets of *S* consisting of 1 object: $\{a\}, \{b\}, \{c\}, \{d\}$, and $\{e\}$; there are 5 subsets of *S* consisting of 4 objects: $\{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}$, and $\{b, c, d, e\}$.

Clearly, the particular 5 objects in *S* do not change the fact that the number of subsets of *S* consisting of 1 object is equal to the number of subsets of *S* consisting of 4 objects. The correct answer is **Choice C**.

Quantity A	Quantity B	
9. The length of line segment *AC*	3	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the length of line segment *AC* with 3.

Note that in the figure, the vertical line segment divides triangle *ABC* into two right triangles. Based on the fact that the sum of the measures of the angles in a triangle is 180°, you can conclude that the triangle to the left of the vertical line is a 45°-45°-90° right triangle and the triangle to the right of the vertical line is a 30°-60°-90° right triangle. The following figure shows all of these angle measures, along with a new label *D* at the vertex of the right angles.
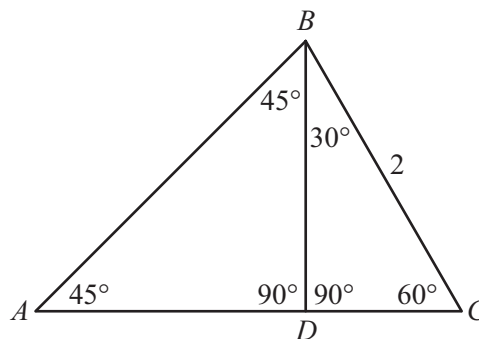

Note that the length of *AC* is equal to the length of *AD* plus the length of *DC*. Also note that *AD* is a leg of the 45°-45°-90° triangle and *DC* is a leg of the 30°-60°-90° triangle.

In the figure, you are given that the length of *BC*, the hypotenuse of the 30°-60°-90° triangle, is 2. No other lengths are given. Recall that if the length of the hypotenuse of a 30°-60°-90° triangle is 2, then the length of the side opposite the 30° angle is 1, and the length of the side opposite the 60° angle is $\sqrt{3}$. So, in the 30°-60°-90° triangle *BDC*, the length of *DC* (the side opposite the 30° angle) is 1, and the length of *BD* (the side opposite the 60° angle) is $\sqrt{3}$.

Now consider the 45°-45°-90° triangle *ABD*. Since this is an isosceles right triangle, its legs, *AD* and *BD*, have equal length. Since the length of *BD* is $\sqrt{3}$, the length of *AD* is also $\sqrt{3}$.

As noted above, the length of *AC* is equal to the length of *AD* plus the length of *DC*. Since the length of *AD* is $\sqrt{3}$ and the length of *DC* is 1, it follows that the length of *AC* is equal to $\sqrt{3} + 1$.

Recall that in the question you were asked to compare the length of *AC* with 3. Because $\sqrt{3}$ is less than 2, it follows that the length of *AC*, which is equal to $\sqrt{3} + 1$, is less than $2 + 1$, or 3. Hence, Quantity B is greater than Quantity A, and the correct answer is **Choice B**.

10. What is the slope of line *k* in the *xy*-plane above?

    Ⓐ −5

    Ⓑ −1

    Ⓒ   0

    Ⓓ   1

    Ⓔ   5
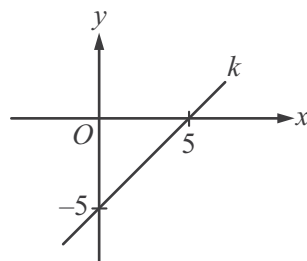
Explanation

Recall that if a line passes through the points with coordinates (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$, then the slope of the line is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

From the graph of line *k* in the *xy*-plane, you can conclude that the *x*-intercept of line *k* has coordinates $(5, 0)$ and the *y*-intercept of line *k* has coordinates $(0, -5)$. Thus, the slope of line *k* is

$$\frac{-5 - 0}{0 - 5} = \frac{-5}{-5} = 1$$

The correct answer is **Choice D**.

$$b - 3, \ b - 1, \ b + 2, \ b + 3, \ b + 4$$

11. The median of the five terms listed above is 5, where *b* is a constant. What is the average (arithmetic mean) of the five terms?

   Ⓐ 3

    Ⓑ 4

    Ⓒ 5

    Ⓓ 6

    Ⓔ 7

Explanation

To calculate the average of the five terms, you first need to use the information given in the question to determine the value of b.

You are given that 5 is the median of the five terms $b - 3$, $b - 1$, $b + 2$, $b + 3$, and $b + 4$, where b is a constant. The median of five terms is the middle, or third, term when the terms are listed in increasing order. Observe that the five terms are already given in increasing order. Since $b + 2$ is the third term, you can conclude that $b + 2 = 5$ and thus $b = 3$.

Since $b = 3$, it follows that the values of the five terms are 0, 2, 5, 6, and 7, and the average of the five terms is $\dfrac{0 + 2 + 5 + 6 + 7}{5} = \dfrac{20}{5} = 4$. The correct answer is **Choice B**.
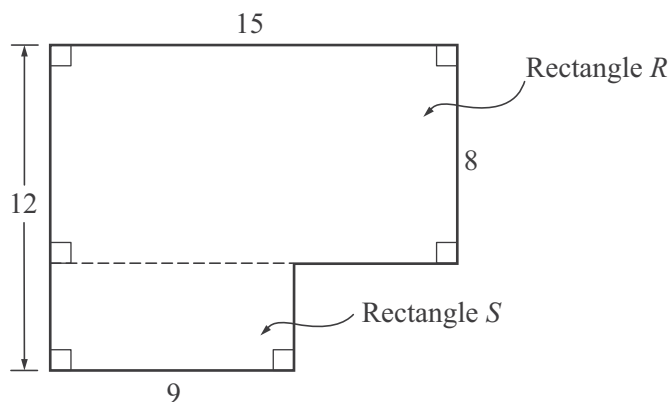
For the following question, enter your answer in the box.
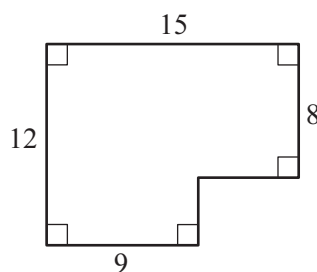
12. What is the area of the region shown above?

Explanation

In this question, you are asked to determine the area of the given region. One approach to solving this problem is to split the region into two rectangles, rectangle R and rectangle S, as follows.

The area of the region is the sum of the areas of the two rectangles. The area of rectangle R is $(15)(8)$, or 120. Since the width of rectangle S is $12 - 8$, or 4, the area of rectangle S is $(9)(4)$, or 36. Thus, the area of the region is $120 + 36$, or 156. The correct answer is **156**.

Another approach to solving this problem is to form the region by removing a small rectangle from a rectangle with length 15 and width 12, as follows.
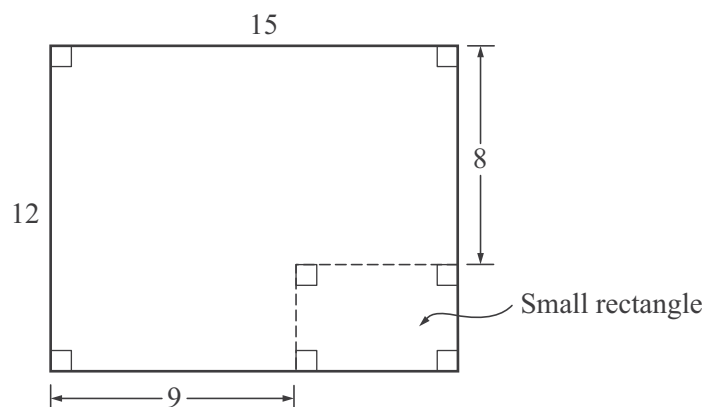
The area of the region is the area of the rectangle with length 15 and width 12 minus the area of the small rectangle. The area of the rectangle with length 15 and width 12 is (15)(12), or 180. Since the length of the small rectangle is 15 − 9, or 6, and the width of the small rectangle is 12 − 8, or 4, the area of the small rectangle is (6)(4), or 24. Thus, the area of the region is 180 − 24, or 156. The correct answer is **156**.

13.   During a one-year study, biologists observed the number of fish in a certain pond as well as the percent of the fish that were catfish. At the beginning of the year, there were 300 fish in the pond, of which 15 percent were catfish; and at the end of the year, there were 400 fish in the pond, of which 10 percent were catfish. From the beginning of the year to the end of the year, the number of catfish in the pond

   Ⓐ   decreased by more than 5%

   Ⓑ   decreased by 5%

   Ⓒ   did not change

   Ⓓ   increased by 5%

   Ⓔ   increased by more than 5%

Explanation
The answer choices indicate that the question is asking about the percent change in the number of catfish. The number of catfish in the pond at the beginning of the year was 15% of 300, or (0.15)(300), which is 45. The number of catfish in the pond at the end of the year was 10% of 400, or (0.10)(400), which is 40. Thus, the number of catfish decreased by 5.

   The percent by which the number of catfish decreased from the beginning of the year to the end of the year is

$$\frac{\text{the decrease in the number of catfish over the year}}{\text{the number of catfish at the beginning of the year}} \times (100\%) = \left(\frac{5}{45}\right)(100\%) \approx 11\%$$

Thus, the number of catfish decreased by about 11%. This is a decrease of more than 5%, so the correct answer is **Choice A**.

For the following question, enter your answer in the box.

14. On a radio tower, a red light flashes every 6 seconds and a blue light flashes every 10 seconds. If both lights flash together at a certain time, how many seconds later will both lights flash together the next time?

<div style="text-align:center">☐ seconds</div>

Explanation

One way to approach this question is to look at the "flash times" for both lights to see what times they have in common. The following lists show the flash times for both lights as the numbers of seconds after the time at which both lights flashed together.

Red light: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, . . .
Blue light: 10, 20, 30, 40, 50, 60, 70, 80, 90, . . .

Note that 30 is the first number that is common to both lists. Therefore, if both lights flash together, they will flash together again 30 seconds later. The correct answer is **30**.

Alternatively, you may realize that if the lights flash together, the number of seconds that will elapse before they flash together the next time is the least common multiple of 6 and 10. To find the least common multiple of 6 and 10, begin by writing each integer as the product of its prime factors.

$$6 = (2)(3)$$
$$10 = (2)(5)$$

Since 2 is a factor in both products, but 3 and 5 are factors of only one of the products, the least common multiple of 6 and 10 is (2)(3)(5), or 30. Therefore, if both lights flash together, the next time they will flash together is 30 seconds later. The correct answer is **30**.

For the following question, select all the answer choices that apply.

15. If $a < b < 0$, which of the following numbers must be positive?

Indicate all such numbers.

A $a - b$
B $a^2 - b^2$
C ab
D $a^2 b$
E $a^2 b + ab^2$

Explanation

In this question, you are given that $a < b < 0$ and are asked to determine which of the answer choices must be positive. Note that the condition $a < b < 0$ means that a and b are negative and that $a < b$.

Choice A: $a - b$. In the question, it is given that $a < b$. Subtracting b from both sides of the inequality $a < b$ gives the inequality $a - b < 0$. Therefore, $a - b$ must be negative.

Choice B: $a^2 - b^2$. Since a and b are negative, you can square both sides of the inequality $a < b$ to get the inequality $a^2 > b^2$. Then you can subtract b^2 from both sides of the inequality $a^2 > b^2$ to conclude that $a^2 - b^2 > 0$. So $a^2 - b^2$ must be positive. Alternatively, note that $a^2 - b^2$ can be factored as $(a - b)(a + b)$. The factor $a - b$ is Choice A, which must be negative, and the factor $a + b$ is the sum of two negative numbers, which also must be negative. Thus, $a^2 - b^2$ is the product of two negative numbers, so it must be positive.

Choice C: ab. Because a and b are negative, you can conclude that their product ab must be positive.

Choice D: a^2b. Because a^2b can be written as $(a)(a)(b)$, which is the product of three negative numbers, you can conclude that a^2b must be negative.

Choice E: $a^2b + ab^2$. By the reasoning in the explanation of Choice D, Choice E is the sum of two negative numbers. Therefore, you can conclude that $a^2b + ab^2$ must be negative.

Choices B and C must be positive, and Choices A, D, and E must be negative. The correct answer consists of **Choices B and C**.
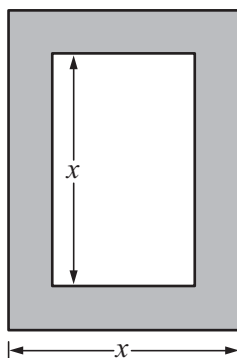
16. A flat rectangular picture, represented by the unshaded region in the figure above, is mounted in a flat rectangular frame, represented by the shaded region. The frame is 1 inch wide on all sides. For what value of x, in inches, is the area of the frame equal to the area of the picture?

    (A) 4
    (B) 5
    (C) 6
    (D) 7
    (E) 8

Explanation
In this question, you are asked to determine the value of x for which the area of the frame is equal to the area of the picture. To do this, you need to express both the area of the frame and the area of the picture in terms of x and then find the value of x for which the two expressions are equal.

The area of the picture is the area of the inner rectangle, and the area of the frame is the area of the outer rectangle minus the area of the inner rectangle. Since the area of a rectangle is the length times the width, you need to know the length and width of the inner and outer rectangles.

In the figure, you are given that the length of the inner rectangle is x inches, but the width is not given. However, since you know that the width of the frame

is 1 inch, it follows that the width of the inner rectangle is equal to the width of the outer rectangle minus 2 inches, or $x - 2$ inches. Thus, the area of the inner rectangle is $x(x - 2)$ square inches.

In the figure, you are given that the width of the outer rectangle is x inches, but the length is not given. However, since you know that the width of the frame is 1 inch, it follows that the length of the outer rectangle is equal to the length of the inner rectangle plus 2 inches, or $x + 2$ inches. Thus, the area of the outer rectangle is $x(x + 2)$ square inches.

Since the area of the frame is the area of the outer rectangle minus the area of the inner rectangle, the area of the frame is
$x(x + 2) - x(x - 2) = x^2 + 2x - x^2 + 2x = 4x$ square inches.

Now you are ready to set up the equation. Set the expression for the area of the picture equal to the expression for the area of the frame and solve the resulting equation for x, as follows.

$$x(x - 2) = 4x$$
$$x^2 - 2x = 4x$$
$$x^2 - 6x = 0$$
$$x(x - 6) = 0$$

There are two solutions to the equation, $x = 0$ and $x = 6$. Since x represents the length of a picture, in inches, the solution $x = 0$ does not make sense in this context. Therefore, when $x = 6$, the area of the picture equals the area of the frame. The correct answer is **Choice C**.

Questions 17 to 20 are based on the following data.

PERCENT OF THE 300 PEOPLE IN GROUP 1 AND THE 400 PEOPLE
IN GROUP 2 WHO HAVE SELECTED AILMENTS

Respiratory Ailment	Percent of People in Group 1 Who Have Ailment	Percent of People in Group 2 Who Have Ailment
Allergic sensitivity to endotoxins	14%	21%
Asthma (allergic)	3%	4%
Asthma (nonallergic)	2%	3%
Hay fever	4%	10%
Sneezing and itchy eyes	8%	11%
Wheezing (allergic)	5%	6%
Wheezing (nonallergic)	2%	5%

17. The number of people in group 2 who have hay fever is how much greater than the number of people in group 1 who have hay fever?

    (A) 37
    (B) 35
    (C) 32
    (D) 28
    (E) 24

Explanation

In group 1, there are 300 people, 4% of whom have hay fever. Therefore, in group 1, there are (0.04)(300) people, or 12 people, who have hay fever. In group 2, there are 400 people, 10% of whom have hay fever. Therefore, in group 2, there are (0.10)(400) people, or 40 people, who have hay fever. Since $40 - 12 = 28$, it follows that there are 28 more people in group 2 who have hay fever than there are in group 1. The correct answer is **Choice D**.

18. For the seven ailments, what is the median of the numbers of people in group 2 who have the ailments?

    (A) 20
    (B) 22
    (C) 24
    (D) 26
    (E) 28

Explanation

The median of the seven numbers of people in group 2 is the middle number when the numbers are listed in increasing order. To find the middle number easily, first order from least to greatest the percents of people in group 2 who have the seven ailments, then find the median of the percents, and finally calculate the number of people corresponding to that percent.

The following table shows the percents of people in group 2 who have the seven ailments, ordered from least to greatest.

Respiratory Ailment	Percent of People in Group 2 Who Have Ailment
Asthma (nonallergic)	3%
Asthma (allergic)	4%
Wheezing (nonallergic)	5%
Wheezing (allergic)	6%
Hay fever	10%
Sneezing and itchy eyes	11%
Allergic sensitivity to endotoxins	21%

The median percent, which is the fourth number in the list, is 6%. Thus, the median number of people in group 2 who have the ailments is (0.06)(400), or 24, and the correct answer is **Choice C**.

19. The number of people in group 1 who have the ailment wheezing (allergic) is what percent greater than the number of people in group 1 who have the ailment wheezing (nonallergic)?

     Ⓐ   50%

     Ⓑ   75%

     Ⓒ 150%

     Ⓓ 200%

     Ⓔ 300%

Explanation

In group 1, there are 300 people, 5% of whom have allergic wheezing and 2% of whom have nonallergic wheezing. That is, (0.05)(300) people, or 15 people, have allergic wheezing and (0.02)(300) people, or 6 people, have nonallergic wheezing. Therefore, in group 1, the number of people who have allergic wheezing exceeds the number who have nonallergic wheezing by 9, which is $\left(\frac{9}{6}\right)$(100%) greater than 6, or 150% greater than 6. The correct answer is **Choice C**.

For the following question, enter your answer in the boxes.

20. What is the ratio of the number of people in group 2 with the ailment sneezing and itchy eyes to the total number of people in both groups with the ailment sneezing and itchy eyes?

    Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

Explanation

In group 2, there are 400 people, 11% of whom have sneezing and itchy eyes. Therefore, in group 2, there are (0.11)(400) people, or 44 people, who have sneezing and itchy eyes. In group 1, there are 300 people, 8% of whom have sneezing and itchy eyes. Therefore, in group 1, there are (0.08)(300) people, or 24 people, who have sneezing and itchy eyes. So the total number of people in both groups who have sneezing and itchy eyes is 24 + 44, or 68. Thus, the ratio of the number of people in group 2 who have sneezing and itchy eyes to the total number of people in both groups who have sneezing and itchy eyes is $\frac{44}{68}$. The correct answer is $\frac{44}{68}$ (or any equivalent fraction).

21. Of the people in a certain survey, 58 percent were at most 40 years old and 70 percent were at most 60 years old. If 252 of the people in the survey were more than 40 years old and at most 60 years old, what was the total number of people in the survey?

   Ⓐ 1,900
   Ⓑ 2,100
   Ⓒ 2,400
   Ⓓ 2,700
   Ⓔ 3,000

Explanation

In this question, it is given that of the people surveyed, 58% were at most 40 years old and 70% were at most 60 years old. Therefore, 70% − 58%, or 12%, of the people surveyed were more than 40 years old and at most 60 years old, and you are given that 252 people are in this group. Let x be the total number of people in the survey. Then 12% of x is 252, that is, $0.12x = 252$, and so $x = \dfrac{252}{0.12} = 2{,}100$. Therefore, the total number of people in the survey was 2,100, and the correct answer is **Choice B**.

22. If $x > 0$, which of the following is equal to 1.25 percent of x ?

   Ⓐ $\dfrac{x}{80}$
   Ⓑ $\dfrac{x}{8}$
   Ⓒ $\dfrac{x}{4}$
   Ⓓ $\dfrac{5x}{8}$
   Ⓔ $\dfrac{3x}{4}$

Explanation

Since the answer choices are fractional expressions, to answer the question you need to convert 1.25% of x to a fractional expression. Note that 1.25% is equivalent to the decimal 0.0125. Converting the decimal 0.0125 to a fraction and simplifying gives $0.0125 = \dfrac{125}{10{,}000} = \dfrac{1}{80}$. Thus, 1.25% of x is equal to $\dfrac{1}{80}$ of x, or $\dfrac{x}{80}$, and the correct answer is **Choice A**.

23. Alice earns d dollars and has t percent of what she earns deducted for taxes. How much of what she earns does Alice have left after taxes?

   Ⓐ $d(1 - 100t)$ dollars
   Ⓑ $d(1 - 10t)$ dollars
   Ⓒ $d(1 - t)$ dollars
   Ⓓ $d(1 - 0.1t)$ dollars
   Ⓔ $d(1 - 0.01t)$ dollars

Explanation

Recall that t percent can be expressed as $\dfrac{t}{100}$, or $0.01t$. Therefore, the amount that Alice has deducted for taxes, which is t percent of d dollars, can be expressed as $0.01td$ dollars. The amount that Alice has left after taxes is the amount that she earns minus the amount that she has deducted for taxes, or $d - 0.01td$ dollars. Note that $d - 0.01td$ is an algebraic expression with two terms, each containing d as a factor. Factoring out d from each term results in the algebraic expression $d(1 - 0.01t)$. The correct answer is **Choice E**.

For the following question, select all the answer choices that apply.

24. A student made a conjecture that for any integer n, the integer $4n + 3$ is a prime number. Which of the following values of n could be used to disprove the student's conjecture?

   Indicate <u>all</u> such values.

   A   1
   B   3
   C   4
   D   6
   E   7

Explanation
Recall that a prime number is an integer greater than 1 that has no positive divisors other than 1 and itself.

The answer choices for this question are integer values of n. Any of the answer choices for which the integer $4n + 3$ is <u>not</u> a prime number could be used to disprove the conjecture that for any integer n, the integer $4n + 3$ is a prime number.

To answer this question, you must determine for each of the answer choices whether the integer $4n + 3$ is a prime number. The evaluations are as follows:

   Choice A: For $n = 1$, the integer $4n + 3$ is $4(1) + 3$, or 7, which is a prime number.

   Choice B: For $n = 3$, the integer $4n + 3$ is $4(3) + 3$, or 15, which is not a prime number.

   Choice C: For $n = 4$, the integer $4n + 3$ is $4(4) + 3$, or 19, which is a prime number.

   Choice D: For $n = 6$, the integer $4n + 3$ is $4(6) + 3$, or 27, which is not a prime number.

Choice E: For $n = 7$, the integer $4n + 3$ is $4(7) + 3$, or 31, which is a prime number.

Therefore, of the answer choices, only Choices B and D, that is, $n = 3$ and $n = 6$, result in integers $4n + 3$ that are not prime numbers. Thus, the correct answer consists of **Choices B and D**.

25. Eight points are equally spaced on a circle. If 4 of the 8 points are to be chosen at random, what is the probability that a quadrilateral having the 4 points chosen as vertices will be a square?

    (A) $\frac{1}{70}$

    (B) $\frac{1}{35}$

    (C) $\frac{1}{7}$

    (D) $\frac{1}{4}$

    (E) $\frac{1}{2}$

Explanation

For questions involving geometry, it is often helpful to draw a figure representing the information in the question as accurately as possible. The figure below shows a circle with 8 equally spaced points, labeled *A* through *H*, and quadrilateral *BCDH*, which is one of the many quadrilaterals that have 4 of the 8 equally spaced points as vertices.
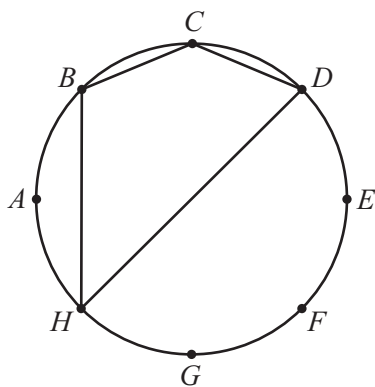
The probability that a quadrilateral having the 4 points chosen as vertices will be a square is equal to the following fraction.

$$\frac{\text{the number of squares that can be drawn using 4 of the 8 points as vertices}}{\text{the number of quadrilaterals that can be drawn using 4 of the 8 points as vertices}}$$

To calculate the desired probability, you need to determine the number of squares and the number of quadrilaterals that can be drawn using 4 of the 8 points as vertices.

To determine the number of quadrilaterals, first note that since the 8 points lie on a circle, every subset of 4 of the 8 points determines a unique quadrilateral. Therefore, the number of quadrilaterals that can be drawn using 4

of the 8 points as vertices is equal to the number of ways of choosing 4 points from the 8 points shown. The number of ways of choosing 4 points from the 8 points shown (also called the number of combinations of 8 objects taken 4 at a time) is equal to $\dfrac{8!}{4!(8-4)!}$. You can calculate the value of this expression as follows.

$$\frac{8!}{4!(8-4)!} = \frac{(8)(7)(6)(5)(4!)}{(4)(3)(2)(1)(4!)}$$
$$= \frac{(8)(7)(6)(5)}{(4)(3)(2)}$$
$$= 70$$

Thus, there are 70 quadrilaterals that can be drawn using 4 of the 8 points as vertices.

Because the points are equally spaced around the circle, there are only 2 squares that can be drawn using 4 of the 8 points as vertices, namely *ACEG* and *BDFH*, as shown in the following figures.
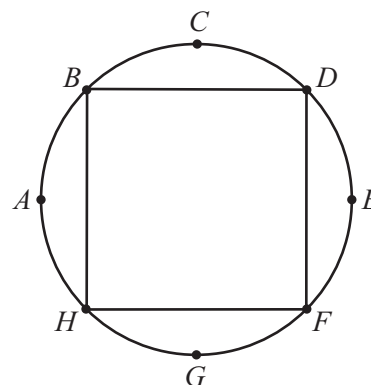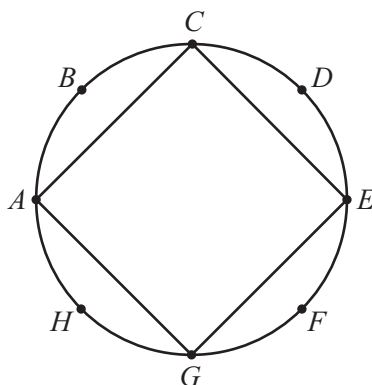
Therefore, the probability that the quadrilateral will be a square is $\dfrac{2}{70}$, or $\dfrac{1}{35}$, and the correct answer is **Choice B**.