Ejercicio 20

$$\frac{d^3x}{dt^3} - \frac{d^2x}{dt^2} - 4x = 0$$

$x = e^{rt}$
$x' = re^{rt}$
$x'' = r^2e^{rt}$
$x''' = r^3e^{rt}$

$r^3e^{rt} - r^2e^{rt} - 4e^{rt} = 0$
$e^{rt}(r^3 - r^2 - 4) = 0$

$r^3 - r^2 - 4 = 0$
$(r-2)(r^2+r+2) = 0$
$r - 2 = 0$
$r = 2$
$r^2 + r + 2 = 0$

$$r = -1 \pm \frac{\sqrt{1-4(1)(2)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

$x_1 = e^{2t}$
$x_2 = e^{(-\frac{1}{2} + \frac{\sqrt{7}}{2}i)t}$
$x_3 = e^{(-\frac{1}{2} - \frac{\sqrt{7}}{2}i)t}$

$$x = C_1e^{2t} + C_2e^{(-\frac{1}{2} + \frac{\sqrt{7}}{2}i)t} + C_3e^{(-\frac{1}{2} - \frac{\sqrt{7}}{2}i)t}$$

Ejercicio 22

$$y''' - 6y'' + 12y' - 8y = 0$$

$y = e^{rx}$
$y' = re^{rx}$
$y'' = r^2 e^{rx}$
$y''' = r^3 e^{rx}$

$$r^3 e^{rx} - 6r^2 e^{rx} + 12re^{rx} - 8e^{rx} = 0$$
$$e^{rx}(r^3 - 6r^2 + 12r - 8) = 0$$
$$r^3 - 6r^2 + 12r - 8 = 0$$
$$r^3 - 2r^2 - 4r^2 + 8r + 4r - 8 = 0$$
$$r^2(r-2) - 4r(r-2) + 4(r-2) = 0$$
$$(r(-2)(r-2)^2 = 0$$
$$(r-2)^3 = 0$$
$$r = 2$$

$$y = C_1 e^{2x} + C_2 x e^{2x} + C_3 x^2 e^{2x}$$

Ejercicio 24

$$y^{(4)} - 2y'' + y = 0$$

$$r^4 e^{rx} - 2r^2 e^{rx} + e^{rx} = 0$$
$$e^{rx}(r^4 - 2r^2 + 1) = 0$$
$$r^4 - 2r^2 + 1 = 0$$
$$(r^2 - 1)^2 = 0$$
$$(r-1)(r+1)(r-1)(r+1) = 0$$
$$(r-1)^2 \cdot (r+1)^2 = 0$$

$$r - 1 = 0 \Rightarrow r = 1$$
$$r + 1 = 0 \Rightarrow r = -1$$

$$y = C_1 e^x + C_2 x e^x + C_3 e^{-x} + C_4 x e^{-x}$$

Ejercicio 26

$$\frac{d^4y}{dx^4} - 7\frac{d^2y}{dx^2} - 18y = 0$$

$$r^4e^{rx} - 7r^2e^{rx} - 18e^{rx} = 0$$
$$e^{rx}(r^4 - 7r^2 - 18) = 0$$
$$r^4 - 7r^2 - 18 = 0$$
$$(r-3)(r^3 + 3r^2 + 2r + 6) = 0$$
$$(r-3)(r+3)(r^2+2) = 0$$

$r - 3 = 0 \Rightarrow r = 3$
$r + 3 = 0 \Rightarrow r = -3$
$r^2 + 2 = 0 \Rightarrow r = \pm\sqrt{-2} = \pm\sqrt{2}\,i$

$$y = C_1e^{3x} + C_2e^{-3x} + C_3e^{\sqrt{2}\,ix} + C_4e^{-\sqrt{2}\,ix}$$